Quantisation Across Bubble Walls and Friction

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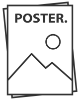
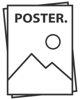
Outline

1. FOPT: Why, What, bubbles dynamic & **friction**



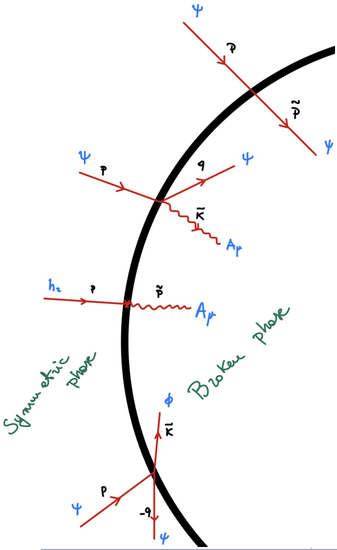
2. Toolkit for Quantisation across the wall (scalar example)
   1. Complete basis of solutions to the spatially dependent EOM
   2. Quantisation & construction of 'In' and 'Out' asymptotic states
   3. Amplitudes
   4. Approximations (Step wall and WKB)

3. **Gauge-fixing** and spin-interpolation



   1. EOM + gauge fixing
   2. Interpolation between Higgs and longitudinal polarisation

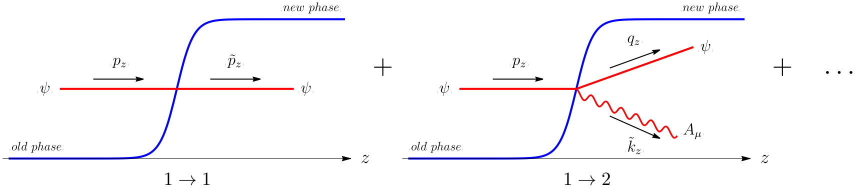
4. Results & Conclusions



Computing the **friction**

- $\gamma_w \gg 1 \quad \longrightarrow \quad$ **interacting with individual particles**.
- Translational symmetry broken $\longrightarrow$ $z-$momentum not conserved!

Incoming flux
$\gamma_w n \, v_w$



$$\mathcal{P} = \underbrace{\int \frac{d^3p}{(2\pi)^3}\frac{p^z}{p_0} f_i^{\rm eq}}_{\text{incoming flux}} \times \underbrace{\sum_f \int d\mathbb{P}_{i\to f}\,\Delta p^z}_{\langle \Delta p^z \rangle}$$

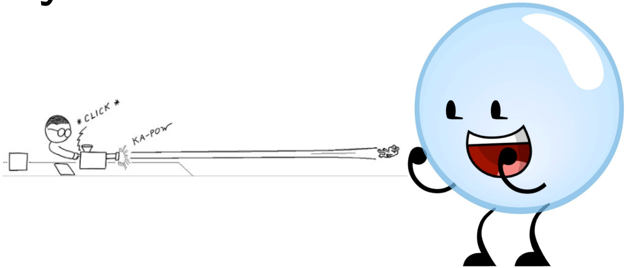
Toolkit for Quantisation across the wall

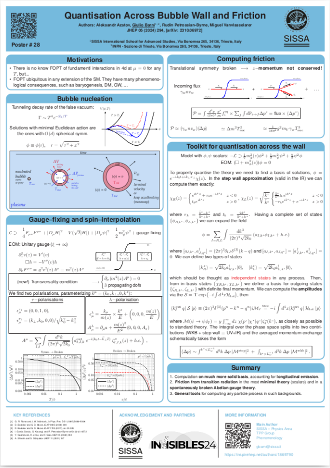
1. Specify a model: $-\mathcal{L} \supset \frac{1}{2}m_\phi^2(z)\phi^2 + \frac{1}{2}m_\psi^2\psi^2 + \frac{y}{2}\psi^2\phi$
2. Solve EOM and find (non-trivial) basis of eigenmodes (for outgoing states): $\{\zeta_R, \zeta_L\}$
3. Quantisation: $\phi = \sum_{I=R,L} \int \frac{dk^3}{(2\pi)^3\sqrt{2k_0}} \left(a_{I,k^z}\phi_{I,k^z} + h.c.\right)$
4. Compute the amplitude: $\mathcal{M}_I \equiv \mathcal{M}(\psi \to \psi\phi_I) = y\int_{-\infty}^{\infty} dz\ \chi(p^z)\chi^*(q^z)\zeta_I^*(k^z)$
5. Phase space integration: $\begin{cases} \text{IR} & \longrightarrow \quad \text{step wall approx } (k^z < L_w^{-1}) \\ \text{UV} & \longrightarrow \quad \text{WKB approx } (k^z > L_w^{-1}) \end{cases}$
6. Computation of the averaged exchanged momentum

$$\langle \Delta p \rangle \sim \sum_{L,R} \int^{k^z < L_w^{-1}} d^3k\ \Delta p\ |\mathcal{M}^{\text{step}}|^2 + \int_{k^z > L_w^{-1}} d^3k\ \Delta p\ |\mathcal{M}^{\text{wkb}}|^2 \ .$$

...and come to my poster!

Thanks for your attention...





