Gravitational Waves in Einstein-Cartan Theory

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OUTLINE

1. GRAVITATIONAL WAVES IN GENERAL RELATIVITY

2. EINSTEIN-CARTAN THEORY

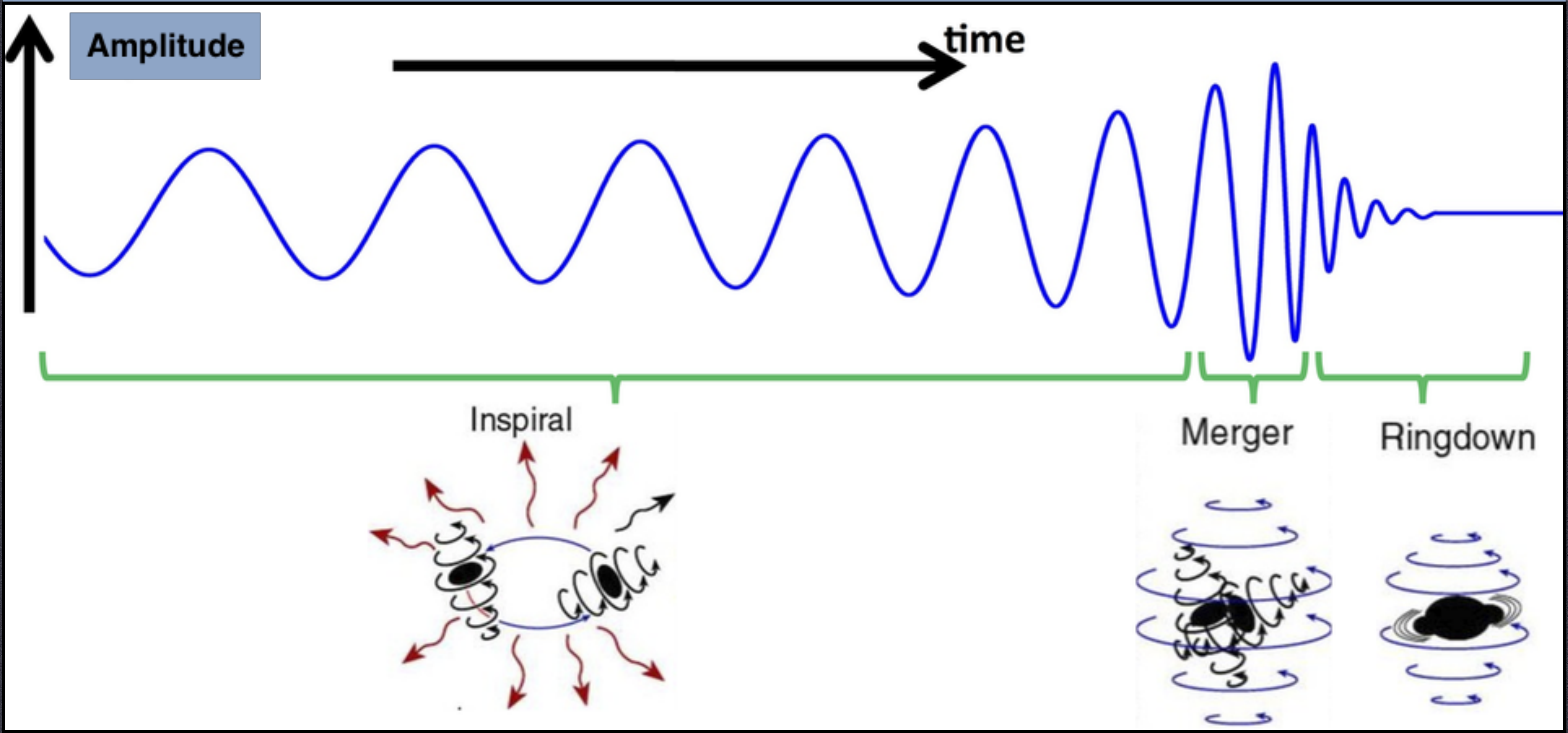
3. BLANCHET-DAMOUR APPROACH IN EINSTEIN-CARTAN THEORY

4. A FIRST APPLICATION TO BINARY NEUTRON STAR AND BLACK HOLE SYSTEMS

5. CONCLUSIONS

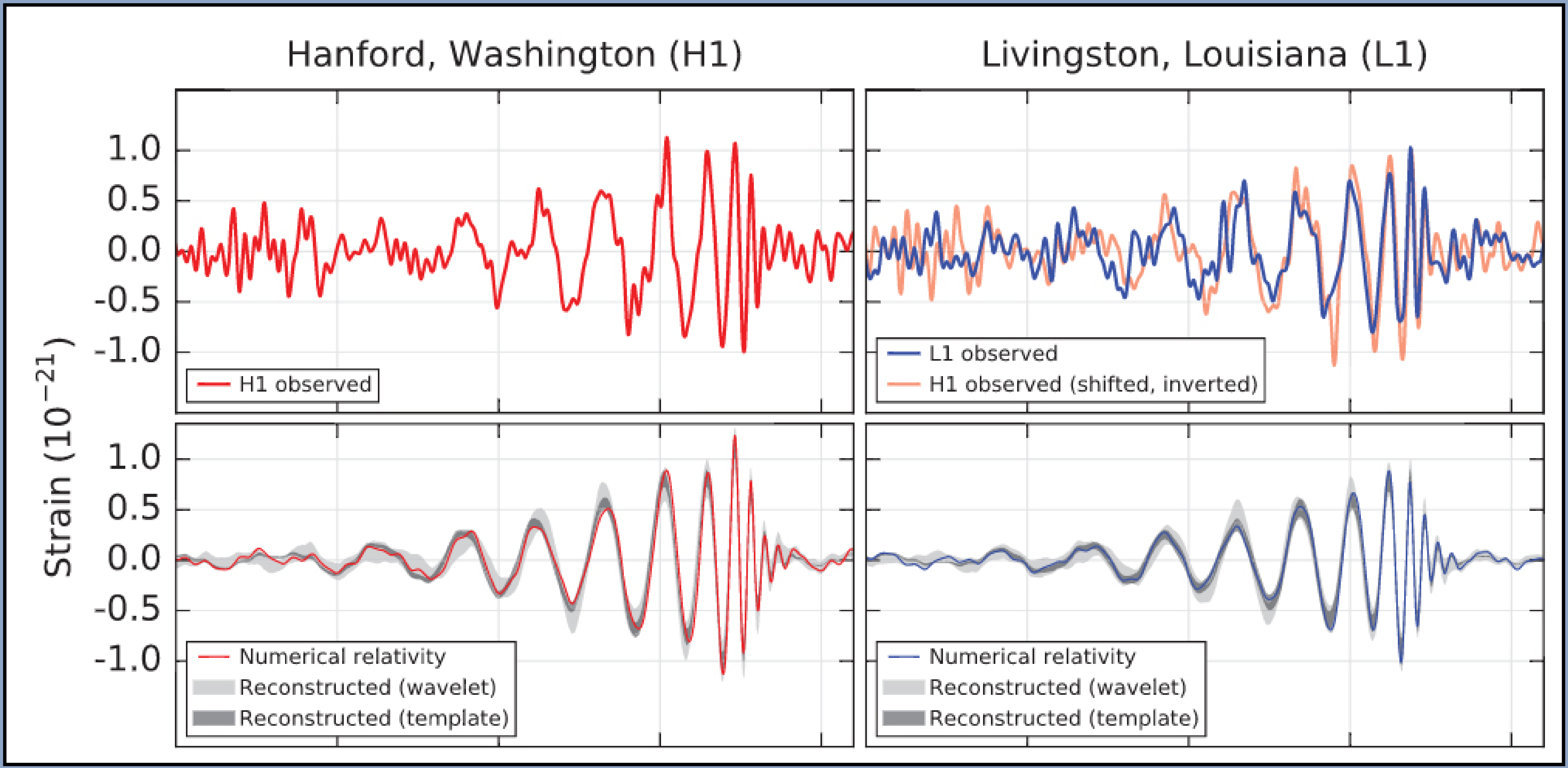
GRAVITATIONAL WAVES IN GR (1)

