CORRELATORS AND OPE COEFFICIENTS IN ARGYRES-DOUGLAS THEORIES

Andrea Cipriani, Gong Show at Meeting PRIN ‘String Theory as a bridge between Gauge Theories and Quantum Gravity’, 22/02/2024

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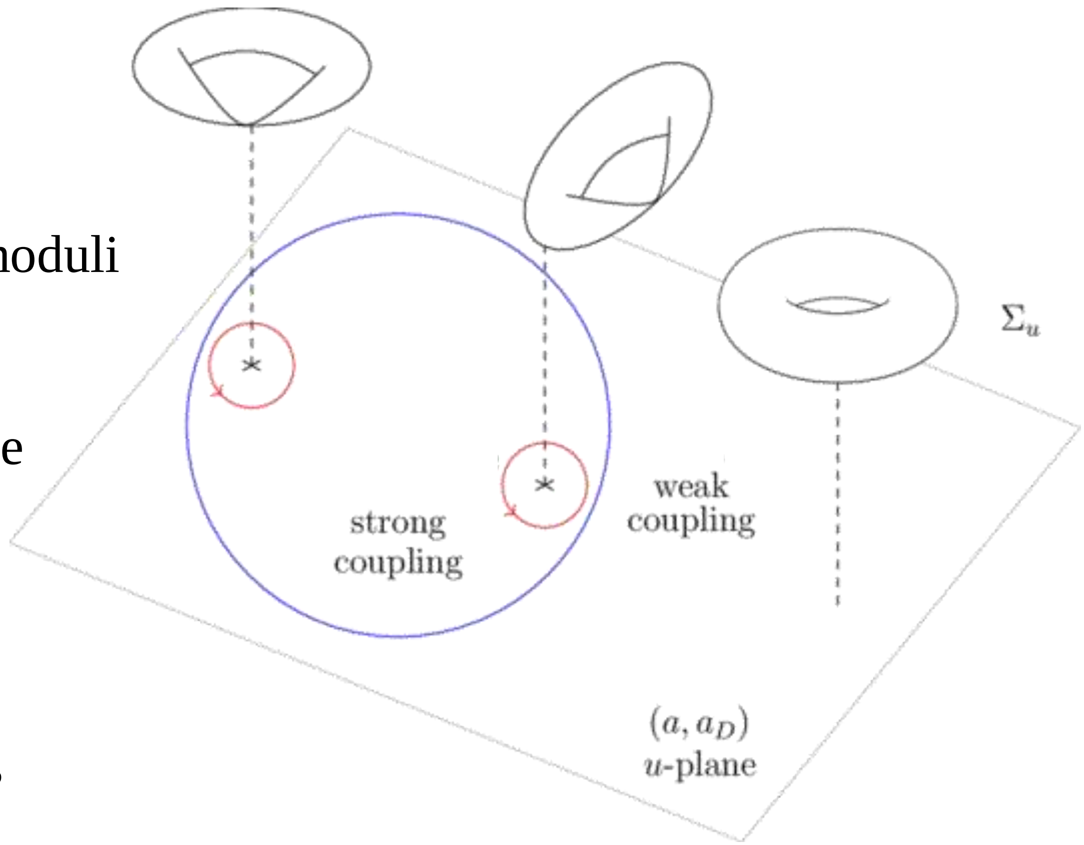
ARGYRES-DOUGLAS THEORIES

- These are 4-dimensional $\mathcal{N} = 2$ superconformal field theories and
  - without a Lagrangian description
  - strongly coupled;
  - isolated;

- We focus on the Coulomb Branch (SSB of $U(1)_R$) of moduli space

- It is parametrized by the VEVs of CB operators (that are scalar chiral superconformal primaries)

- The study is devoted to rank-1 theories, meaning
  - the CB has complex dimension 1 (u is the coordinate)
  - the SW curve associated to each point of CB is a torus



ARGYRES-DOUGLAS THEORIES

- Argyres-Douglas (AD) theories are very special points on the CB, because:

  - from a geometrical side, the SW curve associated to them has both 1-cycles simultaneously shrinking

  - from a physical side, these points describe theories with mutually non-local degrees of freedom that are simultaneously massless

- This makes a local Lagrangian that could describe their interactions not possible

- At points where mutually non-local objects become simultaneously massless the theory is interacting and conformal

