What is the wave packet size of neutrinos?

Gustavo F. S. Alves

To appear soon (I hope)....

In collaboration with:
Shirley Weishi Li
Pedro Machado
Ningqiang Song

Neutrinos oscillations in a nutshell

- The oscillation formula is usually derived similarly a two level QM oscillator.
- So easy that it fits in here:

$$e^{-iHt}\,|\,\nu_\alpha\rangle = \sum_i U^*_{\alpha i} e^{-iE_i t}\,|\,\nu_i\rangle$$

$$\mathscr{A}_{\alpha\to\beta}(L) = \langle\nu_\beta\,|\,\nu_\alpha(L)\rangle = \sum_i U^*_{\alpha i} U_{\beta i} e^{-iE_i L}$$

$$\mathscr{P}_{\alpha\to\beta}(L) = |\,\mathscr{A}_{\alpha\to\beta}(L)\,|^2 = \langle\nu_\beta\,|\,\nu_\alpha(L)\rangle = \sum_{ij} U^*_{\alpha i} U_{\beta j} U_{\alpha i} U^*_{\beta j} e^{-i(E_i - E_j)L}$$

The Devil is in the details

- ~~The oscillation formula is usually derived similarly a two level QM oscillator.~~
- ~~So easy that it fits in here:~~

Position propagation?

$$e^{-iHt}|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} e^{-iE_i t}|\nu_i\rangle$$

How to move from t to L?

Plane waves x localization?

$$\mathscr{A}_{\alpha\to\beta}(L) = \langle\nu_\beta|\nu_\alpha(L)\rangle = \sum_i U^*_{\alpha i} U_{\beta i} e^{-iE_i L}$$

Relativistic approximation equal energy or equal momentum?

$$\mathscr{P}_{\alpha\to\beta}(L) = |\mathscr{A}_{\alpha\to\beta}(L)|^2 = \langle\nu_\beta|\nu_\alpha(L)\rangle = \sum_{ij} U^*_{\alpha i} U_{\beta j} U_{\alpha i} U^*_{\beta j} e^{-i(E_i - E_j)L}$$

What should we do? Wave packets as a resort

- The neutrino should have localization properties.
- Gaussian wave packets have been extensively used as an ansatz. Probability formula is modified to

Necessary input: The wave packet width

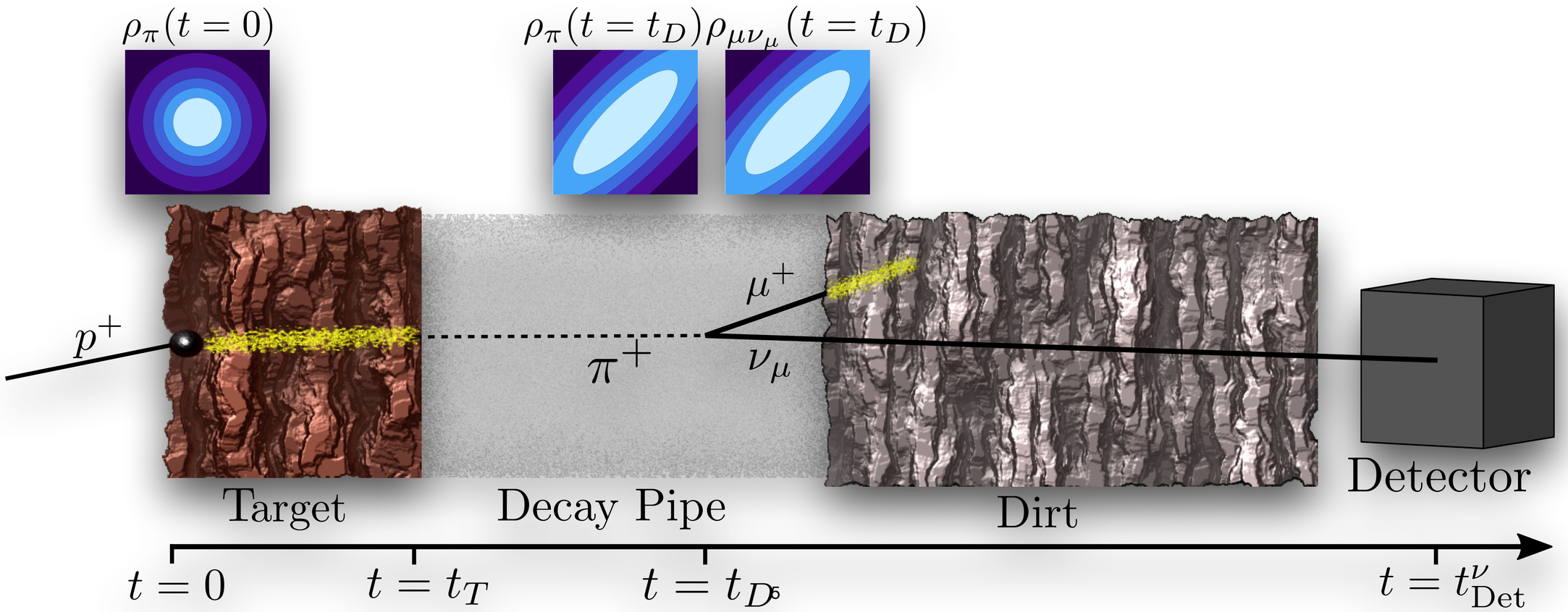
$$\mathscr{P}_{\alpha\to\beta}(L) = \mathscr{P}^{\text{p.w.}}_{\alpha\to\beta}(L) \times \exp\left[-\left(\frac{L}{L^{\text{coh}}}\right)^2 - \left(\frac{\sigma_x}{L^{\text{osc}}}\right)^2 \right]$$

