Introduction to Beam-Driven Plasma Wakefield Acceleration

Livio Verra

EuPRAXIA – DN School on Plasma Accelerators
23.04.2024

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Laboratori Nazionali di Frascati

Summary

0a. Few Reminders on Plasma Physics
0b. Few Reminders on Beam Physics

I. Plasma Wakefields: Linear Regime

II. Plasma Wakefields: Non-Linear Regime

III. Beam-plasma instabilities

Purpose:
→ Provide basic tools and understanding of PWFA
→ Stimulate curiosity

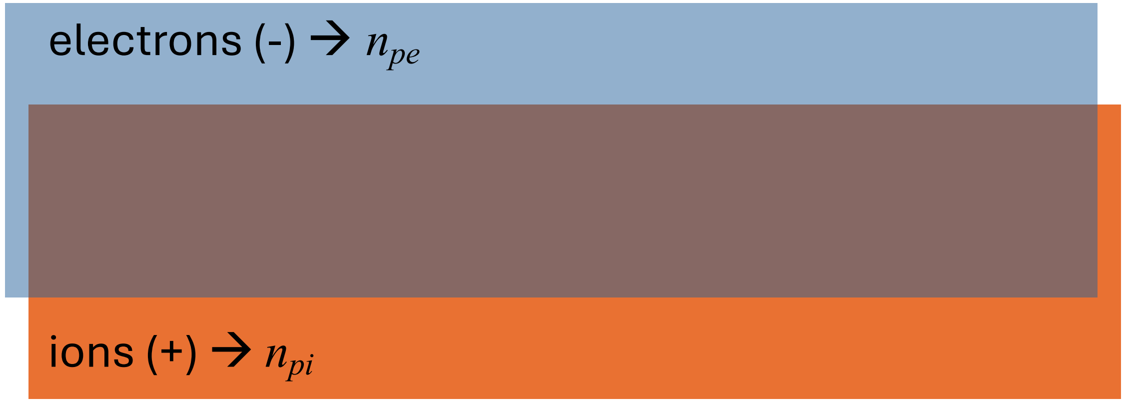
Disclaimers:

- We will barely scratch the surface of the topic
- None of the following is my invention –still learning..

0a. Plasma Physics

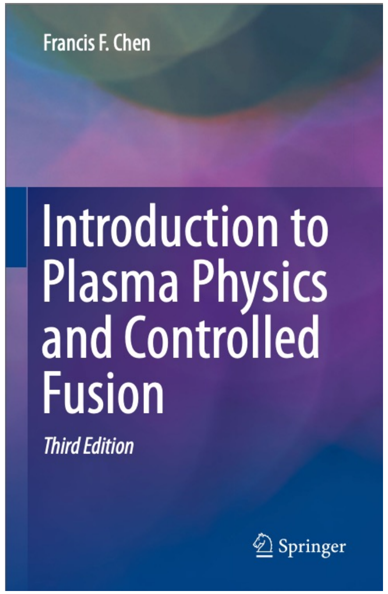
Plasma:

- Ionized gas
- Collisions can be (most of time) neglected
  → Electromagnetic interaction dominates
- Large number of particles ➔ **collective** behavior
- Quasi-neutral ($n_{pe} \sim n_{pi}$)



electrons (-) → n_{pe}

ions (+) → n_{pi}



https://edisciplinas.usp.br/pluginfile.php/5913606/course/section/6090129/Chen.pdf

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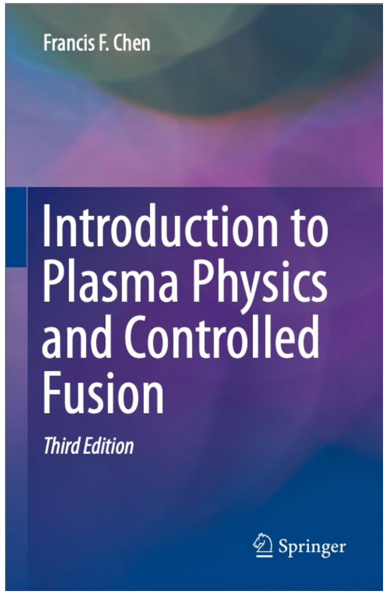
neutral plasma



Francis F. Chen

Introduction to Plasma Physics and Controlled Fusion

Third Edition

Springer

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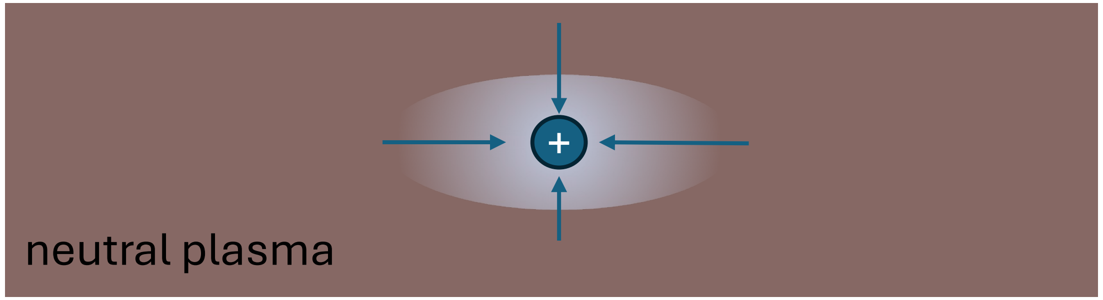
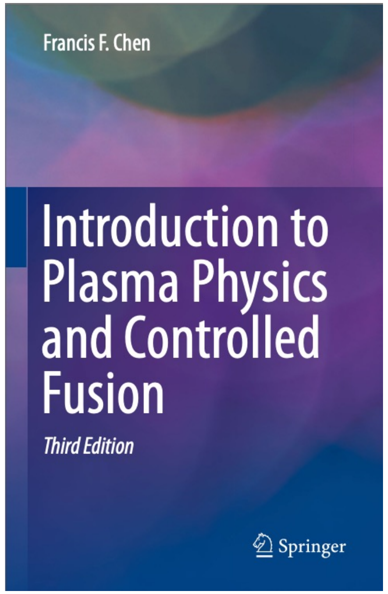
  → It tends to keep the charge and current neutrality:
  Plasma electrons ($m_{ip} >> m_{pe}$) move to compensate for the disturbance
  → Plasma screens electromagnetic fields

