Tutorial Questions

High Power Lasers (Leo Gizzi)

1. List three properties of gain media for high power lasers that are relevant for ultrashort pulse amplification and repetitive operation at high average, and discuss their impact in amplifier specifications.

2. Identify properties of a compressed beam that are relevant for laser-plasma interaction and discuss their impact on plasma formation in laser-plasma acceleration.

Plasma Physics (Pablo San Miguel Claveria)

1. *Basic scaling and order of magnitudes*
   Part (a): What is the H_2 gas pressure needed to have a plasma density $n_p = 10^{16}\ cm^{-3}$? How does this compare to atmospheric pressure? Assume room temperature $T=20^o\ C$

   Part (b): What is the plasma frequency ω_p and wavelength λ_p for $n_p = 10^{16}\ cm^{-3}$? How does this compare to the radio frequencies used in RF cavities where $f \approx 3000\ MHz$?

2. *Current Filamentation Instability*
   After the discovery of the Two-Stream Instability, a second type of beam-plasma instability was postulated in 1959 by Fried [1]. He discovered that when two charged streams counterflow through each other, electro-magnetic unstable waves can also grow transversely to the flow direction. Since the current profile of each specie is also modulated transversely, breaking the beams into filaments, this instability is usually referred as Current Filamentation Instability (CFI) or just "filamentation" instability.

   Using the same fluid formalism as in the lecture for the Two-Stream instability, derive the growth rate of a purely magnetic transverse mode $A_x \sim \exp(ik \cdot x - i\omega t)$

   Part (a): Write the linearised equation of momentum conservation for the particle beam

   Part (b): Write the linearised continuity equation for beam and plasma electrons and use the formula obtained in part (a) to express the perturbed densities $n_b^{(1)}$ and $n_p^{(1)}$ as a function of $A_x^{(1)}$

   Part (c): Linearise the electromagnetique wave equation $(\nabla^2 - \partial_t^2)\mathbf{A} = \mathbf{j}$

   Part (d): Derive the dispersion relation $\omega(k)$ for a purely transverse mode $A_x^{(1)} \sim \exp(i\mathbf{k} \cdot \mathbf{x} - i\omega \mathbf{t})$

   Part (e): Derive the growthrate of the CFI instability

   Part (f): Taking EuPRAXIA initial electron beam and plasma parameters ($\alpha = n_b/n_p \approx 1, \gamma_{0b} \approx 1000, n_p \approx 10^{16}\,\mathrm{cm}^{-3}$), how quickly the electron beam would break-up into filaments due to the instability?

[1] Burton D. Fried. Mechanism for instability of transverse plasma waves. The Physics of Fluids, 2(3):337–337, 1959, doi:10.1063/1.1705933

Linear Accelerators (Marco Bellaveglia)

Exercise 1- Transit time factor

1. Derive the general expression of the transit time factor T of an accelerating gap of length L, with a constant accelerating field in which the field is oscillating at f_{RF} and that accelerates particles with relativistic factor β.

2. Remembering that the light wavelength in free space is given by $\lambda_{RF}=c/f_{RF}$, for which value of the accelerating gap length L is T equal to zero?

3. Calculate the numerical value of T for $L=12\ cm$, $f_{RF}=800\ MHz$ and ultra-relativistic electrons, i.e. $\beta=1$.

4. Calculate the accelerating voltage as a function of the gap length L, assuming an injection phase on crest ($\phi_{inj}=0$). Find an expression for the gap length that provides the maximum energy gain.

Exercise 2 – DTL (Alvarez structures)

A proton beam is injected into a drift tube linac (Alvarez structure) working at $f_{RF}=400\ MHz$, with a kinetic energy $W_{in}=5\ MeV$.

1. The distance between the first two centers of the accelerating gaps (L_{gaps}), assuming a constant velocity of the proton beam between the first two gaps and a negligible increase of the velocity due to the accelerating field.

2. If the structure is composed of 50 accelerating gaps (N_{gaps}) and the average accelerating voltage per gap is $V_{acc}=0.5\ MV$, calculate the final proton beam kinetic energy.

The proton rest energy is 938 MeV, the speed of light is c = 2.998e8 m/s.

Beam Diagnostics (Joe Wolfenden)

Emittance measurement

1. The beam matrix for a particle bunch with Twiss parameters α, β, and γ is defined as: $\Sigma = \epsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix}$,

   where, ϵ is the geometric emittance and $\langle x^2 \rangle, \langle x'^2 \rangle$ and $\langle xx' \rangle$ are the second order moments of the particle distribution in position, angle, and correlation term, respectively.

