Double-Copy Supertranslations

Dario Francia
Roma Tre U & INFN

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Napoli - Accademia Pontaniana

Sept 23-25

Based on:

P. Ferrero, D.F., C. Heissenberg & M. Romoli

[2402.11595]

THE DOUBLE COPY (DC)

$$H_{\mu\nu} = A_\mu * \tilde{A}_\nu$$

GR = (YM)^2

[KLT 1986, BCJ 2008-2010]

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$$A_n^{\text{TREE}} = g^{n-2} \sum_{i \in \Gamma_3} \frac{n_i c_i}{\prod_{\alpha_i} s_{\alpha_i}} \rightarrow M_n^{\text{TREE}} = i(2\kappa)^{n-2} \sum_{i \in \Gamma_3} \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

YM — GR

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- ON SHELL

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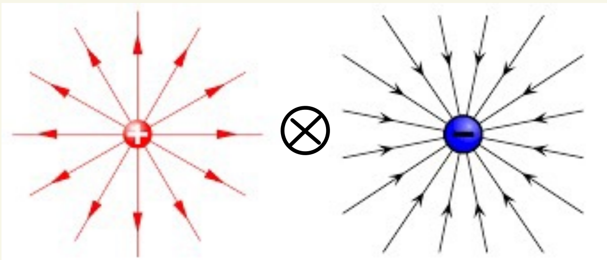
CLASSICAL SOLUTIONS



$\otimes$ =

- GAUGE FIXED
- ON SHELL

PRODUCT OF TWO COULOMB POTENTIALS

SCHWARZSCHILD (KERR-SCHILD FORM)

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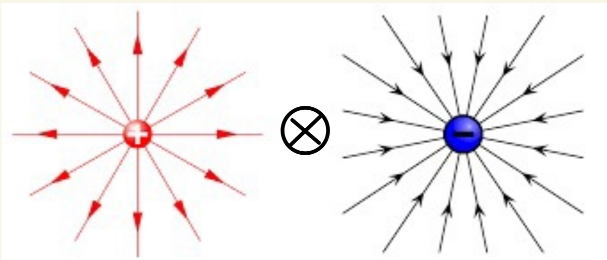

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SCHWARZSCHILD (KERR-SCHILD FORM)

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HOW MUCH DOES THE DC KNOW ABOUT SPACETIME GEOMETRY?

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$$\underbrace{A_m^{\text{TREE}} = g^{m-2} \sum_{i \in \Gamma_3} \frac{n_i \, c_i}{\prod_{\alpha_i} s_{\alpha_i}}}_{\text{YM}} \longrightarrow \underbrace{M_m^{\text{TREE}} = i(2\kappa)^{m-2} \sum_{i \in \Gamma_3} \frac{n_i \, \tilde{n}_i}{\prod_{\alpha_i} s_{\alpha_i}}}_{\text{GR}}$$

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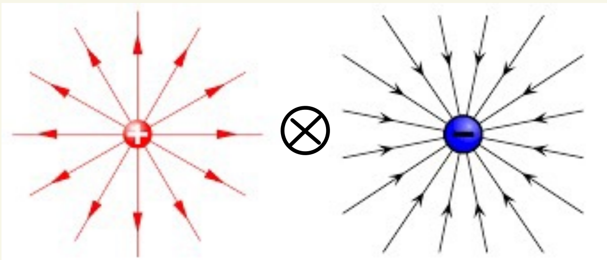

PRODUCT OF TWO COULOMB POTENTIALS

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SCHWARZSCHILD (KERR-SCHILD FORM)

- GAUGE FIXED
- ON SHELL

HOW MUCH DOES THE DC KNOW ABOUT SPACETIME GEOMETRY?

