



University
of Glasgow

Flavour deconstruction: From the electroweak scale to the GUT scale

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Mario Fernández Navarro

Based on:

MFN, Stephen F. King, [[2305.07690](#)] hep-ph, JHEP 08 (2023) 020

MFN, Stephen F. King and Avelino Vicente, [[2311.05683](#)] hep-ph, JHEP 05 (2024) 130

MFN, Stephen F. King and Avelino Vicente, [[2404.12442](#)] hep-ph, to appear in JHEP

Tri-hypercharge: an example of flavour deconstruction

- **Flavour deconstruction:** SM is embedded in a gauge symmetry that contains a separate factor for each fermion family:

$$G_{\text{universal}} \times G_1 \times G_2 \times G_3$$

[[Salam 79'](#), [Rajpoot 81'](#), [Li and Ma 81'](#), [Georgi 82'](#) ...
[Craig et al 11'](#), [Bordone et al 17'](#), [Greljo et al 18'](#),
[Fuentes-Martín et al, 20'](#), [Davighi et al 23'](#) ...]

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A simple example:

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

$$\longrightarrow Y_{\text{SM}} \equiv Y = Y_1 + Y_2 + Y_3$$

Field	$U(1)_{Y_i}$
q_i	1/6
u_i^c	-2/3
d_i^c	1/3
l_i	-1/2
e_i^c	1

q_1	q_2	q_3
u_1^c	u_2^c	u_3^c
d_1^c	d_2^c	d_3^c
l_1	l_2	l_3
e_1^c	e_2^c	e_3^c
		H_3

► Light families are massless in first approximation:

$$\mathcal{L} = y_t q_3 H_3 u_3^c + y_b q_3 H_3 d_3^c + y_\tau l_3 H_3 e_3^c + \text{h.c.}$$

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d_1^c	d_2^c	d_3^c
l_1	l_2	l_3
e_1^c	e_2^c	e_3^c

$$H_3^{u,d}$$

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► Type II 2HDM can take care of $m_{b,\tau}/m_t$ hierarchies via

$$\tan \beta = v_u/v_d \approx \lambda^{-2} \simeq 20$$

Tri-hypercharge: EFT

$$\mathcal{L}_d = (q_1 \quad q_2 \quad q_3) \begin{pmatrix} \approx 0 & \approx 0 & \approx 0 \\ \approx 0 & \approx 0 & \approx 0 \\ \approx 0 & \approx 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

- E.g. $q_2 H_3^d d_2^c \sim (0, \frac{1}{2}, -\frac{1}{2})$ and $q_2 H_3^d d_3^c \sim (0, \frac{1}{6}, -\frac{1}{6})$ are forbidden by tri-hypercharge (**gauge**) symmetry

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$$\mathcal{L}_d = (q_1 \quad q_2 \quad q_3) \begin{pmatrix} \approx 0 & \approx 0 & \approx 0 \\ \approx 0 & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q 23}}{\Lambda_2} \\ \approx 0 & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q 23}}{\Lambda_2} \\ & & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

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- Add “23 hyperons” $\phi_{\ell 23} \sim (0, -\frac{1}{2}, \frac{1}{2})$ and $\phi_{q 23} \sim (0, -\frac{1}{6}, \frac{1}{6})$ $U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$

Tri-hypercharge: EFT

$$\mathcal{L}_d = (q_1 \quad q_2 \quad q_3) \begin{pmatrix} \frac{\phi_{\ell 12}}{\Lambda_1} \frac{\phi_{\ell 23}}{\Lambda_1} & \frac{\phi_{q 12}}{\Lambda_1} \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q 12}}{\Lambda_1} \frac{\phi_{q 23}}{\Lambda_2} \\ \frac{\phi_{\ell 12}}{\Lambda_1} \frac{\tilde{\phi}_{q 12}}{\Lambda_1} \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q 23}}{\Lambda_2} \\ \frac{\phi_{q 12}^2}{\Lambda_1^2} \frac{\phi_{q 23}^2}{\Lambda_2^2} & \frac{\phi_{\ell 23}}{\Lambda_2} \frac{\phi_{q 23}}{\Lambda_2} & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

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- Add “12 hyperons” $\phi_{\ell 12} \sim (-\frac{1}{2}, \frac{1}{2}, 0)$ and $\phi_{q 12} \sim (-\frac{1}{6}, \frac{1}{6}, 0)$

Tri-hypercharge: EFT

$$\mathcal{L}_d = (q_1 \quad q_2 \quad q_3) \begin{pmatrix} \frac{\phi_{\ell 12}}{\Lambda_1} \frac{\phi_{\ell 23}}{\Lambda_1} & \frac{\phi_{q 12}}{\Lambda_1} \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q 12}}{\Lambda_1} \frac{\phi_{q 23}}{\Lambda_2} \\ \frac{\phi_{\ell 12}}{\Lambda_1} \frac{\tilde{\phi}_{q 12}}{\Lambda_1} \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q 23}}{\Lambda_2} \\ \frac{\phi_{q 12}^2}{\Lambda_1^2} \frac{\phi_{q 23}^2}{\Lambda_2^2} & \frac{\phi_{\ell 23}}{\Lambda_2} \frac{\phi_{q 23}}{\Lambda_2} & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

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- When these four hyperons (scalars) get VEVs, **SM flavour structure is dynamically generated** (also in up and charged lepton sectors)

Neutrino masses and mixings also included!

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$$\mathcal{L}_d = (q_1 \quad q_2 \quad q_3) \begin{pmatrix} \frac{\phi_{\ell 12}}{\Lambda_1} & \frac{\phi_{\ell 23}}{\Lambda_1} & \frac{\phi_{q 12}}{\Lambda_1} & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q 12}}{\Lambda_1} & \frac{\phi_{q 23}}{\Lambda_2} \\ \frac{\phi_{\ell 12}}{\Lambda_1} & \frac{\tilde{\phi}_{q 12}}{\Lambda_1} & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q 23}}{\Lambda_2} & \\ \frac{\phi_{q 12}^2}{\Lambda_1^2} & \frac{\phi_{q 23}^2}{\Lambda_2^2} & \frac{\phi_{\ell 23}}{\Lambda_2} & \frac{\phi_{q 23}}{\Lambda_2} & 1 & \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

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- Add “12 hyperons” $\phi_{\ell 12} \sim (-\frac{1}{2}, \frac{1}{2}, 0)$ and $\phi_{q 12} \sim (-\frac{1}{6}, \frac{1}{6}, 0)$ Heavy messengers needed for Λ s!

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Neutrino masses and mixings also included!

Example model

$$Y_d = \left(\begin{array}{ccc} c_{11}^d \frac{\phi_{l12}}{M_{D_{13}}} \frac{\phi_{l23}}{M_{D_{12}}} & c_{12}^d \frac{\phi_{q12}}{M_{D_{13}}} \frac{\phi_{l23}}{M_{D_{23}}} & c_{13}^d \frac{\phi_{q12}}{M_{D_{13}}} \frac{\phi_{q23}}{M_{D_{23}}} \\ c_{21}^d \frac{\phi_{l12}}{M_{D_{12}}} \frac{\tilde{\phi}_{q12}}{M_{D_{13}}} \frac{\phi_{l23}}{M_{D_{23}}} & c_{22}^d \frac{\phi_{l23}}{M_{D_{23}}} & c_{23}^d \frac{\phi_{q23}}{M_{D_{23}}} \\ \approx 0 & \approx 0 & c_{33}^d \end{array} \right) \quad Y_u = Y_d(d \rightarrow u, D \rightarrow U) \quad Y_e = Y_d(d \rightarrow e, q \rightarrow \ell, D \rightarrow E)$$

