# Cosmic rays in a turbulent interstellar medium: Recent progress and open questions

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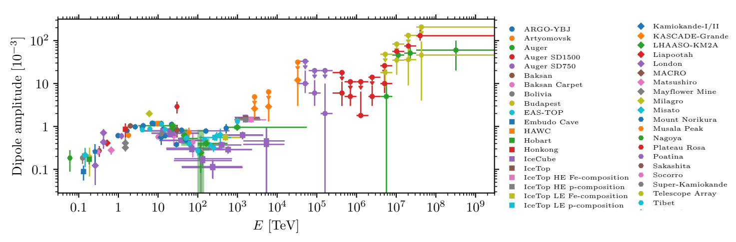
# Cosmic rays diffuse through the Galaxy

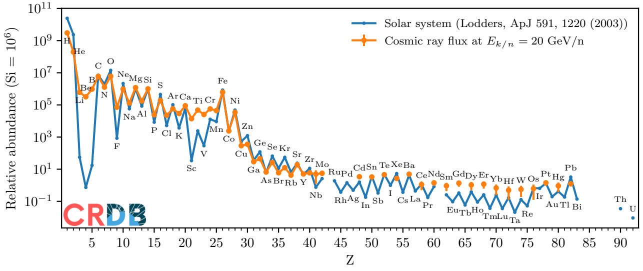
## 1. Anisotropies




- Sources distributed in disk
- Yet, small anisotropy observed

$\rightarrow$ Cosmic rays change direction

## 2. Secondary species




- Some species overabundant in cosmic rays
- Must be produced by

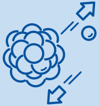
$$\mathrm{CR}_1 + \mathrm{gas} \rightarrow \mathrm{CR}_2 + \ldots$$

$\rightarrow$ Cosmic rays cross gaseous disk many times

# Cosmic rays and turbulence

## Sources of turbulence

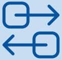
Outflows



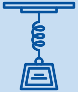
Cosmic ray streaming

## Processes

Advection

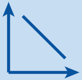


Non-linear interactions



Damping

## Turbulence spectrum



Two-point correlation
$\langle \delta \mathbf{B}(\mathbf{r}) \delta \mathbf{B}(\mathbf{r}') \rangle$

$\downarrow$

Power spectrum
$P(k) \propto k^{-q}$

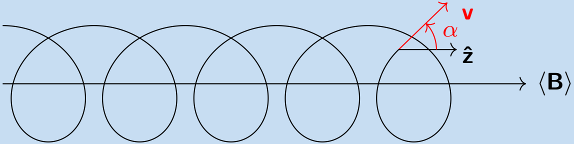
Credit: Noun Project; Ehtisham Abid; Syahrul Hidayatullah; Purwanto; Victoruler

# Parallel transport

- Magnetic field: $\langle \mathbf{B} \rangle + \delta \mathbf{B} \quad \rightarrow \quad$ phase-space density: $\langle f \rangle + \delta f$

$$\Rightarrow \frac{\partial \langle f \rangle}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} \langle f \rangle = \int_0^t \mathrm{d}t \left\langle (\mathbf{v} \times \delta \mathbf{B}) \cdot \nabla_{\mathbf{p}} \Big[ (\mathbf{v} \times \delta \mathbf{B}) \cdot \nabla_{\mathbf{p}} \langle f \rangle \Big]_{\mathbf{r}(t')} \right\rangle$$

Unperturbed trajectory $\mathbf{r}(t)$ characterised by pitch-angle cosine $\mu \equiv \cos \alpha$

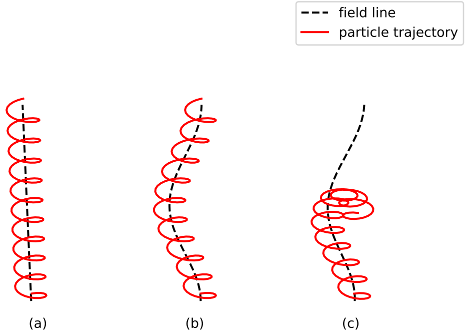



$\Rightarrow$ Pitch-angle scattering $\dfrac{\partial \langle f \rangle}{\partial t} + v \mu \dfrac{\partial \langle f \rangle}{\partial z} = \dfrac{\partial}{\partial \mu} D_{\mu\mu} \dfrac{\partial \langle f \rangle}{\partial \mu}$ with $D_{\mu\mu} \sim \left( \dfrac{\delta B^2}{B_0^2} \right)^{-1} \left( \dfrac{r_g}{L_c} \right)^{q-2} \Omega_g$

$\Rightarrow$ For isotropic phase-space density $\bar{f}$: $\dfrac{\partial \bar{f}}{\partial t} - \dfrac{\partial}{\partial z} \kappa_{\parallel} \dfrac{\partial \bar{f}}{\partial z} = 0$ with $\kappa_{\parallel} = \displaystyle\int_{-1}^{1} \mathrm{d}\mu \frac{(1 - \mu^2)^2}{D_{\mu\mu}}$

# Perpendicular transport

Perpendicular transport = particle transport along field line + transport of field line




(a) Straight field line and gyration
(b) Wandering field line and gyration
(c) Wandering field line and diffusion

## Field-line diffusion coefficient

$$d_{\mathrm{FL}}(z) = \frac{1}{2} \frac{\mathrm{d}\langle (\Delta r_{\perp}^{\mathrm{FL}})^2 \rangle}{\mathrm{d}z}$$

# Open questions

. . . on cosmic rays and turbulence

## Observational anomalies

- Small-scale anisotropies
- Breaks in diffusion coefficient
- Local suppression of diffusion

## Theoretical anomalies

- Different scaling of $\kappa_{\parallel}$ and $\kappa_{\perp}$
- Effect of anisotropic turbulence

# Open questions

...on cosmic rays and turbulence

## Observational anomalies

- Small-scale anisotropies
- Breaks in diffusion coefficient
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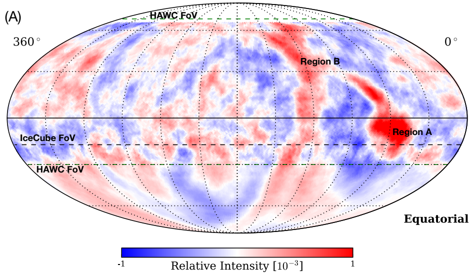
## Theoretical anomalies