- **In high-energy astrophysics, the main sources of gravitational waves (GWs) are compact binary systems: black holes (BHs) and neutron stars (NSs).**



GRAVITATIONAL WAVES IN GR (2)

- **On September 14, 2015 at 09:50:45 UTC, the two Ligo detectors announced the first direct observation of gravitational waves (GWs) resulting from the merging of two black holes**



Event "GW150914"

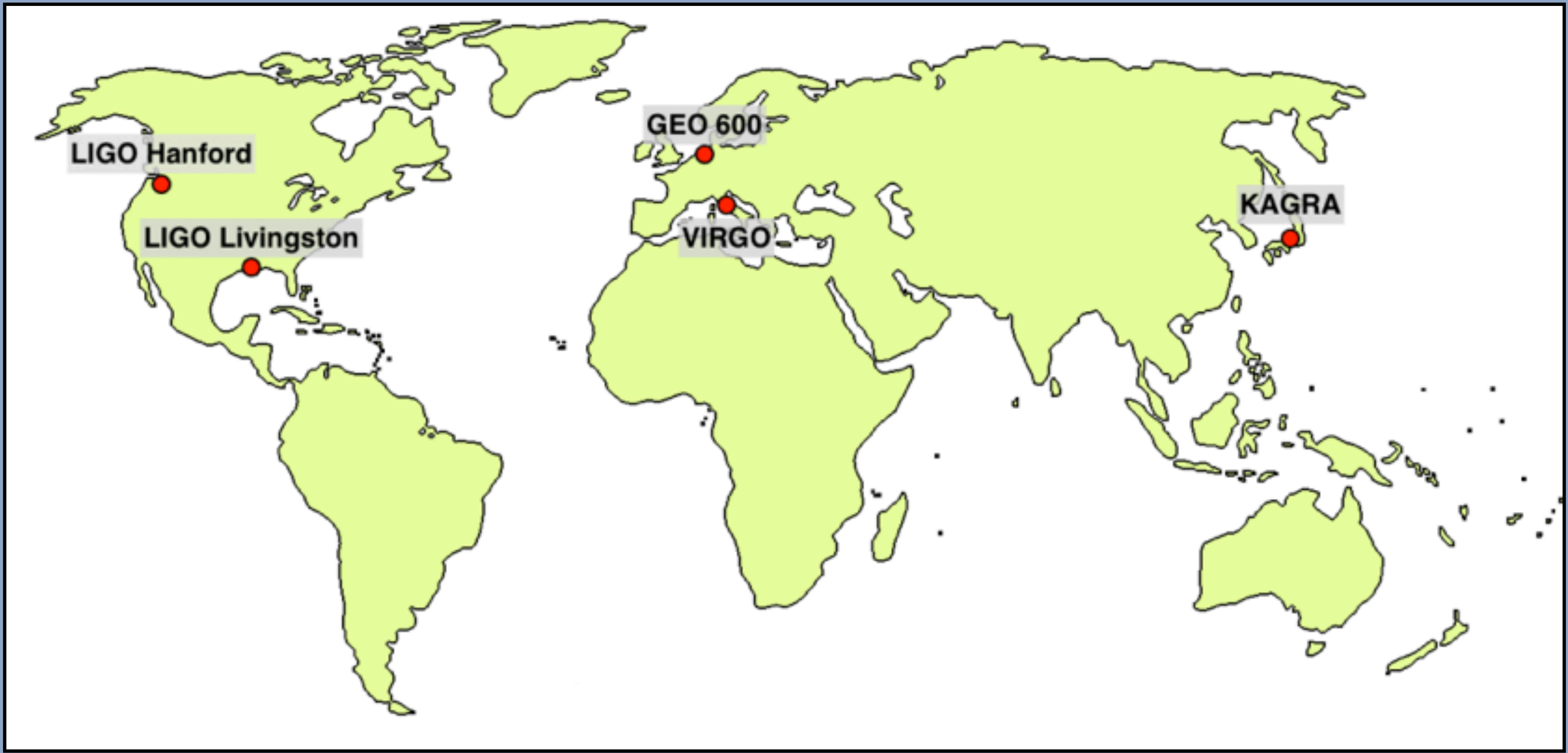
$M_1 \sim 29 M_\odot$

$M_2 \sim 36 M_\odot$

$\mathscr{E} \sim 10^{47}$ J $\sim 3 M_\odot c^2$

GRAVITATIONAL WAVES IN GR (3)

