Fermion-Monopole Scattering

in a Chiral Gauge Theory

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Theories of Fundamental Interactions

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[2406.15552, W.I.P.], with Stefano Bolognesi and Andrea Luzio

Overview

1. Introduction to Callan-Rubakov Problem
   - Standard and less standard examples
   - Proposed solutions
2. A Chiral example
3. Future directions

The Standard CR Problem

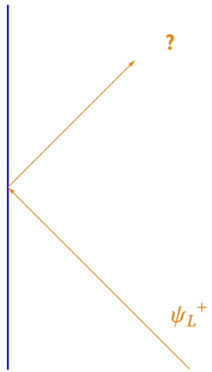
QED with Dirac Fermion $\psi = (\psi_L, \psi_R)$
charge e under $U_e(1)$

Dirac Monopole with magnetic charge m
under $U_e(1)$

Dirac Quantization Condition $em \in \mathbb{Z}$

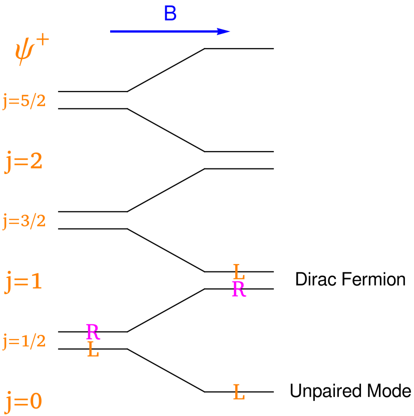
Weak e coupling, so free theory
outside monopole core

Heavy monopole with tiny core ($\sim M_W$)
(background)



Direction-Lock

in-going wave



Decompose modes in angular momentum
$J = L + S - \frac{1}{2} em\hat{r} \leftarrow$ monopole contrib.

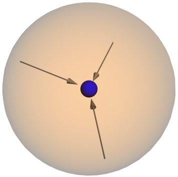
From Dirac equation, fixed
direction of motion (in/out-going wave)

s-wave	direction
ψ_L^+	in
ψ_R^+	out
ψ_L^-	out
ψ_R^-	in

Outside s-wave, both directions exist

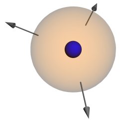
Focus on s-wave

Throw fermion at the monopole..



Focus on s-wave

..and see it bounce back



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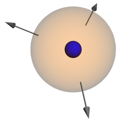


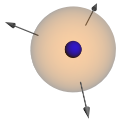
Try to conserve charges:

$U(1)_{\mathrm{e}}$ and $U(1)_{\mathrm{A}}$ (chirality)

s-wave	direction
ψ_L^+	in
ψ_R^+	out
ψ_L^-	out
ψ_R^-	in

p-wave	direction
ψ_L^+	both
ψ_R^+	both
ψ_L^-	both
ψ_R^-	both

Focus on s-wave



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Matching doable outside s-wave
where scattering is trivial at low energies.

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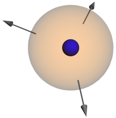

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Matching impossible for the s-wave.

s-wave	direction
ψ_L^+	in
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ψ_R^-	in

p-wave	direction
ψ_L^+	both
ψ_R^+	both
ψ_L^-	both
ψ_R^-	both

What gives way?

Break $U(1)_{\mathrm{e}}$

Because of the gauging, the monopole must absorb the charge and turn into a *dyon*

Break $U(1)_{\mathrm{A}}$

Can be achieved if we have a chiral condensate $\langle\psi\psi\rangle$

We need a UV completion to decide!

$SU(2)_{\mathrm{c}}$ UV completion

Can embed $U(1)_{\mathrm{e}}$ into $SU(2)_{\mathrm{c}}$. Monopole becomes regular!

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$\mathrm{N_f}$ Weyl fermions in $\square$ of $SU(2)_{\mathrm{c}}$ $\longrightarrow$ Flavor symmetry $\square$ of $SU(\mathrm{N_f})$

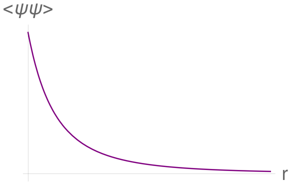
$\psi^{i,a}$, $i = 1, \ldots \mathrm{N_f}$, $a = 1, 2 \in SU(2)_{\mathrm{c}}$. $\mathrm{N_f} = 2\mathbb{Z}$ to avoid Witten anomaly.

