Black hole spectroscopy in Einstein-Maxwell-scalar theory

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Einstein-Maxwell-scalar theory (EMS)

Consider the general **action** for the Einstein-Maxwell-scalar theory

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - 2\partial_\mu \phi \partial^\mu \phi - F[\phi] F_{\mu\nu} F^{\mu\nu}\right] ,$$

where $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$, $F[\phi]$ is a general **coupling function**.

Scalar field equation:

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Classification of the EMS models:

1. **dilatonic-type**: $\phi(r) = 0$ does not solve the field equations, e.g. $F[\phi] = e^{2\alpha\phi}$,
   - $\alpha = \sqrt{3}$ four-dimensional reduction of Kaluza-Klein;
   - $\alpha = 1$ low energy limit of superstring theory.

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2. **scalarized-type**: $\phi(r) = 0$ is a solution of the field equations.

Spontaneous scalarization

The **linearized scalar field equation** for a small $\delta\phi$ perturbation is

$$(\Box - \mu_{eff}^2)\delta\phi = 0 \,, \qquad \mu_{eff}^2 = \frac{F_{\mu\nu}F^{\mu\nu}}{4}\frac{\delta^2 F[\phi]}{\delta\phi^2}\bigg|_{\phi=0} \,.$$

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Types of scalarization [Doneva+ arXiv:2211.01766]

- Induced by curvature (e.g. *scalar-tensor-Gauss-Bonnet theory*);
- Induced by spin ;
- Induced by **matter or coupling to other field** (**EMS theory**);

A specific EMS model

If we consider a spherically symmetric ansatz for the scalarized BH solution, i.e.

$$ds^2 = -N(r)e^{-2\delta(r)}dt^2 + \frac{1}{N(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \ ,$$
$$A(r) = V(r)dt, \qquad \phi = \phi(r).$$

we get the field equations (with $N(r) = 1 - 2m(r)/r$)

$$\delta' + r\phi'^2 = 0 \ ,$$

$$V' = -\frac{Q}{r^2 F[\phi]\, e^{\delta}} \ ,$$

$$r(r-2m)\phi'^2 + r^2 V'^2 e^{2\delta} F[\phi] - 2m' = 0 \ ,$$

$$r(r-2m)\phi'' - [2(m + rm' - r) + (r^2 - 2mr)\delta']\phi' + \frac{r^2 V'^2 e^{2\delta}}{2}\frac{\delta F[\phi]}{\delta\phi} = 0 \ .$$

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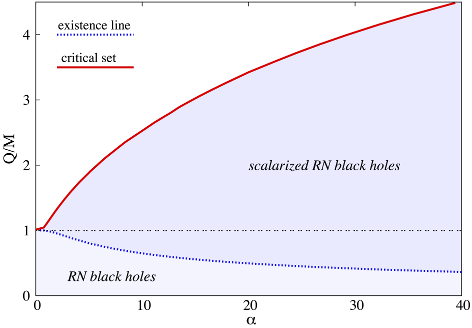
An example of EMS models that exhibits ***spontaneous scalarization*** is given by the following coupling function

$$F[\phi] = e^{\alpha\phi^2}$$

The $F[\phi] = e^{\alpha\phi^2}$ EMS model is characterized by a **domain of scalarization.**

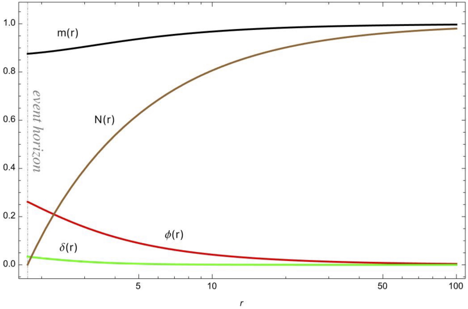
- for $Q/M \leq 1$ the RN solutions is not unique.
- the scalarized solution can be **overcharged**

Herdeiro+ PRL **121**, 101102 (2018)



The field equations are solved numerically requiring asymptotic flatness at infinity and regularity at the black hole horizon.

Example of *scalarized* BH solution with $\alpha = 20$ and $q = Q/M = 0.7$.



Why black hole spectroscopy?

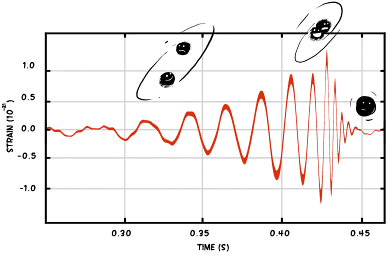


Illustration for GW150914 by Nutsinee Kijbunchoo
https://www.ligo.org/magazine/LIGO-magazine-issue-8.pdf

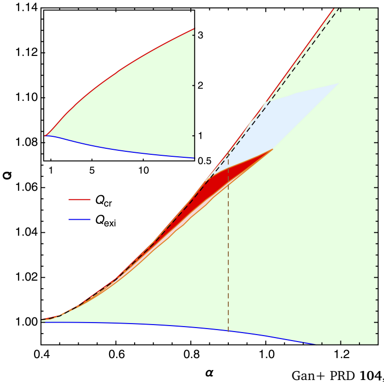
The **ringdown** produced by the final BH of a merger is described by superposition of **quasi-normal modes (QNMs)**:

$$\omega_{lmn} = \omega_{R,lmn} + i\,\omega_{I,lmn}.$$

- The QNMs strongly depend on the **photon sphere**.