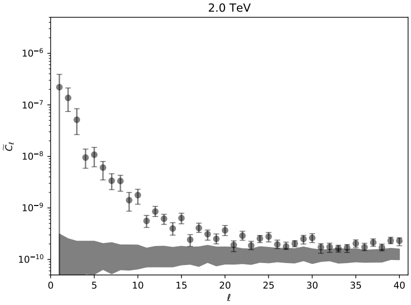
- Different scaling of $\kappa_{\|}$ and $\kappa_{\perp}$
- Effect of anisotropic turbulence

# Outline

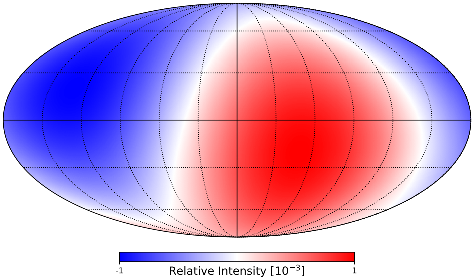
# Small-scale anisotropies

Abeysekara *et al.*, ApJ 796 (2014) 108 Aartsen *et al.*, ApJ 826 (2016) 220; Abeysekara *et al.*, ApJ 865 (2018) 57; Abeysekara *et al.*, ApJ 871 (2019) 96




Diffusion models predict only large-scale anisotropy



# Vlasov equation

Frisch (1968), Pelletier (1977)

- Liouville equation:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \dot{\mathbf{r}} \cdot \nabla_{\mathbf{r}} f + \dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}} f = 0$$

- Lorentz force:

$$\dot{\mathbf{p}} = \frac{q}{c} \mathbf{v} \times \left( \langle \mathbf{B} \rangle + \delta \mathbf{B} \right)$$

deterministic $\gg$ stochastic

- Vlasov equation:

$$\frac{\partial f}{\partial t} + \dot{\mathbf{r}} \cdot \nabla_{\mathbf{r}} f + \frac{q}{c} \mathbf{v} \times \left( \langle \mathbf{B} \rangle + \delta \mathbf{B} \right) \cdot \nabla_{\mathbf{p}} f = 0$$

- Ignoring the gradient:

$$\frac{\partial f_{\oplus}}{\partial t} + \underbrace{\left( \frac{q}{c} (\mathbf{v} \times \langle \mathbf{B} \rangle) \cdot \nabla_{\mathbf{p}} \right)}_{\mathcal{L}_0} f_{\oplus} + \underbrace{\left( \frac{q}{c} (\mathbf{v} \times \delta \mathbf{B}) \cdot \nabla_{\mathbf{p}} \right)}_{\delta \mathcal{L}} f_{\oplus} = 0$$

# QM analogy

- Liouville's theorem:

$$\partial_t f + \mathcal{L}_0 f(t) = -\delta\mathcal{L}(t) f(t) \qquad\qquad \imath\hbar\partial_t|\psi(t)\rangle - H_0|\psi(t)\rangle = -H_1(t)|\psi(t)\rangle$$

- Formally solved as

$$f(\mathbf{r}, \mathbf{p}, t) = U_{t,t_0} f(\mathbf{r}, \mathbf{p}, t_0) \qquad\qquad |\psi(t)\rangle = U(t, t_0)|\psi(t_0)\rangle$$

- With time evolution operator:

$$U_{t,t_0} = \mathcal{T}\exp\left[-\int_{t_0}^{t} \mathrm{d}t' \left(\mathcal{L}_0 + \delta\mathcal{L}(t')\right)\right] = U^{(0)}_{t,t_0}\mathcal{T}\exp\left[-\int_{t_0}^{t} \mathrm{d}t' \underbrace{\left(U^{(0)}_{t',t_0}\right)^{-1}\delta\mathcal{L}(t')U^{(0)}_{t',t_0}}_{\sim\text{interaction picture Hamiltonian}}\right]$$

and free propagator:

$$U^{(0)}_{t,t_0} = \exp\left[-\mathcal{L}_0(t - t_0)\right] \qquad\qquad U^{(0)}(t, t_0) = \exp\left[-\imath H_0(t - t_0)/\hbar\right]$$

# Perturbative expansion

Frisch (1968), Pelletier (1977)

- Series expansion:

$$U_{t,t_0} = U^{(0)}_{t,t_0} + \sum_{n\geq 1} (-1)^n \int_{t_0}^{t} \mathrm{d}t_n \int_{t_0}^{t_n} \mathrm{d}t_{n-1} \ldots \int_{t_0}^{t_2} \mathrm{d}t_1 \, U^{(0)}_{t,t_n} \delta\mathcal{L}(t_n) U^{(0)}_{t_n,t_{n-1}} \delta\mathcal{L}(t_{n-1}) \ldots \delta\mathcal{L}(t_1) U^{(0)}_{t_1,t_0}$$

- But can only make predictions for ensemble-averaged quantities, e.g. $\langle U^{(0)}_{t,t_0} \rangle$

→ Correlation functions, e.g. $\langle \delta\mathcal{L}(t_2)\delta\mathcal{L}(t_1) \rangle \quad \to \quad \langle \delta\mathbf{B}(t_2)\delta\mathbf{B}(t_1) \rangle$

- Diagrammatic representation:

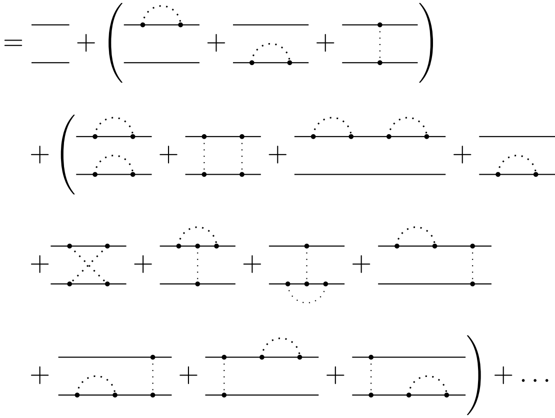
$\langle U_{t,t_0} \rangle \quad =$ 

# Double propagator

Mertsch & Ahlers (2019)