- AD theories are in particular superconformal and, since they are interacting and isolated, they are intrinsically strongly coupled

MOTIVATION AND EXTREMAL CORRELATORS

- We want to compute observable quantities, in particular OPE coefficients between CB operators

- It is a challenge: the **ideal goal** is finding an explicit expression for these quantities in terms of geometric objects (maybe not possible); at the moment we settle for **improving** the results I am going to show

- We indicate the CB operators as $\mathcal{O}_i$ ($i \in \mathbb{N}_0$ related to the R-charge)

- The OPE coefficients we are interested in are determined from the 2-points extremal correlators

$$G_{ij}(x) = \langle \mathcal{O}_i(x)\, \bar{\mathcal{O}}_j(0) \rangle$$

(notice that from the selection rule coming from the conservation of $U(1)_R$ part of R-symmetry at the superconformal point, the two-point functions involving only chiral primaries are trivial)

COMPUTATION WITH LOCALIZATION ON THE 4-SPHERE

- This technique furnishes a formula for the 2-points extremal correlator on the 4-sphere of radius R, $G_{ij}(2\pi R)$, for any rank

- It turns out that if $i \neq j$, then $G_{ij} = 0$, while for $i = j = n \geq 1$ there is the following expression

$$G_{nn}^{\text{Loc}}(2\pi R) = \frac{\det\limits_{0\leq k,l\leq n} C_{kl}}{\det\limits_{0\leq k,l\leq n-1} C_{kl}}$$

[A.Grassi, Z.Komargodski, L.Tizzano, 'Extremal correlators and random matrix theory', JHEP 04 (2021) 214, [1908.10306]]

- The matrix C (two-point matrix model integral) is a $(n+1) \times (n+1)$ whose elements are

$$C_{kl} = \frac{\int_{\mathbb{R}} da\, O_k(a)\, \bar{O}_l(a)\, |Z_{\mathbb{R}^4}(a,R)|^2}{\int_{\mathbb{R}} da\, |Z_{\mathbb{R}^4}(a,R)|^2}$$

[A.Bissi, F.Fucito, A.Manenti, J.F.Morales, R.Savelli, 'OPE coefficients in Argyres-Douglas theories', JHEP 06 (2022) 085, [2112.11899]]

where

- a is related to u as $u \propto a^d$, where d is the conformal dimension of the CB operator
- O_k is the 1-point function on $\mathbb{R}^4$ deformed in a particular way dictated by the localization itself
- $Z_{\mathbb{R}^4}$ is the partition function on this space. We write it as $Z_{\mathbb{R}^4}(a,R) = e^{R^2\,\mathcal{F}(a,R)}$

COMPUTATION WITH LOCALIZATION ON THE 4-SPHERE

- At this point the OPE coefficient can be computed in the following way

$$\lambda_{ij,i+j} = \sqrt{\frac{G^{\text{Loc}}_{i+j,i+j}}{G^{\text{Loc}}_{ii}G^{\text{Loc}}_{jj}}}$$

- So, from this procedure, it is clear that everything consists in computing the matrix C_{kl},

- Following the passages in a particular 'approximation' that we are about to discuss, we get

$$C_{kl} = \frac{1}{(\alpha\, R)^{d(k+l)}} \frac{\Gamma\left(\frac{d}{2}(k+l) + \frac{3}{2}d - 1\right)}{\Gamma\left(\frac{3}{2}d - 1\right)} \qquad (1)$$

[A.Bissi, F.Fucito, A.Manenti, J.F.Morales, R.Savelli, 'OPE coefficients in Argyres-Douglas theories', JHEP 06 (2022) 085, [2112.11899]]

where α is a constant that depends from the theory, but it is not important in the determination of the OPE coefficients

LARGE RADIUS EXPANSION

- The prepotential can be written using the **large radius expansion**, according to which the radius of the 4-sphere is taken very 'large' (approaching the flat space)

$$\mathcal{F}(a,R) = \sum_{g=0}^{\infty} \mathcal{F}_g(a) R^{-2g} = \sum_{g=0}^{\infty} f_g\, a^{2-2g}\, R^{-2g}$$

[A.Bissi, F.Fucito, A.Manenti, J.F.Morales, R.Savelli, 'OPE coefficients in Argyres-Douglas theories', JHEP 06 (2022) 085, [2112.11899]]

- The result (1) is obtained including only $\mathcal{F}_0$ and $\mathcal{F}_1$, since they are explicitly known