Example model

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- Fit SM flavour structure via three naturally small parameters (includes also up-quarks and charged leptons):

$$\frac{m_2}{m_3} = \frac{\langle \phi_{23} \rangle}{M_{23}} \sim \lambda^3,$$

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$$\frac{m_2}{m_3} = \frac{\langle \phi_{23} \rangle}{M_{23}} \sim \lambda^3, \quad \sin \theta_c = \frac{V_{ub}}{V_{cb}} = \frac{\langle \phi_{12} \rangle}{M_{12,13}} \sim \lambda$$

Example model

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$$\frac{m_1}{m_3} = \frac{\langle \phi_{12} \rangle}{M_{12,13}} \frac{\langle \phi_{23} \rangle}{M_{12,13}} \sim \lambda^5 \Rightarrow \frac{\langle \phi_{23} \rangle}{M_{12,13}} \sim \lambda^4$$

Example model

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- Fixed relation between VEVs is predicted:

$$\boxed{\frac{\langle \phi_{23} \rangle}{\langle \phi_{12} \rangle} \sim \lambda^3 \approx 0.01} \longrightarrow \text{Highly non-generic}$$

Example model

$$Y_u = Y_d(d \rightarrow u, D \rightarrow U) \quad Y_e = Y_d(d \rightarrow e, q \rightarrow \ell, D \rightarrow E)$$

$$Y_d = \begin{pmatrix} c_{11}^d \frac{\phi_{\ell 12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{12}}} & c_{12}^d \frac{\phi_{q 12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{13}^d \frac{\phi_{q 12}}{M_{D_{13}}} \frac{\phi_{q 23}}{M_{D_{23}}} \\ c_{21}^d \frac{\phi_{\ell 12}}{M_{D_{13}}} \frac{\tilde{\phi}_{q 12}}{M_{D_{13}}} \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{22}^d \frac{\phi_{\ell 23}}{M_{D_{23}}} & c_{23}^d \frac{\phi_{q 23}}{M_{D_{23}}} \\ \approx 0 & \approx 0 & c_{33}^d \end{pmatrix}$$

Spectrum up to $\mathcal{O}(1)$ variations

E

$$\mathcal{O}(10^4 \text{ TeV}) \quad M_{U_{12,13}, D_{12,13}, E_{12,13}}$$

$$\mathcal{O}(1000 \text{ TeV}) \quad v_{12} \sim \langle \phi_{q 12} \rangle, \langle \phi_{\ell 12} \rangle, M_{U_{23}, D_{23}, E_{23}}$$

$$\mathcal{O}(10 \text{ TeV}) \quad v_{23} \sim \langle \phi_{q 23} \rangle, \langle \phi_{\ell 23} \rangle$$

$$174 \text{ GeV} \quad v_{\text{SM}} \sim \langle H_3^{u,d} \rangle$$

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$



$$SU(3)_c \times SU(2)_L \times U(1)_{Y_{12} \equiv Y_1 + Y_2} \times U(1)_{Y_3}$$



$$SU(3)_c \times SU(2)_L \times U(1)_{Y \equiv Y_1 + Y_2 + Y_3}$$



► Fit SM flavour structure via three naturally small parameters (includes also up-quarks and charged leptons):

$$\frac{m_2}{m_3} = \frac{\langle \phi_{23} \rangle}{M_{23}} \sim \lambda^3, \quad \sin \theta_c = \frac{V_{ub}}{V_{cb}} = \frac{\langle \phi_{12} \rangle}{M_{12,13}} \sim \lambda$$

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► Fixed relation between VEVs is predicted:

$$\frac{\langle \phi_{23} \rangle}{\langle \phi_{12} \rangle} \sim \lambda^3 \approx 0.01$$

Highly non-generic


Correlations and lots of pheno!

Take home messages

✓ A simple option for flavour deconstruction is the tri-hypercharge group:

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

- Translates SM flavour structure into **three simple and correlated NP scales** that carry **meaningful pheno.** The lowest scale may be close to TeV.
- Flavour deconstruction may arise from **gauge unified frameworks!**
- See all this and more in my poster!
- Also [CERN BSM forum](#) on 1st August



**Flavour deconstruction:
from the electroweak scale to the GUT scale**

Mario Fernández Navarro[†] †Mario.FernandezNavarro@glasgow.ac.uk

MFN and Steve King: [2305.07690] [JHEP.08.(2023).020]; MFN, Steve King and Avelino Vicente:[2311.05683] [JHEP.05.(2024).130], [2404.12442]

Tri-hypercharge: a simple example of flavour deconstruction

Field	$U(1)_{Y_i}$	q_1	q_2	q_3
q_i	1/6	u_1^c	u_2^c	u_3^c
u_i^c	-2/3	d_1^c	d_2^c	d_3^c
d_i^c	1/3	ℓ_1	ℓ_2	ℓ_3
ℓ_i	-1/2	e_1^c	e_2^c	e_3^c
e_i^c	1	$H_3^{u,d}$		

Spectrum up to $\mathcal{O}(1)$ variations in complete model with vector-like fermions

Light families are massless in first approximation:
 $\mathcal{L} = y_1 q_3 H_3^u u_3^c + y_2 q_3 H_3^d d_3^c + y_3 \ell_3 H_3^d e_3^c + \text{h.c.}$

Four "hyperon" scalars dynamically generate flavour structure after getting VEVs:
 $\phi_{r23} \sim \left(0, -\frac{1}{2}, \frac{1}{2}\right)$ $\phi_{q23} \sim \left(0, -\frac{1}{6}, \frac{1}{6}\right)$
 $\phi_{r12} \sim \left(-\frac{1}{2}, \frac{1}{2}, 0\right)$ $\phi_{q12} \sim \left(-\frac{1}{6}, \frac{1}{6}, 0\right)$

E.g. in the down sector:

$$\mathcal{L}_d = (q_1 \ q_2 \ q_3) \begin{pmatrix} \phi_{12} & \phi_{23} & \phi_{12} & \phi_{23} \\ M_{13} & M_{12} & M_{13} & M_{23} \\ M_{12} & M_{13} & M_{23} & M_{23} \\ \approx 0 & \approx 0 & \approx 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

Heavy cut-offs M_{23} and $M_{12,13}$ provided by vector-like fermions or heavy scalars in the UV.

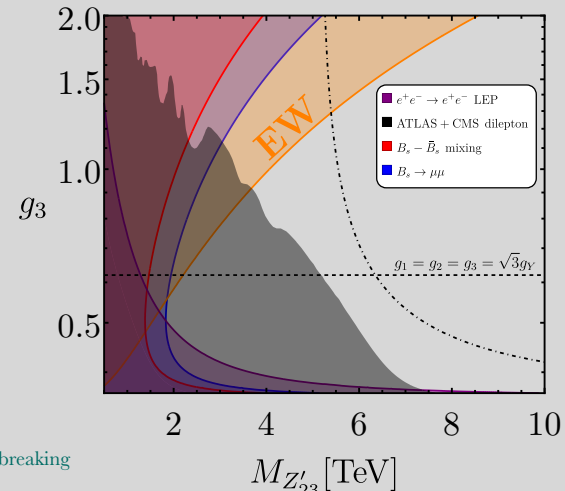
Explains SM flavour structure via three naturally small parameters:
 $\frac{m_2}{m_3} = \frac{\langle \phi_{23} \rangle}{M_{23}} \sim \lambda^3$, $\sin \theta_c = \frac{V_{ub}}{V_{cb}} = \frac{\langle \phi_{12} \rangle}{M_{12,13}} \sim \lambda$
 $\frac{m_1}{m_3} = \frac{\langle \phi_{12} \rangle}{M_{12,13}} \frac{\langle \phi_{23} \rangle}{M_{12,13}} \sim \lambda^5 \Rightarrow \frac{\langle \phi_{23} \rangle}{M_{12,13}} \sim \lambda^4$

$\frac{\langle \phi_{23} \rangle}{\langle \phi_{12} \rangle} \sim \lambda^3 \approx 0.01$

Two consecutive steps of symmetry breaking correlated to the flavour hierarchies

• Z_{23} tested by dilepton tails at the LHC and electroweak precision observables, few TeV is allowed.

