Gauge theory meets cosmology

Naples, TFI 2024

Based on:

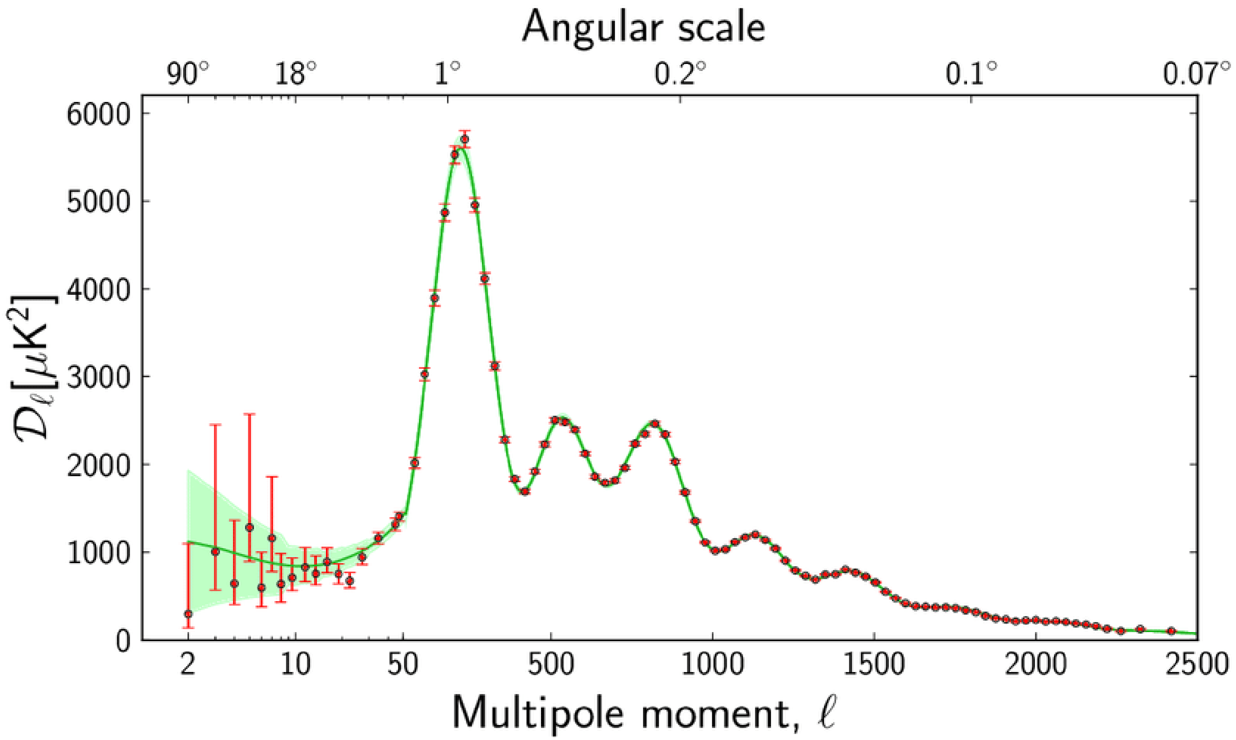
2408.03243 [hep-th]

