

NEWS ON LEPTOGENESIS

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GREATEST HITS

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DISCLAIMER: this is NOT a
comprehensive review of leptogenesis

See for instance:

Buchmüller, Peccei, Yanagida, 2005; Davidson, Nardi, Nir, 2008; Fong, Nardi, Riotto, 2012; Garbrecht, Molinaro et al., 2018

And also Talks by Miguel Vanvlasselaer and Ninetta Saviano

Outline

- Introduction
- Leptogenesis via HNL decay
- Low scale leptogenesis beyond the seesaw
- ν electroweak baryogenesis
- Leptogenesis via heavy neutrino oscillations
- Summary and outlook

1. Introduction

- Baryon number density: determined from
 - Big Bang Nucleosynthesis: primordial abundances of light elements (D, ^3He , ^4He , ^7Li) mainly depend on one parameter, n_B/n_γ
 - CMB anisotropies

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} \simeq \frac{n_B}{s} = (8.66 \pm 0.01) \times 10^{-11}$$

Impressive consistency between both determinations, completely independent !

- **Baryon asymmetry**
 - nucleons and antinucleons were in thermal equilibrium up to $T_{fo} \approx 22 \text{ MeV}$, when $\Gamma_{ann} < H$
 - If the Universe were locally baryon-symmetric:
 $Y_{Bfo} < 10^{-20} \rightarrow$ there was a baryon asymmetry
- **Sakharov's conditions** to dynamically generate the baryon asymmetry (BAU)
 - Baryon number violation
 - C and CP violation
 - Departure from thermal equilibrium

2. Leptogenesis via HNL decay

- BAU generated in the decay of heavy Majorana neutrinos:
Fukugita, Yanagida, 1986
 - Out of equilibrium decay
 - L and CP violating interactions \rightarrow lepton asymmetry, ΔL
 - (B+L)-violating, but (B-L) conserving, non-perturbative sphaleron interactions $\Delta L \rightarrow \Delta B$

- Non-equilibrium process \rightarrow Boltzmann eqs.

$$\frac{dY_{N_1}}{dz} = - \left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) (D + S)$$

$$\frac{dY_{B-L}}{dz} = -\epsilon \left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) D - Y_{B-L}W$$

$$z \equiv M_1/T$$

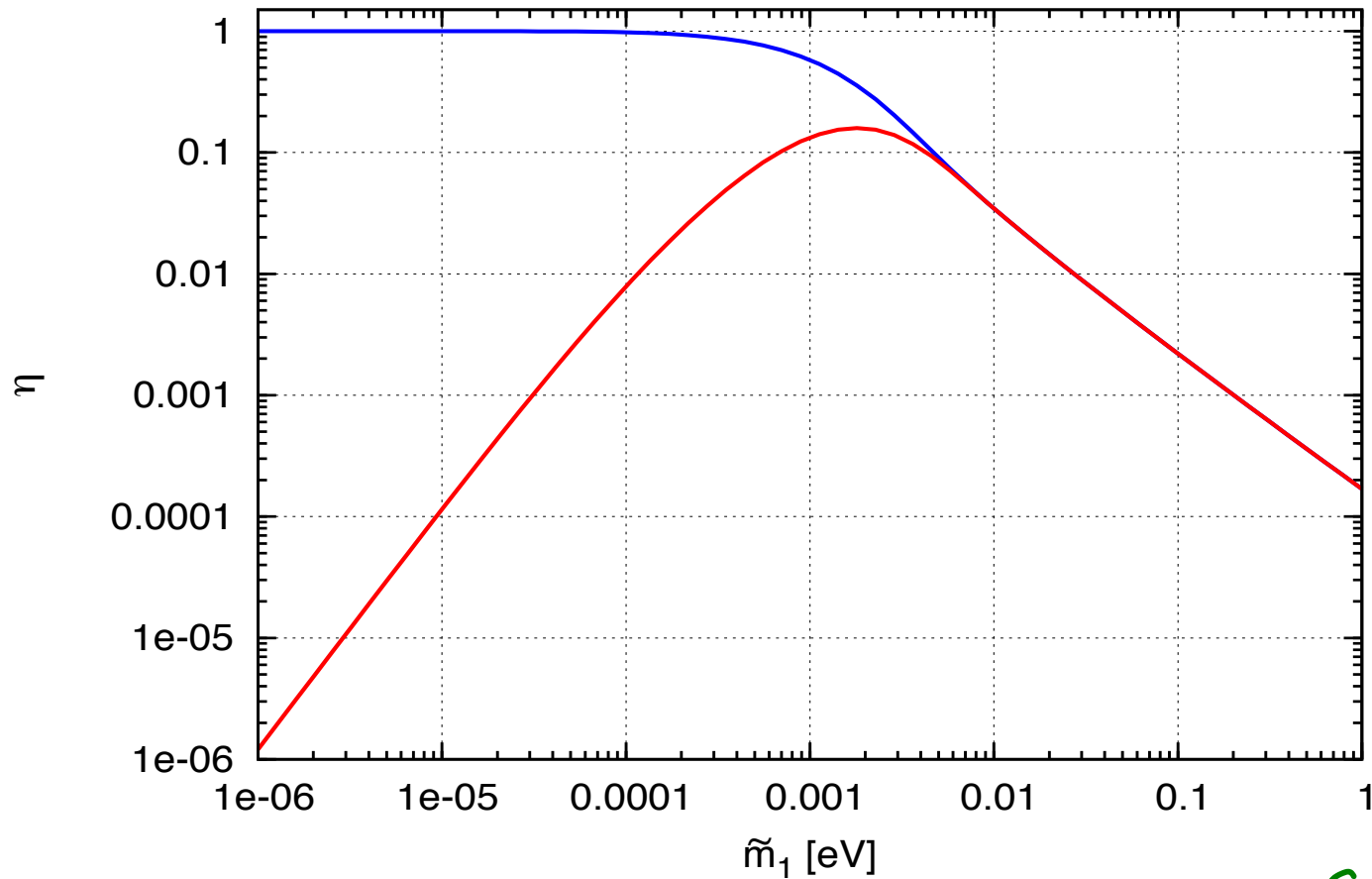
$$\epsilon = \sum_{\alpha} \epsilon_{\alpha 1} = \sum_{\alpha} \frac{\Gamma(N_1 \rightarrow \ell_{\alpha} h) - \Gamma(N_1 \rightarrow \bar{\ell}_{\alpha} \bar{h})}{\sum_{\beta} \Gamma(N_1 \rightarrow \ell_{\beta} h) + \Gamma(N_1 \rightarrow \bar{\ell}_{\beta} \bar{h})}$$

- Final baryon asymmetry:

$$Y_B = -\kappa \epsilon \eta$$

$$\kappa = \frac{28}{79} Y_{N_1}^{eq}(T \gg M_1) \sim 10^{-3}$$

$$\eta = \text{efficiency} : \quad 0 \leq \eta \leq 1$$



— $Y_N^i = 0$

— $Y_N^i = Y_N^{eq}$

Credit:
J. Racker

$$\tilde{m}_1 \equiv \frac{(\lambda^\dagger \lambda)_{11} v^2}{M_1}$$

Related to the contribution of N_1 to light neutrino masses

- η maximum for

$$\tilde{m}_1 = m_* = \frac{16}{3\sqrt{5}} \pi^{5/2} \sqrt{g_*} \frac{v^2}{M_P} \sim 10^{-3} \text{ eV}$$

m_* , defined by:

$$\frac{\Gamma_N}{H(T = M_1)} = \frac{\tilde{m}_1}{m_*}$$

determines the amount of **departure from thermal equilibrium** and the strength of the **washouts**:

$\tilde{m}_1 \gg m_*$ \rightarrow **strong washout:**

- **independence of initial conditions, $\eta \propto 1/\tilde{m}_1$**

$\tilde{m}_1 \ll m_*$ \rightarrow **weak washout:**

- **depends on initial conditions, if $Y_N^i = 0 \rightarrow \eta \propto \tilde{m}_1^2$**

- Hierarchical heavy neutrinos: $\epsilon \sim \frac{3}{16\pi} \frac{\lambda_{\alpha 2}^2}{M_2} M_1$
- Connection to light neutrino masses (type I seesaw):
 $|\epsilon| \leq \epsilon_{DI} = \frac{3}{16\pi} \frac{M_1}{v^2} (m_3 - m_1)$ Davidson, Ibarra, 2002
 → $M_1 \gtrsim 10^9 \text{ GeV}$

Detailed numerical analysis solving BEs:

- $M_1 \gtrsim (4 \times) 10^8 \text{ GeV}$ for fine-tuned regions
Hambye et al. 2004
- bound on light neutrino masses, $m_\nu < 0.15 \text{ eV}$

Buchmüller, Di Bari, Plümacher, 2004

Flavour effects

- At $T \leq 10^{12}$ GeV, the τ Yukawa interaction is fast, and there are (in general) **2 lepton flavour asymmetries** evolving almost independently
- At $T \leq 10^9$ GeV, both τ and μ Yukawa interactions are in equilibrium \rightarrow **3 independent lepton flavour asymmetries, $Y_{\Delta(B/3-L\alpha)}$**

Barbieri et al. 2000; Endoh et al. 2004; Abada et al. 2006;
Nardi et al. 2006

Some consequences:

- ★ Flavoured asymmetries ϵ_α depend on U_{PMNS} phases although in general leptogenesis is “insensitive” to them, even in SUSY Davidson, Garayoa, Palorini, NR, 2007
- ★ Bound on light neutrino masses $m_\nu < 0.15 \text{ eV}$ evaded
- ★ N_2 leptogenesis can survive N_1 washouts more easily:
 - relevant for SO(10) models which predict $M_1 \ll 10^9 \text{ GeV}$
- ★ Leptogenesis possible with $\epsilon = \sum_\alpha \epsilon_\alpha = 0$
 - relevant for models with small $B-L$ violation (inverse seesaw)

$$\epsilon_{\alpha i} \equiv \frac{\Gamma(N_i \rightarrow \ell_\alpha h) - \Gamma(N_i \rightarrow \bar{\ell}_\alpha \bar{h})}{\sum_{\beta} \Gamma(N_i \rightarrow \ell_\beta h) + \Gamma(N_i \rightarrow \bar{\ell}_\beta \bar{h})} = \epsilon_{\alpha i}^{\cancel{L}} + \epsilon_{\alpha i}^L$$

where:

Covi, Roulet, Vissani, 1996

$$\epsilon_{\alpha i}^{\cancel{L}} = \sum_{j \neq i} f(a_j) \text{Im}[\lambda_{\alpha j}^* \lambda_{\alpha i} (\lambda^\dagger \lambda)_{ji}]$$

$$a_j \equiv M_j^2 / M_i^2$$

$$\epsilon_{\alpha i}^L = \sum_{j \neq i} g(a_j) \text{Im}[\lambda_{\alpha j}^* \lambda_{\alpha i} (\lambda^\dagger \lambda)_{ij}]$$



related to L conserving d=6 operators → escape the

DI bound because they are not linked to neutrino masses (LNV d=5 Weinberg operator)

Resonant leptogenesis

- Enhancement of the CP asymmetry for degenerate neutrinos, $M_2 - M_1 \approx \Gamma_2$

$$|\epsilon| \sim \frac{1}{2} \frac{\text{Im}[(\lambda^\dagger \lambda)_{21}^2]}{(\lambda^\dagger \lambda)_{11}(\lambda^\dagger \lambda)_{22}} \leq \frac{1}{2}$$

Covi, Roulet, 1997; Pilaftsis, 1997; Pilaftsis, Underwood 2004

→ EW scale e^- , μ^- , τ^- leptogenesis with observable

LFCV

Pilaftsis 2005; Deppisch, Pilaftsis 2011

- Approximate flavour symmetries + universal RHN masses at the GUT scale → heavy neutrino mass splittings radiatively generated

Non-equilibrium QFT:

Kadanoff-Baym equations for spectral functions and statistical propagators of leptons and Majorana neutrinos
Anisimov et al., 2011; Drewes et al. 2013

★ Free from double counting problem

★ Relevant in the weak washout regime

★ Very important for resonant leptogenesis: suppression wrt classical Boltzmann result.

Garny, et al., 2013

- If $M_2 - M_1 \approx \Gamma_2 \rightarrow$ sterile neutrino oscillations
- Taken into account using “Flavour covariant transport equations” \rightarrow density matrix formalism

$$\dot{\rho} = -i[H, \rho] \quad \text{Dev et al., 2014 (109 p.)}$$

- Identify mixing contribution from diagonal ρ_N and heavy neutrino oscillation contribution from off-diagonal $(\rho_N)_{12}$
- Also in the Kadanoff–Baym approach Dev et al., 2015

3. Low scale leptogenesis beyond the seesaw

Baryogenesis from particle decays or annihilations at low T (TeV scale)

- Potentially testable mechanisms for baryogenesis
- Avoid hierarchy and gravitino (in SUSY) problems
- High scale baryogenesis disfavored if L violating processes were observed at LHC
- Common origin of dark and baryon matter?

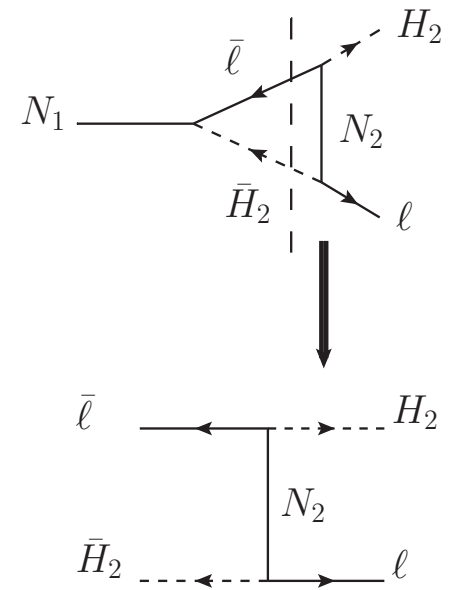
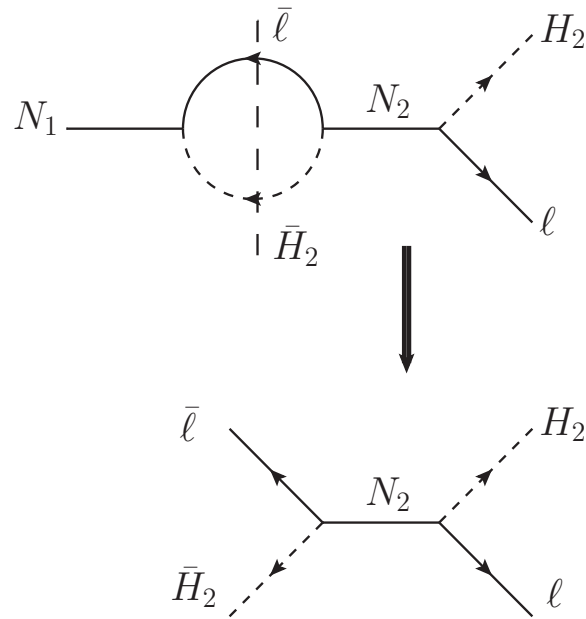
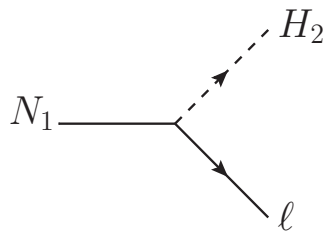
$$\frac{\Omega_{DM}}{\Omega_B} \approx 5 \quad (\text{see A. Hook's talk})$$

- Radiative neutrino mass models (like scotogenic model) → no DI bound on ϵ
- Generic problem of baryogenesis at low scales, independent of the connection to neutrino masses: fast washout of the asymmetries

$$\epsilon \propto \frac{\lambda_{\alpha 2}^2}{M_2} M_1$$

if $M_2 \gg M_1$

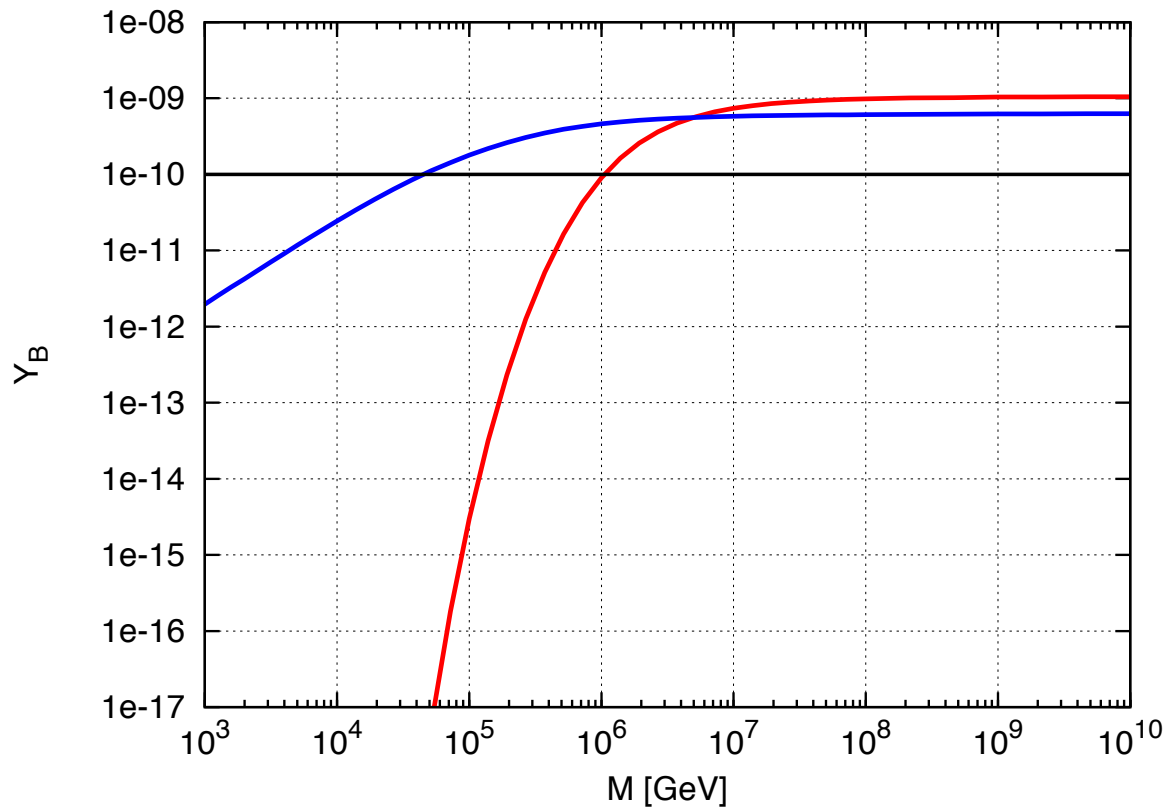
washouts $\propto \left(\frac{\lambda_{\alpha 2}^2}{M_2} \right)^2$



Credit: J. Racker, 2014

_____ without washouts

_____ with washouts



$$\frac{\Gamma_N}{H(T = M_N)} = \text{const.}$$

$$\epsilon_{CP} = \text{const.}$$

Courtesy of J. Racker, 2014

Ways out:

- Late decay: new interactions needed to produce N_1
[Plumacher 1997; Racker, Roulet 2009, Cui, Sundrum 2012]
- Resonant leptogenesis
- Massive decay products:
[Cui, Randall, Shuve, 2012; J. Racker, 2014]
- ALP Leptogenesis [talk by Martina Cataldi]

