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XIX Vulcano Workshop
Frontier Objects in Astrophysics and Particle Physics

Testing Gravity with LAGEOS, LARES and Galileo

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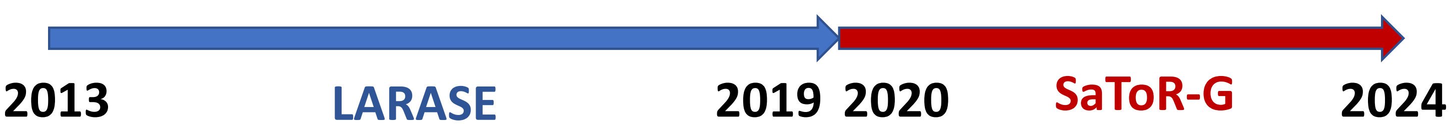
OUTLINE

- The LARASE and SaToR-G experiments
- Geodetic satellites and Satellite Laser Ranging
- Precise Orbit Determination
- Measurements
  - Lense Thirring
  - Preliminary measurement to constrain Yukawa-like interactions
  - Preliminary limit to α_1
- GNSS Galileo constellation
  - Measurement of gravitational red-shift
- Conclusions

LARASE and SATOR_G

The **LARASE** (2013-2019) and **SaToR-G** (started on 2020) are two experiments, funded by the Italian National Institute for Nuclear Physics (**INFN-CSN2**), devoted to measurements of the gravitational interaction in the **Weak-Field** and **Slow-Motion** limit of **General Relativity** to obtain **constraints** on the **parametrized post-Newtonian** (**PPN*)* parameters** and their combinations by means of **laser tracking to geodetic passive satellites** orbiting around the Earth.

The final goal is to perform **precise and accurate measurements**, i.e. to valuate the systematic errors affecting the measurements to get meaningful constrains on the different theories. A main point of these activity is therefore the **modeling of both gravitational and non-gravitational perturbations**.

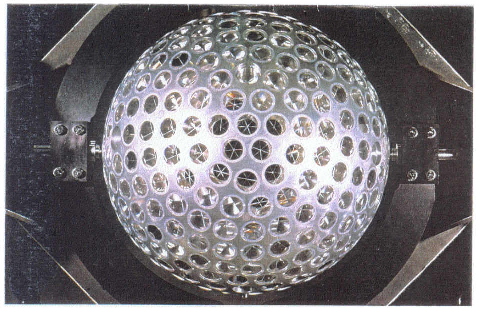


GEODETIC PASSIVE SATELLITES

The **geodetic passive satellites** play the role of **test masses** to test predictions of **GR**

Parameter	Unit	Symbol	LAGEOS	LAGEOS II	LARES
Semi-major axis	km	a	12 270.00	12 162.08	7 820.31
Eccentricity	-	e	0.0044	0.0138	0.0012
Inclination	deg.	i	109.84	52.66	69.49
Radius	cm	R	30.0	30.0	18.2
Mass	kg	M	406.9	405.4	383.8
Area/Mass	m^2/kg	A/M	6.94×10^{-4}	6.97×10^{-4}	2.69×10^{-4}

LAGEOS (NASA, 1976)



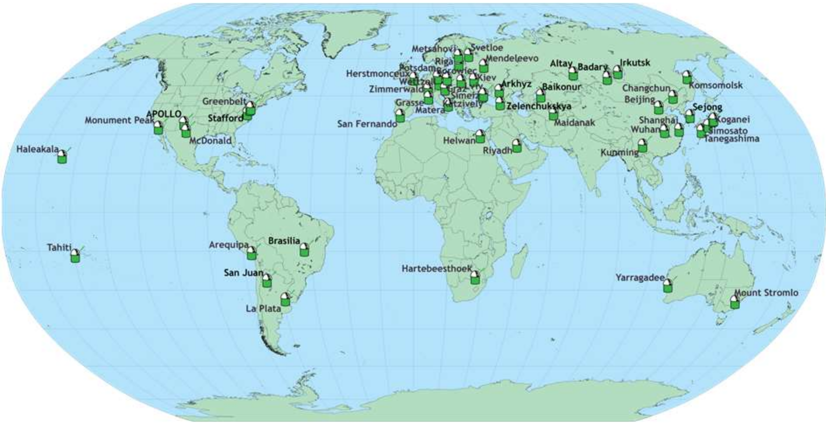
LAGEOS II (ASI/NASA, 1992)

LARES (ASI, 2012)

ILRS STATIONS

There is an international network of laser tracking stations coordinated by the **International Laser Ranging Service**

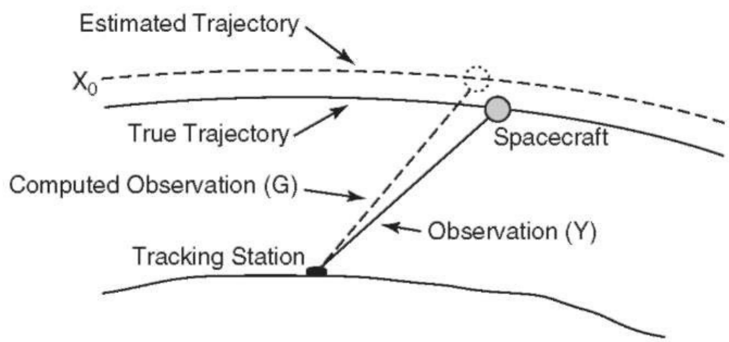
- The stations send laser pulses and observe the reflected signals with a telescope measuring the round–trip time between Earth–bound laser Stations and orbiting satellites The SLR represents a very impressive and powerful technique **to determine the round–trip time** between Earth–bound laser Stations and orbiting passive (and not passive) satellites.



