



## Quantisation Across Bubble Walls and Friction

SISSA & INFN TRIESTE

Giulio Barni

gbarni@sissa.it

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with *Aleksandr Azatov, Rudin Petrossian-Byrne and Miguel Vanvlasselaer*

# Outline

## 1 FOPT: Why, What, bubbles dynamic & **friction**



## 2 Toolkit for Quantisation across the wall (scalar example)

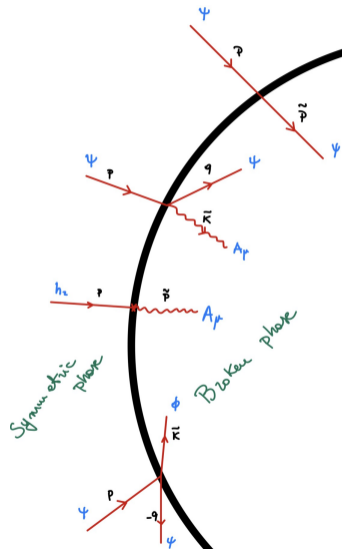
- 1 Complete basis of solutions to the spatially dependent EOM
- II Quantisation & construction of 'In' and 'Out' asymptotic states
- III Amplitudes
- IV Approximations (Step wall and WKB)

## 3 Gauge-fixing and spin-interpolation



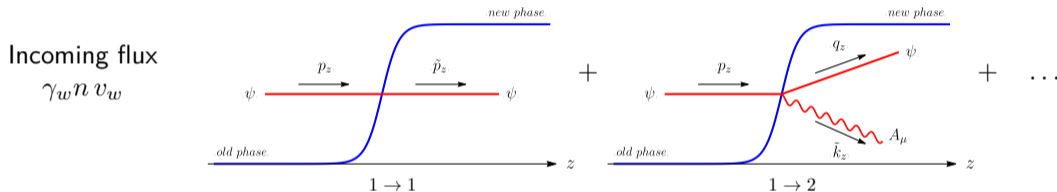
- 1 EOM + gauge fixing
- II Interpolation between Higgs and longitudinal polarisation

## 4 Results & Conclusions



# Computing the friction

- $\gamma_w \gg 1 \rightarrow$  interacting with individual particles.
- Translational symmetry broken  $\rightarrow z$ -momentum not conserved!



$$\mathcal{P} = \underbrace{\int \frac{d^3 p}{(2\pi)^3} \frac{p^z}{p_0} f_i^{\text{eq}}}_{\text{incoming flux}} \times \underbrace{\sum_f \int d\mathbb{P}_{i \rightarrow f} \Delta p^z}_{\langle \Delta p^z \rangle}$$

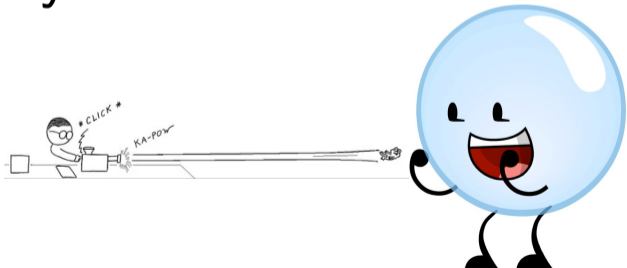
# Toolkit for Quantisation across the wall

- 1 Specify a model:  $-\mathcal{L} \supset \frac{1}{2}m_\phi^2(z)\phi^2 + \frac{1}{2}m_\psi^2\psi^2 + \frac{y}{2}\psi^2\phi$
- 2 Solve EOM and find (non-trivial) basis of eigenmodes (for outgoing states):  $\{\zeta_R, \zeta_L\}$
- 3 Quantisation:  $\phi = \sum_{I=R,L} \int \frac{dk^3}{(2\pi)^3\sqrt{2k_0}} (a_{I,k^z}\phi_{I,k^z} + h.c.)$
- 4 Compute the amplitude:  $\mathcal{M}_I \equiv \mathcal{M}(\psi \rightarrow \psi\phi_I) = y \int_{-\infty}^{\infty} dz \chi(p^z)\chi^*(q^z)\zeta_I^*(k^z)$
- 5 Phase space integration:  $\begin{cases} \text{IR} & \longrightarrow \text{step wall approx } (k^z < L_w^{-1}) \\ \text{UV} & \longrightarrow \text{WKB approx } (k^z > L_w^{-1}) \end{cases}$
- 6 Computation of the averaged exchanged momentum

$$\langle \Delta p \rangle \sim \sum_{L,R} \int^{k^z < L_w^{-1}} d^3k \Delta p |\mathcal{M}^{\text{step}}|^2 + \int_{k^z > L_w^{-1}} d^3k \Delta p |\mathcal{M}^{\text{wkb}}|^2 .$$

...and come to my poster!