4. ν electroweak baryogenesis

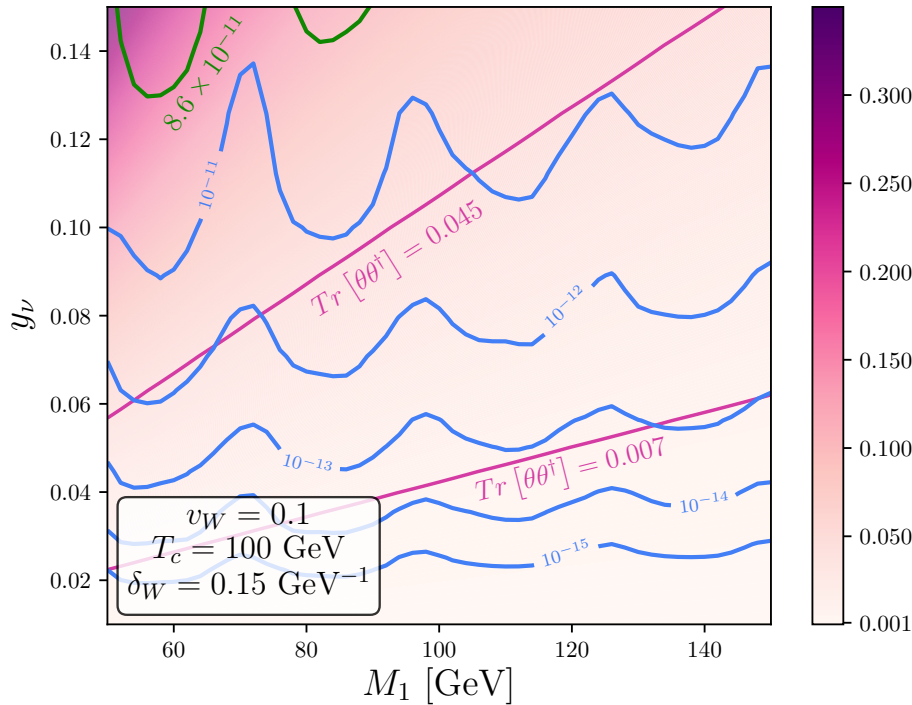
- Extra SM singlet scalar provides both: strongly first order EW phase transition (SFOPT) and sterile neutrino (Dirac) masses
- Inverse or linear seesaw \rightarrow large neutrino Yukawa couplings

$$\mathcal{L} = -\bar{L}_L \tilde{H} Y_\nu N_R - \bar{N}_L \phi Y_N N_R + h.c. - V(\phi^* \phi, H^\dagger H)$$

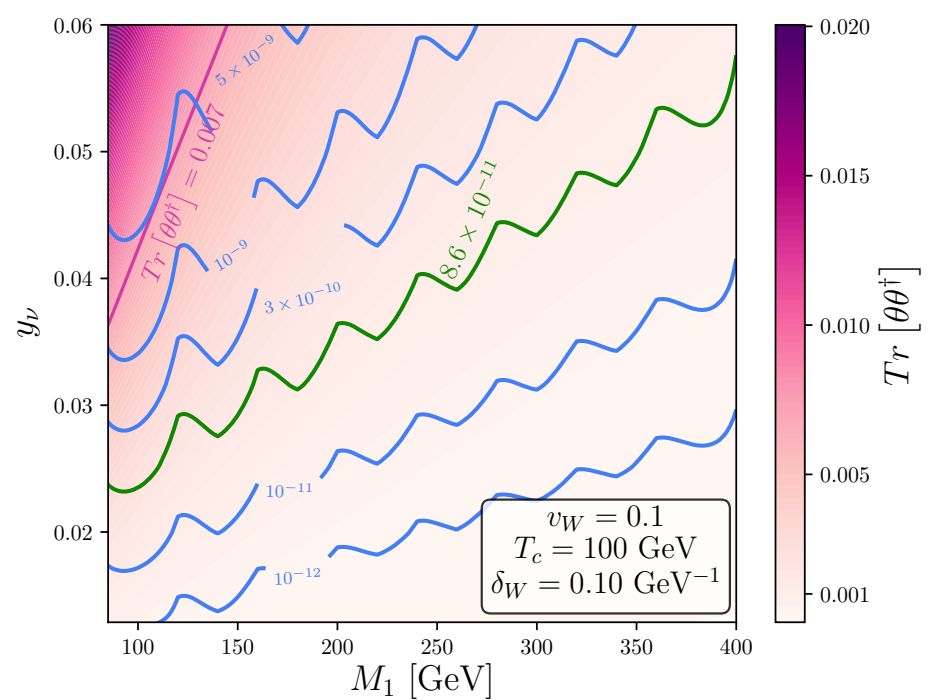
- Profiles of the vevs $v_H(z)$ and $v_\phi(z)$ along the bubble wall must be different

P. Hernández, NR, 1997

Importance of flavour effects: $\theta \equiv m_D M_N^{-1} = \frac{1}{\sqrt{2}} Y_\nu v_H Y_N^{-1} v_\phi$



Unflavoured



With flavour

Regions of SFOPT consistent with current experimental bounds also identified

5. Leptogenesis via sterile neutrino oscillations

Akhmedov, Rubakov, Smirnov, 1998; Asaka, Shaposhnikov 2005; Antusch et al. 2018; Hernández et al. 2016; Drewes et al. 2018; Abada et al. 2019; Domcke et al. 2020; etc

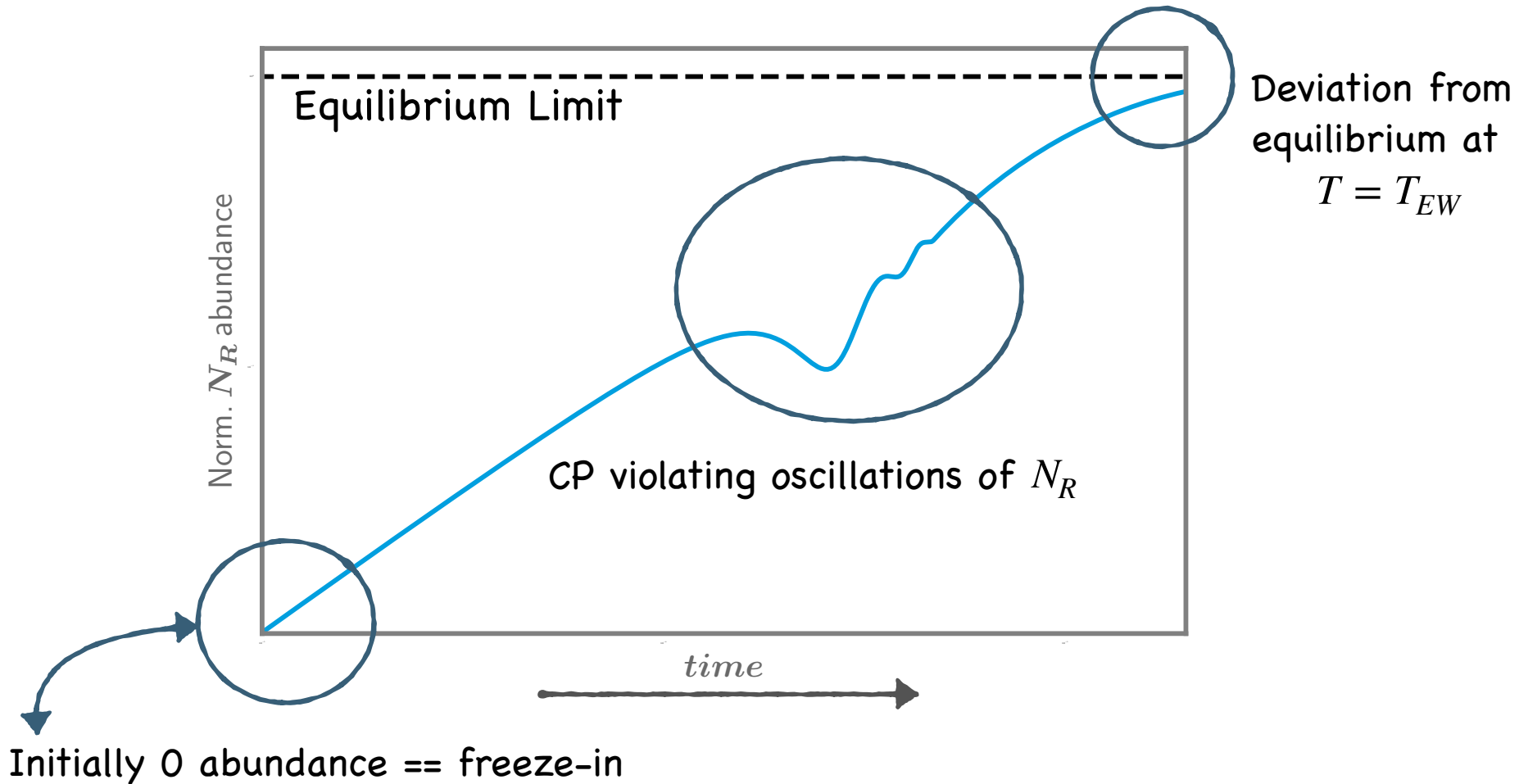
Sakharov conditions for baryogenesis:

- CP violating phases in Y, M
- B violated by sphaleron processes at $T > T_{EW}$
- At least one sterile neutrino does **NOT** equilibrate by T_{EW} , i.e. for some rate

$$\Gamma_i(T_{EW}) \leq H_u(T_{EW}) = T_{EW}^2 / M_p^*$$

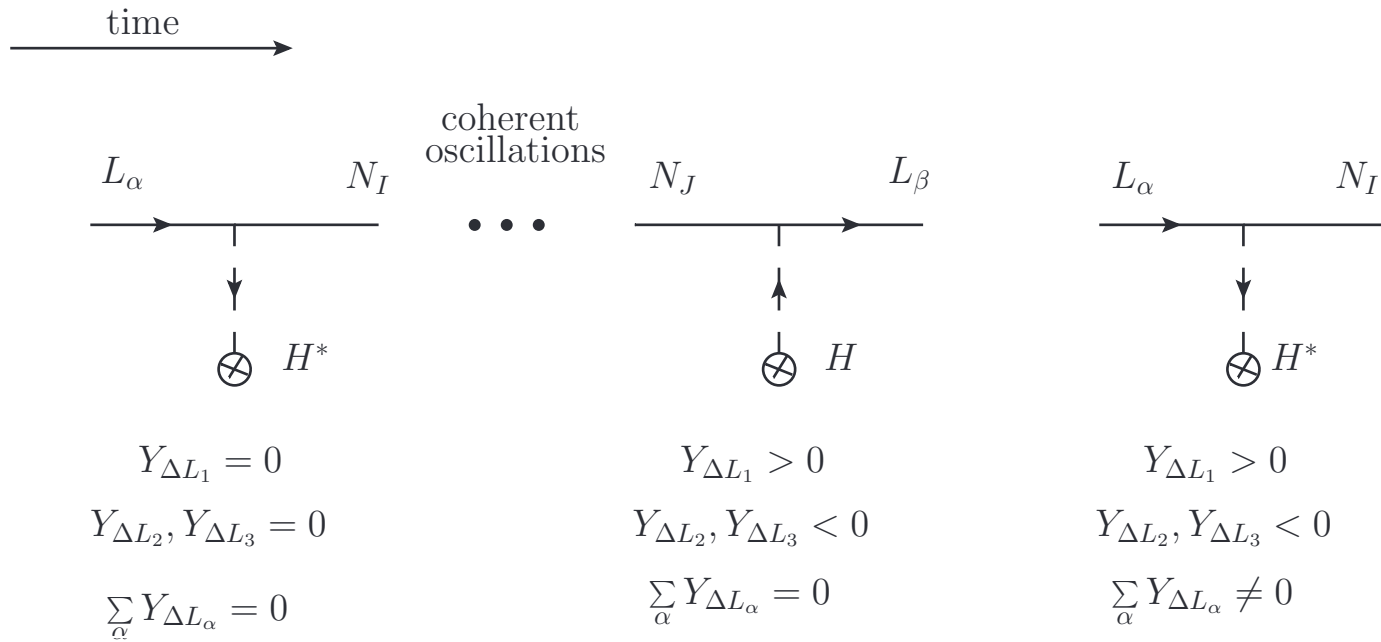
Fulfilled for $M = O(\text{GeV})$, $Y \sim 10^{-6} - 10^{-7}$, in the correct range to explain neutrino masses ! Freeze-in baryogenesis

