Black Hole and Stellar Superradiance

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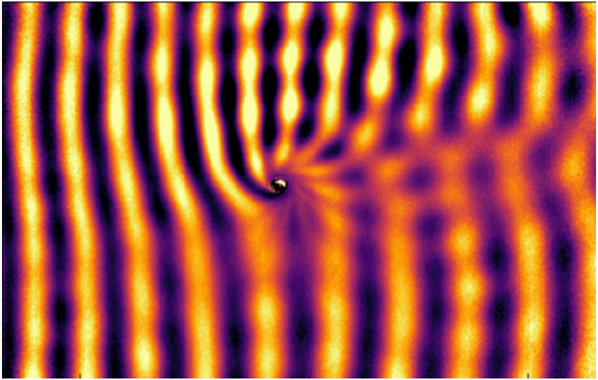
Outline

Superradiance

Superradiance is the amplification or enhancement of radiation in a dissipative system.



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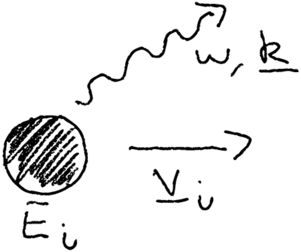
Radiation from a moving particle



$$E_f = E_i - \omega,\ \mathbf{p}_f = \mathbf{p}_i - \mathbf{k}$$

Find the particle's rest mass by moving to comoving frame:

$$m_i = \gamma_i(E_i - \mathbf{v}_i \cdot \mathbf{p_i}),\ m_f = \gamma_f(E_f - \mathbf{v}_f \cdot \mathbf{p_f})$$

$$\Delta m = -\gamma_i(\omega - \mathbf{v_i} \cdot \mathbf{k}) + \mathcal{O}(\delta \mathbf{v}).$$

Brito, Cardosa & Pani, 1501.06570

Bekenstein & Schiffer, gr-qc/9803033

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- This can occur with tachyons or from medium effects giving $\omega(k) < k$.

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- When $v_{\rm ph} > v_i$, an absorption effect can become a spontaneous radiation effect, taking energy from the particle's kinetic energy.

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- Superradiance requires that the rotating body be dissipative.

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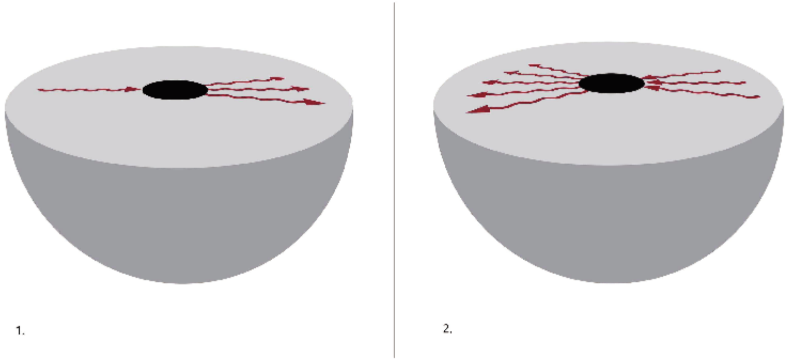
- Black holes can trap massive radiation.
- Could get exponential amplification of this trapped radiation - a superradiant instability.
- Black hole superradiance is effective for Beyond the Standard Model bosons such as axions.

Black Hole Superradiance



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- Similar to Hydrogen atom wavefunctions $\psi_{nlm}(r)$.
- The eigen-energies will have an imaginary component, corresponding to the axion being eaten by the black hole, or to superradiant amplification of the axion field.

Black Hole Superradiance

$$S = \int d^4x\sqrt{-g}(-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}m_a\phi^2)$$

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- Time domain analysis has also been performed.

Zouros & Eardley, Annals of Physics, 1979
Detweiler, Phys Rev D, 1980
Dolan, 0705.2880 & 1212.1477

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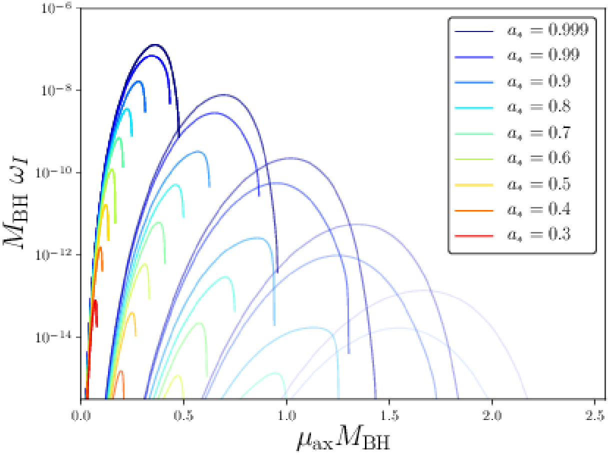

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- The instability is most efficient when the black hole's gravitational radius is similar to the axion's compton radius: $GMm_a \sim 1$.
- The instability is less efficient for higher l and m modes.

