Constraints from Superconformal Symmetry

SCWI for higher point functions in $\mathcal{N} = 4$ SYM

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Based on WIP w Carlo Meneghelli[1]

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Outline

1. **Motivation:**
   Why higher point functions? Why SCWI?
2. **4pt functions:**
   Obtaining the SCWI, comparison with known results
3. **5pt functions:**
   some results, some difficulties, some WIP

Motivation

Why higher point correlators?

CFT data : $\{\Delta, \lambda_{ijk}\}$

$$\langle \mathcal{O}_{\Delta_i}(x_i)\, \mathcal{O}_{\Delta_j}(x_j) \rangle = \frac{1}{|x_{ij}|^{2\Delta}}\,, \quad \Delta_i = \Delta_j = \Delta$$

$$\langle \mathcal{O}_{\Delta_i}(x_i)\, \mathcal{O}_{\Delta_j}(x_j)\, \mathcal{O}_{\Delta_k}(x_k) \rangle = \frac{\lambda_{ijk}}{|x_{ij}|^{\Delta_i+\Delta_j-\Delta_k}\, |x_{jk}|^{\Delta_j+\Delta_k-\Delta_i}\, |x_{ki}|^{\Delta_k+\Delta_i-\Delta_j}}$$

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$$\mathcal{O}(x)\mathcal{O}(0) \sim \sum_{\mathcal{O}'_{\Delta,l}} \lambda_{\mathcal{O}\mathcal{O}\mathcal{O}'}\, \mathcal{C}_{\mathcal{O}'}(x, \partial)\, \mathcal{O}'_{\Delta,l}$$

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$$\langle \mathcal{B}\mathcal{B}\mathcal{B}\mathcal{B}\mathcal{B}\rangle \sim \sum_{\mathcal{A}} \lambda_{\mathcal{B}\mathcal{B}\mathcal{A}}\,\langle \mathcal{B}\mathcal{B}\mathcal{B}\mathcal{A}\rangle$$

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1. Constraints of underlying symmetry group - Ward identities
2. Consistency conditions, e.g crossing symmetry

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Set the stage

$\mathfrak{psu}(2,2|4)$ and its analytic superspace

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- $\mathfrak{su}(2,2) \times \mathfrak{su}(4)_R$
- $\begin{cases} Q_{i\,\alpha}, \bar{Q}^{i}{}_{\dot\alpha} \\ S^{i}{}_{\alpha}, \bar{S}_{i\,\dot\alpha} \end{cases}$, $i = 1, ..., 4;\ \alpha, \dot\alpha = 1, 2$

$$\begin{pmatrix} D, P_\mu, K_\mu, M_{\mu,\nu} & Q, \bar{S} \\ \bar{Q}, S & R^{I}{}_{J} \end{pmatrix}$$

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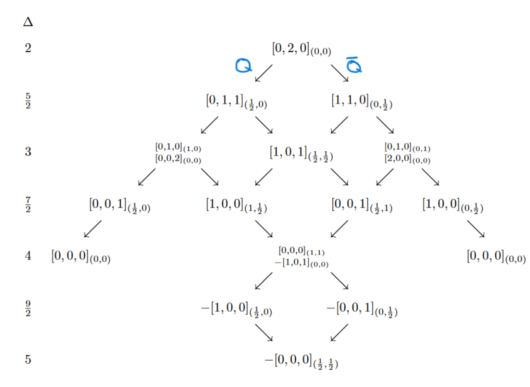
Analytic superspace

$$X_{A\dot{A}} = \begin{pmatrix} x^{\alpha\dot\alpha} & \rho^{\alpha\dot a} \\ \bar\rho^{a\dot\alpha} & y^{a\dot a} \end{pmatrix} \in \mathrm{Mat}(2|2)$$

- $\alpha, \dot\alpha = 1, 2, \ a, \dot a = 1, 2$
- $x^{\alpha\dot\alpha} = (x^\mu \sigma_\mu)^{\alpha\dot\alpha}$: Minkowski
- $y^{a\dot a}$: Internal space
- $\rho^{\alpha\dot a}$, $\bar\rho^{\dot\alpha a}$: Grassmann-odd coord.

Stress tensor multiplet

[0,2,0]- multiplet of $\mathfrak{su}(4)_R$



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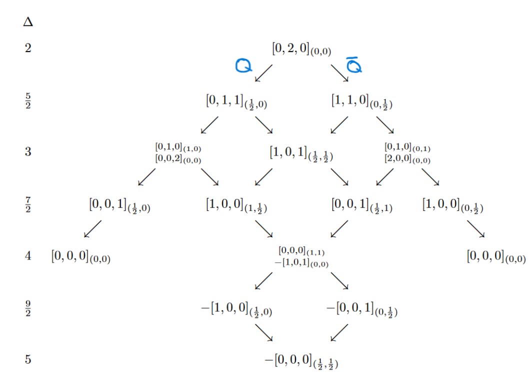