Matera Station

PRECISE ORBIT DETERMINATION

Thanks to the accurate modelling of both gravitational and non–gravitational perturbations on the orbit of these satellites to a range accuracy of less than 1 cm (rms ~1 mm) we are able **to determine their Keplerian elements** with about the same accuracy. We perform the data reduction of the satellites orbit using GEODYN II (NASA/GSFC), **SATAN** (NSGF, UK) in collaboration with "Observatorio de YEBES" (Spain) and **Bernese** (Univ. Berna, CH)

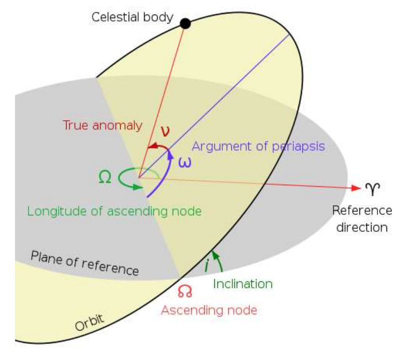


Residuals to minimized:

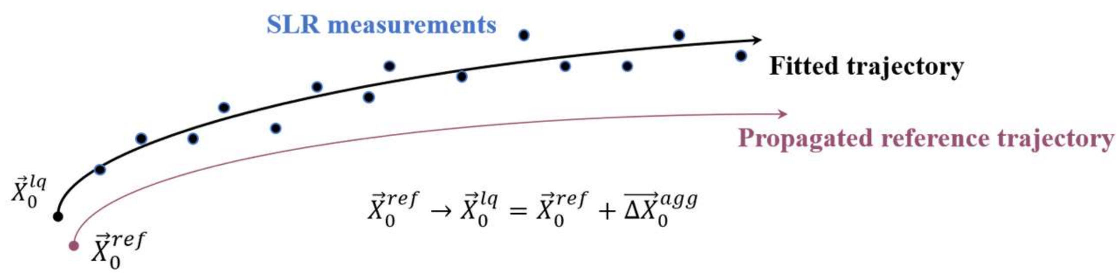
$$R_i = O_i - C_i = \sum \frac{\partial C_i}{\partial P_j} \delta P_j + \delta O_i$$

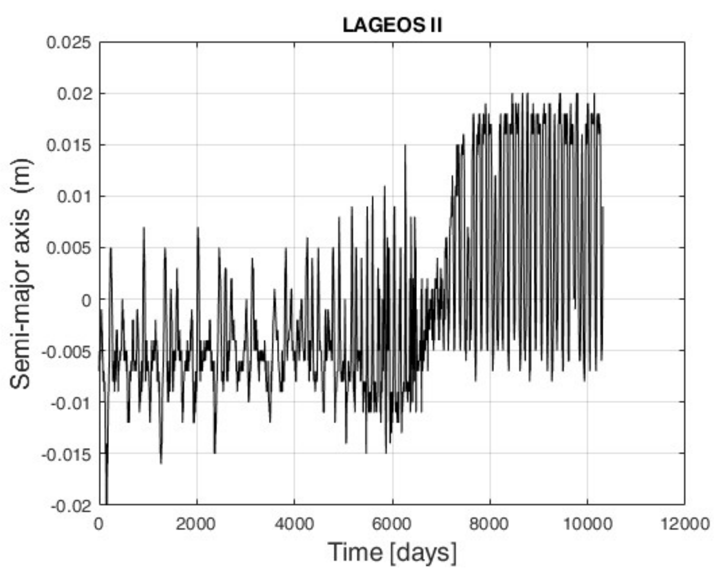
O_i	observed range (form SLR)
C_i	calculated range (from dynamical model of forces)
P_j	element in the parameters vector (status vector + other parameters)
δP_j	correction to j-th parameter
δO_i	error in the observation (noise + defect in the model)

ORBITAL RESIDUALS

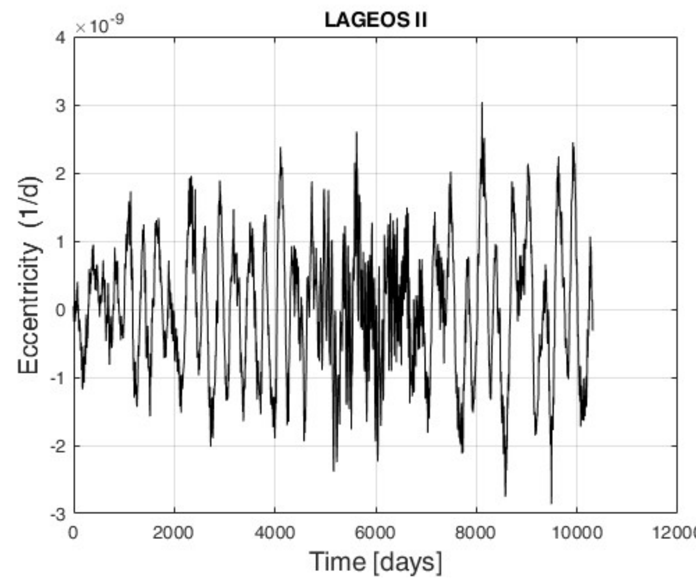


The orbitals residuals contain the effect of forces acting on the satellite that are not modeled or are poorly modeled.





