Topological Portal to the Dark Sector

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Topological Interactions

Terms not involving the spacetime metric.

- Example: Wess-Zumino-Witten term in QCD.

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- There exists an $SU(3)_L \times SU(3)_R$ invariant term (a 5-form) on $X = SU(3)$:

$$\left.\begin{aligned} \omega_5 &= -\frac{iN_c}{480\pi^3}\mathrm{Tr}\left[\left(U^{-1}\mathrm{d}U\right)^5\right] \\ &= -\frac{iN_c}{480\pi^3}\,\mathrm{d}x^5\,\epsilon^{\mu\nu\rho\sigma\tau}\,\mathrm{Tr}\left[U^\dagger\left(\partial_\mu U\right)U^\dagger\left(\partial_\nu U\right)U^\dagger\left(\partial_\rho U\right)U^\dagger\left(\partial_\sigma U\right)U^\dagger\left(\partial_\tau U\right)\right] \end{aligned}\right\} \quad \boxed{S_{\mathrm{WZW}} = \int_{\Sigma_5}\omega_5}$$

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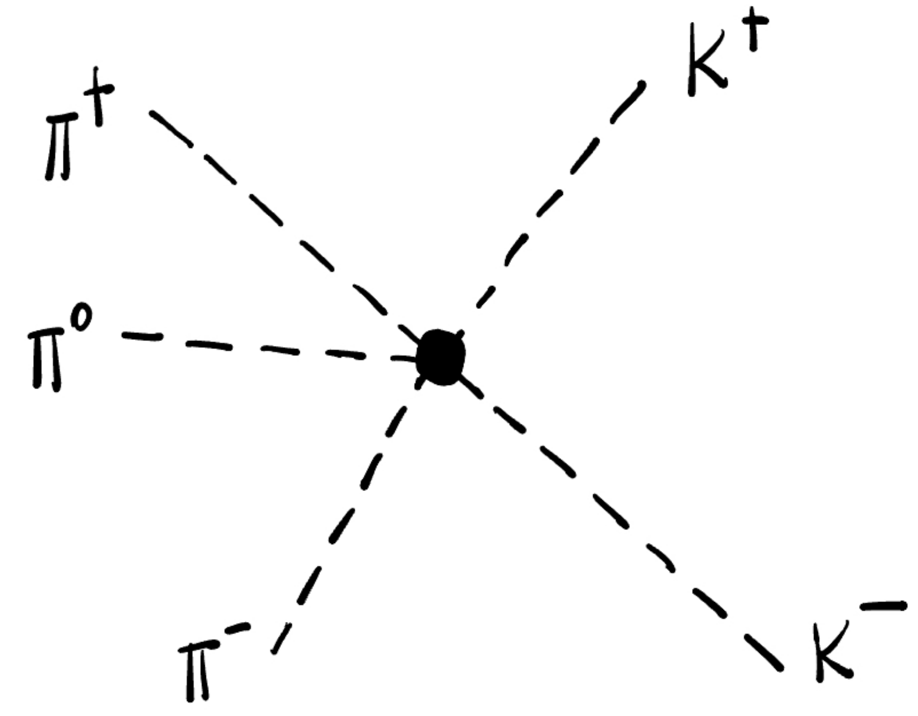

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- Expanding locally:

$$U^\dagger \partial_\mu U = \frac{2i}{f_\pi}\partial_\mu \pi + \mathcal{O}(\pi^2)$$

- Results in new interactions: $$\boxed{S_{\mathrm{WZW}} = \frac{2}{15\pi^2 f_\pi^5}\int_{\Sigma_4 = \partial\Sigma_5} \mathrm{d}^4x\,\epsilon^{\nu\rho\sigma\tau}\,\mathrm{Tr}\left[\pi\left(\partial_\nu\pi\right)\left(\partial_\rho\pi\right)\left(\partial_\sigma\pi\right)\left(\partial_\tau\pi\right)\right] + \mathcal{O}(\pi^6)}$$

QCD $\times$ Dark Topological Portal

EFT description.