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stress-tensor multiplet

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\begin{aligned}
\mathcal{T}(X) &= \left(\exp\left(\rho^{\alpha\dot{a}}\mathcal{Q}_{\alpha\dot{a}} + \bar{\rho}^{a\dot{\alpha}}\bar{\mathcal{Q}}_{a\dot{\alpha}}\right)\right)\mathcal{O}_{20'} \\
&= \mathcal{O}_{20'}(x,y) \\
&\quad + \rho_{\alpha\dot{a}}\Psi_{\alpha\dot{a}}(x,y) + \bar{\rho}^{a\dot{\alpha}}\bar{\Psi}_{a\dot{\alpha}}(x,y) \\
&\quad + \rho^{\alpha\dot{a}}\bar{\rho}^{a\dot{\alpha}}\hat{\mathcal{J}}_{\alpha\dot{\alpha};a\dot{a}}(x,y) \\
&\quad + \rho^2\bar{F}(x,y) + \bar{\rho}^2 F(x,y) \\
&\quad + \rho^2\bar{\rho}^{a\dot{\alpha}}B_{a\dot{\alpha}}(x,y) + \ldots
\end{aligned}
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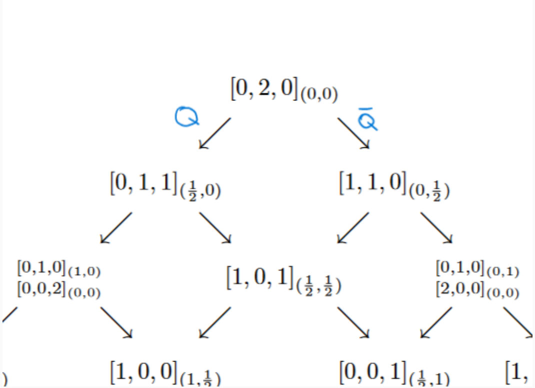

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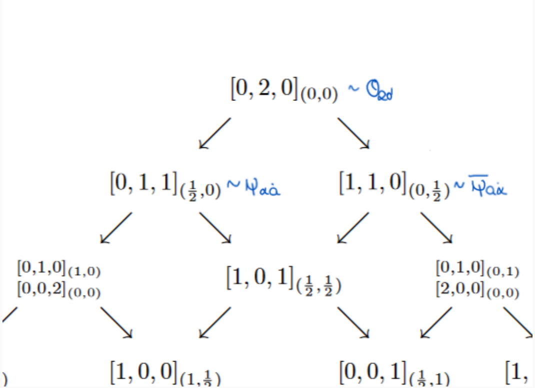

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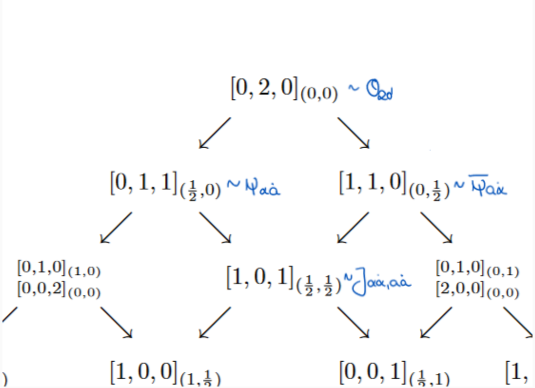

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\end{aligned}
$$

$$
\begin{aligned}
\hat{\mathcal{J}}_{\alpha\dot{\alpha},a\dot{a}} =& \mathcal{J}_{\alpha\dot{\alpha},a\dot{a}}(x,y) \\
& - \frac{1}{2}\frac{\partial}{\partial y^{a\dot{a}}}\frac{\partial}{\partial x^{\alpha\dot{\alpha}}}\mathcal{O}_{20'}(x,y)
\end{aligned}
$$

Correction term at order $\rho\bar{\rho}$ - Origin

- **Very schematically** : $\left\{\mathcal{Q},\bar{\mathcal{Q}}\right\} \sim P \sim \frac{\partial}{\partial x} \Rightarrow \left\{\boldsymbol{\mathcal{Q}_{\alpha\dot{a}}},\boldsymbol{\bar{\mathcal{Q}}_{a\dot{\alpha}}}\right\} \sim \frac{\boldsymbol{\partial}}{\boldsymbol{\partial x_{\alpha\dot{\alpha}}}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial y_{a\dot{a}}}}$
- **Constraint on multiplet:**

$$\partial^{p+1}\mathcal{W}_p(X) = 0$$

$$\partial \equiv (-1)^{|A|}\bar{\xi}^{\dot{A}}\frac{\partial}{\partial X^{A\dot{A}}}\xi^{A}$$

- Covariantization of $\left(\frac{\partial}{\partial y^{a\dot{a}}}\right)^{p+1}\mathcal{W}_p = 0$
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$$\Rightarrow \hat{\mathcal{J}}_{\alpha\dot{\alpha},a\dot{a}} = \mathcal{J}_{\alpha\dot{\alpha},a\dot{a}}(x,y) - \frac{1}{2}\frac{\partial}{\partial y^{a\dot{a}}}\frac{\partial}{\partial x^{\alpha\dot{\alpha}}}\mathcal{O}_{20'}(x,y)$$

