Decomposition and (-1)-form symmetries in the Symmetry TFT

Ling Lin

University and INFN Bologna

Based on w.i.p
with Daniel G. Robbins and Subham Roy

TFI 2024
Napoli, 25/09/2024

Plan of the talk

1. Motivation
2. Decomposition and (-1)-form symmetries
3. Description via Symmetry TFT (SymTFT)
4. Applications / Examples

Motivation

- Global p-form symmetries are powerful generalization of symmetry principle.
  [See previous talk by Francesco Benini]

Motivation

- Global p-form symmetries are powerful generalization of symmetry principle.
  [See previous talk by Francesco Benini]

- Unifying framework to describe global symmetries & anomalies: SymTFT
  [Gaiotto/Kulp ’20, Apruzzi et al ’21, Freed/Moore/Teleman ’22, …]

Motivation

- Global p-form symmetries are powerful generalization of symmetry principle.
  [See previous talk by Francesco Benini]

- Unifying framework to describe global symmetries & anomalies: SymTFT
  [Gaiotto/Kulp ’20, Apruzzi et al ’21, Freed/Moore/Teleman ’22, …]

  - Democratic description of charged objects and symmetry generators.
  - Gauging and quantum-dual symmetries via boundary conditions.

Motivation

- Global p-form symmetries are powerful generalization of symmetry principle. [See previous talk by Francesco Benini]

- Unifying framework to describe global symmetries & anomalies: SymTFT [Gaiotto/Kulp '20, Apruzzi et al '21, Freed/Moore/Teleman '22, …]

  - Democratic description of charged objects and symmetry generators.
  - Gauging and quantum-dual symmetries via boundary conditions.

- Less explored extreme case: $p = -1 \ \rightarrow$ related to θ-angles and axions.

- Quantum-dual (d-1)-form symmetry → "decomposition" [Pantev/Sharpe '05, Hellerman et al '06]

Decomposition from topological point operators

[Sharpe ’22 (review) + references therein]

Decomposition from topological point operators

[Sharpe '22 (review) + references therein]

- Theories with topological point operators $W_\nu(x) \equiv W_\nu$, i.e., (d-1)-form symmetry, have projection operators $\Pi_R = \sum_\nu \chi_R(\nu)\, W_\nu$, such that

Decomposition from topological point operators

[Sharpe '22 (review) + references therein]

- Theories with topological point operators $W_\nu(x) \equiv W_\nu$, i.e., (d-1)-form symmetry, have projection operators $\Pi_R = \sum_\nu \chi_R(\nu)\, W_\nu$, such that
  - $\sum_R \Pi_R = \mathrm{id}$, $[\Pi_R\,, \mathcal{O}] = 0$ for all (codim ≥ 2) operators $\mathcal{O}$, $\Pi_R\,\Pi_S = \delta_{RS}\Pi_S$.

Decomposition from topological point operators

[Sharpe '22 (review) + references therein]

- Theories with topological point operators $W_\nu(x) \equiv W_\nu$, i.e., (d-1)-form symmetry, have projection operators $\Pi_R = \sum_\nu \chi_R(\nu)\, W_\nu$, such that
    - $\sum_R \Pi_R = \mathrm{id},\ [\Pi_R\,, \mathcal{O}] = 0$ for all (codim ≥ 2) operators $\mathcal{O}$, $\Pi_R \Pi_S = \delta_{RS}\Pi_S$.
- Consequence: $\mathscr{H} = \bigoplus_R \mathscr{H}_R\,,\ \langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle = \sum_R \langle \mathcal{O}_1^{(R)} \ldots \mathcal{O}_n^{(R)} \rangle\,, \quad \mathcal{O}_m^{(R)} = \Pi_R \mathcal{O}_m$, $\mathscr{Z} = \sum_R \mathscr{Z}_R = \sum_R \langle \Pi_R \rangle$

Decomposition from topological point operators

[Sharpe '22 (review) + references therein]

- Theories with topological point operators $W_\nu(x) \equiv W_\nu$, i.e., (d-1)-form symmetry, have projection operators $\Pi_R = \sum_\nu \chi_R(\nu)\, W_\nu$, such that
  - $\sum_R \Pi_R = \mathrm{id}$, $[\Pi_R, \mathcal{O}] = 0$ for all (codim ≥ 2) operators $\mathcal{O}$, $\Pi_R \Pi_S = \delta_{RS}\Pi_S$.
- Consequence: $\mathcal{H} = \bigoplus_R \mathcal{H}_R$, $\langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle = \sum_R \langle \mathcal{O}_1^{(R)} \ldots \mathcal{O}_n^{(R)} \rangle$, $\quad \mathcal{O}_m^{(R)} = \Pi_R \mathcal{O}_m$,
  $\mathcal{Z} = \sum_R \mathcal{Z}_R = \sum_R \langle \Pi_R \rangle$
  $\Rightarrow$ decomposition into "universes" [Komargodski et al '20]: $\mathcal{T}^{(d-1)} = \coprod_R \mathcal{T}_R$

Decomposition from topological point operators

[Sharpe '22 (review) + references therein]

- Theories with topological point operators $W_\nu(x) \equiv W_\nu$, i.e., (d-1)-form symmetry, have projection operators $\Pi_R = \sum_\nu \chi_R(\nu)\, W_\nu$, such that
  - $\sum_R \Pi_R = \mathrm{id}$, $[\Pi_R\,, \mathcal{O}] = 0$ for all (codim ≥ 2) operators $\mathcal{O}$, $\Pi_R \Pi_S = \delta_{RS}\Pi_S$.
- Consequence: $\mathcal{H} = \bigoplus_R \mathcal{H}_R\,,\ \langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle = \sum_R \langle \mathcal{O}_1^{(R)} \ldots \mathcal{O}_n^{(R)} \rangle\,, \quad \mathcal{O}_m^{(R)} = \Pi_R \mathcal{O}_m$,
  $\mathcal{Z} = \sum_R \mathcal{Z}_R = \sum_R \langle \Pi_R \rangle$
  $\Rightarrow$ decomposition into "universes" [Komargodski et al '20]: $\mathcal{T}^{(d-1)} = \coprod_R \mathcal{T}_R$
- "Stronger" than super-selection: universes separated by infinite-energy domain walls ("eternal false vacua" [Cherman/Jacobson '20]).

Examples

Examples

- d=2 G Yang—Mills: $\mathscr{Z}(\Sigma_g, A) = \sum_R \dim(R)^{2-2g} \exp(-c_2(R)A) = \sum_{n\leq|Z|} \mathscr{Z}[(G/Z)_n]$ [Witten '91, Tachikawa '13].