with M. Bianchi & J.F. Morales

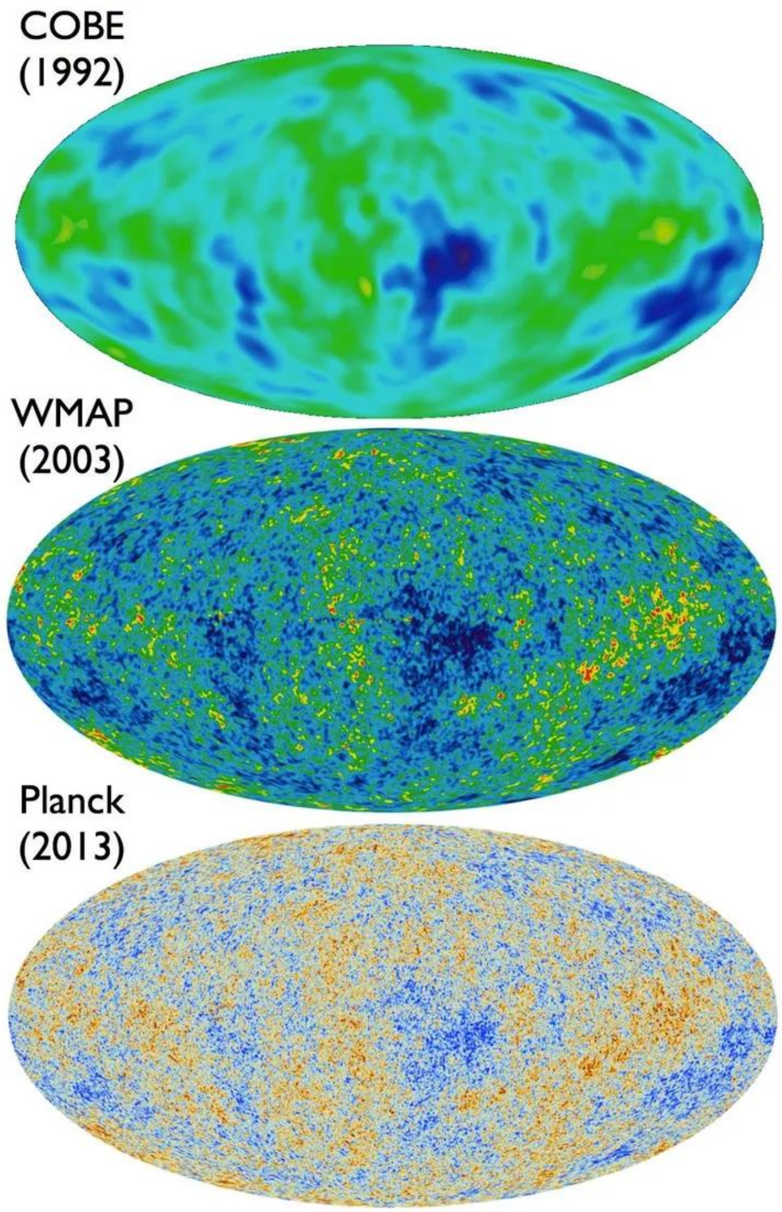
Outline

- Introduction & Motivation
- Review of FLRW cosmologies
- Linearized perturbations of FLRW universes
- Cosmological perturbations within ΛCDM
- Seiberg-Witten/Cosmology correspondence
- Concluding Remarks

