Wilson loops and defect RG flows in ABJM

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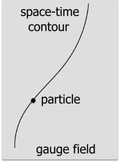
Based on [2211.16501], [2305.01647], [2312.13283], [24xx.xxxx]

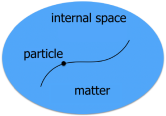
with L. Castiglioni, S. Penati, D. Trancanelli + M. Bianchi

Introduction & motivation

- WLs are fundamental objects in any gauge theory
- Supersymmetric gauge theory: susy WLs

$$W = \mathrm{Tr}\,\mathcal{P}\exp\left[i\int_{\mathcal{C}}\left(A_\mu \dot{x}^\mu + \text{matter}\right)dt\right]$$





  - may be computed **exactly** via localization
    - non-trivial tests of the AdS/CFT correspondence
- Defects
  - 1d superconformal group: superconformal bootstrap
  - Generalized symms: WLs are charged under 2-form symm
    - may provide topological objects to study non-invertible symms

Introduction & motivation

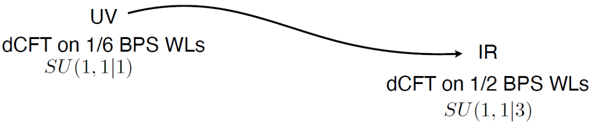
- 1d defects with non-trivial RG flows
  - $\mathcal{N}$=4 SYM

$$W = \mathrm{Tr}\,\mathcal{P}\exp\left[i\oint \left(A_\mu \dot{x}^\mu + \zeta|\dot{x}|\theta^m\Phi^m\right)dt\right], \qquad \theta^m\theta^m = 1$$

    - $\zeta = 0$: "ordinary" non-BPS WL
      UV fixed point
    - $\zeta = 1$: 1/2 BPS Wilson-Maldacena loop
      IR fixed point

[Polchinski-Sully, '11]
[Beccaria-Giombi-Tseytlin, '17...]

  - ABJM
    - 3d theories display a much **richer structure** of WLs
    - also connected via RG flows. E.g.,



Outline

Wilson loops in ABJM

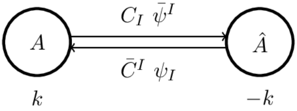
Defect RG flows

Interpolating Bremsstrahlung function

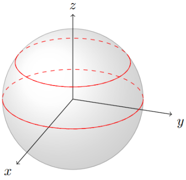
Ongoing: Cohomological equivalence & framing

Future directions

WLs in ABJM



$$W = \mathrm{Tr}\,\mathcal{P}\exp\left[i\oint_{\mathcal{C}} \left(A_\mu \dot{x}^\mu + \text{matter}\right)\,dt\right]$$



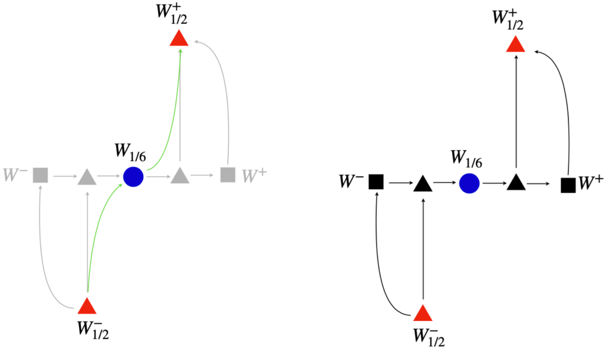
- 1-node loops (bosonic, at most 1/6 BPS)

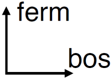
$$W^{\mathrm{bos}} = \mathrm{Tr}\,\mathcal{P}\exp\left[i\oint \left(A_1 + \star\ C\bar{C}\right)\,dt\right]$$