Examples

- d=2 G Yang—Mills: $\mathscr{Z}(\Sigma_g, A) = \sum_R \dim(R)^{2-2g} \exp(-c_2(R)A) = \sum_{n\leq|Z|} \mathscr{Z}[(G/Z)_n]$ [Witten '91, Tachikawa '13].
    - Due to *non-invertible* $V_{[U]}$; invertible for $U \in Z \equiv Z(G)$. [Nguyen/Tanizaki/Ünsal '21]

Examples

- d=2 G Yang—Mills: $\mathscr{Z}(\Sigma_g, A) = \sum_R \dim(R)^{2-2g} \exp(-c_2(R)A) = \sum_{n\leq|Z|} \mathscr{Z}[(G/Z)_n]$ [Witten '91, Tachikawa '13].
  - Due to *non-invertible* $V_{[U]}$; invertible for $U \in Z \equiv Z(G)$. [Nguyen/Tanizaki/Ünsal '21]

❖ d=4 generalization: G-YM with *instanton restriction* [Seiberg '10, Tanizaki/Ünsal '19]:

Examples

- d=2 G Yang—Mills: $\mathscr{Z}(\Sigma_g, A) = \sum_R \dim(R)^{2-2g} \exp(-c_2(R)A) = \sum_{n\leq|Z|} \mathscr{Z}[(G/Z)_n]$ [Witten '91, Tachikawa '13].
  - Due to *non-invertible* $V_{[U]}$; invertible for $U \in Z \equiv Z(G)$. [Nguyen/Tanizaki/Ünsal '21]
- ❖ d=4 generalization: G-YM with *instanton restriction* [Seiberg '10, Tanizaki/Ünsal '19]:
  - Yang—Mills[θ] + top. sector $i\int b(\mathrm{Tr}(F^2) - \frac{N}{2\pi}d\lambda_3) \Rightarrow \int \mathrm{Tr}(F^2) = 0 \mod N$.

Examples

- d=2 G Yang—Mills: $\mathscr{Z}(\Sigma_g, A) = \sum_R \dim(R)^{2-2g} \exp(-c_2(R)A) = \sum_{n\leq|Z|} \mathscr{Z}[(G/Z)_n]$ [Witten '91, Tachikawa '13].
  - Due to *non-invertible* $V_{[U]}$; invertible for $U \in Z \equiv Z(G)$. [Nguyen/Tanizaki/Ünsal '21]

❖ d=4 generalization: G-YM with *instanton restriction* [Seiberg '10, Tanizaki/Ünsal '19]:

  - Yang—Mills[θ] + top. sector $i\int b(\mathrm{Tr}(F^2) - \frac{N}{2\pi} d\lambda_3) \Rightarrow \int \mathrm{Tr}(F^2) = 0 \mod N$.
  - Topological point ops $e^{ib(x)} \Rightarrow \mathscr{Z}[\theta] = \sum_{k=0}^{N-1} \mathscr{Z}^{\mathrm{YM}}[\theta + 2\pi \frac{k}{N}]$.

Examples

- d=2 G Yang—Mills: $\mathscr{Z}(\Sigma_g, A) = \sum_R \dim(R)^{2-2g} \exp(-c_2(R)\,A) = \sum_{n \le |Z|} \mathscr{Z}[(G/Z)_n]$ [Witten '91, Tachikawa '13].
  - Due to *non-invertible* $V_{[U]}$; invertible for $U \in Z \equiv Z(G)$. [Nguyen/Tanizaki/Ünsal '21]

❖ d=4 generalization: G-YM with *instanton restriction* [Seiberg '10, Tanizaki/Ünsal '19]:

  - Yang—Mills[θ] + top. sector $i\int b(\mathrm{Tr}(F^2) - \frac{N}{2\pi} d\lambda_3) \Rightarrow \int \mathrm{Tr}(F^2) = 0 \mod N$.
  - Topological point ops $e^{ib(x)} \Rightarrow \mathscr{Z}[\theta] = \sum_{k=0}^{N-1} \mathscr{Z}^{\mathrm{YM}}[\theta + 2\pi \frac{k}{N}]$.

- Many more examples in orbifold models [Sharpe + collaborators, '06 — now].

Universes and (-1)-form symmetries

- Inserting $\Pi_R = \sum_k \chi_R(k)\, W_k$ projects onto universe labelled by $\chi_R \equiv R$, corresponds to gauging the (d-1)-form sym with discrete torsion χ_R [Sharpe '19].

Universes and (-1)-form symmetries

- Inserting $\Pi_R = \sum_k \chi_R(k)\, W_k$ projects onto universe labelled by $\chi_R \equiv R$, corresponds to gauging the (d-1)-form sym with discrete torsion χ_R [Sharpe ’19].
- Expectation: $\mathcal{T}_R$’s have $d-(d-1)-2=(-1)$-form symmetry; described for orbifolds in [Vandermeulen ’22].

Universes and (-1)-form symmetries

- Inserting $\Pi_R = \sum_k \chi_R(k)\, W_k$ projects onto universe labelled by $\chi_R \equiv R$, corresponds to gauging the (d-1)-form sym with discrete torsion χ_R [Sharpe '19].
- Expectation: $\mathcal{T}_R$'s have $d-(d-1)-2=(-1)$-form symmetry; described for orbifolds in [Vandermeulen '22].
- Equivalently: gauging (-1)-form symmetry = "summing" over $\mathcal{T}_R$.

Universes and (-1)-form symmetries

- Inserting $\Pi_R = \sum_k \chi_R(k)\, W_k$ projects onto universe labelled by $\chi_R \equiv R$, corresponds to gauging the (d-1)-form sym with discrete torsion χ_R [Sharpe '19].
- Expectation: $\mathcal{T}_R$'s have $d - (d-1) - 2 = (-1)$-form symmetry; described for orbifolds in [Vandermeulen '22].
- Equivalently: gauging (-1)-form symmetry = "summing" over $\mathcal{T}_R$.
- (-1)-form symmetry "acts on labels (couplings [Codorva/Freed/Lam/Seiberg '19]) R".

Universes and (-1)-form symmetries

- Inserting $\Pi_R = \sum_k \chi_R(k)\, W_k$ projects onto universe labelled by $\chi_R \equiv R$, corresponds to gauging the (d-1)-form sym with discrete torsion χ_R [Sharpe '19].
- Expectation: $\mathscr{T}_R$'s have $d-(d-1)-2=(-1)$-form symmetry; described for orbifolds in [Vandermeulen '22].
- Equivalently: gauging (-1)-form symmetry = "summing" over $\mathscr{T}_R$.
- (-1)-form symmetry "acts on labels (couplings [Codorva/Freed/Lam/Seiberg '19]) R".

➡ As quantum-dual pairs of symmetry, should have natural description in Symmetry TFT (SymTFT).

Lightning review of SymTFT

[see F. Benini's talk]

Lightning review of SymTFT

[see F. Benini's talk]

