Locality & Operator Algebras in Quantum Gravity

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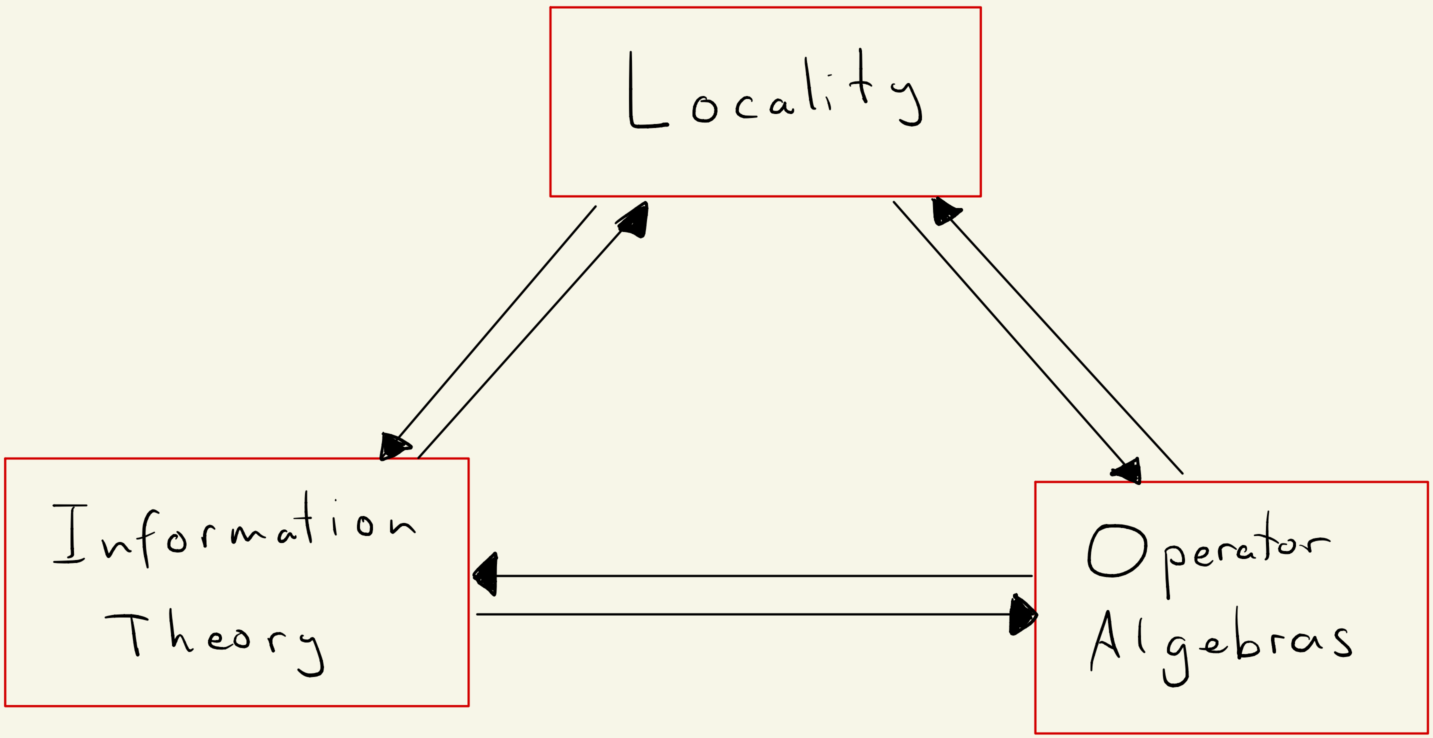
INFN

- How is information localized in Q.G?

- What type of operator algebras exist?



