D-branes backreaction from String Field Theory

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February 22th 2024

Based on:
[arXiv:2305.02843][arXiv:2305.02844]

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Motivations

The central idea is to approach the problem of open-closed duality through SFT techniques.

The Feynman diagrams in 't Hooft's double line notation of a gauge theory in the large N limit organize into an expansion in genus and boundaries $\longrightarrow$ **open string theory**.

$$\log Z^{\text{open}} = \sum_{g,b} \left(g_{\text{YM}}^2\right)^{2g-2} \lambda^b F_{g,b}, \qquad \lambda := g_{\text{YM}}^2 N.$$

The conjecture is that summing on the boundaries gives an expansion in g referring to a **closed string theory** on a different background.

One of the most fundamental realization of this idea is the Maldacena conjecture [Maldacena, 1998].

In simpler setup the open-closed duality can be proved exactly, for example: [Gaiotto-Rastelli, 2004].

SFT approach

We approach one side of the open-closed duality problem through string field theory techniques, outlining a procedure to describe the backreaction of D-branes in 't Hooft's large N limit.

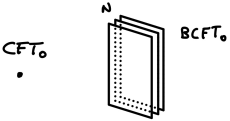
Four steps:

1. Define a consistent **quantum open-closed string field action**

$$S_{\text{oc}}[\Phi, \Psi].$$

2. Perform the **large N limit**, ending up on a theory composed by a purely classical closed string sector and a quantum but planar open string sector

$$S_{\text{oc}}[\Phi, \Psi] \longrightarrow S_{\text{pl}}[\Phi, \Psi].$$



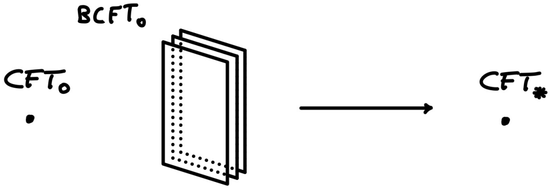
SFT approach

3. **Integrate out** the open string sector, obtaining a classical closed string field theory with tadpole

$$S_{\rm pl}[\Phi, \Psi] \longrightarrow S'_{\rm eff}[\Phi] + \langle \Phi, T \rangle.$$

4. Do the **vacuum shift**, so as to eliminate the tadpole and describe the dynamics in the new background

$$S'_{\rm eff}[\Phi] + \langle \Phi, T \rangle \longrightarrow S_{\rm BR}[\varphi] + \Lambda$$



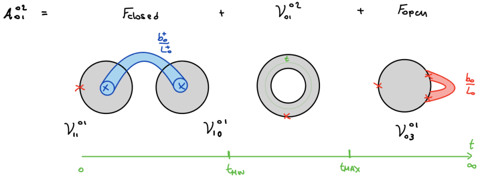
Open-Closed Action

The BV quantum master action ($\hbar = 1$) is given by the sum of **fundamental vertices** weighted by the string coupling constant κ according to the worldsheet topological expansion [Zwiebach, 1997]

$$
\begin{aligned}
S_{\mathrm{oc}}[\Phi, \Psi] &= \frac{1}{2\kappa}\omega_{\mathrm{o}}\left(\Psi, Q_{\mathrm{o}}\Psi\right) + \frac{1}{2\kappa^2}\omega_{\mathrm{c}}\left(\Phi, Q_{\mathrm{c}}\Phi\right) + S_{\mathrm{int}}[\Phi, \Psi] \\
&= \sum_{g=0}^{\infty}\sum_{b=0}^{\infty}\kappa^{2g+b-2}\sum_{k=0}^{\infty}\sum_{\{l_1,\ldots,l_b\}=0}^{\infty}\frac{1}{b!k!(l_1)\cdots(l_b)}\mathcal{V}^{g,b}_{k;\{l_1,\ldots,l_b\}}
\end{aligned}
$$

The interacting part has an implicit integral on moduli space with cut off to avoid surface degenerations. The missing region is recovered through propagators:

$$
\mathcal{A}^{g,b}_{k;\{l_1,\ldots,l_b\}} = \mathcal{V}^{g,b}_{k;\{l_1,\ldots,l_b\}} + \mathcal{F}^{g,b}_{k;\{l_1,\ldots,l_b\}}
$$