The Early Universe…



CMB



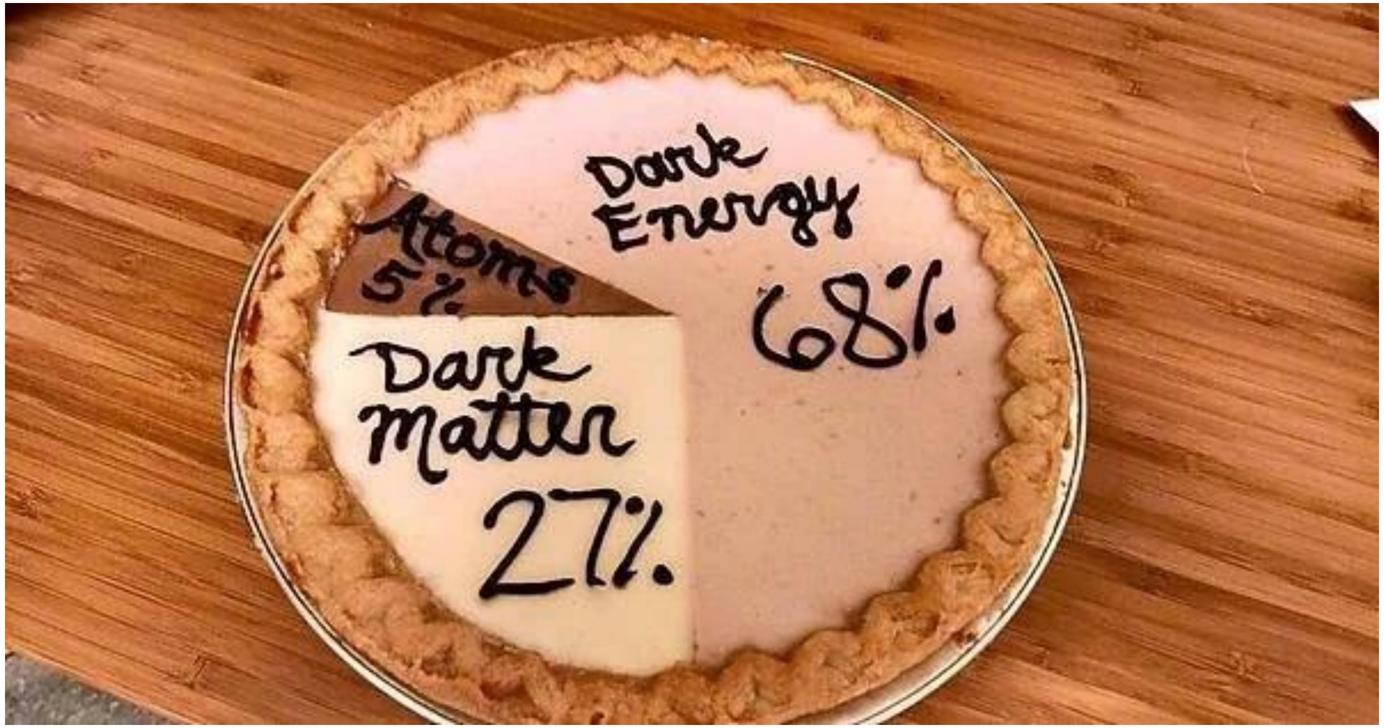
The Late Universe…

BAO

+

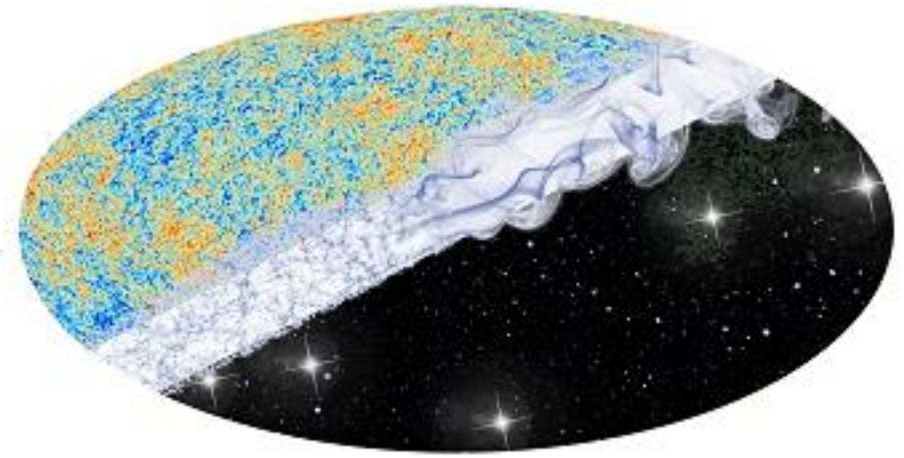
SN Ia

→

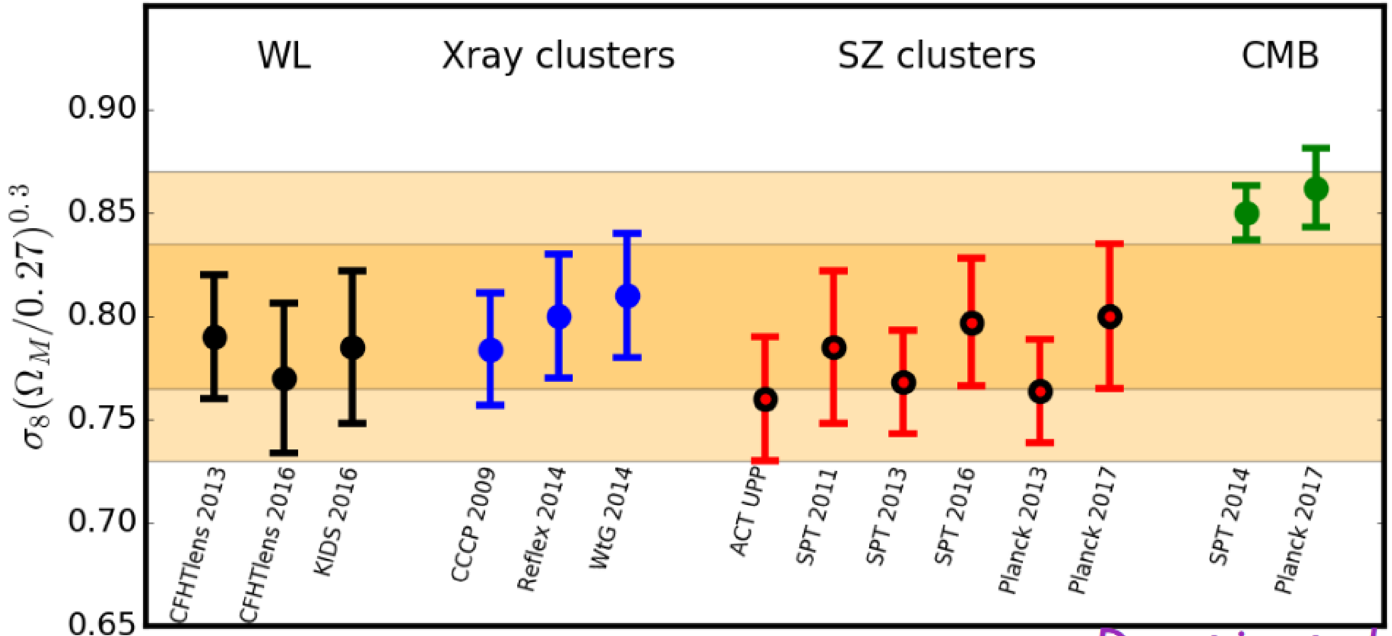


ΛCDM model → The «cheapest» description BUT…

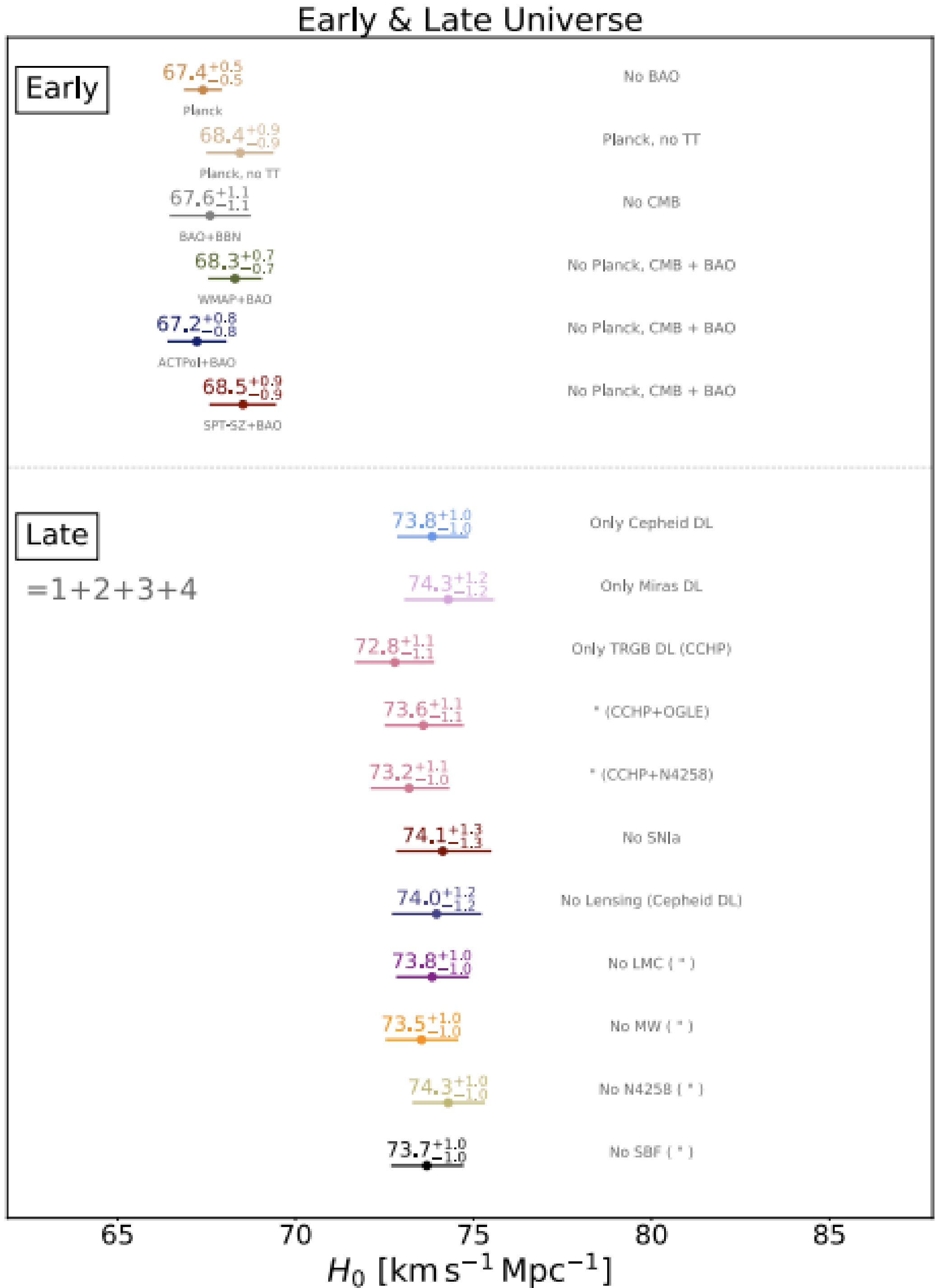
Early/Late Universe puzzles appear…



[image credit: Krzysztof Bolejko]



Douspis et al.



Why cosmological perturbations?

A HARD question because… →

- Different sorts of perturbations are **intrinsically coupled** and analytic treatments are beyond reach…

A RELEVANT question because… →

- The **spectrum** of cosmological perturbations crucially depends on the details of a model!

- It may shed a light on **refined models for the early & late universe** beyond ΛCDM

Novel «semi-analytic» treatments? → Insights from String Theory tools?

Today:

Motivate & introduce the **SW/cosmology** dictionary

FLRW cosmologies revisited

$$ds_4^2 = -dt^2 + a(t)^2 ds_{\mathcal{M}_3}^2,$$

$\mathbb{R}^3$, S^3 or $\mathbb{H}^3$.