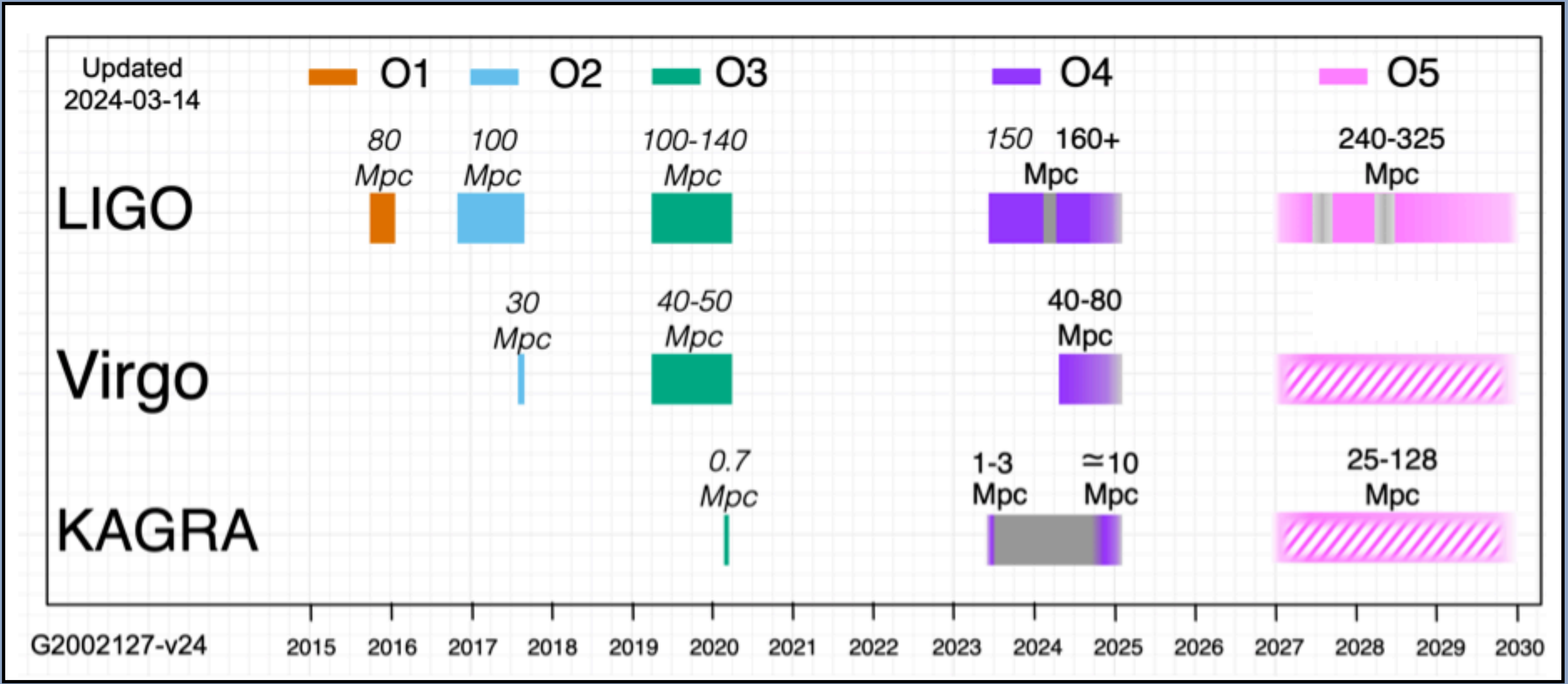
- **Map of current GW detectors**



Ground-based interferometers:
sensitivity ranges from some tens of hertz to about one kilohertz

GRAVITATIONAL WAVES IN GR (4)

- **Observing runs**



Sensitivity expressed in megaparsecs

1 Mpc= 3.26 million light years

Andromeda Galaxy is about 0.78 Mpc from the Earth

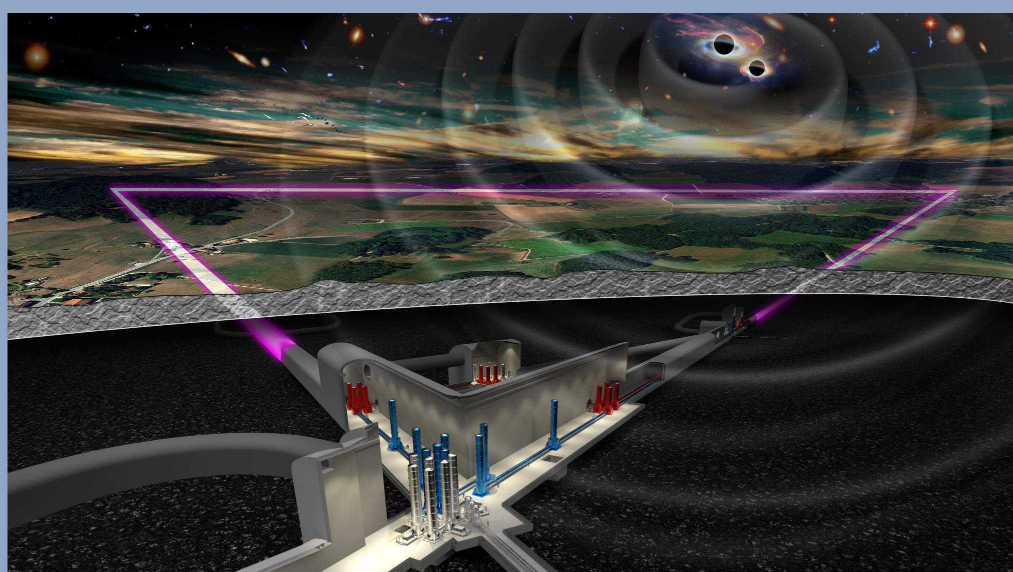
As of May 2024, GW observatories have detected more than 90 GW events from BH-BH, NS-NS, BH-NS mergers

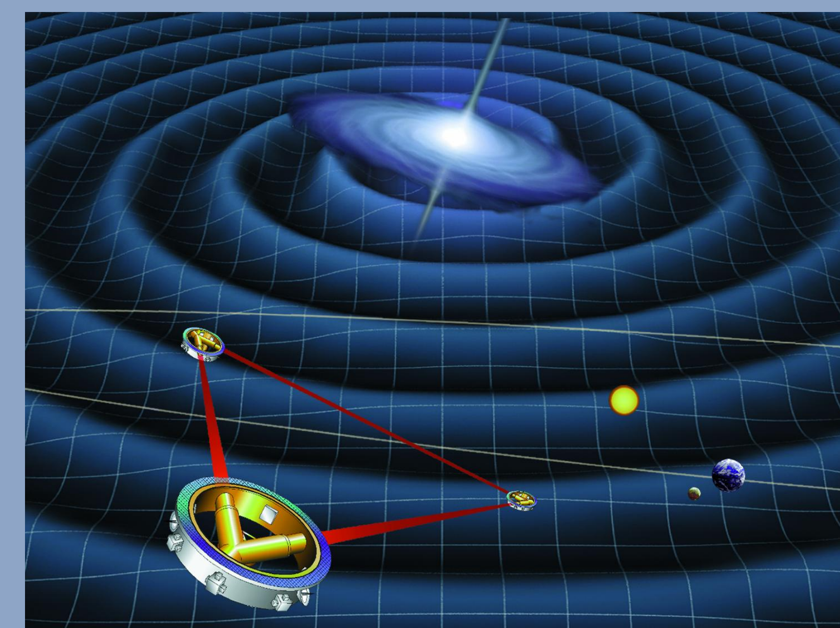
GRAVITATIONAL WAVES IN GR (5)

- **Second-generation: LIGO-India interferometer**

- **Third-generation: Einstein Telescope and Cosmic Explorer**

(sensitivity: from about 5 Hz to several kHz)





- **Space-borne low-frequency detectors: LISA and TianQin**

(sensitivity: from some μHz to about one-tenth of a hertz)

- **Radio telescope pulsar timing arrays (PTAs)**

(frequency band goes from 100 to 1 nHz)

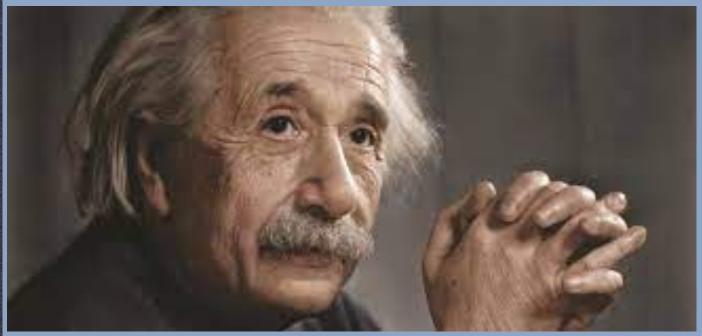
EINSTEIN-CARTAN THEORY (1)

- Einstein-Cartan (EC) theory has been formulated to extend the concepts of general relativity (GR) to the microphysical realm.

- Quantum intrinsic spin carried by elementary particles is described geometrically by means of the torsion tensor.