- The fact is that this expansion is only **formal**, due to the conformal nature of our original theory

- From a mathematical point of view, it means that the series is not perturbative, but asymptotic

- In principle, it is not true that $\mathcal{F}_{g\geq 2}$ terms are less important than $\mathcal{F}_0$ and $\mathcal{F}_1$

- The same argument is valid also for 1-point functions O_k, whose higher-order corrections are not known

EXAMPLES AND APPLICATIONS

- Three examples of rank-1 AD theories: $\mathcal{H}_0, \mathcal{H}_1, \mathcal{H}_2$ with $d = \frac{6}{5}, \frac{4}{3}, \frac{3}{2}$ respectively
- They are particular points of the moduli space of $\mathcal{N} = 2$ $SU(2)$ SQCD with $N_f = 1,2,3$ respectively
- At this point we can use **localization** formula (1) and all the other formulae in order to get the OPE coefficients. The first ones are reported in the table
- Another technique that can be used for this study is the **conformal bootstrap**
- This last one furnishes the window within which the OPE coefficients have to fall in
- Except for the smallest coefficient in $\mathcal{H}_0$, results obtained with the first method are inside the window

OPE COEFFICIENT	METHOD	$\mathcal{H}_0\left(d = \frac{6}{5}\right)$	$\mathcal{H}_1\left(d = \frac{4}{3}\right)$	$\mathcal{H}_2\left(d = \frac{3}{2}\right)$
λ^2_{112}	Loc. Conf. Boost.	2,098 2,142 ÷ 2,167	2,241 2,215 ÷ 2,359	2,421 2,298 ÷ 2,698
λ^2_{123}	Loc. Conf. Boost	3,300 3,192 ÷ 3,637	3,674 3,217 ÷ 4,445	4,175

[A.Bissi, F.Fucito, A.Manenti, J.F.Morales, R.Savelli, 'OPE coefficients in Argyres-Douglas theories', JHEP 06 (2022) 085, [2112.11899]]

LARGE R-CHARGE LIMIT

- We study G_{nn}^{Loc} with only $\mathcal{F}_0$ and $\mathcal{F}_1$ in the large R-charge limit (that is large n)

- The reasons to do it are

  1) the large radius expansion of above becomes a real perturbative expansion: from the saddle point method to the integral for C_{kl}, it can be seen that the largest part of the contribution derives from $a \gg \frac{1}{R}$

  2) we can compare the results of this limit with those obtained using the EFT dictionary

- This last strategy gives a formula for the extremal correlator that is perturbatively exact in n^{-1}

$$G_{nn}^{\mathrm{EFT}} = e^{nA}\, B\, \Gamma\left(dn + \frac{3}{2}d - \frac{1}{2}\right)$$

where A and B are theory-dependent constants that cannot be captured by the EFT technique

UNIVERSAL QUANTITIES

- In order to get rid of these constants, we have focused on the following universal quantities

$$G_{nn}^{U,\text{Loc}} = \frac{G_{n+1,n+1}^{\text{Loc}} G_{n-1,n-1}^{\text{Loc}}}{\left(G_{nn}^{\text{Loc}}\right)^2} \qquad G_{nn}^{U,\text{EFT}} = \frac{G_{n+1,n+1}^{\text{EFT}} G_{n-1,n-1}^{\text{EFT}}}{\left(G_{nn}^{\text{EFT}}\right)^2}$$

- Nowadays it is not possible to get an analytical expression of the correlator G_{nn}^{Loc} for AD theories: the integrals that come out using the Andréief identity for the determinant cannot be solved exactly

- Only a numerical study is reachable (another reason to eliminate the constants in our study)

- We expected that the difference between the two methods for the universal quantities would start from n^{-3} term