- Collective non-linear sigma model on a product coset:

$$X = \frac{SU(3)_L \times SU(3)_R \times G_{\mathrm{D}}}{SU(3)_{L+R} \times H_{\mathrm{D}}} \simeq SU(3) \times \frac{G_{\mathrm{D}}}{H_{\mathrm{D}}}$$

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- There is another $SU(3)_L \times SU(3)_R$ invariant term (a 3-form) on $SU(3)$, and no more than that!

$$\omega_3^{\mathrm{QCD}} \sim \mathrm{Tr}\left[\left(U^{-1}\mathrm{d}U\right)^3\right] \sim f_{abc}\,\mathrm{d}\pi^a\,\mathrm{d}\pi^b\,\mathrm{d}\pi^c$$

- Can be used to construct:

$$\omega_5^{\mathrm{portal}} \sim \omega_3^{\mathrm{QCD}} \times \omega_2^{\mathrm{D}} \longrightarrow \omega_2^{\mathrm{D}} = \frac{1}{4\pi f_D^2}\epsilon_{ij}\,\mathrm{d}\chi_i\,\mathrm{d}\chi_j$$ G_D-invariant 2-form on $G_\mathrm{D}/H_\mathrm{D}$.

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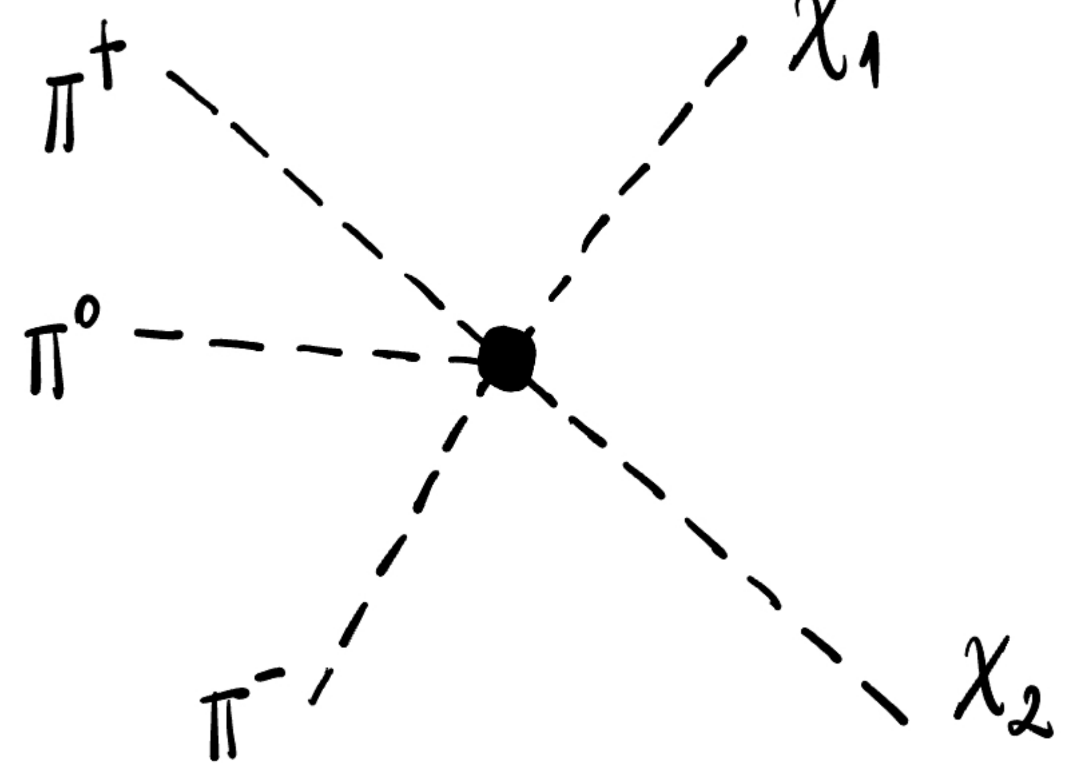

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- Locally:

$$\boxed{\mathcal{L}_{\mathrm{portal}}^{e=0} = \frac{in\epsilon^{\mu\nu\rho\sigma}}{48\pi^2 f_\pi^3 f_D^2} f_{abc}\epsilon_{ij}\pi_a\partial_\mu\pi_b\partial_\nu\pi_c\partial_\rho\chi_i\partial_\sigma\chi_j}$$

Relic Abundance

Can be robustly set.