For $\langle f(\hat{\mathbf{p}}_1) f(\hat{\mathbf{p}}_2) \rangle$ we need correlated evolution of two particles:

$$\langle U^{A}_{t,t_0} U^{B*}_{t,t_0} \rangle =$$

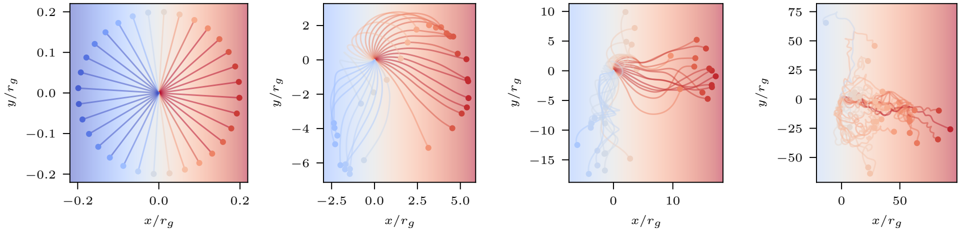


Formulate differential equation of $\langle C_\ell(t) \rangle$ and solve for steady-state

# Test particle simulations

Kuhlen, Mertsch, Phan (2022)

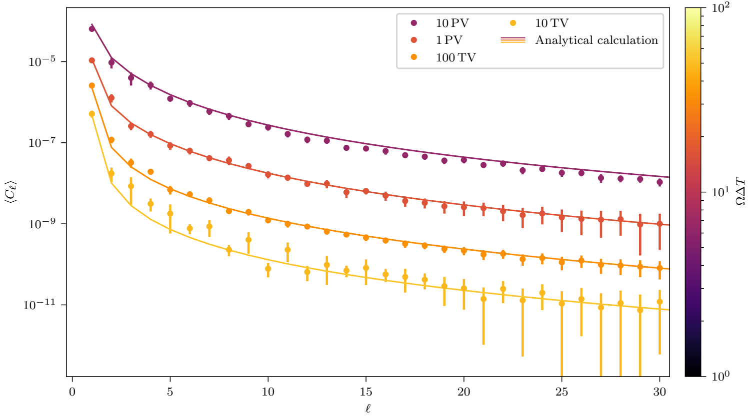

- Need to check analytical results with simulations
- Set up turbulent magnetic field on computer
- Solve the equations of motion for $\mathcal{O}(10^7)$ particles numerically




- Rinse and repeat
- Compute diffusion coefficients, angular power spectra, . . .

# Results

Kuhlen, Mertsch, Phan (2022)




When applying to observational data:

- Independent measurement of scattering time
- Constraints on details of magnetised turbulence

# Outline

# Different scaling of $\kappa_{\parallel}$ and $\kappa_{\perp}$

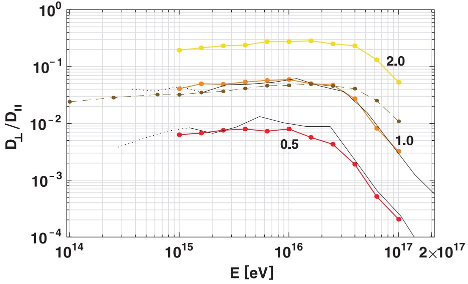
- Regular and turbulent field: $\mathbf{B}(\mathbf{r}) = \mathbf{B}_0 + \delta\mathbf{B}(\mathbf{r})$
- Isotropic turbulence (!)
- Kolmogorov power spectrum with largest turbulent scale $L_c$

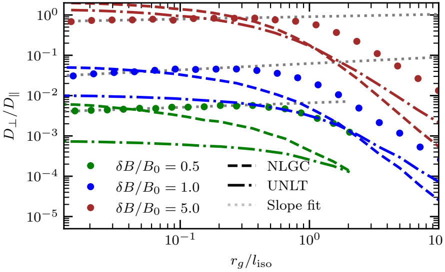
- Naive expectation:

$$\kappa_{\parallel} \sim \left(\frac{r_g}{L_c}\right)^{1/3} \left(\frac{\delta B^2}{B_0^2}\right)^{-1}$$

$$\kappa_{\perp} \sim \left(\frac{r_g}{L_c}\right)^{1/3} \left(\frac{\delta B^2}{B_0^2}\right)$$

DeMarco *et al.* (2007)




Dundovic *et al.* (2020)

# Test particle simulations

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05881]; also Mertsch (2019)


1. Set up realisation of $\delta\mathbf{B}$ on computer
2. Propagate a large number of particles for long times
3. Rinse and repeat
4. Running diffusion coefficients:

$$d_{\parallel}(t) \equiv \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\langle(\Delta z)^2\rangle$$

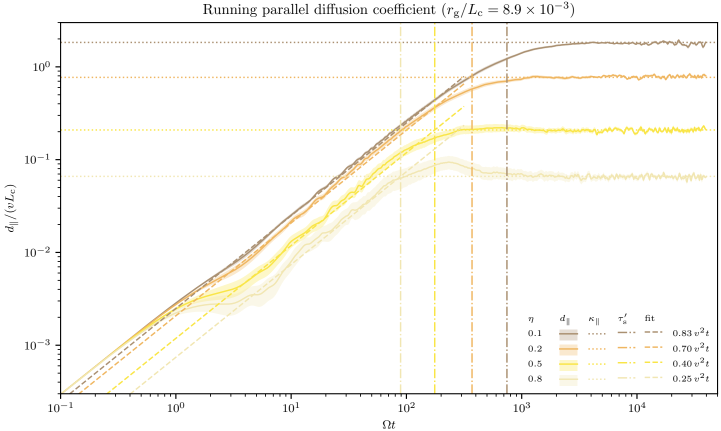
$$d_{\perp}(t) \equiv \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\langle(\Delta r_{\perp})^2\rangle$$

Results depend on:

- Reduced time: $\Omega t$
- Reduced rigidity: $\frac{r_{\mathrm{g}}}{L_{\mathrm{c}}}$
- Turbulence level: $\eta = \frac{\delta B^2}{B_0^2 + \delta B^2}$

# Running parallel diffusion coefficient

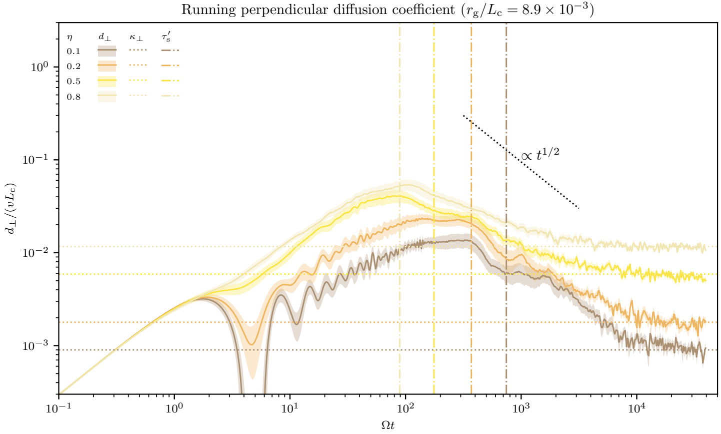
Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05881]




