

Type IIB S-folds: New solutions and consistent truncations

Colin Sterckx

INFN Padova

TFI meeting Napoli

04/09/24

Based on : [2407.11593] Guarino, Rudra, Trigiante & CS
[To appear] Guarino, Rudra, Trigiante & CS

Universal black hole

In M-theory and Type IIA

The universal black hole

[Romans '92]

[Caldarelli, Klemm, '98]

[Romans '92]

[Caldarelli, Klemm, '98]

The universal black hole

$$\mathcal{L}_{N=2} = \left(\frac{R}{2} - \Lambda \right) \star 1 - \frac{1}{2} H \wedge \star H$$

The universal black hole

$$\mathcal{L}_{N=2} = \left(\frac{R}{2} - \Lambda \right) \star 1 - \frac{1}{2} H \wedge \star H$$

$$ds^2 = -f(r) dt^2 + f(r)^{-1} dr^2 + r^2 ds_{\Sigma_g}^2$$

$$f(r) = \left(\frac{r}{L_{AdS_4}} - \frac{L_{AdS_4}}{2r} \right)^2$$

The universal black hole

$$\mathcal{L}_{N=2} = \left(\frac{R}{2} - \Lambda \right) \star 1 - \frac{1}{2} H \wedge \star H$$

$$ds^2 = -f(r) dt^2 + f(r)^{-1} dr^2 + r^2 ds_{\Sigma_g}^2$$

$$f(r) = \left(\frac{r}{L_{AdS_4}} - \frac{L_{AdS_4}}{2r} \right)^2$$

$$H = p \text{vol}_{\Sigma_g}$$

The universal black hole

$$\mathcal{L}_{N=2} = \left(\frac{R}{2} - \Lambda \right) \star 1 - \frac{1}{2} H \wedge \star H$$

$$ds^2 = -f(r) dt^2 + f(r)^{-1} dr^2 + r^2 ds_{\Sigma_g}^2$$

$$f(r) = \left(\frac{r}{L_{AdS_4}} - \frac{L_{AdS_4}}{2r} \right)^2$$

$$H = p \text{vol}_{\Sigma_g}$$

$$s = \frac{A(\Sigma_g)}{4} = L_{AdS_4}^2 \frac{(g-1)\pi}{2}$$

Field theory dual

Deep UV:

AdS_4

CFT_3

Field theory dual

Azzurli, Benini, Bobev,
Crichigno, Hristov, Min,
Zaffaroni,... '15 – '17

Deep UV:

flow



Deep IR

AdS_4

Universal BH

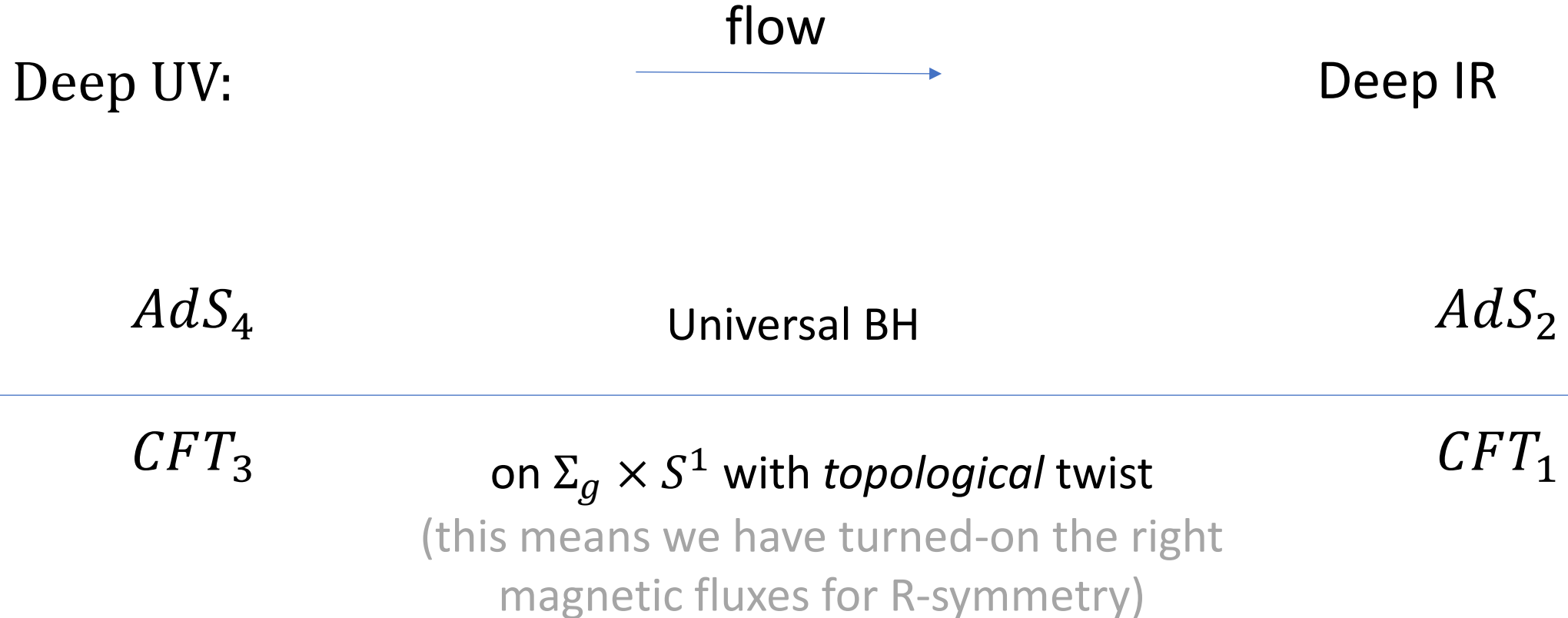
AdS_2

CFT_3

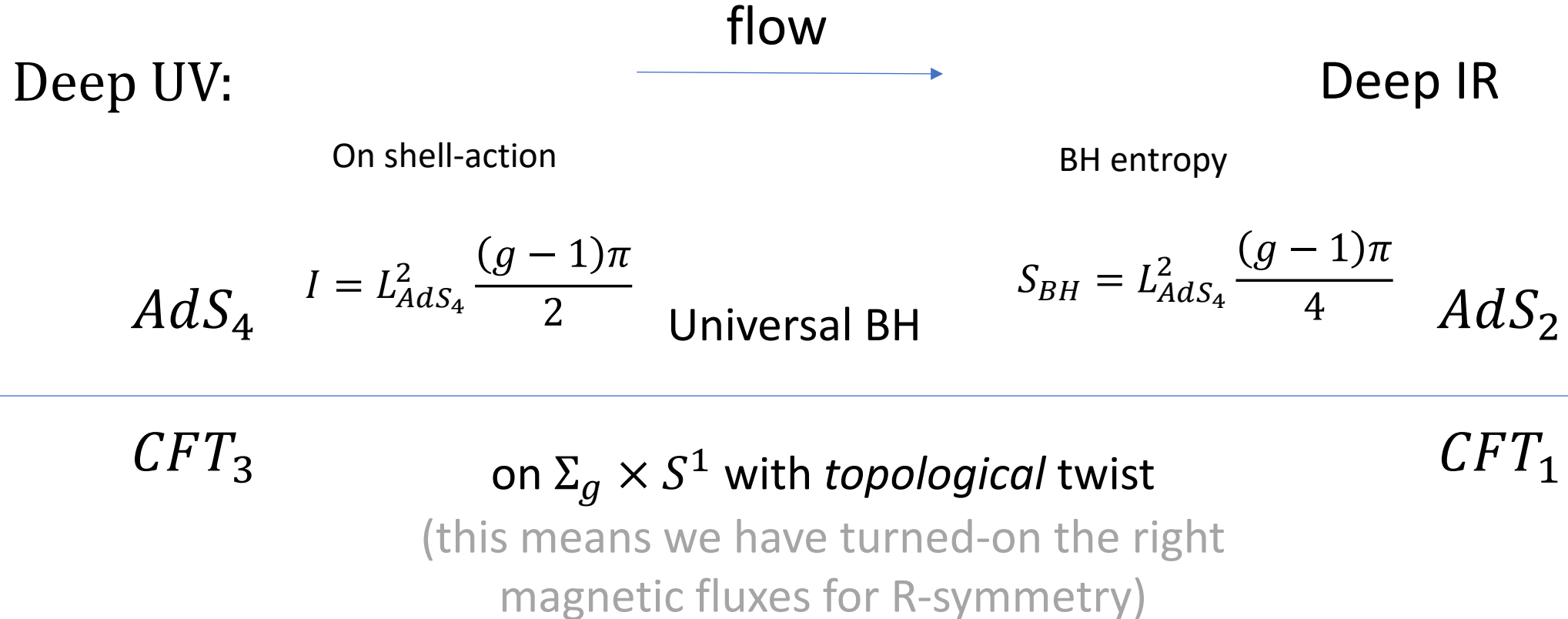
CFT_1

Field theory dual

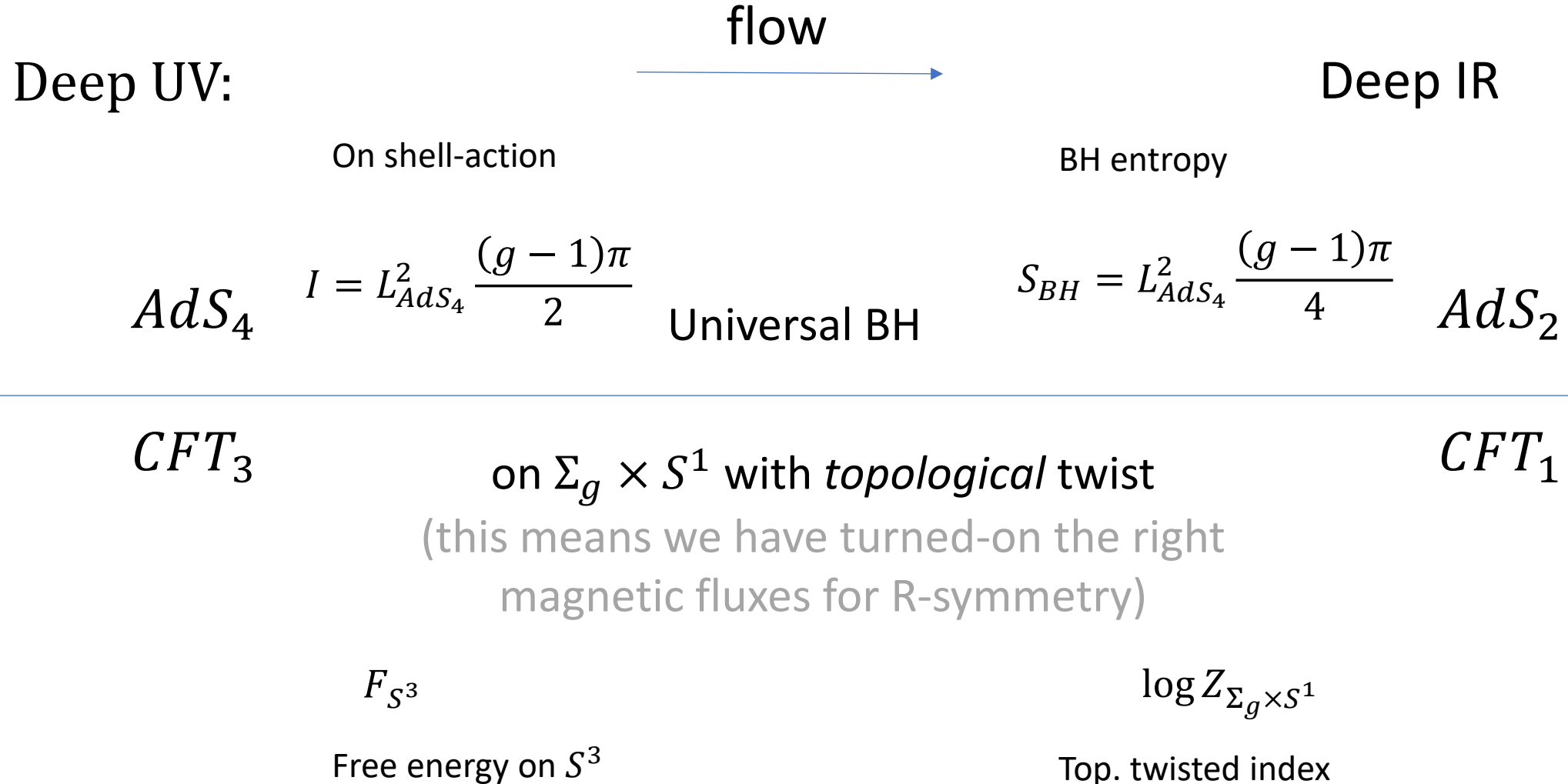
Azzurli, Benini, Bobev,
Crichigno, Hristov, Min,
Zaffaroni,... '15 – '17



Field theory dual



Field theory dual



The universal black-hole embedded

M-theory

Massive type IIA

[Guarino, Varela 15]

[Guarino, Tarrío 17]

[Benin, Hristov, Zaffaroni '15]

[Hosseini, Hristov, Passias17]

[Benini, Khachatryan, Milan 17]

The universal black-hole embedded

M-theory

- S^7 compactification
- $\mathcal{N} = 8$ $SO(8)$ -gauged

[Benin, Hristov, Zaffaroni '15]

Massive type IIA

[Guarino, Varela 15]

[Guarino, Tarrío 17]

[Hosseini, Hristov, Passias17]

[Benini, Khachatryan, Milan 17]

The universal black-hole embedded

M-theory

- S^7 compactification
- $\mathcal{N} = 8$ $SO(8)$ -gauged
- Restrict to Cartan:
 $\mathcal{N} = 2 \quad n_v = 3 \quad n_h = 0$

[Benin, Hristov, Zaffaroni '15]

Massive type IIA

[Guarino, Varela 15]

[Guarino, Tarrío 17]

[Hosseini, Hristov, Passias17]

[Benini, Khachatryan, Milan 17]

