QC for gravitational waves

Cristiano Palomba – INFN Roma

- Gravitational waves in a nutshell
- Data analysis issues
- Why Quantum Computing
- Promising quantum algorithms

Where we are: studying the basics of quantum computing

- GWs are perturbative solutions of the equations of General Relativity:
  Qualitatively: ripples of the space-time fabric, travelling at c

$$h_{ij}(t,\vec{x}) = \frac{2G}{rc^4}\ddot{Q}_{ij}\,(t-\frac{r}{c})$$

GWs in the quadrupole approximation

$$Q_{ij} = \int \rho(t,\vec{x})\left(x_i x_j - \frac{1}{3}r^2\delta_{ij}\right)d^3x$$

Source mass quadrupole moment

- GW signals are – typically - deeply embedded in detector noise

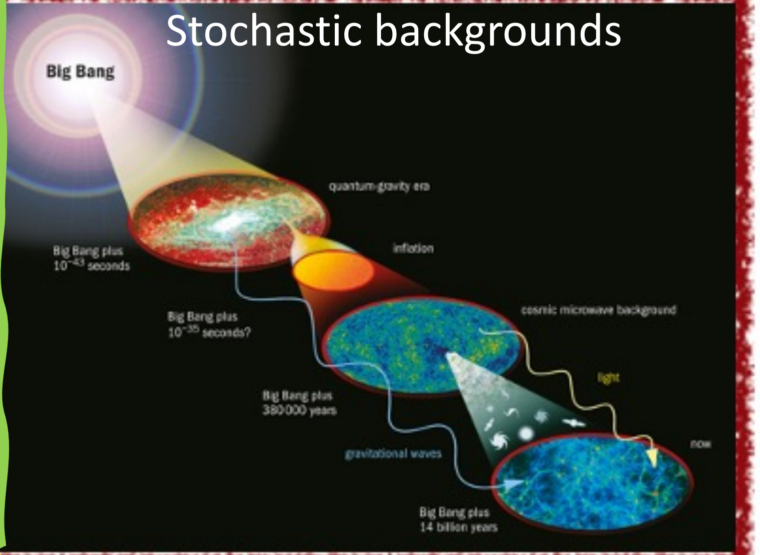
- Some kinds of signals can be modeled in a robust way, others not

Principali classi di sorgenti per gli interferometri terrestri

Modeled | **Unmodeled**

Long-lived

T ~ months or years



Transients

T ~ up to minutes

Credit: Ornella J Piccinni

Stelle di neutroni e buchi neri sono tra i target principali della ricerca delle OG

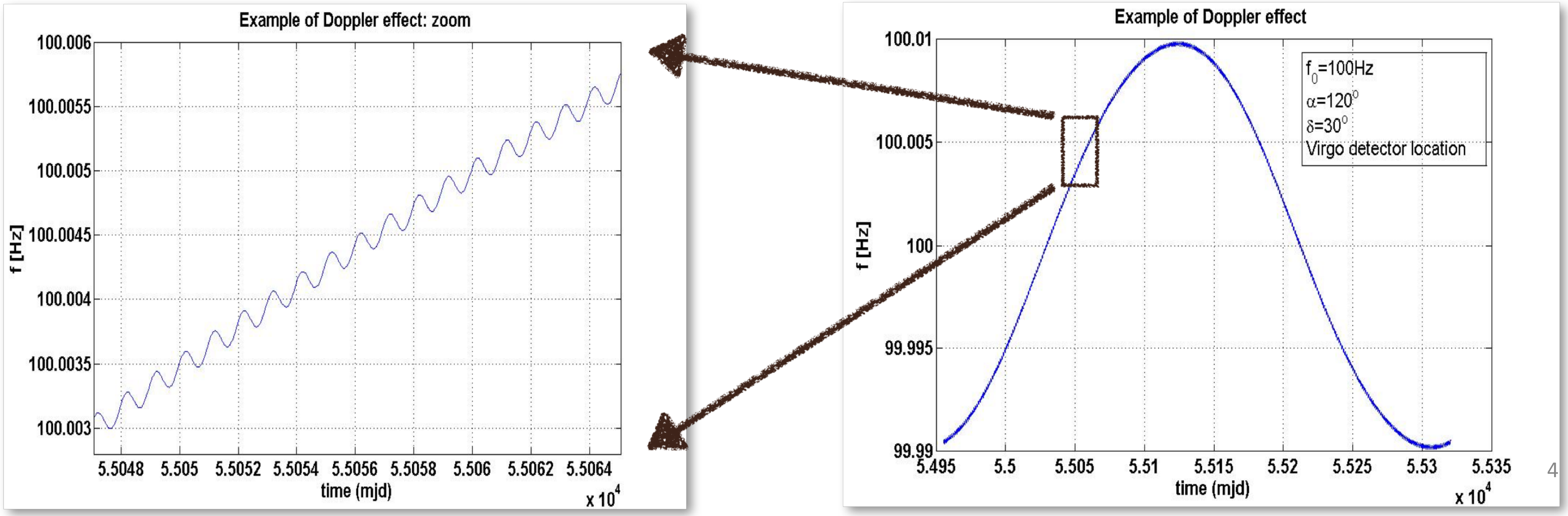
❖ Modeled waveforms – **at the detector** - require several parameters