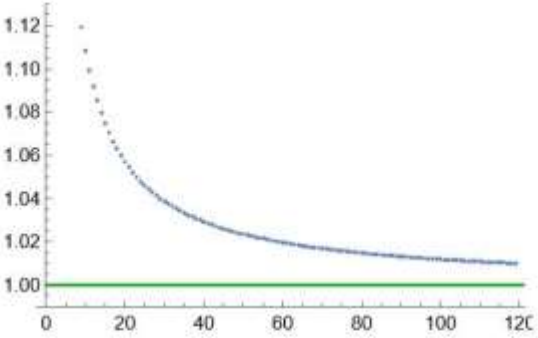
$$G_{nn}^{U,\text{Loc}} = 1 + \frac{\alpha}{n} + \frac{\beta}{n^2} + \frac{\gamma}{n^3} + \mathcal{O}(n^{-4}) \qquad G_{nn}^{U,\text{EFT}} = 1 + \frac{\alpha}{n} + \frac{\beta}{n^2} + \frac{\gamma_1}{n^3} + \mathcal{O}(n^{-4})$$

$$\alpha = d \qquad \beta = \frac{2 - 3d + d^2}{2} \qquad \gamma_1 = \frac{(d-1)^2\,(11 - 14d + 2d^2)}{12\,d}$$

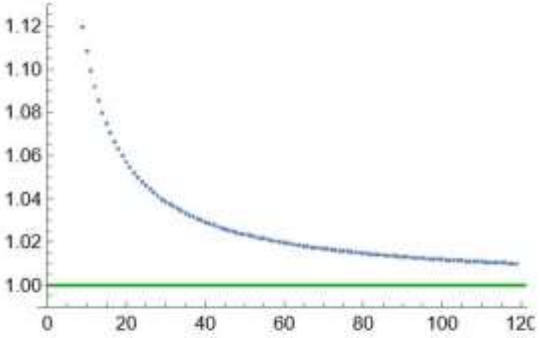
NUMERICAL STUDY FOR $\mathcal{H}_0$

[AC, F.Fucito, J.F.Morales, R.Savelli, In preparation]

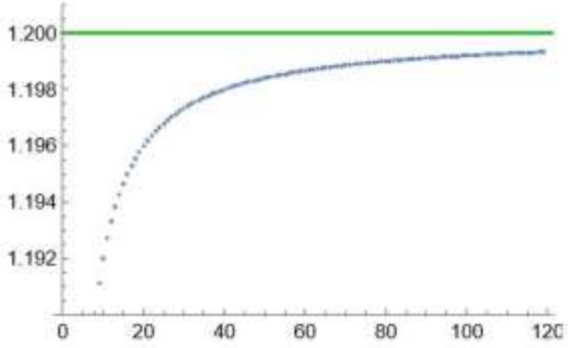
$$G_{nn}^{U,\text{Loc}}$$



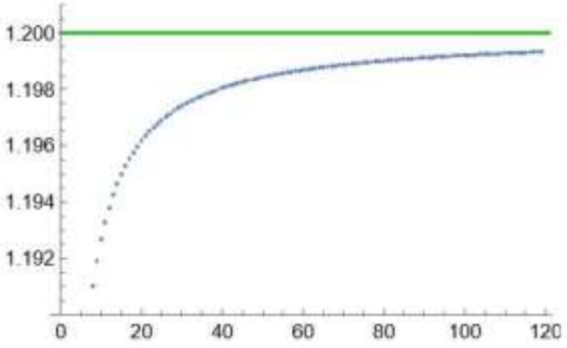
$$G_{nn}^{U,\text{EFT}}$$



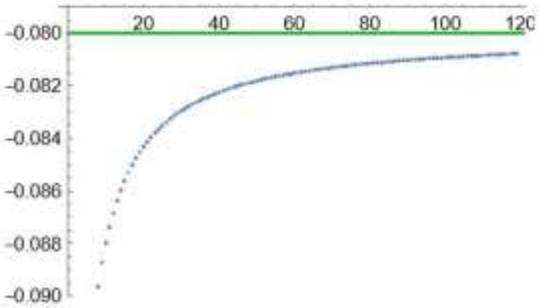
$$\left(G_{nn}^{U,\text{Loc}} - 1\right) \cdot n$$



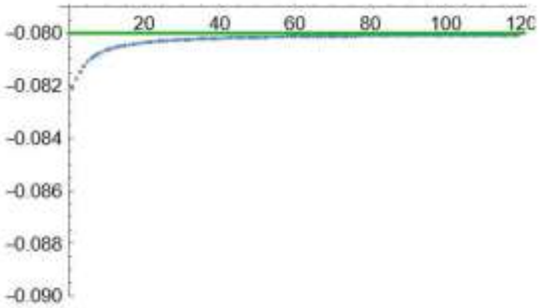
$$\left(G_{nn}^{U,\text{EFT}} - 1\right) \cdot n$$