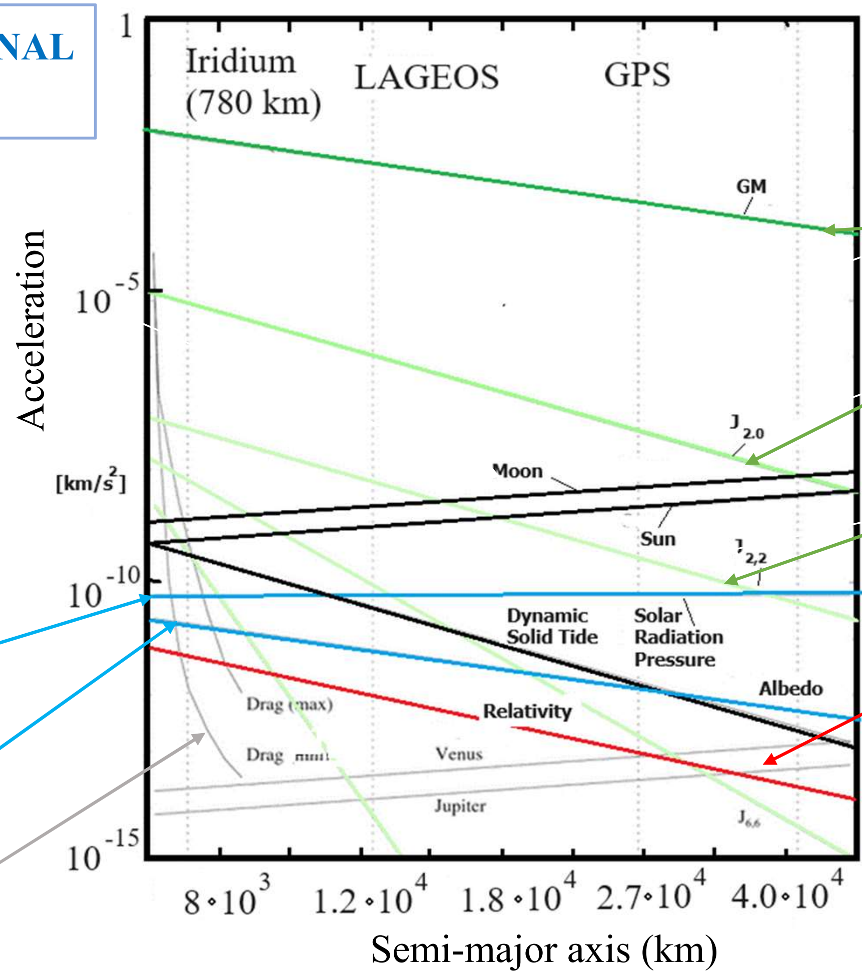
Rates over 7 days



PERTURBATION EFFECTS ON SATELLITE ORBIT

NON-GRAVITATIONAL PERTUBATION

GRAVITATIONAL PERTUBATION



Earth monopole

Earth quadrupole

Earth octopole

Solar radiation pressure

Albedo

Drag

Relativity

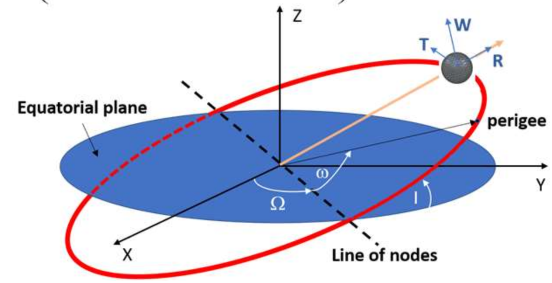
O. Montenbruck, E. Gill, Satellite Orbits, Springer, Berlino, 2005

THE LENSE-THIRRING PRECESSION

The so-called **Lense-Thirring** effect consists of a precession of the orbit of a satellite around a primary produced by its rotation, i.e. by its **angular momentum *J*** (**mass currents**).

This precession produces a secular effect in two orbital elements:

- the right ascension of the ascending node (RAAN), **Ω**
- the argument of pericenter, ω



$$\left\langle \frac{d\Omega}{dt} \right\rangle_{sec} = \mu \frac{2G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}}$$

$$\left\langle \frac{d\omega}{dt} \right\rangle_{sec} = -\mu \frac{6G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}} \cos i$$

$$\mu = \begin{cases} 1 \text{ in General Relativity} \\ 0 \text{ in Newtonian physics} \end{cases}$$

Rate (mas/yr)	LAGEOS	LAGEOS II	LARES
$\dot{\Omega}_{LT}$	+30.67	+31.50	+118.48
$\dot{\omega}_{LT}$	+31.23	-57.31	-334.68

30 mas/yr ~ 180 cm/yr at the orbital height of the LAGEOS

$J_\oplus$	-is the source of the effect
G, c	-are two fundamental constants of nature
a, e, i	-mean Keplerian elements

SYSTEMATIC ERRORS

- The **Lense-Thirring** precession is very small compared to the classical precession of the orbit due to the deviation from the spherical symmetry for the distribution of the Earth's mass, or even compared to the same relativistic **Schwarzschild** precession produced by the mass of the primary (≈ 3350 mas/yr for **LAGEOS**)

$$\left\langle \frac{d\Omega}{dt} \right\rangle_{sec} = \mu \frac{2G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}}$$

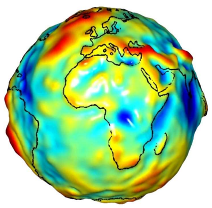
$$\dot{\Omega}^{Class} \cong -\frac{3}{2} n \left(\frac{R_\oplus}{a}\right)^2 \frac{\cos I}{\left(1-e^2\right)^2} \left\{ J_2 + J_4 \left[\frac{5}{8} \left(\frac{R_\oplus}{a}\right)^2 \left(7\sin^2 I - 4\right) \frac{\left(1 + \frac{3}{2} e^2\right)}{\left(1-e^2\right)^2} \right] + \ldots \right\}$$

Lense-Thirring **Classic**

Therefore, the **correct modelling of the even zonal harmonics** (ℓ = even, m = 0) represents the main challenge in this kind of measurements, since they have the same signature of the relativistic effect but much larger amplitudes. **These harmonics are the main sources of systematic errors**

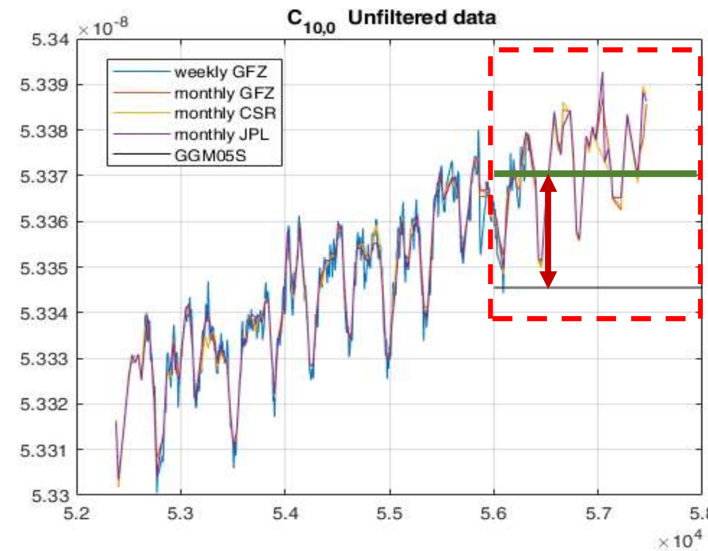
MODELING EARTH GRAVITATIONAL FIELD

The Earth does not have the shape of perfect sphere, therefore its potential cannot be modelled as that of a point mass. The shape of the Geoid can be approximated using spherical harmonics.



$$V = \frac{GM}{r}\left(1 + \sum_{n=2}^{n_{\max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \overline{P}_{nm}(\sin\phi)\left[\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda\right]\right),$$

$\bar{C}_{10,0}$



- We considered several static models for the background gravitational field of the Earth
- To **reduce the impact** of the harmonics, we modeled the first **10** even zonal harmonics exploiting their significant time dependency as well evidenced by their **Temporal Solutions** (**TS**) provided by the (NASA/DLR) **GRACE** and **GRACE –FO** missions.