- 2-node loops (fermionic, at most 1/2 BPS)

$$W^{\mathrm{fer}} = \mathrm{sTr}\,\mathcal{P}\exp\left[i\oint \begin{pmatrix} A_1 + \star_1\ C\bar{C} & \star_2\ \bar\psi \\ \star_3\ \psi & A_2 + \star_4\ \bar{C}C \end{pmatrix}\,dt\right]$$

WLs in ABJM





$\square = SU(4)$

$\triangle = SU(3)$

$\bigcirc = SU(2) \times SU(2)$

— non-BPS

— 1/6 BPS

— 1/2 BPS

— 1/24 BPS

WLs in ABJM

- 1d auxiliary method: $\langle W(t_1,t_2)\rangle = \langle \bar{z}(t_2) z(t_1)\rangle$
  - originally proposed in QCD

$$S_{\text{eff}} = S_{\text{QCD}} + \int \Big[\bar{z}(t)\,(\partial_t + i\,A_\mu \dot{x}^\mu)\,z(t)\Big]\,dt$$

[Samuel, '79]
[Gervais-Neveu, '80]

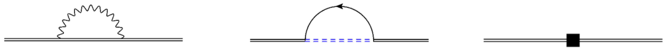
  - adapted to ABJM

$$S_{\text{eff}} = S_{\text{ABJM}} + \int \Big[\bar{\Psi}(t)\,(\partial_t + i\mathcal{L})\,\Psi(t)\Big]\,dt$$

$$\downarrow$$

$$\mathcal{L} = (A + \star\; C\bar{C})\,, \quad \mathcal{L} = \begin{pmatrix} A_1 + \star_1\; C\bar{C} & \star_2\; \bar{\psi} \\ \star_3\; \psi & A_2 + \star_4\; \bar{C}C \end{pmatrix}\,, \quad \cdots$$

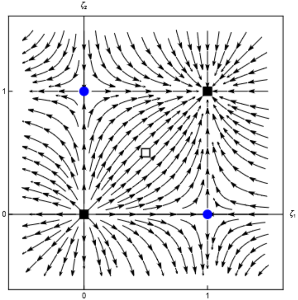
- ▶ used to compute β-functions: $\beta(\star_i) \neq 0$!



Defect RG flows

- $\color{red}{\star_i}$ generic & generically ~~susy~~
- $\langle W \rangle = F(\star_i)$ with non-trivial $\beta(\star_i)$
- Bosonic flows: marginally relevant (bosonic) deformations

$$W = \mathrm{Tr}\,\mathcal{P}\exp\left[i\oint\left(A_\mu \dot{x}^\mu + M_J^I\, C_I \bar{C}^J\right)dt\right]$$



■ "Ordinary" WLs $M = \pm\mathbb{1}_4$

□ Only gauge $M = 0$

● 1/6 BPS $M = \pm\mathrm{diag}(-1,-1,1,1)$

- In contrast with $\mathcal{N} = 4$, in 3d "ordinary" loops include scalars

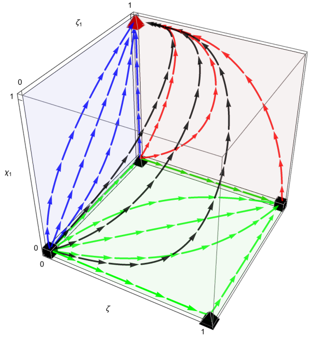
Defect RG flows

- Fermionic flows: marginally relevant (bosonic and fermionic) deformations

$$W = \mathrm{sTr}\,\mathcal{P}\exp\left[i\oint\begin{pmatrix} A_1 + M^I_J C_I \bar{C}^J & 0 \\ 0 & A_2 + M^I_J \bar{C}^J C_I \end{pmatrix} dt\right]$$

$$\downarrow$$

$$W = \mathrm{sTr}\,\mathcal{P}\exp\left[i\oint\begin{pmatrix} A_1 + M^I_J C_I \bar{C}^J & \eta\bar{\psi} \\ \psi\bar{\eta} & A_2 + M^I_J \bar{C}^J C_I \end{pmatrix} dt\right]$$