E.g. CWs waveforms are described by:

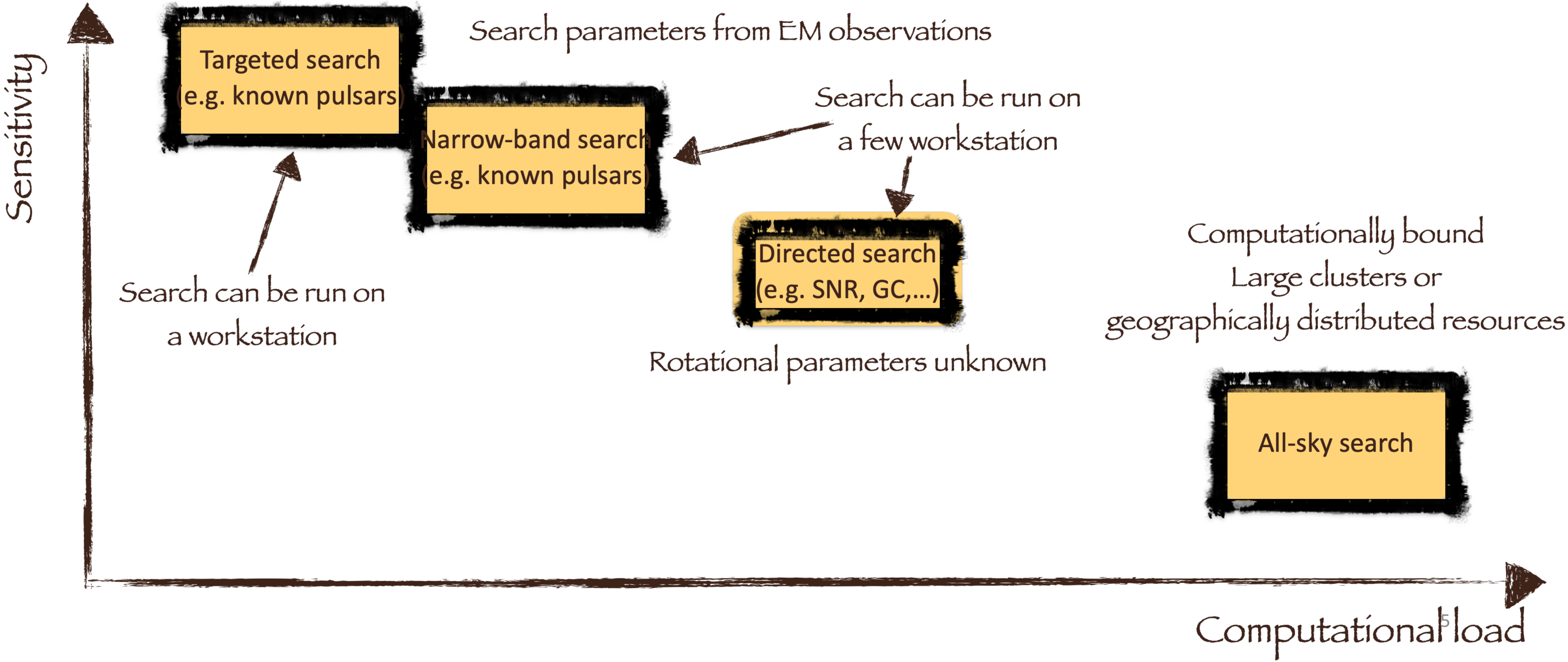
- Frequency
- Frequency derivative(s)
- Sky position