ANALYSIS METHOD

- By solving a linear system of three equations in three unknowns, we can solve for the relativistic precession while reducing the impact in the measurement of the non perfect knowledge of the Earth's gravitational field:

$$\dot{\Omega}_2^{L1}\delta J_2 + \dot{\Omega}_4^{L1}\delta J_4 + \dot{\Omega}_{LT}^{L1}\mu + \cdots = \delta\dot{\Omega}_{res}^{L1}$$
$$\dot{\Omega}_2^{L2}\delta J_2 + \dot{\Omega}_4^{L2}\delta J_4 + \dot{\Omega}_{LT}^{L2}\mu + \cdots = \delta\dot{\Omega}_{res}^{L2}$$
$$\dot{\Omega}_2^{LR}\delta J_2 + \dot{\Omega}_4^{LR}\delta J_4 + \dot{\Omega}_{LT}^{LR}\mu + \cdots = \delta\dot{\Omega}_{res}^{LR}$$

$$(\mu, \delta J_2, \delta J_4)$$

$$\dot{\Omega}_{GR}^{comb} = 50.17\ mas/yr$$

$$\dot{\Omega}^{comb} = \delta\dot{\Omega}_{res}^{L1} + k_1\delta\dot{\Omega}_{res}^{L2} + k_2\delta\dot{\Omega}_{res}^{LR}$$

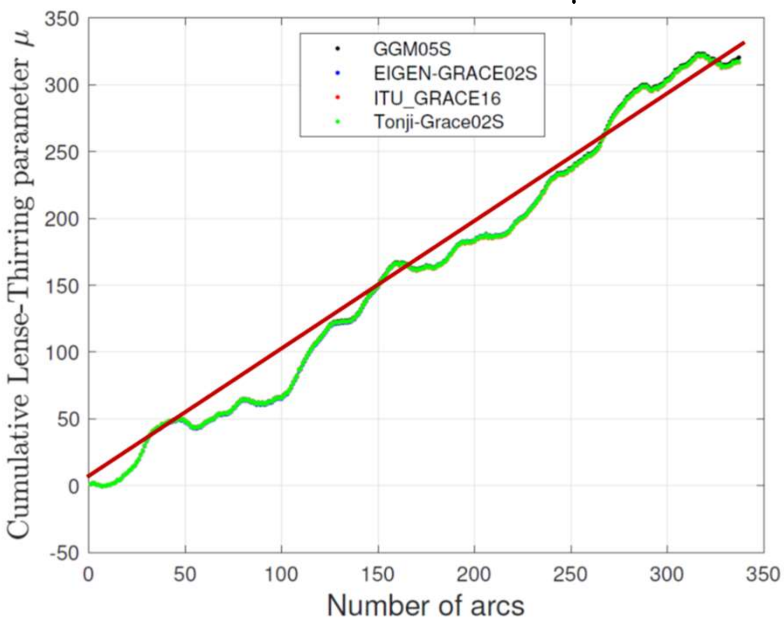
$$\mu = \frac{\dot{\Omega}^{comb}}{\dot{\Omega}_{GR}^{comb}} = \begin{cases} 1 \text{ in General Relativity} \\ 0 \text{ in Newtonian physics} \end{cases}$$

$k_1 \cong 0.345$
$k_2 \cong 0.073$

LENSE-THIRRING MEASUREMENT

We performed an analysis of about 6.5 years (2359 days) from **MJD 56023**, that is from April 6th 2012, and we computed the residuals on the orbit elements of **LAGEOS**, **LAGESOS II** and **LARES**

Cumulative sum for μ



$$\mu_{meas} - 1 = 1.5 \times 10^{-3} \pm 7.4 \times 10^{-3} \pm 16 \times 10^{-3}$$

Model	$\mu \pm \delta\mu$	$\mu - 1$
GGM05S	1.0053 ± 0.0074	$+0.0053$
EIGEN-GRACE02S	1.0002 ± 0.0074	$+0.0002$
ITU_GRACE16	0.9996 ± 0.0074	-0.0004
Tonji-Grace02s	1.0008 ± 0.0074	$+0.0008$

Perturbations	$\delta\mu_{sys}$ [%]
Gravitational field	1.0
Tides	0.6
Periodic effect	1.0
De Sitter effect	0.3
RSS	1.6
SAV	2.9

Errors @ 95% CL

This is indeed a very **precise** and **accurate** measurement

D. Lucchesi et al: *An improved measurement of the Lense-Thirring precession on the orbits of laser-ranged satellites with an accuracy approaching the 1% level,* arXiv:1910.01941, oct 2019

D. Lucchesi et al.: *1% Measurement of the Gravitomagnetic Field of the Earth with Laser-Tracked Satellites,* Universe **2020**, 6, 139

ISL - YUKAWA-LIKE INTERATION

- A **Yukawa-like potential produces a radial acceleration** $\Re = -\frac{G_\infty M_\oplus}{a^2}\left(\frac{a}{r}\right)^2 \alpha\left(1+\frac{r}{\lambda}\right)e^{-\frac{r}{\lambda}}$ that gives secular effect only on two orbital parameters. The effects are function of the mean orbital parameters *a, e*, of the true anomaly *f* and of the mean motion *n*.