- Initially ballistic: $\langle(\Delta z)^2\rangle \propto t^2$
- Ultimately diffusive: $\langle(\Delta z)^2\rangle \propto t$
- Dependence on turbulence level
- Suppression at intermediate times

# Running perpendicular diffusion coefficient

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05881]




- Initially ballistic: $\langle (\Delta r_\perp)^2 \rangle \propto t^2$
- Ultimately diffusive: $\langle (\Delta r_\perp)^2 \rangle \propto t$
- Suppression at intermediate times
- Subdiffusion: $\langle (\Delta r_\perp)^2 \rangle \propto t^{0.5\ldots0.7}$

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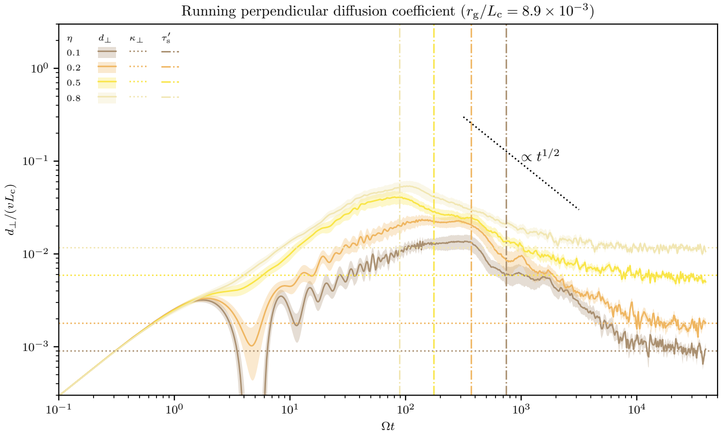


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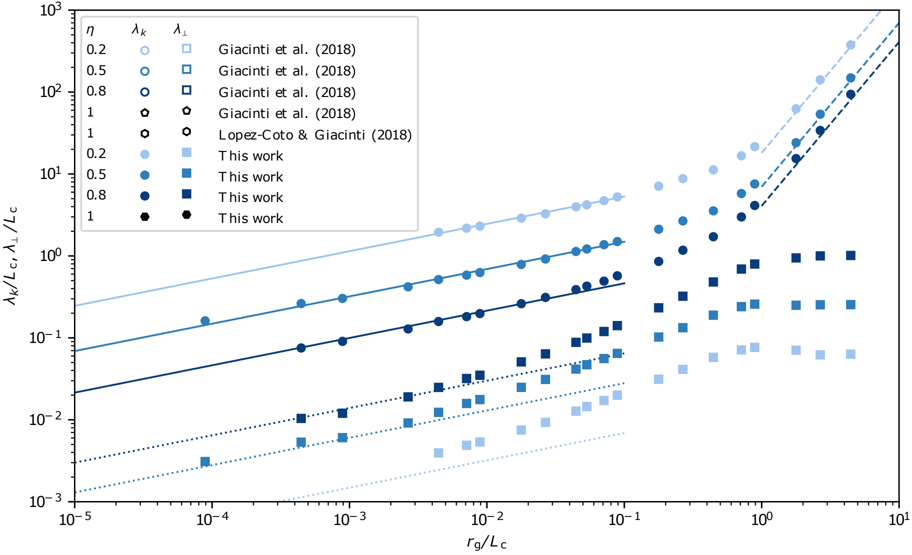
Why subdiffusion?

# Mean-free paths

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05881]



# Heuristic model

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05882]

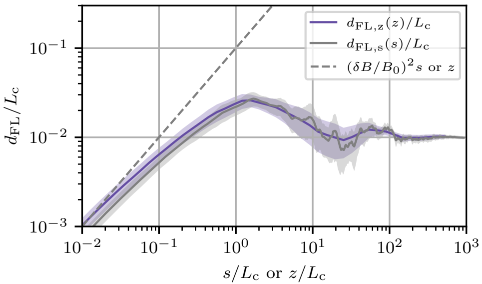
- Assumption:

  Perpendicular transport = particle transport along field line + transport of field line

→ Perpendicular diffusion coefficient:

$$d_{\perp}(t) = \frac{d_{\mathrm{FL}}(\sqrt{\langle(\Delta z)^2\rangle})}{\sqrt{\langle(\Delta z)^2\rangle}} d_{\parallel}(t)$$

- Parametrise $d_{\mathrm{FL}}$ and $d_{\parallel}$ by heuristic models

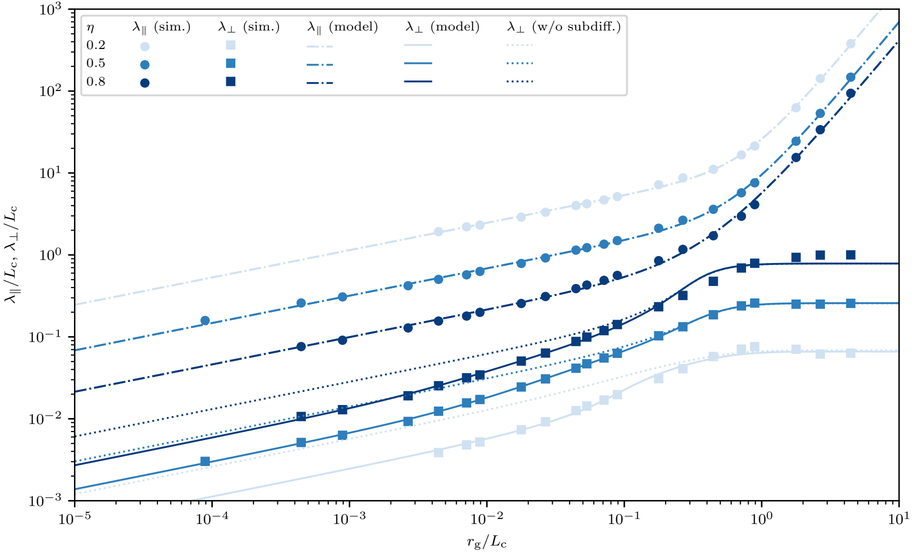



**Field-line transport is subdiffusive at intermediate distances!**

See also Sonsrettee *et al.* (2016)