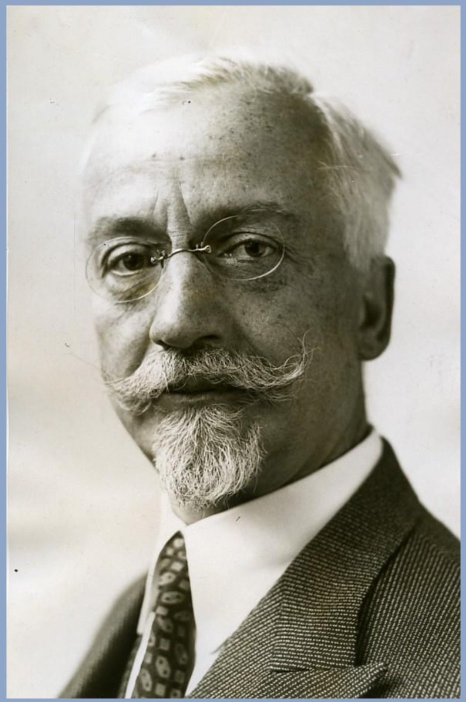
Mass-energy ⟷ Curvature

Energy-momentum tensor



Spin ⟷ Torsion

Spin density tensor



EINSTEIN-CARTAN THEORY (2)

- **EC field equations**

$$\hat{G}^{\alpha\beta} = \frac{\chi}{2}\Theta^{\alpha\beta},$$

$\hat{G}^{\alpha\beta} \equiv$ **Einstein tensor constructed with the Christoffel symbols** $\hat{\Gamma}^{\mu}{}_{\alpha\beta}$

$(\chi = 16\pi G/c^4)$

where

$$\Theta^{\alpha\beta} = T^{\alpha\beta} + \frac{\chi}{2}\mathcal{S}^{\alpha\beta},$$

combined energy-momentum tensor

$$T^{\alpha\beta} = \frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta g_{\alpha\beta}}$$

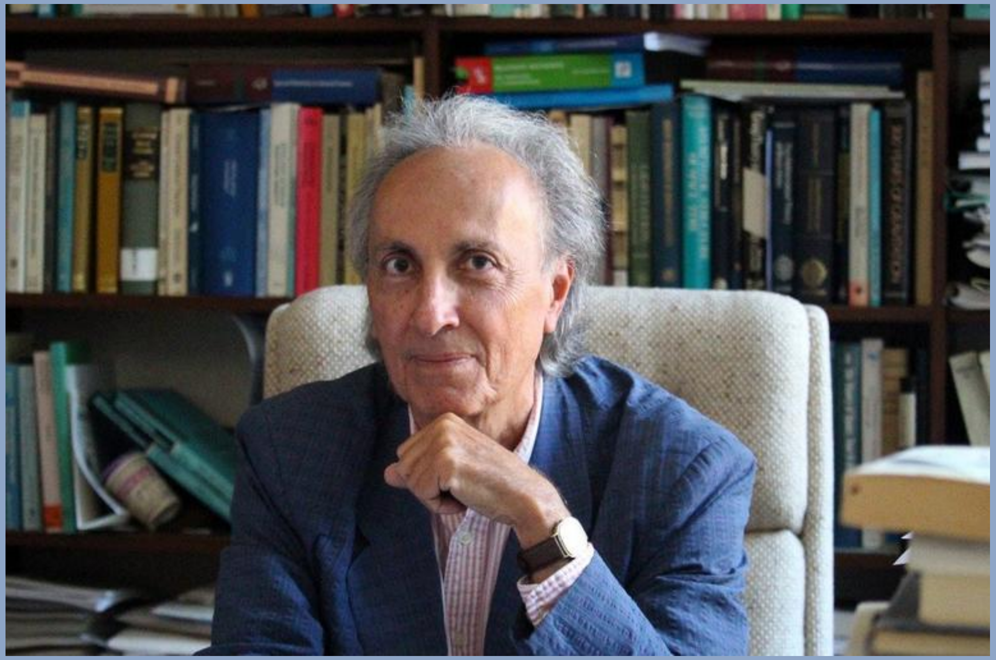
metric energy-momentum tensor

$$\mathcal{S}^{\alpha\beta} \equiv -4\tau^{\alpha\gamma}{}_{[\delta}\tau^{\beta\delta}{}_{\gamma]} - 2\tau^{\alpha\gamma\delta}\tau^{\beta}{}_{\gamma\delta} + \tau^{\gamma\delta\alpha}\tau_{\gamma\delta}{}^{\beta} + \frac{1}{2}g^{\alpha\beta}\left(4\tau_{\mu}{}^{\gamma}{}_{[\delta}\tau^{\mu\delta}{}_{\gamma]} + \tau^{\mu\gamma\delta}\tau_{\mu\gamma\delta}\right),$$

Contribution due to spin ($\tau^{\alpha\beta}{}_{\gamma} \equiv$ **canonical spin angular momentum tensor**)

BLANCHET-DAMOUR APPROACH IN EC THEORY (1)

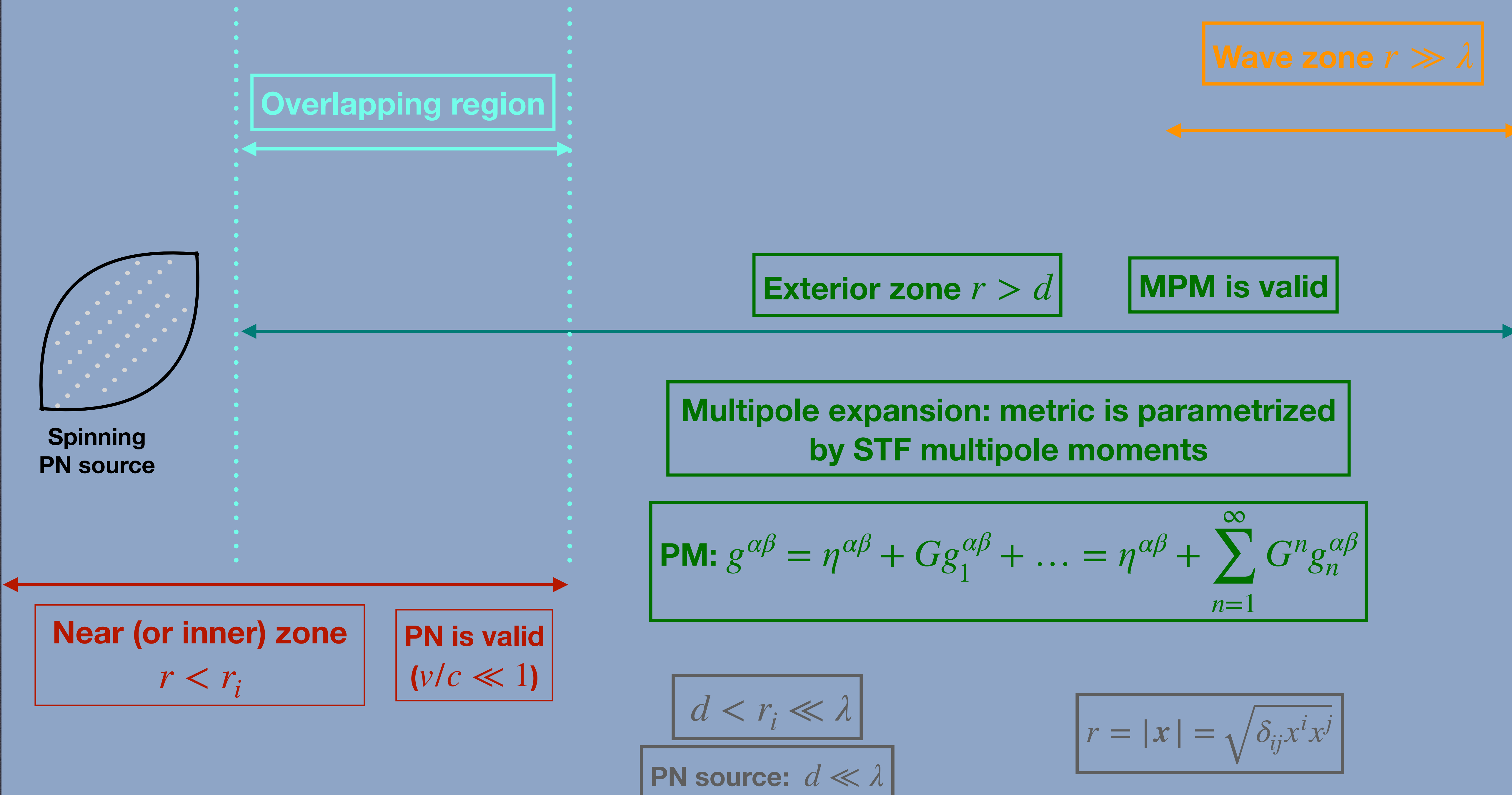
- Spinning, weakly self-gravitating, weakly stressed, and slowly moving sources (i.e., spinning PN sources).
- Motion and radiation of binary systems in their early inspiralling stage.