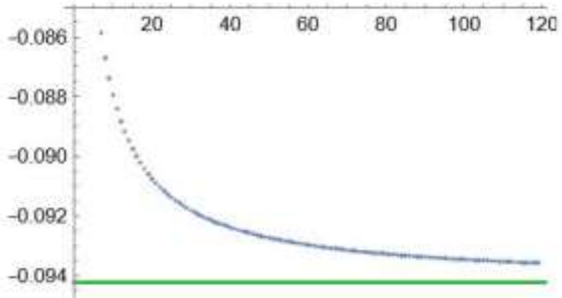
$$\left(G_{nn}^{U,\text{Loc}} - 1 - \frac{6}{5\,n}\right) \cdot n^2$$



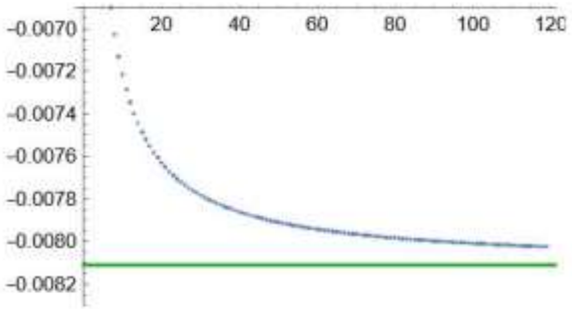
$$\left(G_{nn}^{U,\text{EFT}} - 1 - \frac{6}{5\,n}\right) \cdot n^2$$



$$\left(G_{nn}^{U,\text{Loc}} - 1 - \frac{6}{5\,n} + \frac{2}{25\,n^2}\right) n^3$$



$$\left(G_{nn}^{U,\text{EFT}} - 1 - \frac{6}{5\,n} + \frac{2}{25\,n^2}\right) n^3$$



NUMERICAL RESULTS AND COMPARISON

- We managed to determine the coefficient of the n^{-3} term for $\mathcal{H}_0$ and $\mathcal{H}_1$

$$G_{nn}^{U,\mathrm{Loc}}(\mathcal{H}_0) = 1 + \frac{6}{5\,n} - \frac{2}{25\,n^2} - \frac{106}{1125\,n^3} + \mathcal{O}(n^{-4})$$

$$G_{nn}^{U,\mathrm{Loc}}(\mathcal{H}_1) = 1 + \frac{4}{3\,n} - \frac{1}{9\,n^2} - \frac{7}{324\,n^3} + \mathcal{O}(n^{-4})$$

$$G_{nn}^{U,\mathrm{EFT}}(\mathcal{H}_0) = 1 + \frac{6}{5\,n} - \frac{2}{25\,n^2} - \frac{73}{9000\,n^3} + \mathcal{O}(n^{-4})$$

$$G_{nn}^{U,\mathrm{EFT}}(\mathcal{H}_1) = 1 + \frac{4}{3\,n} - \frac{1}{9\,n^2} - \frac{37}{1296\,n^3} + \mathcal{O}(n^{-4})$$

- From these relations we can also find the perturbative expansion for the $\ln(G_{nn}^{\mathrm{Loc}})$ (easier than G_{nn}^{Loc} for exponential terms), in particular, minus the term proporional to n and the constant one

$$\ln\left(G_{nn}^{\mathrm{Loc}}(\mathcal{H}_0)\right) \simeq \frac{6}{5} n \ln n + \frac{4}{5} \ln n + \frac{17}{90\,n} + \mathcal{O}(n^{-2})$$

$$\ln\left(G_{nn}^{\mathrm{Loc}}(\mathcal{H}_1)\right) \simeq \frac{4}{3} n \ln n + \ln n + \frac{25}{72\,n} + \mathcal{O}(n^{-2})$$

$$\ln\left(G_{nn}^{\mathrm{EFT}}(\mathcal{H}_0)\right) \simeq \frac{6}{5} n \ln n + \frac{4}{5} \ln n + \frac{167}{720\,n} + \mathcal{O}(n^{-2})$$

$$\ln\left(G_{nn}^{\mathrm{EFT}}(\mathcal{H}_1)\right) \simeq \frac{4}{3} n \ln n + \ln n + \frac{11}{32\,n} + \mathcal{O}(n^{-2})$$

- This is something shown in *[A.Grassi, Z.Komargodski, L.Tizzano, 'Extremal correlators and random matrix theory', JHEP 04 (2021) 214, [1908.10306]]* for SQCD with $N_f = 4$ and said by them for the three AD theories under consideration, but the explicit computation of the first coefficient involved by the difference is new