The universal black-hole embedded

M-theory

- S^7 compactification
- $\mathcal{N} = 8$ $SO(8)$ -gauged
- Restrict to Cartan:
$$\mathcal{N} = 2 \quad n_v = 3 \quad n_h = 0$$
- 4 charges

[Benin, Hristov, Zaffaroni '15]

Massive type IIA

[Guarino, Varela 15]

[Guarino, Tarrío 17]

[Hosseini, Hristov, Passias17]

[Benini, Khachatryan, Milan 17]

The universal black-hole embedded

M-theory

- S^7 compactification
- $\mathcal{N} = 8$ $SO(8)$ -gauged
- Restrict to Cartan:
$$\mathcal{N} = 2 \quad n_v = 3 \quad n_h = 0$$
- 4 charges
$$\sum p_i = \frac{1}{2}$$
- ABJM on $S^1 \times \Sigma_g$

[Benin, Hristov, Zaffaroni '15]

Massive type IIA

[Guarino, Varela 15]

[Guarino, Tarrio 17]

[Hosseini, Hristov, Passias17]

[Benini, Khachatryan, Milan 17]

The universal black-hole embedded

M-theory

- S^7 compactification
- $\mathcal{N} = 8$ $SO(8)$ -gauged
- Restrict to Cartan:
$$\mathcal{N} = 2 \quad n_v = 3 \quad n_h = 0$$
- 4 charges
$$\sum p_i = \frac{1}{2}$$
- ABJM on $S^1 \times \Sigma_g$

[Benin, Hristov, Zaffaroni '15]

Massive type IIA

[Guarino, Varela 15]

[Guarino, Tarrío 17]

- S^6 compactification
- $\mathcal{N} = 8$ $ISO(7)$ -gauged

[Hosseini, Hristov, Passias17]

[Benini, Khachatryan, Milan 17]

The universal black-hole embedded

M-theory

- S^7 compactification
- $\mathcal{N} = 8$ $SO(8)$ -gauged
- Restrict to Cartan:
$$\mathcal{N} = 2 \quad n_v = 3 \quad n_h = 0$$
- 4 charges
$$\Sigma p_i = \frac{1}{2}$$
- ABJM on $S^1 \times \Sigma_g$

[Benin, Hristov, Zaffaroni '15]

Massive type IIA

[Guarino, Varela 15]

[Guarino, Tarrio 17]

- S^6 compactification
- $\mathcal{N} = 8$ $ISO(7)$ -gauged
- Restrict to $U(1)^2$ -inv
$$\mathcal{N} = 2 \quad n_v = 3 \quad n_h = 1$$

$$U(1)^3 \times \mathbb{R}\text{-gauging}$$

[Hosseini, Hristov, Passias17]

[Benini, Khachatryan, Milan 17]

The universal black-hole embedded

M-theory

- S^7 compactification
- $\mathcal{N} = 8$ $SO(8)$ -gauged
- Restrict to Cartan:
$$\mathcal{N} = 2 \quad n_v = 3 \quad n_h = 0$$
- 4 charges
$$\sum p_i = \frac{1}{2}$$
- ABJM on $S^1 \times \Sigma_g$

[Benin, Hristov, Zaffaroni '15]

Massive type IIA

[Guarino, Varela 15]

[Guarino, Tarrio 17]

- S^6 compactification
- $\mathcal{N} = 8$ $ISO(7)$ -gauged
- Restrict to $U(1)^2$ -inv
$$\mathcal{N} = 2 \quad n_v = 3 \quad n_h = 1$$

$$U(1)^3 \times \mathbb{R}$$
-gauging
- Univ. BH and horizon solutions
- $SU(N)$ CS_k on $S^1 \times \Sigma_g$

[Hosseini, Hristov, Passias17]

[Benini, Khachatryan, Milan 17]

Universal black hole in type IIB

From 10 to 4 dimensions

D= 10 Type IIB SUGRA

Compactification on
 $S^5 \times S^1$
with a $SL(2, \mathbb{Z})$ monodromy
along the S^1



Type IIB S-folds:

D= 10 Type IIB SUGRA

Compactification on
 $S^5 \times S^1$
with a $SL(2, \mathbb{Z})$ monodromy
along the S^1

D=4 N=8 gauged SUGRA

Type IIB S-folds:

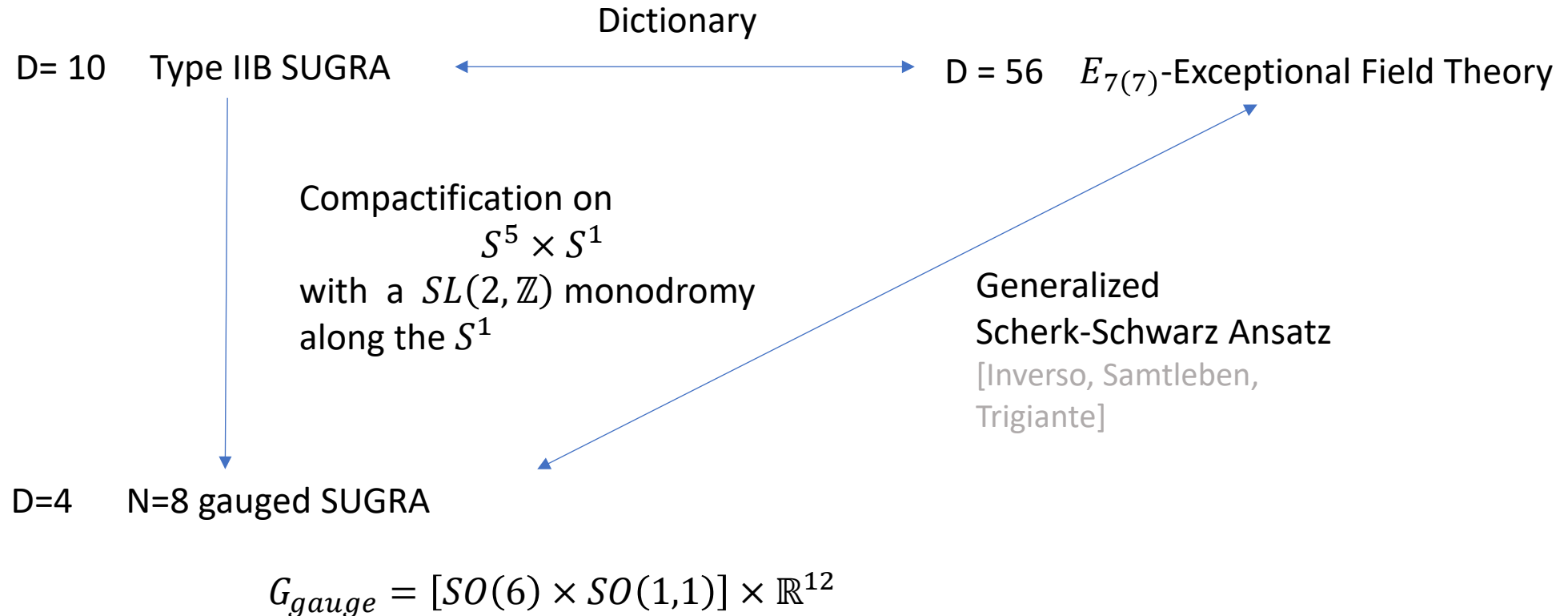
D= 10 Type IIB SUGRA

Compactification on
 $S^5 \times S^1$
with a $SL(2, \mathbb{Z})$ monodromy
along the S^1