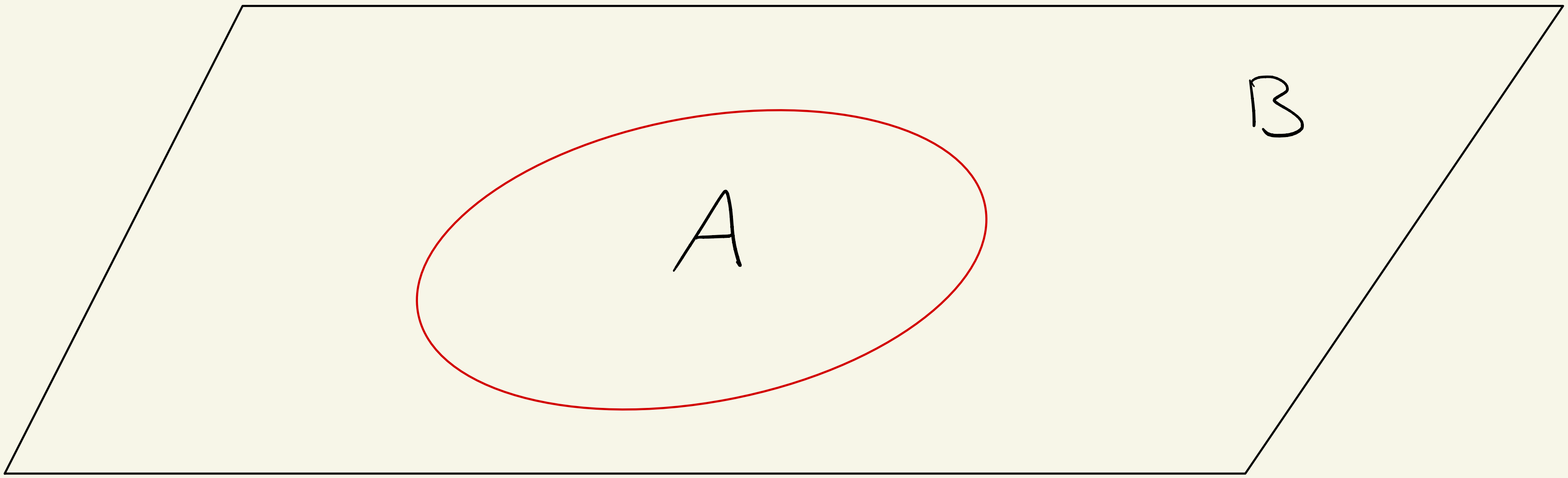
Reminder from QFT

Micro causality: $[O(x^\mu), O(0)] = 0 \quad \forall x^\mu \mid x^\mu x_\mu > 0$

$\Longrightarrow$ Information is strictly localizable in QFT

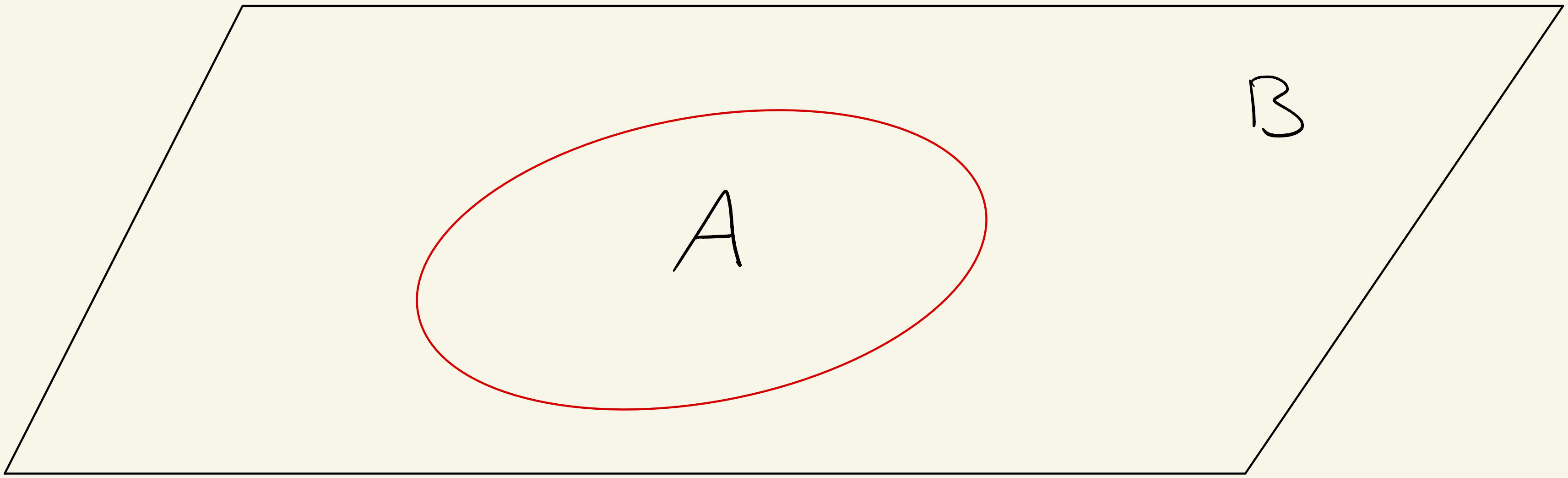
- One can specify the state independently in each subsystem

- $F \sim V_{spatial} \cdot T^d$ for QFT_d



$$|\psi'\rangle = e^{i\,O(x)}\,|\psi\rangle \qquad x \in A$$

Can we tell $|\psi\rangle$ and $|\psi'\rangle$ apart from B ?



$$|\psi'\rangle = e^{iO(x)}|\psi\rangle \qquad x \in A$$

Can we tell $|\psi\rangle$ and $|\psi'\rangle$ apart from B ?