• Z_{12} tested via FCNCs sensitive to the UV completion - typically leading bounds beyond 100 TeV from $\bar{K}-K$ and $D-D$ mixing.



Grand unified origin of flavour deconstruction: tri-unification

$SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times \mathbb{Z}_3$ → Cyclic symmetry \mathbb{Z}_3 ensures single gauge coupling at the GUT scale, and imposes the requirement of having the same matter under each $SU(5)$

For tri-hypercharge, need of deconstructing $SU(3)_c$ and $SU(2)_L$, as well for embedding in $SU(5)^3$. This suggests a possible SM³ intermediate scale, i.e. two options:

$SU(5)^3$

$\xrightarrow{225} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_1 \times U(1)_2 \times U(1)_3$

$\xrightarrow{512} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2} \times U(1)_3$

$\xrightarrow{225} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2} \times U(1)_3$

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$SU(5)^3$, SM₁ × SM₂ × SM₃

$\xrightarrow{512} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_1 \times U(1)_2 \times U(1)_3$

$\xrightarrow{225} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2} \times U(1)_3$

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Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	5	1	1
F_2	1	5	1
F_3	1	1	5
T_1	10	1	1
T_2	1	10	1
T_3	1	1	10
Ω_{12}	24	24	1
Ω_{13}	24	1	24
Ω_{23}	1	24	24
H_1	5	1	1
H_2	1	5	1
H_3	1	1	5




Take home messages

✓ A simple option for flavour deconstruction is the tri-hypercharge group:

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

- Translates SM flavour structure into **three simple and correlated NP scales** that carry **meaningful pheno.** The lowest scale may be close to TeV.
- Flavour deconstruction may arise from **gauge unified frameworks!**
- See all this and more in my poster!
- Also CERN BSM forum on 1st August

Thank you!

University of Glasgow

Flavour deconstruction: from the electroweak scale to the GUT scale

Mario Fernández Navarro[†] Mario.FernandezNavarro@glasgow.ac.uk

MFN and Steve King: [2305.07690] [JHEP.08.(2023).020]; MFN, Steve King and Avelino Vicente:[2311.05683] [JHEP.05.(2024).130], [2404.12442]

Tri-hypercharge: a simple example of flavour deconstruction

Spectrum up to $\mathcal{O}(1)$ variations in complete model with vector-like fermions

Field	$U(1)_{Y_1}$	q_1	q_2	q_3
q_i	1/6	u_1^c	u_2^c	u_3^c
u_i^c	-2/3	d_1^c	d_2^c	d_3^c
d_i^c	1/3	ℓ_1	ℓ_2	ℓ_3
ℓ_i	-1/2	e_1^c	e_2^c	e_3^c
e_i^c	1	$H_3^{u,d}$		

Light families are massless in first approximation:
 $\mathcal{L} = y_1 q_3 H_3^u u_3^c + y_2 q_3 H_3^d d_3^c + y_3 \ell_3 H_3^d e_3^c + \text{h.c.}$

Four "hyperon" scalars dynamically generate flavour structure after getting VEVs:
 $\phi_{r23} \sim \left(0, -\frac{1}{2}, \frac{1}{2}\right)$ $\phi_{q23} \sim \left(0, -\frac{1}{6}, \frac{1}{6}\right)$
 $\phi_{r12} \sim \left(-\frac{1}{2}, \frac{1}{2}, 0\right)$ $\phi_{q12} \sim \left(-\frac{1}{6}, \frac{1}{6}, 0\right)$

E.g. in the down sector:

$$\mathcal{L}_d = (q_1 \ q_2 \ q_3) \begin{pmatrix} \phi_{12} & \phi_{23} & \phi_{12} & \phi_{23} & \phi_{12} & \phi_{23} \\ M_{13} & M_{12} & M_{13} & M_{23} & M_{13} & M_{23} \\ M_{12} & M_{13} & M_{23} & M_{23} & M_{23} & M_{23} \\ \approx 0 & \approx 0 & \approx 0 & 1 & & \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_3^d$$

Heavy cut-offs M_{23} and $M_{12,13}$ provided by vector-like fermions or heavy scalars in the UV.

Explains SM flavour structure via three naturally small parameters:
 $\frac{m_2}{m_3} = \frac{\langle \phi_{23} \rangle}{M_{23}} \sim \lambda^3$, $\sin \theta_c = \frac{V_{ub}}{V_{cb}} = \frac{\langle \phi_{12} \rangle}{M_{12,13}} \sim \lambda$
 $\frac{m_1}{m_3} = \frac{\langle \phi_{12} \rangle}{M_{12,13}} \frac{\langle \phi_{23} \rangle}{M_{12,13}} \sim \lambda^5 \Rightarrow \frac{\langle \phi_{23} \rangle}{M_{12,13}} \sim \lambda^4$

$\frac{\langle \phi_{23} \rangle}{\langle \phi_{12} \rangle} \sim \lambda^3 \approx 0.01$

Two consecutive steps of symmetry breaking correlated to the flavour hierarchies

Translates SM flavour structure into three physical scales of new physics that carry meaningful phenomenology:

- Z_{23} tested by dilepton tails at the LHC and electroweak precision observables, few TeV is allowed.
- Z_{12} tested via FCNCs sensitive to the UV completion - typically leading bounds beyond 100 TeV from $\bar{K}-K$ and $D-D$ mixing.

Grand unified origin of flavour deconstruction: tri-unification

$SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times \mathbb{Z}_3$ → Cyclic symmetry \mathbb{Z}_3 ensures single gauge coupling at the GUT scale, and imposes the requirement of having the same matter under each $SU(5)$

For tri-hypercharge, need of deconstructing $SU(3)_c$ and $SU(2)_L$, as well for embedding in $SU(5)^3$. This suggests a possible SM³ intermediate scale, i.e. two options:

$SU(5)^3$

- $\xrightarrow{SM^2} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_1 \times U(1)_2 \times U(1)_3$
- $\xrightarrow{v_{12}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2} \times U(1)_3$
- $\xrightarrow{v_{23}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2+3}$

$SU(5)^3$, SM₁ × SM₂ × SM₃

- $\xrightarrow{v_{SM^2}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_1 \times U(1)_2 \times U(1)_3$
- $\xrightarrow{v_{23}} SU(3)_{1+2+3} \times SU(2)_{1+2+3} \times U(1)_{1+2} \times U(1)_3$
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Field	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$
F_1	5	1	1
F_2	1	5	1
F_3	1	1	5
T_1	10	1	1
T_2	1	10	1
T_3	1	1	10
Ω_{12}	24	24	1
Ω_{13}	24	1	24
Ω_{23}	1	24	24
H_1	5	1	1
H_2	1	5	1
H_3	1	1	5

+ tri-hypercharge content at low energies
 $F_i \rightarrow d_i^c \oplus \ell_i$ $T_i \rightarrow q_i \oplus u_i^c \oplus e_i^c$

Low v_{SM} → Proton decay!
 Unification is possible! (SUSY not needed)

Backup: Phenomenology

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

$$\xrightarrow{v_{12}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3} + Z'_{12}$$

$$\xrightarrow{v_{23}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3} + Z'_{23} + Z'_{12}$$

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$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