HARD TO TELL, IN GENERAL

ASYMPTOTIC SYMMETRIES (AS)

AS MAY PROVIDE A NON-TRIVIAL, INTERMEDIATE SETUP

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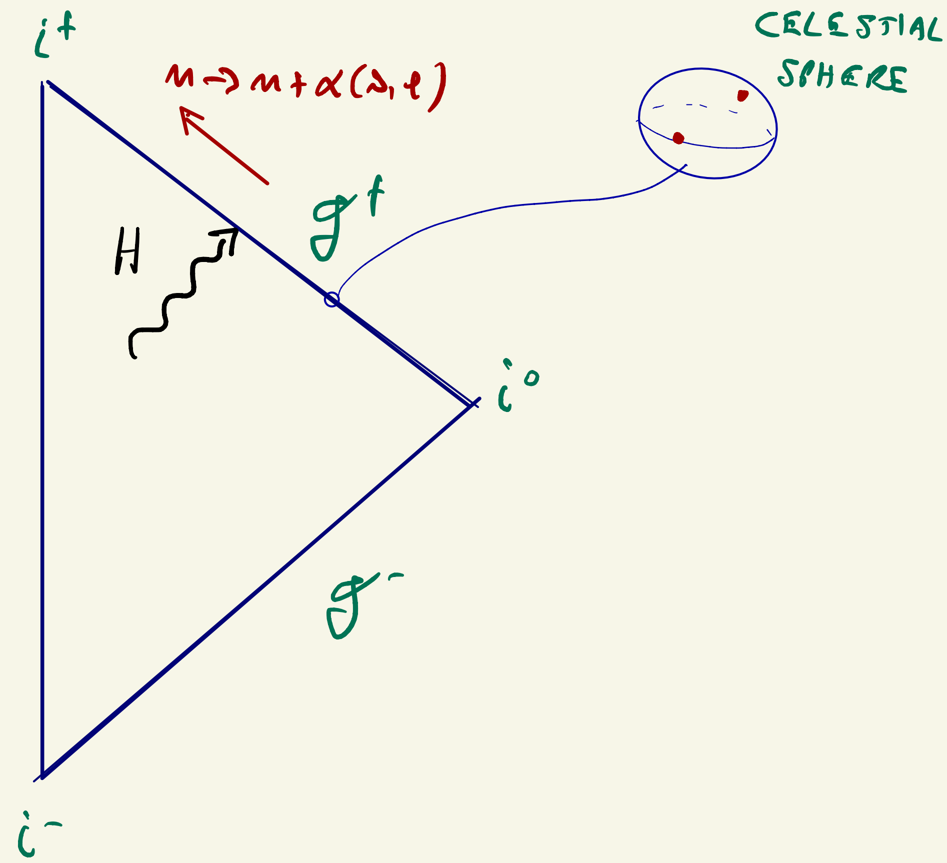
RESIDUAL DIFFEOMORPHISMS AT $\mathscr{I}$ FOR ASYMPTOTICALLY-FLAT SPACES ENCODE DATA ON SOFT THEOREMS AND MEMORY EFFECTS



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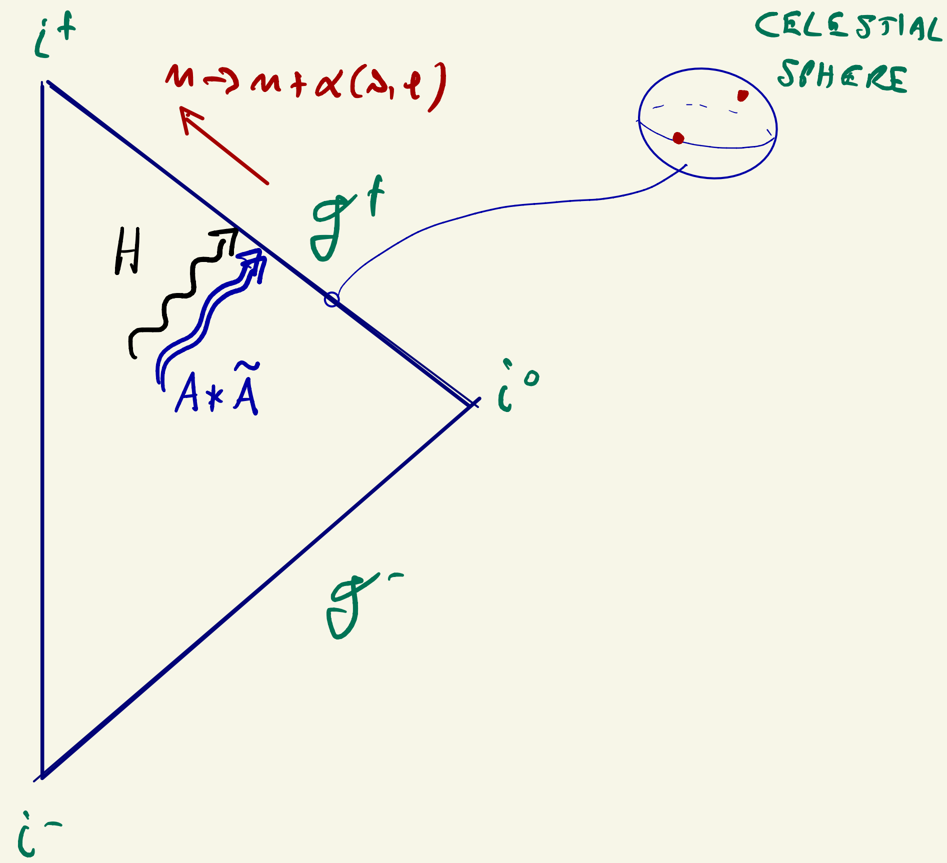