- GW generation problem: relating the asymptotic gravitational-wave form generated by some isolated spinning PN source and which we observe via a detector (located in the wave zone of the source), to the material content of the source, i.e., its tensor $\Theta^{\alpha\beta}$, using some suitable approximation methods.

BLANCHET-DAMOUR APPROACH IN EC THEORY (2)

Let us introduce a set of harmonic coordinates $x^\mu = (ct, \boldsymbol{x})$. The spatial part $\mathbb{R}^3$ of the spacetime manifold is decomposed in the following domains:



BLANCHET-DAMOUR APPROACH IN EC THEORY (3)

- **Blanchet-Damour formalism is based on two approximation schemes: MPM and PN methods. It allows to solve approximately the GW generation problem**

- **Solution of GW generation problem**

$$g^{\mu\nu}_{\text{ext}} = g^{\mu\nu}_{\text{ext}}(U_L, V_L)$$

wave zone

U_L: **mass-type STF radiative multipole moment of order *l***

Physical Observables

V_L: **current-type STF radiative multipole moment of order *l***

- U_L, V_L **are given as integral expressions involving the source variables; in particular, they are given as integrals extending over combined stress-energy tensor** $\Theta^{\alpha\beta}$ **of the material source.**

Multi-index notation, where *L* denotes the multi-index $i_1 i_2 \ldots i_l$ **made of *l* spatial indices. Hence** $I_L = I_{i_1 i_2 \ldots i_l}$

BLANCHET-DAMOUR APPROACH IN EC THEORY (4)

- **1PN-accurate asymptotic gravitational radiation amplitude (or waveform)**

$$\mathscr{H}_{ij} = \frac{2G}{c^4\mathscr{R}}\mathscr{P}_{ijkl}\Bigg\{U_{kl} + \frac{1}{c}\left[\frac{1}{3}N_aU_{kla} + \frac{4}{3}\epsilon_{ab(k}V_{l)a}N_b\right] + \frac{1}{c^2}\left[\frac{1}{12}N_aN_bU_{klab} + \frac{1}{2}\epsilon_{ab(k}V_{l)ac}N_bN_b\right] + \mathrm{O}(c^{-3})\Bigg\},$$

- **Total radiated power (or luminosity or flux) of the source at 1PN order**

$$\mathscr{F} = \frac{G}{c^5}\left\{\frac{1}{5}\overset{(1)}{U}_{ij}\overset{(1)}{U}_{ij} + \frac{1}{c^2}\left[\frac{1}{189}\overset{(1)}{U}_{ijk}\overset{(1)}{U}_{ijk} + \frac{16}{45}\overset{(1)}{V}_{ij}\overset{(1)}{V}_{ij}\right] + \mathrm{O}(c^{-4})\right\}.$$

APPLICATION TO A BINARY NS SYSTEM (1)

- ***N* weakly self-gravitating, slowly moving, widely separated spinning bodies.**

$$U_{ij} = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\sum_{A=1}^{N} m_A \left\{ r_A^{\langle i} r_A^{j\rangle} \left[1 + \frac{1}{c^2}\left(\frac{3}{2} v_A^2 - \sum_{B\neq A} \frac{G m_B}{|\boldsymbol{r}_A - \boldsymbol{r}_B|} \right) \right] + \frac{1}{14c^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(r_A^2 r_A^{\langle i} r_A^{j\rangle} \right) - \frac{20}{21c^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(v_A^k r_A^{\langle i} r_A^j r_A^{k\rangle} \right) \right\}$$

$$+\frac{\mathrm{d}^2}{\mathrm{d}t^2}\sum_{A=1}^{N}\left\{ \frac{4}{c^2}\left[\left(\boldsymbol{v}_A \times \boldsymbol{s}_A\right)^i r_A^j + \left(\boldsymbol{v}_A \times \boldsymbol{s}_A\right)^j r_A^i - \frac{2}{3}\delta^{ij}\left(\boldsymbol{v}_A \times \boldsymbol{s}_A\right)\cdot \boldsymbol{r}_A \right] - \frac{4}{3c^2}\frac{\mathrm{d}}{\mathrm{d}t}\left[\left(\boldsymbol{r}_A \times \boldsymbol{s}_A\right)^i r_A^j + \left(\boldsymbol{r}_A \times \boldsymbol{s}_A\right)^j r_A^i \right] \right\} + \mathrm{O}\left(\mathrm{c}^{-3}\right),$$

1PN accurate

mass quadrupole moment

$$U_{ijk} = \frac{\mathrm{d}^3}{\mathrm{d}t^3}\sum_A m_A r_A^{\langle i} r_A^j r_A^{k\rangle} + \mathrm{O}\left(\mathrm{c}^{-2}\right),$$

mass octupole moment

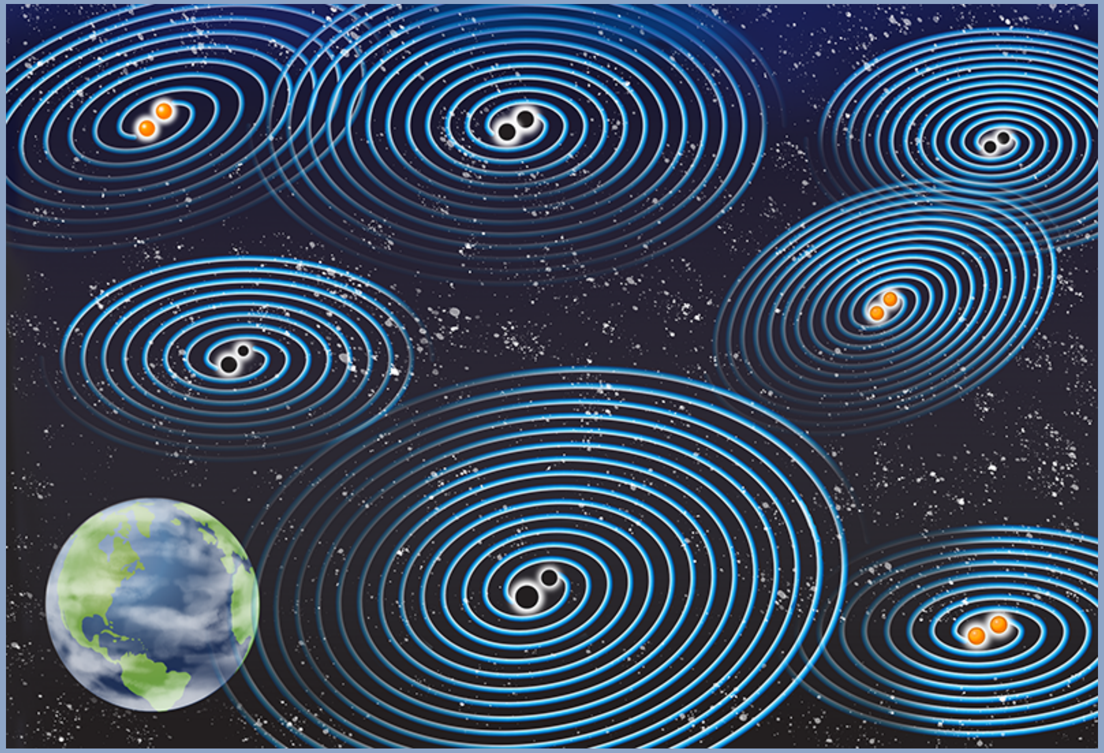
$$V_{ij} = \frac{\mathrm{d}^2}{\mathrm{d}t^2}\left\{ \sum_A m_A \epsilon^{kl\langle i} r_A^{j\rangle} r_A^k v_A^l + \frac{1}{2}\sum_A \left[3\left(s_A^i r_A^j + s_A^j r_A^i \right) - 2\delta^{ij} \boldsymbol{s}_A \cdot \boldsymbol{r}_A \right] \right\} + \mathrm{O}\left(\mathrm{c}^{-2}\right),$$