$$h(t) = A(\alpha, \delta, \psi, \iota; t)e^{j\varphi(t)}$$

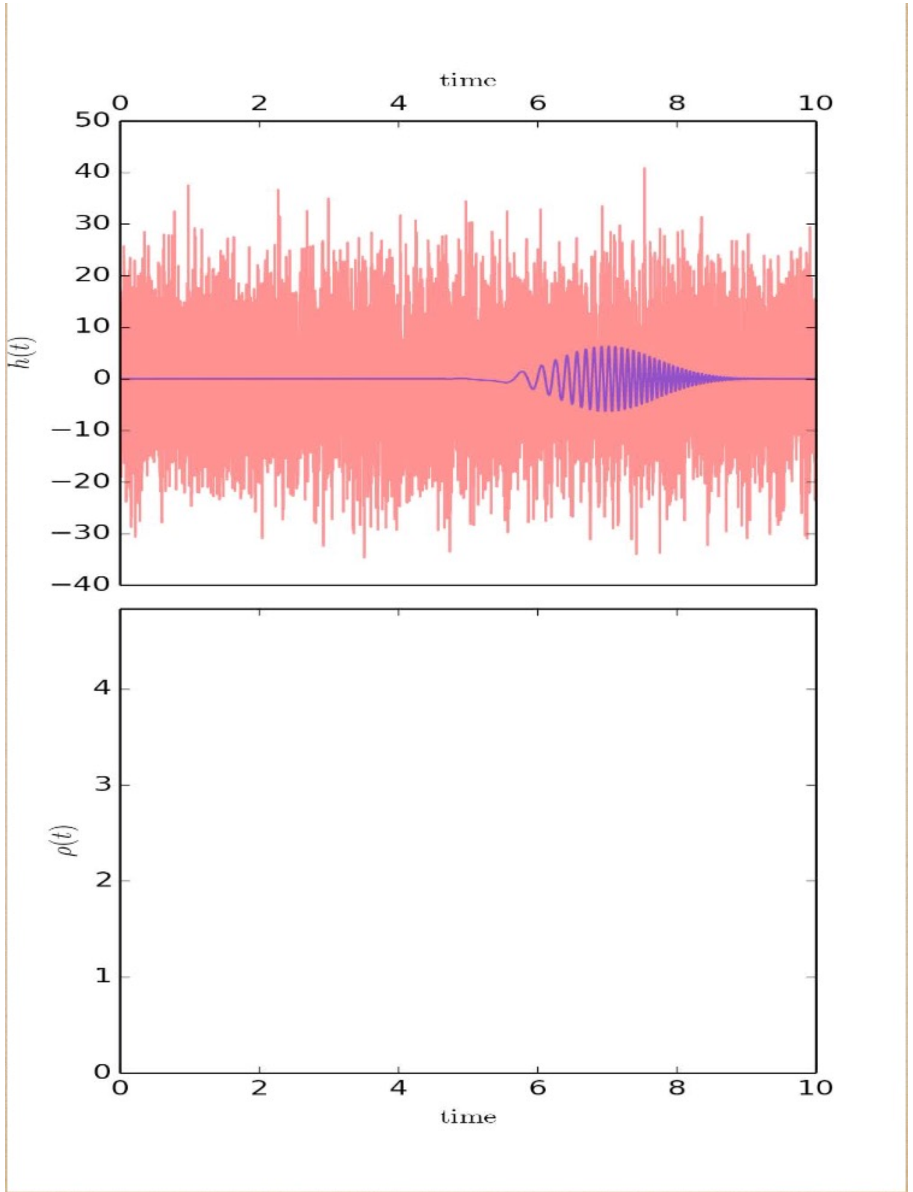
$$\varphi(t) \approx 2\pi \sum_{k=0}^{n-1} \frac{f_k t^k}{(k+1)!}\left(t + \frac{\vec{r} \times \hat{n}}{c}\right)$$