Schematic evolution of N_R abundance



- Basic stages:

Shuve, Yavin 2014



Out of equilibrium
HNL production

Asymmetries in
lepton flavours

Different washout
of flavoured
asymmetries

- Inclusion of LNV (helicity conserving, HC) rates, suppressed by $(M/T)^2$

Density matrix formalism(*)

Raffelt & Sigl, 1993

$$\dot{\rho} = -i[H, \rho] - \frac{1}{2}\{\Gamma^a, \rho\} + \frac{1}{2}\{\Gamma^p, \rho_{eq} - \rho\}$$

- Hamiltonian term: $H = \frac{M^2}{2k^0} + \frac{T^2}{8k^0} Y^\dagger Y$
- Annihilation and production rates of the N's: Γ^a, Γ^p
- For antineutrinos: $\bar{\rho}$, $H \Rightarrow H^*$
- Diagonal density matrix for SM leptons, which are in thermal equilibrium, with chemical potential

$$f_\alpha(k^0) = \frac{1}{e^{(k^0 - \mu_\alpha)/T} + 1}$$

- For antileptons $\mu_\alpha \Rightarrow -\mu_\alpha$

(*) Similar results in Closed-time-path formalism

Time scales and slow modes

$$\dot{\rho} = -i[H, \rho] - \frac{1}{2}\{\Gamma^a, \rho\} + \frac{1}{2}\{\Gamma^p, \rho_{eq} - \rho\}$$

- Annihilation and production rates of the N's: at $T \gg M$,

$$\Gamma(T) \propto \text{Tr}[YY^\dagger] T$$

Ghiglieri, Laine, 2017

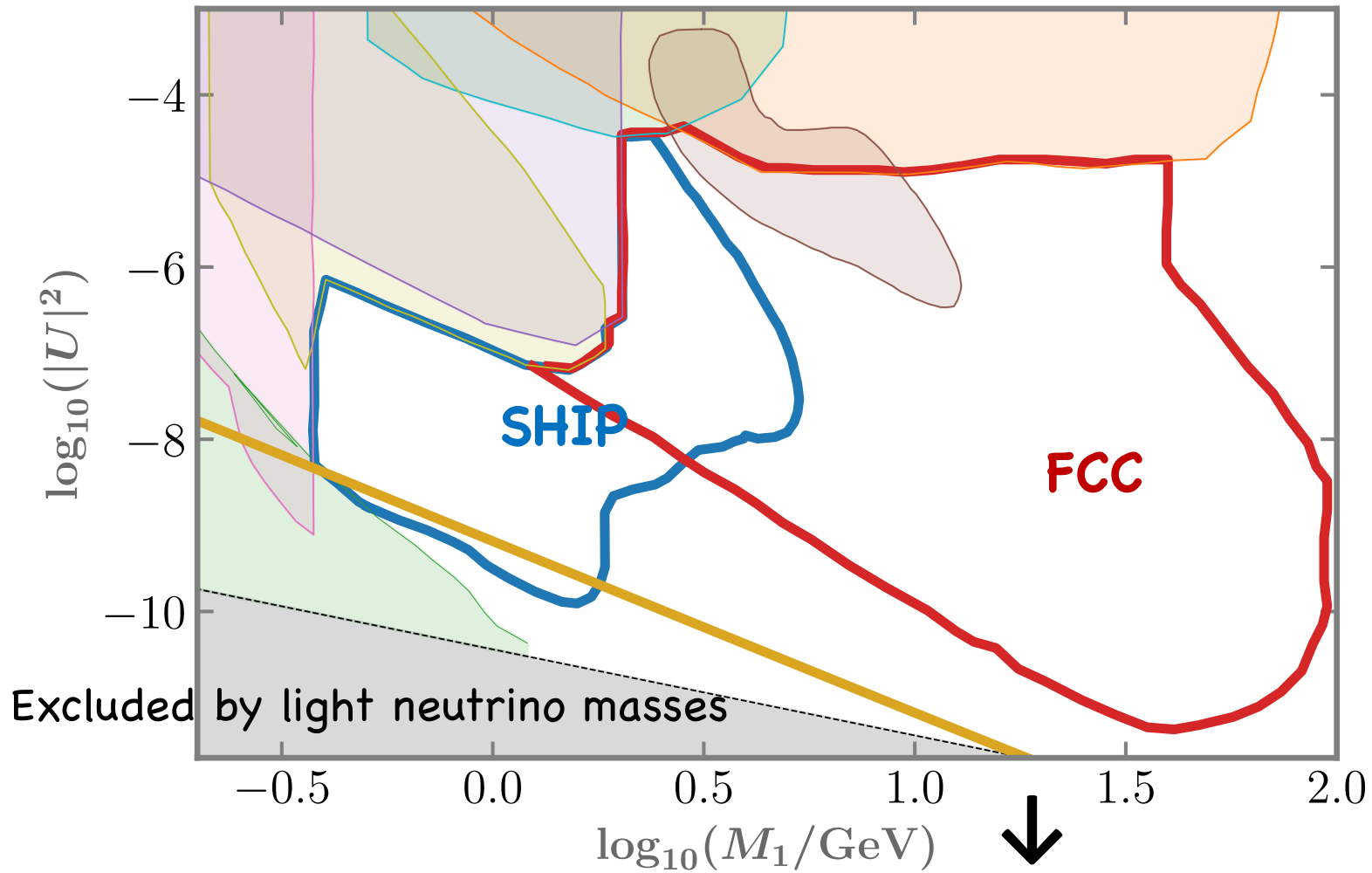
- Flavoured rates: $\Gamma_\alpha(T) \propto \epsilon_\alpha \Gamma(T)$ $\epsilon_\alpha \equiv \frac{(YY^\dagger)_{\alpha\alpha}}{\text{Tr}[YY^\dagger]}$

- Oscillation rate: $\Gamma_{osc}(T) \propto \frac{\Delta M^2}{T}$

- LNV (HC) rate: $\Gamma_M \propto (M/T)^2 \Gamma(T)$

N=2 HNL

Hernández, López-Pavón, NR, Sandner, 2022



$$\Gamma(T_{EW}) = H_u(T_{EW})$$

Approximately conserved lepton number limit

- **Inverse seesaw** Wyler, Wolfenstein 1983; Mohapatra, Valle 1986

$$M = \begin{pmatrix} \mu_1 & \Lambda \\ \Lambda & \mu_2 \end{pmatrix}, \quad Y = \begin{pmatrix} y_e & y'_e e^{i\beta'_e} \\ y_\mu & y'_\mu e^{i\beta'_\mu} \\ y_\tau & y'_\tau e^{i\beta'_\tau} \end{pmatrix}$$

$$y'_\alpha \ll y_\alpha, \quad \mu_i \ll \Lambda$$

- Once neutrino masses and mixings are fixed, there are **6 free parameters**:

- $y^2 \equiv \sum_{\alpha} y_{\alpha}^2$, or, equivalently $U^2 \simeq \frac{y^2 v^2}{2\Lambda^2}$

- Three independent phases ($\mu_1 = \mu_2 \equiv \mu$ can be chosen real)

- In terms of physical HNL masses:

$$\Lambda = (M_1 + M_2)/2 = M, \quad \mu = (M_2 - M_1)/2 = \Delta M/2$$

Analytic approach

- Identify the non-thermal modes and their characteristic time scales
- Solve the equations perturbatively, exploiting the weakly coupled modes
- Identify the CP-invariants that control Y_B

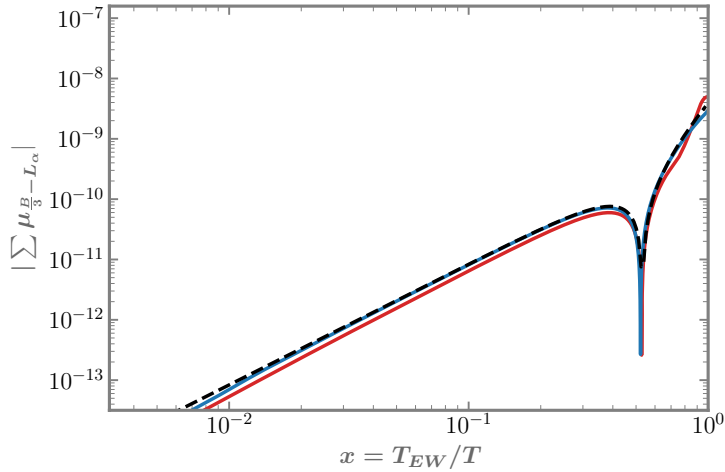
$$I_0 = \text{Im} \left(\text{Tr} \left[Y^\dagger Y M^\dagger M Y^\dagger Y_\ell Y_\ell^\dagger Y \right] \right)$$

$$\rightarrow \sum_{\alpha} y_{\ell\alpha}^2 \sum_{i < j} (M_j^2 - M_i^2) \text{Im} \left[Y_{\alpha j}^* Y_{\alpha i} (Y^\dagger Y)_{ij} \right] \equiv \sum_{\alpha} y_{\ell\alpha}^2 \Delta_{\alpha}$$

$$I_1 = \text{Im} \left(\text{Tr} \left[Y^\dagger Y M^\dagger M M^* Y^T Y^* M \right] \right)$$

$$\rightarrow \sum_{\alpha} \sum_{i < j} (M_j^2 - M_i^2) M_i M_j \text{Im} \left[Y_{\alpha j} Y_{\alpha i}^* (Y^\dagger Y)_{ij} \right] \equiv \sum_{\alpha} \Delta_{\alpha}^M$$