No !

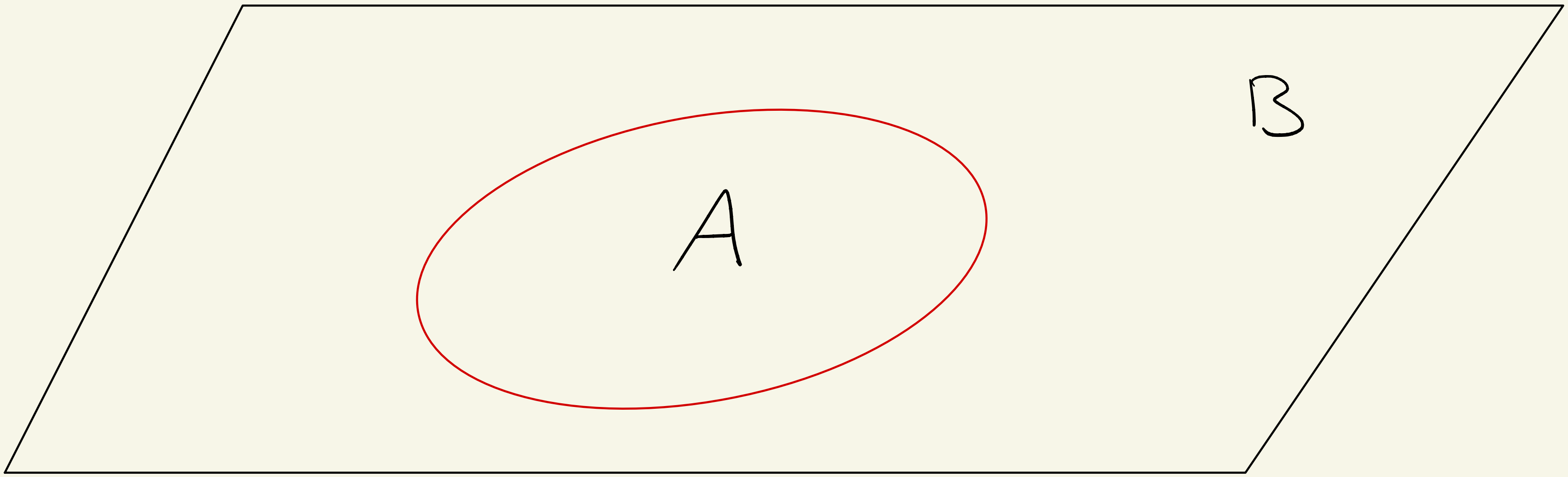
$$\langle\psi'| O_1(y_1) \ldots\ldots O_n(y_n)|\psi'\rangle$$

$$= \langle\psi| e^{-iO(x)} O_1(y_1) \ldots\ldots O_n(y_n)\, e^{iO(x)}|\psi\rangle$$

$$= \langle\psi| O_1(y_1) \ldots\ldots O_n(y_n)|\psi\rangle$$

Each subsystem is independent,

but (usually) <u>highly entangled</u>!



$$S_{EE}(A) = -\mathrm{Tr}\,\rho_A \log \rho_A = \infty$$

The Reeh-Schlieder Theorem

Consider H : $\phi_{f_1} \dots \phi_{f_n} |0\rangle$

supported only in A

$$\phi_{f_i} = \int d^d x \; f_i(x) \, \phi(x)$$

$\Rightarrow$ H is dense in $\mathcal{H}$

This does not contradict my previous statements

$\Rightarrow$ Most states (like $|0\rangle$) are very entangled!

Quantum Gravity

These properties are fundamentally altered in Quantum Gravity!