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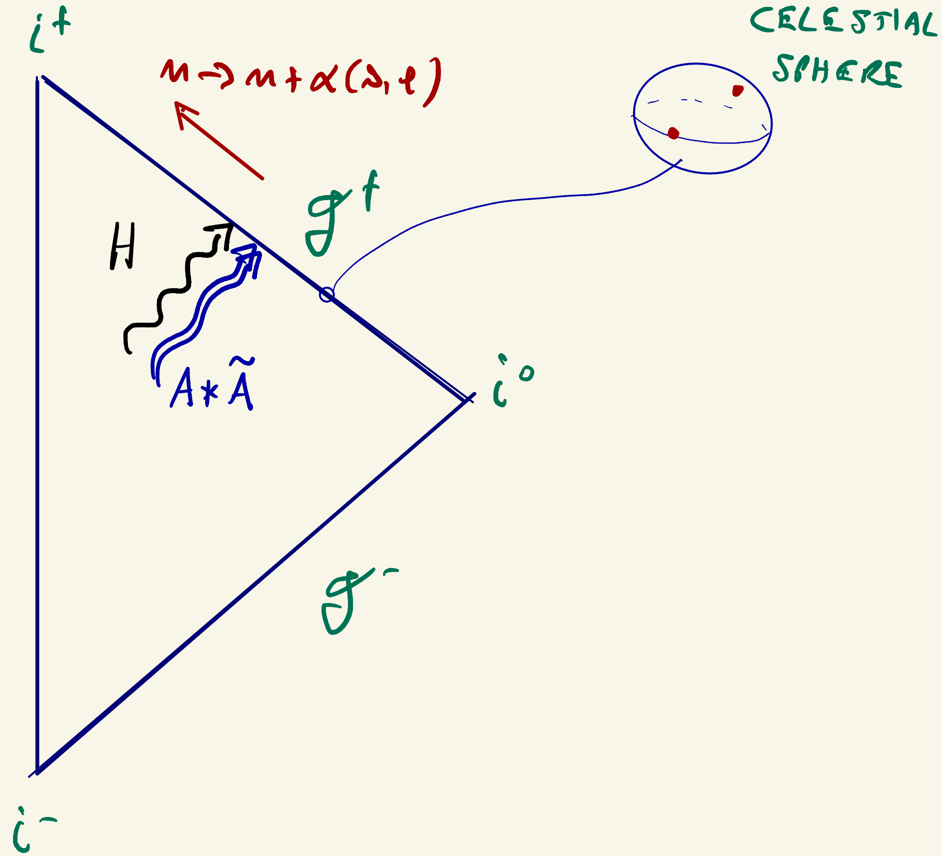


PARTICLE CONTENT OF THE DC:

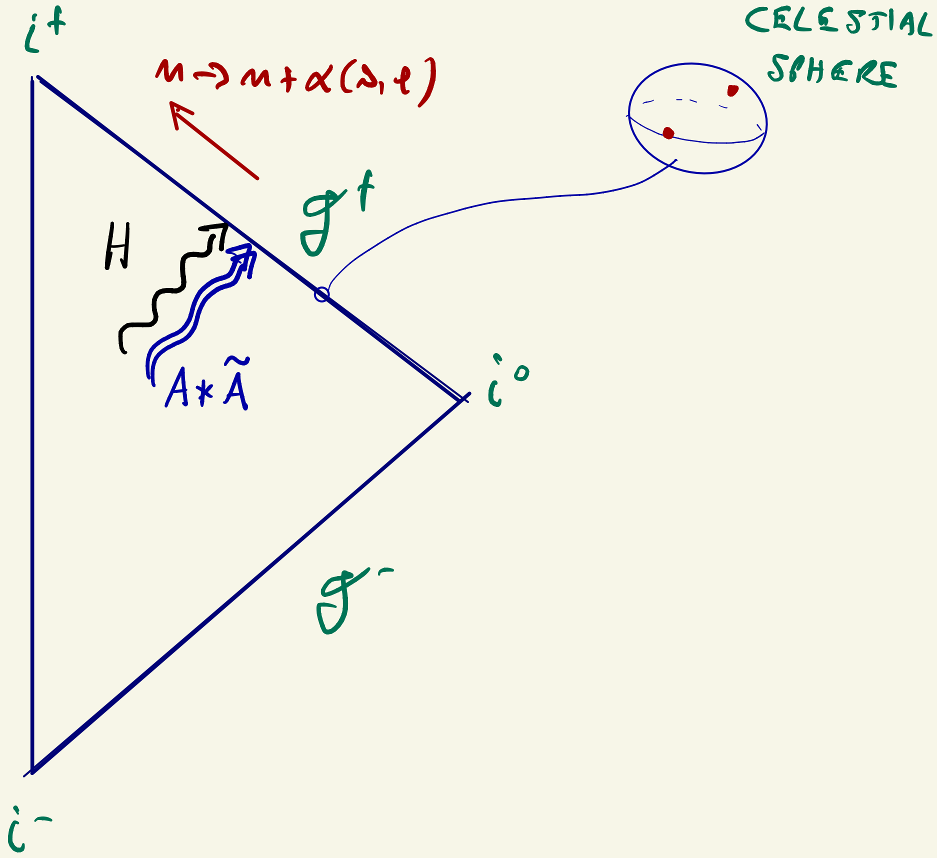
$$\varepsilon_i \varepsilon_j = \square \otimes \square = \square\square \oplus \begin{smallmatrix}\square\\\square\end{smallmatrix} \oplus \bullet$$

$[D(D-2)]$

Asymptotic Symmetries (AS)

AS may provide a non-trivial, intermediate setup

Residual diffeomorphisms at $\mathscr{I}$ for asymptotically-flat spaces encode data on soft theorems and memory effects



Particle content of the DC:

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$[D(D-2)]$

$\square\!\square$: AS well known

$\begin{array}{c}\square\\\square\end{array} \oplus \bullet$: AS less known

ASYMPTOTIC SYMMETRIES (AS)

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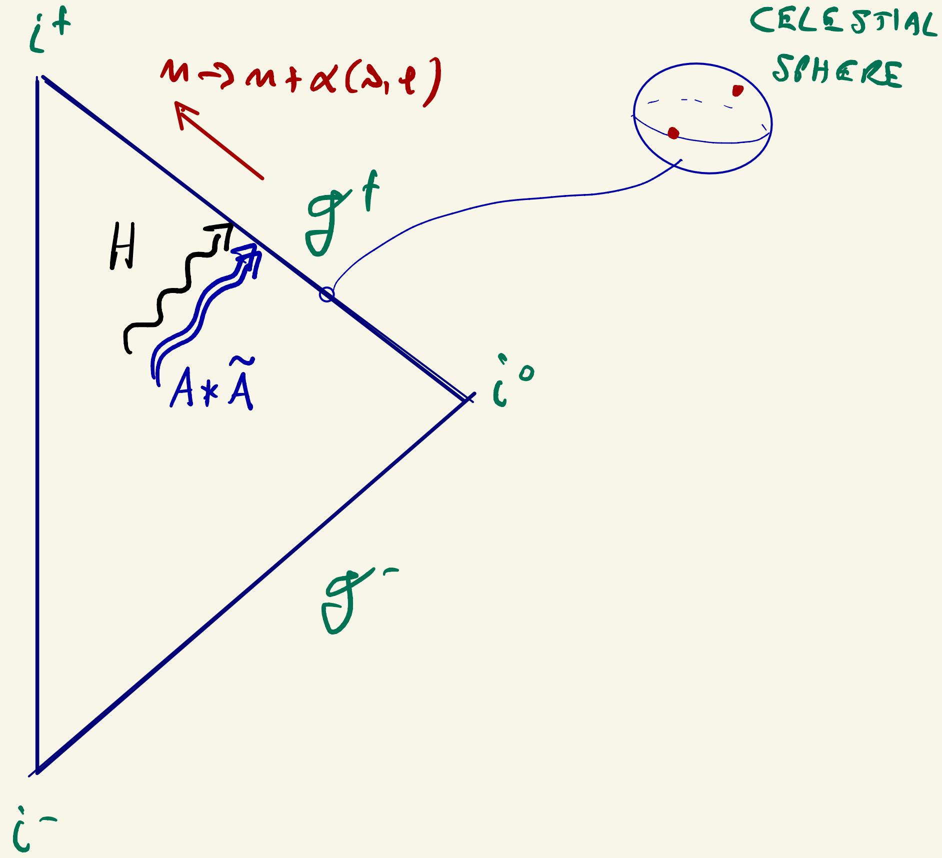