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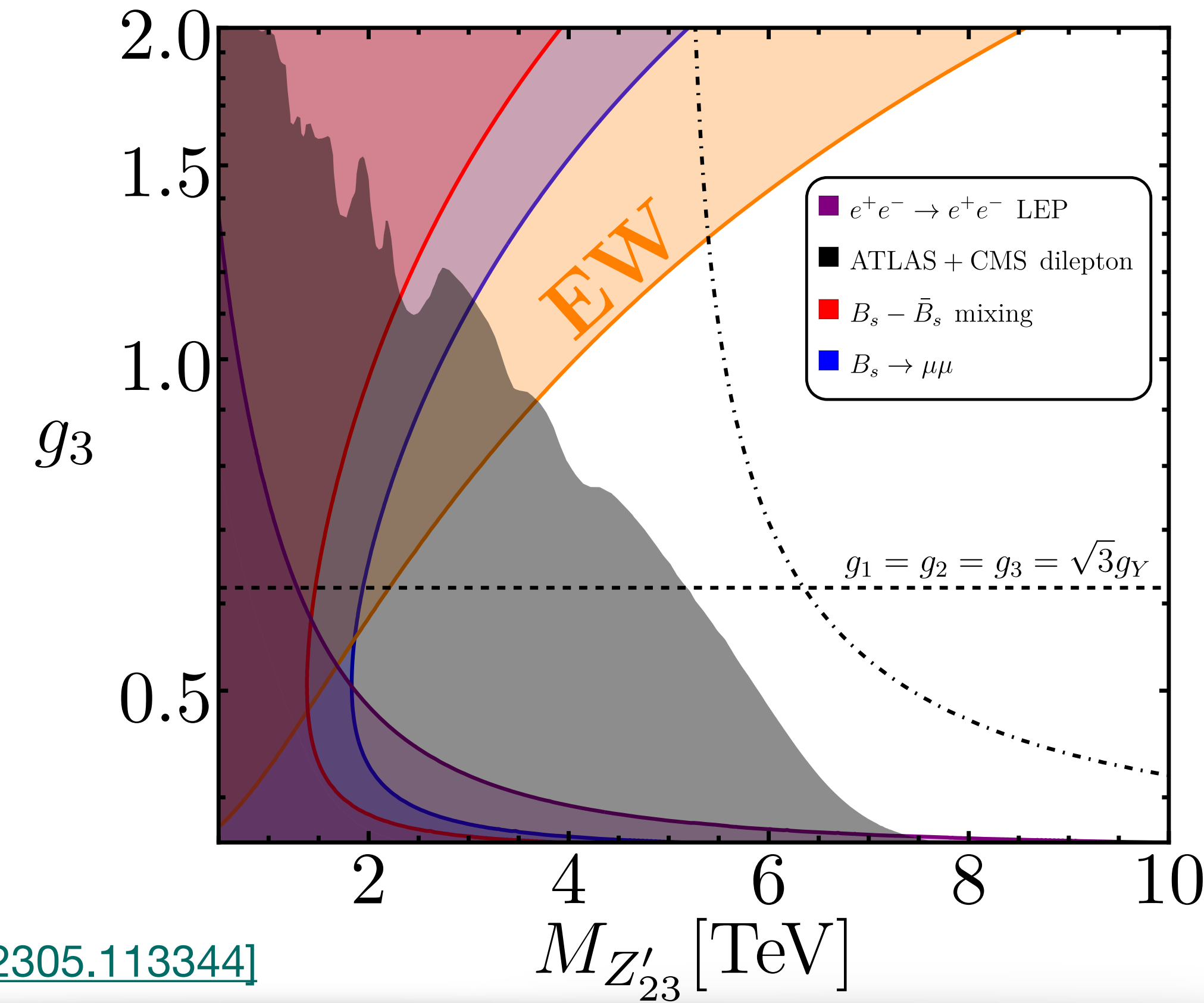
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- Tested by dilepton tails at hadron colliders or EWPOs (independent of UV-completion) - bounds of order TeV

$$g_{12} = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}$$

$$g_Y = \frac{g_{12} g_3}{\sqrt{g_{12}^2 + g_3^2}} \simeq 0.36 (M_Z)$$



[see EW global fit and FCC-ee projections in Davighi and Stefanek, [2305.113344](https://arxiv.org/abs/2305.11334)]

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- ▶ Z'_{12} is heavier and tested via FCNCs sensitive to the different complete models - bounds typically beyond 100 TeV

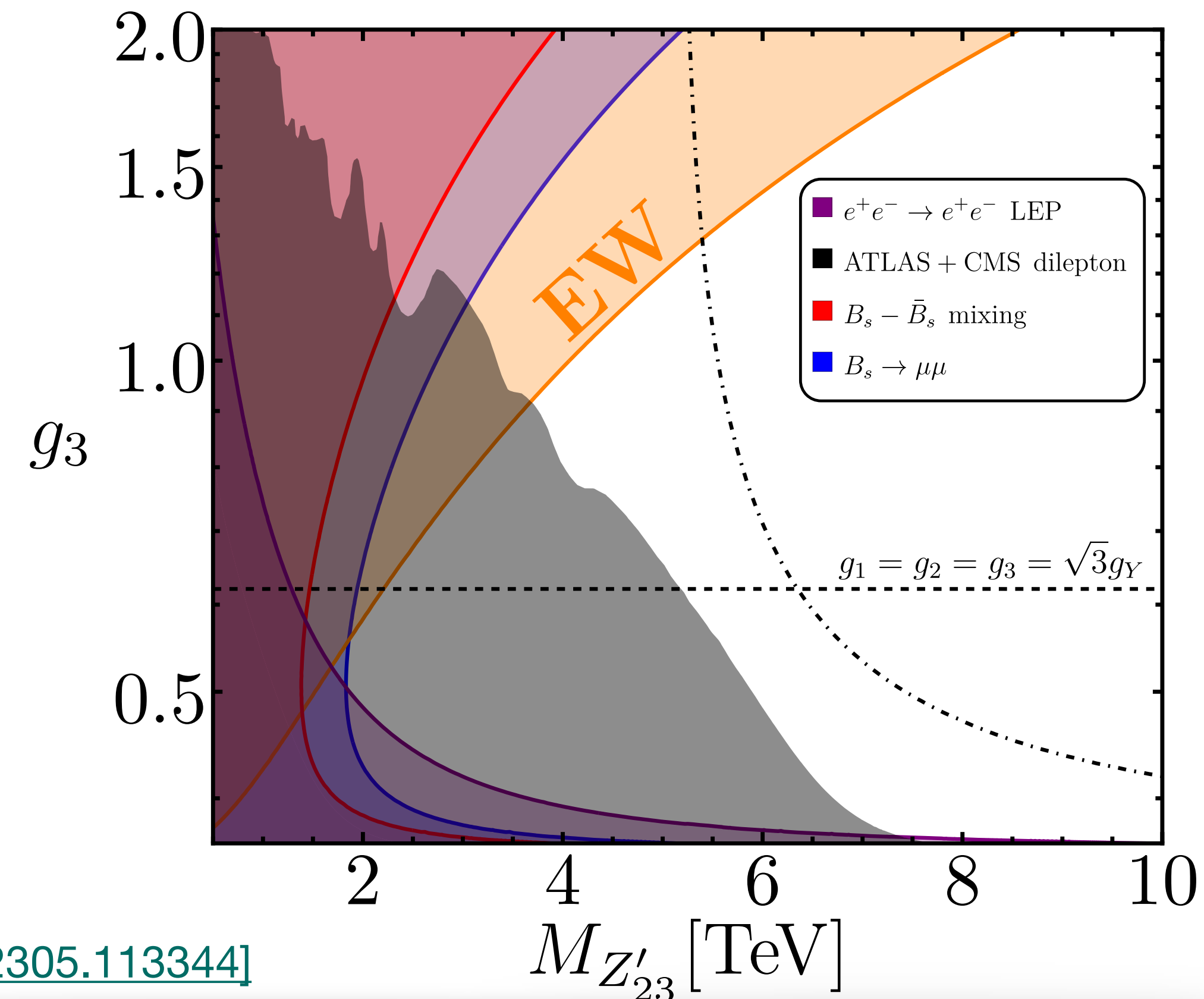
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Model	Observable	Mediator	Bound (TeV)
1	$K - \bar{K}$ (Re)	Z'_{12}	$M_{Z'_{12}}/g_1 > 340 \times \left \text{Re} \left[\frac{c_{12}^d}{c_{22}^d} \frac{c_{21}^d}{c_{22}^d} \right] \right $
	$K - \bar{K}$ (Im)	Z'_{12}	$M_{Z'_{12}}/g_1 > 3 \cdot 10^3 \times \left \text{Im} \left[\frac{c_{12}^d}{c_{22}^d} \frac{c_{21}^d}{c_{22}^d} \right] \right $
	$\mu \rightarrow e\gamma$	Z'_{12}	$M_{Z'_{12}}/g_1 > 30 \times c_{12}^e/c_{22}^e $
		Z'_{23}	$M_{Z'_{23}}/g_3 > 8 \times y_{62}^e (y_{65}^e y_{15}^e)^* $
	$\mu \rightarrow 3e$	Z'_{12}	$M_{Z'_{12}}/g_1 > 30 \times c_{12}^e/c_{22}^e $
2	$D - \bar{D}$ (Re)	Z'_{12}	$M_{Z'_{12}}/g_1 > 150 \times \left \text{Re} \left[\frac{c_{12}^u}{c_{22}^u} \frac{c_{21}^u}{c_{22}^u} \right] \right $
	$D - \bar{D}$ (Im)	Z'_{12}	$M_{Z'_{12}}/g_1 > 500 \times \left \text{Im} \left[\frac{c_{12}^u}{c_{22}^u} \frac{c_{21}^u}{c_{22}^u} \right] \right $