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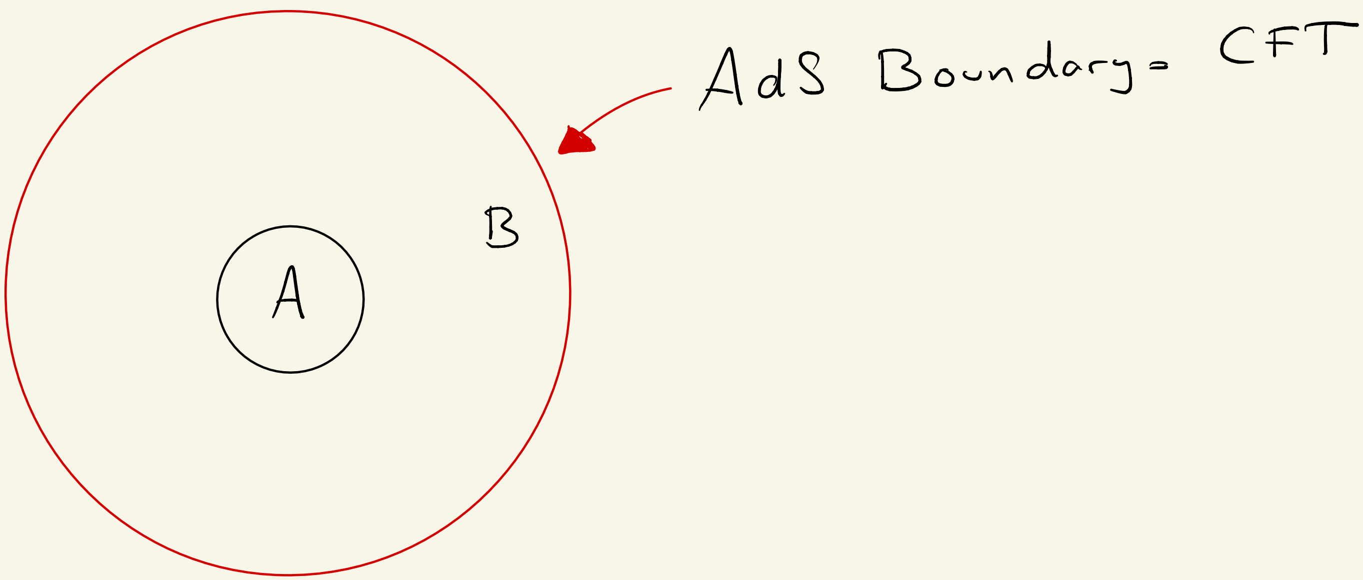
① Locality is destroyed

We cannot hide information in A anymore!



(2) $S_{EE} = \infty \longrightarrow S_{BH} = \underbrace{\frac{A}{4G_N} + S_{out}}_{\text{Finite !}}$

② $S_{EE} = \infty \longrightarrow S_{BH} = \underbrace{\frac{A}{4G_N} + S_{out}}_{\text{Finite!}}$

③ Operator Algebras must take into account Gauss' Law

$$\phi(x) \qquad \times \qquad \Rightarrow \qquad \phi(x) \qquad \times$$

- The idea of using Von Neumann algebras is to touch on these issues.

- Mathematically precise statements can be made in the strict $G_N \to 0$ limit.

- We better recover QFT in the limit $G_N \to 0$.

- There are different ways to take this limit, and we would like to control the breaking for $G_N \neq 0$.

OUTLINE

1. Introduction
2. Von Neumann Algebras in Q. G.
3. Stringy effects ?
4. Localizing information in Q. G.
5. Conclusion

Von Neumann Algebras

[Review Witten]

$\mathcal{A}$: "Weakly closed $\star$-subalgebra" of bounded op. on $\mathcal{H}$

Von Neumann Algebras

[Review Witten]

$\mathcal{A}$: "Weakly closed $\star$ - subalgebra" of bounded op. on $\mathcal{H}$

limits are in the algebra

$a^\dagger \in \mathcal{A}$

Sums and products are in $\mathcal{A}$

Von Neumann Algebras come in different types

Many of these types appear in Q. G.

Type I

$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \qquad \mathcal{A} = \mathcal{B}(\mathcal{H}_1)$

$\Rightarrow$ Type I factor

center of $\mathcal{A} \propto \mathbb{1}$

I_d if $\dim \mathcal{H}_1 = d < \infty$

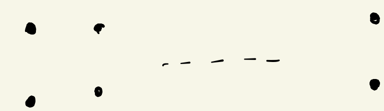
I_∞ if $\dim \mathcal{H}_1 = \infty$

$\Rightarrow \exists$ tr, which may not be defined on all of $\mathcal{A}$ for I_∞

Type II

Consider $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\right)$

$$|\Psi\rangle = \underbrace{|\psi\rangle \otimes \cdots \otimes |\psi\rangle}_{n \text{ times}}, \quad n \to \infty$$



$\mathcal{H}$:= All but finitely many of the qubit pairs are in $|\psi\rangle$.