Overdamped regime:



$$\epsilon \propto \frac{\Delta M^2 / T}{\Gamma(T)} \ll 1$$

---- analytical solution:

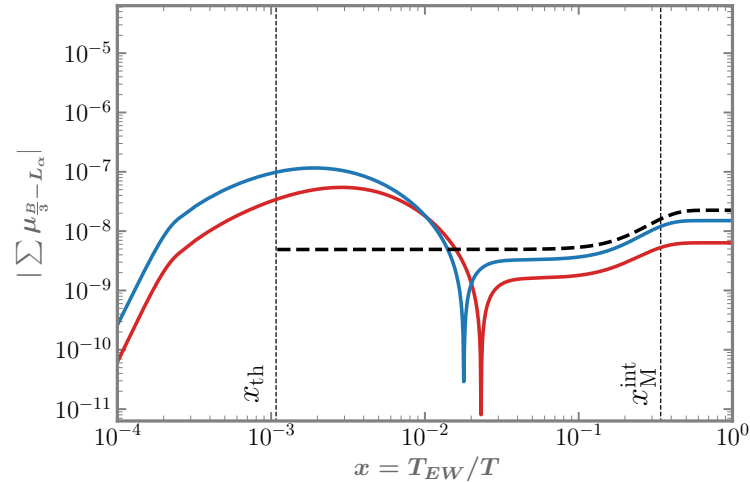
Perturbing in y' and in $(M/T)^2$

Linearized equations

— numerical solution with same approximations

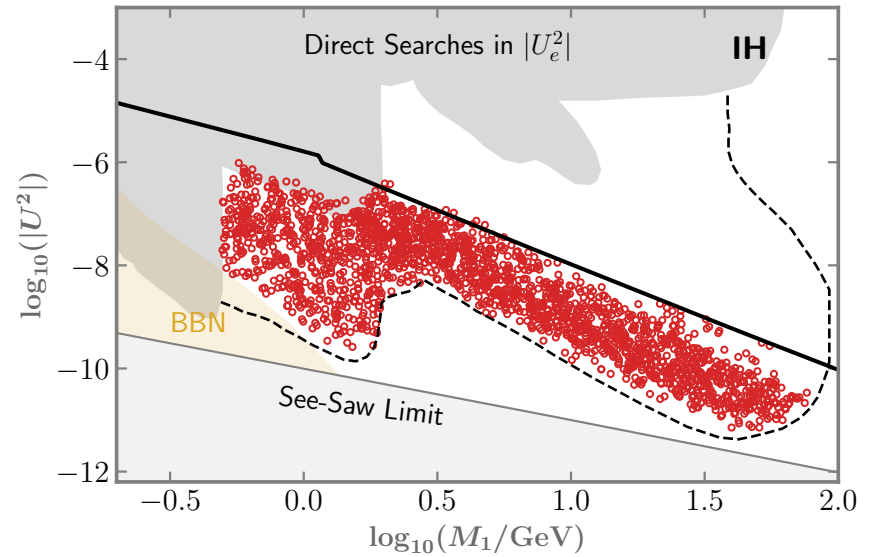
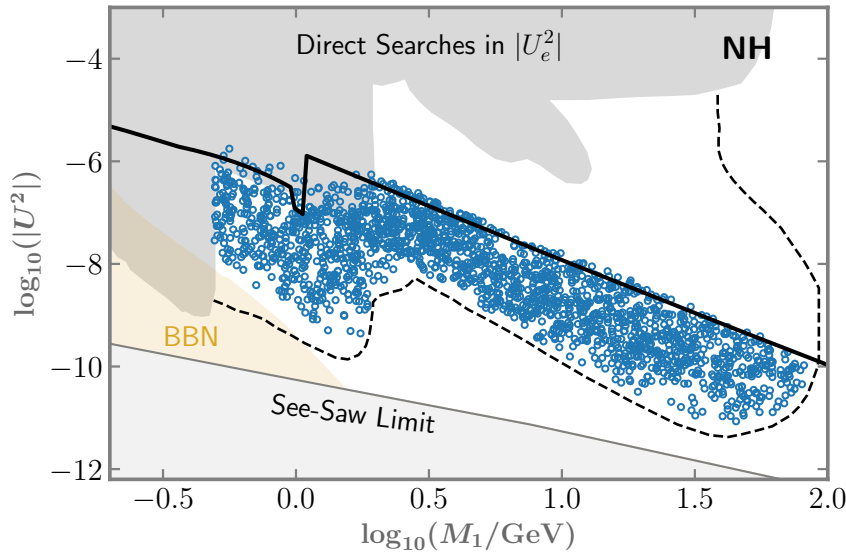
— full numerical solution

Slow flavour α :



$$\Gamma_\alpha(T_{EW}) < H_u(T_{EW}) < \Gamma(T_{EW})$$

Numerical scan



Absolute upper bound on U^2 from the overdamped regime:

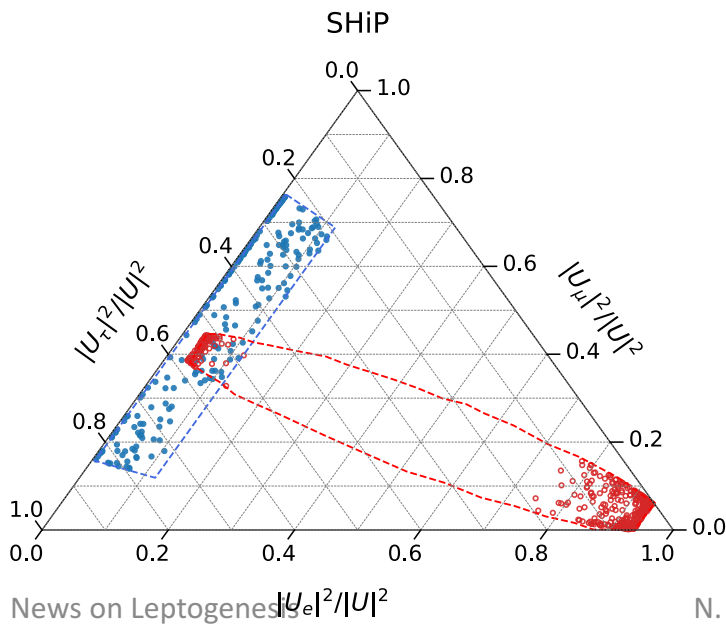
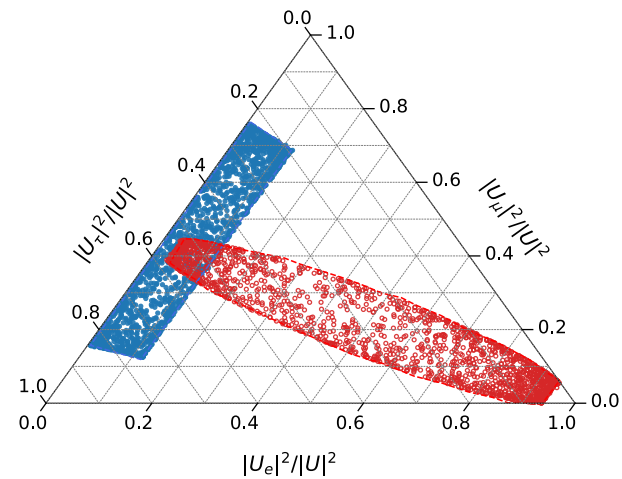
- Weak LNV
 $M \lesssim O(1 \text{ GeV})$ $(U^2)_{\text{ov}}^{\text{wLNV}} \lesssim 4(17) \times 10^{-7} \left(\frac{1 \text{ GeV}}{M}\right)^{4/3}$ NH(IH)
- Strong LNV
 $M \gtrsim O(1 \text{ GeV})$ $(U^2)_{\text{ov}}^{\text{sLNV}} \lesssim 16(2.3) \times 10^{-7} \left(\frac{1 \text{ GeV}}{M}\right)^{28/13}$ NH(IH)