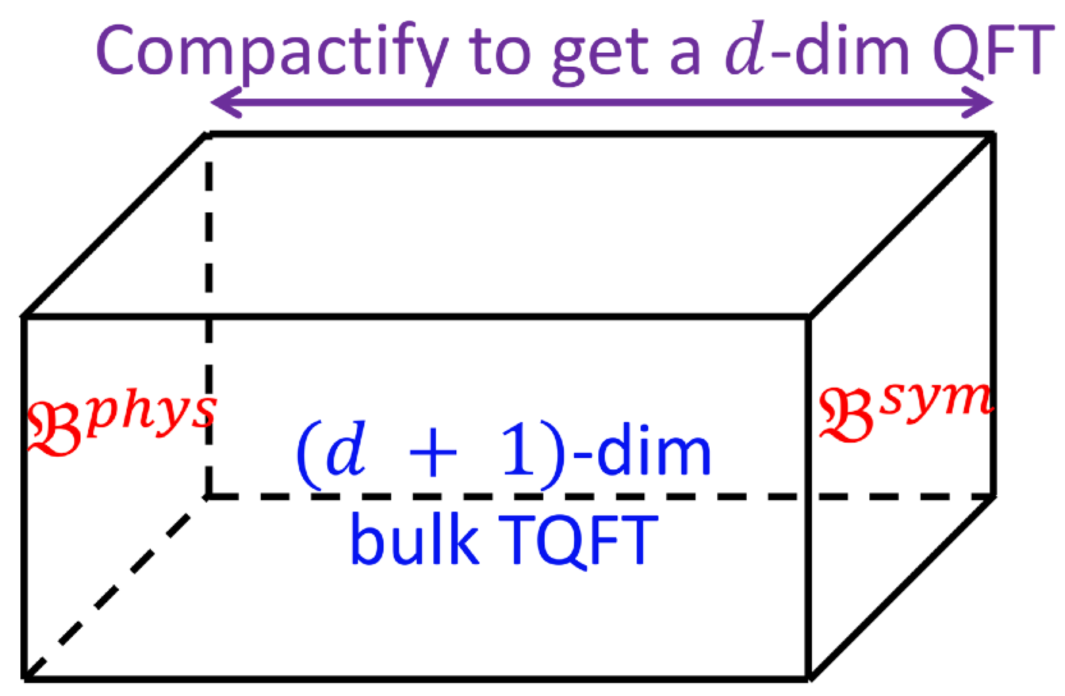
- General idea : separate gapless d.o.f. from topological ones, i.e., symmetry generators [Apruzzi et al '21, Freed/Moore/Teleman '22].

Lightning review of SymTFT

[see F. Benini's talk]

- General idea : separate gapless d.o.f. from topological ones, i.e., symmetry generators [Apruzzi et al '21, Freed/Moore/Teleman '22].

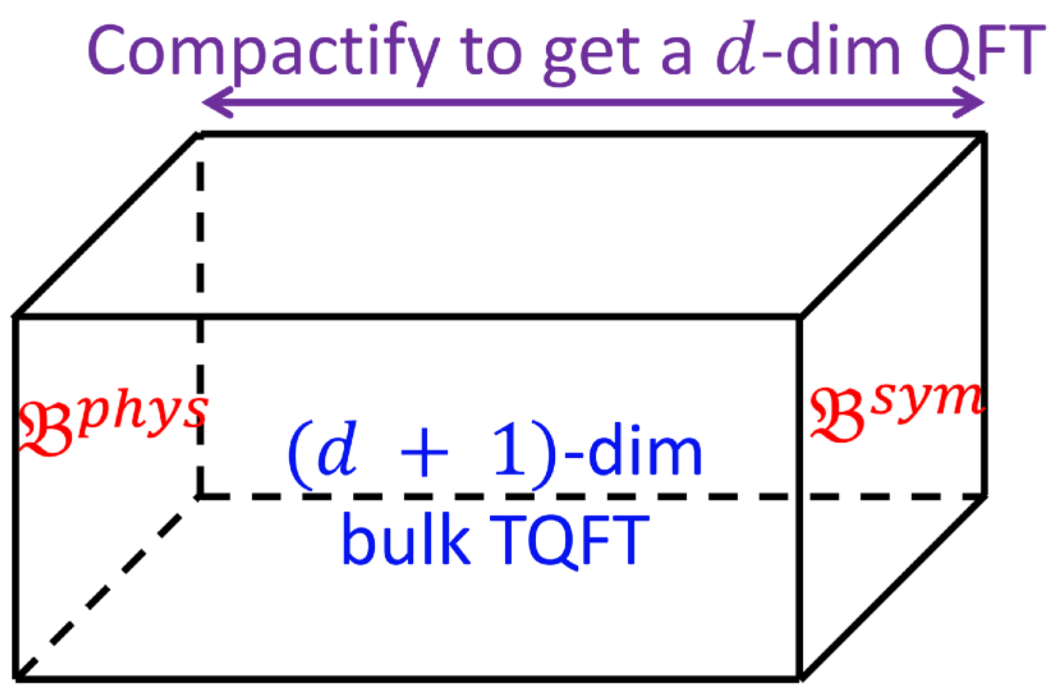
- Topological operators live in (d+1)-dim bulk TQFT;
  imposing boundary conditions at $\mathfrak{B}^{\rm sym}$ + reducing interval
  → QFT with specified global symmetry and charged operators



Lightning review of SymTFT

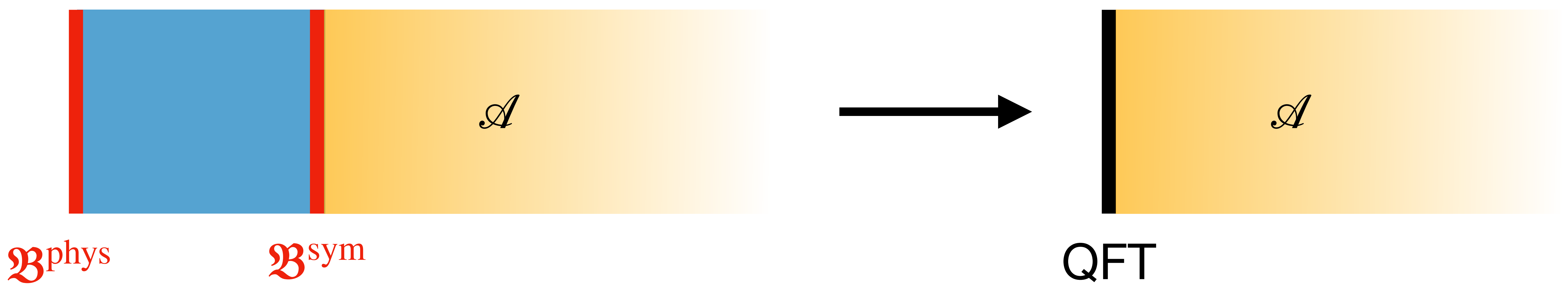
[see F. Benini's talk]

- General idea : separate gapless d.o.f. from topological ones, i.e., symmetry generators [Apruzzi et al '21, Freed/Moore/Teleman '22].



- Topological operators live in (d+1)-dim bulk TQFT; imposing boundary conditions at $\mathfrak{B}^{\text{sym}}$ + reducing interval → QFT with specified global symmetry and charged operators

- Relationship to anomaly theory [Freed '14]: if $\mathfrak{B}^{\text{sym}}$ is a gapped interface, then "RHS" is anomaly theory; without $\mathfrak{B}^{\text{sym}}$: $\mathfrak{B}^{\text{phys}}$ is "relative" theory.



Boundary conditions of $\mathbb{Z}_N$-SymTFT

Boundary conditions of $\mathbb{Z}_N$-SymTFT

- Bulk: $S_{\text{bulk}} = i\int_{d+1} \frac{N}{2\pi} db_{p+1} \wedge c_{d-p-1}$, operators $W(\ell_{p+1}) = \exp(i\oint_\ell b_{p+1})$ and $V(s_{d-p-1}) = \exp(i\oint_s c_{d-p-1})$ with $W(\ell)V(s) = \exp\left(\frac{2\pi i}{N}\ell \circ s\right) V(s)W(\ell)$.