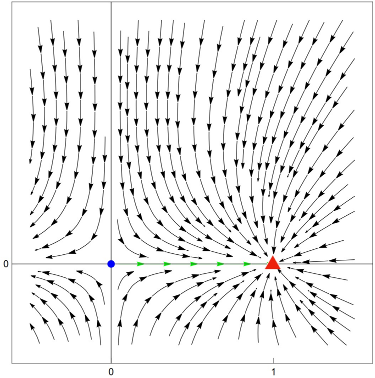
■ "Ordinary" WLs $\quad M = \pm\mathbb{1}_4$

▲ SU(3) bosonic $\quad M = \pm\mathrm{diag}(-1,1,1,1)$

▲ 1/2 BPS $\quad \begin{cases} M = \mathrm{diag}(-1,1,1,1) \\ \eta_I^\alpha = \delta_I^1\,(e^{it/2}, -ie^{-it/2})^\alpha \\ \bar{\eta}^I_\alpha = \delta^I_1 \begin{pmatrix} ie^{-it/2} \\ -e^{it/2} \end{pmatrix}_\alpha \end{cases}$

Defect RG flows

- $\star_i$ constrained & susy preserved $\rightarrow$ **Enriched flows**



- 1/6 BPS bosonic
- 1/24 BPS
- 1/6 BPS fermionic
- 1/2 BPS

- Partition function in presence of the defect $\rightarrow$ g-function
- g-theorem: $g_{\mathrm{UV}} > g_{\mathrm{IR}}$ [Cuomo-Komargodski-Raviv-Moshe, '21]

Bremsstrahlung function

- Latitute WLs: less parameters $\star_i$ allowed & less susy preserved

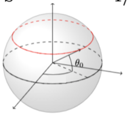
interpolating cusp

$\bigstar = 0$ 1/6 BPS $\longrightarrow$ $\bigstar = 1$ 1/2 BPS

cusp/latitude correspondence

interpolating latitude

$\bigstar = 0$ 1/12 BPS $\longrightarrow$ $\bigstar = 1$ 1/6 BPS



$$B \sim \frac{\partial^2}{\partial \varphi^2} \Gamma_{\text{cusp}} \Big|_{\varphi=0}$$

$$B \sim \frac{\partial}{\partial \theta_0} \log \langle W_{\text{latitude}} \rangle \Big|_{\theta_0=0}$$

interpolating bremsstrahlung

linear dCFT

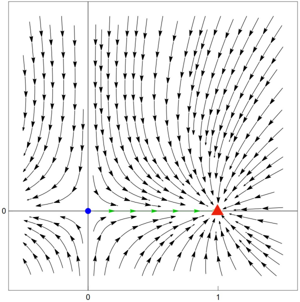
$$B \sim \beta_{g^2} \frac{\partial}{\partial g^2} \int ds_{1>2} \langle\!\langle \mathcal{L}^{(1)}(s_1) \mathcal{L}^{(1)}(s_2) \rangle\!\rangle_{\text{line}}$$

circular dCFT

$$B \sim \int d\tau_{1>2} \langle\!\langle \mathcal{L}^{(1)}(\tau_1) \mathcal{L}^{(1)}(\tau_2) \rangle\!\rangle_{\text{circle}}$$

- Prescriptions agree up to terms $\propto \beta(\star_i)$

Cohomological equivalence



- $1/6$ BPS bosonic ($W^{\text{bos}}_{1/6}$)
- $1/24$ BPS
- $1/6$ BPS fermionic
- $1/2$ BPS

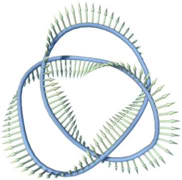
$$W = W^{\text{bos}}_{1/6} + \mathcal{Q}V, \quad \mathcal{Q} \text{ mutually preserved}$$