D=4 N=8 gauged SUGRA

$$G_{gauge} = [SO(6) \times SO(1,1)] \times \mathbb{R}^{12}$$

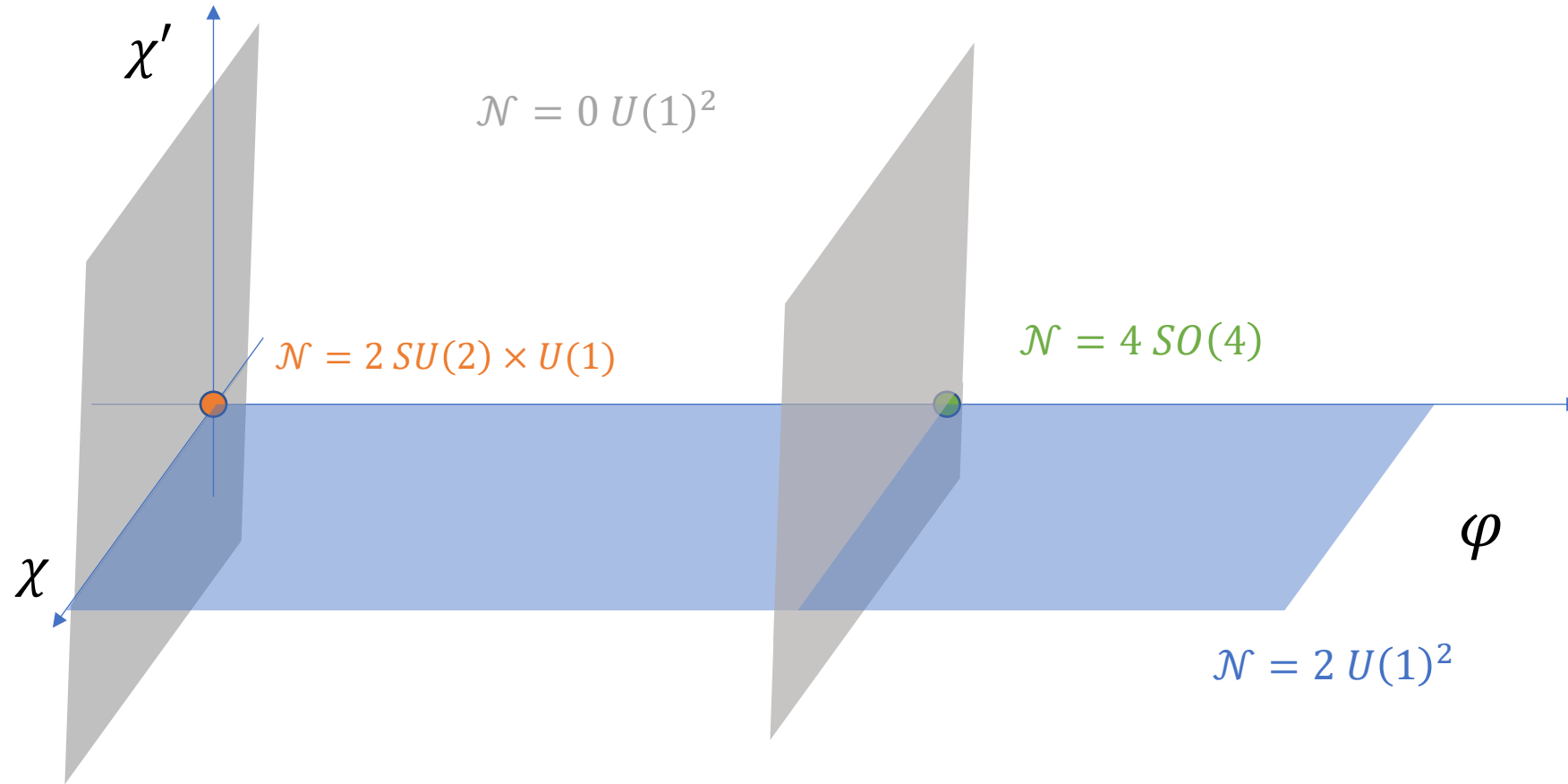
Type IIB S-folds:



The AdS_4 asymptotics

[Guarino, CS]

[Guarino, CS, Trigiante]



For other types of S-folds AdS_4 see also
[Gallerati, Samtleben, Trigiante], [Bobev, Gautasson, van Muiden]

From $\mathcal{N} = 8$ to $\mathcal{N} = 2$

See also [Bobev, Gautasson, van Muiden '24]
for a different 4d truncation

From $\mathcal{N} = 8$ to $\mathcal{N} = 2$

Consistent truncation $\Leftrightarrow G_{inv} \subset G_{gauge} \times G_{global}$

See also [Bobev, Gautasson, van Muiden '24]
for a different 4d truncation

From $\mathcal{N} = 8$ to $\mathcal{N} = 2$

$$\begin{aligned} \text{Consistent truncation} &\Leftrightarrow G_{inv} \subset G_{gauge} \times G_{global} \\ &\subset [SO(6) \times SO(1,1)] \times \Gamma \end{aligned}$$

See also [Bobev, Gautasson, van Muiden '24]
for a different 4d truncation

From $\mathcal{N} = 8$ to $\mathcal{N} = 2$

$$\begin{aligned} \text{Consistent truncation} &\Leftrightarrow G_{inv} \subset G_{gauge} \times G_{global} \\ &\subset [SO(6) \times SO(1,1)] \times \Gamma \end{aligned}$$

- AdS_4 solution is G_{inv}

From $\mathcal{N} = 8$ to $\mathcal{N} = 2$

Consistent truncation $\Leftrightarrow G_{inv} \subset G_{gauge} \times G_{global}$
 $\subset [SO(6) \times SO(1,1)] \times \Gamma$

- AdS_4 solution is G_{inv}
- R-symmetry is G_{inv}

From $\mathcal{N} = 8$ to $\mathcal{N} = 2$

Consistent truncation $\Leftrightarrow G_{inv} \subset G_{gauge} \times G_{global}$
 $\subset [SO(6) \times SO(1,1)] \times \Gamma$

- AdS_4 solution is G_{inv}
- R-symmetry is G_{inv}
- Two gravitini are G_{inv}

From $\mathcal{N} = 8$ to $\mathcal{N} = 2$

Consistent truncation $\Leftrightarrow G_{inv} \subset G_{gauge} \times G_{global}$
 $\subset [SO(6) \times SO(1,1)] \times \Gamma$