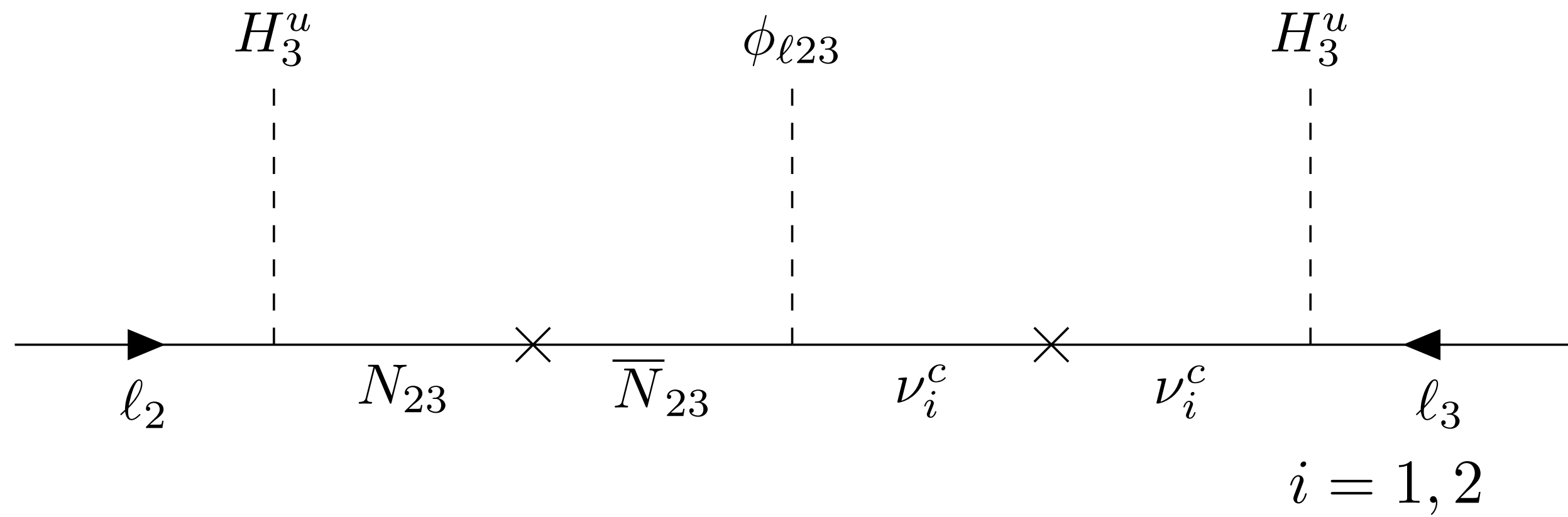
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Backup: Neutrino sector

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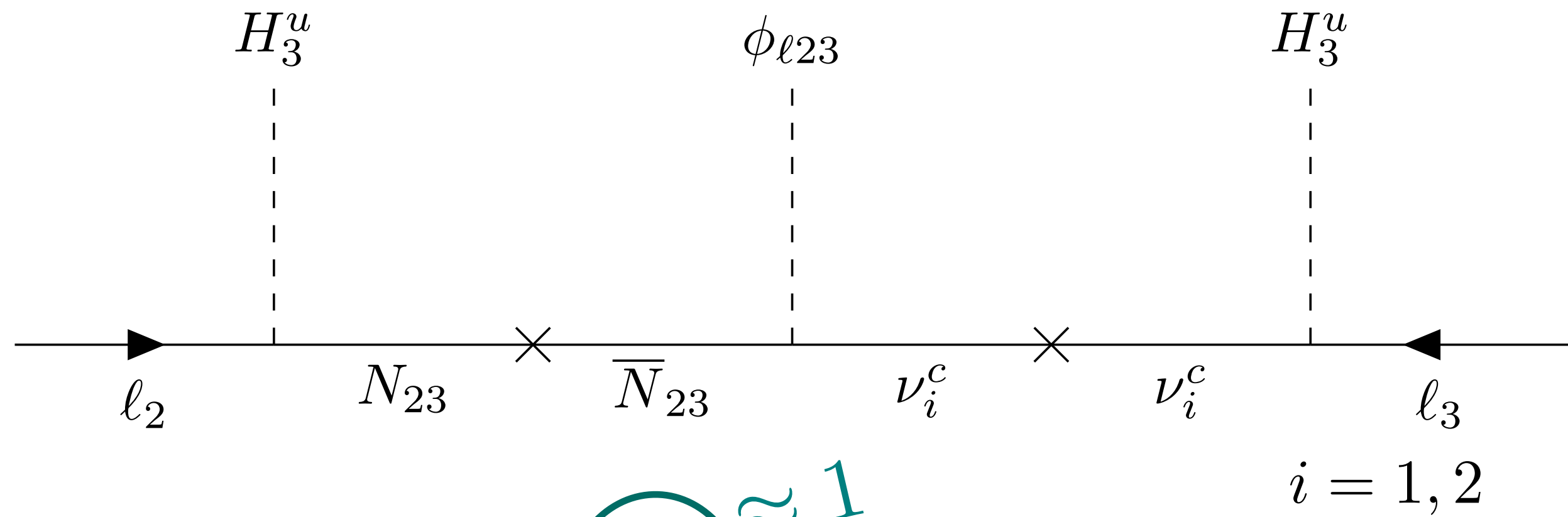


$$\mathcal{L} = c_{3i} l_3 H_3^u \nu_i^c + c_{2i} \frac{\phi_{l23}}{M_{N_{23}}} l_2 H_3^u \nu_i^c + \dots + M_{ij} \nu_i^c \nu_j^c + \text{h.c.}$$