Relation to other observables

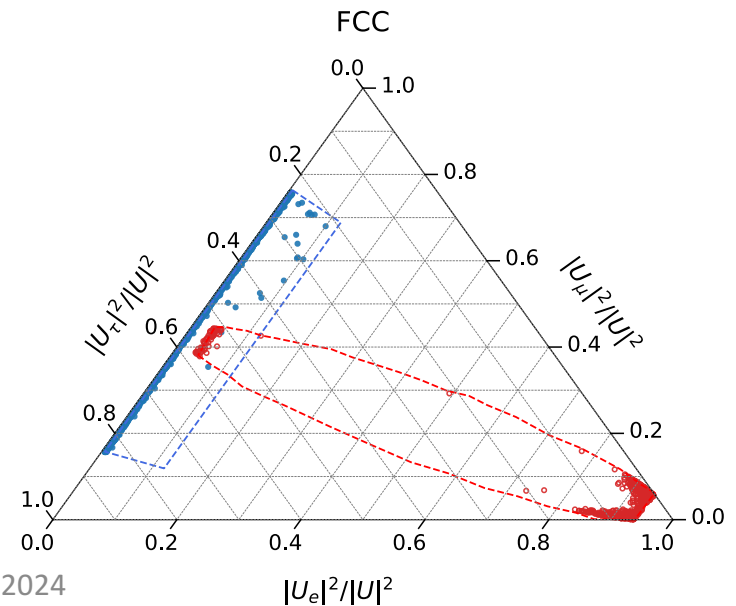
1. HNL flavour mixing

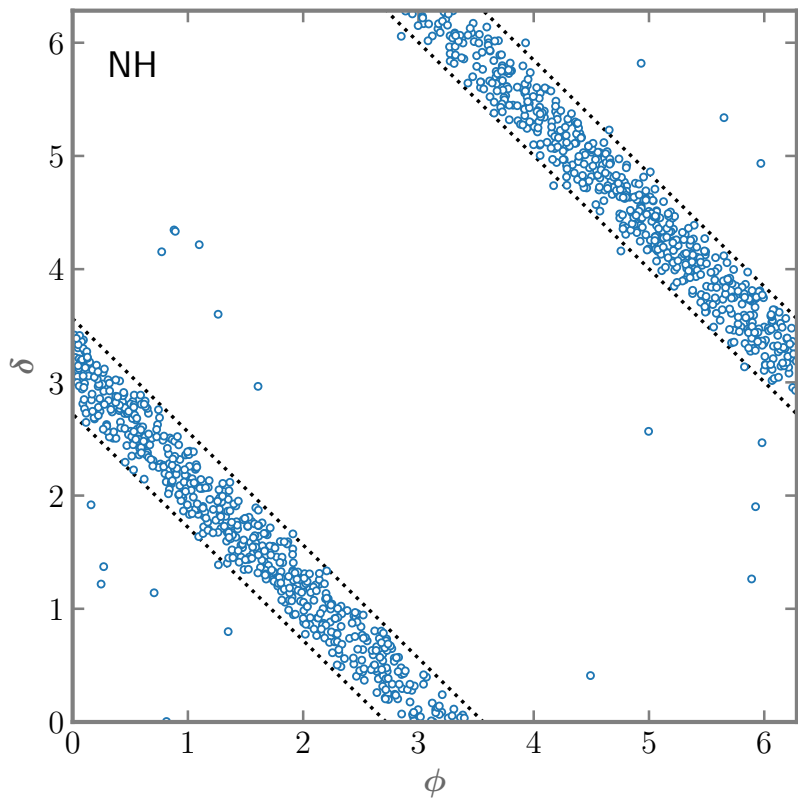
- Full scan: **NH** and **IH**

- $\Delta M/M = 10^{-2}$



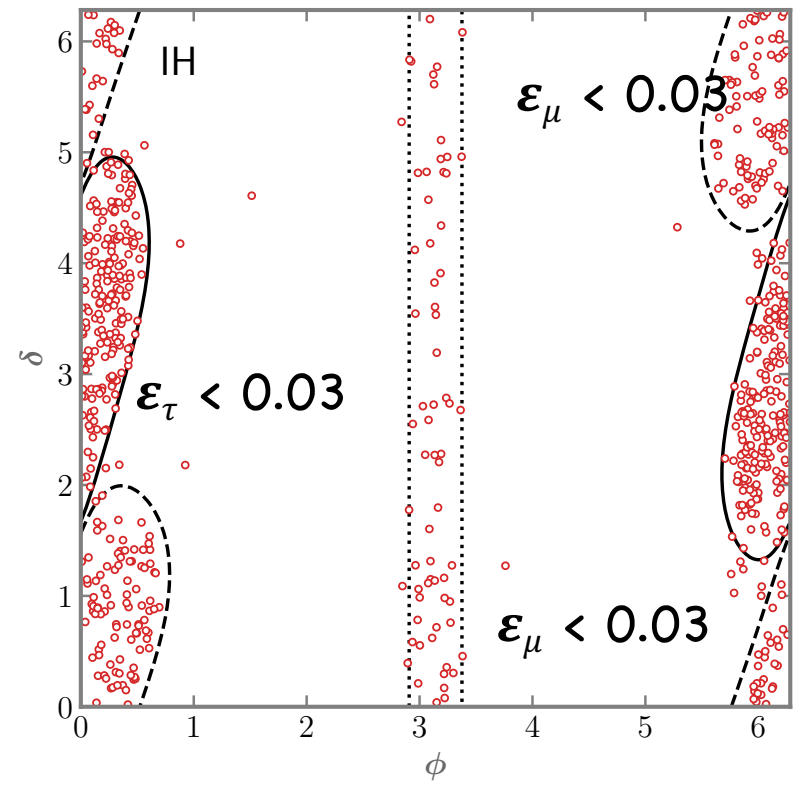
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$\epsilon_e < 0.01$ ($\delta + \phi \approx \pi, 3\pi$)

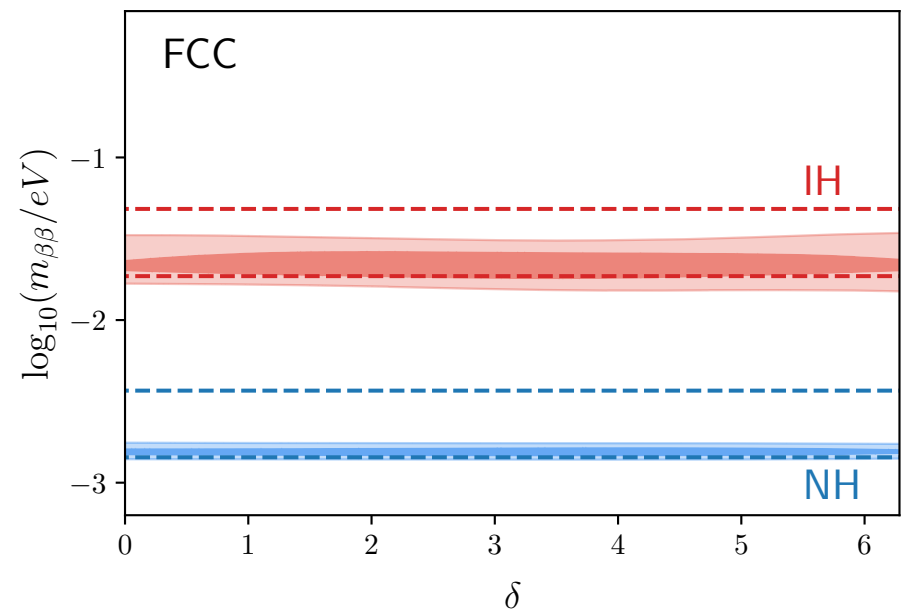
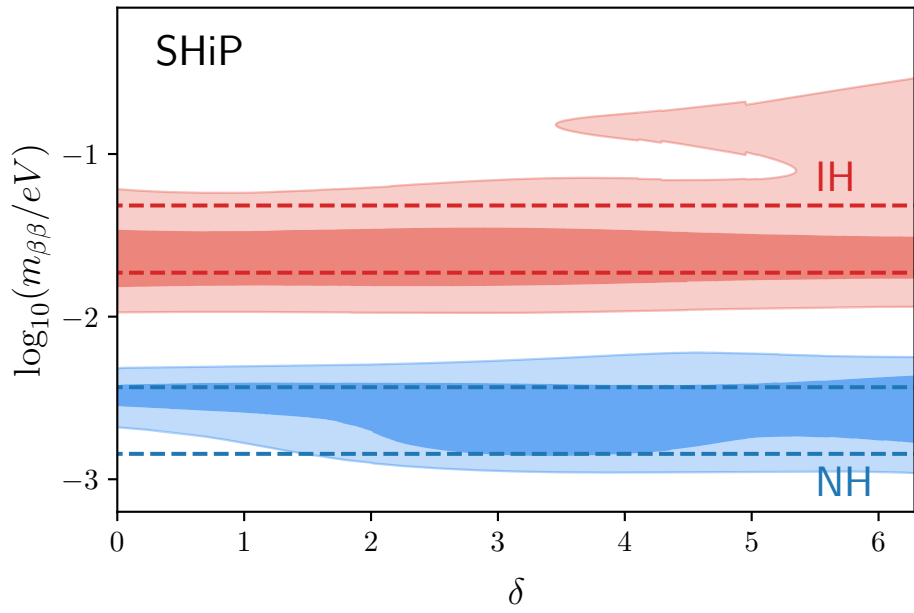
$$\epsilon_\alpha \equiv \frac{(YY^\dagger)_{\alpha\alpha}}{\text{Tr}[YY^\dagger]}$$



$\epsilon_e < 0.05$
 $(\phi \approx \pi)$

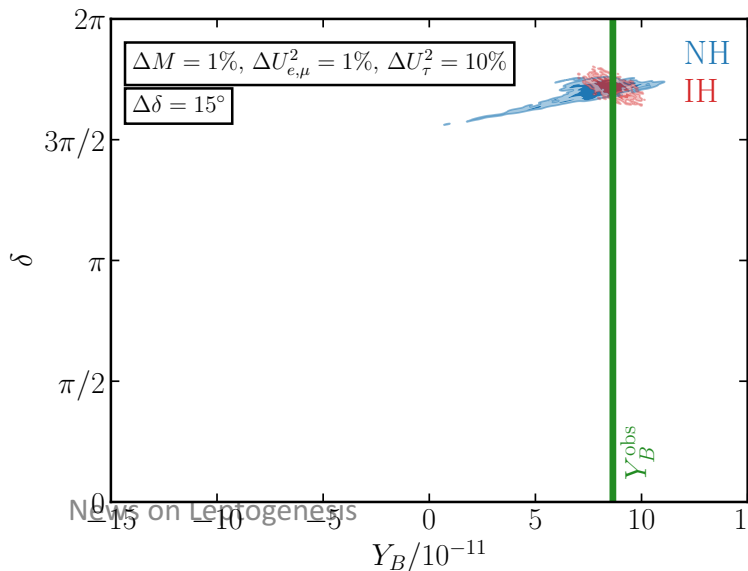
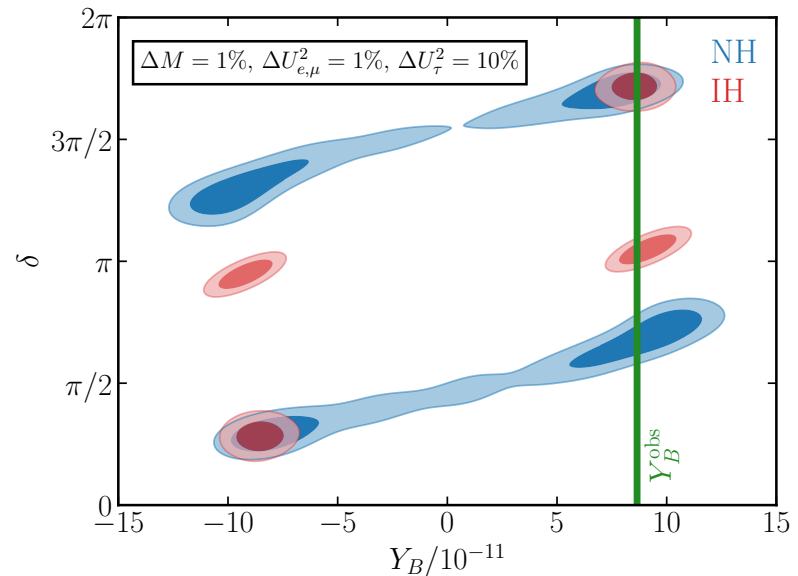
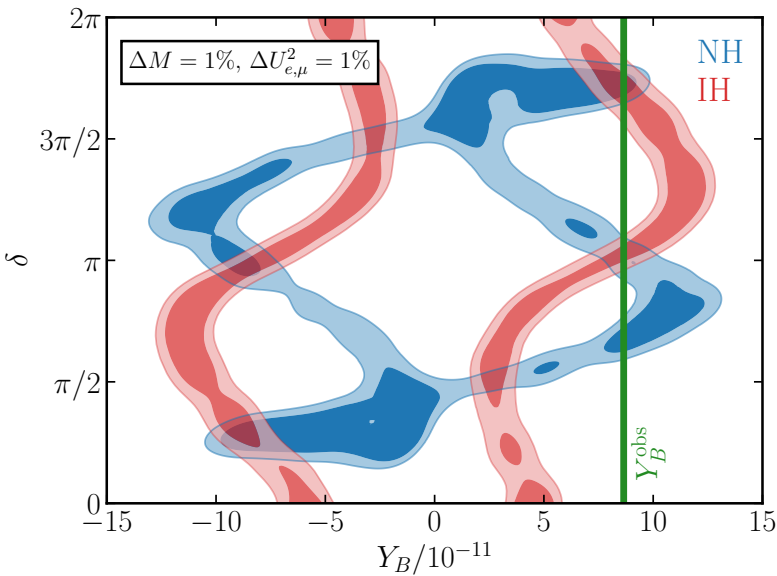
2. Neutrinoless double beta decay: $\Delta M/M = 10^{-2}$