Thanks for  
your attention...



**Quantisation Across Bubble Walls and Friction**

Authors: Alessandro Andreoli, Giulio Barni\* - Paolo Paganini-Morino, Miguel Ángel Rodríguez  
\*INFN, Università di Trieste, I.N.F.N. Sezione di Trieste, I.C.T.P., Trieste, Italy  
\*INFN, Università di Trieste, I.N.F.N. Sezione di Trieste, I.C.T.P., Trieste, Italy

SISSA International School for Advanced Studies, Via Boncompagni 12, 34136, Trieste, Italy  
1976, Università di Trieste, I.N.F.N. Sezione di Trieste, Trieste, Italy

Poster # 20

**Motivations**

- There is no known PCPT of fundamental interactions in 4d at  $\mu = 0$  for any  $\mathcal{F}$  but...
- PCPT stabilises in any extension of the SM. They have many phenomenological consequences, such as baryogenesis, DM, GW...

**Computing friction**

Translational symmetry broken  $\rightarrow$  momentum not conserved!

Incoming flux  $\mathcal{F}_{in}$

$$P = \int_{\Sigma} d^3x \mathcal{E} \cdot \mathbf{v} = \sum_i \int_{\Sigma} \mathcal{E}_i v_i d^3x = \text{Re} \langle \mathcal{E} | \mathcal{F} \rangle$$

$$P = \int_{\Sigma} \mathcal{E} \cdot \mathbf{v} d^3x \Rightarrow \text{Re} \langle \mathcal{E} | \mathcal{F} \rangle = \frac{1}{2} \langle \mathcal{E} | \mathcal{F} + \mathcal{F}^\dagger | \mathcal{E} \rangle$$

**Bubble nucleation**

Tunneling decay rate of the false vacuum:

$$\Gamma \sim e^{-S_B}$$

Solutions with minimal Euclidean action are the ones with (1/2) spherical symmetry.

$\phi = \phi(r), \quad r = \sqrt{t^2 + x^2}$

**Toolkit for quantisation across the wall**

Model with 2-d.o.f. action:  $\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \psi \partial^\mu \psi + V(\phi)$

To properly quantise the theory we need to find a basis of solutions,  $\phi = \sum_i c_i \psi_i(x)$ . In the **wall approximation** (valid in the IR) we can compute their orthonormality:

$$\langle \psi_i | \psi_j \rangle = \int_{-\infty}^{\infty} dx \psi_i(x) \psi_j(x) = \int_{-\infty}^{\infty} dx \left[ \frac{1}{2} \left( \psi_i(x) \psi_j'(x) - \psi_i'(x) \psi_j(x) \right) \right]_{-\infty}^{\infty} = \delta_{ij}$$

where  $\psi_i = \frac{e^{i(kx - \omega t)}}{\sqrt{2\omega}}$  and  $\psi_j = \frac{e^{i(kx - \omega t)}}{\sqrt{2\omega}}$ . Fixing a complete set of states  $\{|\psi_{k,\omega}, \psi_{k,\omega}\rangle\}$  we can expand the field:

$$\phi = \sum_{k,\omega} \frac{a_{k,\omega}}{\sqrt{2\omega}} \psi_{k,\omega}(x) + \text{h.c.}$$

where  $a_{k,\omega} = \int_{-\infty}^{\infty} dx \psi_{k,\omega}^*(x) \phi(x)$  and  $a_{k,\omega}^\dagger = \int_{-\infty}^{\infty} dx \psi_{k,\omega}(x) \phi(x)$ . We can define two types of states:

$$|k, \omega\rangle = \sqrt{2\omega} a_{k,\omega}^\dagger |0\rangle, \quad |k, \omega\rangle = \sqrt{2\omega} a_{k,\omega} |0\rangle$$

which should be thought as **independent** states in any process. Then, from 3d-like states  $\{|\psi_{k,\omega}, \psi_{k,\omega}\rangle\}$  we define a basis for outgoing states  $\{|\psi_{k,\omega}, \psi_{k,\omega}\rangle\}$  with particular final momenta. We can compute the amplitudes via the  $S = T \exp(-i \int d^4x \mathcal{L})$ , then:

$$\langle \psi_{k,\omega} | S | \psi_{k,\omega} \rangle = \langle \psi_{k,\omega} | T \exp(-i \int d^4x \mathcal{L}) | \psi_{k,\omega} \rangle$$

where  $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + \mathcal{L}_I$ , as closely as possible to standard theory. The integral over the phase space splits into two contributions:  $\mathcal{M}^{(1)}$  (loop level) and the so-called **friction** term which schematically takes the form:

$$|\mathcal{M}^{(1)}|^2 \sim \int d^4x \mathcal{L}_I \phi \mathcal{L}_I \phi \sim \int d^4x \mathcal{L}_I \phi \mathcal{L}_I \phi$$

**Summary**

- Computation on-shell near wall basis, according to longitudinal extension.
- Friction from interaction evaluation in the most relevant theory (vacuum) and in a spontaneously broken Abelian gauge theory.
- General tools for computing any particle process in such backgrounds.

**KEY REFERENCES**

- 1. S. Weinberg, *Gravitation and Cosmology*, Wiley, 1973.
- 2. S. Weinberg, *The Quantum Theory of Fields, Volume I*, Cambridge University Press, 2005.
- 3. S. Weinberg, *The Quantum Theory of Fields, Volume II*, Cambridge University Press, 2009.
- 4. S. Weinberg, *The Quantum Theory of Fields, Volume III*, Cambridge University Press, 2013.

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**MORE INFORMATION**

Work Address: [giulio.barni@siissa.it](mailto:giulio.barni@siissa.it)  
 Email: [giulio.barni@siissa.it](mailto:giulio.barni@siissa.it)  
 Website: [www.sissa.it](http://www.sissa.it)  
 Twitter: [giulio\\_barni](https://twitter.com/giulio_barni)  
 LinkedIn: [giulio\\_barni](https://www.linkedin.com/in/giulio-barni)  
 Instagram: [giulio\\_barni](https://www.instagram.com/giulio_barni)  
 Facebook: [giulio.barni](https://www.facebook.com/giulio.barni)  
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https://arxiv.org/abs/2407.10078

# (Non trivial) eigenmodes for step wall approximation

$$\phi, \psi \text{ scalars: } -\mathcal{L} \supset \frac{1}{2}m_\phi^2(z)\phi^2 + \frac{1}{2}m_\psi^2\psi^2 + \frac{y}{2}\psi^2\phi$$

→  $m_\psi = \text{const}$  does not feel the wall

→ while  $m_\phi \equiv m_\phi(z)$  does

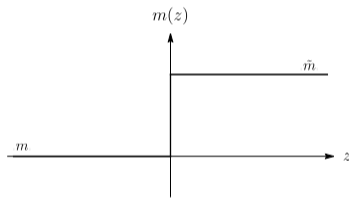
→ EOM:  $(\square + m_\phi^2(z))\phi = 0$  and  $\phi(z) = e^{-ik_0t + ik_\perp x_\perp} \chi(z)$

$$\chi_R(z) = \begin{cases} e^{ik^z z} + r_k e^{-ik^z z} & z < 0 \\ t_k e^{i\tilde{k}^z z} & z > 0 \end{cases}$$

$$\chi_L(z) = \sqrt{\frac{k^z}{\tilde{k}^z}} \begin{cases} \frac{\tilde{k}^z}{k^z} t_k e^{ik^z z} & z < 0 \\ -r_k e^{i\tilde{k}^z z} + e^{-i\tilde{k}^z z} & z > 0 \end{cases}$$



$$\text{where } r_k = \frac{\tilde{k}^z - k^z}{\tilde{k}^z + k^z} \text{ and } t_k = \frac{2k^z}{\tilde{k}^z + k^z}$$