Boundary conditions of $\mathbb{Z}_N$-SymTFT

- Bulk: $S_{\text{bulk}} = i\int_{d+1} \frac{N}{2\pi} db_{p+1} \wedge c_{d-p-1}$, operators $W(\ell_{p+1}) = \exp(i\oint_{\ell} b_{p+1})$ and $V(s_{d-p-1}) = \exp(i\oint_{s} c_{d-p-1})$ with $W(\ell)V(s) = \exp\left(\frac{2\pi i}{N}\ell \circ s\right) V(s)W(\ell)$.

- Dirichlet b.c. for c_{d-p-1} / V: $S_{\text{bulk}+\partial} = \frac{iN}{2\pi}\int_{d+1} db \wedge c + \frac{iN}{2\pi}\int_{\partial \equiv \mathfrak{B}} b \wedge (\hat{c} + d\lambda)$

Boundary conditions of $\mathbb{Z}_N$-SymTFT

- Bulk: $S_{\text{bulk}} = i\int_{d+1} \frac{N}{2\pi} db_{p+1} \wedge c_{d-p-1}$, operators $W(\ell_{p+1}) = \exp(i\oint_\ell b_{p+1})$ and $V(s_{d-p-1}) = \exp(i\oint_s c_{d-p-1})$ with $W(\ell)V(s) = \exp\left(\frac{2\pi i}{N}\ell \circ s\right) V(s)W(\ell)$.

- Dirichlet b.c. for c_{d-p-1} / V: $S_{\text{bulk}+\partial} = \frac{iN}{2\pi}\int_{d+1} db \wedge c + \frac{iN}{2\pi}\int_{\partial \equiv \mathfrak{B}} b \wedge (\hat{c} + d\lambda)$
    - e.o.m.'s $\Rightarrow c\,|_{\mathfrak{B}} = \hat{c} + (-1)^{\#} d\lambda$,
      $\lambda \equiv \lambda_{d-p-2}$ dynamical (topological) "Stückelberg field" on boundary.

Boundary conditions of $\mathbb{Z}_N$-SymTFT

- Bulk: $S_{\text{bulk}} = i\int_{d+1} \frac{N}{2\pi} db_{p+1} \wedge c_{d-p-1}$, operators $W(\ell_{p+1}) = \exp(i\oint_\ell b_{p+1})$ and $V(s_{d-p-1}) = \exp(i\oint_s c_{d-p-1})$ with $W(\ell)V(s) = \exp\left(\frac{2\pi i}{N}\ell \circ s\right) V(s)W(\ell)$.
- Dirichlet b.c. for c_{d-p-1} / V: $S_{\text{bulk}+\partial} = \frac{iN}{2\pi}\int_{d+1} db \wedge c + \frac{iN}{2\pi}\int_{\partial \equiv \mathfrak{B}} b \wedge (\hat{c} + d\lambda)$
  - e.o.m.'s $\Rightarrow c\,|_{\mathfrak{B}} = \hat{c} + (-1)^{\#} d\lambda$,
    $\lambda \equiv \lambda_{d-p-2}$ dynamical (topological) "Stückelberg field" on boundary.
  - After interval reduction, λ_{d-p-2} dynamical $\mathbb{Z}_N$ gauge field in d-dim. theory $\rightarrow \exp(i\oint\lambda)$ "charged object" of $(d-p-2)$-form symmetry.

Boundary conditions of $\mathbb{Z}_N$-SymTFT

- Bulk: $S_{\text{bulk}} = i\int_{d+1} \frac{N}{2\pi} db_{p+1} \wedge c_{d-p-1}$, operators $W(\ell_{p+1}) = \exp(i\oint_\ell b_{p+1})$ and $V(s_{d-p-1}) = \exp(i\oint_s c_{d-p-1})$ with $W(\ell)V(s) = \exp\left(\frac{2\pi i}{N}\ell \circ s\right) V(s)W(\ell)$.
- Dirichlet b.c. for c_{d-p-1} / V: $S_{\text{bulk}+\partial} = \frac{iN}{2\pi}\int_{d+1} db \wedge c + \frac{iN}{2\pi}\int_{\partial\equiv\mathfrak{B}} b \wedge (\hat{c} + d\lambda)$
    - e.o.m.'s $\Rightarrow c\,|_{\mathfrak{B}} = \hat{c} + (-1)^{\#} d\lambda$,
      $\lambda \equiv \lambda_{d-p-2}$ dynamical (topological) "Stückelberg field" on boundary.
    - After interval reduction, λ_{d-p-2} dynamical $\mathbb{Z}_N$ gauge field in d-dim. theory $\rightarrow \exp(i\oint\lambda)$ "charged object" of $(d-p-2)$-form symmetry.
- B.c.'s are states $|\mathfrak{B}_{\text{sym}}; \hat{c}\rangle$ of SymTFT; analogous description of $|\mathfrak{B}_{\text{sym}}; \hat{b}\rangle$.

Changing boundary conditions

b and c are canonically conjugate variables of the (d+1)-dim SymTFT
[Freed/Moore/Teleman '22, Kaidi/Ohmori/Zheng '22]

- $|\mathfrak{B}^{\text{sym}}; \hat{b}_{p+1}\rangle \propto \sum_c e^{i\int_{\mathfrak{B}} \hat{b}\wedge c} |\mathfrak{B}^{\text{sym}}; c\rangle$ and $|\mathfrak{B}^{\text{sym}}; \hat{c}_{d-p-1}\rangle \propto \sum_b e^{i\int_{\mathfrak{B}} b\wedge \hat{c}} |\mathfrak{B}^{\text{sym}}; b\rangle$

Changing boundary conditions

b and c are canonically conjugate variables of the (d+1)-dim SymTFT
[Freed/Moore/Teleman '22, Kaidi/Ohmori/Zheng '22]

- $|\mathfrak{B}^{\text{sym}}; \hat{b}_{p+1}\rangle \propto \sum_c e^{i\int_{\mathfrak{B}} \hat{b}\wedge c} |\mathfrak{B}^{\text{sym}}; c\rangle$ and $|\mathfrak{B}^{\text{sym}}; \hat{c}_{d-p-1}\rangle \propto \sum_b e^{i\int_{\mathfrak{B}} b\wedge \hat{c}} |\mathfrak{B}^{\text{sym}}; b\rangle$
- $Z^{(p)}[\hat{b}] = \langle \mathfrak{B}^{\text{phys}} | \mathfrak{B}^{\text{sym}}; \hat{b}\rangle$

Changing boundary conditions

b and c are canonically conjugate variables of the (d+1)-dim SymTFT
[Freed/Moore/Teleman '22, Kaidi/Ohmori/Zheng '22]

- $|\mathfrak{B}^{\text{sym}}; \hat{b}_{p+1}\rangle \propto \sum_c e^{i\int_{\mathfrak{B}} \hat{b}\wedge c} |\mathfrak{B}^{\text{sym}}; c\rangle$ and $|\mathfrak{B}^{\text{sym}}; \hat{c}_{d-p-1}\rangle \propto \sum_b e^{i\int_{\mathfrak{B}} b\wedge \hat{c}} |\mathfrak{B}^{\text{sym}}; b\rangle$
- $Z^{(p)}[\hat{b}] = \langle \mathfrak{B}^{\text{phys}} | \mathfrak{B}^{\text{sym}}; \hat{b}\rangle \propto \sum_c e^{i\int_{\mathfrak{B}} \hat{b}\wedge c} Z^{(d-p-2)}[c] = \sum_\gamma e^{i\int_\gamma \hat{b}} \int \mathscr{D}[\phi, b] e^{-S[\phi,b;c]}$