For perfect ISOTROPIC fluids…

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + p\, g_{\mu\nu}$$

$$\mathrm{diag}(\rho, p, p, p)$$

$$p = w\rho$$

Eqn of state

Einstein equations…

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = M_{\rm Pl}^{-2} T_{\mu\nu}$$

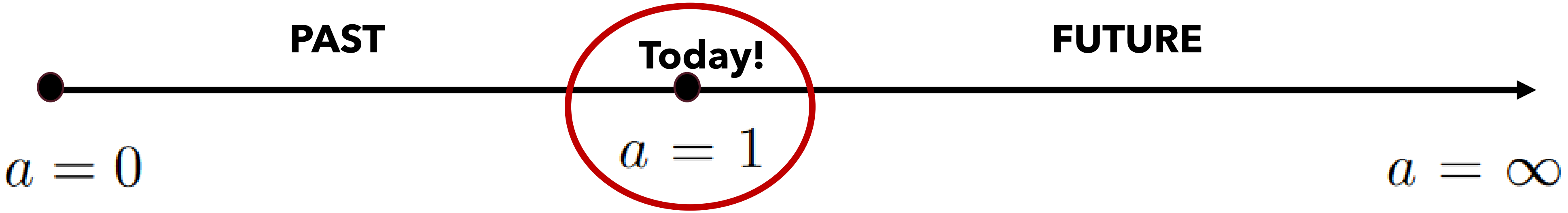
$$a(t) \sim t^{\frac{2}{3(1+w)}}$$

A «new» time variable...

$$ds_4^2 = -\frac{da^2}{a^2 b(a)^2} + a^2\, ds^2_{\mathcal{M}_3},$$

$$dt = \frac{da}{ab(a)}$$

Use time reparametrization invariance!!!



In this way we get **simpler** equations to solve...

Perfect fluid dynamics

$$\rho = 3(M_{\rm Pl}\,H_0)^2\,a^{-n} \quad , \quad p = 3w\,(M_{\rm Pl}\,H_0)^2\,a^{-n} \quad , \quad b^2 = H_0^2\,a^{-n} \quad , \quad n = 3(1+w)$$

Fluid Type	Symbol	w	n	$b(a)$	$\eta(a)$
Vacuum	Λ	-1	0	1	$-1/a$
Strings, Curvature	σ, κ	$-\frac{1}{3}$	2	a^{-1}	$\log(a)$
Matter	m	0	3	$a^{-3/2}$	$2a^{1/2}$
Radiation	γ	$\frac{1}{3}$	4	a^{-2}	a
Stiff	s	1	6	a^{-3}	$a^2/2$

For the «full» universe we have…

$$b^2 = H_0^2\left(\Omega_\gamma\, a^{-4} + \Omega_{\rm m}\, a^{-3} + \Omega_\Lambda + \Omega_\kappa\, a^{-2}\right)$$

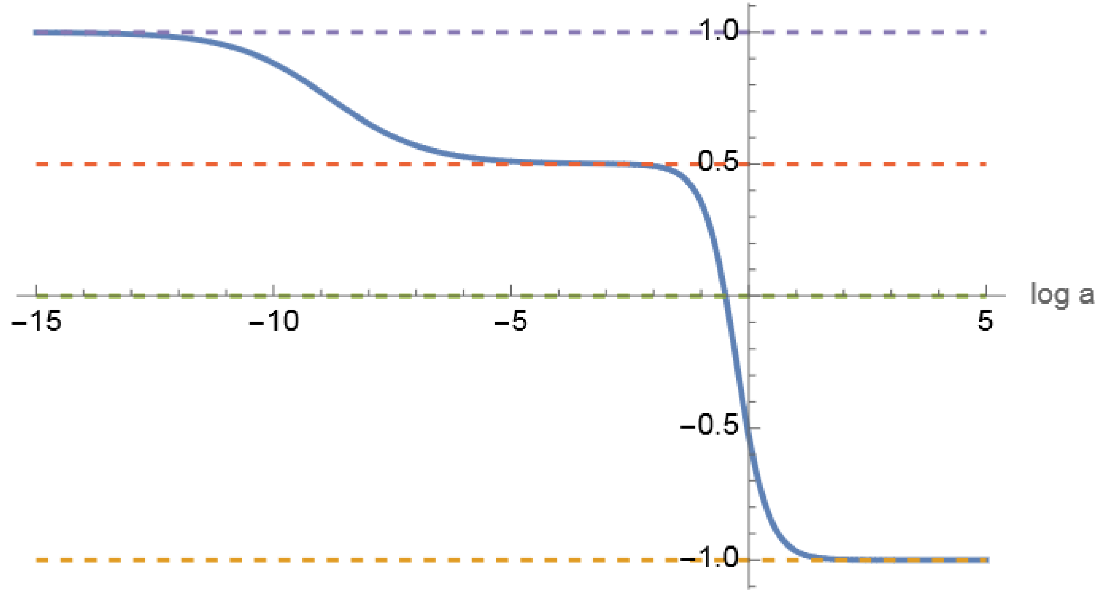
$$\rho(a) = 3(M_{\rm Pl}\, H_0)^2\, \left(\Omega_\Lambda + \Omega_{\rm m} a^{-3} + \Omega_\gamma a^{-4}\right)\,,$$

$$p(a) = 3(M_{\rm Pl}\, H_0)^2\, \left(-\Omega_\Lambda + \tfrac{1}{3}\Omega_\gamma a^{-4}\right)$$