- Leading term obtained after *gauging*:

$$\tilde{\mathcal{L}}_{\rm portal} = \frac{n e\, \epsilon^{\mu\nu\rho\sigma}}{16\pi^2 f_\pi f_D^2}\left(\pi^0 + \frac{\eta}{\sqrt{3}}\right) F_{\mu\nu}\partial_\rho \chi_1 \partial_\sigma \chi_2$$

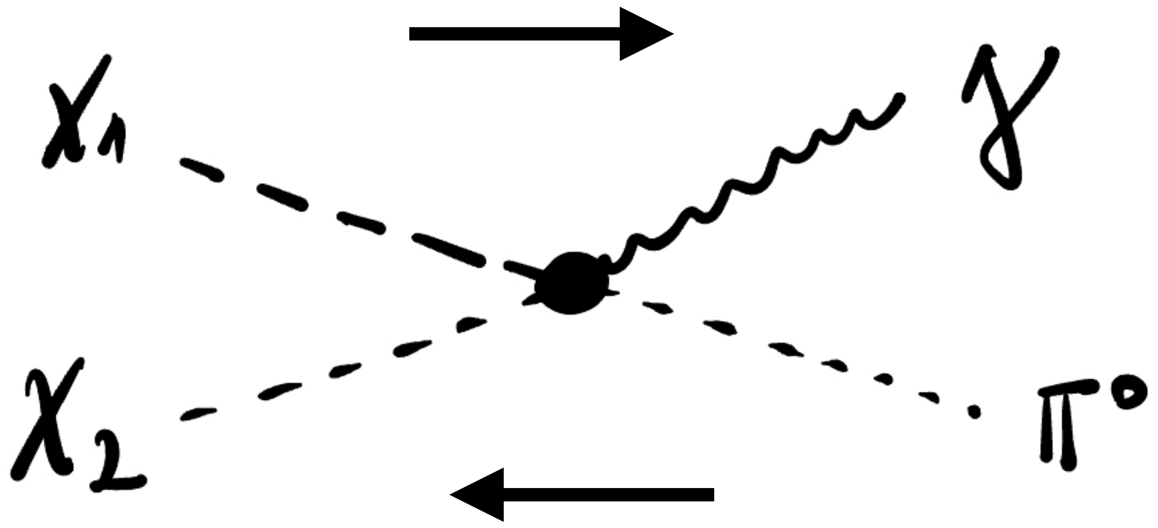
- Portal activates *after* the QCD phase transition.

- *No dependence* on the cosmological history of the dark pions.

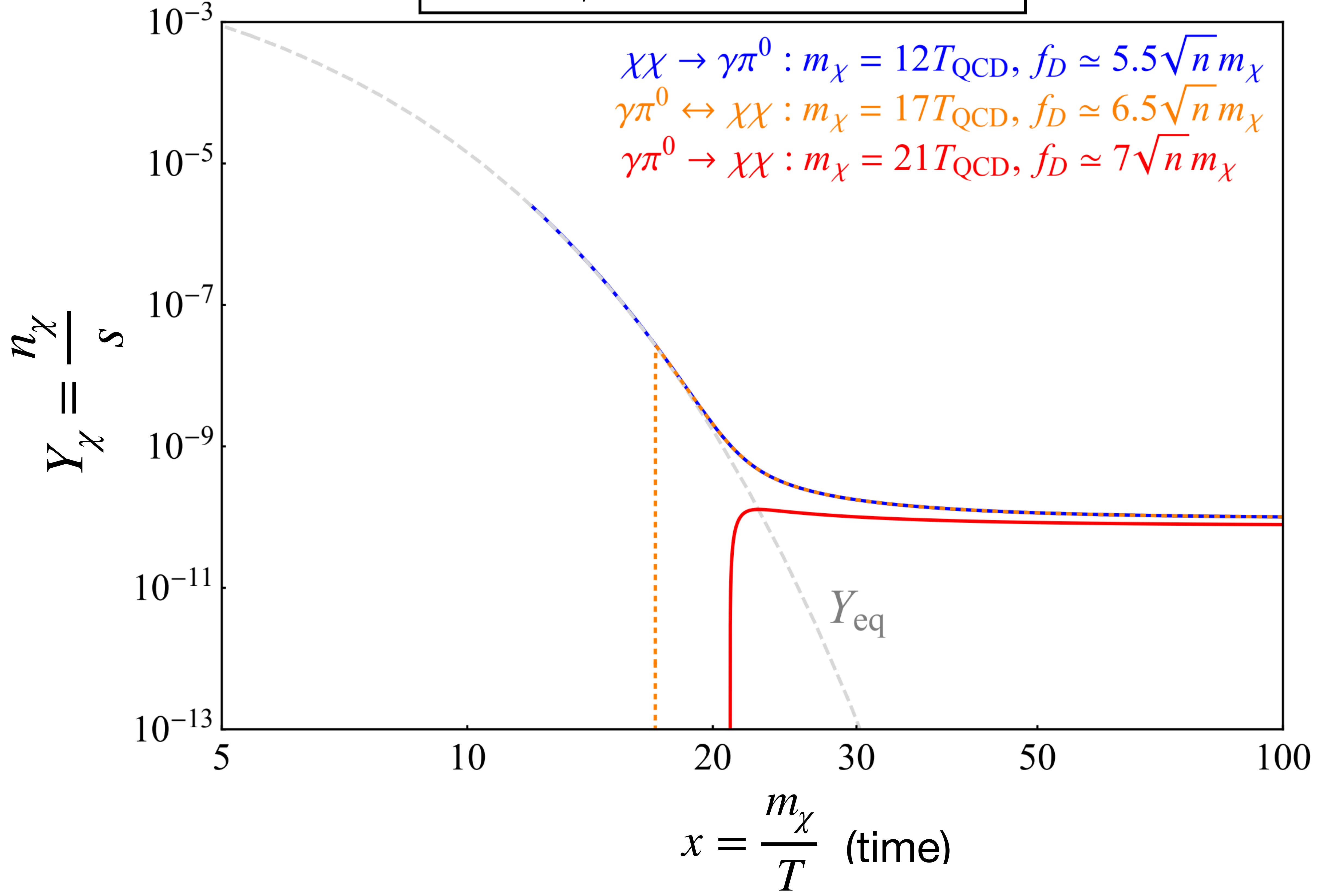
- QCD phase transition should happen *no later* than $x = x_{\rm max} \approx 23$:

$$\boxed{m_\chi \lesssim 3.7\ {\rm GeV}}$$

$$f_D \sim \mathcal{O}(5-7)\sqrt{n}\, m_\chi$$

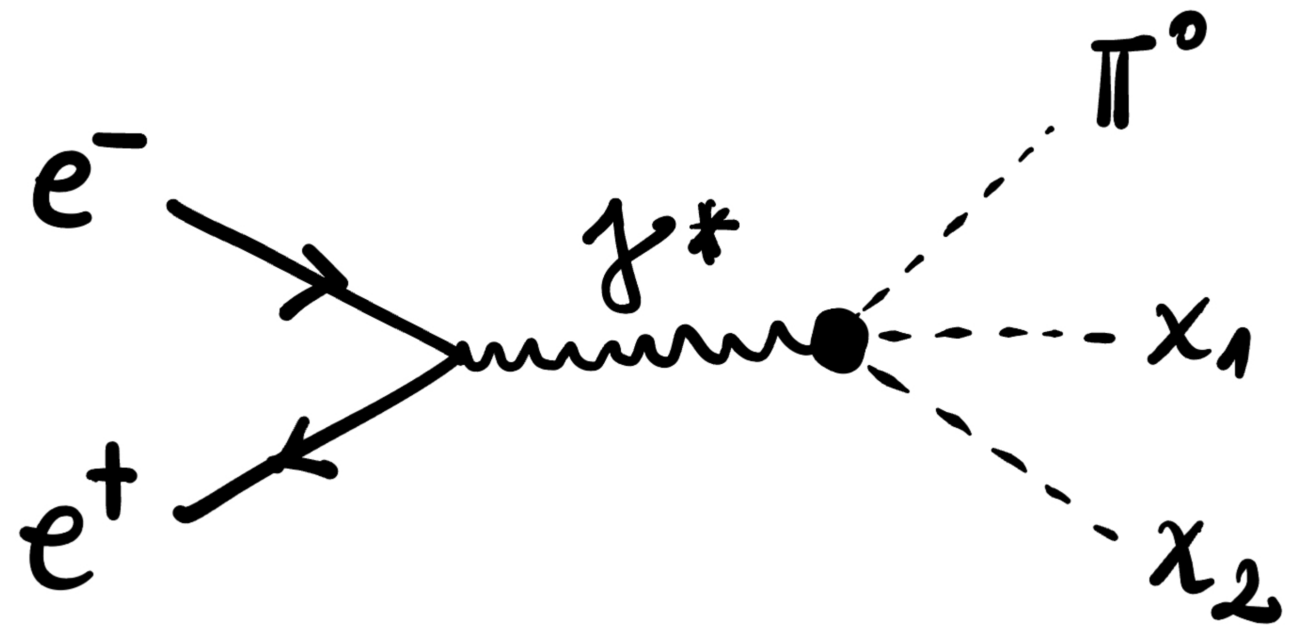


$$\frac{dY_\chi}{dx} = -\sqrt{\frac{\pi g_*}{45}}\frac{M_{\rm P} m_\chi}{x^2}\langle \sigma v\rangle \left(Y_\chi^2 - Y_{\rm eq}^2\right)$$