$(\gamma \equiv \gamma_{p+1} = \text{PD}^{(d)}(c))$

Changing boundary conditions

b and c are canonically conjugate variables of the (d+1)-dim SymTFT
[Freed/Moore/Teleman '22, Kaidi/Ohmori/Zheng '22]

- $|\mathfrak{B}^{\text{sym}}; \hat{b}_{p+1}\rangle \propto \sum_c e^{i\int_{\mathfrak{B}} \hat{b}\wedge c} |\mathfrak{B}^{\text{sym}}; c\rangle$ and $|\mathfrak{B}^{\text{sym}}; \hat{c}_{d-p-1}\rangle \propto \sum_b e^{i\int_{\mathfrak{B}} b\wedge \hat{c}} |\mathfrak{B}^{\text{sym}}; b\rangle$
- $Z^{(p)}[\hat{b}] = \langle \mathfrak{B}^{\text{phys}} | \mathfrak{B}^{\text{sym}}; \hat{b}\rangle \propto \sum_c e^{i\int_{\mathfrak{B}} \hat{b}\wedge c} Z^{(d-p-2)}[c] = \sum_\gamma e^{i\int_\gamma \hat{b}} \int \mathscr{D}[\phi, b] e^{-S[\phi,b;c]}$

[(d-p-2)-form sym-transf.]

$(\gamma \equiv \gamma_{p+1} = \text{PD}^{(d)}(c))$

Changing boundary conditions

b and c are canonically conjugate variables of the (d+1)-dim SymTFT
[Freed/Moore/Teleman '22, Kaidi/Ohmori/Zheng '22]

- $|\mathfrak{B}^{\text{sym}};\hat{b}_{p+1}\rangle \propto \sum_c e^{i\int_{\mathfrak{B}}\hat{b}\wedge c}|\mathfrak{B}^{\text{sym}};c\rangle$ and $|\mathfrak{B}^{\text{sym}};\hat{c}_{d-p-1}\rangle \propto \sum_b e^{i\int_{\mathfrak{B}}b\wedge\hat{c}}|\mathfrak{B}^{\text{sym}};b\rangle$
- $Z^{(p)}[\hat{b}] = \langle\mathfrak{B}^{\text{phys}}|\mathfrak{B}^{\text{sym}};\hat{b}\rangle \propto \sum_c e^{i\int_{\mathfrak{B}}\hat{b}\wedge c} Z^{(d-p-2)}[c] = \sum_\gamma e^{i\int_\gamma \hat{b}} \int \mathscr{D}[\phi,b] e^{-S[\phi,b;c]}$

[(d-p-2)-form sym-transf.] $\quad = \sum_\gamma e^{i\int_\gamma \hat{b}} \int \mathscr{D}\phi\, W(\gamma)^{-1}\, e^{-S[\phi;c\equiv 0]} \qquad (\gamma \equiv \gamma_{p+1} = \text{PD}^{(d)}(c))$

Changing boundary conditions

b and c are canonically conjugate variables of the (d+1)-dim SymTFT
[Freed/Moore/Teleman '22, Kaidi/Ohmori/Zheng '22]

- $|\mathfrak{B}^{\text{sym}}; \hat{b}_{p+1}\rangle \propto \sum_c e^{i\int_{\mathfrak{B}} \hat{b}\wedge c} |\mathfrak{B}^{\text{sym}}; c\rangle$ and $|\mathfrak{B}^{\text{sym}}; \hat{c}_{d-p-1}\rangle \propto \sum_b e^{i\int_{\mathfrak{B}} b\wedge \hat{c}} |\mathfrak{B}^{\text{sym}}; b\rangle$
- $Z^{(p)}[\hat{b}] = \langle \mathfrak{B}^{\text{phys}} | \mathfrak{B}^{\text{sym}}; \hat{b}\rangle \propto \sum_c e^{i\int_{\mathfrak{B}} \hat{b}\wedge c} Z^{(d-p-2)}[c] = \sum_\gamma e^{i\int_\gamma \hat{b}} \int \mathscr{D}[\phi, b] e^{-S[\phi,b;c]}$

[(d-p-2)-form sym-transf.] $= \sum_\gamma e^{i\int_\gamma \hat{b}} \int \mathscr{D}\phi \, W(\gamma)^{-1} \, e^{-S[\phi;c\equiv 0]} \qquad (\gamma \equiv \gamma_{p+1} = \text{PD}^{(d)}(c))$

$= \langle \sum_\gamma e^{i\int_\gamma \hat{b}} W(\gamma)^{-1} \rangle_{(d-p-2)}$

Changing boundary conditions

b and c are canonically conjugate variables of the (d+1)-dim SymTFT
[Freed/Moore/Teleman '22, Kaidi/Ohmori/Zheng '22]

- $|\mathfrak{B}^{\text{sym}}; \hat{b}_{p+1}\rangle \propto \sum_c e^{i\int_{\mathfrak{B}} \hat{b}\wedge c} |\mathfrak{B}^{\text{sym}}; c\rangle$ and $|\mathfrak{B}^{\text{sym}}; \hat{c}_{d-p-1}\rangle \propto \sum_b e^{i\int_{\mathfrak{B}} b\wedge \hat{c}} |\mathfrak{B}^{\text{sym}}; b\rangle$
- $Z^{(p)}[\hat{b}] = \langle \mathfrak{B}^{\text{phys}} | \mathfrak{B}^{\text{sym}}; \hat{b}\rangle \propto \sum_c e^{i\int_{\mathfrak{B}} \hat{b}\wedge c} Z^{(d-p-2)}[c] = \sum_\gamma e^{i\int_\gamma \hat{b}} \int \mathscr{D}[\phi, b] e^{-S[\phi,b;c]}$

[(d-p-2)-form sym-transf.] $\quad = \sum_\gamma e^{i\int_\gamma \hat{b}} \int \mathscr{D}\phi\, W(\gamma)^{-1}\, e^{-S[\phi;c\equiv 0]} \qquad (\gamma \equiv \gamma_{p+1} = \text{PD}^{(d)}(c))$

$$= \langle \sum_\gamma e^{i\int_\gamma \hat{b}} W(\gamma)^{-1}\rangle_{(d-p-2)}$$

➡projector $\sum_\gamma e^{i\int_\gamma \hat{b}} W(\gamma)^{-1} = \sum_\gamma e^{i\int_\gamma (\hat{b}-b)} \equiv \delta_{b,\hat{b}}$ on codim $\geq p+3$ operators.

Features for $p = -1$

- Dirichlet b.c. $b_0|_{\mathfrak{B}} = \hat{b}_0$ has no $d\lambda_{-1}$, no charged objects; "just" parameter.