Summary stress tensor multiplet on analytic superspace

$$
\begin{aligned}
\mathcal{T}(X) =& \mathcal{O}_{20'}(x,y) \\
& + \rho_{\alpha\dot{a}} \Psi_{\alpha\dot{a}}(x,y) + \bar{\rho}^{a\dot{\alpha}} \bar{\Psi}_{a\dot{\alpha}}(x,y) \\
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\end{aligned}
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Correlation functions

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$$\langle \mathcal{T}(\boldsymbol{X}_1)\,\mathcal{T}(\boldsymbol{X}_2)\,\mathcal{T}(\boldsymbol{X}_3)\,\mathcal{T}(\boldsymbol{X}_4)\rangle =$$

$$= \langle \mathcal{O}_{20'}(x_1, y_1)\,\mathcal{O}_{20'}(x_2, y_2)\,\mathcal{O}_{20'}(x_3, y_3)\,\mathcal{O}_{20'}(x_4, y_4)\rangle$$

$$+ \sum_{i=1}^{4} \rho_i^{\alpha\dot{a}}\,\bar{\rho}_i^{a\dot{\alpha}}\,\langle \mathcal{J}_{\alpha\dot{\alpha};a\dot{a}}(x_i, y_i)\prod_{k\neq i}\mathcal{O}_{20'}(x_k, y_k)\rangle$$

$$- \frac{1}{2}\sum_{i=1}^{4} \rho_i^{\alpha\dot{a}}\,\bar{\rho}_i^{a\dot{\alpha}}\,\frac{\partial}{\partial x_i^{\alpha\dot{\alpha}}}\,\frac{\partial}{\partial y_i^{a\dot{a}}}\,\langle \mathcal{O}_{20'}(x_1, y_1)\,\mathcal{O}_{20'}(x_2, y_2)\,\mathcal{O}_{20'}(x_3, y_3)\,\mathcal{O}_{20'}(x_4, y_4)\rangle$$

$$+ \sum_{i=1}^{4}\sum_{j\neq i} \rho_i^{\alpha\dot{a}}\,\bar{\rho}_j^{a\dot{\alpha}}\,\langle \Psi_{\alpha\dot{a}}(x_i, y_i)\,\bar{\Psi}_{a\dot{\alpha}}(x_j, y_j)\,\mathcal{O}_{20'}(x_k, y_k)\,\mathcal{O}_{20'}(x_l, y_l)\rangle$$

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$$- \frac{1}{2}\sum_{i=1}^{4} \rho_i^{\alpha\dot{a}} \bar{\rho}_i^{a\dot{\alpha}} \frac{\partial}{\partial x^{\alpha\dot{\alpha}}} \frac{\partial}{\partial y^{a\dot{a}}} \langle \mathcal{O}_{20'}(x_1, y_1)\,\mathcal{O}_{20'}(x_2, y_2)\,\mathcal{O}_{20'}(x_3, y_3)\,\mathcal{O}_{20'}(x_4, y_4)\rangle$$

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&\quad - \frac{1}{2}\sum_{i=1}^{4} \rho_i^{\alpha\dot{a}}\bar{\rho}_i^{a\dot{\alpha}} \frac{\partial}{\partial x^{\alpha\dot{\alpha}}}\frac{\partial}{\partial y^{a\dot{a}}} \langle \mathcal{O}_{20'}(x_1,y_1)\,\mathcal{O}_{20'}(x_2,y_2)\,\mathcal{O}_{20'}(x_3,y_3)\,\mathcal{O}_{20'}(x_4,y_4)\rangle \\
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correlator $= \sum$ structures $\times$ unspecified function of the invariants

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Conformal invariance: $SU(2,2)$

2pt function: $\langle \mathcal{O}_{\Delta_1}(x_1)\mathcal{O}_{\Delta_2}(x_2)\rangle = \frac{1}{x_{12}^{2\Delta}} \cdot c, \quad \Delta_1 = \Delta_2 \equiv \Delta$

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R-symmetry invariance: $SU(4)_R \leftrightarrow [0,2,0]^{\otimes 4} \supset 6$ singlets

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$+ y_{12}^2 y_{14}^2 y_{23}^2 y_{34}^2 + y_{13}^2 y_{14}^2 y_{23}^2 y_{24}^2$

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$$\langle \mathcal{O}_{20'}(1)\mathcal{O}_{20'}(2)\mathcal{O}_{20'}(3)\mathcal{O}_{20'}(4)\rangle = \frac{y_{12}^4 y_{34}^4}{x_{12}^4 x_{34}^4} f_1(u,v) + \frac{y_{13}^4 y_{24}^4}{x_{13}^4 x_{24}^4} f_2(u,v) + \frac{y_{14}^4 y_{23}^4}{x_{14}^4 x_{23}^4} f_3(u,v)$$
$$+ \frac{y_{12}^2 y_{13}^2 y_{24}^2 y_{34}^2}{x_{12}^2 x_{13}^2 x_{24}^2 x_{34}^2} f_4(u,v) + \frac{y_{12}^2 y_{14}^2 y_{23}^2 y_{34}^2}{x_{12}^2 x_{14}^2 x_{23}^2 x_{34}^2} f_5(u,v) + \frac{y_{13}^2 y_{14}^2 y_{23}^2 y_{24}^2}{x_{13}^2 x_{14}^2 x_{23}^2 x_{24}^2} f_6(u,v)$$