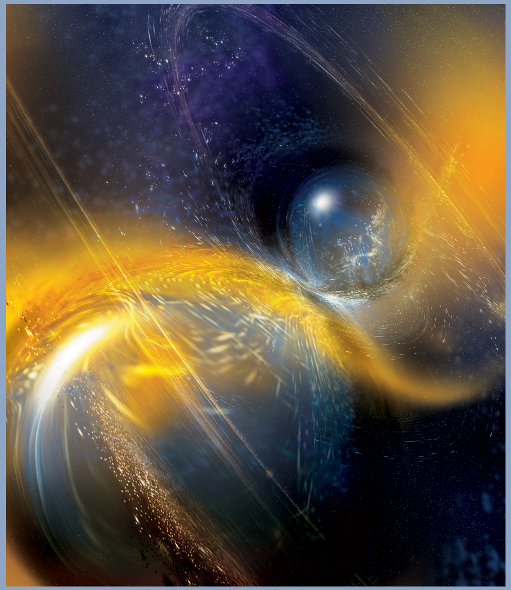
current quadrupole moment

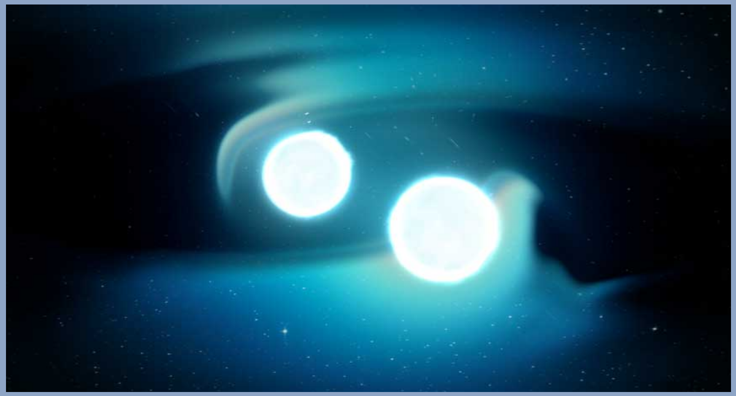
APPLICATION TO A BINARY NS SYSTEM (2)

$$U_{ijkl} = \frac{\mathrm{d}^4}{\mathrm{d}t^4} \sum_{A=1}^{N} m_A r_A^{\langle i} r_A^j r_A^k r_A^{l\rangle} + \mathrm{O}\left(\mathrm{c}^{-2}\right),$$

mass 2^4-pole moment







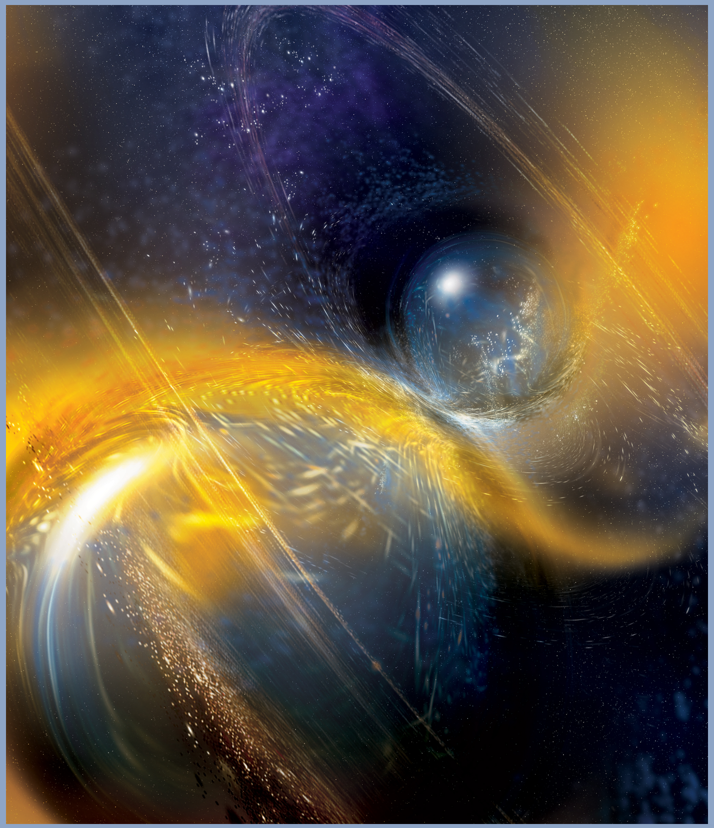
$$V_{ijk} = \frac{\mathrm{d}^3}{\mathrm{d}t^3} \sum_{A=1}^{N} \left[m_A r_A^{\langle i} r_A^j \epsilon^{k\rangle lp} r_A^l v_A^p + 2\left(r_A^n s_A^q \, \delta_n^{\langle i} r_A^j \delta_q^{k\rangle} - \boldsymbol{r}_A \cdot \boldsymbol{s}_A \, \delta_n^{\langle i} r_A^j \delta_n^{k\rangle} + s_A^q \, r_A^{\langle i} r_A^j \delta_q^{k\rangle} \right) \right] + \mathrm{O}\left(\mathrm{c}^{-2}\right).$$

current octupole moment

APPLICATION TO A BINARY NS SYSTEM (3)

- **Let us consider a binary NS system**

$$
\begin{aligned}
m_1 &= 1.60 M_\odot \\
m_2 &= 1.17 M_\odot \\
|\boldsymbol{s}_1| &= 1.21 \times 10^{57} \hbar \\
|\boldsymbol{s}_2| &= 4.73 \times 10^{56} \hbar \\
R_{\mathrm{av}} &= 4.69 \times 10^{8}\,\mathrm{m}
\end{aligned}
$$



$$
\mathcal{E}_{\mathcal{F}}(t) \equiv \left| \frac{\mathcal{F}_{\mathrm{EC}}(t)}{\mathcal{F}_{\mathrm{GR}}(t)} \right|,
$$

$$
\mathcal{E}_{\mathscr{H}}(t) \equiv |\mathscr{H}_{11}^{\mathrm{GR}}(t)| - |\mathscr{H}_{11}^{\mathrm{EC}}(t)|.
$$