$$q(a) \equiv -\frac{\ddot{a}a}{\dot{a}^2}$$

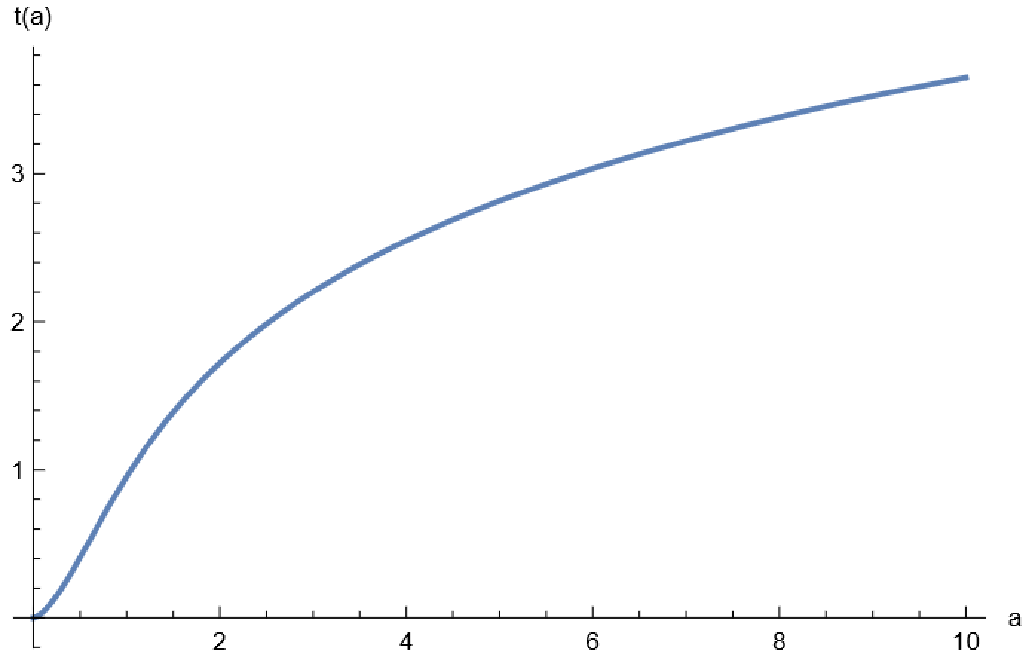
→ **Deceleration parameter**

$$= \frac{\Omega_\gamma a^{-4} + \frac{1}{2}\Omega_{\rm m} a^{-3} - \Omega_\Lambda}{\Omega_\gamma a^{-4} + \Omega_{\rm m} a^{-3} - \Omega_\kappa a^{-2} + \Omega_\Lambda}$$



What's the (cosmic) time?

$$b(a) = \frac{\dot{a}}{a} = H(a)$$



$$H_0 t = \int \frac{da}{\sqrt{\Omega_\Lambda a^2 + \Omega_\kappa + \Omega_\mathrm{m} a^{-1} + \Omega_\gamma a^{-2}}} = \int \frac{a\,da}{\sqrt{\Omega_\Lambda a^4 + \Omega_\kappa a^2 + \Omega_\mathrm{m} a + \Omega_\gamma}}$$

Review of cosmological perturbations

[Mukhanov]

Perturbed metric:

$$g_{\mu\nu} = \underbrace{g^{(0)}_{\mu\nu}}_{\text{hom. + iso.}} + \delta g_{\mu\nu}$$

Small perturbations

Gauge invariant scalar perturbations may be parametrized as …

$$ds_4^2|_{\mathrm{S}} \stackrel{\mathrm{L}}{=} -(1+2\Phi(a,\mathbf{x}))\,\frac{da^2}{a^2\,b(a)^2} + a^2\,(1-2\Psi(a,\mathbf{x}))\,ds^2_{\mathcal{M}_3}$$

Isotropy!

$$\Psi = \Phi$$

After **Fourier transformation…**

$$\Phi(a,\mathbf{x}) = e^{i\mathbf{kx}}\Phi(a) \quad \Rightarrow \quad \Delta\Phi = -k^2\Phi$$