Step 2 : Impose bosonic invariance

$$\langle \mathcal{O}_{20'}(1)\mathcal{O}_{20'}(2)\mathcal{O}_{20'}(3)\mathcal{O}_{20'}(4)\rangle = \frac{y_{12}^4 y_{34}^4}{x_{12}^4 x_{34}^4} f_1(u,v) + \frac{y_{13}^4 y_{24}^4}{x_{13}^4 x_{24}^4} f_2(u,v) + \frac{y_{14}^4 y_{23}^4}{x_{14}^4 x_{23}^4} f_3(u,v)$$
$$+ \frac{y_{12}^2 y_{13}^2 y_{24}^2 y_{34}^2}{x_{12}^2 x_{13}^2 x_{24}^2 x_{34}^2} f_4(u,v) + \frac{y_{12}^2 y_{14}^2 y_{23}^2 y_{34}^2}{x_{12}^2 x_{14}^2 x_{23}^2 x_{34}^2} f_5(u,v) + \frac{y_{13}^2 y_{14}^2 y_{23}^2 y_{24}^2}{x_{13}^2 x_{14}^2 x_{23}^2 x_{24}^2} f_6(u,v)$$

$$\langle \mathcal{J}_{\alpha\dot\alpha,a\dot a}(1)\mathcal{O}_{20'}(2)\mathcal{O}_{20'}(3)\mathcal{O}_{20'}(4)\rangle \ :\sim y_{1i}^2 y_{1j}^2 (y_{1i}^{-1} y_{ij} y_{j1}^{-1})_{\dot a a}\,(x_{1i}^{-1} x_{ij} x_{j1}^{-1})_{\dot\alpha\alpha}\, g(u,v)$$
$$\langle \Psi_{\alpha\dot a}(1)\bar\Psi_{a\dot\alpha}(2)\mathcal{O}_{20'}(3)\mathcal{O}_{20'}(4)\rangle \ :\sim y_{1i}^2 y_{2j}^2 (y_{1i}^{-1} y_{ij} y_{j2}^{-1})_{\dot a a}\,(x_{1i}^{-1} x_{ij} x_{j2}^{-1})_{\dot\alpha\alpha}\, h_1(u,v)$$
$$\sim y_{12}^2 (y_{12}^{-1})_{\dot a a}\,(x_{12}^{-1})_{\dot\alpha\alpha}\, h_2(u,v)$$

Step 2 : Impose bosonic invariance

$$\langle \mathcal{O}_{20'}(1)\mathcal{O}_{20'}(2)\mathcal{O}_{20'}(3)\mathcal{O}_{20'}(4)\rangle = \frac{y_{12}^4 y_{34}^4}{x_{12}^4 x_{34}^4} f_1(u,v) + \frac{y_{13}^4 y_{24}^4}{x_{13}^4 x_{24}^4} f_2(u,v) + \frac{y_{14}^4 y_{23}^4}{x_{14}^4 x_{23}^4} f_3(u,v)$$
$$+ \frac{y_{12}^2 y_{13}^2 y_{24}^2 y_{34}^2}{x_{12}^2 x_{13}^2 x_{24}^2 x_{34}^2} f_4(u,v) + \frac{y_{12}^2 y_{14}^2 y_{23}^2 y_{34}^2}{x_{12}^2 x_{14}^2 x_{23}^2 x_{34}^2} f_5(u,v) + \frac{y_{13}^2 y_{14}^2 y_{23}^2 y_{24}^2}{x_{13}^2 x_{14}^2 x_{23}^2 x_{24}^2} f_6(u,v)$$

$$\langle \mathcal{J}_{\alpha\dot\alpha,a\dot a}(1)\mathcal{O}_{20'}(2)\mathcal{O}_{20'}(3)\mathcal{O}_{20'}(4)\rangle \;:\sim y_{1i}^2 y_{1j}^2 (y_{1i}^{-1} y_{ij} y_{j1}^{-1})_{\dot a a}\,(x_{1i}^{-1} x_{ij} x_{j1}^{-1})_{\dot\alpha\alpha}\, g(u,v)$$

$$\langle \Psi_{\alpha\dot a}(1)\bar\Psi_{a\dot\alpha}(2)\mathcal{O}_{20'}(3)\mathcal{O}_{20'}(4)\rangle \;:\sim y_{1i}^2 y_{2j}^2 (y_{1i}^{-1} y_{ij} y_{j2}^{-1})_{\dot a a}\,(x_{1i}^{-1} x_{ij} x_{j2}^{-1})_{\dot\alpha\alpha}\, h_1(u,v)$$
$$\sim y_{12}^2 (y_{12}^{-1})_{\dot a a}\,(x_{12}^{-1})_{\dot\alpha\alpha}\, h_2(u,v)$$