How to test this?

Novel collider signatures.



- Topological operator (a differential form) couples two *different* dark pions:

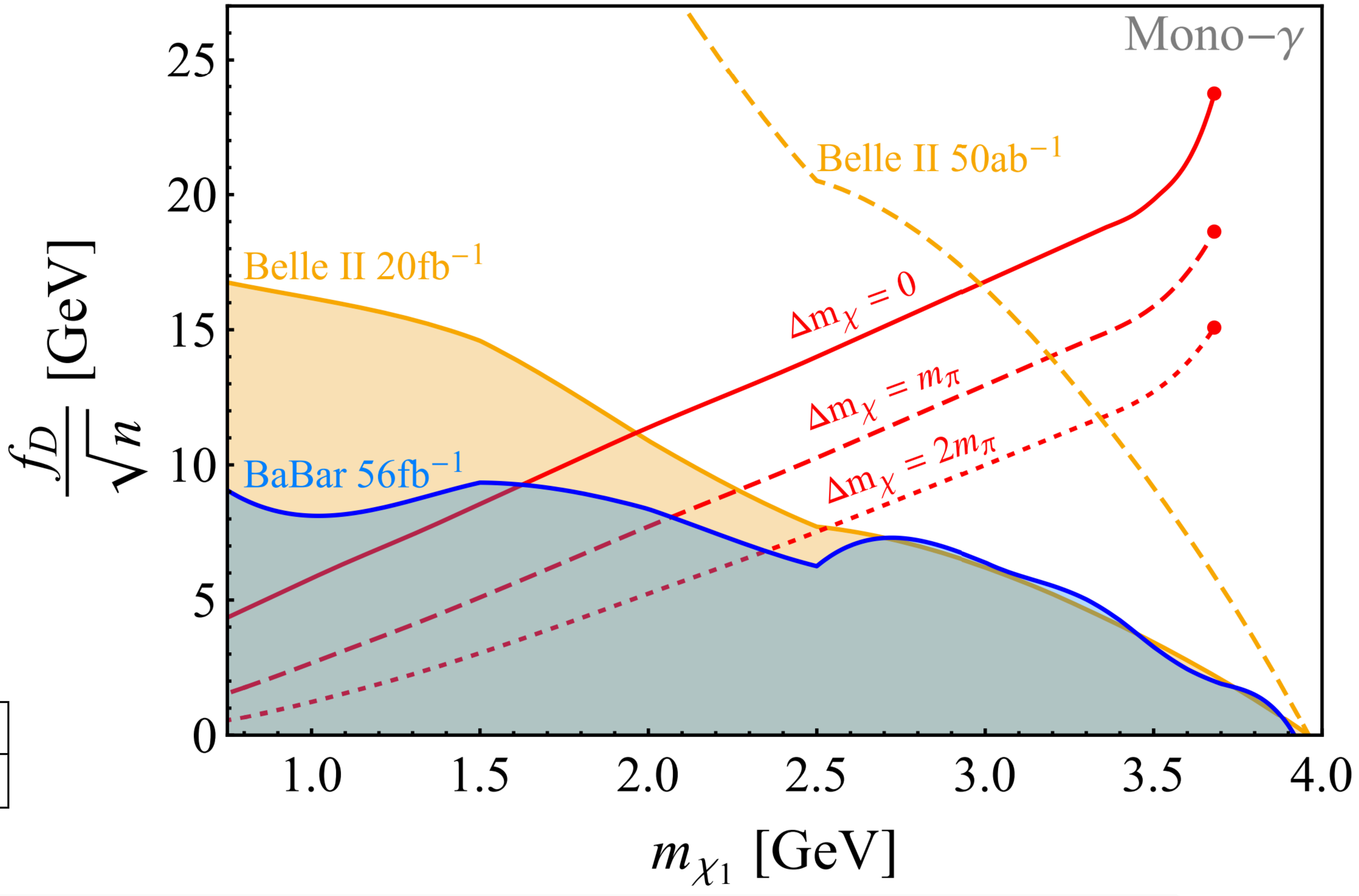
  1. (Indirect detection)
  Annihilation of $\chi_1\chi_1$ highly suppressed.