- AdS_4 solution is G_{inv}
- R-symmetry is G_{inv}
- Two gravitini are G_{inv}

$$\Rightarrow G_{inv} = \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$\mathcal{N} = 2 \quad n_v = 3 \quad n_h = 4$$

$$U(1)^2 \times \mathbb{R}^2$$

See also [Bobev, Gautasson, van Muiden '24]
 for a different 4d truncation

Solutions

Solutions

Universal BH

Solutions

Universal BH

A single BH for each (φ, χ) AdS_4

Solutions

Universal BH

A single BH for each (φ, χ) AdS_4

$$s = L_{AdS_4}^2 \frac{(g-1)\pi}{2}$$

Solutions

Universal BH

A single BH for each (φ, χ) AdS_4

$$s = L_{AdS_4}^2 \frac{(g-1)\pi}{2}$$

Magnetic charge

Solutions

Universal BH

A single BH for each (φ, χ) AdS_4

$$s = L_{AdS_4}^2 \frac{(g-1)\pi}{2}$$

Magnetic charge

$$p_R = \frac{1}{2}$$

Solutions

Universal BH

A single BH for each (φ, χ) AdS_4

$$s = L_{AdS_4}^2 \frac{(g-1)\pi}{2}$$

Magnetic charge

$$p_R = \frac{1}{2}$$

Nothing surprising here

Uplift of universal BH

[Imagine a series of ugly, but reasonable, formulas for IIB fields]

Solutions

Universal BH

A single BH for each (φ, χ) AdS_4

$$s = L_{AdS_4}^2 \frac{(g-1)\pi}{2}$$

Magnetic charge

$$p_R = \frac{1}{2}$$

Nothing surprising here

Scale separated solution $AdS_2 \times \mathbb{H}^2$

Solutions

Universal BH

A single BH for each (φ, χ) AdS_4

$$s = L_{AdS_4}^2 \frac{(g-1)\pi}{2}$$

Magnetic charge

$$p_R = \frac{1}{2}$$

Nothing surprising here

Scale separated solution $AdS_2 \times \mathbb{H}^2$

2-parameters family of solutions

$$(\lambda, \gamma) \in \mathbb{R} \times [0, \pi/2[$$

Solutions

Universal BH

A single BH for each (φ, χ) AdS_4

$$s = L_{AdS_4}^2 \frac{(g-1)\pi}{2}$$

Magnetic charge

$$p_R = \frac{1}{2}$$

Nothing surprising here

Scale separated solution $AdS_2 \times \mathbb{H}^2$

2-parameters family of solutions

$$(\lambda, \gamma) \in \mathbb{R} \times [0, \pi/2[$$

$$\frac{L_{AdS_2}^2}{L_{\mathbb{H}^2}^2} = \frac{1}{1 + \frac{\cosh(2\lambda)}{\sin(\gamma)}}$$

Solutions

Universal BH

A single BH for each (φ, χ) AdS_4

$$s = L_{AdS_4}^2 \frac{(g-1)\pi}{2}$$

Magnetic charge

$$p_R = \frac{1}{2}$$

Nothing surprising here

Scale separated solution $AdS_2 \times \mathbb{H}^2$

2-parameters family of solutions

$$(\lambda, \gamma) \in \mathbb{R} \times [0, \pi/2[$$

$$\frac{L_{AdS_2}^2}{L_{\mathbb{H}^2}^2} = \frac{1}{1 + \frac{\cosh(2\lambda)}{\sin(\gamma)}}$$

$(0, \pi/4) \Rightarrow$ NH limit of univ. BH
with $\mathcal{N} = 2$ $SU(2)$ asymptotics

Uplift of the scale separated solution

Why?

Uplift of the scale separated solution

Why?

- Check the SUGRA approximation

$$g_s \ll 1 \text{ and } \alpha' R \ll 1$$

Uplift of the scale separated solution

Why?

- Check the SUGRA approximation

$$g_s \ll 1 \text{ and } \alpha' R \ll 1$$

- Check that there is scale separation

Uplift of the scale separated solution

Why?

- Check the SUGRA approximation

$$g_s \ll 1 \text{ and } \alpha' R \ll 1$$

- Check that there is scale separation

$$\text{Requires } \gamma = \frac{\pi}{4} \text{ so that } AdS_2 \times M_8 \Rightarrow AdS_2 \times M_8$$

Uplift of the scale separated solution

Why?

- Check the SUGRA approximation

$$g_s \ll 1 \text{ and } \alpha' R \ll 1$$

- Check that there is scale separation

Requires $\gamma = \frac{\pi}{4}$ so that $AdS_2 \times M_8 \Rightarrow AdS_2 \times M_8$

$$\lambda \gg 1 \text{ or } \lambda \ll 1$$

$$L_{AdS_2} \propto \cosh^{-1} \lambda \quad L_{\mathbb{H}^2} \propto \frac{1}{\sqrt{2}} \quad L_{S^1} \propto T \cosh^{-1} \lambda \quad vol_{S^5} \propto \cosh^5 \lambda$$

Exotic consistent truncations

$\mathcal{N} = 4$ $SO(4)$ -gauged truncation of Type IIB

From $\mathcal{N} = 8$ to $\mathcal{N} = 2$

Consistent truncation $\Leftrightarrow G_{inv} \subset G_{gauge} \times G_{global}$
 $\subset [SO(6) \times SO(1,1)] \times \Gamma$

- AdS_4 solution is G_{inv}
- R-symmetry is G_{inv}
- Two gravitini are G_{inv}

$$\Rightarrow G_{inv} = \mathbb{Z}_2 \times \mathbb{Z}_2$$
$$\mathcal{N} = 2 \quad n_v = 3 \quad n_h = 4$$
$$U(1)^2 \times \mathbb{R}^2$$

From $\mathcal{N} = 8$ to $\mathcal{N} = 4$

Consistent truncation $\Leftrightarrow G_{inv} \subset G_{gauge} \times G_{global}$
 $\subset [SO(6) \times SO(1,1)] \times \Gamma$

- AdS_4 solution is G_{inv}
- R-symmetry is G_{inv}
- **Four** gravitini are G_{inv}

$\Rightarrow G_{inv} = ???$

From $\mathcal{N} = 8$ to $\mathcal{N} = 4$

Consistent truncation $\Leftrightarrow G_{inv} \subset G_{gauge} \times G_{global}$
 $\subset [SO(6) \times SO(1,1)] \times \Gamma$

- AdS_4 solution is G_{inv}
- R-symmetry is G_{inv}
- **Four** gravitini are G_{inv}
- $n_\nu = 0$

$\Rightarrow G_{inv} = ???$

From $\mathcal{N} = 8$ to $\mathcal{N} = 4$

Consistent truncation $\Leftrightarrow G_{inv} \in \cancel{G_{gauge}} \times \cancel{G_{global}}$
 $\in \cancel{[SO(6) \times SO(1,1)]} \times \cancel{F}$

- AdS_4 solution is G_{inv}
- R-symmetry is G_{inv}
- **Four** gravitini are G_{inv}
- $n_\nu = 0$

$\Rightarrow G_{inv} = ???$

$SU(4)_S, SU(4)_R, SO(6)_g$ and $SU(8)$

$$\begin{pmatrix} SU(4)_R & 0 \\ 0 & SU(4)_S \end{pmatrix}$$

$SO(6)_g$ is the diagonal subgroup of $SU(4)_R \times SU(4)_S \subset SU(8)$

From $\mathcal{N} = 8$ to $\mathcal{N} = 4$

Consistent truncation $\Leftrightarrow G_{inv} \in \cancel{G_{gauge}} \times \cancel{G_{global}}$
 $\in \cancel{[SO(6) \times SO(1,1)]} \times \mathbb{F}$