Features for $p = -1$

- Dirichlet b.c. $b_0|_{\mathfrak{B}} = \hat{b}_0$ has no $d\lambda_{-1}$, no charged objects; "just" parameter.
- $\gamma_{p+1} = \nu \times \mathrm{pt} \in H_0(\mathfrak{B}; \mathbb{Z}_N) = \mathbb{Z}_N \;\; \Rightarrow \;\; \sum_\gamma e^{i\int_\gamma (\hat{b}-b)} = \sum_{\nu=0}^{N-1} e^{i\nu\hat{b}} e^{-i\nu b} \equiv \sum_\nu \chi_{\hat{b}}(\nu) W^\nu$

Features for $p = -1$

- Dirichlet b.c. $b_0|_{\mathfrak{B}} = \hat{b}_0$ has no $d\lambda_{-1}$, no charged objects; "just" parameter.
- $\gamma_{p+1} = \nu \times \mathrm{pt} \in H_0(\mathfrak{B}; \mathbb{Z}_N) = \mathbb{Z}_N \ \Rightarrow\ \sum_\gamma e^{i\int_\gamma (\hat{b}-b)} = \sum_{\nu=0}^{N-1} e^{i\nu\hat{b}} e^{-i\nu b} \equiv \sum_\nu \chi_{\hat{b}}(\nu) W^\nu$
- Easy generalization to $U(1)^{(-1)}/\mathbb{Z}^{(d-1)}$ symmetries with $S_{\text{bulk}} = \frac{i}{2\pi}\int_{d+1} db_0 \wedge c_d$, with $\oint dc_d = 0$ [Brennan/Sun, Antinucci/Benini, Apruzzi/Bedogna/Dondi, '24]

Features for $p = -1$

- Dirichlet b.c. $b_0|_{\mathfrak{B}} = \hat{b}_0$ has no $d\lambda_{-1}$, no charged objects; "just" parameter.
- $\gamma_{p+1} = \nu \times \mathrm{pt} \in H_0(\mathfrak{B}; \mathbb{Z}_N) = \mathbb{Z}_N \;\Rightarrow\; \sum_\gamma e^{i\int_\gamma (\hat{b}-b)} = \sum_{\nu=0}^{N-1} e^{i\nu\hat{b}} e^{-i\nu b} \equiv \sum_\nu \chi_{\hat{b}}(\nu) W^\nu$
- Easy generalization to $U(1)^{(-1)}/\mathbb{Z}^{(d-1)}$ symmetries with $S_{\mathrm{bulk}} = \frac{i}{2\pi}\int_{d+1} db_0 \wedge c_d$, with $\oint dc_d = 0$ [Brennan/Sun, Antinucci/Benini, Apruzzi/Bedogna/Dondi, '24]
    - $|\mathfrak{B}^{\mathrm{sym}}; \hat{c}_d\rangle : S_\partial = \frac{i}{2\pi}\int_{\mathfrak{B}} b_0 \wedge (dr_{d-1} - \hat{c}_d)$

Features for $p = -1$

- Dirichlet b.c. $b_0|_{\mathfrak{B}} = \hat{b}_0$ has no $d\lambda_{-1}$, no charged objects; “just” parameter.
- $\gamma_{p+1} = \nu \times \mathrm{pt} \in H_0(\mathfrak{B}; \mathbb{Z}_N) = \mathbb{Z}_N \;\Rightarrow\; \sum_\gamma e^{i\int_\gamma (\hat{b}-b)} = \sum_{\nu=0}^{N-1} e^{i\nu\hat{b}} e^{-i\nu b} \equiv \sum_\nu \chi_{\hat{b}}(\nu) W^\nu$
- Easy generalization to $U(1)^{(-1)}/\mathbb{Z}^{(d-1)}$ symmetries with $S_{\text{bulk}} = \frac{i}{2\pi}\int_{d+1} db_0 \wedge c_d$, with $\oint dc_d = 0$ [Brennan/Sun, Antinucci/Benini, Apruzzi/Bedogna/Dondi, ’24]
  - $|\mathfrak{B}^{\text{sym}}; \hat{c}_d\rangle \,:\, S_\partial = \frac{i}{2\pi}\int_{\mathfrak{B}} b_0 \wedge (dr_{d-1} - \hat{c}_d)$
  - $|\mathfrak{B}^{\text{sym}}; \hat{b}_0\rangle \,:\, S_\partial = \frac{i}{2\pi}\int_{\mathfrak{B}} \hat{b}_0 \wedge c_d$

Example 1: Instanton-restriction in 4d Yang—Mills

$\theta \to \theta + b$ of 4d YM is $U(1)^{(-1)}$ symmetry [Codorva/Freed/Lam/Seiberg '19].

Example 1: Instanton-restriction in 4d Yang—Mills

$\theta \to \theta + b$ of 4d YM is $U(1)^{(-1)}$ symmetry [Codorva/Freed/Lam/Seiberg '19].

- $S_{\mathrm{Sym}} = S_{\mathrm{YM}}^{(\theta)}|_{\mathfrak{B}^{\mathrm{phys}}} + \int_{\mathfrak{B}^{\mathrm{phys}}} b\mathrm{Tr}(F^2) + \frac{i}{2\pi}\int_{\mathrm{5d}} b \wedge dc_4 + S_{\mathfrak{B}^{\mathrm{sym}}} \rightsquigarrow S_{\mathrm{4d}} = S_{\mathrm{YM}}^{(\theta)} + S_{\mathrm{top}}$

Example 1: Instanton-restriction in 4d Yang—Mills

$\theta \to \theta + b$ of 4d YM is $U(1)^{(-1)}$ symmetry [Codorva/Freed/Lam/Seiberg '19].

- $S_{\text{Sym}} = S_{\text{YM}}^{(\theta)} |_{\mathfrak{B}^{\text{phys}}} + \int_{\mathfrak{B}^{\text{phys}}} b \text{Tr}(F^2) + \frac{i}{2\pi} \int_{5\text{d}} b \wedge dc_4 + S_{\mathfrak{B}^{\text{sym}}} \rightsquigarrow S_{4\text{d}} = S_{\text{YM}}^{(\theta)} + S_{\text{top}}$
- "Standard" YM: $S_{\mathfrak{B}^{\text{sym}}} = \frac{i}{2\pi} \int \hat{b} \wedge c_4 \rightsquigarrow S_{4\text{d}} = S_{\text{YM}} + \int \hat{b} \text{Tr}(F^2)$

Example 1: Instanton-restriction in 4d Yang—Mills

$\theta \to \theta + b$ of 4d YM is $U(1)^{(-1)}$ symmetry [Codorva/Freed/Lam/Seiberg '19].

- $S_{\mathrm{Sym}} = S^{(\theta)}_{\mathrm{YM}}|_{\mathfrak{B}^{\mathrm{phys}}} + \int_{\mathfrak{B}^{\mathrm{phys}}} b\,\mathrm{Tr}(F^2) + \frac{i}{2\pi}\int_{5\mathrm{d}} b \wedge dc_4 + S_{\mathfrak{B}^{\mathrm{sym}}} \rightsquigarrow S_{4\mathrm{d}} = S^{(\theta)}_{\mathrm{YM}} + S_{\mathrm{top}}$
- "Standard" YM: $S_{\mathfrak{B}^{\mathrm{sym}}} = \frac{i}{2\pi}\int \hat{b} \wedge c_4 \rightsquigarrow S_{4\mathrm{d}} = S_{\mathrm{YM}} + \int \hat{b}\,\mathrm{Tr}(F^2)$
- Gauging $\mathbb{Z}_N^{(-1)} \subset U(1)^{(-1)}$ by $S_{\mathrm{top}}[b, \lambda_3] = i\int b\,[\mathrm{Tr}(F^2) - \frac{N}{2\pi}(d\lambda_3 + \hat{c}_4)]$
  $\Rightarrow \frac{Nb}{2\pi} \in \mathbb{Z}$, and $\int \mathrm{Tr}(F^2) = \int \frac{N}{2\pi}(d\lambda_3 + \hat{c}_4) \equiv \int \frac{N}{2\pi}\hat{c}_4 \mod N$ [Tanizaki/Ünsal '19].