Higgs in adjoint of $SU(2)_{\mathrm{c}}$, $A^3_\mu = A^{\mathrm{IR}}_\mu \begin{pmatrix} \frac{1}{2} & \\ & -\frac{1}{2} \end{pmatrix}$

$2\mathrm{N_f}$ Weyl fermions $\psi^{i,\pm} \to \mathrm{N_f}\ \psi^{\mathrm{i}}_{\mathrm{Dirac}} = \begin{pmatrix} \psi^{i+} \\ (\psi^{i-})^c \end{pmatrix}$.

$SU(2)_{\mathrm{c}}$ UV completion

When $\mathrm{N_f} = 2$, Rubakov found a condensate $\langle \epsilon_{ab}\epsilon_{ij}\epsilon_{\alpha\beta}\psi^{i,a,\alpha}\psi^{j,b,\beta}\rangle$, $\quad U(1)_{\mathrm{A}} \to \mathbb{Z}_2$



$$\psi_L^{1+} \to \psi_{2,R}^{+}, \quad \psi_R^{1-} \to \psi_{2,L}^{-} \quad \psi_L^{2+} \to \psi_{1,R}^{+}, \quad \psi_R^{2-} \to \psi_{1,L}^{-}$$

Effective boundary condition in the IR theory

$\psi_L^{i+}|_{r=0} = \epsilon^{ij}(\psi_L^{j-})^*|_{r=0}$

is fixed by the IR symmetries we want to preserve, and completely determines the scattering

Punchline: Either fix the boundary condition at low energies, or do a UV computation of the condensate.

$\mathrm{N_f} \geq 4$? Fractional Charges and Semitons

Assume we preserve $U(1)_{\mathrm{e}}$ and $SU(\mathrm{N_f})_{\mathrm{V}}$ at low energies.

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$\psi_L^{1+} \to \frac{1}{2}\psi_{1,L}^{-} + \frac{1}{2}\psi_{2,R}^{+} + \frac{1}{2}\psi_{3,R}^{+} + \frac{1}{2}\psi_{4,R}^{+}$

$\big($Contrast with $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e\big)$

Proposed solutions

- Leave s-wave [Csaki et al. 2009.14213, 2109.01145]
- Global symmetries are broken (by condensates)
- Light d.o.f. on monopole worldline (dyons, JR zero-modes)
  [Brennan 2309.00680]
- Need massive fermions/non-Fock representations and soft radiation
  [Brennan 2109.11207]
- UV completion does not exist?
- Generalized Symmetries [Marieke van Beest, Philip Boyle Smith, Diego Delmastro, Zohar Komargodski, David Tong 2306.07318]

Kitano&Matsudo ['21], Hamada&al.['22], Loladze&Okui['24], Hook&Ristow['24], . . .

P.B. Smith & D. Tong

A Chiral Example

We want a UV asymptotically free $SU(N)_{\rm c}$ chiral gauge theory.

To get a $U(1)$ theory in the IR, one possibility is to break $SU(N)_{\rm c}$ to the Cartan.

The $(N-1)$ $U(1)$'s can embed into $SU(N)_{\rm c}$ as $\text{Diag}(\ldots, 0, 1, -1, 0, \ldots)$, same as for $SU(2)_{\rm c}$ example earlier! *Locally vectorial* theory.

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same as for $SU(2)_{\mathrm{c}}$ example earlier! *Locally vectorial* theory.

$\psi\chi\eta$ model $\square\!\square \oplus \overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}} \oplus 8\,\overline{\square}$ $\quad \psi^{(ab)}$, $\chi^{[ab]}$, η^{a}_{i}. Adj Higgs Φ^{a}_{b} $SU(N)_{\mathrm{c}} \to U(1)^{N-1}$

Gauge anomaly free. Yukawa $\mathrm{y}\Phi^{a}_{b}\psi^{(bc)}\chi_{[ac]}$ gaps many modes.

We can check..

Leveraging the regularity of the core, we can check..

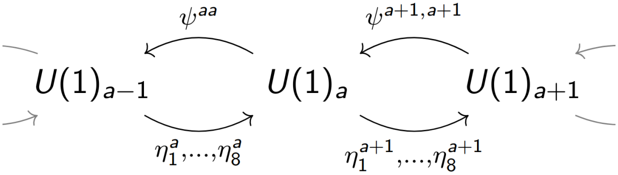
- Absence of dyons with $M_{dyon} - M_{monopole} \sim E_{scattering}$
- Absence of JR zero-modes
- Presence of a single condensate, which only breaks anomalous global symmetries
- Check that boundary conditions at $r = 0$ preserve the remaining symmetries

Idea: Go to regular gauge, study fluctuations regular at the origin.

This forces the correct boundary conditions!