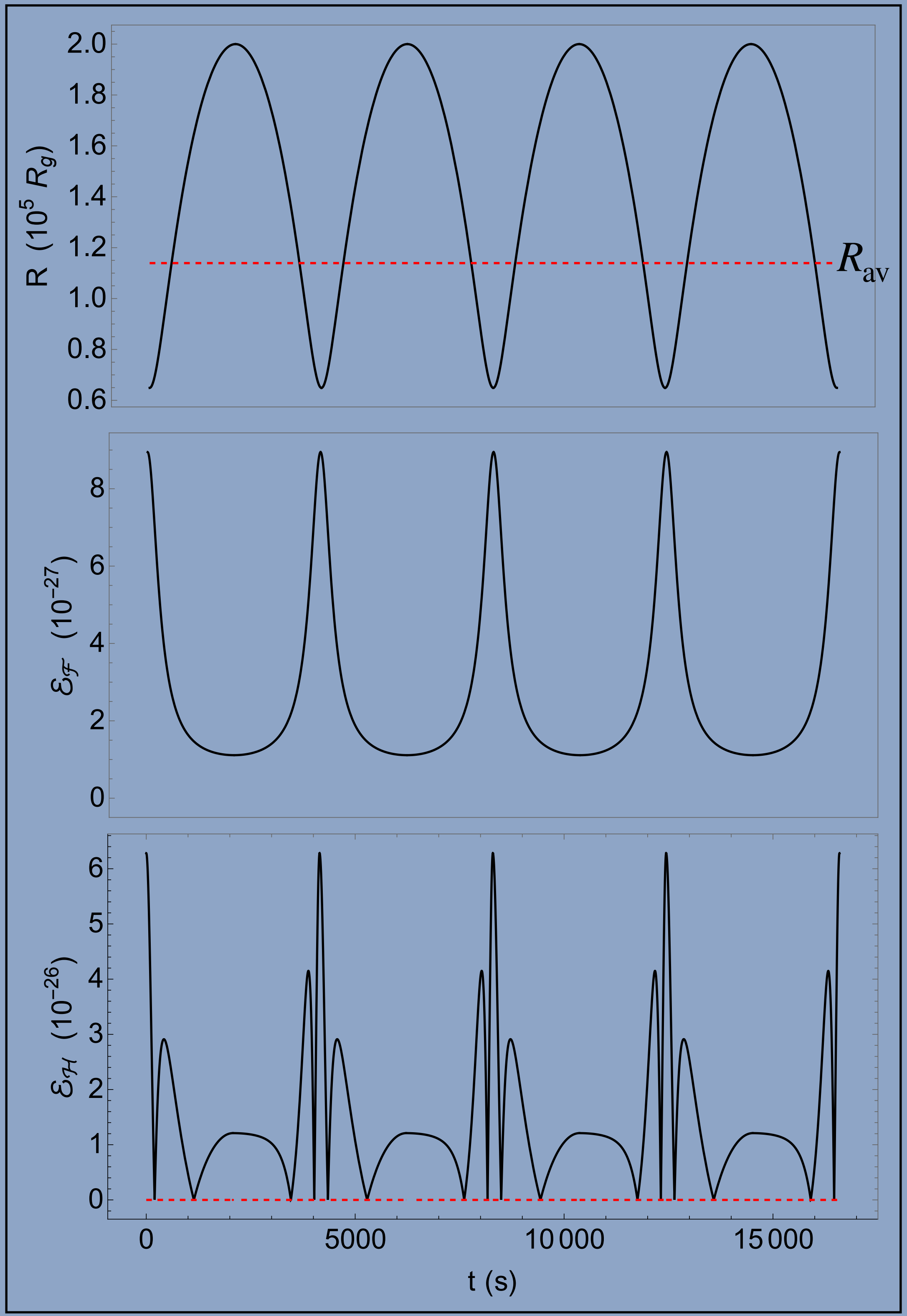
EC contribution to GR flux

EC contribution to GR waveform

APPLICATION TO A BINARY NS SYSTEM (4)

- **Plots**

The average EC contributions are smaller than GR ones by a factor 10^{-23}

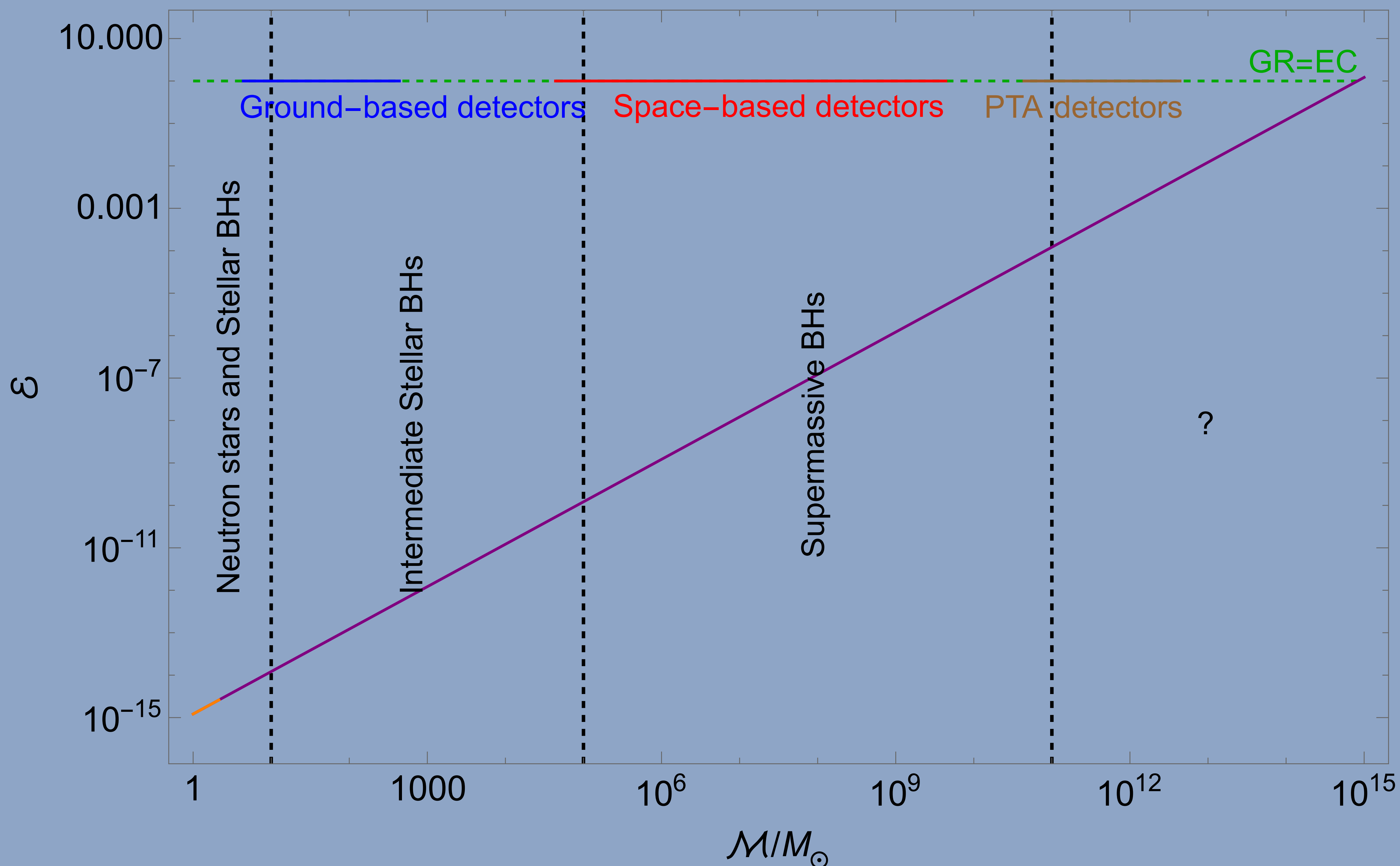


Function $R(t)$

Function $\mathcal{E}_{\mathcal{F}}(t)$

Function $\mathcal{E}_{\mathcal{H}}(t)$

APPLICATION TO BINARY BH SYSTEMS



GR and EC effects are comparable for masses of the order of $10^{14}M_\odot$, which corresponds to a GW frequency of the order of 10^{-12} Hz.