$$L^{\text{coh}} = \frac{4\sqrt{2}E^2\sigma_x}{|\Delta m^2|} \qquad L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}$$

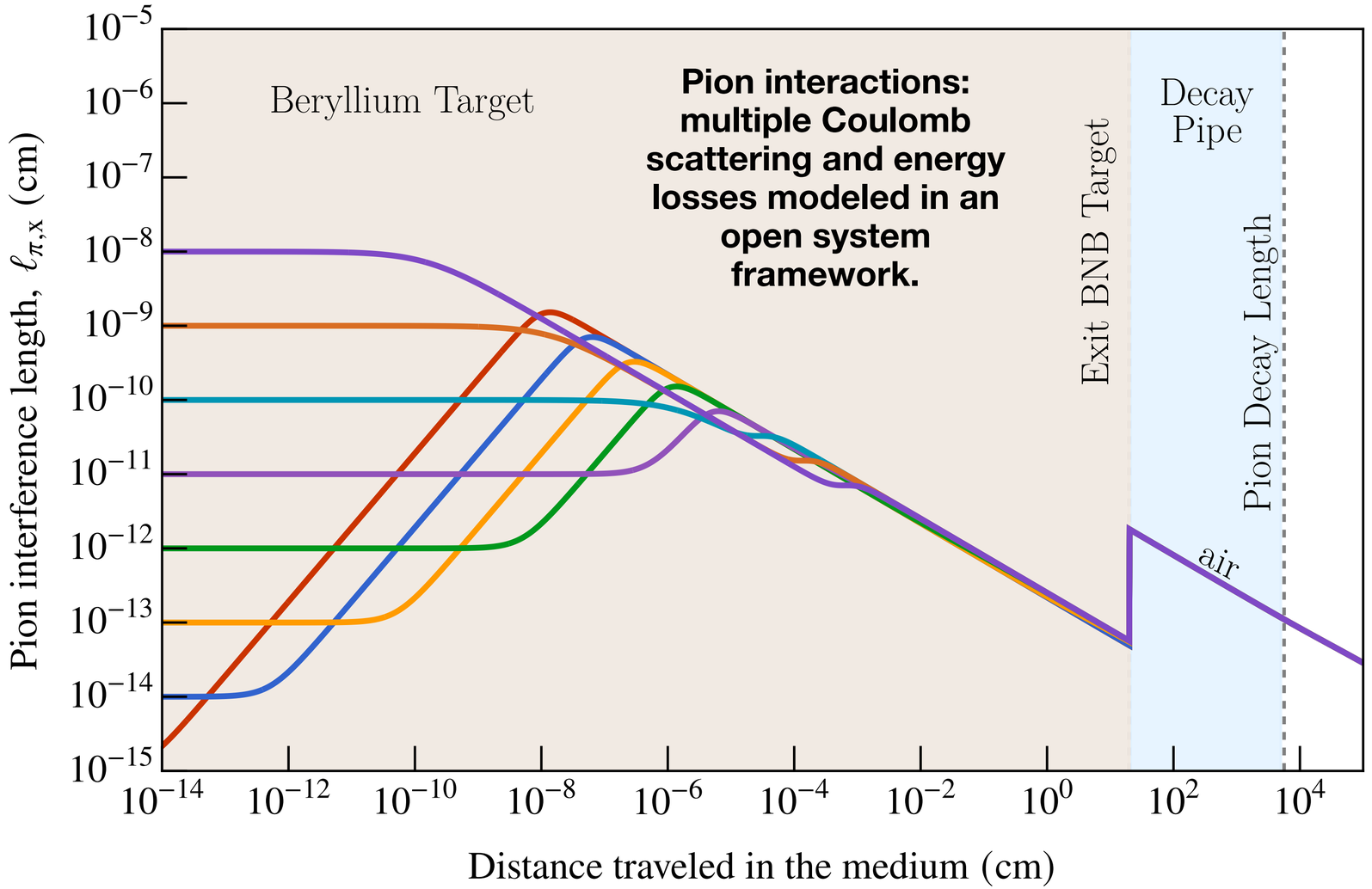
Are there alternatives to estimates?