# Mean-free paths

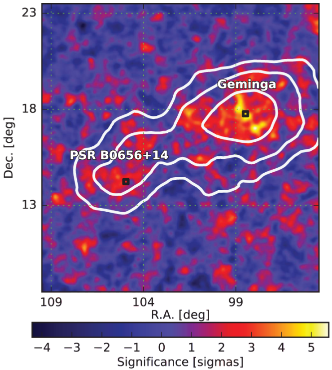
Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05882]

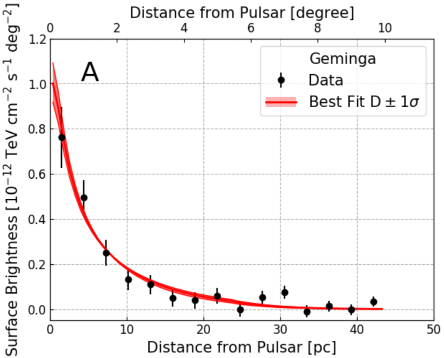


# Outline

# Gamma-ray halos

Abeysekara *et al.* (2017)




- Gamma-ray emission around two nearby pulsars
- Emission from $\mathrm{e}^{\pm}$ much more confined than expected

$\rightarrow$ Ambient diffusion coefficient suppressed by factor $\sim 100$

- Also evidence of suppressed diffusion around some supernova remnants

# Gamma-ray halos

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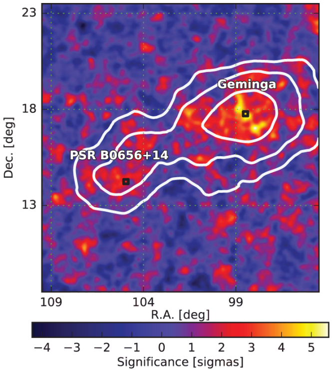


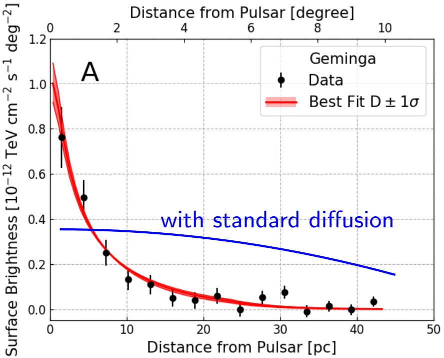


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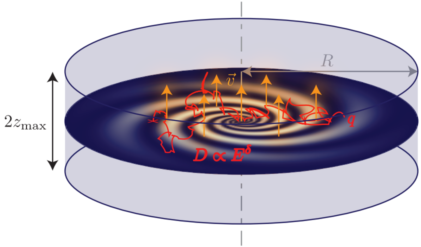
# A swiss cheese Galaxy

Jacobs, Mertsch, Phan, arXiv:2305.10337



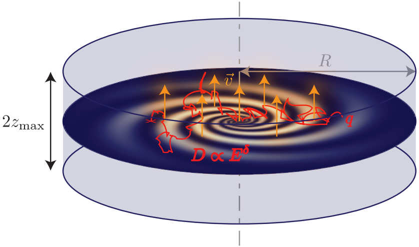

- Diffusion suppressed in bubbles around sources

$\rightarrow$ Transport of cosmic rays affected on large scales?

- Different diffusion coefficient in disk and halo: $\kappa_{\text{disk}} = \alpha\, \kappa_{\text{halo}}$

# A swiss cheese Galaxy

Jacobs, Mertsch, Phan, arXiv:2305.10337

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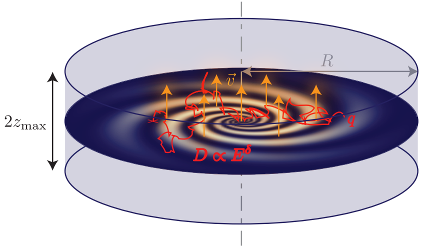
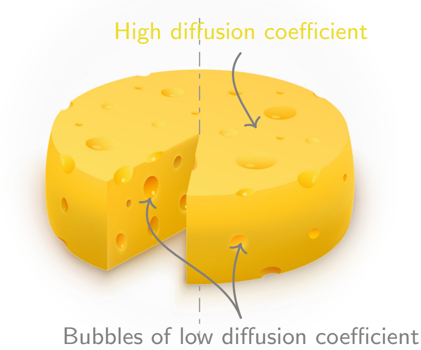
Jacobs, Mertsch, Phan, arXiv:2305.10337

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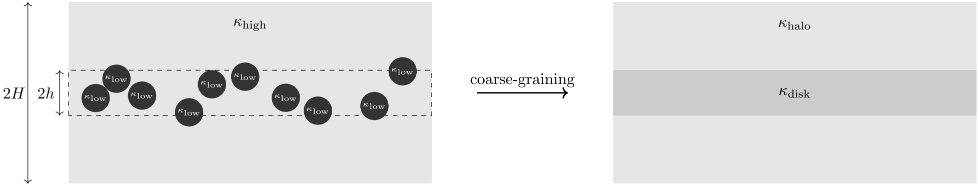
# Coarse-graining

Jacobs, Mertsch, Phan (2023)

- Filling fraction $f \lesssim$ (a few) % $\rightarrow$ negligible?
- Difficult to model numerically

$\rightarrow$ Adopt course-grained $\kappa_{\mathsf{disk}} = \alpha\kappa_{\mathsf{high}}$ and $\kappa_{\mathsf{halo}} = \kappa_{\mathsf{high}}$

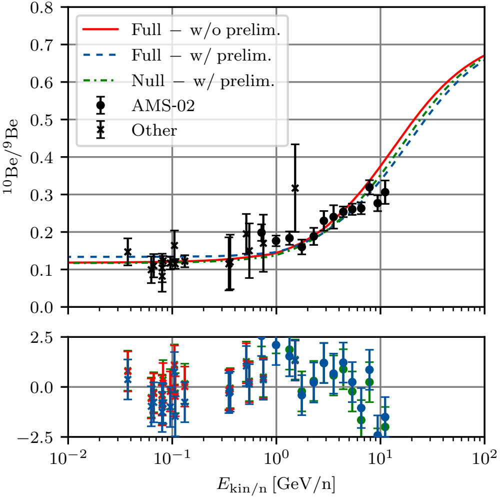



- Study impact of $\alpha < 1$ on cosmic ray observables
- The coarse-grained $\kappa_{\mathsf{disk}}$ can only depend on $\kappa_{\mathsf{high}}$, $\kappa_{\mathsf{low}}$ and the filling fraction

$\rightarrow$ Can infer filling fraction from data?