Effect of HNL only in SHiP range



$\mu \approx 0$ case:

Once active neutrino masses and mixings are fixed, only 4 free parameters: M , U^2 (or y^2), and PMNS phases, (δ, ϕ)



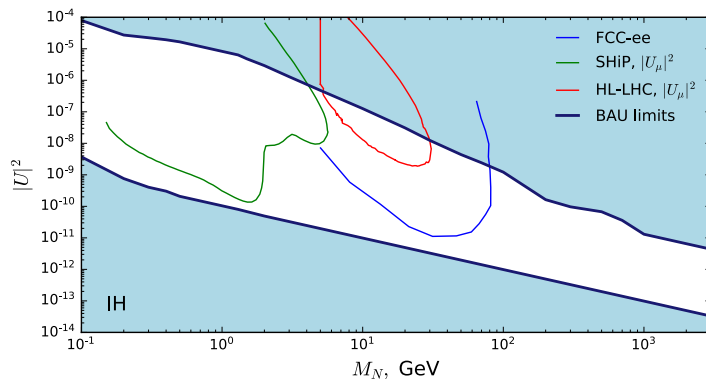
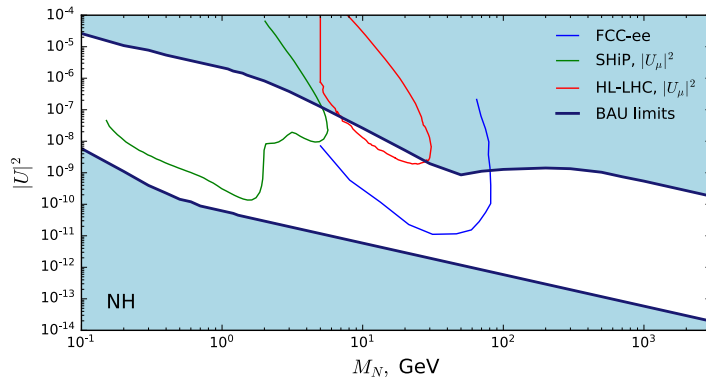
ϕ determined by Y_B

$$\Delta M \gg \Gamma$$

HNL oscillations can not be observed at FCC but LNV processes can

Extension to larger HNL masses:

unifying resonant leptogenesis and baryogenesis via neutrino oscillations



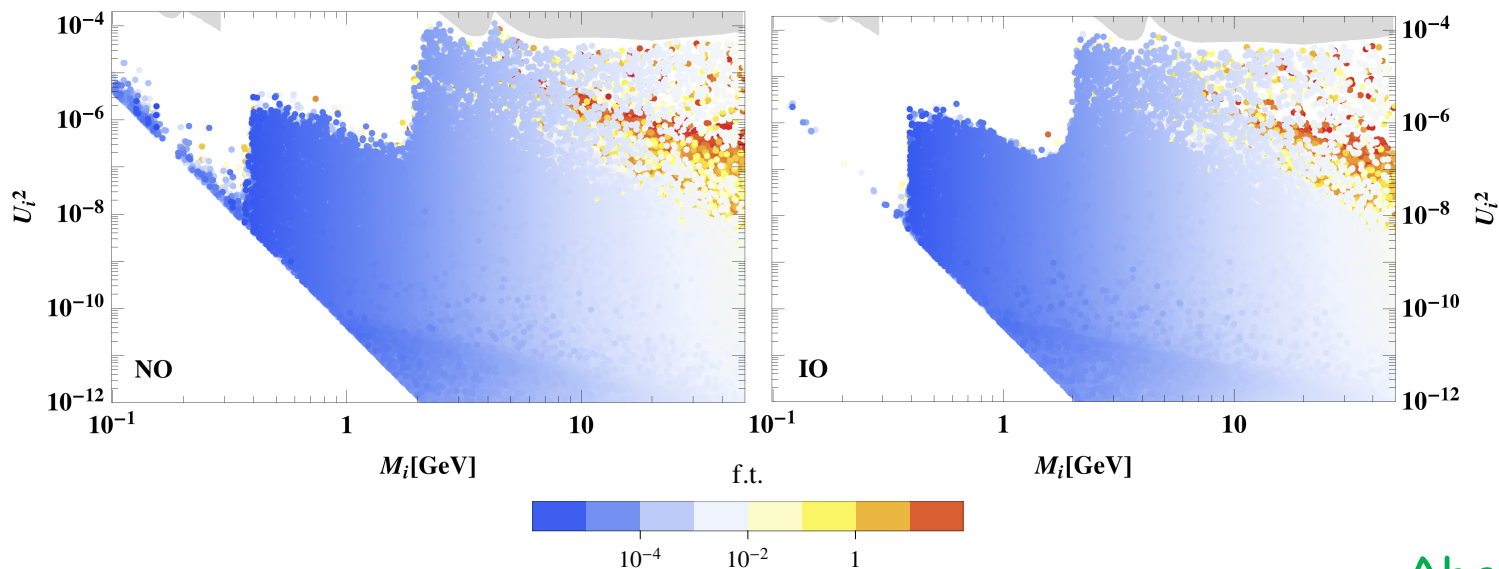
Klaric et al. 2020, 21

N=3 HNL

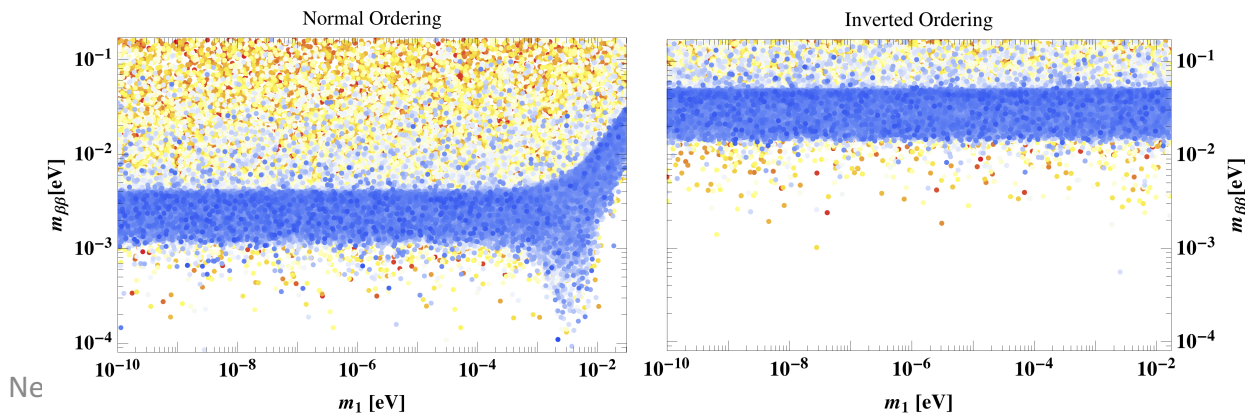
Larger values of HNL mass splittings allowed

NH

IH



Abada et al. 2019



6. Summary and outlook

- Thermal leptogenesis in generic type I seesaw: simple, appealing ... but difficult to test.
- Explore low energy alternatives:
 - ν electroweak baryogenesis
 - Progress understanding leptogenesis via sterile neutrino oscillations, interesting implications for low energy observables
 - Unified description of resonant leptogenesis and baryogenesis via neutrino oscillations under way

Backup slides

Parameter scan

Antusch et al. 2018; Abada et al. 2019; Klaric' et al. 2020, 2021; Drewes et al. 2022

- Nested sampling algorithm UltraNest

$$\log(\mathcal{L}) = -\frac{1}{2} \left(\frac{Y_B(T_{EW}) - Y_B^{\text{exp}}}{\sigma_{Y_B^{\text{exp}}}} \right)^2 \quad Y_B^{\text{exp}} = (8.66 \pm 0.05) \times 10^{-11}$$

- Priors:

$\log_{10}(M_1)$	$\log_{10}(\Delta M/M_1)$	$\log_{10}(y)$	θ	δ	α
$[-1, 2]$	$[-14, -1]$	$[-8, -4]$	$[0, 2\pi]$	$[0, 2\pi]$	$[0, 2\pi]$