- AdS_4 solution is G_{inv}
- R-symmetry is G_{inv}
- **Four** gravitini are G_{inv}

$$\Rightarrow G_{inv} = SU(4)_S$$
$$\mathcal{N} = 4 \quad SO(4)\text{-gauging}$$

Technical statement

Technical statement

Consistent truncation to $\mathcal{N} = 8$



Generalised Id-structure on M_6

Technical statement

Consistent truncation to $\mathcal{N} = 8$



Generalised Id-structure on M_6

The D=4 gauging



Torsion \sim embedding tensor

Technical statement

Consistent truncation to $\mathcal{N} = 8$

\Leftrightarrow

Generalised Id-structure on M_6

Consistent truncation to $\mathcal{N} = 4$

\Leftrightarrow

Gen. $SU(4)_S$ -structure on M_6

The D=4 gauging

\Leftrightarrow

Torsion \sim embedding tensor

Technical statement

Consistent truncation to $\mathcal{N} = 8$

\Leftrightarrow

Generalised Id-structure on M_6

The D=4 gauging

\Leftrightarrow

Torsion \sim embedding tensor

Consistent truncation to $\mathcal{N} = 4$

\Leftrightarrow

Gen. $SU(4)_S$ -structure on M_6

The D=4 gauging

\Leftrightarrow

Intrinsic torsion

Technical statement

Consistent truncation to $\mathcal{N} = 8$

\Leftrightarrow

Generalised Id-structure on M_6

The D=4 gauging

\Leftrightarrow

Torsion \sim embedding tensor

$T_{\mathcal{N}=8}$

Consistent truncation to $\mathcal{N} = 4$

\Leftrightarrow

Gen. $SU(4)_S$ -structure on M_6

The D=4 gauging

\Leftrightarrow

Intrinsic torsion

Technical statement

Consistent truncation to $\mathcal{N} = 8$

\Leftrightarrow

Generalised Id-structure on M_6

The D=4 gauging

\Leftrightarrow

Torsion \sim embedding tensor

Not $SU(4)_S$ -inv $\longrightarrow T_{\mathcal{N}=8}$

Consistent truncation to $\mathcal{N} = 4$

\Leftrightarrow

Gen. $SU(4)_S$ -structure on M_6

The D=4 gauging

\Leftrightarrow

Intrinsic torsion

Technical statement

Consistent truncation to $\mathcal{N} = 8$

\Leftrightarrow

Generalised Id-structure on M_6

The D=4 gauging

\Leftrightarrow

Torsion \sim embedding tensor

$$\text{Not } SU(4)_S\text{-inv} \longrightarrow T_{\mathcal{N}=8} = T_{\mathcal{N}=4}^{int} \oplus T^{non\ int}$$

Consistent truncation to $\mathcal{N} = 4$

\Leftrightarrow

Gen. $SU(4)_S$ -structure on M_6

The D=4 gauging

\Leftrightarrow

Intrinsic torsion

Technical statement

Consistent truncation to $\mathcal{N} = 8$

\Leftrightarrow

Generalised Id-structure on M_6

The D=4 gauging

\Leftrightarrow

Torsion \sim embedding tensor

$$\text{Not } SU(4)_S\text{-inv} \longrightarrow T_{\mathcal{N}=8} = T_{\mathcal{N}=4}^{int} \oplus T^{non\ int}$$