$\langle \mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\rangle$	:	6 structures
$\langle \mathcal{J}_{\alpha\dot\alpha,a\dot a}\mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\rangle$	:	$4\cdot 3\cdot 2 = 24$ structures
$\langle \Psi_{\alpha\dot a}\bar\Psi_{a\dot\alpha}\mathcal{O}_{20'}\mathcal{O}_{20'}\rangle$	:	$4\cdot 3\cdot 6\cdot 2 = 144$ structures

$\Rightarrow$ **$6 + 168$ unknown functions**

Step 3 : Impose supersymmetric invariance

$$\langle \mathcal{T}(X_1)\,\mathcal{T}(X_2)\,\mathcal{T}(X_3)\,\mathcal{T}(X_4)\rangle =$$

$$= \langle \mathcal{O}_{20'}(1)\,\mathcal{O}_{20'}(2)\,\mathcal{O}_{20'}(3)\,\mathcal{O}_{20'}(4)\rangle$$

$$+ \sum_{i=1}^{4} \rho_i^{\alpha\dot{a}}\bar{\rho}_i^{a\dot{\alpha}} \langle \mathcal{J}_{\alpha\dot{\alpha};a\dot{a}}(i) \prod_{k\neq i} \mathcal{O}_{20'}(k)\rangle$$

$$- \frac{1}{2}\sum_{i=1}^{4} \rho_i^{\alpha\dot{a}}\bar{\rho}_i^{a\dot{\alpha}} \frac{\partial}{\partial x^{\alpha\dot{\alpha}}}\frac{\partial}{\partial y^{a\dot{a}}} \langle \mathcal{O}_{20'}(1)\,\mathcal{O}_{20'}(2)\,\mathcal{O}_{20'}(3)\,\mathcal{O}_{20'}(4)\rangle$$

$$+ \sum_{i=1}^{4}\sum_{j\neq i} \rho_i^{\alpha\dot{a}}\bar{\rho}_j^{a\dot{\alpha}} \langle \Psi_{\alpha\dot{a}}(i)\,\bar{\Psi}_{a\dot{\alpha}}(j)\,\mathcal{O}_{20'}(k)\,\mathcal{O}_{20'}(l)\rangle$$

$$+ \ldots$$

$$0 = \sum_{i=1}^{4} \frac{\partial}{\partial \rho_i^{\alpha\dot{a}}} \langle \ldots \rangle$$

$$0 = \sum_{i=1}^{4} \frac{\partial}{\partial \bar{\rho}_i^{a\dot{\alpha}}} \langle \ldots \rangle$$

Step 3 : Impose supersymmetric invariance

$$
\begin{aligned}
&\langle \mathcal{T}(X_1)\,\mathcal{T}(X_2)\,\mathcal{T}(X_3)\,\mathcal{T}(X_4)\rangle = \\
&\quad = \langle \mathcal{O}_{20'}(1)\,\mathcal{O}_{20'}(2)\,\mathcal{O}_{20'}(3)\,\mathcal{O}_{20'}(4)\rangle \\
&\qquad + \sum_{i=1}^{4} \rho_i^{\alpha\dot{a}} \bar{\rho}_i^{a\dot{\alpha}} \langle \mathcal{J}_{\alpha\dot{\alpha};a\dot{a}}(i) \prod_{k\neq i} \mathcal{O}_{20'}(k)\rangle \\
&\qquad - \frac{1}{2}\sum_{i=1}^{4} \rho_i^{\alpha\dot{a}} \bar{\rho}_i^{a\dot{\alpha}} \frac{\partial}{\partial x^{\alpha\dot{\alpha}}} \frac{\partial}{\partial y^{a\dot{a}}} \langle \mathcal{O}_{20'}(1)\,\mathcal{O}_{20'}(2)\,\mathcal{O}_{20'}(3)\,\mathcal{O}_{20'}(4)\rangle \\
&\qquad + \sum_{i=1}^{4} \sum_{j\neq i} \rho_i^{\alpha\dot{a}} \bar{\rho}_j^{a\dot{\alpha}} \langle \Psi_{\alpha\dot{a}}(i)\,\bar{\Psi}_{a\dot{\alpha}}(j)\,\mathcal{O}_{20'}(k)\,\mathcal{O}_{20'}(l)\rangle \\
&\qquad + \ldots
\end{aligned}
$$

$$
0 = \sum_{i=1}^{4} \frac{\partial}{\partial \rho_i^{\alpha\dot{a}}} \langle \ldots \rangle
$$

$$
0 = \sum_{i=1}^{4} \frac{\partial}{\partial \bar{\rho}_i^{a\dot{\alpha}}} \langle \ldots \rangle
$$

- Apply supersymmetric constraints to relate correlators.
- Insert bosonic expressions for correlators.
- (Linear) independence of structures $\Rightarrow$ get many equations relating the unknown functions in an algebraic way.