  2. (Direct detection)
  $\chi_1 \to \chi_2$ kinematically forbidden,
  $\chi_1 \to \chi_1$ highly suppressed.

Light thermal inelastic DM scenario.

- *Belle II* could tell us a lot:

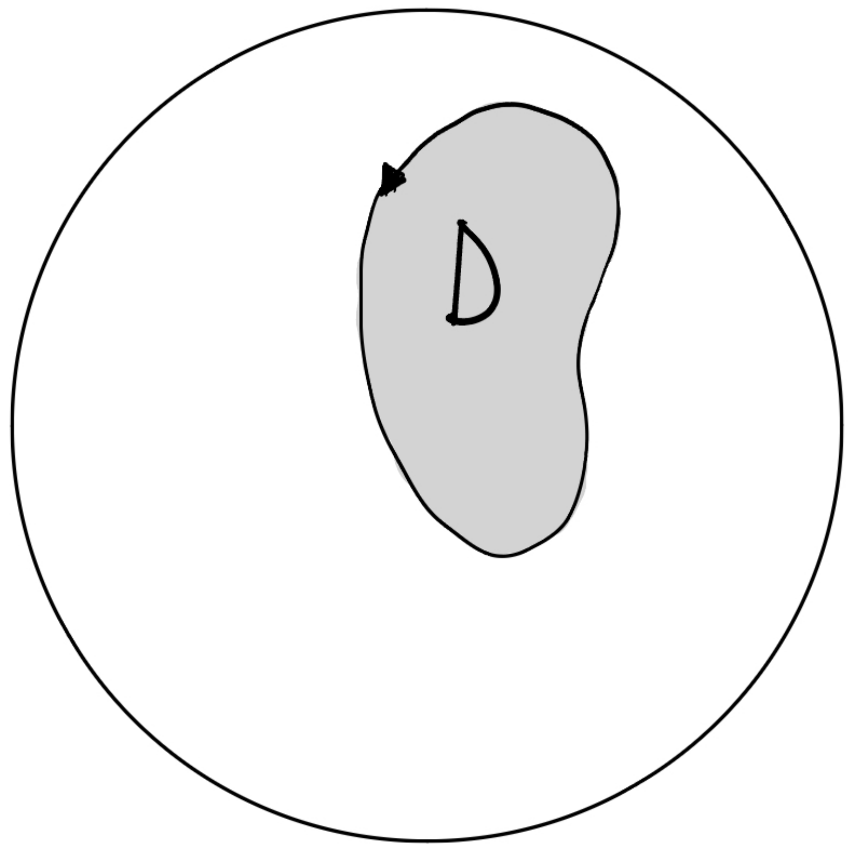
Δm_χ	$\lesssim 1.7 m_{\pi^0}$	$\gtrsim 1.7 m_{\pi^0}$
Signature	$\pi^0 + \not{E}_T$	$\pi^0 + \not{E}_T + \mathrm{DV}(\pi^0 \gamma \not{E}_T)$