	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$	$SU(3)_c \times SU(2)_L$
ν_1^c	0	0	0	(1, 1)
ν_2^c	0	0	0	(1, 1)
N_{12}	1/2	-1/2	0	(1, 1)
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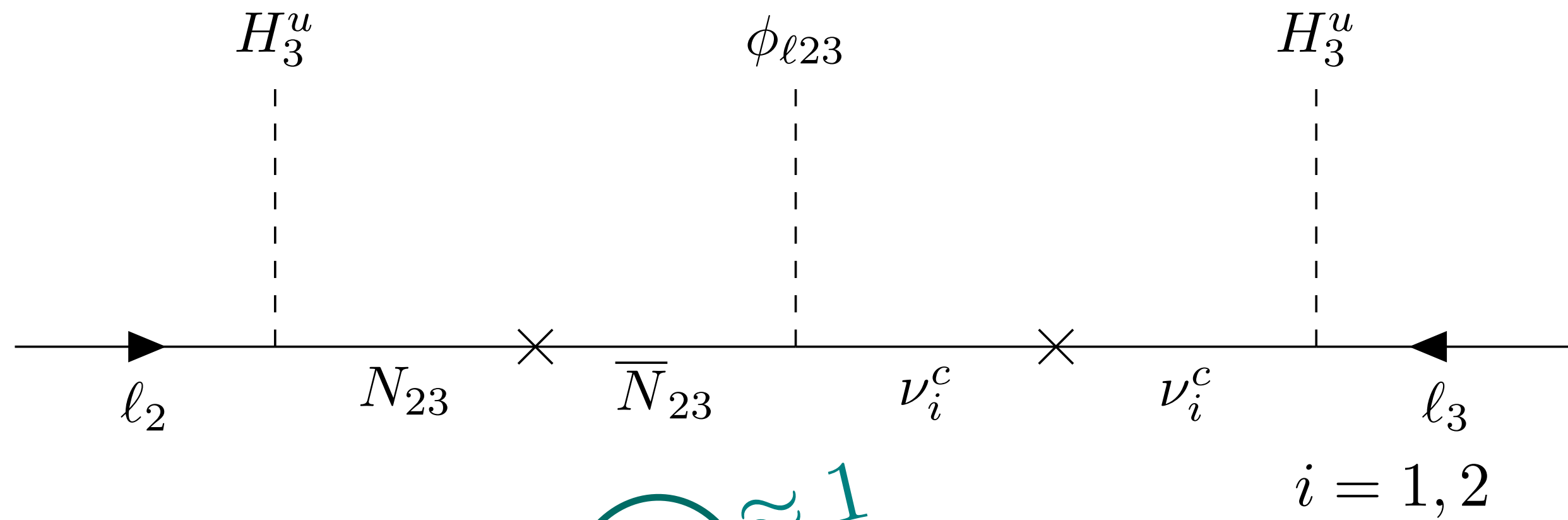


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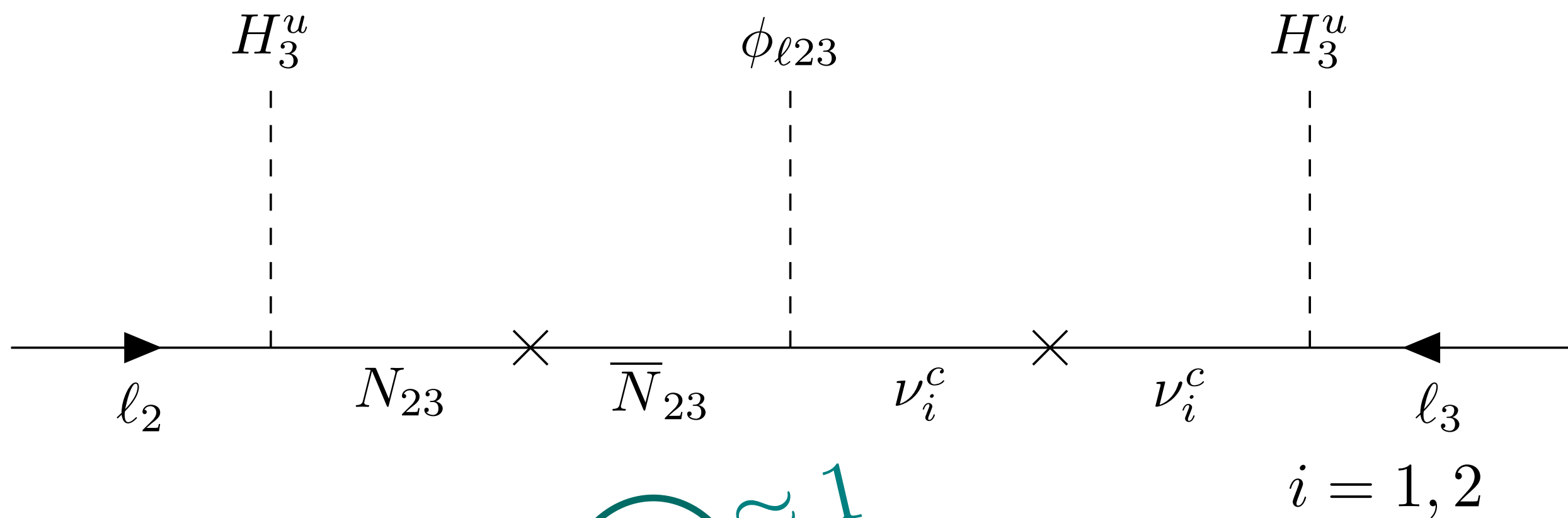
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$$m_D = \left(\begin{array}{c|cc} & \nu_1^c & \nu_2^c \\ \hline l_1 & c_{11} & c_{12} \\ l_2 & c_{21} & c_{22} \\ l_3 & c_{31} & c_{32} \end{array} \right) H_3^u, \quad M_R = \left(\begin{array}{c|cc} & \nu_1^c & \nu_2^c \\ \hline \nu_1^c & M_{11} & M_{12} \\ \nu_2^c & M_{21} & M_{22} \end{array} \right)$$

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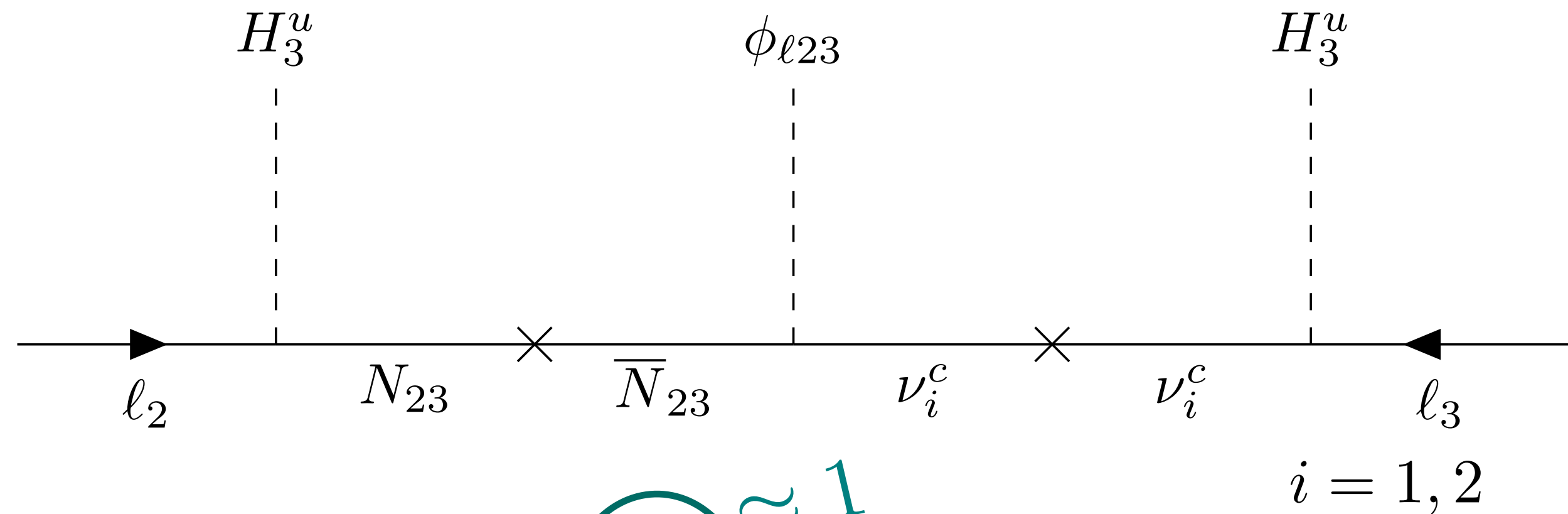
$$m_D = \begin{pmatrix} & \nu_1^c & \nu_2^c \\ l_1 & c_{11} & c_{12} \\ l_2 & c_{21} & c_{22} \\ l_3 & c_{31} & c_{32} \end{pmatrix} H_3^u, \quad M_R = \begin{pmatrix} & \nu_1^c & \nu_2^c \\ \nu_1^c & M_{11} & M_{12} \\ \nu_2^c & M_{21} & M_{22} \end{pmatrix}$$

$$m_\nu \simeq m_D M_R^{-1} m_D^T$$

Seesaw mechanism!