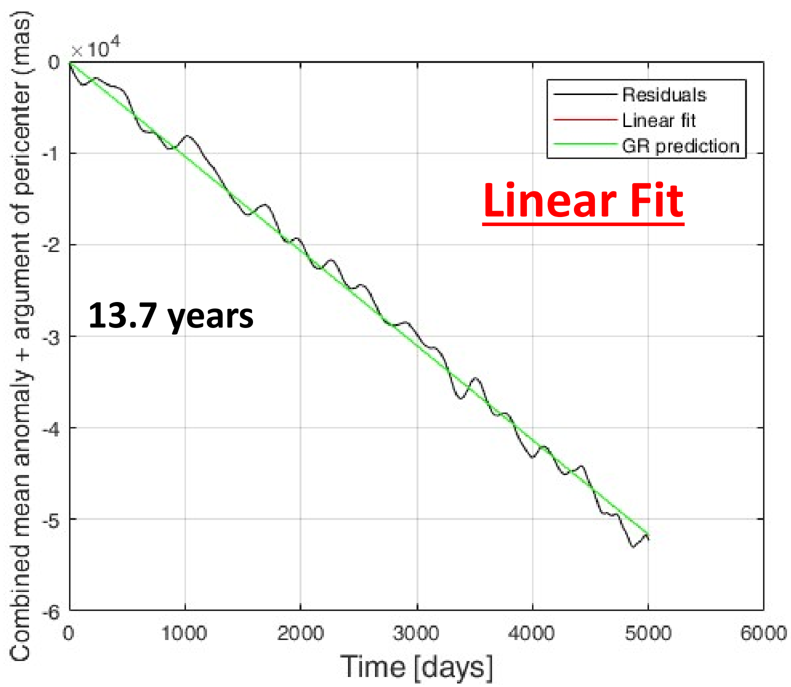
Argument of pericenter
$$\dot{\omega}(\alpha,\lambda) = -\frac{\sqrt{1-e^2}}{n\,a\,e}\Re\cos f$$

Mean anomaly
$$\dot{M}(\alpha,\lambda) = n + \frac{1}{na}\Re\left(\frac{\cos u(f,e)}{e(1-e^2)} - \sqrt{1-e^2}\sin f\sin u(f,e) + 2\frac{(1-e^2)}{(1+e\cos f)}\right)$$

- The effect of this interaction must be **compared with the precession predicted by General Relativity**

PRELIMINARY ANALYSIS: LAGEOS II ARGUMENT OF PERICENTER AND MEAN ANOMALY

- The analysis was done using argument of **pericenter and the mean anomaly of LAGEOS II** on a time span of 13.7 years to reduce systematic errors introduced by gravitational field
- The use of two observables $\boldsymbol{obs = M_{res}^{L2} + k\dot{\omega}_{res}^{L2}\ (k \cong -0.1235)}$ allows to *cancel* the errors due to J_2



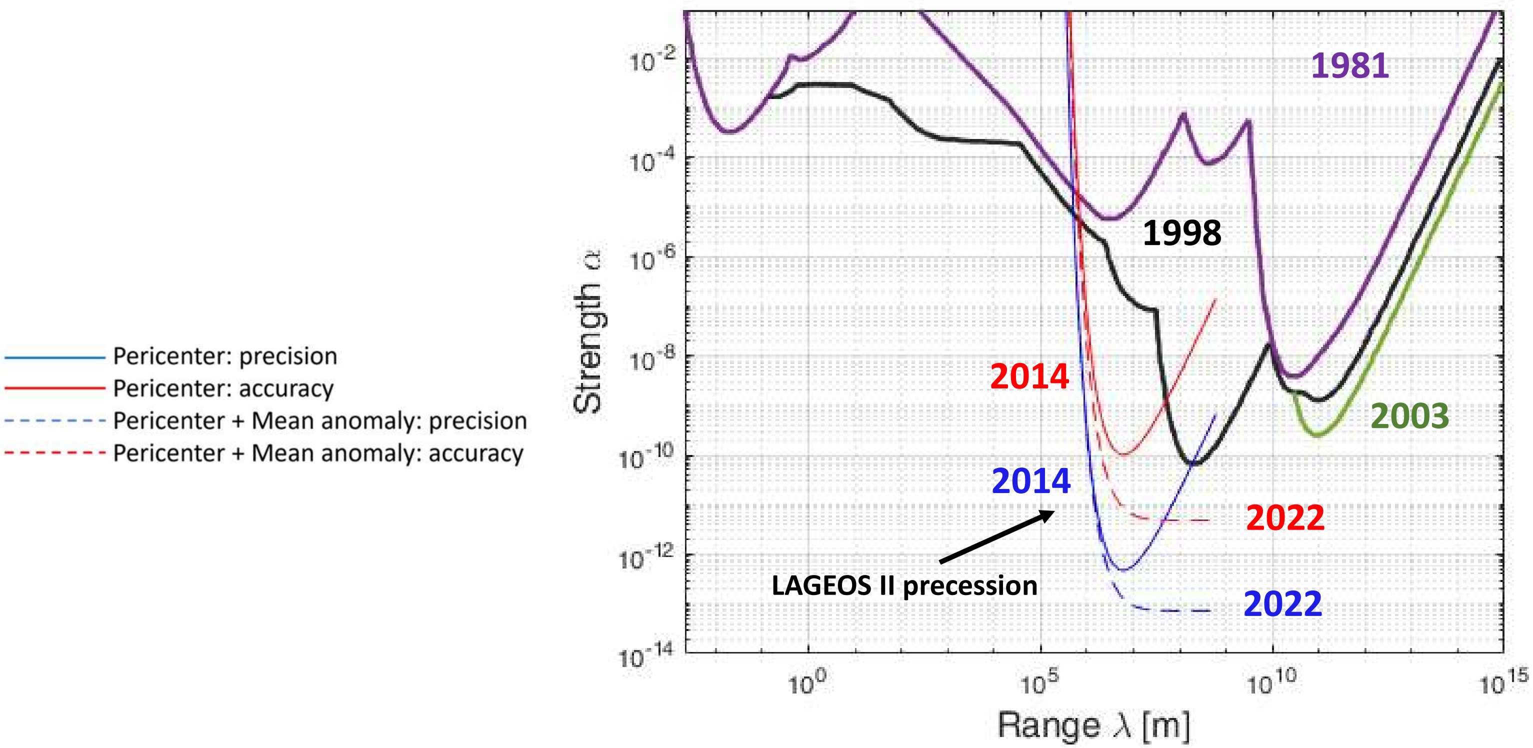
$$obs_{tot} = \boldsymbol{\varepsilon}\, obs_{GR} + obs_{GP} + obs_{NGP} + \cdots$$

$$\boldsymbol{\varepsilon - 1 \cong (+0.35 \pm 2.42) \times 10^{-3} \pm 0.8 \cdot 10^{-2}}$$

A previous measurement in 2014 was made using a non-linear fit:

$$\boldsymbol{\varepsilon - 1 = (-0.12 \pm 2.10) \cdot 10^{-3} \pm 2.5 \cdot 10^{-2}}$$