- $y'/y < 0.1$, to ensure approximate LNC limit
- Restricted to region testable at SHIP, FCC-ee.
- Additional scan for NH with log priors in all angles, in range

4. $\mu \approx 0$ case

$$M = \begin{pmatrix} 0 & \Lambda \\ \Lambda & 0 \end{pmatrix} \quad Y = \begin{pmatrix} y_e & y'_e e^{i\beta'_e} \\ y_\mu & y'_\mu e^{i\beta'_\mu} \\ y_\tau & y'_\tau e^{i\beta'_\tau} \end{pmatrix}$$

- Sterile neutrinos degenerate at $T > T_{EW}$, except for small loop correction

$$\Delta\mu \propto yy' \rho M / (4\pi)^2 \ll yy' \rho T^2 / (8M)$$

- At $T=0$:
$$\Delta M_{NH} = |m_3| - |m_2| = \sqrt{\Delta m_{atm}^2} - \sqrt{\Delta m_{sol}^2}$$

$$\Delta M_{IH} = |m_2| - |m_1| = \sqrt{\Delta m_{atm}^2} - \sqrt{\Delta m_{atm}^2 - \Delta m_{sol}^2}$$

- Once active neutrino masses and mixings are fixed, only 4 free parameters: M , U^2 (or y^2), and PMNS phases, (δ, ϕ)

CP violating flavour basis invariants

- All previous CP invariants vanish in the $\mu \approx 0$ limit
- Higher order in the Yukawa couplings:

$$\tilde{I}_0 \equiv \text{Im} \left(\text{Tr} \left[Y^\dagger Y M_R^* Y^T Y^* M_R Y^\dagger Y_\ell Y_\ell^\dagger Y \right] \right) \equiv \sum_\alpha y_{\ell_\alpha}^2 \Delta_\alpha$$

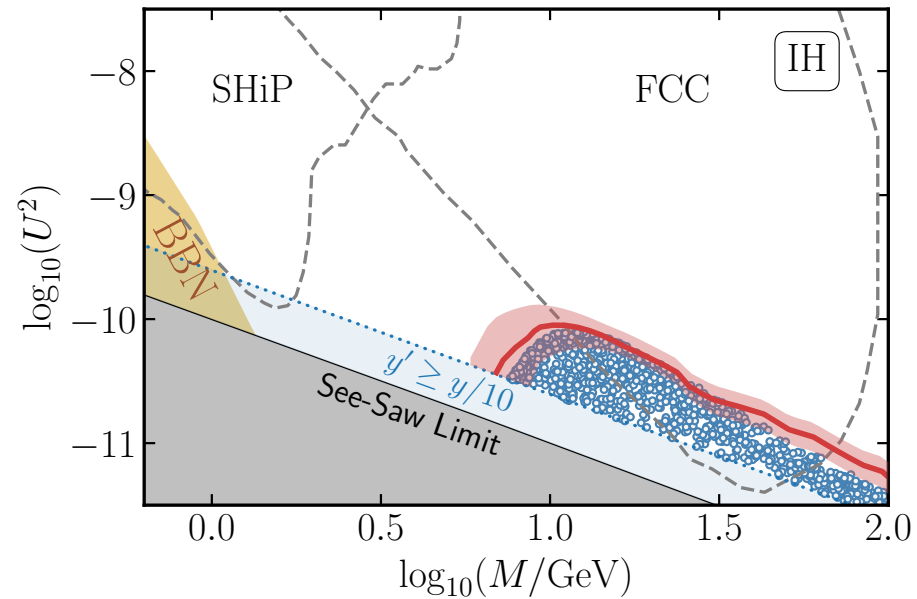
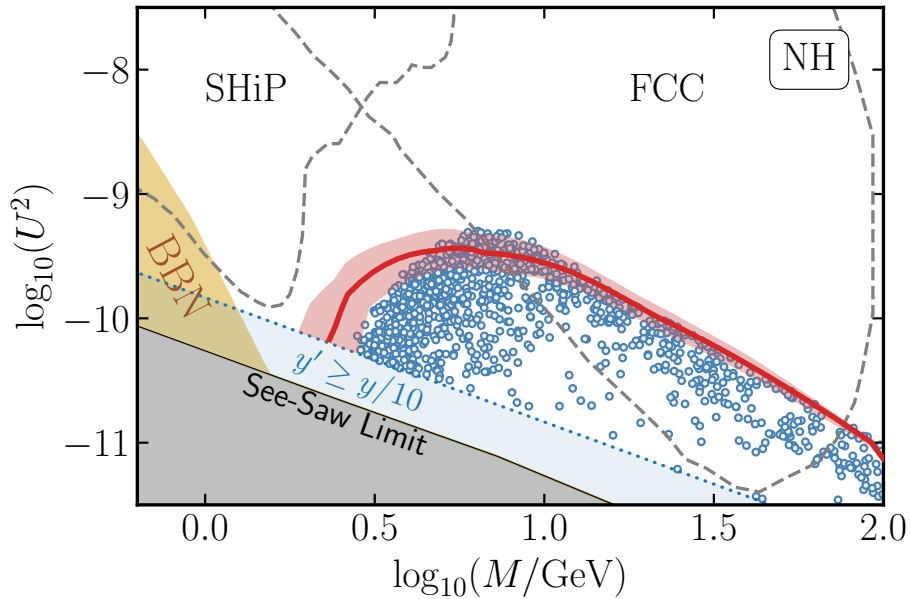
with

$$\Delta_\alpha = \text{Im} \left[\left(Y Y^\dagger Y M_R^* Y^T Y^* M_R Y^\dagger \right)_{\alpha\alpha} \right] \quad \sum_\alpha \Delta_\alpha = 0$$

We find contribution from weak flavour:

$$\Delta_\alpha^{\text{fw}} = \frac{1}{\text{Tr} (Y^\dagger Y)^2} \Delta_\alpha ,$$

$\mu \approx 0$ case

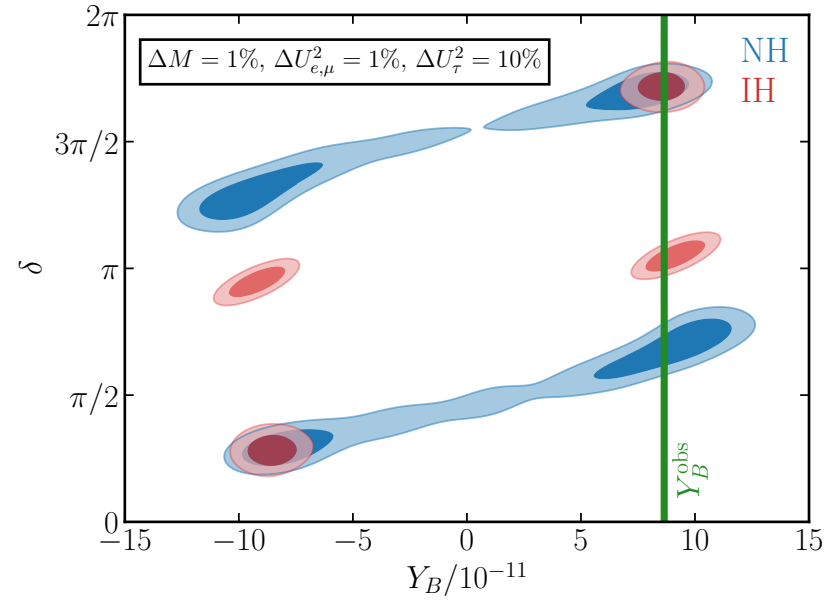
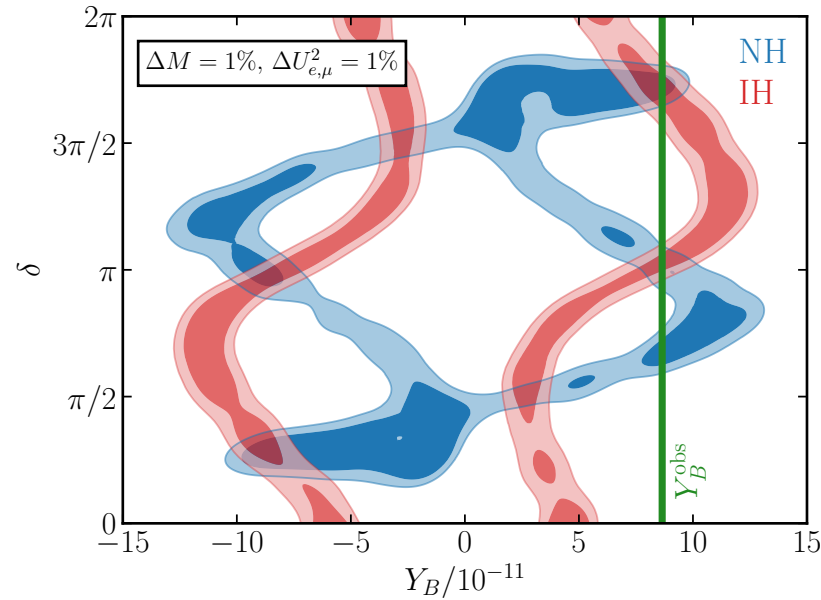


Observable at FCC only in sHC regime

$$Y_B = -1.5 \times 10^{-25} \left(\frac{\text{GeV}}{M} \right)^2 \left(\frac{1}{U^2} \right)^2 f_\alpha^{\text{H}} \quad U^2 \geq 1 \times 10^{-6} \left(\frac{1 \text{ GeV}}{M} \right)^4$$

$$f_e^{\text{NH}} = -\frac{\sqrt{r}}{2} \theta_{13} s_{12} \sin(\delta + \phi), \quad f_\mu^{\text{IH}} = f_\tau^{\text{IH}} = -\frac{r^2}{8} c_{12} s_{12} \sin \phi$$

Numerical likelihood inference in the case of measuring HNL-active neutrino mixings



	$M^{\text{true}}/\text{GeV}$	$(U_e^2)_{\text{true}}$	$(U_\mu^2)_{\text{true}}$	$(U_\tau^2)_{\text{true}}$	$\delta^{\text{true}}/\text{rad}$
NH	31.60	2.843×10^{-12}	1.087×10^{-11}	1.234×10^{-11}	5.396
IH	20.731	3.291×10^{-11}	4.823×10^{-12}	3.465×10^{-12}	5.402