# $^{10}\mathrm{Be}/^{9}\mathrm{Be}$

Jacobs, Mertsch, Phan, *in prep.*

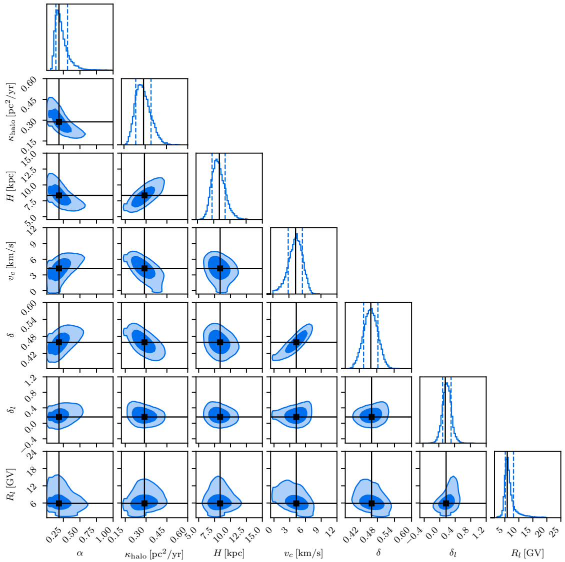


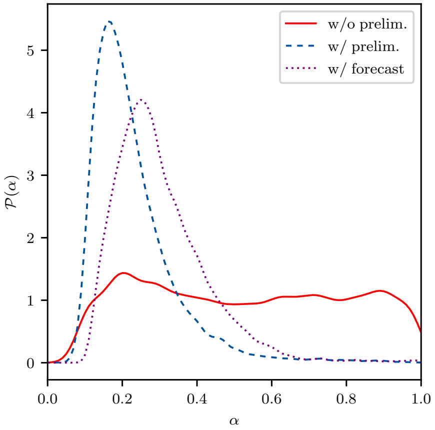

- Unstable $^{10}\mathrm{Be}$ created in disk
- At high energies: essentially stable
- At low energies: decays while diffusing

If diffusion is suppressed, $\kappa_{\mathrm{disk}} < \kappa_{\mathrm{halo}}$,
$^{10}\mathrm{Be}/^{9}\mathrm{Be}$ is increased at low energies

# Posterior distributions

Jacobs, Mertsch, Phan (2023)



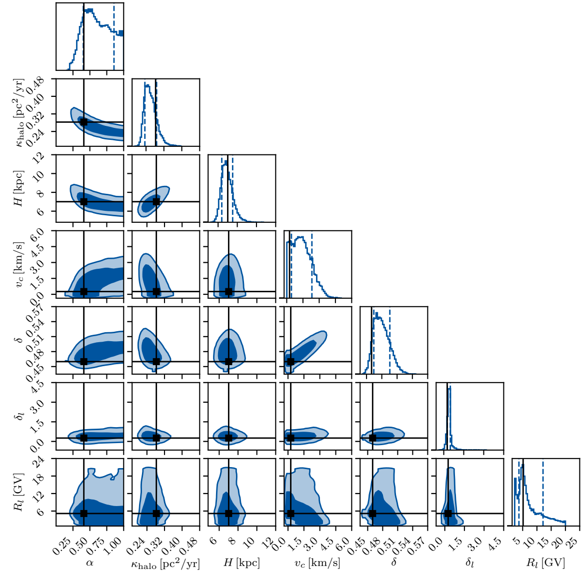




## With prelim. AMS-02 data

- Best fit for $\alpha \simeq 0.2$
- $\alpha = 1$ excluded at $\sim 4\sigma$
- Implies very large filling fraction $f \sim 0.5$

# Cross-section

Jacobs, Mertsch, Phan, *in prep.*

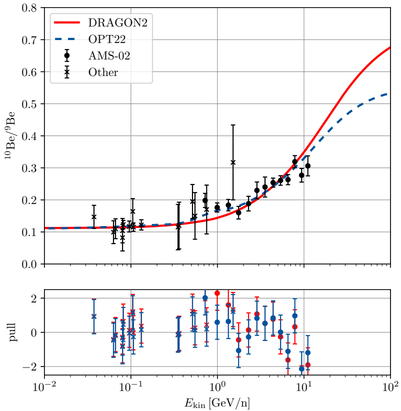



## With alternative cross-sections

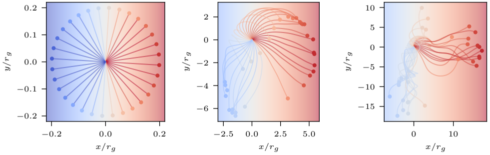
- OPT22 has weaker energy-dependence

→ Marginal preference for $\alpha < 1$

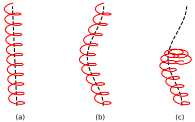
→ Need better cross-section data!



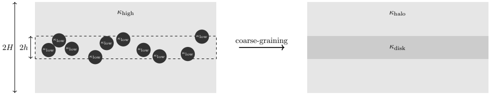
# Summary




Small-scale anisotropies
→ Transition from ballistic to diffusive motion




Different scaling of $\kappa_{\|}$ and $\kappa_{\perp}$
→ Subdiffusion in field line transport



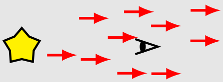

Suppressed diffusion
→ Can be investigated with $^{10}\mathrm{Be}/^{9}\mathrm{Be}$

# Outline

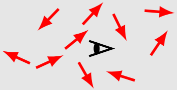
6. Backup slides
   - Anisotropy
   - Small-scale anisotropies
   - Different scaling of $\kappa_{\|}$ and $\kappa_{\perp}$
   - Suppressed diffusion

# Anisotropy

Between a few GeV and a PeV: $a = \mathcal{O}(10^{-4} \ldots 10^{-3})$



- Sources are discrete
- If CRs were travelling balistically, would expect $\mathcal{O}(1)$ anisotropy
- See, e.g., electro-magnetic radiation



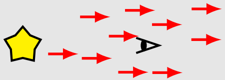
- CRs are distributed very isotropically