COMPARISATION WITH PREVIOUS RESULTS



MEASURE OF α_1

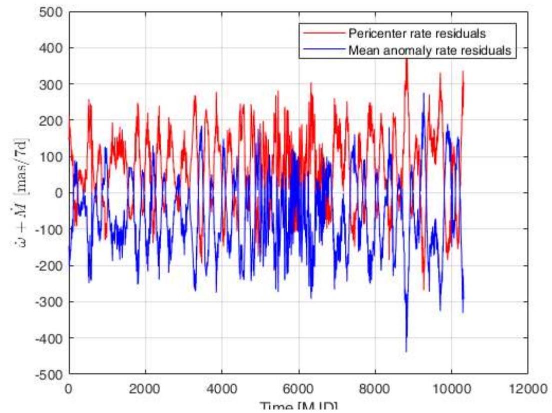
- In some theories that contain vector fields or other tensor fields, in addition to the metric tensor $g_{\mu\nu}$, the global distribution of matter in the Universe **could select a preferred rest frame** for the local gravitational interaction.
- **Damour** and **Esposito-Farese** have shown that the orbits of some **artificial satellites** have the potential to provide improvements in the limit of the α_1 parameter down to the **10^{-6}** level.
- As gravitationally **preferred rest frame** we consider that of the **cosmic background radiation**.
- Yearly effects are expected on **argument of the pericenter** ω and **mean anomaly MA,** it is convenient to concentrate on the observable $(\omega + M)$

$$\left(\dot{\omega} + \dot{M}\right)_{\alpha_1} = -\alpha_1 \, k \sin(n_{\oplus} t - \lambda_{PF}) + \cdots \qquad k = -\, 2n \frac{w \, v_{\oplus}}{c^2} \cos \beta_{PF}$$

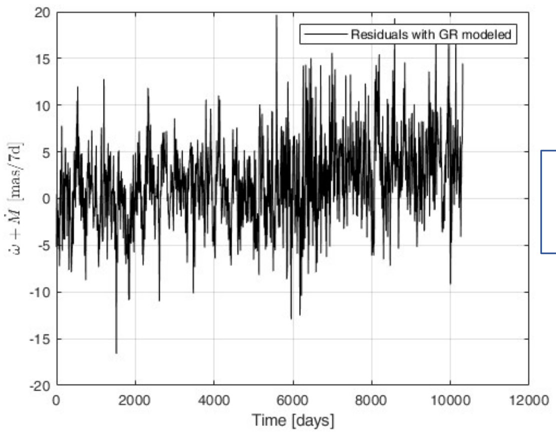
$w = (368 \pm 2)$ km/s is the speed of the Sun with respect to this preferred frame with orientation given by the ecliptic coordinates $(\lambda_{PF} = 171°.55,\ \beta_{PF} = -11°.13)$

ANALYSIS WITH LAGEOS II

- We used the data of LAGEOS II. The analysis covers a timespan of about 28.3 years, starting from 31 October 1992 (i.e. MJD=48925)

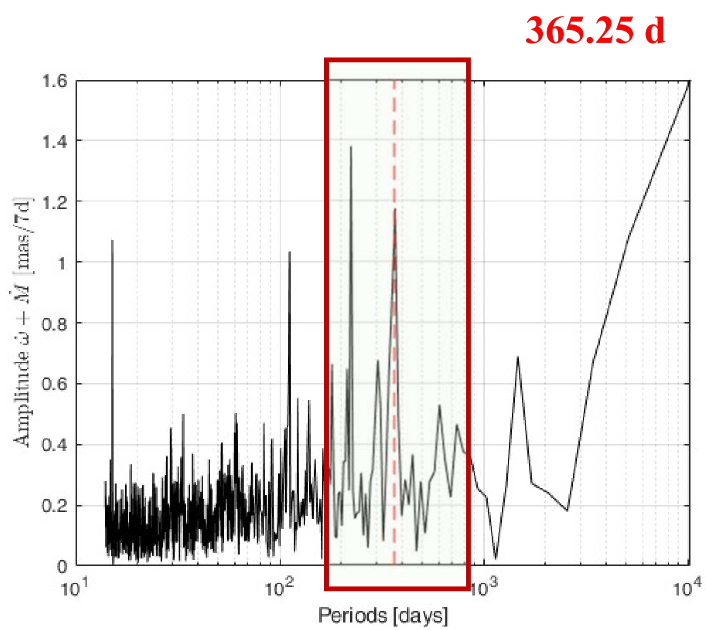


Residuals in the **observable** $\dot{\omega}$ and $\dot{M}$



Residuals in the **observable** $\dot{\omega} + \dot{M}$

FFT of the **Residuals** in the **observable**



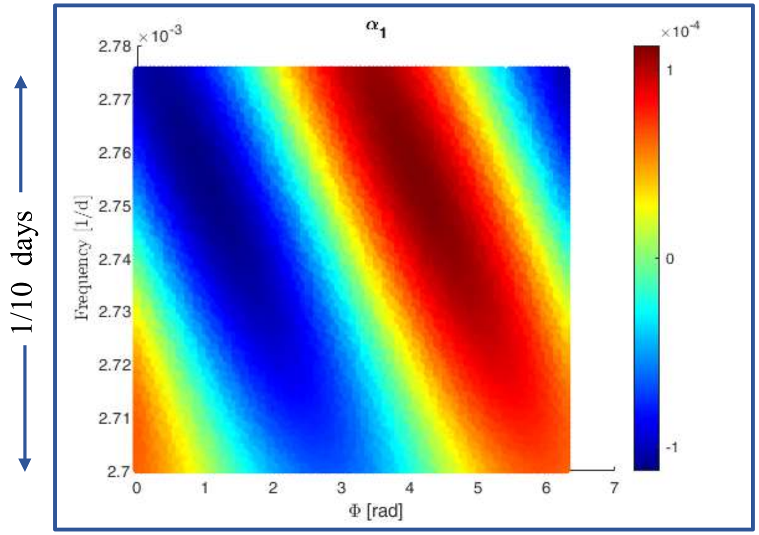
ANALYSIS WITH LAGEOS II

- To analyze the data we used a lock-in with $f_{\oplus}$ 2.738 × 10^{-3} day^{-1} and phase $\lambda_0 - \lambda_{PF}$ with λ_0= 223.83° (MJD=48932).
- This preliminary result represents the best constraint in α_1 in the field of the Earth based on a pure gravitational experiment

$$\alpha_1 = \mathbf{1.6 \cdot 10^{-6}} \pm 7 \cdot 10^{-5} \pm 1.6 \cdot 10^{-6}$$