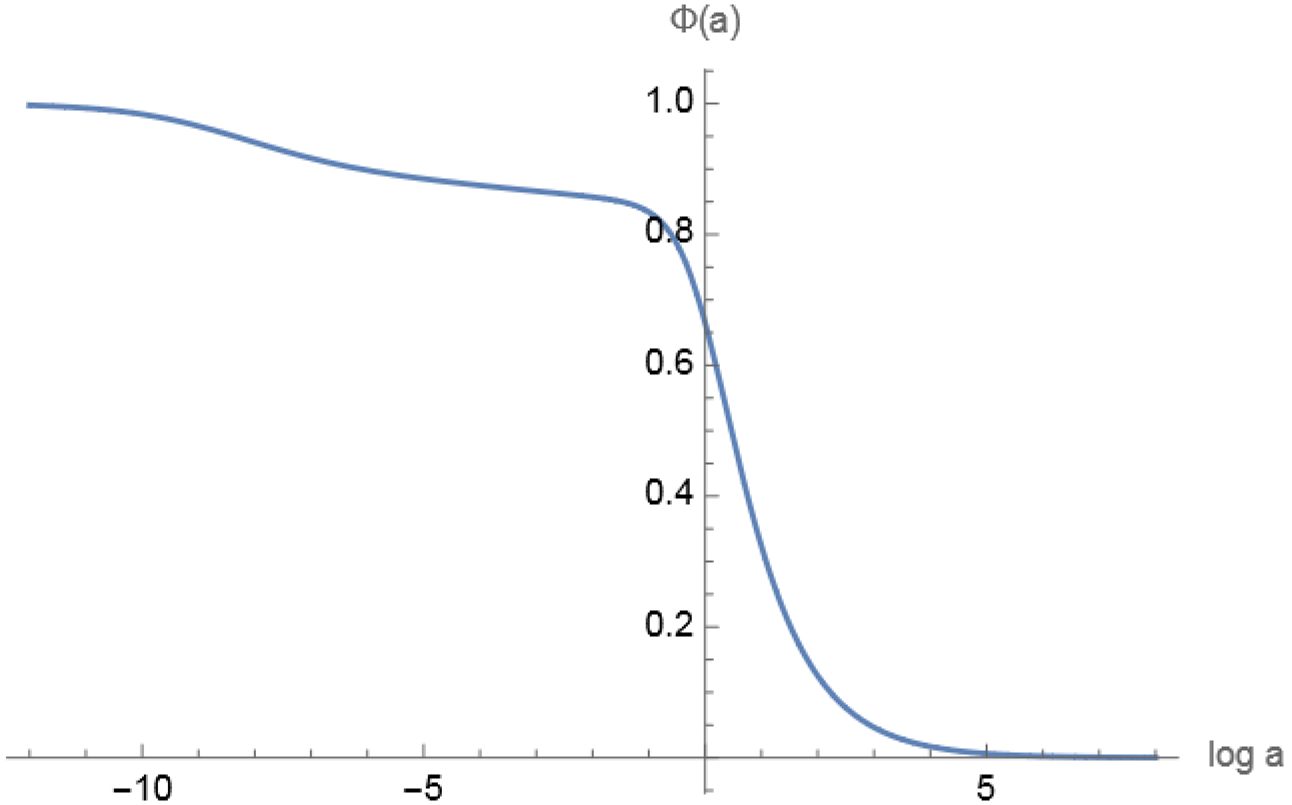
ODE for time evolution!

After taking ODE into its **canonical form…**

$$\Psi''(a) + Q(a)\Psi(a) = 0$$

Rational function of time!