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$\square\square$ → AS WELL KNOWN

$\begin{array}{c}\square\\ \square\end{array} \oplus \bullet$ → AS LESS KNOWN

HOW MUCH DOES THE DC KNOW OF THE FULL RESULTING AS?

MOTIVATIONS

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* A STEP TOWARDS UNDERSTANDING THE DC AT THE LEVEL OF SYMMETRIES

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* MAY (HOPEFULLY) HELP SIMPLIFYING COMPUTATIONS
* UNIFIED FRAMEWORK ENCOMPASSING GAUGE SYMMETRIES OF DIFFERENT THEORIES

PLAN

§ SETUP: THE FIELD THEORETICAL DC

§ TECHNICALIA: SQUARING ASYMPTOTICS

§ DC SUPERTRANSLATIONS

§ Setup: the field theoretical DC

Focus on the symmetry aspects

Three building blocks:

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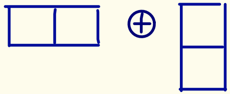
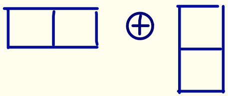
(1) $H_{\mu\nu} := A^{a}_{\mu} \circ \Phi^{-1}_{aa'} \circ \tilde{A}^{a'}_{\nu} := A_\mu * \tilde{A}_\nu =$ 

FOCUS ON THE SYMMETRY ASPECTS

THREE BUILDING BLOCKS:

(1) $H_{\mu\nu} := A_\mu^a \circ \Phi^{-1}_{aa'} \circ \tilde{A}_\nu^{a'} := A_\mu * \tilde{A}_\nu =$ 

* $A_\mu, \tilde{A}_\nu$ YM fields of groups G and $\tilde{G}$

* $[f \circ g](x) = \int d^D y\, f(y)\, g(x-y)$

* $\Phi_{aa'}$ BIADJOINT "SPECTATOR" scalar field

* The $*$-product is s.t. $\partial_\mu (A_\rho * \tilde{A}_\sigma) = (\partial_\mu A_\rho) * \tilde{A}_\sigma = A_\rho * (\partial_\mu \tilde{A}_\sigma)$

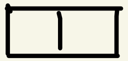
[Anastasiou, Borsten, Duff, Hughes & Nagy 2014, 2018]

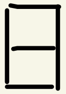
The DC field $H_{\mu\nu}$

(2) $$\delta_0 H_{\mu\nu} = \partial_\mu \alpha_\nu + \partial_\nu \tilde{\alpha}_\mu$$

$$\alpha_\mu = \epsilon * \tilde{A}_\mu \,, \quad \tilde{\alpha}_\mu = A_\mu * \tilde{\epsilon}$$

[$\epsilon, \tilde{\epsilon}$ gauge parameters for $A_\mu, \tilde{A}_\mu$]

* δ_0  = linearised diff

* δ_0  = two-form transf

[Anastasiou, Borsten, Duff, Hughes & Nagy 2014, 2018]

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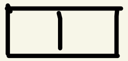
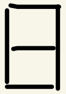
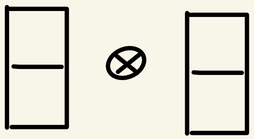
* δ_0  = linearised diff

* δ_0  = two-form transf

(3) $R_{\mu\nu\rho\sigma} \sim F_{\mu\nu} * \tilde{F}_{\rho\sigma}$

[$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$]