Nilpotent structure

The full BV master action can be written as

$$\boxed{\begin{aligned} S_{\rm oc}[\Phi,\Psi] &= \int_0^1 dt\left(\frac{\omega_{\rm c}}{\kappa^2}\left(\pi_{10}\boldsymbol{\partial}_t\,\mathcal{G}\,,\,\pi_{10}\boldsymbol{l}\,\mathcal{G}\right)+\frac{\omega_{\rm o}}{\kappa}\left(\pi_{01}\,\boldsymbol{\partial}_t\,\mathcal{G}\,,\,\pi_{01}\boldsymbol{m}\,\mathcal{G}\right)\right)\\ &= \int_0^1 dt\,\hat{\omega}\left(\pi_1\boldsymbol{\partial}_t\,\mathcal{G}\,,\,\pi_1\,\boldsymbol{n}\,\mathcal{G}\right).\end{aligned}}$$

Consistency condition:
the entire moduli space is covered through the Feynman diagram expansion $\longleftrightarrow$ the action satisfies the **BV quantum master equation**:

$$\boxed{\frac{1}{2}(S_{oc},S_{oc})+\Delta S_{oc}=0\longleftrightarrow(\boldsymbol{U}+\boldsymbol{n})^2=0}$$

[Maccaferri-AR-Vosmera, 2023]

where $\boldsymbol{U}$ is the so-called **Poisson bi-vector**

$$\boldsymbol{U}=\kappa^2\boldsymbol{U}_{\rm c}+\kappa\boldsymbol{U}_{\rm o}^{(\rm p)}+\kappa\boldsymbol{U}_{\rm o}^{(\rm np)}.$$

Large N limit

Let us describe the theory for a large number N of identical D-branes. The fundamental vertex $\mathcal{V}^{g,b}$ contains a number of traces over CP factors which equals the number of boundaries. We can thus define **normalized vertices** as

$$\mathcal{V'}^{g,b} := \frac{1}{N^b}\,\mathcal{V}^{g,b} \longrightarrow \text{finite}, \qquad \text{as} \quad N \longrightarrow \infty.$$

The action written in terms of normalized vertices becomes

$$S_{\rm oc}[\Phi,\Psi] = \sum_{g=0}^{\infty}\sum_{b=0}^{\infty} N^{2-2g}\lambda^{2g+b-2}\sum_{k=0}^{\infty}\sum_{\{l_1,\dots,l_b\}=0}^{\infty} \times \frac{1}{b!k!(l_1)\cdots(l_b)}\mathcal{V'}^{g,b}_{k;\{l_1,\dots,l_b\}}\left(\Phi^{\wedge k}\otimes'\Psi^{\odot l_1}\wedge'\dots\wedge'\Psi^{\odot l_b}\right),$$

where we defined the **'t Hooft coupling** $\lambda := \kappa N$.

$$\boxed{S_{\rm pl}\,[\Phi,\Psi] = \lim_{N\to\infty}\frac{1}{N^2}S_{\rm oc}(\Phi,\Psi) = \int_0^1 dt\,\hat{\omega}'\left(\pi_1\boldsymbol{\partial}_t\,\mathcal{G}\,,\,\pi_1\,\boldsymbol{n}^{(p)}\,\mathcal{G}\right)}$$

$$\boxed{\left(\boldsymbol{U}^{(p)}+\boldsymbol{n}^{(p)}\right)^2 = 0}$$

Open string integrating out

Now, we want to integrate out the open string sector and compute the corresponding effective action

$$e^{-S_{\text{eff}}[\Phi]} := \int_{\text{g.f.}} \mathcal{D}_\Psi \, e^{-S_{pl}[\Phi,\Psi]},$$

this can be formally done using the **homological perturbation lemma**

[Doubek et al, 2019][Kajiura, 2002] [Erbin et al, 2020][Okawa, 2022]

Effective products:

$$\begin{aligned}\tilde{\boldsymbol{n}}^{(p)} =& \boldsymbol{P}_c \boldsymbol{n}^{(p)} \frac{1}{1+\boldsymbol{h}_{\text{o}}(\delta \boldsymbol{n}^{(p)} + \boldsymbol{U}^{(p)})} \boldsymbol{P}_c := \tilde{\boldsymbol{l}},\\ \tilde{\boldsymbol{U}}^{(p)} =& 0.\end{aligned}$$

Effective action:

$$\boxed{S_{\text{eff}}\,[\Phi] = \int_0^1 dt \, \frac{\omega_{\text{c}}}{\lambda^2} \left(\dot{\Phi}\,,\, \pi_1 \tilde{\boldsymbol{l}}\, e^{\wedge\Phi}\right) + \Lambda_{\text{open}}.}$$