Decoherence Models

The idea:



Stage 1: Pion interactions fix its state coherence properties.



A taste of the physics at play:

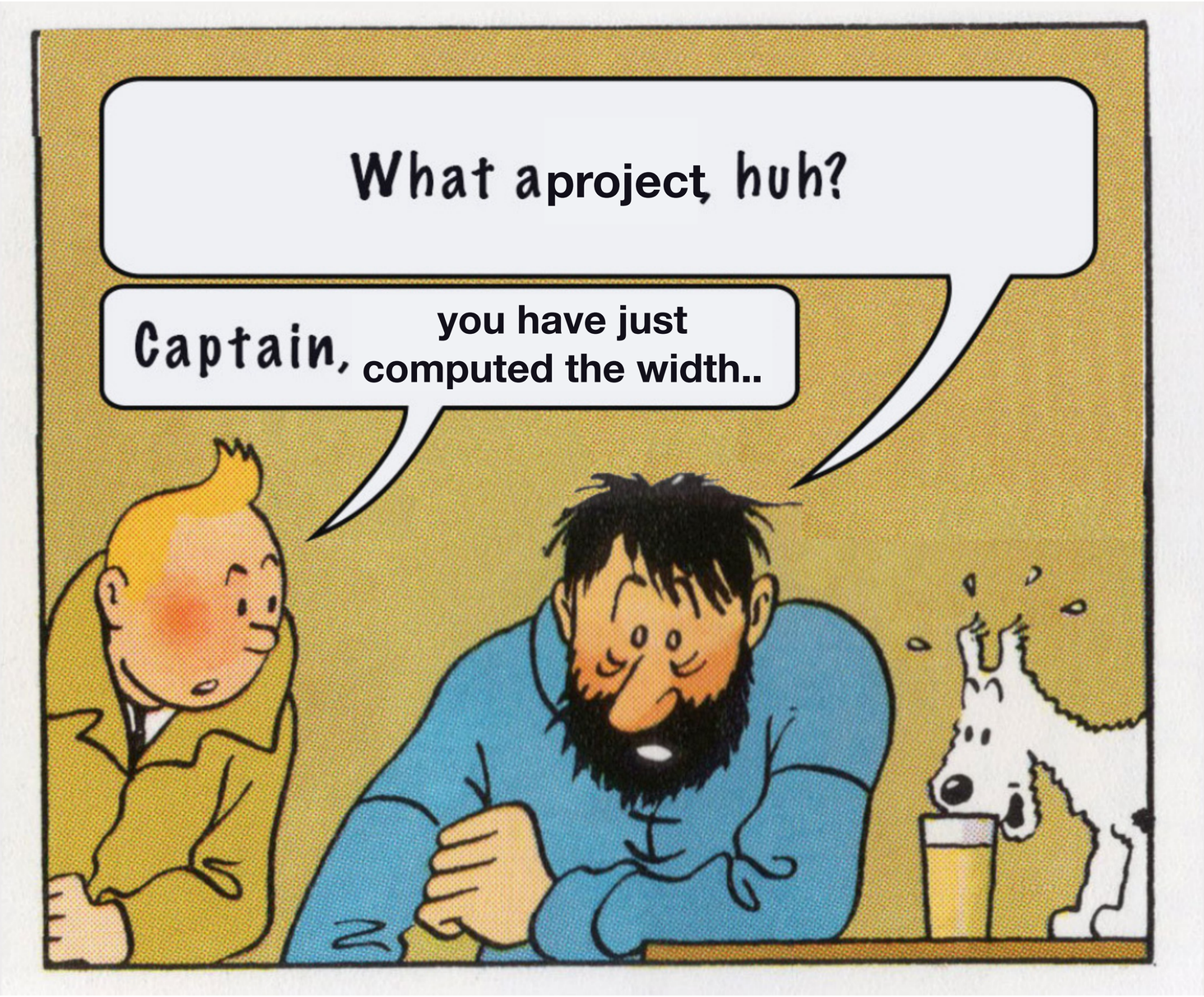
Scattering governed by parameter Λ that affects the state as:

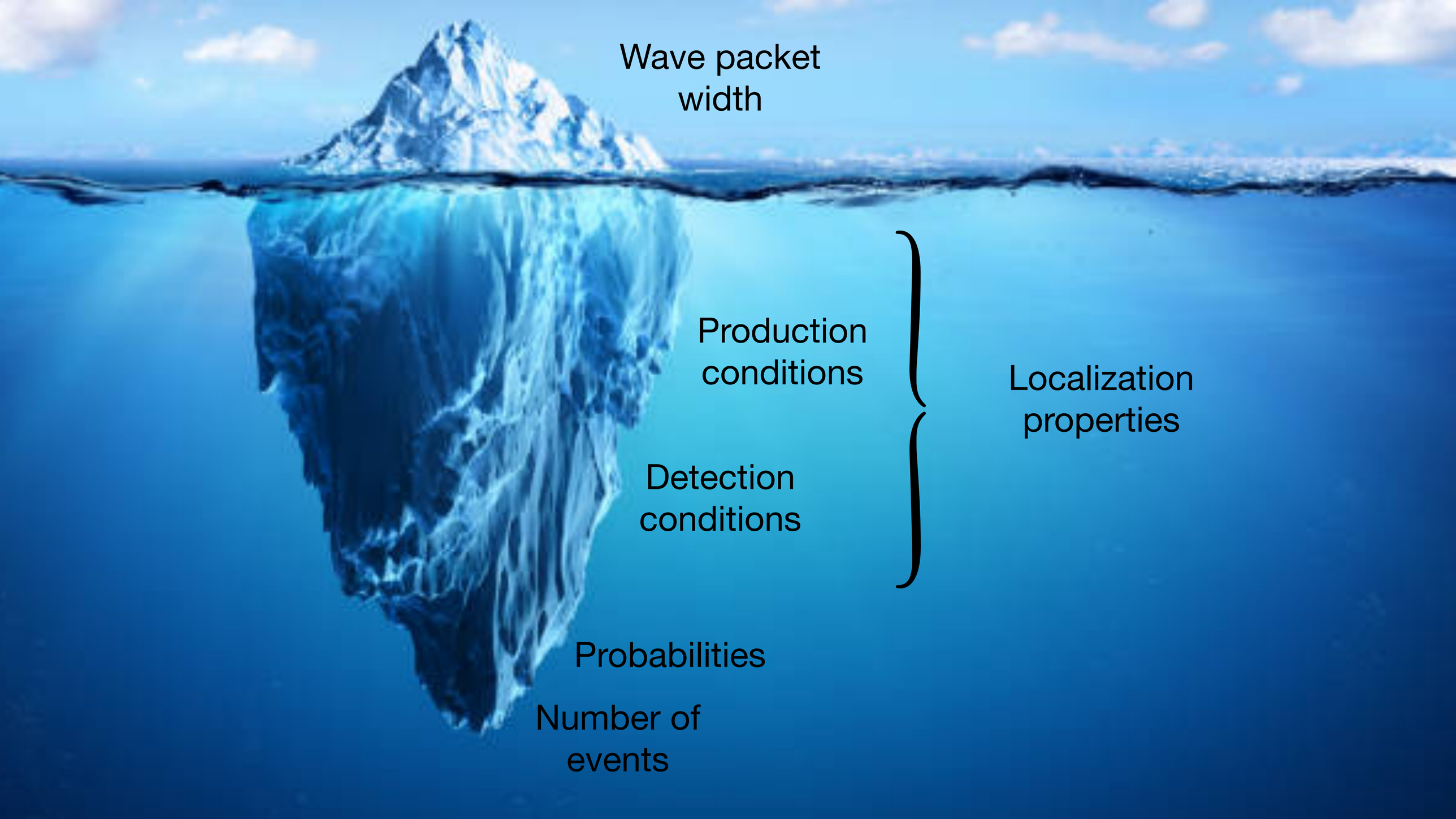
$$\rho(x, x', t) = e^{-\Lambda(x-x')^2 t}\rho(x, x', 0)$$