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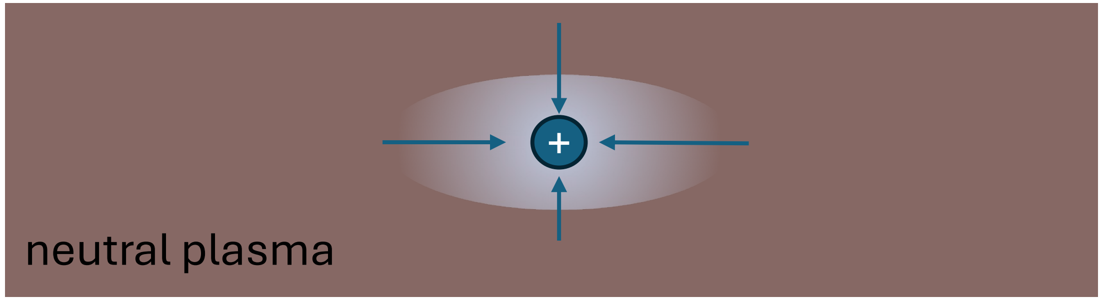
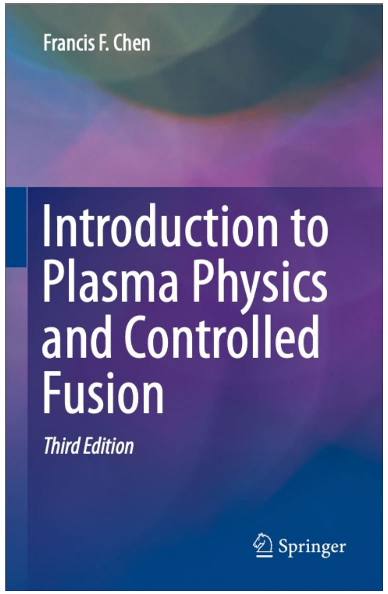


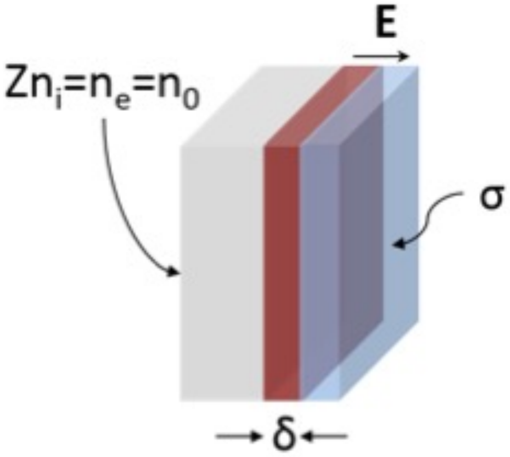
When the equilibrium is perturbed:

- Electrons oscillate with angular frequency $\omega_{pe} = \sqrt{\frac{n_{pe}e^2}{m_e\varepsilon_0}}$
- Ions with $\omega_{pi} = \sqrt{\frac{n_{pi}e^2}{m_i\varepsilon_0}} \ll \omega_{pe}$ (ions considered immobile for short time-scales)



0a. Plasma Physics

- Fields in plasmas are sustained by the charge separation
  - As high as the cold wave-breaking field: $E_{WB} = \frac{m_e\, c\, \omega_{pe}}{q}$ → oscillation length cannot exceed plasma wavelength
  - E.g. for n_{pe} = (10^{14} – 10^{18}) cm^{-3}, $E_{WB} \sim 100\,\frac{V}{m}\,\sqrt{n_{pe}[\,cm^{-3}]}$ = (1 – 100 GV /m)



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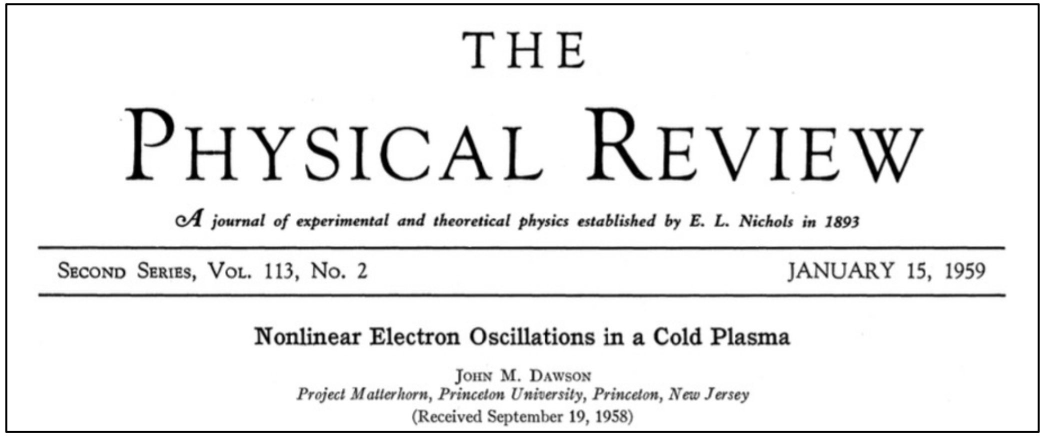