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Bosenova

Energy of a cloud of size R with N axions:

$$V(R) \sim N\frac{l(l+1)+1}{2m_a R^2} - N\frac{GMm_a}{R} + \frac{N^2}{32\pi f_a^2 R^3}$$

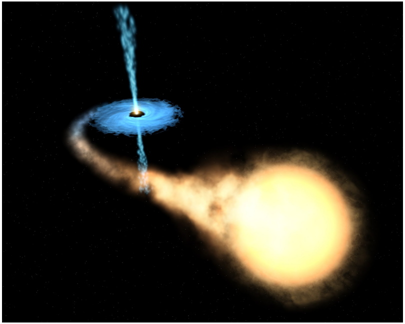
At large N, the gradient energy of the axion field makes the cloud unstable. The collapse may be observed as a gravitational wave and potentially γ-ray burst.

Arvanitaki & Dubovsky, 1004.3558

Black hole spin depletion

We can measure black hole spins:

- X-ray spectra of black hole X-ray binaries
- Gravitational wave emission from mergers



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- Advanced Ligo will be sensitive to $m_a \lesssim 10^{-10}\,\mathrm{eV}$. (Arvanitaki *et al*, 1604.03958).

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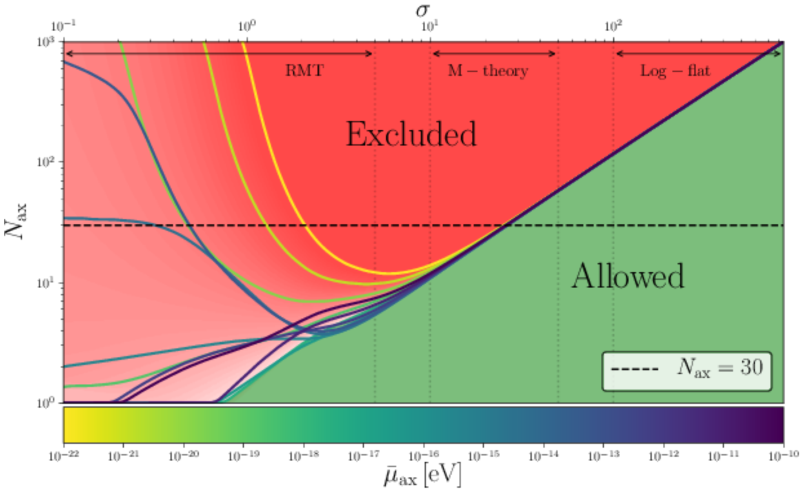

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- Orbits in binary systems (Kavic *et al*, 1910.06977)

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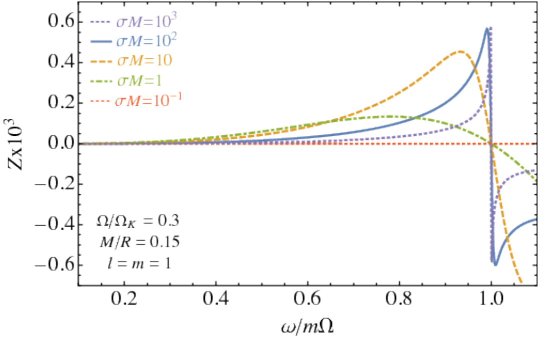
- Axion annihilations could decrease the superradiance rate.
- For large initial seeds, if both superradiant and non-superradiant modes are populated, the instability may not occur (Ficarra, Pani & Witek, 1812.02758.).

Superradiance in Stars

No horizion - superradiance in stars relies on non-gravitational dissipative dynamics, which become amplifying due to the star's rotation (Zel'dovich, 1971).

Example: Dark photons in neutron stars

Massive dark photons with dissipation from a hidden sector conductivity (V Cardoso, P Pani and T Yu, 1704.06151).



$$Z := \frac{|A_{\text{out}}|^2}{|A_{\text{in}}|^2} - 1.$$

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- This gives the required dissipative interaction.