STEP WALL APPROX.  
(valid in the IR)

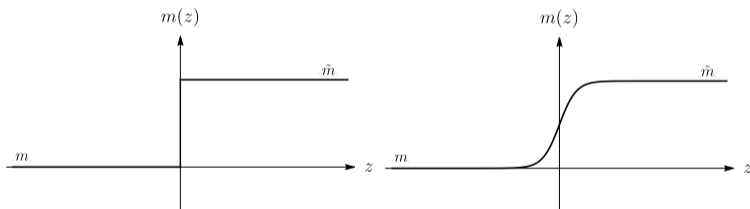
# Beyond step wall $\rightarrow$ WKB

When does the step wall approximation break?

- If the  $z$  **momentum is large enough** ( $k^z L_w \gtrsim 1$ ) there will be **mostly transmission!**  $\rightarrow$  WKB

$$k^z \lesssim L_w^{-1}$$

$$L_w^{-1} \lesssim k^z \leq k_{\max}^z$$



Step wall:  $\zeta_{R,L}$

$$\text{WKB: } \chi_R(z) = \sqrt{\frac{k^z}{k^z(z)}} e^{-i \int_0^z dz' k^z(z') z'}$$

- When  $\Delta p L_w \gg 1$  then  $\mathcal{M} \rightarrow 0$  ( $z$ -momentum conservation is restored!)

# Amplitudes & Phase Space (for step wall)

We are ready to compute the **amplitudes**

$$\mathcal{S} = \text{T exp} \left( -i \int d^4x \mathcal{H}_{\text{Int}} \right) \quad \mathcal{H}_{\text{Int}} = -iy\psi^2(x)\phi(x)$$

$$\langle k_I^{\text{out}} q | \mathcal{S} | p \rangle \equiv (2\pi)^3 \delta^{(3)}(p^n - k^n - q^n) i\mathcal{M}_I \stackrel{\text{tree}}{=} -i \int d^4x \langle k_I^{\text{out}} q | \mathcal{H}_{\text{Int}} | p \rangle$$

$$\mathcal{M}_I \equiv \mathcal{M}(\psi \rightarrow \psi\phi_I) = y \int_{-\infty}^{\infty} dz \chi(p^z) \chi^*(q^z) \zeta_I^*(k^z)$$

Then the **averaged exchanged momentum**

$$\begin{aligned} \langle \Delta p \rangle &= \langle \Delta p_R \rangle + \langle \Delta p_L \rangle \\ &= \int d\mathbb{P}_{\psi \rightarrow \psi\phi_{\zeta_R}} (p^z - q^z - \tilde{k}^z) + \int d\mathbb{P}_{\psi \rightarrow \psi\phi_{\zeta_L}} (p^z - q^z + k^z) \\ \int d\mathbb{P}_{\psi \rightarrow \psi\phi_I} \Delta p_I^z &= \int_{k_{\min}^z}^{k_{\max}^z} \frac{dk_z}{2\pi} \frac{1}{2k_0} \int_0^{k_{\perp, \max}^2} \frac{dk_{\perp}^2}{4\pi} \cdot \frac{1}{2p^z} \left[ \frac{1}{2|q^z|} |\mathcal{M}_I|^2 \Delta p_I^z \right]_{q^z = \pm q_k^z} \end{aligned}$$



# Vector boson emission: Abelian Higgs model

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu H|^2 - V(\sqrt{2}|H|) + |D_\mu \psi|^2 - \frac{1}{2}m_\psi^2\psi^2 + \text{gauge fixing}, \quad D_\mu = \partial_\mu + igA_\mu$$

EOM: Unitary gauge  $\xi \rightarrow \infty$

$$\partial_z^2 v(z) = V'(v)$$

$$\square h = -V''(v)h$$

$$\partial_\nu F^{\mu\nu} = g^2 v^2(z) A^\mu \equiv m^2(z) A^\mu$$

(new!) Transversality condition  $\begin{cases} \partial_\mu(m^2(z)A^\mu) = 0 \\ 3 \text{ propagating dofs} \end{cases}$