Homotopy relations: **weak L_∞ algebra**→ $\boxed{\left(\tilde{\boldsymbol{l}}\right)^2 = 0.}$

Vacuum shift

To describe the dynamics of the system in the new background, it is necessary to eliminate the tadpole by implementing the so-called **vacuum shift**:

1. Solve the tadpole-sourced closed string field theory equation of motion

$$\sum_{k=1}^{\infty} \frac{1}{k!}\tilde{l}_k(\Phi^{\wedge k}) = -\tilde{l}_0 \Longrightarrow \Phi_*(\lambda).$$

2. Use the fluctuations around that solution as dynamical fields, thus obtaining the backreacted action that describes the theory in the new background $\Phi = \Phi_* + \varphi$:

$$\boxed{S_{\mathrm{BR}}[\varphi] = \int_0^1 dt\, \frac{\omega_{\mathrm{c}}}{\lambda^2}\left(\dot{\varphi}\,,\, \pi_1 \tilde{\boldsymbol{l}}_*\, e^{\wedge\varphi}\right) + \Lambda_{\mathrm{open}} + \Lambda_{\mathrm{closed}}}$$

Homotopy relations: **strong L_∞ algebra**→ $\boxed{\left(\tilde{\boldsymbol{l}}_*\right)^2 = 0.}$

Summarizing

$$\boxed{S_{\rm oc}\,[\Phi,\Psi] = \int_0^1 dt\,\hat{\omega}\,(\pi_1\boldsymbol{\partial}_t\,\mathcal{G}\,,\,\pi_1\,\boldsymbol{n}\,\mathcal{G})\,, \qquad (\boldsymbol{U}+\boldsymbol{n})^2 = 0}$$

$\Downarrow$ Large N limit $\Downarrow$

$$\boxed{S_{\rm pl}\,[\Phi,\Psi] = \int_0^1 dt\,\hat{\omega}'\left(\pi_1\boldsymbol{\partial}_t\,\mathcal{G}\,,\,\pi_1\,\boldsymbol{n}^{(p)}\,\mathcal{G}\right), \qquad \left(\boldsymbol{U}^{(p)}+\boldsymbol{n}^{(p)}\right)^2 = 0}$$

$\Downarrow$ Open string integrating out $\Downarrow$

$$\boxed{S_{\rm eff}\,[\Phi] = \int_0^1 dt\,\frac{\omega_{\rm c}}{\lambda^2}\left(\dot{\Phi}\,,\,\pi_1\tilde{\boldsymbol{l}}\,e^{\wedge\Phi}\right) + \Lambda_{\rm open}\,, \qquad \left(\tilde{\boldsymbol{l}}\right)^2 = 0}$$

$\Downarrow$ Vacuum shift $\Downarrow$

$$\boxed{S_{\rm BR}[\varphi] = \int_0^1 dt\,\frac{\omega_{\rm c}}{\lambda^2}\left(\dot{\varphi}\,,\,\pi_1\tilde{\boldsymbol{l}}_*\,e^{\wedge\varphi}\right) + \Lambda_{\rm open} + \Lambda_{\rm closed}\,, \qquad \left(\tilde{\boldsymbol{l}}_*\right)^2 = 0}$$

[Maccaferri-AR-Vosmera, 2023]

Conclusion and outlook

The next step is to apply this procedure to an explicit example $\longrightarrow (2,1)$ bosonic minimal string model with a large number of $FZZT$ branes:

- Total bulk CFT:

$$CFT_0 = CFT^{c=-2}_{\text{matter}} \oplus CFT^{c=28}_{\text{Liuville}} \oplus CFT^{c=-26}_{\text{ghost}}.$$

- Total BCFT:

$$|\mathcal{B}(\mu_B)\rangle = |\mathcal{B}^{\text{Dirichlet}}_{\text{matter}}\rangle \otimes |\mathcal{B}^{\text{FZZT}}_{\text{Liuville}}\rangle \otimes |\mathcal{B}_{ghost}\rangle.$$

Gaiotto and Rastelli proved open-closed duality exactly for this model. In particular, they described the backreaction of this $(2,1)$ model with a large number of $FZZT$ branes into a $(2,2k+1)$ minimal string model without D-branes.

Thank you for your attention!