THE

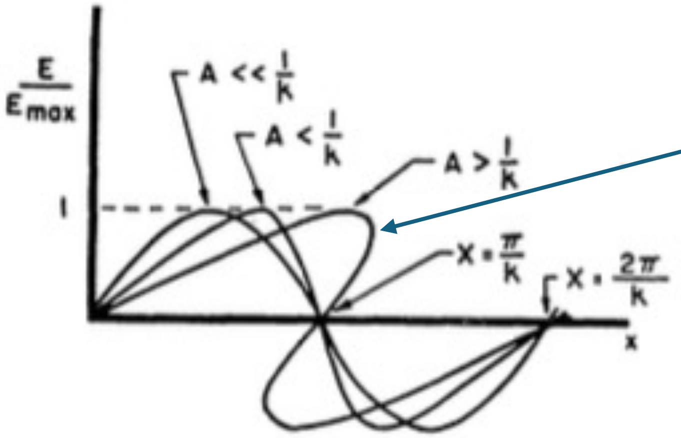
PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 113, NO. 2 JANUARY 15, 1959

Nonlinear Electron Oscillations in a Cold Plasma

JOHN M. DAWSON
Project Matterhorn, Princeton University, Princeton, New Jersey
(Received September 19, 1958)



Wave «breaks» when the maximum amplitude is reached

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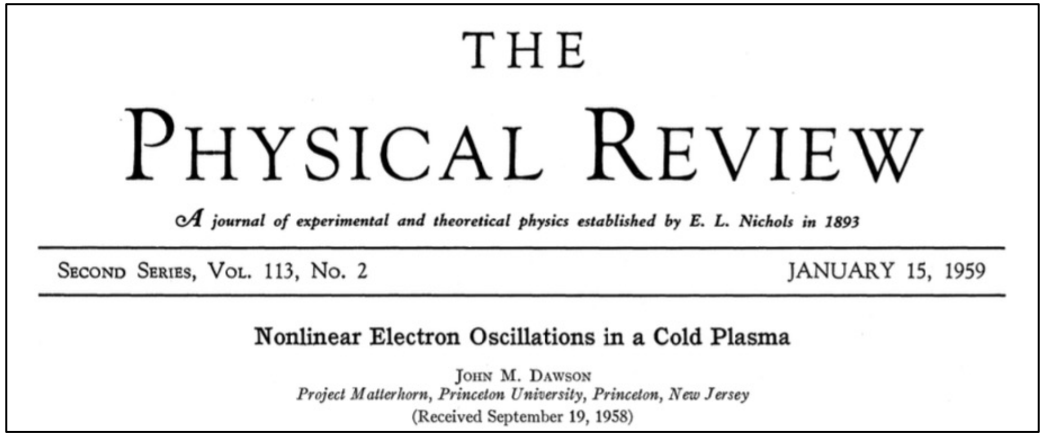

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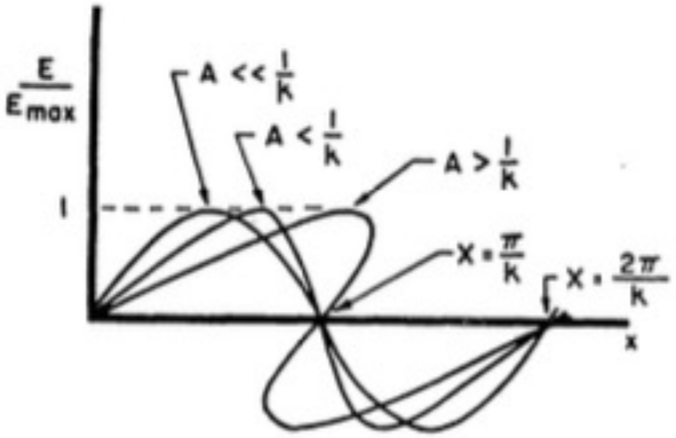
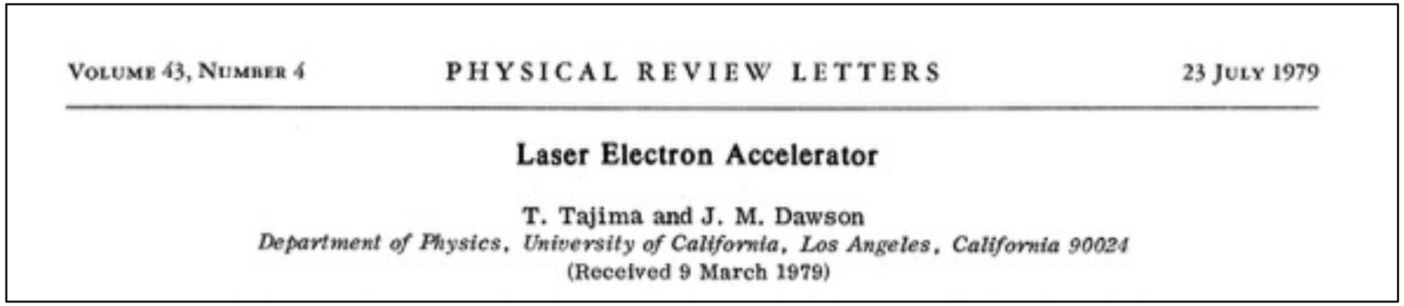


- RF cavities limited to 100MV/m by breakdown, caused e.g. by fatigue, pulse heating, etc..