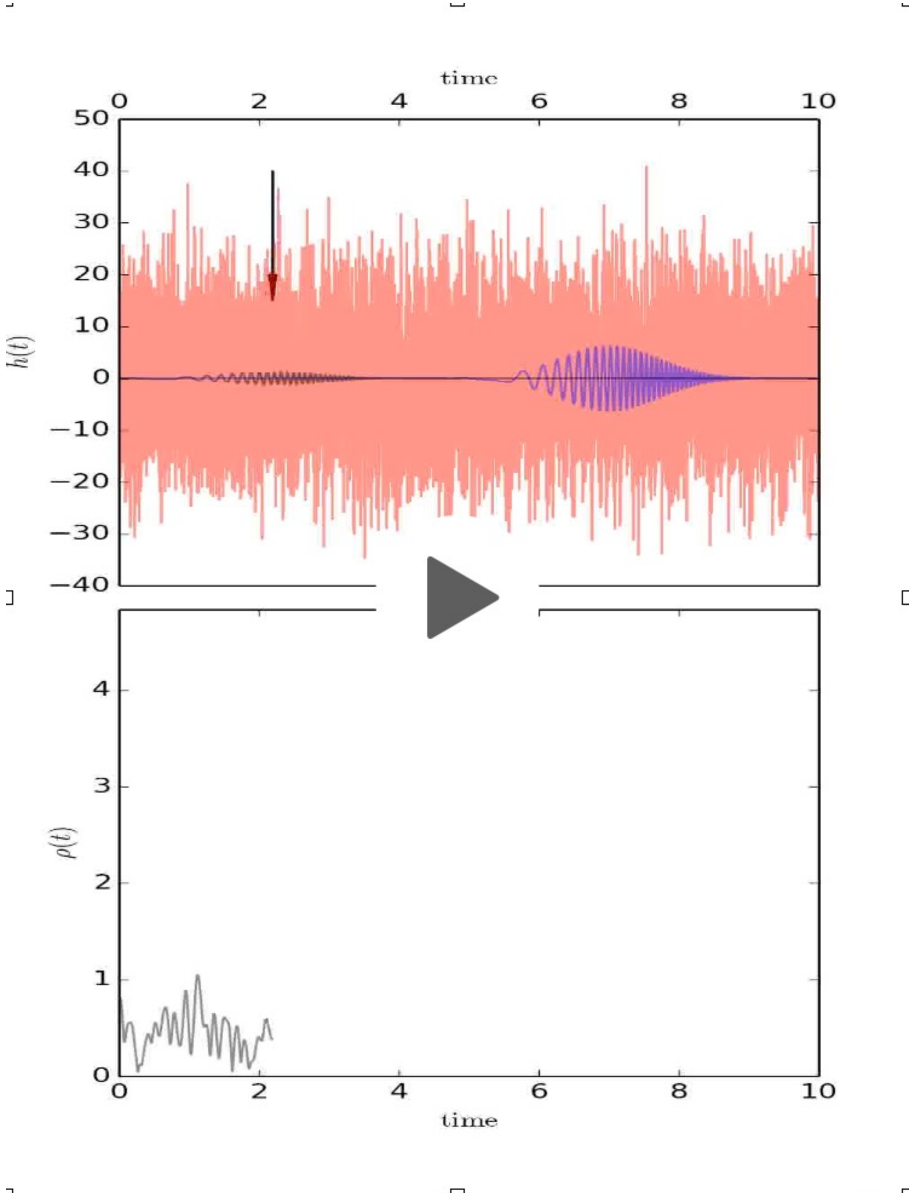


- Some parameters may be known from EM observations
- The volume of the parameter space impacts on the search sensitivity and computational cost