$\rightarrow$ Need to isotropise CRs

- (Coulomb) collisions with interstellar matter too infrequent

# Anisotropy

Between a few GeV and a PeV: $a = \mathcal{O}(10^{-4} \ldots 10^{-3})$



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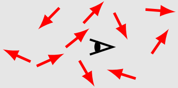


- CRs are distributed very isotropically

→ Need to isotropise CRs

- (Coulomb) collisions with interstellar matter too infrequent

- Scattering of charged particles with turbulent magnetic field isotropises particle directions

→ Particles perform a random walk in space:

$$\langle (\Delta r)^2 \rangle \propto \Delta t$$

- The constant of proportionality is called the **diffusion coefficient** $\kappa$

# Mixing matrices

Mertsch & Ahlers (2019), Kuhlen, Mertsch, Phan (2022)

- Define propagator:

$$U_{t,t_0} = \mathcal{T} \exp\left[ -\int_{t_0}^{t} \mathrm{d}t' \left( \mathcal{L}' + \delta\mathcal{L}(t') \right) \right]$$

- Formal solution of Vlasov equation:

$$f_\oplus(\mathbf{p}, t) = U_{t,t_0} f_\oplus(\mathbf{p}, t_0) + \int_{t_0}^{t} \mathrm{d}t' \, U_{t,t'} \, c \, \hat{\mathbf{p}} \cdot \mathbf{G}$$

→ Differential equation for $\langle C_\ell \rangle$,

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t} \langle C_\ell \rangle(t) + \left( \lim_{t_0 \to t} \frac{\delta_{\ell\ell_0} - M_{\ell\ell_0}(t, t_0)}{t - t_0} \right) \langle C_{\ell_0} \rangle(t) = \frac{8\pi}{9} K |\mathbf{G}|^2 \delta_{\ell 1}}$$

mixing $\ell_0 \to \ell$ sourcing $\ell$

where

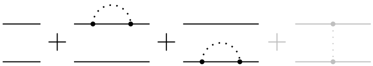
$$M_{\ell\ell_0}(t, t_0) = \frac{1}{4\pi} \int \mathrm{d}\hat{\mathbf{p}}_A \int \mathrm{d}\hat{\mathbf{p}}_B \mathrm{P}_\ell(\hat{\mathbf{p}}_A \cdot \hat{\mathbf{p}}_B) \langle U^A_{t,t_0} U^{B*}_{t,t_0} \rangle \frac{2\ell_0 + 1}{4\pi} \mathrm{P}_{\ell_0}(\hat{\mathbf{p}}_A \cdot \hat{\mathbf{p}}_B)$$

# Ignoring correlations

Mertsch & Ahlers (2019), Kuhlen, Mertsch, Phan (2022)

- Without "interactions":

$$\langle U^{A}_{t,t_0} U^{B*}_{t,t_0} \rangle \simeq$$



- Mixing matrix diagonal:

$$M_{\ell\ell_0}(t, t_0) \sim \delta_{\ell\ell_0}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle C_\ell \rangle(t) + \left( \lim_{t_0 \to t} \frac{\delta_{\ell\ell_0} - M_{\ell\ell_0}(t, t_0)}{t - t_0} \right) \langle C_{\ell_0} \rangle(t) = \frac{8\pi}{9} K |\mathbf{G}|^2 \delta_{\ell 1} ,$$

→ Only dipolar anisotropy:

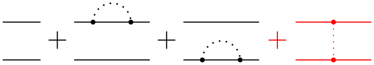
$$\langle C_\ell \rangle \propto \delta_{\ell 1} ,$$

## **With** correlations

Mertsch & Ahlers (2019), Kuhlen, Mertsch, Phan (2022)

- With "interactions"

$$\langle U^{A}_{t,t_0} U^{B*}_{t,t_0} \rangle \simeq$$



- Mixing matrix **not** diagonal:

$$M_{\ell\ell_0}(t,t_0) \sim \delta_{\ell\ell_0} + \sum_{\ell_A} \kappa_{\ell_A}(t-t_0) \begin{pmatrix} \ell & \ell_A & \ell_0 \\ 0 & 0 & 0 \end{pmatrix}^2 (2\ell_0+1)\ell_0(\ell_0+1)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle C_\ell \rangle(t) + \left( \lim_{t_0 \to t} \frac{\delta_{\ell\ell_0} - M_{\ell\ell_0}(t,t_0)}{t-t_0} \right) \langle C_{\ell_0} \rangle(t) = \frac{8\pi}{9} K |\mathbf{G}|^2 \delta_{\ell 1}\,,$$

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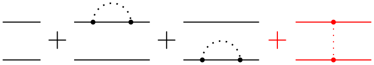

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→ Gradient source term is mixing into higher harmonics!

# Results

- Expect scaling $\Omega \Delta T \propto (\Omega \tau_s)^{1/3}$
  $\rightarrow$ Confirmed by numerical simulations
- Observations of small scale anisotropies can constrain $\Omega \tau_s$
  $\rightarrow$ Constraints on the turbulent magnetic field!

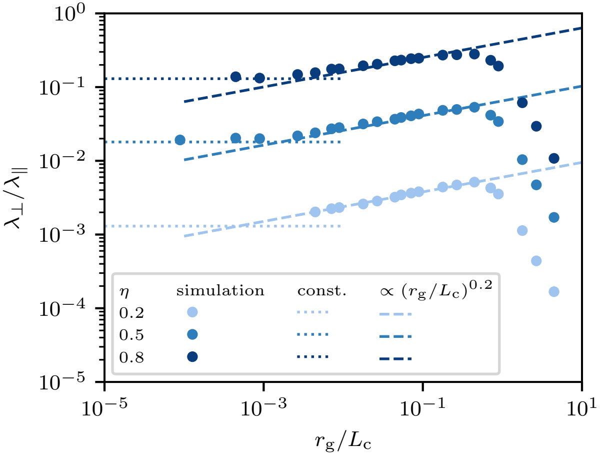
- Numerical and analytic angular power spectra become steeper for smaller energies
- Observed angular power spectra become flatter for smaller energies

### Possible reasons for different scaling

- A feature in the power spectrum
- A very small outer scale
- Slab turbulence does not describe data well
- Finite energy resolution

# Ratio of mean-free paths

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05881]




$\kappa_\parallel$ and $\kappa_\perp$ scale differently at medium rigidites,
but they scale the same at low rigidities