$SU(4)_S$ -inv

Consistent truncation to $\mathcal{N} = 4$

\Leftrightarrow

Gen. $SU(4)_S$ -structure on M_6

The D=4 gauging

\Leftrightarrow

Intrinsic torsion

Technical statement

Consistent truncation to $\mathcal{N} = 8$

\Leftrightarrow

Generalised Id-structure on M_6

The D=4 gauging

\Leftrightarrow

Torsion \sim embedding tensor

Not $SU(4)_S$ -inv \longrightarrow

$$T_{\mathcal{N}=8} = T_{\mathcal{N}=4}^{int}$$

$SU(4)_S$ -inv

Consistent truncation to $\mathcal{N} = 4$

\Leftrightarrow

Gen. $SU(4)_S$ -structure on M_6

The D=4 gauging

\Leftrightarrow

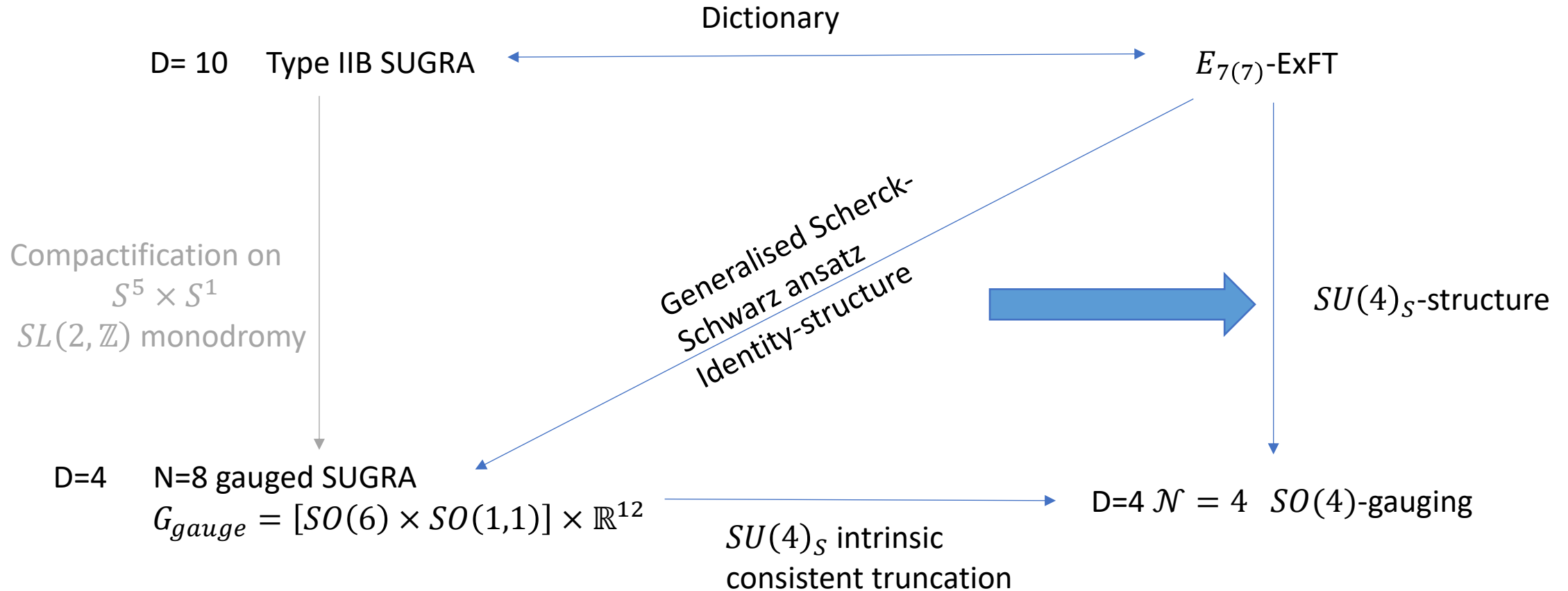
Intrinsic torsion

$$\bigoplus T^{non\ int}$$

Not $SU(4)_S$ -inv

Truncations of truncations

Consistent of course



Uplift of the $U(1)^2$ subsector

4 dimensions

$$\{\tau = \chi + i e^{-\phi}, A_1, A_2\}$$

Type IIB metric

$$ds^2 = \Delta^{-1} \left(\frac{1}{2} ds_{ext}^2 + g_{mn} D x^M D x^N \right)$$
$$g_{mn} dx^m dx^n = d\alpha^2 + d\eta^2 + f_1^{-1} vol_1 + f_2^{-1} vol_2$$
$$vol_i = d\theta_i^2 + \sin^2 \theta_i d\varphi_i$$
$$f_1 = 1 + 2 e^{-\phi} \cos^2 \alpha$$
$$f_2 = 1 + 2 e^{\phi} |\tau|^{-2} \sin^2 \alpha$$
$$D\varphi_i = d\varphi_i + A_i$$

Conclusion and outlook

Summary and outlook

- We have built
 - A universal BH that asymptotes any $\mathcal{N} = 2$ S-fold
 - A scale separated $AdS_2 \times \mathbb{H}^2$ solution (with uplift)
Can be understood as AdS_2 S-folds
- A consistent truncation of Type IIB SUGRA to $\mathcal{N} = 4$ SO(4)-gauged SUGRA
 - New IIB solutions
 - A laboratory for non-geometric solutions and AdS/CFT

Thank you

Uplift of a universal BH

Does it match what you had in mind?

$$ds_{10}^2 = \Delta^{-1} \left(\frac{1}{2} ds_4^2 + g_{mn} Dy^m Dy^n \right)$$

$$Dy^n = dy^n + A_\mu{}^n dx^\mu$$

$$g_{mn} dy^m dy^n = d\eta^2 + d\alpha^2 + \frac{\cos^2 \alpha}{2 + \cos(2\alpha)} ds_{S_1^2}^2 + \frac{\sin^2 \alpha}{2 - \cos(2\alpha)} ds_{S_2^2}^2$$

$$\mathbb{B}^1 = \frac{1}{\sqrt{2}} \frac{1}{r} dt \wedge d(e^{-\eta} \sin \alpha \cos \theta_2) + \frac{\cosh \theta}{2\sqrt{2}} d\phi \wedge d(e^{-\eta} \cos \alpha \cos \theta_1) - 2\sqrt{2} e^{-\eta} \cos \alpha \widetilde{\text{vol}}_1 ,$$

$$\mathbb{B}^2 = -\frac{1}{\sqrt{2}} \frac{1}{r} dt \wedge d(e^\eta \cos \alpha \cos \theta_1) - \frac{\cosh \theta}{2\sqrt{2}} d\phi \wedge d(e^\eta \sin \alpha \cos \theta_2) - 2\sqrt{2} e^\eta \sin \alpha \widetilde{\text{vol}}_2 ,$$

$$\widetilde{F}_5 = 6 \widetilde{\text{vol}}_5 - 4 \sin(2\alpha) d\eta \wedge \widetilde{\text{vol}}_1 \wedge \widetilde{\text{vol}}_2$$

$$+ \frac{r^2}{4} \sinh \theta (2 d(\cos^2 \alpha) - 3d\eta) \wedge dt \wedge dr \wedge d\theta \wedge d\phi$$

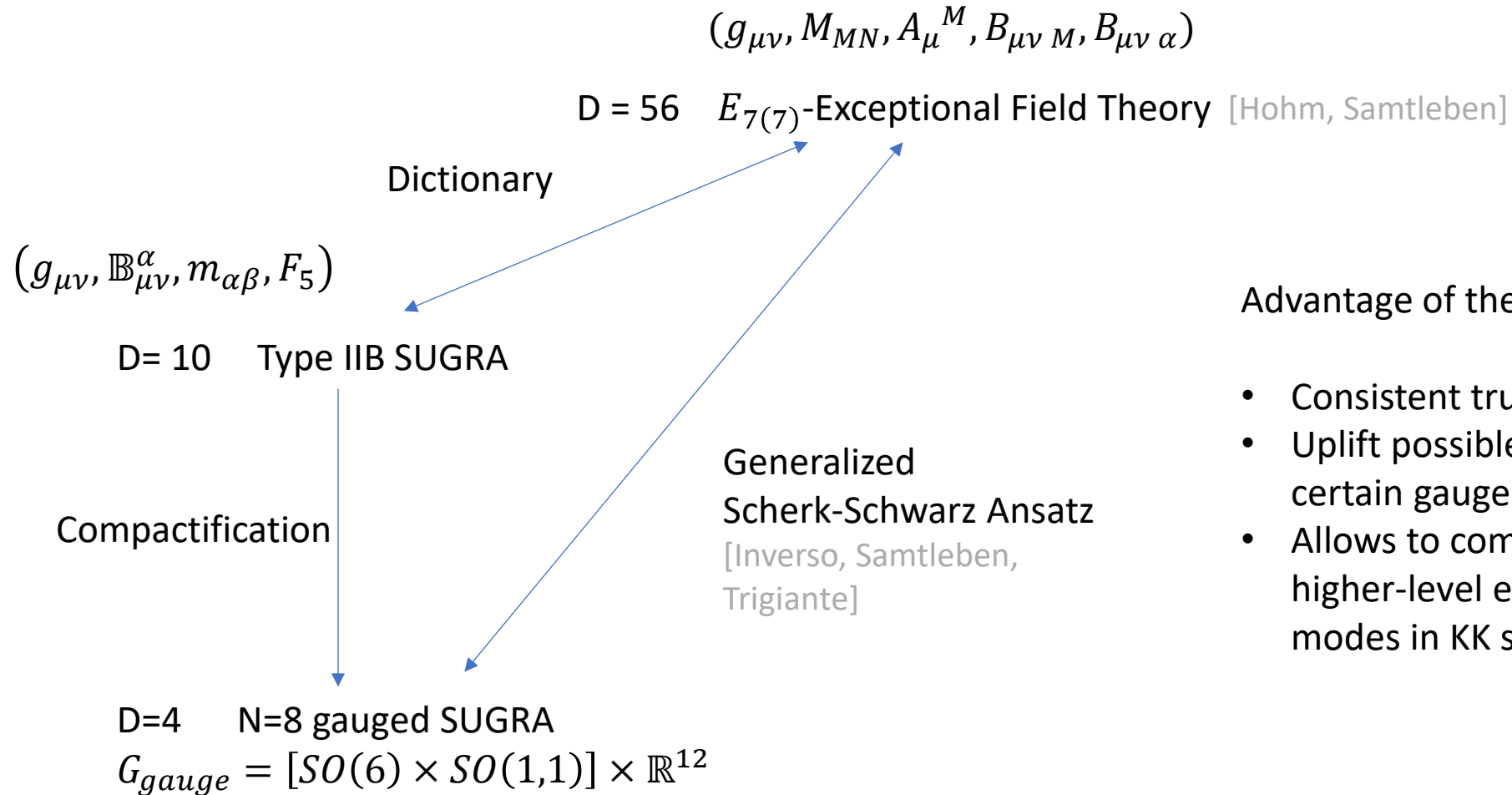
$$+ \frac{1}{r^2} dt \wedge dr \wedge (\cos \theta_2 \widetilde{\text{vol}}_2 - \cos \theta_1 \widetilde{\text{vol}}_1) \wedge (d(\cos^2 \alpha) - d\eta)$$

$$- \frac{1}{r^2} dt \wedge dr \wedge (\sin^2 \theta_1 D(\alpha) + \sin^2 \theta_2 D(\alpha)) \wedge d(\cos^2 \alpha) \wedge d\eta$$

Back-up slides

How to compactify?