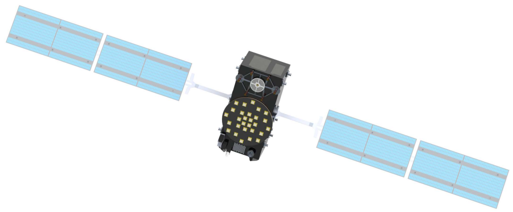
From the distribution

Systematic evaluation



- Gravitational field (quadrupole): $\delta\alpha_1 \cong 2.47 \times 10^{-6}$
- Solid tides: $7 \times 10^{-9} < \delta\alpha_1 < 7 \times 10^{-8}$
- Ocean tides: $1.5 \times 10^{-6} < \delta\alpha_1 < 1.5 \times 10^{-5}$
- Non-Gravitational Perturbations: $\delta\alpha_1 \cong 0$

The G4S.2 PROJECT



- The **G4S_2.0** project, founded by the **Italian Space Agency** (**ASI**), aims to perform a set of measurements in the field of **gravitation** with the satellites of **Galileo Full Operational Capability** (**FOC**) constellation and, in particular with **GSAT0201** and **GSAT0202,** exploiting their relatively high eccentricity ($\cong$0.16) with respect to that ($\cong$ 0) of the other satellites taking advantage of the **accuracy of their on-board atomic clocks**
- Three research centers in Italy are involved in this project:
  - ASI-CGS (Center for Space Geodesy) in Matera
  - Istituto di Astrofisica e Planetologia Spaziali (IAPS/INAF) in Roma and OATO/INAF in Torino
  - Politecnico (POLITO) in Torino
- Today I will speak about a new measurement of the Gravitational Red-shift exploiting the orbits of the Galileo satellites GSAT0201 and GSAT0202

GRAVITATIONAL RED SHIFT

- The Gravitational Redshift GRS is the change in frequency of e.m. waves travelling in a variable gravitational field: i.e. the relative **frequency change in two clocks** operating in different gravitational potentials .

potential difference

$$z = \frac{\Delta \upsilon}{\upsilon} = \frac{\Delta U}{c^2}$$

speed of light

$$z = (1+\alpha)\frac{\Delta U}{c^2}$$

$\alpha = 0$ in GR

- Our goal is to improve present limit α **G**alileo gravitational **R**edshift **E**xperiment with eccentric s**AT**ellites (**GREAT**), 2018
  - **SYRTE**: $\boldsymbol{\alpha = (0.19 \pm 2.48) \times 10^{-5}}$ *P. Delva, et al., Phys. Rev. Letter, 121, 231101 (2018)*
  - **ZARM**: $\boldsymbol{\alpha = (4.5 \pm 3.1) \times 10^{-5}}$ *S. Herrmann, et al., Phys. Rev Lett. , 121, 231102 (2018)*

- A careful reconstruction of **time dependence of the gravitational field** is needed.

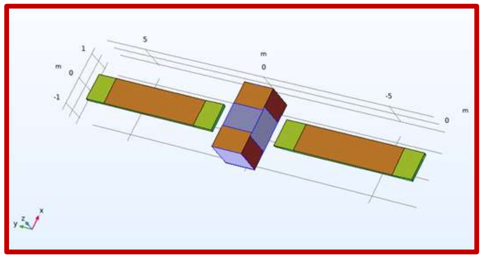
MAIN PERTURBATIONS

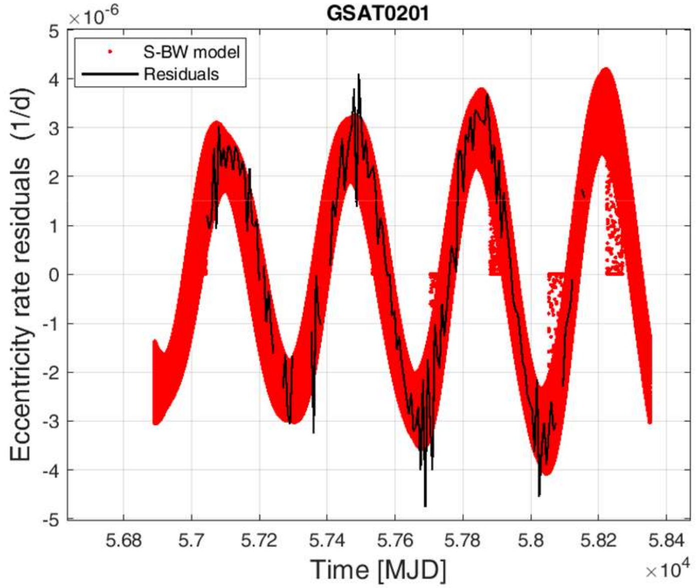
- Also in this analysis Precise Orbit Determination plays a main role.
- Galileo satellites have a complex structure and the Non-Gravitational perturbations are important in particular the reduction of the Solar Radiation Pressure effect is a main challenge.