# Heuristic model

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05882]

Perpendicular transport = particle transport along field line + transport of field line

1. Start from $d_{\mathrm{FL}}(z) = \frac{1}{2} \frac{\mathrm{d}\langle (\Delta r_\perp^{\mathrm{FL}})^2 \rangle}{\mathrm{d}z}$
2. Integrate: $\langle (\Delta r_\perp^{\mathrm{FL}})^2 \rangle (z) = 2 \int_0^z \mathrm{d}z' \, d_{\mathrm{FL}}(z')$
3. Assume that particles follow field lines: $\langle (\Delta r_\perp^{\mathrm{CR}})^2 \rangle (z) = \langle (\Delta r_\perp^{\mathrm{FL}})^2 \rangle (z)$
4. Subsitute into $d_\perp(t) \equiv \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} \left( \langle (\Delta r_\perp^{\mathrm{CR}})^2 \rangle \right) = \frac{\mathrm{d}}{\mathrm{d}t} \int_0^{z(t)} \mathrm{d}z' \, d_{\mathrm{FL}}(z')$
5. Evaluate $z(t)$ as $\sqrt{\langle z^2 \rangle (t)} \quad \Rightarrow \quad d_\perp(t) = \frac{\mathrm{d}}{\mathrm{d}t} \int_0^{\sqrt{\langle z^2 \rangle}} \mathrm{d}z' \, d_{\mathrm{FL}}(z') = \frac{d_{\mathrm{FL}}\left(\sqrt{\langle (\Delta z)^2 \rangle}\right)}{\sqrt{\langle (\Delta z)^2 \rangle}} d_\parallel(t)$

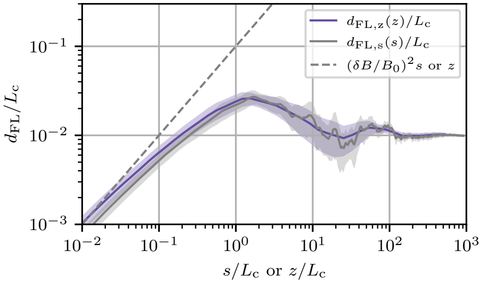
N.B.: This can also be derived from a microscopic model of particle transport.

# Heuristic model

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05882]

$$d_\perp(t) = \frac{d_{\mathrm{FL}}(\sqrt{\langle(\Delta z)^2\rangle})}{\sqrt{\langle(\Delta z)^2\rangle}} d_\parallel(t)$$

Parametrise $d_{\mathrm{FL}}$ and $d_\parallel$ by broken power laws

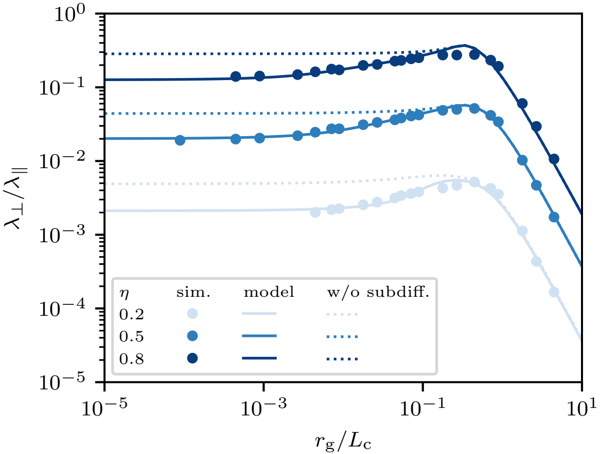



**Field-line transport is subdiffusive at intermediate distances!**

See also Sonsrettee *et al.* (2016)

# Ratio of mean-free paths

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05882]

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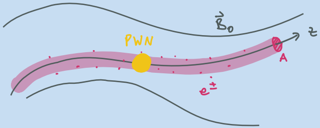
(More details → Appendix)

# NOT the conflict

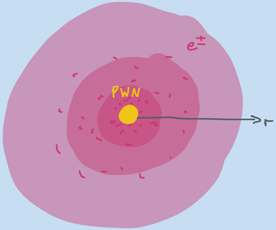
Evoli, Linden, Morlino (2018), Linden & Mukhopadhyay (2022)

## 1D




- Growth rate: $\Gamma_{1D} \propto \frac{\partial f}{\partial z}$
- Volume occupied: $V_{1D} \sim A \left\langle (\Delta z)^2 \right\rangle^{1/2}$

## 3D




- Growth rate: $\Gamma_{3D} \propto \frac{\partial f}{\partial r}$
- Volume occupied: $V_{3D} \sim \left\langle (\Delta r)^2 \right\rangle^{3/2}$

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Evoli, Linden, Morlino (2018), Linden & Mukhopadhyay (2022)

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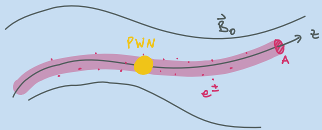


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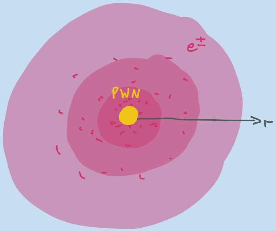


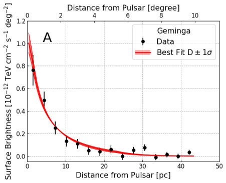

- Growth rate: $\Gamma_{3D} \propto \frac{\partial f}{\partial r}$
- Volume occupied: $V_{3D} \sim \left\langle (\Delta r)^2 \right\rangle^{3/2}$

Locally, transport is **not** isotropic

See Lopez-Coto and Giacinti (2019) though

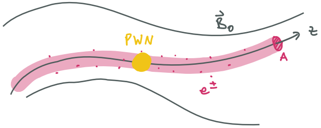
# The conflict

## Profile



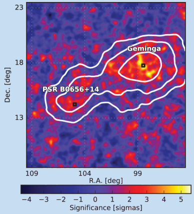

→

Suppression of diffusion



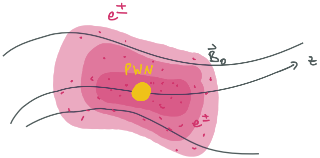

$$\kappa_\perp \ll \kappa_\| \quad \Leftrightarrow \quad \delta B^2 \ll B_0^2$$

## Symmetry




→

Spherical distribution




$$\kappa_\perp \sim \kappa_\| \quad \Leftrightarrow \quad \delta B^2 \gg B_0^2$$