   Given a beamline consisting of a quadrupole, a drift space, and a scintillation screen show how a quadrupole scan is used to calculate beam emittance.

   Hint: $M_{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ and $M_{Quad} = \begin{pmatrix} 1 & 0 \\ -K & 1 \end{pmatrix}$, where L is the drift length, l is the length of the quadrupole, k is the quadrupole strength, and $K = kl$.

2. Describe the issues which can arise when trying to produce a quadrupole scan when using a beam accelerated by a plasma accelerator.
3. The pepper-pot method is often considered a single-shot alternative to a quadrupole scan. What are the main limitations of this technique for multi-GeV beams produced in plasma accelerators, and what are the viable alternatives to this?

Beam diagnostics methods

1. Why are streak cameras not a viable option to measure the longitudinal profile of a plasma accelerated bunch? What is the cause of this limitation?

2. What is the bunch form factor and how is it related to the bunch distribution?

3. Describe how interferometers can be used to sample the bunch form factor? What are some limitations to these methods?

Technology of Plasma Sources (Angelo Biagioni)

1. What are the fundamental problems that need to be addressed to bring current plasma sources to work at high repetition?

2. How would you implement a code (Mathlab or Python) to obtain the longitudinal diagnostics from the following spectral images (Stark method)? Provide at least the algorithm

3. Calculate the optical path to transport the plasma light, which is used for the plasma diagnostics, in order to include all dimension of the capillary on the CCD of the intensified camera

   Input parameters:
   - CCD dimension = 6mm
   - Capillary longitudinal dimension = 50 mm
   - $M = q/p$, Optical Magnification = 6/50 = 1/8
   - Lenses with following focal lengths:
     - f_1=150 mm
     - f_2=75 mm

Introduction to Particle-driven Acceleration (Livio Verra)

1. **Drawing Exercise**
   a. Schematically show in 1D the accelerating gradient driven by a square bunch with length equal to the plasma wavelength λ_{pe}. Assume transformer ratio $R = 2$.
   b. Show that a train of identical bunches spaced by λpe drives resonantly enhanced wakefields.
   Hint: draw the wakefields driven by each bunch and then superpose them.
   c. Show that a train of three bunches with increasing charge and spaced by $1.5\ \lambda_{pe}$ drives wakefields with $R > 2$.
   What is the scaling of bunch charge ratio that maximizes R?

2. **Design your PWFA**
   You have a beamline providing Gaussian electron bunches with transverse size $\sigma_r = 50$ μm.
   a. What is the maximum plasma electron density npe that can be used without current filamentation instability occurring?
   b. For the obtained value of n_{pe}, what is the optimal bunch length σ_z for effective wakefields excitation?
   c. What is the maximum charge Q in the bunch to “safely” drive wakefields in the linear regime?
   d. What is the minimum Q to drive fully non-linear wakefields?

3. **Ion Column**
   Consider a column of ion with density n_{pi}.
   a. Show that the radial electric field E_r increases linearly with the radial distance r from the axis.
   b. Show that the matching condition for an electron bunch entering the ion column at a waist is $\beta = \sqrt{\frac{2\gamma mc^2\varepsilon_0}{n_{pi}q^2}}$.
      Hint: start from Newton's law for a single electron, then leap to envelope equation.
   c. What is the value of the matched transverse size for $\gamma = 100$, normalized emittance $\varepsilon_N = 1$ mm-mrad, $n_{pi}=10^{15}$ cm^{-3}?

4. **Shape your Driver (OPTIONAL)**
   An infinitely short thin disk of charge eσ drives longitudinal wakefields
   $E_z(\xi) = \frac{e\sigma}{\varepsilon_0}\cos(k_{pe}\xi)$
   a. Consider a bunch with finite length and longitudinal charge density distribution $\rho(\xi)$. Write the equation for $E_z(\xi)$, neglecting the $R(r)$ term.
      Hint: Green's function.
   b. Show that a bunch with linearly increasing density $\rho(\xi) = \rho_0 k_{pe}\xi$, for $0 \leq k_{pe}\xi \leq 2\pi N$, drives wakefields with $R > 2$.

Introduction to Laser-driven Heavy Ion Acceleration (Josefine Metzkes-Ng)

1. What is the dominant source of energetic protons and ions in laser-driven ion acceleration and how can heavier ions be accelerated efficiently?

2. How can the maximum accelerating field in the sheath be measured or estimated?