  ➔ one could dream of shrinking down the size of accelerators by orders of magnitude!



VOLUME 43, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JULY 1979

Laser Electron Accelerator

T. Tajima and J. M. Dawson

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Seminal paper on plasma wakefield acceleration:
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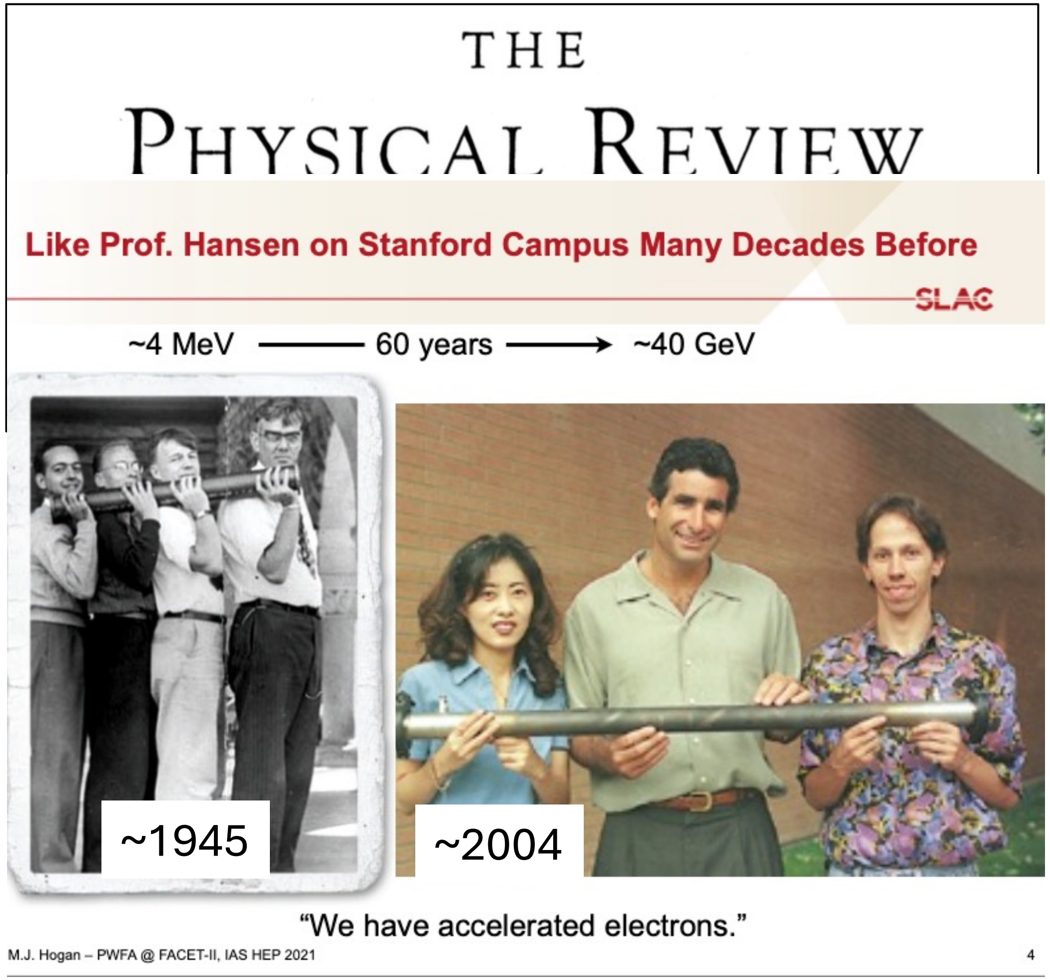

THE PHYSICAL REVIEW

Like Prof. Hansen on Stanford Campus Many Decades Before

SLAC

~4 MeV ——— 60 years ———> ~40 GeV

~1945

~2004

“We have accelerated electrons.”

M.J. Hogan – PWFA @ FACET-II, IAS HEP 2021

4

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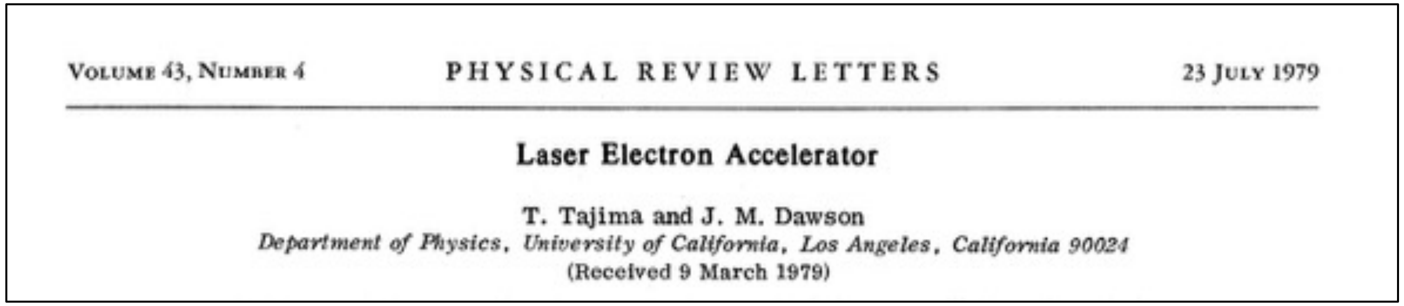