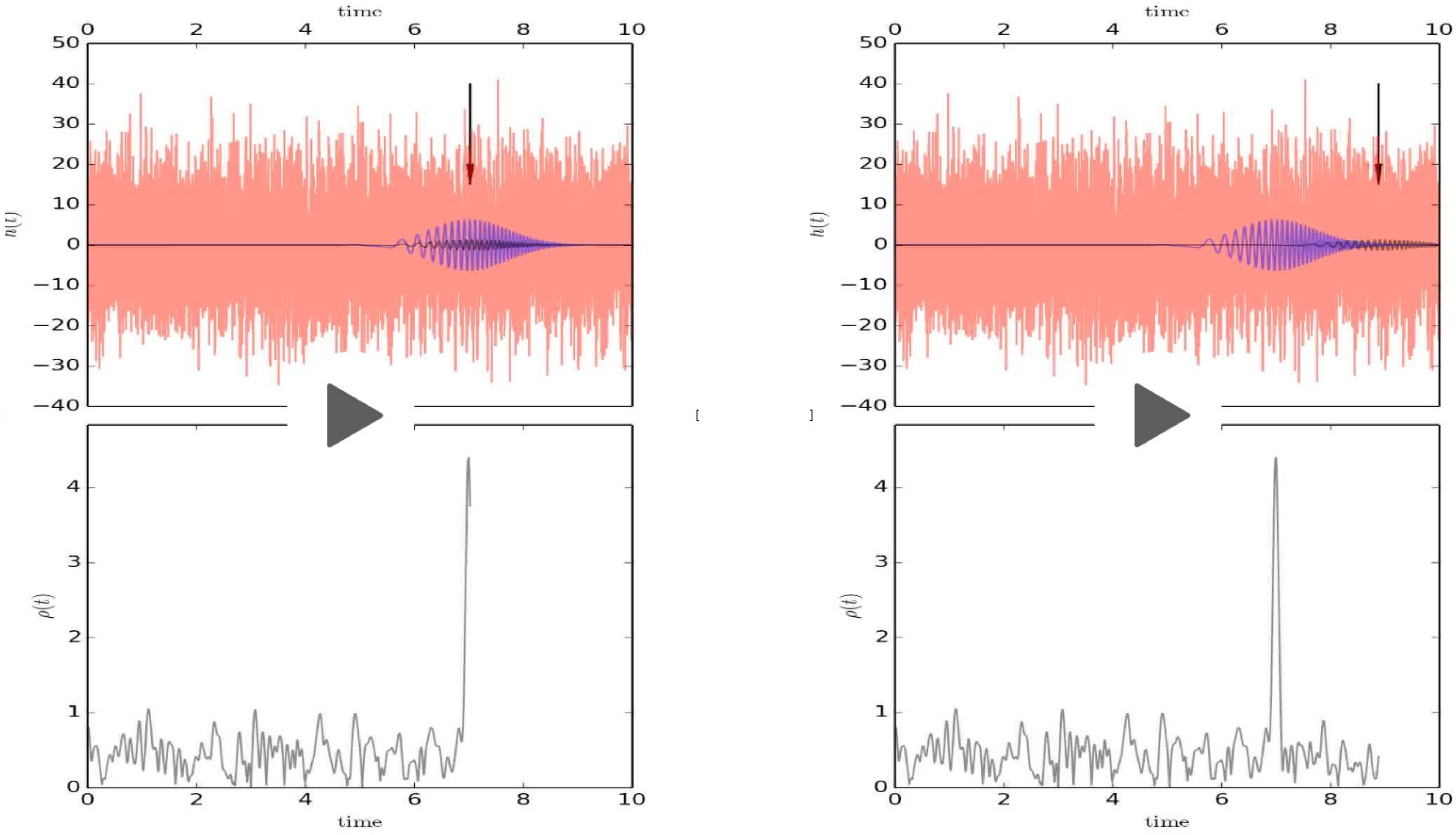


In principle, matched filtering – based on the cross-correlation among the data and signal templates – provides the best sensitivity





Best matching template maximizes SNR



The number of templates to be built and cross-correlated with the data is dictated by the number of cells in the parameters space

For an all-sky full-coherent search this number is

total observation time

$$N_{\rm tot} = 10^{-8}\pi \frac{T_{\rm obs}^8}{\Delta t^5 \tau_{\rm min}^3} = 1.33 \times 10^{29} \left(\frac{T_{\rm obs}}{4\ \rm months}\right)^8 \left(\frac{0.001\rm s}{\Delta t}\right)^5 \left(\frac{10^4\ \rm years}{\tau_{\rm min}}\right)^3$$

sampling time

minimum spin-down decay time

Template length: $T_{obs}/\Delta t \approx 10^{10}$

Various optimization schemes, e.g. based on band-sampled data, have been developed but, nevertheless,