$E_{7(7)}$ -ExFT as a guide



Advantage of the method :

- Consistent truncation
- Uplift possible (and not so hard for certain gauge groups)
- Allows to compute the masses of higher-level excitation (i.e. higher modes in KK spectrum)

Uplift of the $\mathcal{N} = 4$ S-fold

Solution on $AdS_4 \times S_\eta^1 \times S^5$

The S^5 is deformed and preserves an $SO(4) \sim SO(3) \times SO(3)$ and is understood as :

$$S^5 \sim S_1^2 \times S_2^2 \times I_\alpha$$

$$\text{Metric : } ds_{10}^2 = \Delta^{-1} \left[\frac{1}{2} ds_{AdS_4}^2 + d\eta^2 + d\alpha^2 + \frac{\cos^2(\alpha)}{2+\cos(2\alpha)} ds_{S_1}^2 + \frac{\sin^2(\alpha)}{2-\cos(2\alpha)} ds_{S_2}^2 \right]$$

$$ds_{S_i}^2 = d\theta^2 + \cos\theta^2 d\varphi_i^2$$

$$\text{Warp factor : } \Delta^{-4} = 4 - \cos^2(2\alpha)$$

$$\text{Five-form : } F_5 = (1 + *) f(\alpha) Vol_{S^5}$$

$$\text{Dilaton : } e^\phi = e^{-\eta} \frac{\sqrt{2-\cos(2\alpha)}}{\sqrt{2+\cos(2\alpha)}}$$

$$\text{2-Form : } B_2 = -2\sqrt{2} e^{-\eta} \frac{\cos^3(\alpha)}{2+\cos(2\alpha)} vol_{S_1}$$

$$\text{Axion : } C_0 = 0$$

$$C_2 = 2\sqrt{2} e^\eta \frac{\sin^3(\alpha)}{2-\cos(2\alpha)} vol_{S_2}$$

Generating a family of solutions :

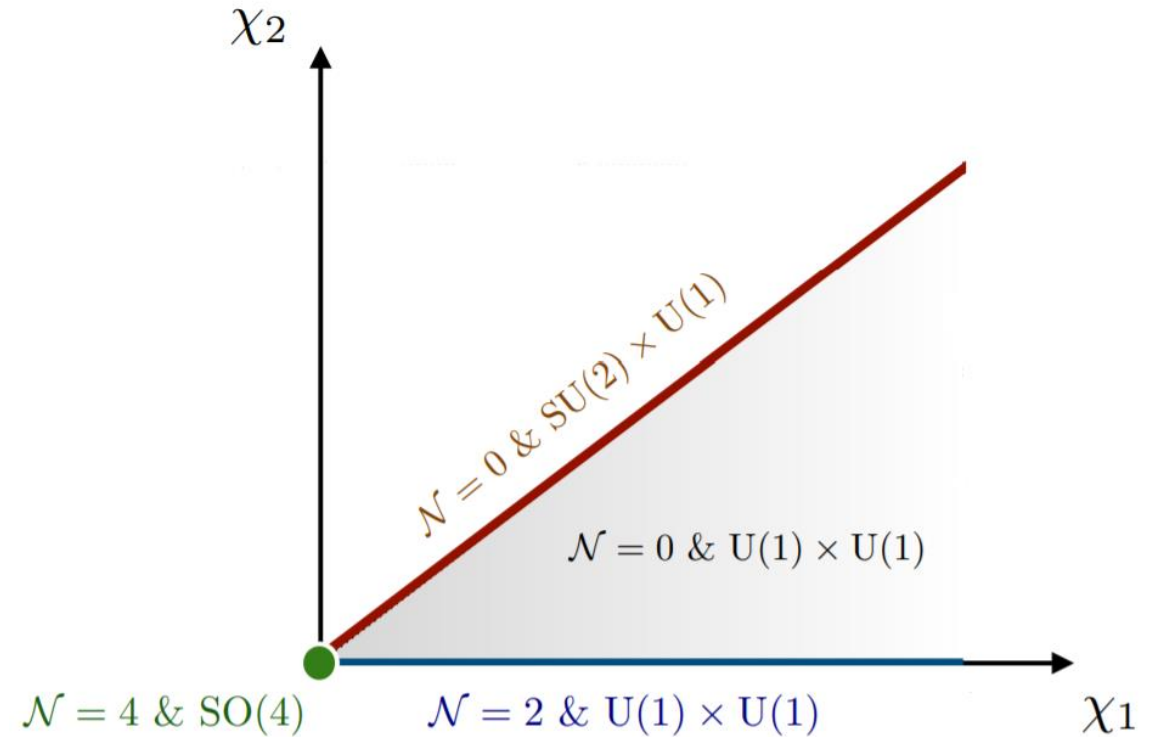
The « $\mathcal{N} = 4$ family » [Guarino, CS]

$\mathcal{N} = 4$ locus with $SO(4) \sim SO(3) \times SO(3)$ symmetry

$\text{Rank}(SO(4)) = 2 \Rightarrow$ 2D moduli space (χ_1, χ_2) .

Discrete symmetries : $\chi_1 \leftrightarrow \chi_2$ and $\chi_i \leftrightarrow -\chi_i$

The values of χ_i modify the residual symmetry.
modify the masses of excitations



Deformations of the S-fold

What is the uplift of the N=4 family ? [Giambrone, Guarino, Malek, Samtleben, Trigiante, CS]

Equivalent uplift up to *local* change of coordinates :

$$\varphi'_i = \varphi_i + \chi_i \eta$$

Concretely :

$$\text{Metric : } ds_{10}^2 = \Delta^{-1} \left[\frac{1}{2} ds_{AdS_4}^2 + d\eta^2 + d\alpha^2 + \frac{\cos^2(\alpha)}{2+\cos(2\alpha)} ds_{S_1}^2 + \frac{\sin^2(\alpha)}{2-\cos(2\alpha)} ds_{S_2}^2 \right]$$

$$ds_{S_i}^2 = d\theta^2 + \cos\theta^2 d\varphi_i^2 \rightarrow ds_{S_i}'^2 = d\theta^2 + \cos\theta^2 d\varphi_i'^2 \\ = d\theta^2 + \cos\theta^2 d(\varphi_i + \chi_i \eta)^2$$