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✓ $M \approx 10^{15}$ GeV

✓ No need of small couplings nor v_{12}, v_{23} being very heavy

✓ No need of adding extra scalars

✓ $M_{N_{23}} \approx v_{23} \gtrsim \mathcal{O}(10 \text{ TeV})$

Backup: GUT

- Gauge sector of flavour deconstructed models may contain up to 9 gauge couplings:

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}$$

[This talk]

$$SU(3)_c \times SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \times U(1)_Y$$

[Li and Ma, [PRL 81](#)'; Muller and Nandi, [hep-ph/9602390](#) ...
Chiang *et al*, [0911.1480](#); Allwicher *et al*, [2011.01946](#);
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- “Deconstructed” theories seem to preserve an approximate \mathbb{Z}_3 (cyclic permutation symmetry) relating the three sites (i.e. approx. same matter content under the three sites):

► E.g. $\{\phi_{\ell 12}^{(\frac{1}{2}, -\frac{1}{2}, 0)}, \phi_{\ell 13}^{(\frac{1}{2}, 0, -\frac{1}{2})}, \phi_{\ell 23}^{(0, \frac{1}{2}, -\frac{1}{2})}\}$, $\{H_1^{(\frac{1}{2}, 0, 0)}, H_2^{(0, \frac{1}{2}, 0)}, H_3^{(0, 0, \frac{1}{2})}\}$, $\{D_{12}^{(-\frac{1}{6}, \frac{1}{2}, 0)}, D_{13}^{(-\frac{1}{6}, 0, \frac{1}{2})}, D_{23}^{(0, -\frac{1}{6}, \frac{1}{2})}\}$

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- ▶ If \mathbb{Z}_3 is exact at very high energies, then:

$$SU(5)_1 \times SU(5)_2 \times SU(5)_3 \times \mathbb{Z}_3$$

[Salam 79', Rajpoot 81', Georgi 82',

de Rújula, Georgi, Glashow 84', $SU(3)_c \times SU(3)_L \times SU(3)_R \times \mathbb{Z}_3 \dots$]

with \mathbb{Z}_3 permuting the three $SU(5)$, contains a single gauge coupling in the UV.

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✓ Deconstructed GUTs may be the origin of low energy flavour deconstructed models.

Backup: Gauge coupling unification

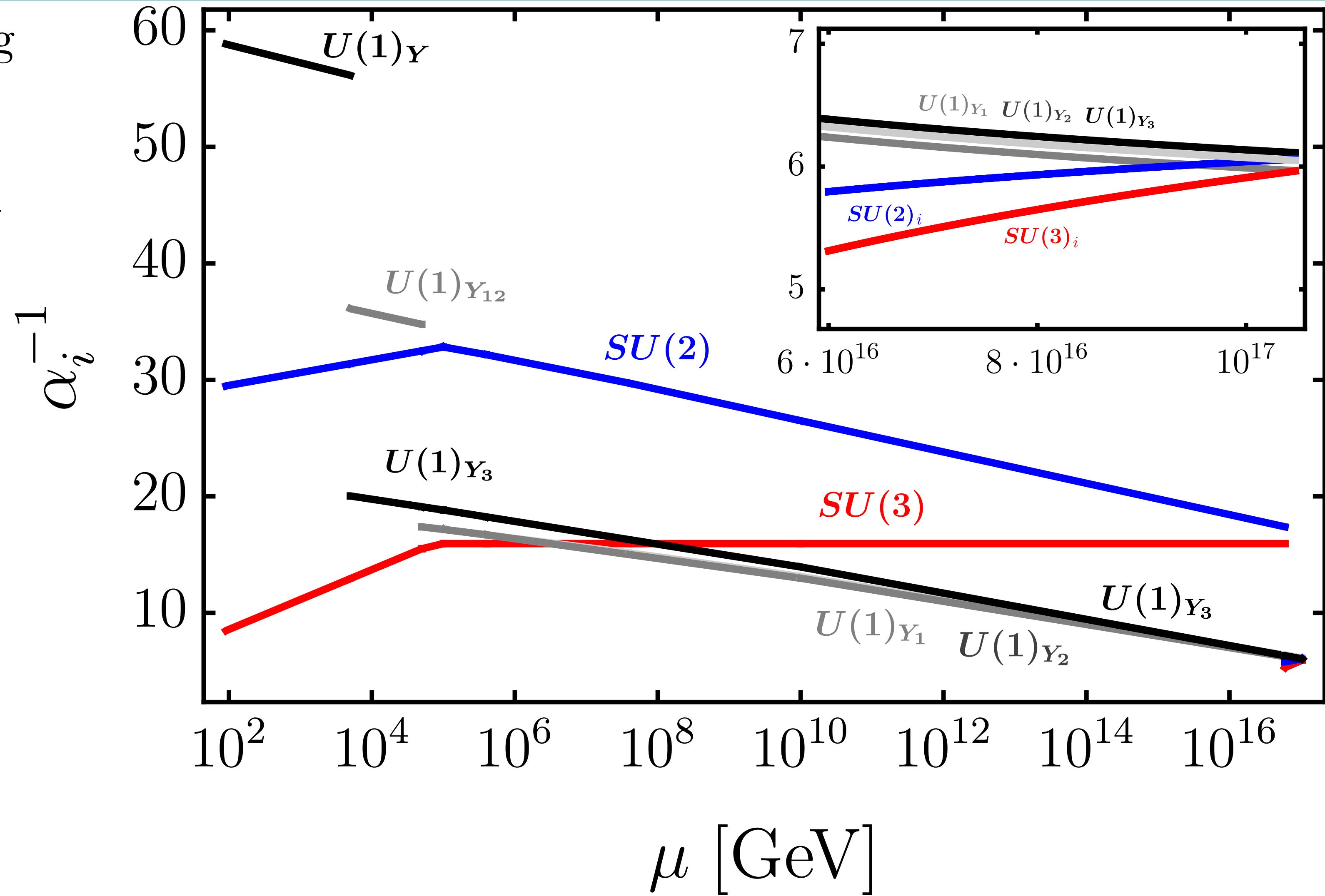
- Discontinuities due to gauge coupling matching conditions:

$$\alpha_{Y_{12}}^{-1} + \alpha_{Y_3}^{-1} = \alpha_Y^{-1}(v_{23})$$

$$\alpha_{Y_1}^{-1} + \alpha_{Y_2}^{-1} = \alpha_{Y_{12}}^{-1}(v_{12})$$

$$\alpha_{s,L,1}^{-1} + \alpha_{s,L,2}^{-1} + \alpha_{s,L,3}^{-1} = \alpha_{s,L}^{-1}(v_{SM^3})$$