IR Theory, Monopole in $U(1)_a$

$U(1)^{N-1}$ gauge theory. ψ^{aa}, $a = 1, \ldots, N$, η_i^a, $i = 1, \ldots 8$.



	$U(1)_a$	$U(1)_{a-1}$	$U(1)_{a+1}$	$U(1)'_a$	$SU(8)_{\mathrm{F}}$	$SU(2)_J$	dir.
$\psi^{a,a}$	2	-2	0	2	$(\cdot)$	1/2	in
$\psi^{a+1,a+1}$	-2	0	2	2	$(\cdot)$	1/2	out
η_i^a	-1	1	0	-1	$\square$	0	out
η_i^{a+1}	1	0	-1	-1	$\square$	0	in

Bosonization

$$\psi^{a,a,m=\frac{1}{2}} \to \frac{1}{2}\psi^{a+1,a+1,m=\frac{1}{2}} - \frac{1}{2}\psi^{a+1,a+1,m=-\frac{1}{2}} - \frac{1}{4}\eta_i^a$$

Still fractional charges..

Bosonization

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In 2D, Dirac fermions $\psi^m = \begin{pmatrix} \psi^{a,a,m} \\ \psi^{a+1,a+1,m} \end{pmatrix}$, $\eta_i = \begin{pmatrix} \eta_i^{a+1} \\ \eta_i^a \end{pmatrix}$

that bosonize as $\chi_{\ell/r} =: e^{i\phi_{\ell/r}} :, \quad \chi = \psi^{\pm\frac{1}{2}}, \eta_{1\ldots8}$

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Conserved charges fix boundary condition at $r = 0$ for ϕ's

$\Rightarrow$ Symmetries fix the S matrix

$$Q_\ell \phi_\ell|_{r=0} = Q_r \phi_r|_{r=0}, \quad \phi_r^i|_{r=0} = \sum_j S^i{}_j \phi_\ell^j|_{r=0}, \quad S = Q_r^{-1} Q_\ell$$

$$\psi^{a,a,m=\frac{1}{2}} =: e^{i\phi_\ell^{\psi,1}} :\to: e^{i\left(\frac{1}{2}\phi_r^{\psi,1} - \frac{1}{2}\phi_r^{\psi,2} - \frac{1}{4}\sum_{i=1}^{8}\phi_r^{\eta,i}\right)} :=??$$

Future Direction

Discrete gauge invariance $\phi^i_{\ell,r} \to \phi^i_{\ell,r} + 2\pi\mathbb{Z}^i_{\ell,r}$ must be restored!

Gauging the shift symmetry, we obtain Wilson lines, which act as symmetry operators [van Beest et al.]. Topological operators, except at end-points.

Discrete symmetry $\mathbb{Z}^n_\ell \times \mathbb{Z}^n_r \leftrightarrow$ Continuous charge $U(1)^n_\ell \times U(1)^n_r$

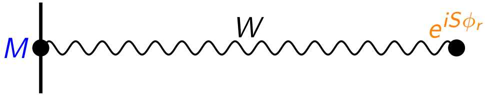
Boundary condition is not invariant under generic shift symmetry

$$\phi^i_r = S^i{}_j \phi^j_\ell \longrightarrow \phi^i_r = S^i{}_j \phi^j_\ell + \theta^i$$

This restricts the allowed gauge transformations on the boundary.

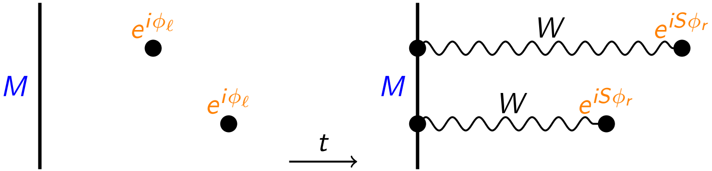
Future Direction

A Wilson line of the shift sym. can connect the OUT fermions to the monopole.



This extended object is fully gauge invariant except at monopole, where shift sym. is broken by boundary condition anyway.

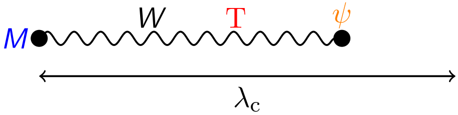
Gauge invariant correlators have no monodromy.



Thank You!

Adding a Mass Term

What happens to the Wilson line if we are in QED
and we give mass m_i to the fermions?



Decay rate of string to simple fermions $\propto \langle \text{ferm.}_i | m_i \bar{\psi}^i \psi^i | \text{string}_i \rangle \propto m_i$

At $r \gg \lambda_c$, fewer symmetries can be imposed and $\psi^i_\ell \to \psi^i_r$.