$\mathcal{A}$:= All operators acting on finitely many qubits

The Hilbert Space does not factorize

$$\mathcal{H} = \mathcal{H}_{top} \otimes \mathcal{H}_{bottom}$$

But : There is a trace!

$$\mathrm{tr}\, a := \langle \Psi | a | \Psi \rangle \qquad \text{finite}$$

$$\mathrm{tr}(a^\dagger a) > 0$$

$$\mathrm{tr}(ab) = \mathrm{tr}(ba)$$

So states in $\mathcal{H}$ all have infinite entanglement entropy, But, we are looking at states with a finite difference.

This was really a II_1 V.N. algebra.

$\exists$ II_∞ V.N. algebra

$$\mathrm{II}_1 \otimes B(\mathcal{H}_\infty)$$

$$\mathrm{tr}(\mathbb{1}) = \infty$$

This type of algebra also appears in Q. 6.

Type III

This is what happens in QFT.

Replace $|\psi\rangle \longrightarrow |\phi\rangle$ only partially entangled

$$|\phi\rangle = \sqrt{\lambda}\,|\uparrow\uparrow\rangle + \sqrt{1-\lambda}\,|\downarrow\downarrow\rangle$$

$$\frac{1}{2} < \lambda < 1$$

Also here $\mathcal{H} \neq \mathcal{H}_{top} \otimes \mathcal{H}_{bottom}$,

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type II_1 type I

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Where do these algebras appear?

Type I_∞ $\Rightarrow$ $\mathcal{N}=4$ SYM at finite N

Put the theory on S^3

$\Rightarrow$ QM with $\dim \mathcal{H} = \infty$

⚠ If you consider a subsystem of S^3 $\Rightarrow$ III

Type III_1

$\mathcal{N}=4$ SYM, as $\begin{matrix} G_N \to 0 \\ N \to \infty \end{matrix}$, large λ

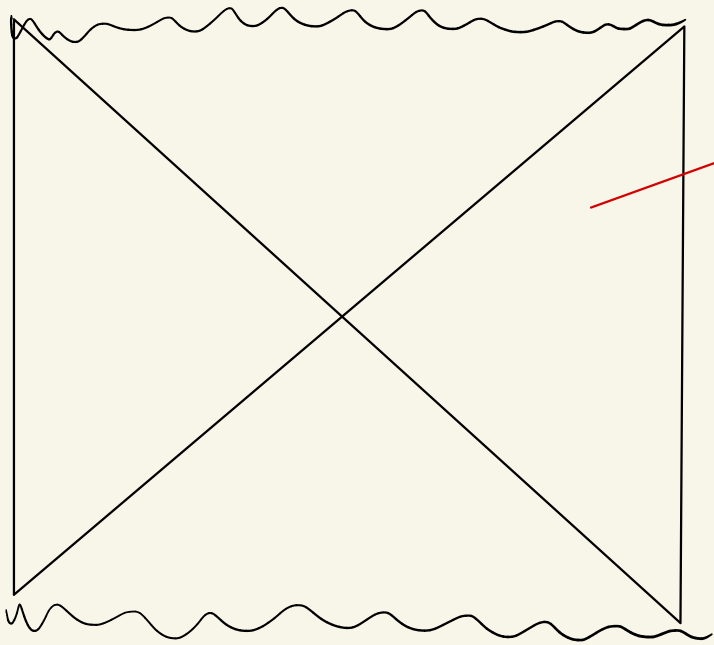
$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |n\rangle_L \otimes |n\rangle_R$$

$\beta \leqslant \beta_{HP} \Rightarrow$ Black Hole

The algebra of S.T. operators acting on $\mathcal{H}_L$

is a type III_1 V.N. algebra.

[Leutheusser, Liu]



$G_N \to 0$

QFT on exterior of AdS BH

Horizon $\sim$ Rindler Horizon

$\Rightarrow$ continuous spectral density

The large N limit has washed away the discrete spectrum of the type I algebra at finite N.

$\Rightarrow$ Connection to information loss

[Maldacena; Furuya, Lashkari, Moosa, Ouseph]

$$S_{BH} = \frac{A}{4G_N} + S_{out}$$

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frozen

$\hookrightarrow = \infty$ in type III_1

What about $1/N$ or G_N corrections?