$$\alpha_i = \frac{g_i^2}{4\pi}$$



Backup: Gauge coupling unification

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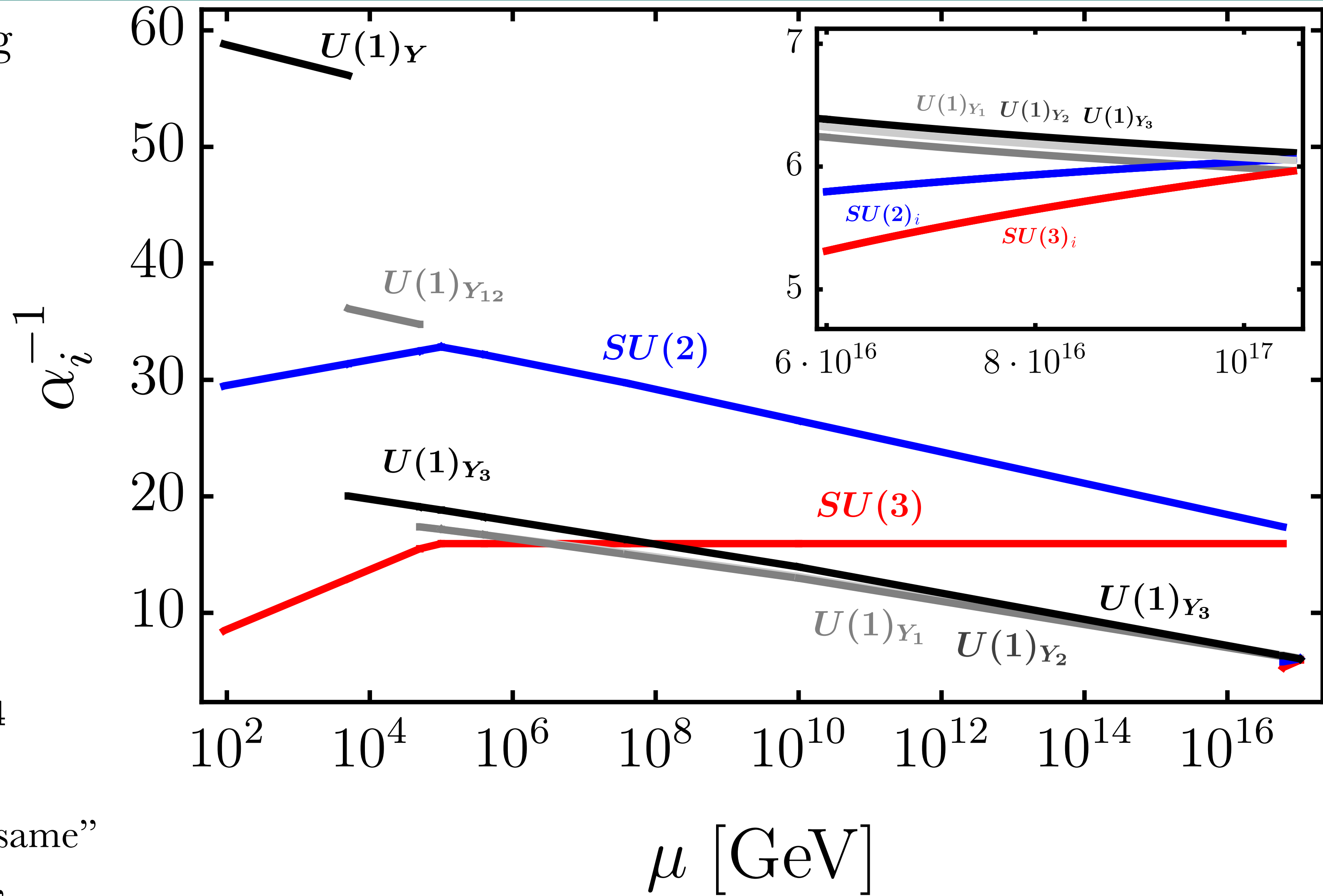
$$\alpha_{Y_{12}}^{-1} + \alpha_{Y_3}^{-1} = \alpha_Y^{-1}(v_{23})$$

$$\alpha_{Y_1}^{-1} + \alpha_{Y_2}^{-1} = \alpha_{Y_{12}}^{-1}(v_{12})$$

$$\alpha_i = \frac{g_i^2}{4\pi}$$

$$\alpha_{s,L,1}^{-1} + \alpha_{s,L,2}^{-1} + \alpha_{s,L,3}^{-1} = \alpha_{s,L}^{-1}(v_{SM^3})$$

- VL quarks Q_i help bend SU(2).
- Colour octet $\Theta_i \sim (\mathbf{8}, \mathbf{1}, 0)_i$ from cyclic $\mathbf{24}$ at v_{12} scale to bend SU(3) (non-SUSY).
- Gauge couplings approximately “run the same” thanks to approximate \mathbb{Z}_3 at low energies, which becomes **exact at high energies**.



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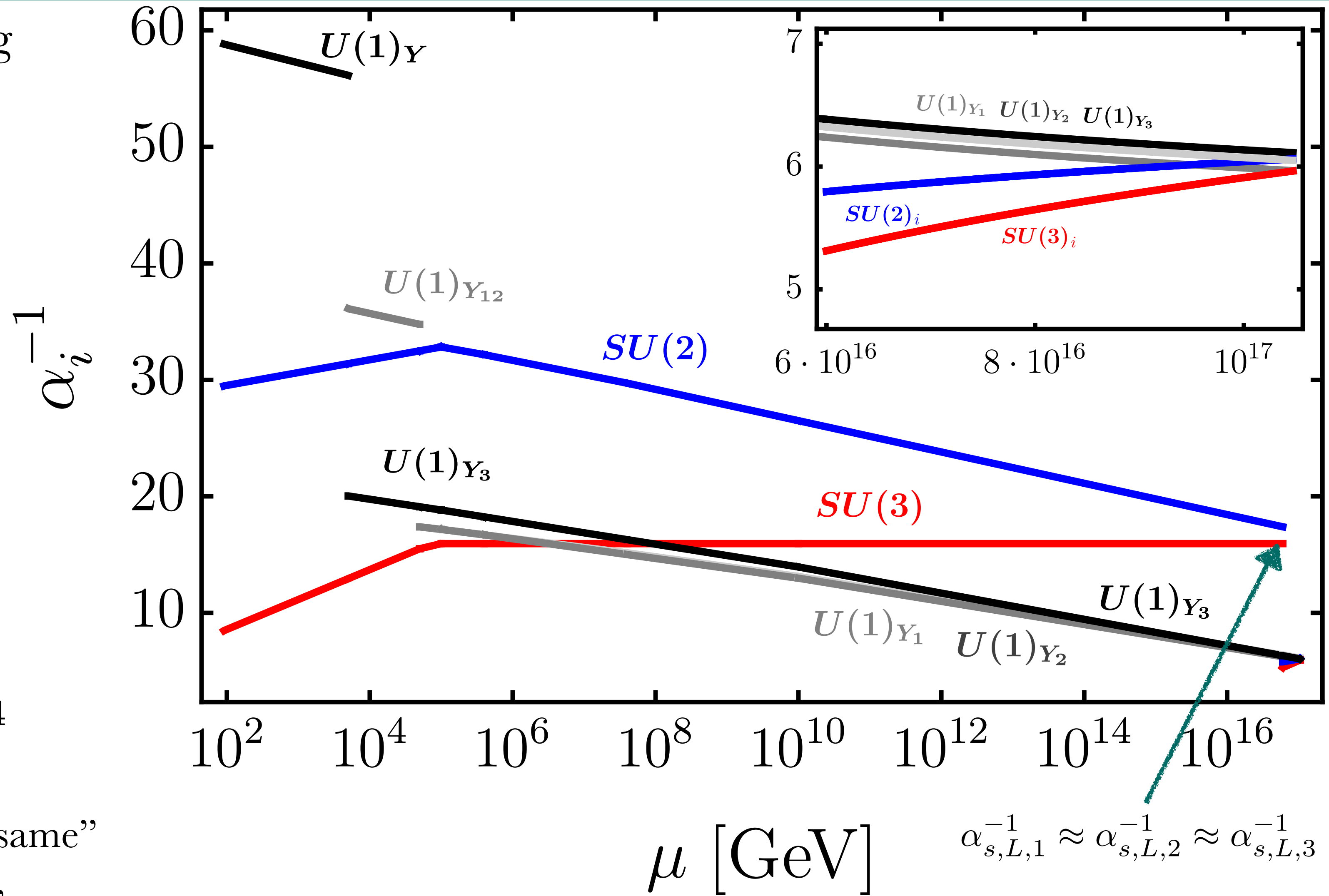
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$$\mu \text{ [GeV]} \quad \alpha_{s,L,1}^{-1} \approx \alpha_{s,L,2}^{-1} \approx \alpha_{s,L,3}^{-1}$$

$$v_{SM^3} = 6 \times 10^{16} \text{ GeV} \longrightarrow M_{GUT} = 10^{17} \text{ GeV}$$