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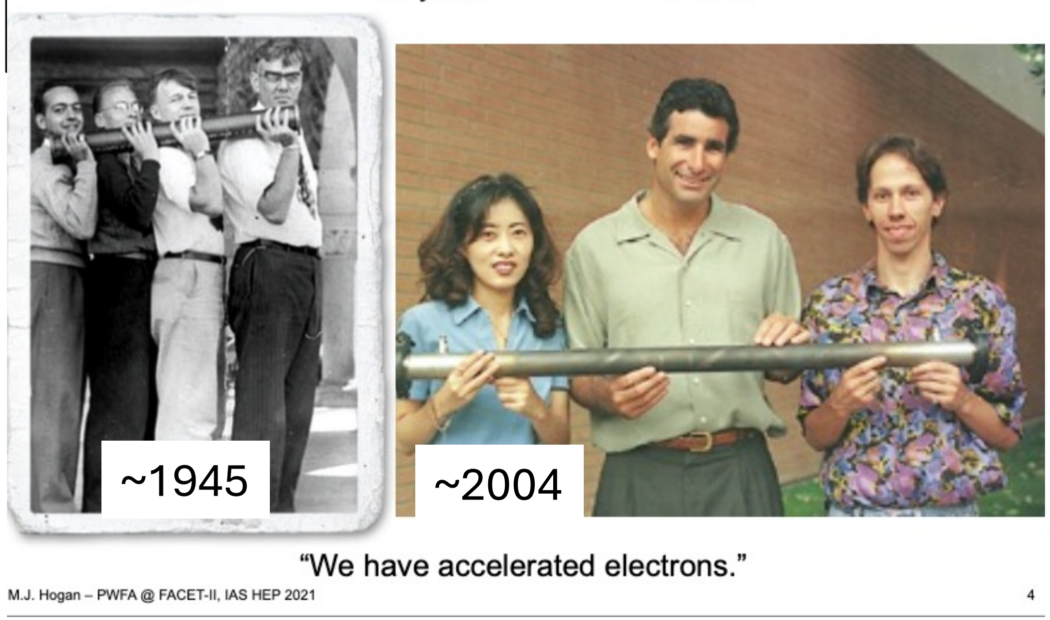

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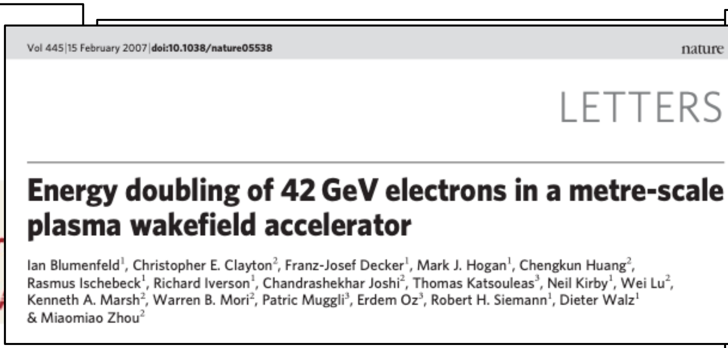


Vol 445 | 15 February 2007 | doi:10.1038/nature05538 nature

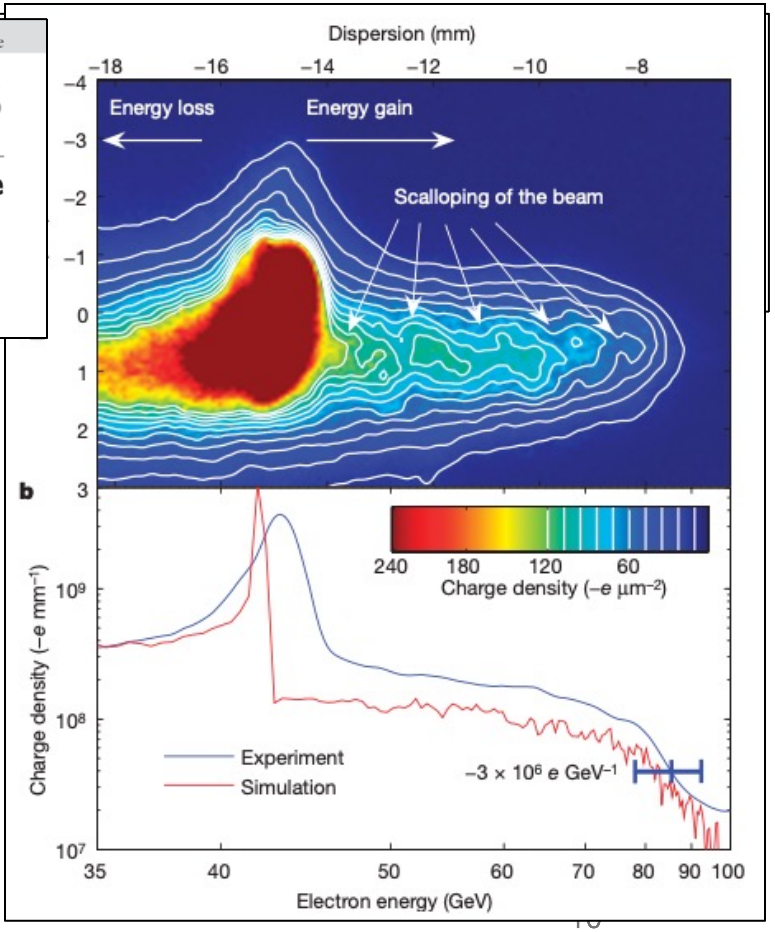
LETTERS

Energy doubling of 42 GeV electrons in a metre-scale plasma wakefield accelerator

Ian Blumenfeld[1], Christopher E. Clayton[2], Franz-Josef Decker[1], Mark J. Hogan[1], Chengkun Huang[2], Rasmus Ischebeck[1], Richard Iverson[1], Chandrashekhar Joshi[2], Thomas Katsouleas[3], Neil Kirby[1], Wei Lu[2], Kenneth A. Marsh[2], Warren B. Mori[2], Patric Muggli[3], Erdem Oz[3], Robert H. Siemann[1], Dieter Walz[1] & Miaomiao Zhou[2]