Example 1: Instanton-restriction in 4d Yang—Mills

$\theta \to \theta + b$ of 4d YM is $U(1)^{(-1)}$ symmetry [Codorva/Freed/Lam/Seiberg '19].

- $S_{\text{Sym}} = S^{(\theta)}_{\text{YM}}|_{\mathfrak{B}^{\text{phys}}} + \int_{\mathfrak{B}^{\text{phys}}} b\text{Tr}(F^2) + \frac{i}{2\pi}\int_{5\text{d}} b \wedge dc_4 + S_{\mathfrak{B}^{\text{sym}}} \rightsquigarrow S_{4\text{d}} = S^{(\theta)}_{\text{YM}} + S_{\text{top}}$
- "Standard" YM: $S_{\mathfrak{B}^{\text{sym}}} = \frac{i}{2\pi}\int \hat{b} \wedge c_4 \rightsquigarrow S_{4\text{d}} = S_{\text{YM}} + \int \hat{b}\text{Tr}(F^2)$
- Gauging $\mathbb{Z}_N^{(-1)} \subset U(1)^{(-1)}$ by $S_{\text{top}}[b, \lambda_3] = i\int b\,[\text{Tr}(F^2) - \frac{N}{2\pi}(d\lambda_3 + \hat{c}_4)]$
  $\Rightarrow \frac{Nb}{2\pi} \in \mathbb{Z}$, and $\int \text{Tr}(F^2) = \int \frac{N}{2\pi}(d\lambda_3 + \hat{c}_4) \equiv \int \frac{N}{2\pi}\hat{c}_4 \mod N$ [Tanizaki/Ünsal '19].
  - Project back: $S_{\text{top}}[b, \lambda_3, c_4] = i\int b\,[\text{Tr}(F^2) - \frac{N}{2\pi}(d\lambda_3 + c_4)] + \int \frac{N\hat{b}}{2\pi} c_4$, and path-integrate over $c_4 \Rightarrow b = \hat{b}$ and $\int \text{Tr}(F^2) = \int \frac{N}{2\pi}(d\lambda_3 + c_4)$ can be any integer.

Example 1: Instanton-restriction in 4d Yang—Mills

$\theta \to \theta + b$ of 4d YM is $U(1)^{(-1)}$ symmetry [Codorva/Freed/Lam/Seiberg '19].

- $S_{\text{Sym}} = S^{(\theta)}_{\text{YM}}|_{\mathfrak{B}^{\text{phys}}} + \int_{\mathfrak{B}^{\text{phys}}} b\text{Tr}(F^2) + \frac{i}{2\pi}\int_{5\text{d}} b \wedge dc_4 + S_{\mathfrak{B}^{\text{sym}}} \rightsquigarrow S_{4\text{d}} = S^{(\theta)}_{\text{YM}} + S_{\text{top}}$
- "Standard" YM: $S_{\mathfrak{B}^{\text{sym}}} = \frac{i}{2\pi}\int \hat{b} \wedge c_4 \rightsquigarrow S_{4\text{d}} = S_{\text{YM}} + \int \hat{b}\text{Tr}(F^2)$
- Gauge full $U(1)^{(-1)}$ (flat) by $S_{\text{top}}[b, r_3] = \frac{i}{2\pi}\int b\,(\text{Tr}(F^2) - (dr_3 + \hat{c}_4))$
  $\Rightarrow \int \text{Tr}(F^2) = \int (dr_3 + \hat{c}_4) = \int \hat{c}_4$, so no instanton sum at all!

Example 1: Instanton-restriction in 4d Yang—Mills

$\theta \to \theta + b$ of 4d YM is $U(1)^{(-1)}$ symmetry [Codorva/Freed/Lam/Seiberg '19].

- $S_{\text{Sym}} = S^{(\theta)}_{\text{YM}}|_{\mathfrak{B}^{\text{phys}}} + \int_{\mathfrak{B}^{\text{phys}}} b\text{Tr}(F^2) + \frac{i}{2\pi}\int_{5\text{d}} b \wedge dc_4 + S_{\mathfrak{B}^{\text{sym}}} \rightsquigarrow S_{4\text{d}} = S^{(\theta)}_{\text{YM}} + S_{\text{top}}$
- "Standard" YM: $S_{\mathfrak{B}^{\text{sym}}} = \frac{i}{2\pi}\int \hat{b} \wedge c_4 \rightsquigarrow S_{4\text{d}} = S_{\text{YM}} + \int \hat{b}\text{Tr}(F^2)$
- Gauge full $U(1)^{(-1)}$ (flat) by $S_{\text{top}}[b, r_3] = \frac{i}{2\pi}\int b\,(\text{Tr}(F^2) - (dr_3 + \hat{c}_4))$
  $\Rightarrow \int \text{Tr}(F^2) = \int (dr_3 + \hat{c}_4) = \int \hat{c}_4$, so no instanton sum at all!
- Conveniently describes mixed anomaly / 4-group with 1-form symmetry:
  $S_{\text{bulk}} \supset \frac{i\alpha}{2\pi}\int db \wedge B_2 \wedge B_2$, incorporates / generalizes findings of [Tanizaki/Ünsal '19]

Example 2: dimensional reduction

Example 2: dimensional reduction

Reduce 4d theory with $U(1)_F^{(0)} \times U(1)^{(1)}$ on S^2 [Nardoni/Sacchi/Sela/Zafrir/Zheng '24]

Example 2: dimensional reduction

Reduce 4d theory with $U(1)_F^{(0)} \times U(1)^{(1)}$ on S^2 [Nardoni/Sacchi/Sela/Zafrir/Zheng '24]

- $S_{\text{bulk}} = \int_{5\text{d}} \left(\frac{1}{2\pi} c_3 \wedge da_1 + \frac{1}{2\pi} f_2 \wedge dc_2 + \frac{K}{8\pi^2} a_1 \wedge f_2{}^2\right)$ (e.g., 4d SQCD + gauged $U(1)_B^{(0)}$)

Example 2: dimensional reduction

Reduce 4d theory with $U(1)_F^{(0)} \times U(1)^{(1)}$ on S^2 [Nardoni/Sacchi/Sela/Zafrir/Zheng '24]

- $S_{\text{bulk}} = \int_{5\text{d}} \left(\frac{1}{2\pi} c_3 \wedge d a_1 + \frac{1}{2\pi} f_2 \wedge d c_2 + \frac{K}{8\pi^2} a_1 \wedge {f_2}^2 \right)$ (e.g., 4d SQCD + gauged $U(1)_B^{(0)}$)
  if $K \neq 0$ (ABJ-anom.) $U(1)_F^{(0)} \to \mathbb{Z}_K \boxplus (\mathbb{Q}/\mathbb{Z})_K{}^{\text{n.i.}}$ [Cordova/Ohmori, Choi/Lam/Shao '22]