With first order corrections: [Witten]

$$\mathrm{III}_1 \longrightarrow \mathrm{II}_\infty$$

$\Rightarrow$ Careful treatment of the Hamiltonian mode

$$U = \frac{1}{N}\left(H_L - \langle H_L \rangle\right)$$

A nice fact about type II algebras

$\Rightarrow$ Entanglement entropies are defined up to an additive constant, state-independent

$$|\Psi\rangle = O_{S.T.} \cdots O_{S.T.} |TFD\rangle$$

$$\Delta S_{EE} = \frac{\Delta A}{4G_N} + \Delta S_{out}$$

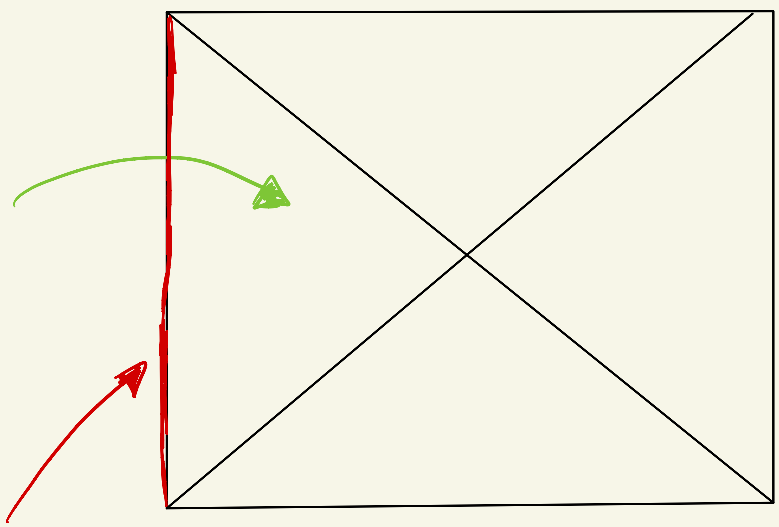
These differences are captured by the type II_∞ algebra

II_1 V.N. algebras

Appears in dS space, in the presence of an observer

[Chandrasekaran, Longo, Penington, Witten]

Static Patch



Worldline of an observer

The observer is crucial, to define diff-invariant operators, they need to be anchored somewhere

$$\hat{H} = H + H_{obs} = H + q \qquad q \geq 0$$

$$\mathcal{A} = \{ e^{ipH} a e^{-ipH} , q \} \qquad p = -i \frac{d}{dq}$$

Simplest toy-model for an observer, 1 d.o.f.

CLPW showed that the algebra is II_1

Gives an algebraic interpretation to the dS Entropy (or really differences between states)

Many generalizations

- Observers in AdS, closed regions

  [Jensen, Sorce, Speranza]

- Observer $\longrightarrow$ Inflaton

  [Chen, Penington]

- ......

3 Stringy effects

The previous picture was valid as $N \to \infty$, and also at large λ.

What happens at finite λ (still large N)?

③ Stringy effects

The previous picture was valid as $N \to \infty$, and also at large λ.

What happens at finite λ (still large N)?

Many properties still hold:

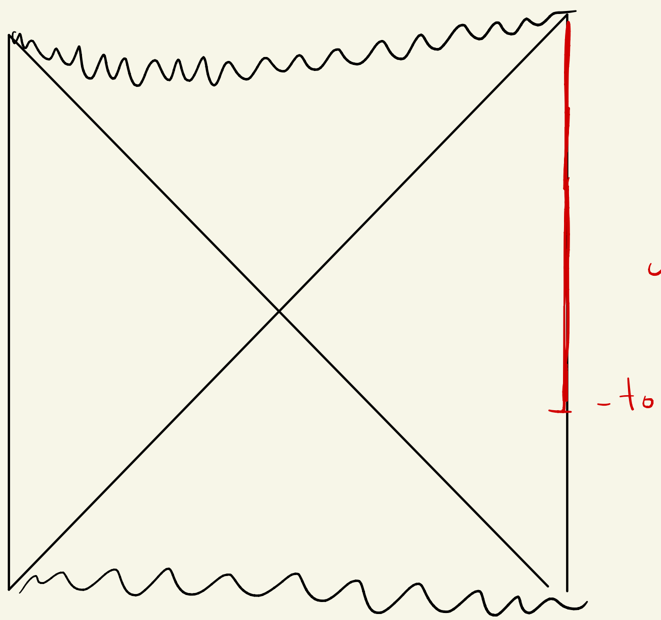
- Deconfinement transition at some T_{HP}
- We expect a continuous spectral density in $\langle O(t)\, O(0)\rangle_\beta$ ($\forall \lambda \neq 0$)
- Algebra satisfies large N factorization

However, the bulk picture should drastically change at finite α'.

Does the BH horizon become fuzzy?

Does it become probe-dependent?