Kaplan & Rajendran, 1908.10440

Example: Axion-photon superradiance

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- Bound states - i.e. massive particle for gravitational bound state

Can the massive particle and the dissipation be in different sectors that talk to eachother?

FCD & McDonald, 1904.08341

Axion-photon superradiance

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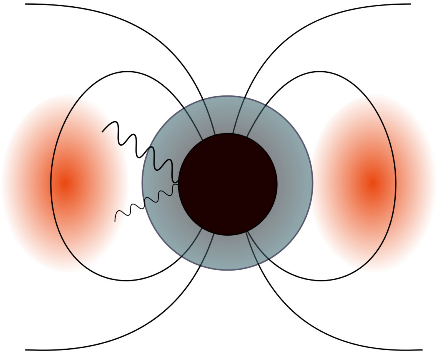
- *Photons* dissipate energy into the neutron star magnetosphere via the magnetosphere's bulk conductivity.
- Axions and photons mix, so these effects together lead to a superradiant instability in the neutron star magnetosphere.

Axion-photon superradiance in neutron stars



Schematic illustration of the instability. The axion boundstate (orange) mixes with a photon mode which is then amplified by scattering off the rotating magnetosphere (grey). The photon energy is then deposited back into the axion sector.

Axion-photon superradiance in neutron stars

- We typically find superradiant timescales $\tau = \frac{1}{\mathrm{Im}[\omega_{\ell mn}]}$ a few orders of magnitude higher than the neutron star spin down time.
- Therefore, we do not expect this process to be observable.
- Our result is an example of a more general phenomenon which can arise when there is an instability in the plasma sector.
- For axion modes which couple to an unstable mode of the neutron star, one could in principle find similar instabilities.

Stellar superradiance: A general approach

- Many different Beyond the Standard Model interactions could lead to stellar superradiance.
- Stellar environments are complex, with many more degrees of freedom than black holes.
- Spin down from stellar superradiance can be observed directly.
- Can we find a general method for computing stellar superradiance rates from a BSM Lagrangian?

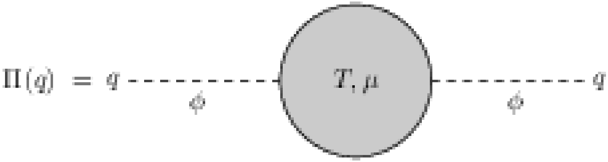
See FCD, Garbrecht & McDonald, 2207.07662.

Stellar superradiance: Damping

Stellar superradiance depends on the **damping rate** of the field into the star. We can find this with thermal field theory.

$$\partial^2\phi + \mu^2\phi + \Gamma_\phi\dot{\phi} = 0,$$

$$\Gamma_\phi = -\lim_{p\to 0} \mathrm{Im}\,\Pi(p)/p^0.$$



Stellar superradiance: Worldline Effective Field Theory

- Describe interaction of a field ϕ and the star by expanding in $\frac{R}{\lambda}$.
- The extended nature of the star is described by an infinite series of interactions between ϕ and a point-like object.

$$
\begin{aligned}
H_{\text{int}}(t, \mathbf{x}) = & \partial^I \phi(x) \mathcal{O}_I^{(1)}(x) \delta^{(3)}(\mathbf{x} - \mathbf{y}(t)) + \\
& \partial^I \partial^J \phi(x) \mathcal{O}_{IJ}^{(2)}(x) \delta^{(3)}(\mathbf{x} - \mathbf{y}(t)) + \ldots
\end{aligned}
$$

Stellar superradiance: Worldline Effective Field Theory

If the star is rotating:

$$H_{\text{int}}(t) = \partial^I \phi(t) R_I^J(t) \mathcal{O}_J^{(1)}(t) + \partial^I \partial^J \phi(t) R_I^K(t) R_J^L(t) \mathcal{O}_{KL}^{(2)}(t) + \dots$$

See S. Endlich and R. Penco, 1609.06723

Superradiant scattering in the worldline EFT

Superradiant scattering from a rotating star:

$$P_{\rm abs} = \sum_{X_f} \frac{|\langle X_f; 0|S|X_i; \omega, \ell, m\rangle|^2}{\langle \omega, \ell, m \mid \omega, \ell, m\rangle}.$$

Amplification factor:

$$Z_{\ell m} = \frac{\Phi_{\rm out} - \Phi_{\rm in}}{\Phi_{\rm in}} = \frac{\ell! q^{2\ell+2}}{4\pi(2\ell+1)!! v\omega}\rho_\ell(m\Omega - \omega),$$

$\rho_\ell(m\Omega - \omega)$ is related to the worldline EFT operators and must be found with a matching calculation.