$\delta_0 R_{\mu\nu\rho\sigma} = 0$

* $R_{\mu\nu\rho\sigma}$ =  → geometry of $H_{\mu\nu}$ involves a torsion

* From $R_{\mu\nu\rho\sigma}$ linear EOM for $H_{\mu\nu}$, with non-locally coupled sources

EXPLOITING $H_{\mu\nu}$ OFF-SHELL MEETS A SUBTLETY:

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$\rightsquigarrow$ DC FIELD

$$\underset{[GL(D)]}{H_{\mu\nu}} = \square \otimes \square = \overset{H^{S}_{\mu\nu}}{\square\!\square} \oplus \underset{H^{A}_{\mu\nu}}{\begin{array}{c}\square\\[-4pt]\square\end{array}}$$

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$H^{A}_{\mu\nu}$ → SYMMETRIES OF AN OFF-SHELL TWO-FORM

$H^{S}_{\mu\nu}$ → SYMMETRIES OF AN OFF-SHELL GRAVITON

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$[GL(D)]$

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ALSO PROVIDING A PROPAGATING SCALAR

→ THE APPARENT SYMMETRIES NOTWITHSTANDING,

ONE CANNOT USE LINEARISED EINSTEIN + TWO-FORM

WHAT DO WE NEED? A LAGRANGIAN FOR THE REDUCIBLE GL(D)- TENSOR $H_{\mu\nu}$

WHOSE EOM PROPAGATE GRAVITON + KALB-RAMOND + SCALAR :

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WHOSE EOM PROPAGATE GRAVITON + KALB-RAMOND + SCALAR:

$$\mathcal{L}_{ML} = \frac{1}{2} H^{\alpha\beta}\left\{\eta_{\alpha\mu}\eta_{\beta\nu}\Box - \eta_{\alpha\mu}\partial_\beta\partial_\nu - \eta_{\beta\nu}\partial_\alpha\partial_\mu\right\} H^{\mu\nu}$$

[A. CAMPOLEONI & D.F. 2013, P. FERRERO & D.F. 2020]

- LAGRANGIAN CONSTRUCTION EXTENDED TO CUBIC LEVEL
- FIRST-ORDER DEFORMATION OF GAUGE SYMMETRY COMPUTED

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- FIRST-ORDER DEFORMATION OF GAUGE SYMMETRY COMPUTED

HERE FOCUS ON THE LINEAR THEORY:

$$\Box H_{\mu\nu} - \partial_\mu \partial^\alpha H_{\alpha\nu} - \partial_\nu \partial^\alpha H_{\mu\alpha} = 0$$

LOOKING INSIDE:

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$$A_\mu * \tilde{A}_\nu$$

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$$A_\mu * \tilde{A}_\nu$$

$$\Box A^a_\mu - \partial_\mu \partial \cdot A^a = 0$$

$$\delta A^a_\mu = \partial_\mu \epsilon^a$$

$$\Box \Phi_{aa'} = 0$$

$$\Box \tilde{A}^{a'}_\mu - \partial_\mu \partial \cdot \tilde{A}^{a'} = 0$$

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WE WORK IN

SINGLE-COPY LORENZ GAUGE:

$$\partial^\alpha A_\alpha = 0 = \partial^\beta \tilde{A}_\beta$$

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WE WORK IN SINGLE-COPY LORENZ GAUGE:

$$\partial^\alpha A_\alpha = 0 = \partial^\beta \tilde{A}_\beta \quad \Longrightarrow$$

ALL FIELDS (SINGLE- AND DOUBLE-COPY) AND ALL PARAMETERS (IDEM) SATISFY:

$$\Box \begin{Bmatrix} \text{FIELD} \\ \text{PARAMETER} \end{Bmatrix} = 0$$

$$\partial \cdot \begin{Bmatrix} \text{FIELD} \\ \text{PARAMETER} \end{Bmatrix} = 0$$

§ TECHNICALIA: SQUARING ASYMPTOTICS

ALTERNATIVE PERSPECTIVE : [CAMPIGLIA AND NAGY 2021]

* TECHNICALIA : BASICS

1. (RETARDED) BONDI COORDS: $(u = t - r,\ r,\ \{z_i\})$

2. $A_\mu \sim \frac{1}{r}$ (LEADING ORDER = RADIATION)