[Gesteau, Liu]



$\mathcal{A}$ of S.T. op in $[-t_0, \infty[$

$\mathcal{T} = t_0$ s.t. $\mathcal{A}$ strict subalgebra of $\mathcal{A}_R$.

$\mathcal{T} = \infty$ at $\lambda = \infty$

τ is extractable from $\langle O(t)\, O(0) \rangle_\beta$,

i.e. from its spectral density $\mathcal{S}(\omega)$.

At any $\lambda \neq 0$, we expect $\mathcal{S}(\omega)$ to have continuous support.

[Gesteau, Liu] studied various prototypes of $\mathcal{S}(\omega)$, and their implications for $\tilde{\tau}$.

But the spectral density of a fixed set of operators cannot be the whole story.

In $d=2$, in the D1D5 CFT, $\lambda = 0$ is the symmetric orbifold point.

$$\mathcal{C} = (\mathbb{T}^4)^{\otimes N} / S_N$$

One can show:

$$\boxed{\langle O(t)\, O(0) \rangle_\beta = \langle O(t)\, O(0) \rangle_{BTZ} + \mathcal{O}(1/N) \quad \forall\, O}$$

[AB, Bintanja, Castro, Knop]

So for any fixed S.T. operator, we have a type III_1 V.N. algebra.

But we have an infinite tower of them!

$$\rho_{S.T.}(\Delta) \sim e^{2\pi\Delta}$$ [Keller]

How do we resum this tower?

Can the algebra still be understood?

There have been computations of Entanglement Entropy in string theory [Dabholkar, Moitra]

The computations proceed via the replica trick, and there are some subtle steps to deal with the analytic continuation, but they find:

$$\boxed{S_{EE}^{1\text{-loop}} = \text{finite}} \implies$$

This suggests the algebra is type I already at finite λ, infinite N.

④ Localizing Information in Q.G.

We already discussed that non-perturbatively (i.e. finite N), Q.G. localizes information drastically differently

Does this breakdown of locality occur only via non-perturbative (i.e. e^{-N^2}) effects?

Or does locality break down in G_N-pert. thy?

The idea of holography of information is that this breakdown happens already in G_N-pert. theory, because of the grav. Gauss law.

This has been shown explicitly around the AdS vacuum

[Chowdhury, Godet, Papadoulaki, Raju]

But the AdS vacuum is a very special state, preserving all symmetries.

Already in classical GR, there are no local diff-invariant observables around maximally-symmetric spaces.

$\Rightarrow$ I would like to show an explicit construction of localized information in AdS/CFT

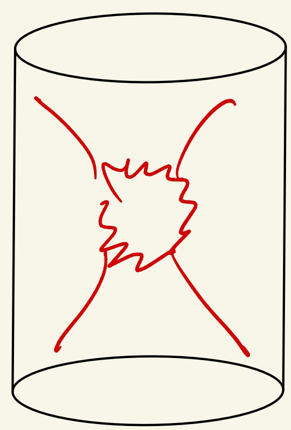
Setup

Consider a CFT state $|\psi\rangle$:

$\langle\psi| H |\psi\rangle \sim N^2 \longrightarrow$ strong backreaction

$\langle\psi| \Delta H^2 |\psi\rangle \sim N^2 \longrightarrow$ classical time-dependence