Acceleration of electrons from 42 to 84 GeV in 80 cm

Blumenfeld et al.,
Nature 445, 741 (2007)



0b. Beam Physics

- Relativistic particle bunches:
    - Propagate at v_b~c
    - Not affected by index of refraction
    - Large inertia (γm >> m)

https://arxiv.org/pdf/2007.04102.pdf

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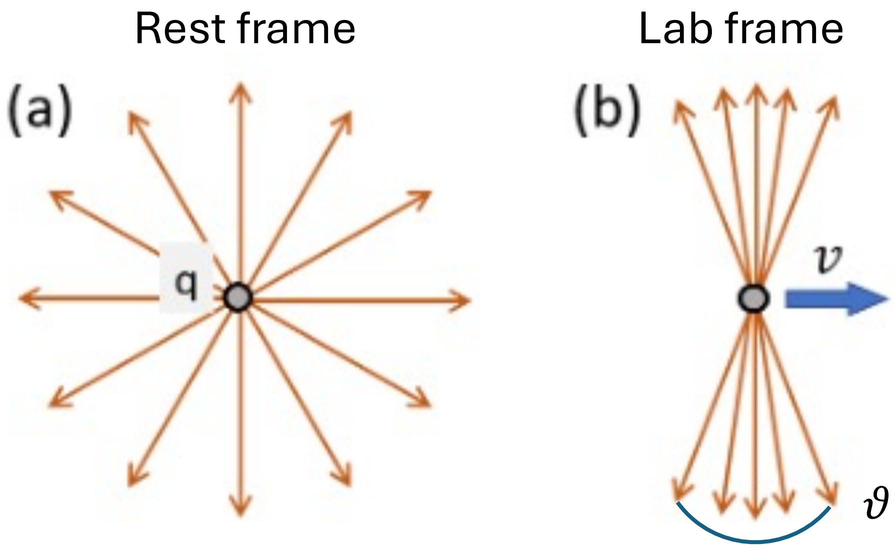

$$\vartheta \sim \frac{1}{\gamma}$$

$$\overrightarrow{E_0} = \frac{1}{4\pi\varepsilon_0} - \frac{e}{r^2}\frac{\vec{r}}{|r|}$$

$$E'_{\parallel} = E_0$$
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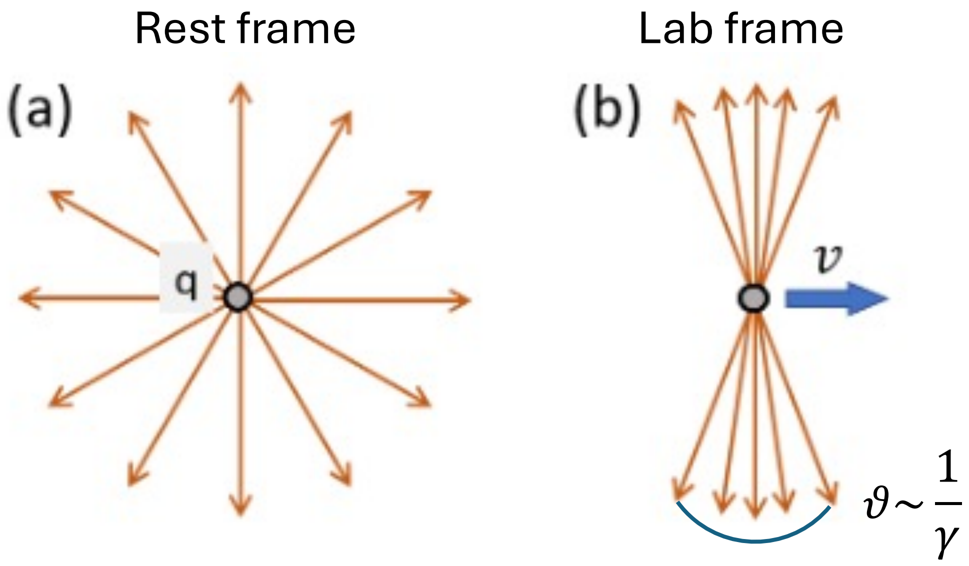

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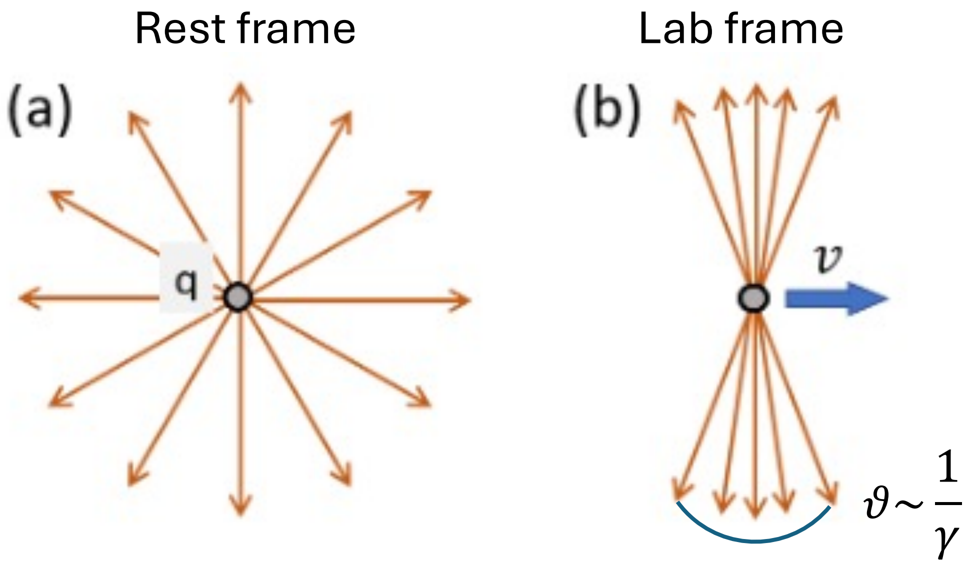

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→ Azimuthal magnetic field associated with beam current: $B_\theta = \frac{\beta}{c} E_0$

→ Net force acting on each electron: $F_r = e\,(E_r - \beta c B_\theta) = \frac{eE_r}{\gamma^2}$ ➔ Negligible at high energies!