[Drukker-MT-Trancanelli et al, '19]
[Drukker-MT-Trancanelli, '20]

- Cohomologically equivalent
- VEVs localize to the same matrix model
- However, we have seen that $\langle W \rangle = F(\star_i)$

Framing

- $\color{red}{\star_i}$-dependence possibly cancelled by suitably **framing the WL**
  - Perturbation theory usually performed at $f = 0$
  - Exact result holds at $f = 1$
- CS topological but $\langle W \rangle$ topologically invariant iff $\mathcal{C}$ is framed



$$\langle W^{\mathrm{CS}} \rangle_f = \exp\left(\frac{i\pi N}{k} f\right) \langle W^{\mathrm{CS}} \rangle_{f=0}$$

$$f = \frac{1}{4\pi} \int_{\mathcal{C}} dx_1^\mu \int_{\mathcal{C}_f} dx_2^\nu \, \epsilon_{\mu\nu\rho} \frac{(x_1 - x_2)^\rho}{|x_1 - x_2|^3}$$

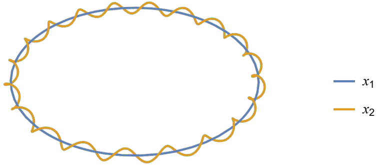
- ABJM not topological but $\langle W \rangle$ sensitive to framing

Ongoing direction: cohomological equivalence & framing

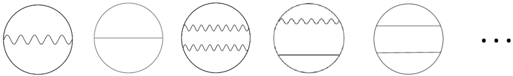
1. **Can we compute $\langle W \rangle$ perturbativelly at generic f?**

- Point-splitting: define an helix going around the circle n times

$$x^\mu(t) \to x^\mu(t) + \delta n^\mu(t)$$
$$|n(t)| = 1$$

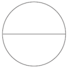


- $\langle W \rangle$ sensitive to framing via gauge and matter contributions
- Compute each Feynman diagram using point-splitting

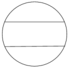


Ongoing direction: cohomological equivalence & framing

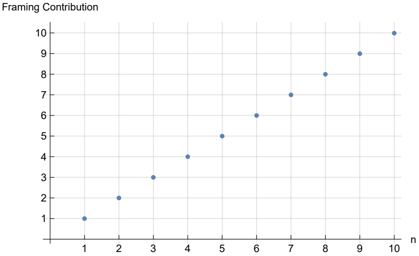
- E.g. fermion exchanges at 1 and 2-loops

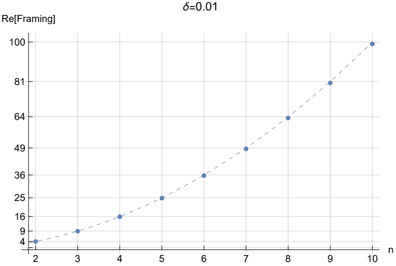


$\langle (\star\bar{\psi})_1 \, (\star\psi_2) \rangle$



$\langle ((\star\bar{\psi})_1) \, (\star\psi)_2) \rangle \, \langle ((\star\bar{\psi})_3) \, (\star\psi)_4) \rangle$





2. **What about the dependence on $\star$?**

   - It seems to drop out for $f = 1$!

Future directions

- Breaking of cohomological equivalence at $f = 0$
- Gravity dual of ABJM is M-theory on $\mathrm{AdS}_4 \times S^7/Z_k$ or, for large enough k, type IIA string theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$
  - Strong coupling description of WLs not completely known
  - Interpolating boundary conditions on $\mathbb{CP}^3$?

[Polchinski-Sully, '11] [Correa-Faraggi-Garay-Silva, '22]

string/brane configuration	**Wilson loop**
Dirichlet b.c.	1/2 BPS WL
Newmann b.c. on 2 of the $\mathbb{CP}^3$ coord. & Dirichlet on the rest	1/6 BPS bosonic WL