Example 2: dimensional reduction

Reduce 4d theory with $U(1)^{(0)}_F \times U(1)^{(1)}$ on S^2 [Nardoni/Sacchi/Sela/Zafrir/Zheng '24]

- $S_{\text{bulk}} = \int_{5\text{d}} \left(\frac{1}{2\pi} c_3 \wedge da_1 + \frac{1}{2\pi} f_2 \wedge dc_2 + \frac{K}{8\pi^2} a_1 \wedge f_2{}^2 \right)$ (e.g., 4d SQCD + gauged $U(1)^{(0)}_B$)
  if $K \neq 0$ (ABJ-anom.) $U(1)^{(0)}_F \to \mathbb{Z}_K \boxplus (\mathbb{Q}/\mathbb{Z})_K{}^{\text{n.i.}}$ [Cordova/Ohmori, Choi/Lam/Shao '22]

- On S^2: $c_3 = c_1 \wedge \text{vol}_{S^2}$, $c_2 = \bar{c}_2 \wedge 1 + \varphi \wedge \text{vol}_{S^2}$, $f_2 = \bar{f}_2 \wedge 1 + b \wedge \text{vol}_{S^2}$

Example 2: dimensional reduction

Reduce 4d theory with $U(1)_F^{(0)} \times U(1)^{(1)}$ on S^2 [Nardoni/Sacchi/Sela/Zafrir/Zheng '24]

- $S_{\text{bulk}} = \int_{5\text{d}} \left(\frac{1}{2\pi} c_3 \wedge d a_1 + \frac{1}{2\pi} f_2 \wedge d c_2 + \frac{K}{8\pi^2} a_1 \wedge {f_2}^2\right)$ (e.g., 4d SQCD + gauged $U(1)_B^{(0)}$)
  if $K \neq 0$ (ABJ-anom.) $U(1)_F^{(0)} \to \mathbb{Z}_K \boxplus (\mathbb{Q}/\mathbb{Z})_K{}^{\text{n.i.}}$ [Cordova/Ohmori, Choi/Lam/Shao '22]

- On S^2: $c_3 = c_1 \wedge \text{vol}_{S^2}$, $c_2 = \bar{c}_2 \wedge 1 + \varphi \wedge \text{vol}_{S^2}$, $f_2 = \bar{f}_2 \wedge 1 + b \wedge \text{vol}_{S^2}$
  $\to \bar{S}_{\text{bulk}} = \int_{3\text{d}} \left(\frac{1}{2\pi} c_1 \wedge d a_1 + \frac{1}{2\pi} b \wedge d\bar{c}_2 + \frac{1}{2\pi} \bar{f}_2 \wedge d\varphi + \frac{K}{8\pi^2} b \wedge a_1 \wedge \bar{f}_2\right)$

Example 2: dimensional reduction

Reduce 4d theory with $U(1)_F^{(0)} \times U(1)^{(1)}$ on S^2 [Nardoni/Sacchi/Sela/Zafrir/Zheng '24]

- $S_{\text{bulk}} = \int_{5\text{d}} \left(\frac{1}{2\pi} c_3 \wedge da_1 + \frac{1}{2\pi} f_2 \wedge dc_2 + \frac{K}{8\pi^2} a_1 \wedge f_2{}^2 \right)$ (e.g., 4d SQCD + gauged $U(1)_B^{(0)}$)
  if $K \neq 0$ (ABJ-anom.) $U(1)_F^{(0)} \to \mathbb{Z}_K \boxplus (\mathbb{Q}/\mathbb{Z})_K{}^{\text{n.i.}}$ [Cordova/Ohmori, Choi/Lam/Shao '22]

- On S^2: $c_3 = c_1 \wedge \text{vol}_{S^2}$, $c_2 = \bar{c}_2 \wedge 1 + \varphi \wedge \text{vol}_{S^2}$, $f_2 = \bar{f}_2 \wedge 1 + b \wedge \text{vol}_{S^2}$
  $\to \bar{S}_{\text{bulk}} = \int_{3\text{d}} \left(\frac{1}{2\pi} c_1 \wedge da_1 + \frac{1}{2\pi} b \wedge d\bar{c}_2 + \frac{1}{2\pi} \bar{f}_2 \wedge d\varphi + \frac{K}{8\pi^2} b \wedge a_1 \wedge \bar{f}_2 \right)$

- $b = \hat{b} \in \mathbb{Z}$ [Nardoni et al '24]: $\mathbb{Z}_{\hat{b}K}^{(0)} \times U(1)^{(1)}$ global symmetry in 2d

Example 2: dimensional reduction

Reduce 4d theory with $U(1)_F^{(0)} \times U(1)^{(1)}$ on S^2 [Nardoni/Sacchi/Sela/Zafrir/Zheng '24]

- $S_{\text{bulk}} = \int_{5\text{d}} \left(\frac{1}{2\pi} c_3 \wedge da_1 + \frac{1}{2\pi} f_2 \wedge dc_2 + \frac{K}{8\pi^2} a_1 \wedge f_2{}^2 \right)$ (e.g., 4d SQCD + gauged $U(1)_B^{(0)}$)
  if $K \neq 0$ (ABJ-anom.) $U(1)_F^{(0)} \to \mathbb{Z}_K \boxplus (\mathbb{Q}/\mathbb{Z})_K{}^{\text{n.i.}}$ [Cordova/Ohmori, Choi/Lam/Shao '22]

- On S^2: $c_3 = c_1 \wedge \text{vol}_{S^2}$, $c_2 = \bar{c}_2 \wedge 1 + \varphi \wedge \text{vol}_{S^2}$, $f_2 = \bar{f}_2 \wedge 1 + b \wedge \text{vol}_{S^2}$
  $\to \bar{S}_{\text{bulk}} = \int_{3\text{d}} \left(\frac{1}{2\pi} c_1 \wedge da_1 + \frac{1}{2\pi} b \wedge d\bar{c}_2 + \frac{1}{2\pi} \bar{f}_2 \wedge d\varphi + \frac{K}{8\pi^2} b \wedge a_1 \wedge \bar{f}_2 \right)$

- $b = \hat{b} \in \mathbb{Z}$ [Nardoni et al '24]: $\mathbb{Z}_{\hat{b}K}^{(0)} \times U(1)^{(1)}$ global symmetry in 2d

- What about shifts of b? $\rightsquigarrow$ non-invertible 0-form in 2d?

Summary & Outlook

- SymTFT naturally incorporates (-1)-form symmetries and decomposition with all bells and whistles.

  - May enhance symmetry category (4d $\rightarrow$ 2d example).

  - Dynamical gauging of (-1)-form gives axions $\Rightarrow$ new approach to Strong-CP-problem? [see also Aloni/García-Valdecasas/Reece/Suzuki '24]

  - Topological point operators omnipresent in geometric engineering of string theory, but not studied so far…

Summary & Outlook

- SymTFT naturally incorporates (-1)-form symmetries and decomposition with all bells and whistles.
  - May enhance symmetry category (4d → 2d example).
  - Dynamical gauging of (-1)-form gives axions ⇒ new approach to Strong-CP-problem? [see also Aloni/García-Valdecasas/Reece/Suzuki ’24]
  - Topological point operators omnipresent in geometric engineering of string theory, but not studied so far…

Thank you!