(Non trivial) eingenmodes for step wall approximation

ϕ, ψ scalars: $-\mathcal{L} \supset \frac{1}{2} m_\phi^2(z)\phi^2 + \frac{1}{2} m_\psi^2 \psi^2 + \frac{y}{2}\psi^2\phi$

$\rightarrow$ $m_\psi = const$ does not feel the wall

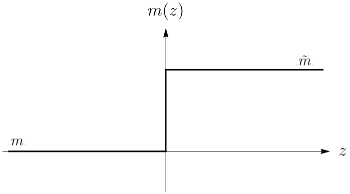
$\rightarrow$ while $m_\phi \equiv m_\phi(z)$ does

$\rightarrow$ EOM: $(\Box + m_\phi^2(z))\phi = 0$ and $\phi(z) = e^{-ik_0 t + ik_\perp x_\perp}\chi(z)$

$$\chi_R(z) = \begin{cases} e^{ik^z z} + r_k e^{-ik^z z} & z < 0 \\ t_k e^{i\tilde{k}^z z} & z > 0 \end{cases}$$

$$\chi_L(z) = \sqrt{\frac{k^z}{\tilde{k}^z}} \begin{cases} \frac{\tilde{k}^z}{k^z} t_k e^{ik^z z} & z < 0 \\ -r_k e^{i\tilde{k}^z z} + e^{-i\tilde{k}^z z} & z > 0 \end{cases}$$

where $r_k = \frac{\tilde{k}^z - k^z}{\tilde{k}^z + k^z}$ and $t_k = \frac{2k^z}{\tilde{k}^z + k^z}$



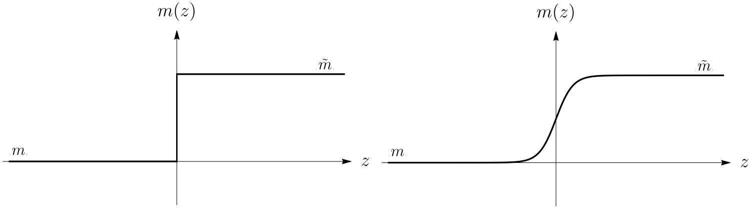
STEP WALL APPROX.
(valid in the IR)

Beyond step wall $\to$ WKB

When does the step wall approximation break?

- If the z **momentum is large enough** $(k^z L_w \gtrsim 1)$ there will be **mostly transmission**! $\to$ WKB

$$k^z \lesssim L_w^{-1} \qquad\qquad L_w^{-1} \lesssim k^z \leq k^z_{\text{max}}$$



Step wall: $\zeta_{R,L}$

WKB: $\chi_R(z) = \sqrt{\dfrac{k^z}{k^z(z)}}\, e^{-i\int_0^z dz' k^z(z')z'}$

- When $\Delta p L_w \gg 1$ then $\mathcal{M} \to 0$ $\qquad$ ($z-$momentum conservation is restored!)

Amplitudes & Phase Space (for step wall)

We are ready to compute the **amplitudes**

$$\mathcal{S} = \mathrm{T}\exp\left(-i\int d^4x\mathcal{H}_{\mathrm{Int}}\right) \qquad \mathcal{H}_{\mathrm{Int}} = -iy\psi^2(x)\phi(x)$$

$$\langle k_I^{\mathrm{out}}\, q|\,\mathcal{S}\,|p\rangle \equiv (2\pi)^3\delta^{(3)}(p^n - k^n - q^n)i\mathcal{M}_I \overset{\mathrm{tree}}{=} -i\int d^4x\langle k_I^{\mathrm{out}}\, q|\,\mathcal{H}_{\mathrm{Int}}\,|p\rangle$$

$$\boxed{\mathcal{M}_I \equiv \mathcal{M}(\psi\to\psi\phi_I) = y\int_{-\infty}^{\infty}dz\;\chi(p^z)\chi^*(q^z)\zeta_I^*(k^z)}$$

Then the **averaged exchanged momentum**

$$\begin{aligned}\langle\Delta p\rangle &= \langle\Delta p_R\rangle + \langle\Delta p_L\rangle \\ &= \int d\mathbb{P}_{\psi\to\psi\phi_{\zeta_R}}(p^z - q^z - \tilde{k}^z) + \int d\mathbb{P}_{\psi\to\psi\phi_{\zeta_L}}(p^z - q^z + k^z)\end{aligned}$$

$$\int d\mathbb{P}_{\psi\to\psi\phi_I}\Delta p_I^z = \int_{k_{\mathrm{min}}^{z,I}}^{k_{\mathrm{max}}^z}\frac{dk_z}{2\pi}\frac{1}{2k_0}\int_0^{k^2_{\perp,\mathrm{max}}}\frac{dk_\perp^2}{4\pi}\cdot\frac{1}{2p^z}\left[\frac{1}{2|q^z|}|\mathcal{M}_I|^2\Delta p_I^z\right]_{q^z=\pm q_k^z}$$

Vector boson emission: Abelian Higgs model

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu H|^2 - V(\sqrt{2}|H|) + |D_\mu \psi|^2 - \frac{1}{2}m_\psi^2\psi^2 + \text{gauge fixing}, \qquad D_\mu = \partial_\mu + igA_\mu$$

EOM: Unitary gauge $\xi \to \infty$

$$\partial_z^2 v(z) = V'(v)$$
$$\Box h = -V''(v)h$$
$$\partial_\nu F^{\mu\nu} = g^2 v^2(z) A^\mu \equiv m^2(z) A^\mu$$

(new!) Transversality condition $\begin{cases} \partial_\mu(m^2(z)A^\mu) = 0 \\ 3 \text{ propagating dofs} \end{cases}$