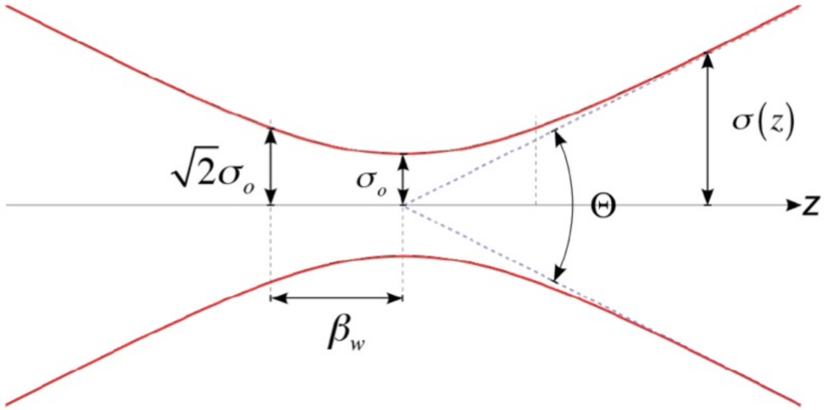
https://arxiv.org/pdf/2007.04102.pdf

0b. Beam Physics

- Propagation of the beam distribution is dominated by the emittance (at high energies)

$$\sigma'' = \frac{\epsilon_{rms}^2}{\sigma^3}$$

$$\sigma(z) = \sqrt{\left(\sigma_0 + \sigma_0'(z - z_0)\right)^2 + \frac{\epsilon_{rms}^2}{\sigma_0^3}(z - z_0)^2}.$$



(M. Ferrario)

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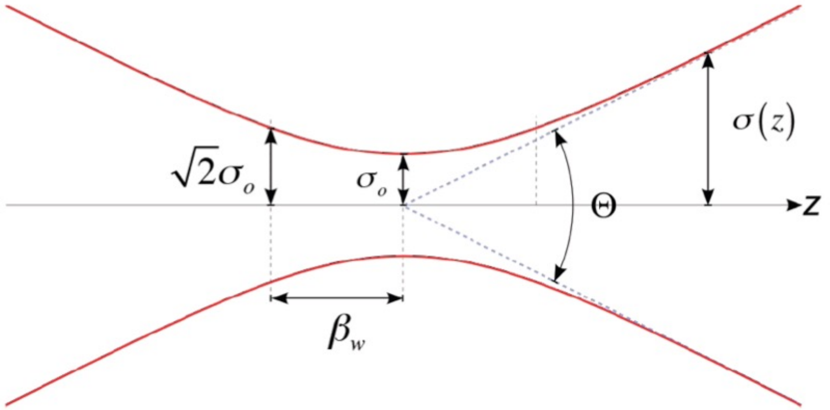

(M. Ferrario)

- If an external focusing force is applied:

$$\sigma'' + k_{ext}^2\sigma = \frac{\epsilon_{rms}^2}{\sigma^3}$$

- Emittance → parameter describing how small a beam can be focused
  Normally expressed in terms of **normalized** emittance: $\epsilon_N = \beta\gamma\epsilon_{rms}$ → preserved upon acceleration

- For delivery to applications we need:
  - Injectors that generate beams with small emittance
  - Accelerators that preserve emittance

$\left.\right\}$ beam quality

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I. Wakefields Excitation (linear regime)