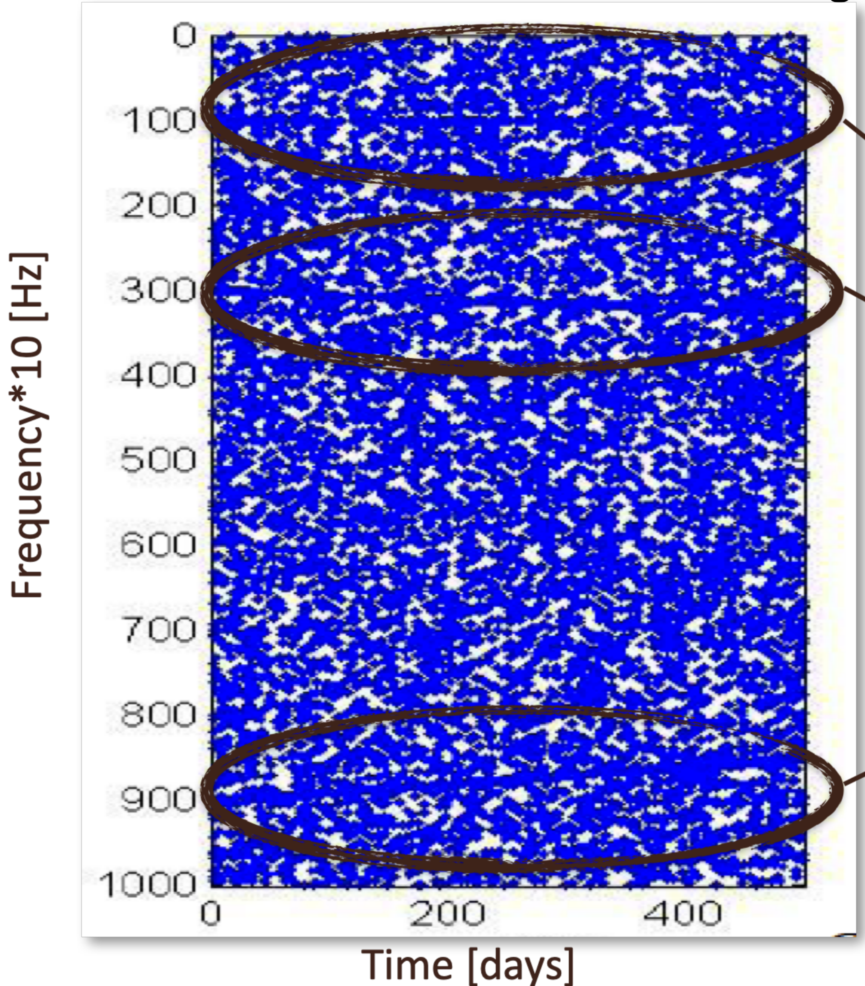
Such searches are not feasible with classical computation

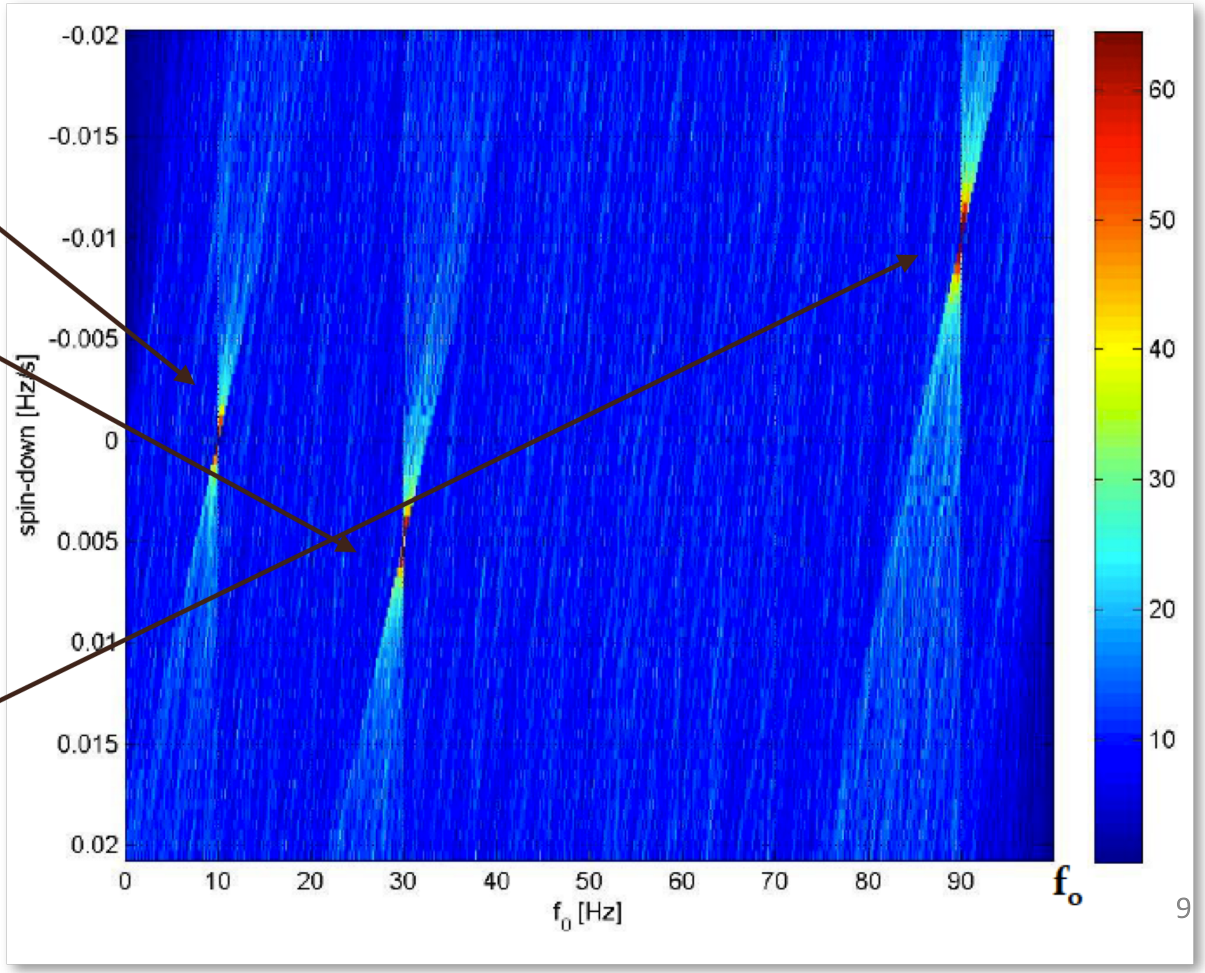
Alternative semi-coherent approaches have been developed: less computing power at the cost of a sensitivity reduction