Related to Φ

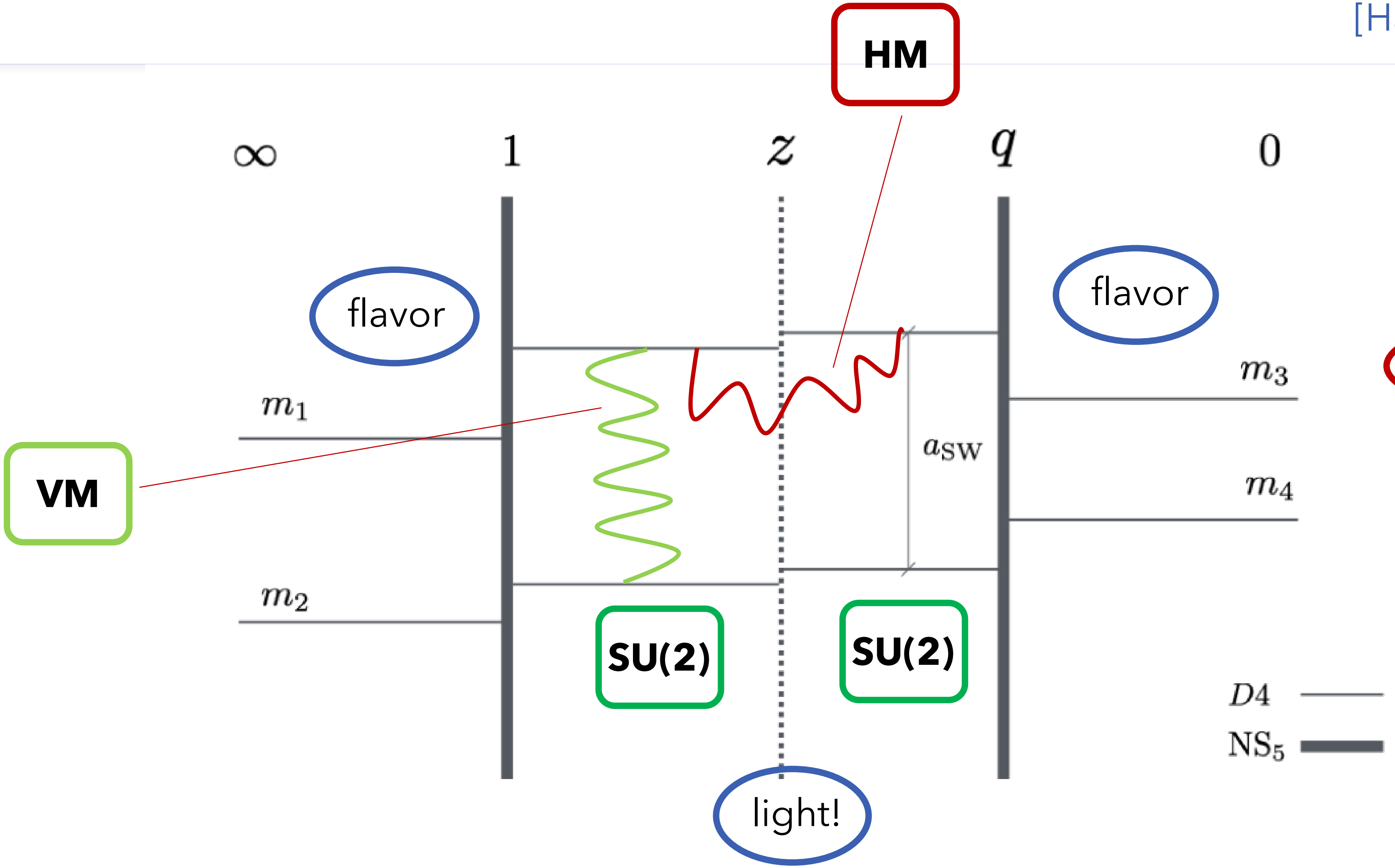


N	Type	Components
3	Hypergeometric	$\Lambda\ \kappa,\ \Lambda\ \mathrm{m},\ \ \kappa\ \mathrm{m},\ \kappa\ \gamma$
4	Heun	$\Lambda\ \gamma,\ \ \mathrm{m}\gamma,\ \ \Lambda\ \kappa\ \gamma$
5	Gen. Heun	$\kappa\ \mathrm{m}\ \Lambda,\ \ \kappa\ \mathrm{m}\ \gamma$
7	Gen. Heun	$\Lambda\ \mathrm{m}\ \gamma,\ \ \Lambda\ \kappa\ \mathrm{m}\ \gamma$

Numerical solution for **radiation** & **matter** (Heun eqn)

SW/cosmology correspondence

[Hanany, Witten 1996]



$$q_i = z_{i+1}/z_i$$

$$q = e^{2\pi i \tau}$$

Complexified coupling

From classical to quantum geometry…

$$(x,z) \in \mathbb{C}^2 : \qquad P_0(x) z^2 - P_1(x) z + q P_2(x) = 0$$

(Classical SW curve)

Quadratic fct's of $m_i \quad q$

«Quantum» $x = -z\partial_z$

$$\left[P_0(-z\partial_z + \tfrac{1}{2}) - P_1(-z\partial_z) z^{-1} + q P_2(-z\partial_z - \tfrac{1}{2}) z^{-2}\right] W(z) = 0$$

with

$$W(z) = z^{1-\frac{m_3+m_4}{2}} (1-z)^{-\frac{m_1+m_2+1}{2}} (z-q)^{\frac{m_3+m_4-1}{2}} \Psi(z)$$

A complete example: radiation & matter

The associated **Schroedinger-like** equation is specified by

$$Q(a) = \frac{64a^2\zeta k^2\left(3a^2+7a\zeta+4\zeta^2\right)-3\left(189a^4+924a^3\zeta+1820a^2\zeta^2+1600a\zeta^3+512\zeta^4\right)}{48a^2(a+\zeta)^2(3a+4\zeta)^2}$$

$$\zeta = \frac{\Omega_\gamma}{\Omega_m}$$

which fits the **SW** induced Heun equation, once the **dictionary** is fixed…

$$z = -\zeta\, a^{-1} \quad , \quad q = \frac{3}{4} \quad , \quad u = \frac{4\hat{k}^2\zeta^2}{3} + \frac{33}{16}$$

$$m_1 = \frac{7}{4} \quad , \quad m_2 = -\frac{5}{4} \quad , \quad m_{3,4} = 1 \pm \tfrac{1}{12}\sqrt{225 - 64\hat{k}^2\zeta}$$

Conclusions & Outlook

- Studying the dynamical evolution of cosmological perturbations is an important challenge for cosmology
- We can tackle the problem by using tools borrowed from SUSY QFT's
- This analysis offers the possibility of a «semi-analytic» treatment (important for parametric control!)
- In our setup it would be interesting to consider more involved scenarios, like inflationary dynamics, early dark energy, quintessence…

→ Hopefully more to come, so stay tuned…

Thank you for your attention!