Superradiant instabilities in the worldline EFT

Superradiant instabilities arise from *bound states*:

$$P_{\rm abs} = \sum_{X_f} \left| \langle X_f; 0 | S | X_i; n\ell m \rangle \right|^2 .$$

The superradiance rate is:

$$\Gamma_{n\ell m} = \Gamma_{\rm em} - \Gamma_{\rm abs} = \frac{A_{n\ell m}}{2\omega_{\ell n}} \left(\frac{1}{r_{n\ell}} \right)^{2\ell+3} \rho_\ell \left(m\Omega - \omega_{\ell n} \right),$$

where $r_{n\ell} = (n + \ell + 1)/(2GM\mu^2)$.

Matching the worldline EFT

- We can also find the amplification factor $Z_{\ell m}$ directly from the equation of motion $\partial^2\phi + \mu^2\phi + \Gamma_\phi\dot{\phi} = 0$ (see 1505.05509).
- Matching these results we can obtain $\rho_\ell\,(m\Omega - \omega_{\ell n})$ (or equivalently the EFT coefficients).

This gives a superradiance rate:

$$\Gamma_{n\ell m} = C_{n\ell m}\left(\frac{R}{r_{n\ell}}\right)^{(2\ell+3)}\frac{(m\Omega - \omega)}{\omega}\Gamma_\phi,$$

where the damping Γ_ϕ of the field ϕ into the star can be calculated in thermal field theory.

Stellar superradiance: Axion Example

Consider an axion damping into a neutron star via the interactions

$$\mathcal{L}_{aNN} = \frac{g_{an}}{2m}\partial_\mu a \bar{N}\gamma^\mu\gamma_5 N,$$

$$\mathcal{L}_{\pi NN} = \frac{2m}{m_\pi} f \pi_0 \bar{N}\gamma^5 N,$$

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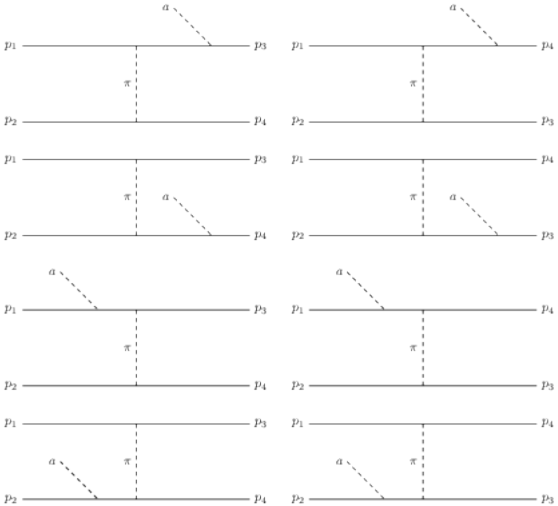
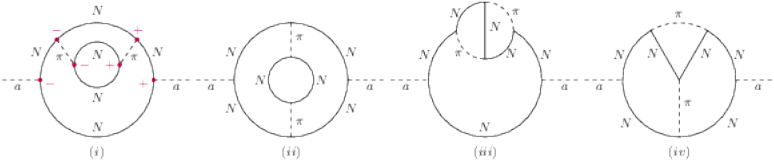


Stellar superradiance: Axion Example

To find the superradiance rate, we must compute Γ_ϕ from the self-energy:



The self-energy is related to the axion mean free path

$$\lambda^{-1} = \frac{\text{Im}\Pi}{\omega}$$

calculated in e.g. S. Harris, 2005.09618 - we find a superradiance time somewhat larger than the age of the universe!

Stellar superradiance: A general approach

- We find the damping rate from the in-medium self energy.
- We find the superradiant instability rate from the damping rate using **worldline effective field theory** (S Endlich & R Penco, 1609.06723).
- Match calculations at low energy to obtain the superradiant instability rate:

$$\Gamma_{n\ell m} = C_{n\ell m} \left(\frac{R}{r_{n\ell}} \right)^{(2\ell+3)} \frac{(m\Omega - \omega)}{\omega} \Gamma_{\phi}$$

This allows us to calculate the stellar superradiance rate for any interaction between a bosonic field and a star.
See FCD, Garbrecht & McDonald, 2207.07662.

Conclusions

- Black hole superradiance offers a purely gravitational probe of Beyond the Standard Model bosons.
- Stellar superradiance can probe additional interactions between new bosons and the Standard Model.