$$N_{\rm tot} \approx 5.6\pi \times 10^{-9} K_f K_{\rm sky} \left(\frac{T_{\rm FFT}}{\delta t}\right)^{3+j_{\max}} \prod_{j \le j_{\max}} \left(\frac{T_{\rm obs}}{\tau_{\min}}\right)^{j} \approx 10^{21}$$

for T_{FFT} =1 day (and the other parameters as in the previous case)

(for a semi-coherent search based on the **Hough Transform**)





In perspective, three kinds of quantum algorithms look promising for GW searches:

- quantum Matched Filter
- quantum Radon Transform
- quantum Machine Learning

To my knowledge, the only published work on QC for GWs describes a quantum implementation of Matched Filter for **transient signals** (Gao+, arXiv: 2109.01535)

Quantum Radon Transform

- Radon Transform is strictly related to the Hough Transform
- Operates on bi-dimensional representation of the data
- Interpolation-based Discretized Radon Transform for line detection (Ma+, arXiv: 2107.05524)
- Polynomial speed-up w.r.t. classical version
- Open problem: how to take into account Doppler modulation? (Not straight lines to be detected)

Quantum Matched Filter

- Templates are computed as part of the processing: no use of qRAM
- Grover's algorithm (quantum counting) used to return wheter one (or more) templates match with the data above a threshold
- Complexity $\mathcal{O}(MlogM \cdot \sqrt{N})$, with M: number of data samples, N: number of points in the parameter space (i.e. the number of templates)
- Speed-up $\propto \sqrt{N}$ w.r.t. the classical computation
- Needed number of qbits: $\mathcal{O}(M)$, much larger than available in the near future

Quantum Machine Learning

- Classical ML promising for some kinds of GW searches
  - Current efforts mostly focused on the classification task
  - Bayesian inference looks well suited → computation of posterior probability distributions
- $\mathcal{O}(10^6)$ BH-BH events per year are expected for third generation GW detectors (like Einstein Telescope)
- Quantum-enanched ML could provide the speed-up needed to make (real-time) Bayesian inference feasible

Conclusions

- Quantum Computing could be a game-changer for GW Data Analysis in third generation detectors (namely, Einstein Telescope)
- We expect an increasing interest of the GW experimental community in the next years (in Virgo initial trigger by Piero Rapagnani)
- We are beginners in the field, and warmly welcome collaborations with more expert people