For scalarized BH in the EMS model $F[\phi] = e^{\alpha\phi^2}$ there exist **two unstable photon spheres** outside the horizon in a small region of the parameter space.

Two photon spheres may trigger ***long-lived modes***!



Gan+ PRD **104**, 044049 (2021)

Linear and spherical perturbations

Consider **spherically symmetric** and **linear perturbations** of the fields

$$ds^2 = -\tilde{N}(t,r)\, e^{-2\tilde{\delta}(t,r)} dt^2 + \frac{dr^2}{\tilde{N}(t,r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \ ,$$

$$A = \tilde{V}(t,r)dt \ , \qquad \phi = \tilde{\phi}(t,r) \ ,$$

$$\tilde{N}(t,r) = N(r) + \epsilon N_1(r) e^{-i\Omega t} \ , \qquad \tilde{\delta}(t,r) = \delta(r) + \epsilon\delta_1(r) e^{-i\Omega t} \ ,$$

$$\tilde{\phi}(t,r) = \phi(r) + \epsilon\phi_1(r) e^{-i\Omega t} \ , \qquad \tilde{V}(t,r) = V(r) + \epsilon V_1(r) e^{-i\Omega t} \ .$$

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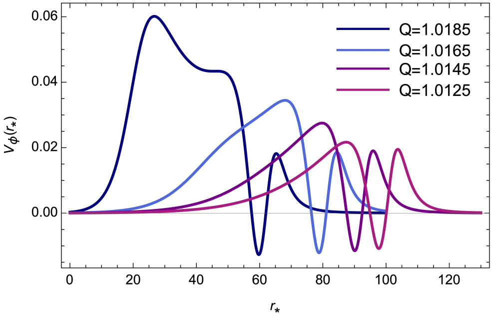
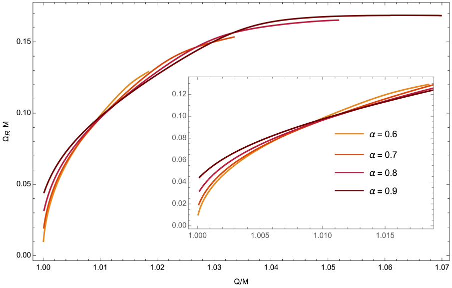
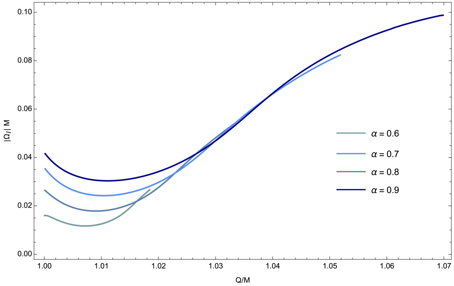


Scalar QNMs

For the QNMs computation we use ***direct integration*** with proper **boundary conditions**:

- *outgoing wave* at infinity ($r_* \to +\infty$),
- *ingoing wave* at the horizon ($r_* \to -\infty$).





Smaller the coupling constant α less damped the QNMs.

Linear and non-spherical perturbations

Derivation of the **linearized perturbed equations**

- Consider *perturbed fields* as $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad A_\mu = \bar{A}_\mu + \delta A_\mu,$ $\phi = \bar{\phi} + \delta\phi;$
- Decompose the field perturbations in terms of **scalar**, **vector** and **tensor spherical harmonics**;
- The perturbations are split into "**axial**" $(-1)^{l+1}$ and "**polar**" $(-1)^l$.

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The linearized perturbed equations divide into two sectors.

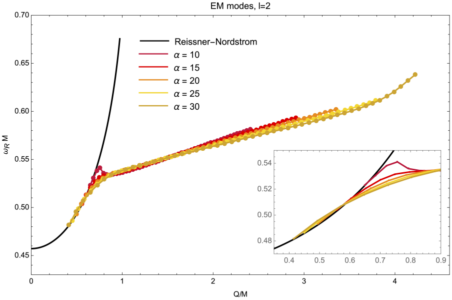
Axial sector: *system of two coupled ODEs of the second order*

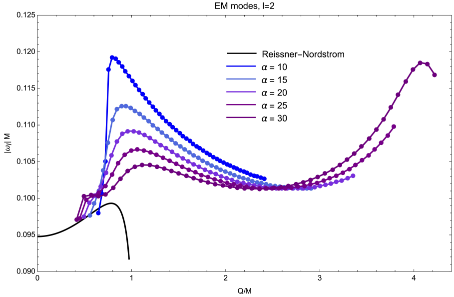
$$\left(\frac{d^2}{dr_*^2} + \omega^2\right)U(r) = V_{UU}U(r) + V_{UH}H(r)\ , \quad U(r) \text{ gravitational perturbation}$$