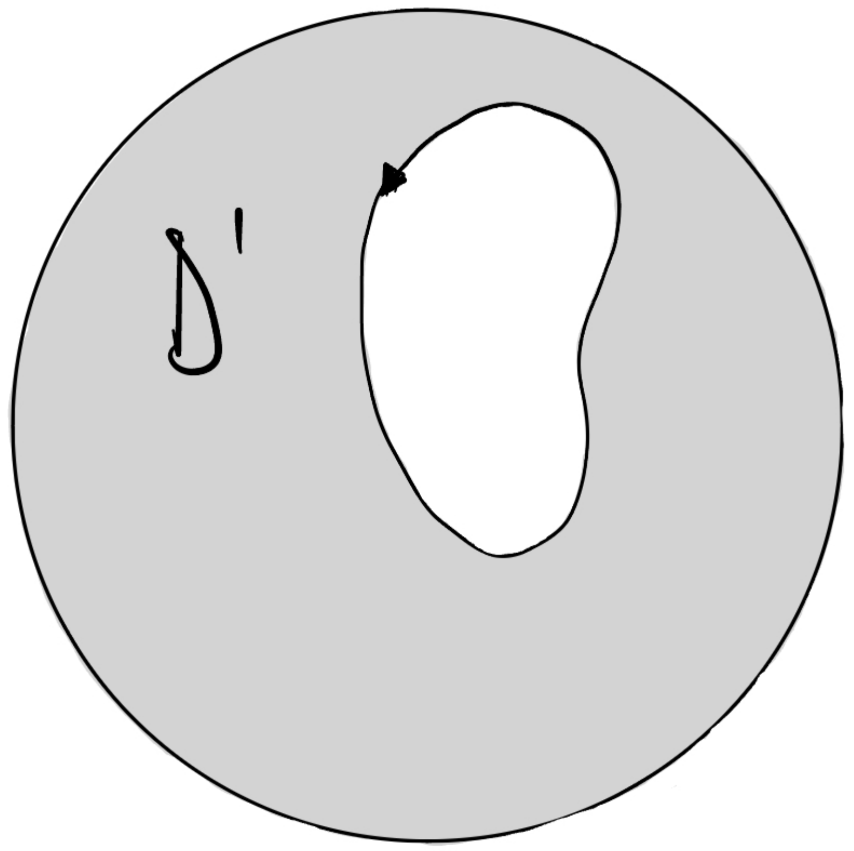
Thank you!

Some properties of WZW terms

- Integer coefficients:



or



$$\exp\left(ik\int_D \omega_5\right) = \exp\left(ik\int_{D'} \omega_5\right)$$

$\exp\left(ik\int_{DUD'=\Sigma_5} \omega_5\right) = 1$ and $\int_{\Sigma_5} \omega_5 = 2\pi n \longrightarrow k \in \mathbb{Z}$

- Gauging:

$$U(1)_{\text{QED}} \supset SU(3)_{L+R}$$

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} \qquad U \to U + i\epsilon\left[Q, U\right]$$

Non-trivial derivation using Noether method:

$$J^\mu = \frac{1}{48\pi^2}\,\epsilon^{\mu\rho\sigma\tau}\text{tr}\left(\{Q, U^\dagger\}\,\partial_\rho U\,U^\dagger\partial_\sigma U\,U^\dagger\partial_\lambda U\right)$$

$$L_{\text{WZW}}^{\text{gauged}} \supset \frac{ke^2}{96\pi^2 f_\pi}\pi^0\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

$$\downarrow$$

$$k = N_c$$

Some properties of our portal ω_5^{portal}

$$\omega_5^{\text{portal}} \sim \omega_3^{\text{QCD}} \times \omega_2^{\text{D}} \quad \text{on} \quad X = \frac{SU(3)_L \times SU(3)_R \times G_{\text{D}}}{SU(3)_{L+R} \times H_{\text{D}}} \simeq SU(3) \times \frac{G_{\text{D}}}{H_{\text{D}}}$$

1. Closed: $\mathrm{d}\omega_5^{\text{portal}} = 0$ implies $\mathrm{d}\omega_2^{\text{D}} = 0$ since $\mathrm{d}\omega_3^{\text{QCD}} = 0$.
2. $SU(3)_L \times SU(3)_R \times G_{\text{D}}$ - invariance: Product structure implies ω_2^{D} is G_{D} - invariant.
3. Integrality: Cycles factorise; normalise ω_3^{QCD} and ω_2^{D} separately.

- Unique choice for QCD-like theories: $\frac{G_{\text{D}}}{H_{\text{D}}} = \left\{ \frac{SU(N)_L \times SU(N)_R}{SU(N)_{L+R}}, \frac{SU(N)}{SO(N)}, \frac{SU(2N)}{Sp(2N)} \right\}$
- Theorem: G_{D} - invariant forms on $G_{\text{D}}/H_{\text{D}}$ are in 1-to-1 with cohomology classes:

  The portal exists iff $H^2(G_{\text{D}}/H_{\text{D}}) \neq 0 \longrightarrow G_{\text{D}}/H_{\text{D}} = SU(2)/SO(2)$.