Ex: a supernova explosion in AdS



These states can be built with the Euclidean path integral

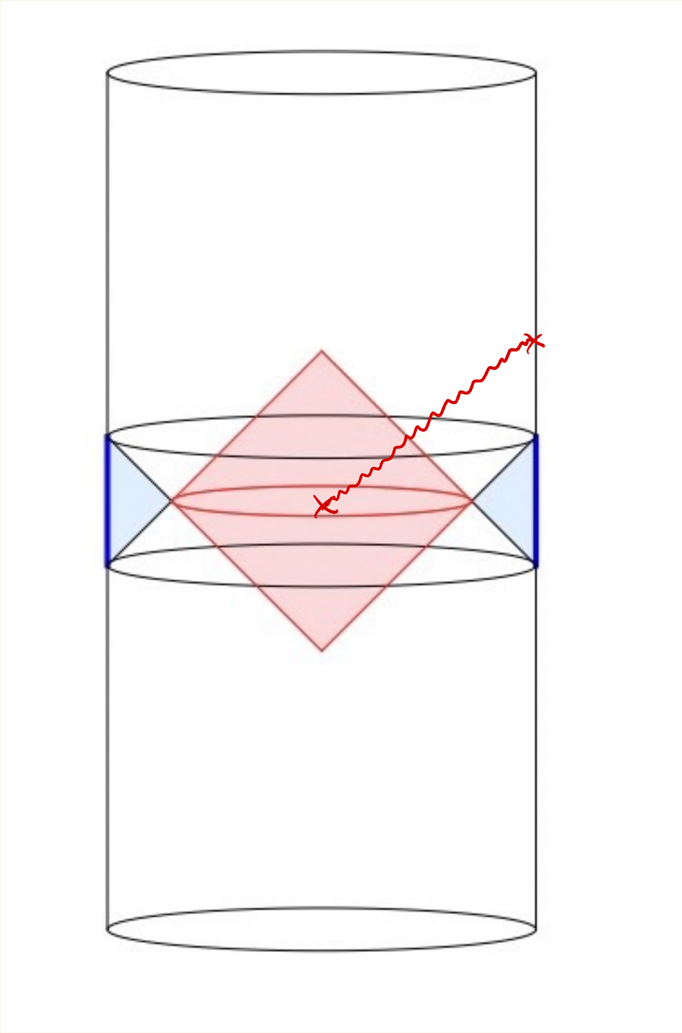
Question

$\mathcal{A}$: the "algebra" of S.T. operators in a time band $t \in [-t_0, t_0]$

Can we find O such that

1. $[O, a] = 0$ to all orders in $1/N$, $\forall a \in \mathcal{A}$

2. O creates a particle that we detect at some $t > t_0$



Answer : Yes ?

Procedure :

Start with $\phi_{HKLL} = \int dt' dx' \, K(t, t', x, x', z) \, O(t', z')$

Problem: $[\phi_{HKLL}, H_{CFT}] \sim \frac{1}{N} \neq 0$

$\hookrightarrow$ Boundary dressed.

$$\boxed{\hat{\underline{\Phi}} = \int_{-t^*}^{t^*} dT \, e^{-iTH} P_0 \, \phi_{HKLL} \, P_0 \, e^{iTH}}$$

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t^* → $O(1)$ timescale

P_0 → Projector onto code subspace of $|\psi\rangle$

36 [Bahiru, AB, Papadodimas, Sarosi, Vardian]

Using $|\langle\psi(t)|\psi\rangle|^2 \sim e^{-t^2 \Delta H^2} \sim e^{-t^2 N^2}$

One can show:

1. $\langle\psi|[H, \hat{\underline{\Phi}}]|\psi\rangle \sim \mathcal{O}(e^{-N^2})$

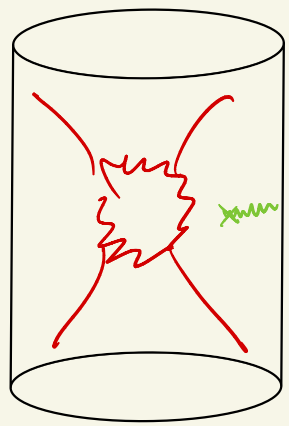
2. $\langle\psi| O_1 \ldots \hat{\underline{\Phi}} \ldots O_n |\psi\rangle = \langle\psi| O_1 \ldots \phi_{HKLL} \ldots O_n |\psi\rangle$
$$+ \mathcal{O}(1/N)$$

$\Rightarrow$ to leading order in $1/N$, $\hat{\underline{\Phi}}$ also creates a particle that will be detectable at $t > t_0$

The interpretation:

ϕ_{HKLL} was a bulk operator that was dressed to the boundary.

$\hat{\phi}$ is an operator that is dressed to a feature of the state. It is "state-dressed".



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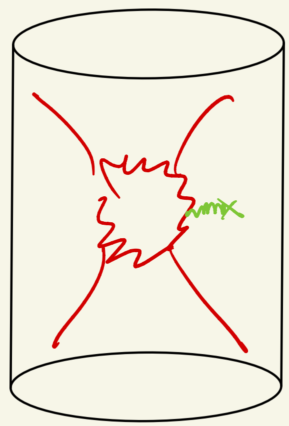

Comments

- This does not contradict Chowdury et al, because $|0\rangle$ is not in our class of states

- This can only work if the state breaks all the symmetries, like in classical GR.

- It would be nice to know if the timeband algebra can be made into a proper V.N. algebra.

Conclusion

- Operator algebras are an interesting framework to probe semi-classical Q. G.

- Provide an algebraic interpretation of the Bekenstein-Hawking formula for BH entropy.

- They make mathematically precise statements on how locality emerges as $G_N \to 0$, in agreement with our intuition that we live in a local world.

Thank You!

And long Live fried Pizza!