4pt constraints of superconformal symmetry

- All 168 descendent functions are fixed in terms of the superprimary ones.
- 6 further PDEs for the unknown primary functions:

Drukker-Plefka twist [Drukker, Plefka, 2009]

$$0 = \sum_{i=1}^{6} \frac{\partial}{\partial u} f_i(u,v),\; 0 = \sum_{i=1}^{6} \frac{\partial}{\partial v} f_i(u,v)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}$$

$$v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$

4 further equations equivalent to chiral algebra twist:

$$0 = \frac{f_1^{(1,0)}(z,\bar{z})}{\bar{z}} + \bar{z} f_2^{(1,0)}(z,\bar{z}) + f_4{}^{(1,0)}(z,\bar{z}),$$

$$0 = \frac{f_1{}^{(0,1)}(z,\bar{z})}{z} + z f_2^{(0,1)}(z,\bar{z}) + f_4{}^{(0,1)}(z,\bar{z}),$$

$$0 = \frac{(\bar{z}-1) f_1^{(1,0)}(z,\bar{z})}{\bar{z}} + \frac{\bar{z} f_3^{(1,0)}(z,\bar{z})}{\bar{z}-1} + f_5^{(1,0)}(z,\bar{z}),$$

$$0 = \frac{(z-1) f_1^{(0,1)}(z,\bar{z})}{z} + \frac{z f_3^{(0,1)}(z,\bar{z})}{z-1} + f_5^{(0,1)}(z,\bar{z})$$

[Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees, 2013]

4pt constraints of superconformal symmetry

- All 168 descendent functions are fixed in terms of the superprimary ones.
- 6 further PDEs for the unknown primary functions:

Drukker-Plefka twist [Drukker, Plefka, 2009]

$$0 = \sum_{i=1}^{6} \frac{\partial}{\partial z} f_i(z,\bar{z}),\ 0 = \sum_{i=1}^{6} \frac{\partial}{\partial \bar{z}} f_i(z,\bar{z})$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}$$

$$v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$

[Dolan, Osborn, 2002]

4 further equations equivalent to chiral algebra twist:

$$0 = \frac{f_1^{(1,0)}(z,\bar{z})}{\bar{z}} + \bar{z} f_2^{(1,0)}(z,\bar{z}) + f_4{}^{(1,0)}(z,\bar{z}),$$

$$0 = \frac{f_1{}^{(0,1)}(z,\bar{z})}{z} + z f_2^{(0,1)}(z,\bar{z}) + f_4{}^{(0,1)}(z,\bar{z}),$$

$$0 = \frac{(\bar{z}-1) f_1^{(1,0)}(z,\bar{z})}{\bar{z}} + \frac{\bar{z} f_3^{(1,0)}(z,\bar{z})}{\bar{z}-1} + f_5^{(1,0)}(z,\bar{z}),$$

$$0 = \frac{(z-1) f_1^{(0,1)}(z,\bar{z})}{z} + \frac{z f_3^{(0,1)}(z,\bar{z})}{z-1} + f_5^{(0,1)}(z,\bar{z})$$

[Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees, 2013]

4pt result

$$\langle \mathcal{T}\mathcal{T}\mathcal{T}\mathcal{T}\rangle = \text{prefactor} \cdot \mathcal{H}(z,\bar{z}|y,\bar{y})$$

$$\mathcal{H}(z,\bar{z}|y,\bar{y}) = a + \left[\left(\frac{(z-y)(z-\bar{y})(\bar{z}-y)}{(z-\bar{z})(y-\bar{y})}c(z,y) + z \leftrightarrow \bar{z}\right) + y \leftrightarrow \bar{y}\right] + (z-y)(z-\bar{y})(\bar{y}-z)(\bar{z}-\bar{y})h(z,\bar{z})$$

[P. Heslop, 2023]

- Method works!
- For 4pt, there is no more than DP and CA twist conditions from Supersymmetry.

What about five points?

5pt correlator of stress tensor multiplets

$$\langle \mathcal{T}(1)\mathcal{T}(2)\mathcal{T}(3)\mathcal{T}(4)\mathcal{T}(5)\rangle$$

Strategy:

1. Insert same multiplet field expansion of $\mathcal{T}(X)$.
2. Use bosonic symmetries to express correlators in structures and functions of cross ratios.
3. Relate them by imposing supersymmetric invariance.

5pt correlator of stress tensor multiplets

$$\langle \mathcal{T}(1)\mathcal{T}(2)\mathcal{T}(3)\mathcal{T}(4)\mathcal{T}(5)\rangle$$

Number of structures:

$$\left.\begin{array}{lcc} \langle \mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\rangle & : & 22 \text{ structures } \Rightarrow f_i(\{u_a\}), i=1,...,22 \\ \langle \mathcal{J}_{\alpha\dot{\alpha},a\dot{a}}\mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\rangle & : & 5\cdot 21\cdot 4 = 420 \text{ structures} \\ \langle \Psi_{\alpha\dot{a}}\bar{\Psi}_{a\dot{\alpha}}\mathcal{O}_{20'}\mathcal{O}_{20'}\mathcal{O}_{20'}\rangle & : & 5\cdot 4\cdot 28\cdot 4 = 2240 \text{ structures} \end{array}\right\} \begin{array}{c} \mathbf{22+2660} \\ \textbf{unknown functions} \end{array}$$

Cross ratios:

[V. Schomerus et al., 2021]

$$u_1 = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z_1\bar{z}_1,\ v_1 = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-z_1)(1-\bar{z}_1),$$

$$u_2 = \frac{x_{23}^2 x_{45}^2}{x_{24}^2 x_{35}^2} = z_2\bar{z}_2,\ v_2 = \frac{x_{25}^2 x_{34}^2}{x_{24}^2 x_{35}^2} = (1-z_2)(1-\bar{z}_2),$$

$$U_1^{(5)} = \frac{x_{15}^2 x_{23}^2 x_{34}^2}{x_{24}^2 x_{13}^2 x_{35}^2} = w_1(z_1-\bar{z}_1)(z_2-\bar{z}_2) + (1-z_1-z_2)(1-\bar{z}_1-\bar{z}_2)$$

5pt correlator : Constraints from superconformal symmetry

- From 2660 descendent functions, SUSY fixed 2554. $\Rightarrow \mathcal{O}(\rho\bar{\rho})$: **6 unfixed functions.**
- **35 PDEs for the superprimary functions:**

$$5\text{x:} \sum_{i=1}^{22} \frac{\partial}{\partial u_1} f_i(u_1, v_1, u_2, v_2, U_1^{(5)}) = 0, \ \ldots \ \text{(DP type)} \tag{1}$$

$$10\text{x:} \ u_a \frac{\partial}{\partial u_a} G_b = u_b \frac{\partial}{\partial u_b} G_a \ \text{for all pairs of} \ u_a, u_b \tag{2}$$

$$20\text{x: others} \tag{3}$$

The equations are obeyed both, at one-loop order [Drukker, Plefka, 2008], as well as on the SUGRA side [Gonçalves, Pereira, Zhou, 2019].

Further WIP

- Further simplifying the SCWI. $\Rightarrow$ Solving them?
- Constraining the 5pt blocks.

Thank you!

Backup Slides

Work In Progress . Solve 11211.

$$\langle 11\rangle \langle 11211\rangle \langle 11\rangle = \frac{y_{12}^4 y_{34}^2 y_{45}^2 y_{53}^2}{x_{12}^4 x_{34}^2 x_{45}^2 x_{53}^2} f_1(\{u_a\}) + \frac{y_{45}^4 y_{12}^2 y_{23}^2 y_{31}^2}{x_{45}^4 x_{12}^2 x_{23}^2 x_{31}^2} f_2(\{u_a\}) + \frac{y_{12}^2 y_{23}^2 y_{34}^2 y_{45}^2 y_{51}^2}{x_{12}^2 x_{23}^2 x_{34}^2 x_{45}^2 x_{51}^2} f_3(\{u_a\})$$
$$+ \frac{y_{12}^2 y_{23}^2 y_{35}^2 y_{54}^2 y_{41}^2}{x_{12}^2 x_{23}^2 x_{35}^2 x_{54}^2 x_{41}^2} f_4(\{u_a\}) + \frac{y_{12}^2 y_{24}^2 y_{45}^2 y_{53}^2 y_{31}^2}{x_{12}^2 x_{24}^2 x_{45}^2 x_{53}^2 x_{31}^2} f_5(\{u_a\}) + \frac{y_{12}^2 y_{25}^2 y_{54}^2 y_{43}^2 y_{31}^2}{x_{12}^2 x_{25}^2 x_{54}^2 x_{43}^2 x_{31}^2} f_6(\{u_a\})$$

Constraints on primary:

5x: $\sum_{i=1}^{6} \frac{\partial}{\partial u_1} f_i(u_1, v_1, u_2, v_2, U_1^{(5)}) = 0$, ... (DP type)

10x: $u_a \frac{\partial}{\partial u_a} f_b = u_b \frac{\partial}{\partial u_b} f_a$ for all pairs of u_a, u_b

5x: others

Solutions:

- The correlator $\langle 11211\rangle$ is parametrised by a single function $\mathcal{H}$ of five variables and a constant.
- This function further satisfies 2nd order PDEs.

$\mathbf{psu(1,1|2)}$

Analytic superspace:

$$X = \begin{pmatrix} x & \rho \\ \bar{\rho} & y \end{pmatrix}$$

Multiplet expansion:

$$\mathcal{W} = \left(1 - \rho\bar{\rho}\frac{\partial}{\partial x}\frac{\partial}{\partial y}\right) J(x,y) + \rho\tilde{G}(x,y) + \bar{\rho}G(x,y) + \rho\bar{\rho}T(x)$$

$J = \mathbf{3}$; $\tilde{G}, G = \mathbf{2}$; $T = \mathbf{1}$ of $\mathrm{psu}(1,1|2)$

Correlator:

6+95 functions after bosonic symmetries.