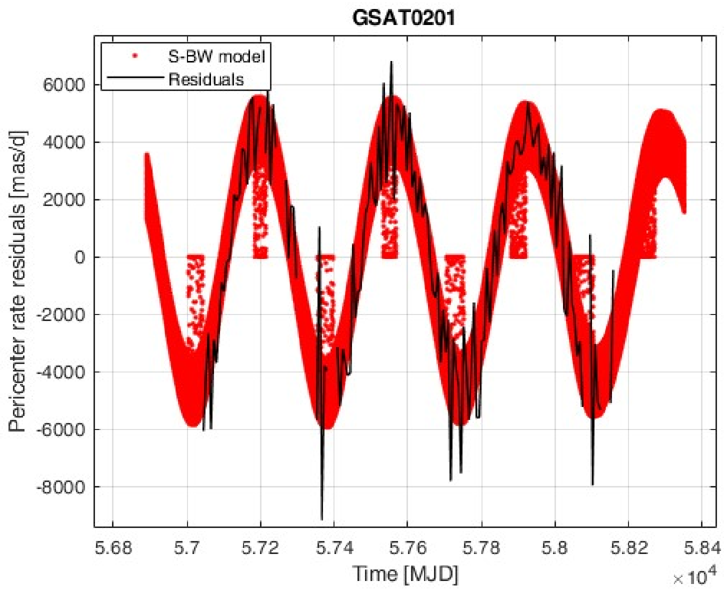
Physical effects	Formula	LAGEOS II (m/s^2)	Galileo FOC (m/s^2)
Earth's monopole	$G\frac{M_{\oplus}}{r^2}$	2.6948	0.4549
Direct SRP	$C_R\frac{A}{M}\frac{\Phi_{\odot}}{c}$	3.2×10^{-9}	1.0×10^{-7}
Earth's Albedo	$2\frac{A}{M}\frac{\Phi_{\odot}}{c}A_{\oplus}\frac{\pi R_{\oplus}^2}{4\pi r^2}$	1.3×10^{-1}	7.0×10^{-1}
Earth's infrared radiation	$\frac{A}{M}\frac{\Phi_{IR}}{c}\frac{R_{\oplus}^2}{r^2}$	1.5×10^{-10}	1.1×10^{-9}
Power from antennas	$\frac{P}{Mc}$	—	1.2×10^{-9}
Thermal effect solar panels	$\frac{2}{3}\frac{\sigma}{c}\frac{A}{M}(\epsilon_1 T_1^4 - \epsilon_2 T_2^4)$	—	1.9×10^{-10}

MODEL TO REDUCE PERTURBATION

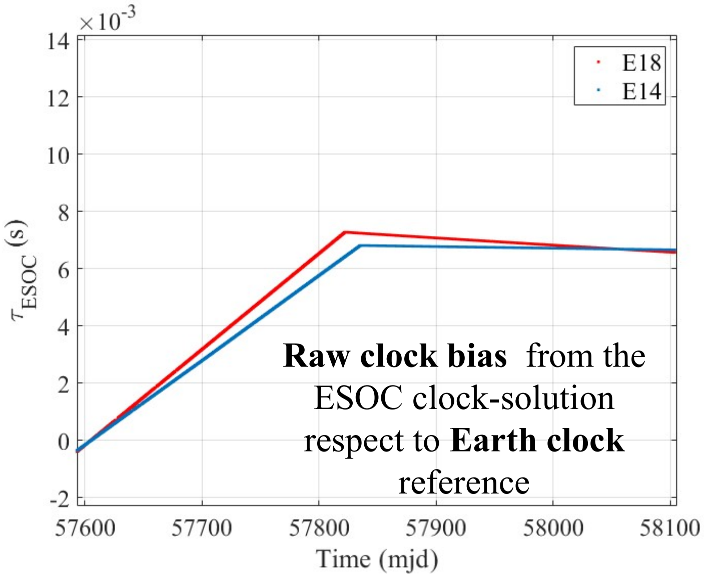
Here we show the residuals for two keplerian elements (in black) and the predicted effect (in red) of a preliminary model for the Direct Solar Radiation Pressure.



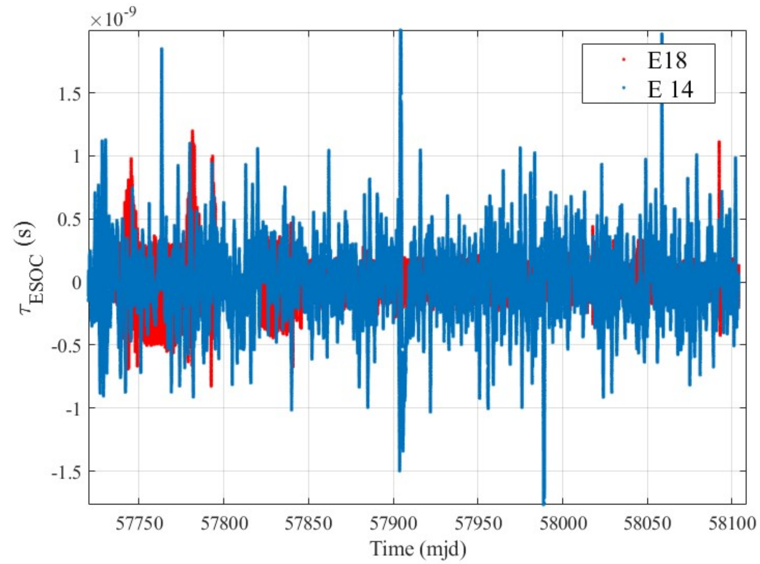




PRELIMINARY ANALYSIS



Removing long term drift and daily corrections (GREAT data)



Proper time from General Relativity

$$\tau_{GR} = \int \frac{d\tau}{dt} dt = \int [1 - \frac{v^2}{2c^2} - \frac{\Delta U}{c^2}] dt$$

Doppler effect — Potential difference

The corrected clock bias is calculated as

$$\tau_{corr} = \tau_{ESOC} + \tau_{Kepler} - \tau_{GR}$$

The raw time shift is reconstructed removing the correction for the eccentricity routinely applied by ESOC

MEASUREMENT OF GRAVITATIONAL RED-SHIFT

- We are ready to repeat the measurements carried out under GREAT project, but the **information from ESA** on routine clock bias correction (*keplerian* correction) is **missing**
- We used the data from ESA to study how to remove unwanted disturbance from data of the satellite clock.
- ILRS Central Bureau has approved an **observation campaign for G4S** which will last 24 months
- We are completing an **independent analysis pipeline** that we hope will allow us to improve the **Gravitational Red Shift** measurement.

CONCLUSIONS

- Satellite Laser Ranging technique represents a powerful tool to study Gravitation in the Weak-Field and Slow-Motion Limit of GR in the Field of the Earth.

- The gravitational effects are measured as residuals in the orbital elements, i.e. the difference between the measured and the calculated evolution of the satellite.

- A crucial point to obtain valuable measurements is to estimate with precision gravitational and non-gravitational effects acting on the satellite. Very often the quality of the model used by the software for Precise Orbit Determination is not sufficient.

- The satellite best tracked are the passive geodetic ones, but the GNSS satellite have the advantage of the microwave positioning and on-board atomic clocks.