3. $D = 4$

4 $\forall$ FIELD $\varphi(x)$: $\Box \varphi = 0$

* Technicalia: Basics

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2. $A_\mu \sim \frac{1}{r}$ (leading order = radiation)
3. $D = 4$
4. $\forall$ field $G(x)$: $\Box G = 0$

$$G_+(x) = \int_0^{+\infty} d\omega \, \frac{\omega}{2} \int d^2\vec{w} \; e^{-i\omega q^0 u - \frac{i\omega r |\vec{w} - \vec{z}|^2}{1 + |\vec{z}|^2}} \, Q(\omega q(\vec{w}))$$

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$$\omega r |\vec{w} - \vec{z}|^2 \sim O(1)$$

LEADING BEHAVIOUR AT LARGE r

EXPLORING $r \to +\infty$

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LEADING BEHAVIOUR AT LARGE r

RELEVANT FOR REGIONS INVOLVING BEHAVIOUR IN ω

$$\omega \left| \vec{w} - \vec{z} \right|^2 \sim O(1/r)$$

TWO RELEVANT REGIONS

$$\omega\,|\vec{w}-\vec{z}|^2 \sim O(1/r)$$

TWO RELEVANT REGIONS

$$|\vec{w}-\vec{z}|^2 \sim 1/r$$

COLLINEAR REGION

always brings about leading RADIATION behaviour

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$$G \sim \frac{1}{r} \, \eta^{C}(u, \vec{z})$$

$$\omega \sim 1/r$$

REGULAR REGION

overall behaviour depends on $Q(\omega \eta(\vec{w}))$:

with $Q(\omega \eta(\vec{w})) \sim \omega^{\beta-2} Q(\eta(\vec{w}))$

$$G \sim \frac{1}{r^{\beta}} \, \eta^{R}(\vec{z})$$

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ALTOGETHER, $\forall$ FIELD OR PARAMETER

$$G \sim \frac{1}{r}\,\eta^{c}(u,\vec{z}) + \frac{1}{r^{\beta}}\,\eta^{R}(\vec{z})$$

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ALTOGETHER, $\forall$ FIELD OR PARAMETER

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IN PARTICULAR: THE LEADING BEHAVIOUR DEPENDS ON β (>0)

~ ASYMPTOTICS OF SINGLE-COPY FIELDS A_μ, $\tilde{A}_\mu$, Φ

* SOFT THEOREMS $\Rightarrow$ $\beta_A = \beta_{\tilde{A}} = \beta_\Phi = 1$

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POLYHOMOGENEOUS EXPANSION

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→ CAN COMPUTE THE PREDICTION OF THE DC FOR THE ASYMPTOTIC SYMMETRIES AND CHARGES OF GRAVITON, TWO-FORM AND SCALAR

§ DC SUPERTRANSLATIONS

Preamble : Charges & Curvatures

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* For the DC

$$R_{\mu\nu\rho\sigma} = -\frac{1}{2} F_{\mu\nu} * \tilde{F}_{\rho\sigma} = R^S_{\mu\nu\rho\sigma} + R^A_{\mu\nu\rho\sigma}$$

$$H^S_{\mu\nu} = \frac{1}{2} H_{(\mu\nu)} \qquad H^A_{\mu\nu} \sim \frac{1}{2} H_{[\mu\nu]}$$

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DC - BMS SUPERTRANSLATIONS

$$Q^S = \frac{1}{8\pi G} \int d^2z \, \gamma_{z\bar{z}} \, T(z,\bar{z}) \, R^{S\,(3)}_{urur}$$

$$= \frac{-1}{32\pi G} \int d^2z \, \gamma_{z\bar{z}} \left(\varepsilon * \tilde{A}^u + A^u * \tilde{\varepsilon}\right)^{(0)} \left(F_{ur} * \tilde{F}_{ur}\right)^{(3)}$$

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THE DC IMPLEMENTS THE CORRECT ASYMPTOTIC EXPANSION OF THE PARAMETERS

OUTLOOK

- (EXTENDED) SUPERROTATIONS

  [are all gravitational AS double copies?]

- NON-LINEAR CORRECTIONS

- $D > 4$

- MORE GENERAL GAUGE THEORIES, MULTIPLE COPIES, HIGHER SPINS.