- After imposing SUSY, there is only **one unfixed descendent function**.
- Among the superprimary functions, we get the following relations:

$$0 = \sum_{i=1}^{6} f_i^{(1,0)}(z_1, z_2)\,,\ 0 = \sum_{i=1}^{6} f_i^{(0,1)}(z_1, z_2)\,;\ \frac{x_{12}x_{34}}{x_{13}x_{24}} = z_1,\ \frac{x_{23}x_{45}}{x_{24}x_{35}} = z_1,$$

$$\begin{aligned}
0 =& \frac{(1-z_1)}{z_2} f_1^{(0,1)}(z_1,z_2) + \frac{(-z_1+z_2+1)}{z_2} f_2^{(0,1)}(z_1,z_2) + 2f_3^{(0,1)}(z_1,z_2) \\
&+ f_4^{(0,1)}(z_1,z_2) + f_5^{(0,1)}(z_1,z_2) + \frac{(z_1-1)}{z_2} f_4^{(1,0)}(z_1,z_2) \\
&- \frac{(z_1-1)z_1}{(z_2-1)z_2} f_1^{(1,0)}(z_1,z_2) - \frac{(z_1-1)(z_1-z_2+1)}{(z_2-1)z_2} f_2^{(1,0)}(z_1,z_2) \\
&- \frac{(z_1-1)(z_1-z_2+1)}{(z_2-1)z_2} f_3^{(1,0)}(z_1,z_2) - \frac{(z_1-1)z_1}{(z_2-1)z_2} f_5^{(1,0)}(z_1,z_2)
\end{aligned}$$

$\mathbf{psu(1,1|2)}$ - Solution

Analytic superspace:

$$X = \begin{pmatrix} x & \rho \\ \bar{\rho} & y \end{pmatrix}$$

Multiplet expansion:

$$\mathcal{W} = \left(1 - \rho\bar{\rho}\frac{\partial}{\partial x}\frac{\partial}{\partial y}\right) J(x,y) + \rho\tilde{G}(x,y) + \bar{\rho}G(x,y) + \rho\bar{\rho}T(x)$$

$J = \mathbf{3}$; $\tilde{G}, G = \mathbf{2}$; $T = \mathbf{1}$ of $\mathrm{psu}(1,1|2)$

Correlator:

6+95 functions after bosonic symmetries.

$$\begin{aligned}
&\langle J(1)J(2)J(3)J(4)J(5)\rangle\,|_{\text{bare}} \\
=&\frac{z_1 z_2}{y_1 y_2}\Big((z_1 - y_1)A_1(z_1,z_2) + (z_2 - y_2)A_2(z_1,z_2) \\
&+ \left(\frac{1-y_1}{y_2} - \frac{1-z_1}{z_2}\right) A_3(z_1,z_2) \\
&+ \left(\frac{1-y_2}{y_1} - \frac{1-z_2}{z_1}\right) A_4(z_1,z_2)\Big) \\
&+ \frac{(-1+y_1+y_2)(y_1-z_1)(y_2-z_2)}{y_1^2 y_2^2 z_1 z_2} A_5(z_1,z_2)
\end{aligned}$$

with $A_1(z_1,z_2) = H^{(1,0)}(z_1,z_2) + c(z_1) + \kappa$

$A_2(z_1,z_2) = H^{(0,1)}(z_1,z_2) + c(z_2) + \kappa$

$psu(1,1|2)$: more details

$$\langle J(1)J(2)J(3)J(4)J(5)\rangle = \frac{y_{12}y_{13}y_{23}y_{45}^2}{x_{12}x_{13}x_{23}x_{45}^2} f_1 + \frac{y_{12}y_{23}y_{43}y_{15}y_{45}}{x_{12}x_{23}x_{43}x_{15}x_{45}} f_3 + \frac{y_{31}y_{32}y_{41}y_{25}y_{45}}{x_{31}x_{32}x_{41}x_{25}x_{45}} f_5$$
$$+ \frac{y_{12}y_{13}y_{43}y_{25}y_{45}}{x_{12}x_{13}x_{43}x_{25}x_{45}} f_2 + \frac{y_{13}y_{34}y_{42}y_{25}y_{51}}{x_{13}x_{34}x_{42}x_{25}x_{51}} f_6 + \frac{y_{12}y_{24}y_{43}y_{35}y_{51}}{x_{12}x_{24}c_{43}x_{35}x_{51}} f_4;$$

$$\mathcal{L}^{22222} = \frac{x_2 - x_4}{(x_1 - x_2)^2 (x_2 - x_3)(x_3 - x_4)(x_4 - x_5)^2} \frac{(y_1 - y_2)^2 (y_2 - y_3)(y_3 - y_4)(y_4 - y_5)^2}{y_2 - y_4}$$

$$\langle J(1)J(2)J(3)J(4)J(5)\rangle |_{\text{bare}}$$
$$= \frac{z_1^2 z_2^2}{y_1^2 y_2^2} \left(\frac{y_1 y_2^2}{z_1 z_2^2} f_1(z_1, z_2) + \frac{y_1(y_2-1)y_2}{z_1(z_2-1)z_2} f_2(z_1,z_2) + \frac{y_1 y_2 (y_1+y_2-1)}{z_1 z_2 (z_1+z_2-1)} f_3(z_1,z_2) \right.$$
$$\left. + \frac{(y_1-1)(y_2-1)y_2}{(z_1-1)(z_2-1)z_2} f_5(z_1,z_2) + \frac{y_1(y_1+y_2-1)}{z_1(z_1+z_2-1)} f_4(z_1,z_2) + \frac{(y_2-1)(y_1+y_2-1)}{(z_2-1)(z_1+z_2-1)} f_6(z_1,z_2) \right)$$