Hence superpositions get dampened.

What did we learn?

1. **Pion interference properties is fixed from interactions.**
2. **Its interference width at decay is $\ell_{\pi,x} = 10^{-13}$ cm**


What a project, huh?
Captain, you have just computed the width..


Wave packet width
Production conditions
Detection conditions
Localization properties
Probabilities
Number of events

Come to my poster to hear about other details

Take home message:

1. We could compute the wave packet size from the pion interactions.
2. We could not find any scenario where we can probe the wave packet separation (yet).
3. Detection/production provides the biggest challenge to be overcomed.

(Almost) First-principle calculation of the accelerator neutrino wave packet size

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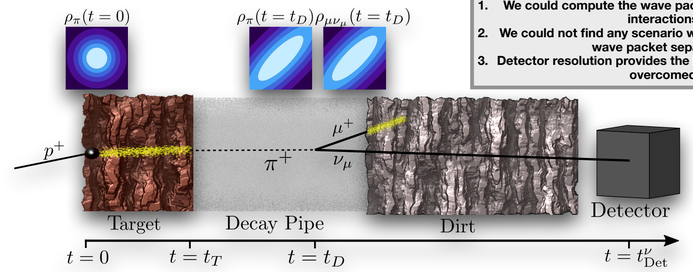
Summary

Version for people in a hurry:

1. We could compute the wave packet size from the pion interactions.
2. We could not find any scenario where we can probe the wave packet separation.
3. Detector resolution provides the biggest challenge to be overcomed.



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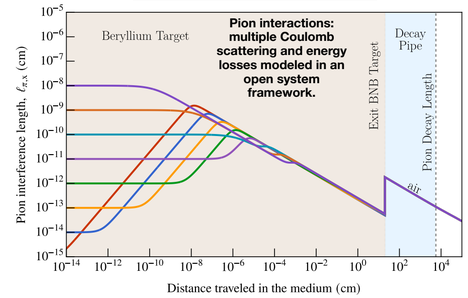

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Stage 2:

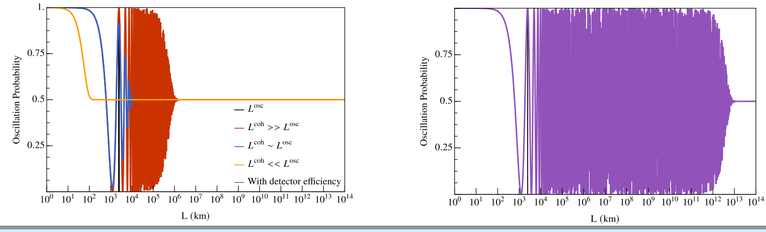
Pion decays producing the kinematically entangled neutrino-muon state with coherence properties inherited from the mother particle.

Stage 3:

Get the reduced neutrino state by tracing over the muon degrees of freedom and study how the muon state can affect the coherence properties of the neutrino state.

Stage 4:

Study neutrino oscillations with a neutrino state unambiguously determined from the mother particle state.



What did we learn?

1. Neutrino coherence length from our model is ~2000 km for 1 GeV neutrinos.
2. We must compare it to the oscillation length, their ratio is 0.43 and this is a mass splitting independent statement.
3. Wave packet separation is a necessary but not sufficient condition. Detector resolutions is key to proper interpret the results.