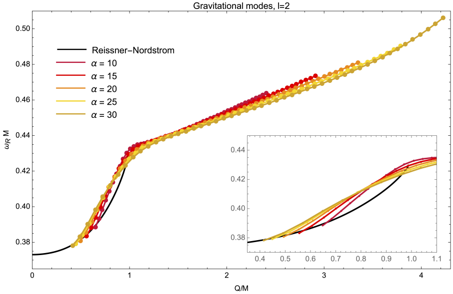
$$\left(\frac{d^2}{dr_*^2} + \omega^2\right)H(r) = V_{UH}U(r) + V_{HH}H(r)\ , \quad H(r) \text{ EM perturbation .}$$

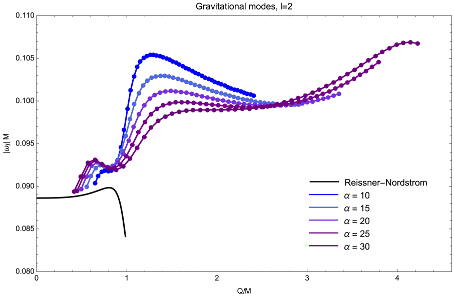
Polar sector: *system of six coupled ODEs*.

Axial sector QNMs (*EM and Gravitational perturbations*)









Summary and next steps

- Einstein-Maxwell-scalar theory admits black hole solutions with **scalar hair**: two photon spheres in a region of the parameter space!
- The *effective potential* of linear and spherical perturbations presents a **double peak** in the same region of the parameter space.
- The associated **QNMs** present a small imaginary part: less damped than RN QNMs.
- We studied *linear non-spherical perturbations* and computed the QNMs for the **axial sector**.

- Study **time evolution** of linear and spherical perturbation for the effective potential with a double peak.
- Compute the QNMs for the **polar sector**.
- Search for **long-lived QNMs** (axial and polar) in the parameter space of the BH with two photon spheres.

Thank you for the attention!

Additional material:
Spherical harmonics decomposition and Regge-Wheeler gauge

Expansion of the metric tensor, vector potential and scalar field perturbations in scalar, vector and tensor spherical harmonics using the **Regge-Wheeler gauge**.

$$h_{\mu\nu} = h^A_{\mu\nu} + h^P_{\mu\nu} \ ,$$

$$h^A_{\mu\nu} = \sum_{l,m} \int d\omega \, e^{-i\omega t} \begin{bmatrix} 0 & 0 & -\frac{h_0(r)\partial_\varphi Y_l^m}{\sin\theta} & h_0(r)\sin\theta\partial_\theta Y_l^m \\ * & 0 & -\frac{h_1(r)\partial_\varphi Y_l^m}{\sin\theta} & h_1(r)\sin\theta\partial_\theta Y_l^m \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} ,$$

$$h^P_{\mu\nu} = \sum_{l,m} \int d\omega \, e^{-i\omega t} Y_l^m \begin{bmatrix} e^{-2\delta(r)}N(r)H_0(r) & H_1(r) & 0 & 0 \\ * & \frac{H_2(r)}{N(r)} & 0 & 0 \\ * & * & r^2K(r) & 0 \\ * & * & * & r^2\sin^2\theta K(r) \end{bmatrix} ,$$

$$\delta F_{\mu\nu} = \delta F^A_{\mu\nu} + \delta F^B_{\mu\nu} \, ,$$

$$\delta F^A_{\mu\nu} = \sum_{l,m} \int d\omega \, e^{-i\omega t} \begin{bmatrix} 0 & 0 & -\frac{i\omega u_4(r)\partial_\varphi Y_l^m}{\sin\theta} & i\omega u_4(r)\sin\theta\partial_\theta Y_l^m \\ * & 0 & \frac{u_4'(r)\partial_\varphi Y_l^m}{\sin\theta} & -u_4'(r)\sin\theta\partial_\theta Y_l^m \\ * & * & 0 & l(l+1)u_4(r)\sin\theta Y_l^m \\ * & * & * & 0 \end{bmatrix} \, ,$$

$$\delta F^P_{\mu\nu} = \sum_{l,m} \int d\omega \, e^{-i\omega t} \begin{bmatrix} 0 & f_{01}(r)Y_l^m & f_{02}(r)\partial_\theta Y_l^m & f_{02}(r)\partial_\varphi Y_l^m \\ * & 0 & f_{12}(r)\partial_\theta Y_l^m & f_{12}(r)\partial_\theta Y_l^m \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \, ,$$

$$\delta\phi = \sum_{l,m} \int d\omega \, e^{-i\omega t} z(r) Y_l^m \, .$$