PROPOSAL FOR IMPROVEMENT OF THE RESULTS

- In order to fix this mismatch, we must include in the computation from localization also all the other terms in the prepotential

- Ansatz for the partition function that interpolates between the behavior for large a (known) and small a (new contribution)

- The ansatz cannot change the coefficients of n^{-1} and n^{-2} in the universal quantity

- The first idea that has come in our mind is (setting $R = 1$)

$$Z_{\mathbb{R}^4} = e^{\mathcal{F}_0}\, e^{\mathcal{F}_1}\; a^{-d\, f_\infty}\left(t + a^d\right)^{f_\infty}$$

with $t > 0$. Here we are studying what happens in this situation. Work in progress…

CONCLUSIONS AND GOALS

- Concluding, the main goal of this study is finding a better ansatz for the partition function (we are not still touching the insertions) in order to reproduce the right coefficients

- Consequently, the goodness of the ansatz, provided that the previous point is satisfied, can be also seen through the fact that the minimal OPE coefficient for $\mathcal{H}_0$ theory falls within the conformal bootstrap window

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THANK YOU SO MUCH FOR YOUR ATTENTION!

APPLICATION OF ANDREIEF IDENTITY

- Andrèief identity states that, given two sets of n functions $\{f_k(y); g_k(y)\}_{k=0}^{n-1}$ and a measure $d\mu(y)$, then

$$\det_{ab} \int d\mu(y)\, f_a(y) g_b(y) = \frac{1}{n!} \int \prod_{i=0}^{n-1} d\mu(y_i) \ \det_{ab}\big(f_a(y_b)\big) \ \det_{cd}\big(g_c(y_d)\big) \qquad (\#)$$

that is the identity relates a determinant of integrals to a multivariate integral over determinants

- In our case, we have to compute (modulo some constants that do not care in the comparison with the EFT formula)

$$\det_{kl} \int_{\mathbb{R}} da\, \left(a^d\right)^{k+l} a^{3(d-1)}\, e^{-a^2}$$

- Hence, by comparing with (#), we identify $d\mu(y) \leftrightarrow da\, e^{-a^2}\, a^{3(d-1)}$, $f_k(y) \leftrightarrow a^{dk}$, $g_l(y) \leftrightarrow a^{dl}$ (and, roughly, we replace every a with y_i) and hence we get, from the identity of the Vandermonde determinant

$$\det_{kp}\Big(f_k\big(y_p\big)\Big) = \det_{kp}\Big(\big(y_p^d\big)^k\Big) = \prod_{j<k}\big(y_j^d - y_k^d\big) \qquad \det_{ls}\big(g_l(y_s)\big) = \det_{ls}\Big(\big(y_s^d\big)^l\Big) = \prod_{j<k}\big(y_j^d - y_k^d\big)$$

APPLICATION OF ANDREIEF IDENTITY

- So our determinant becomes

$$\det_{kl} \int_{\mathbb{R}} da \, \left(a^d\right)^{k+l} a^{3(d-1)} \, e^{-a^2} = \frac{1}{n!} \int_{\mathbb{R}^n} \prod_{j=0}^{n-1} dy_i \; e^{-y_j^2} \, y_j^{3(d-1)} \prod_{j<k} \left(y_j^d - y_k^d\right)^2$$

- Applying the following change of variable (I will be sloppy on the interval of integration, which should be $\mathbb{R}^{n,+}$), $x_i = y_i^d$, then we get

$$\det_{kl} \int_{\mathbb{R}} da \, \left(a^d\right)^{k+l} a^{3(d-1)} \, e^{-a^2} = \frac{1}{n!} \int_{\mathbb{R}^{n,+}} \prod_{j=0}^{n-1} dx_i \; e^{-x_j^{\frac{2}{d}}} \, x_j^{2-\frac{2}{d}} \prod_{j<k} \left(x_j - x_k\right)^2$$

- If $d = 2$ these integrals can be solved in an analytical way, finding the known result for SQCD with $N_f = 4$ of *[A.Grassi, Z.Komargodski, L.Tizzano, 'Extremal correlators and random matrix theory', JHEP 04 (2021) 214, [1908.10306]]*; for generic d nowadays we cannot solve these integrals analytically