CONCLUSIONS (1)

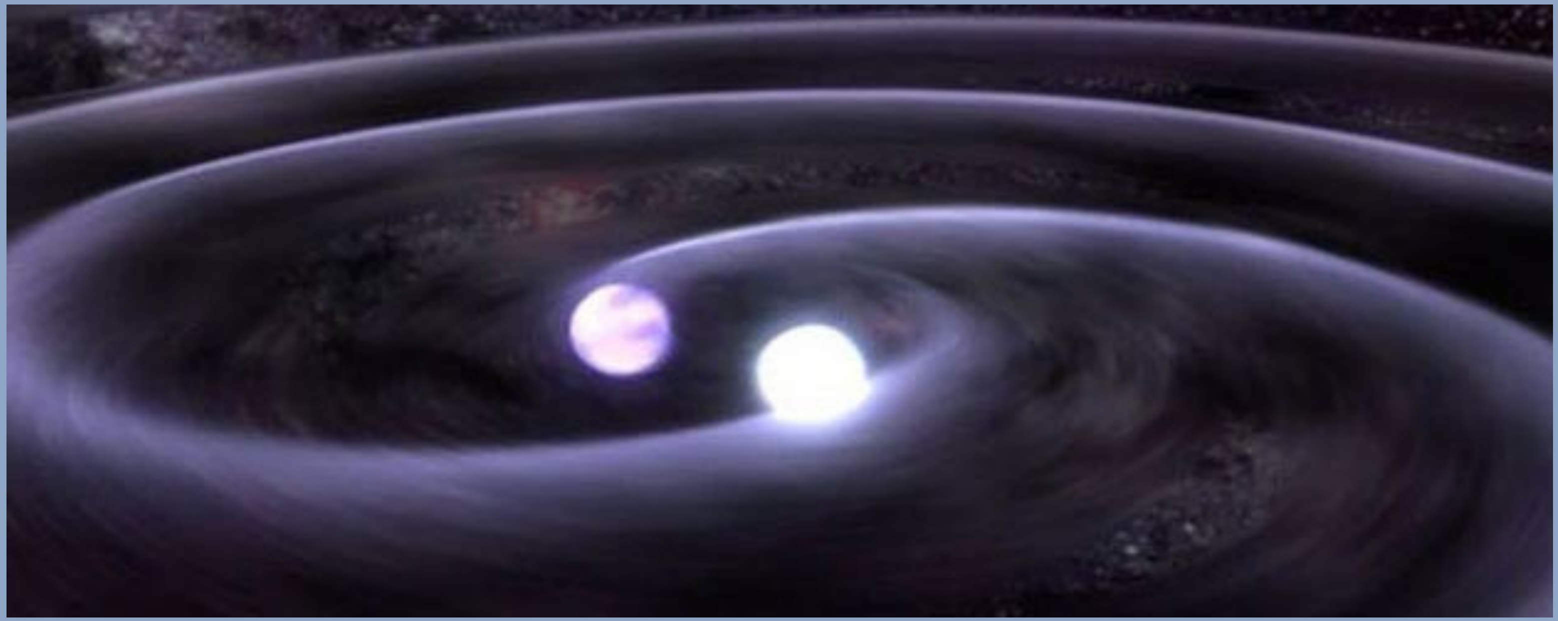
- **The research activity underlying this seminar aims at understanding possible quantum imprints in the propagation of GWs produced by spinning PN sources in EC theory, namely spinning, weakly self-gravitating, slowly moving, and weakly stressed sources.**

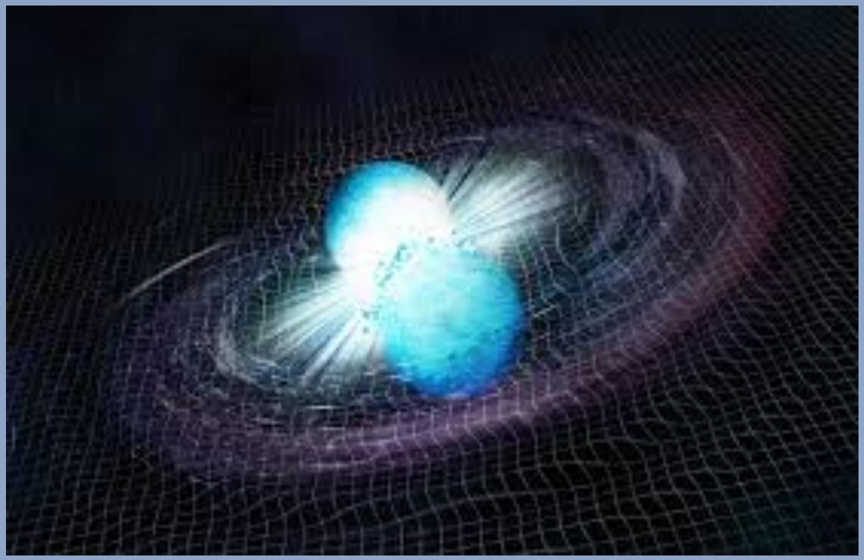
- **We have seen how the GW generation problem can be solved in the presence of torsion by extending the Blanchet-Damour approach to EC theory.**

- **We have provided a concrete application by applying the Blanchet-Damour method to a binary NS system**

CONCLUSIONS (2)

- The case of binary BH systems has also been considered. We have seen that EC corrections imprinted in their gravitational-wave signal can be potentially detected by means of the pulsar timing array technique.





- Future work: analysis of the behavior of compact binaries in their later evolution phases (i.e., plunge, merger, ringdown)

CONCLUSIONS (3)

- **Further details can be found in:**

- “*First post-Newtonian generation of gravitational waves in Einstein-Cartan theory*” (Emmanuele Battista and Vittorio De Falco), Phys. Rev. D 104, 084067 (2021)

- “*Gravitational waves at the first post-Newtonian order with the Weyssenhoff fluid in Einstein-Cartan theory*” (Emmanuele Battista and Vittorio De Falco), Eur. Phys. J. C 82, 628 (2022)

- “*First post-Newtonian N-body problem in Einstein-Cartan theory with the Weyssenhoff fluid: equations of motion*” (Emmanuele Battista and Vittorio De Falco), Eur. Phys. J. C 82, 782 (2022)

- “*First post-Newtonian N-body problem in Einstein-Cartan theory with the Weyssenhoff fluid: Lagrangian and first integrals*” (Emmanuele Battista, Vittorio De Falco, and Davide Usseglio), Eur. Phys. J. C 83, 112 (2023)

- *”Analytical results for binary dynamics at the first post-Newtonian order in Einstein-Cartan theory with the Weyssenhoff fluid”* (Vittorio De Falco and Emmanuele Battista), Phys. Rev. D 108, 064032 (2023)

- *”Radiative losses and radiation-reaction effects at the first post-Newtonian order in Einstein-Cartan theory”* (Vittorio De Falco, Emmanuele Battista, Davide Usseglio, and Salvatore Capozziello), Eur. Phys. J. C 84, 137 (2024)