- Let’s take a plasma with density n_{pe}

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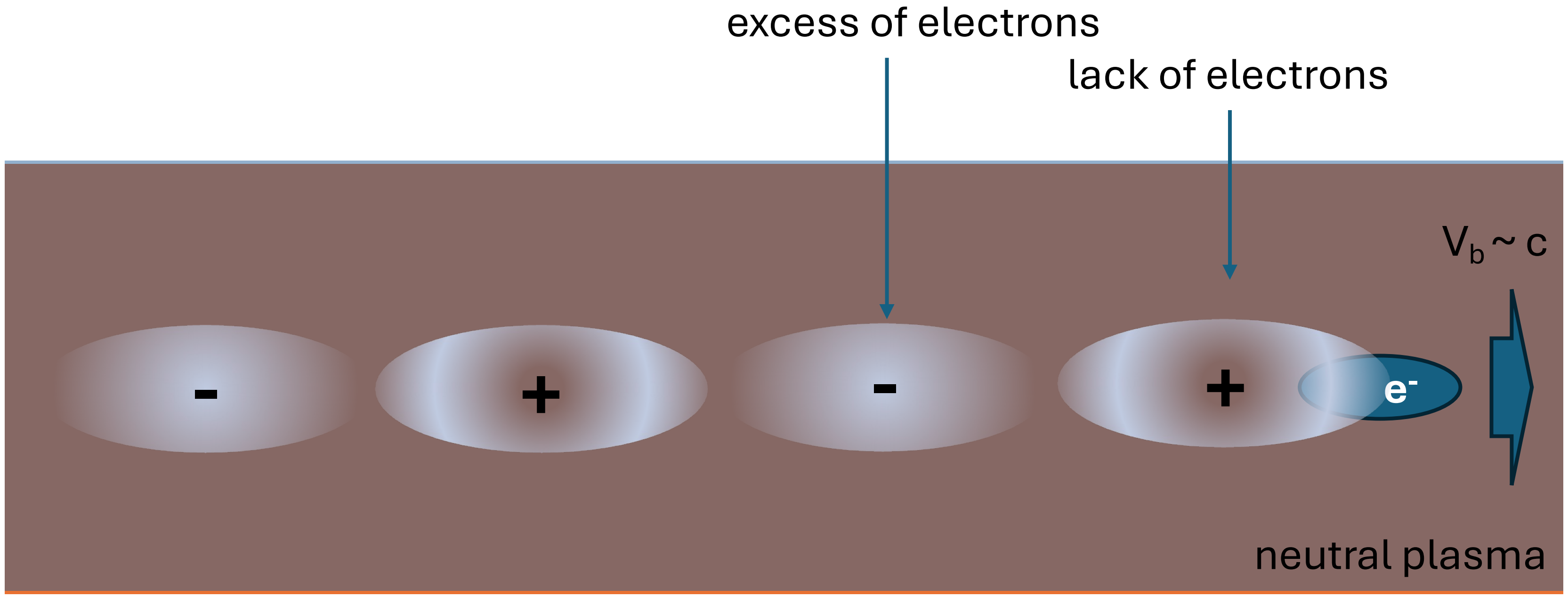

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Blowout (non-linear) regime: electrons DO cross the axis

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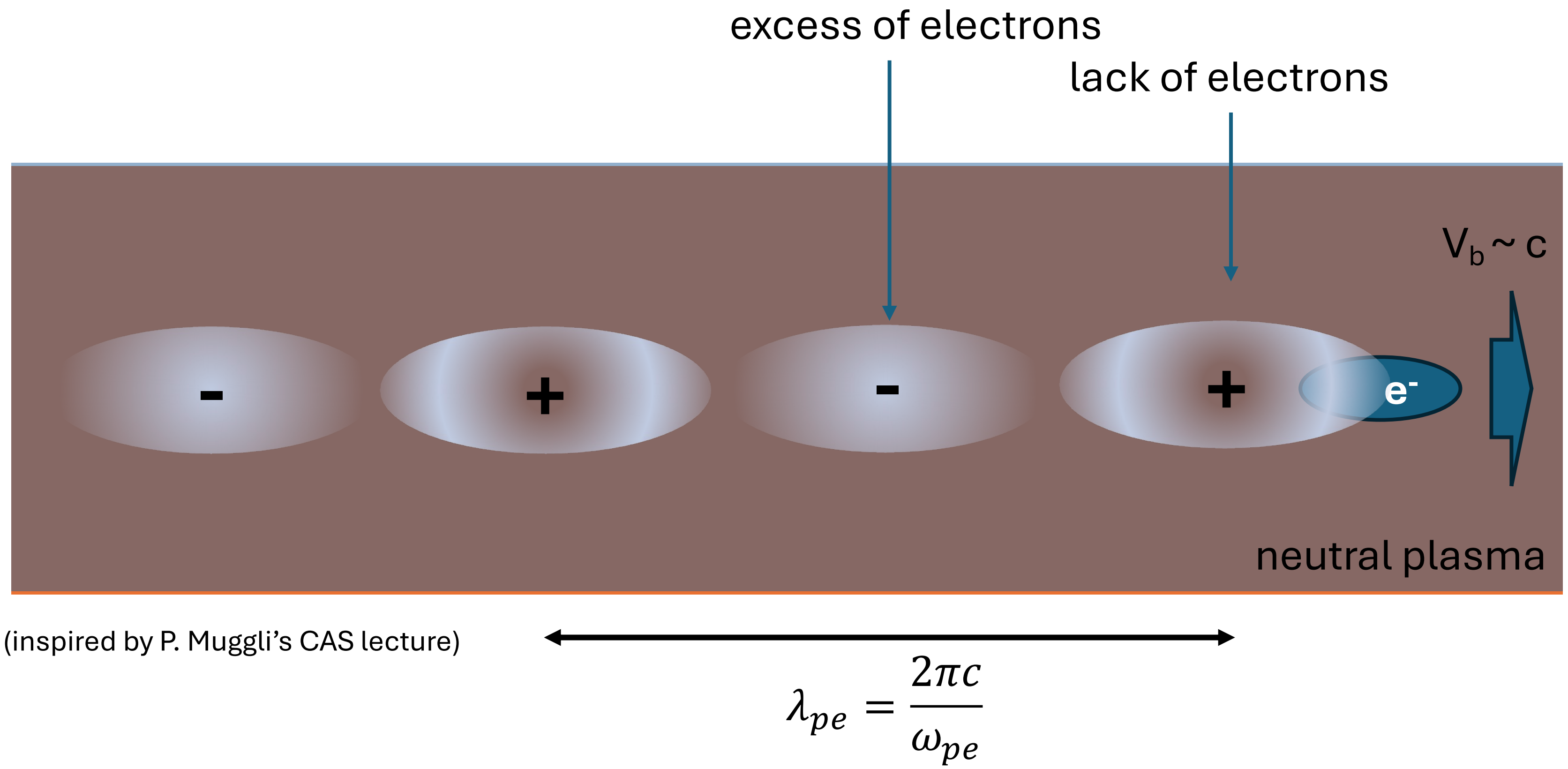

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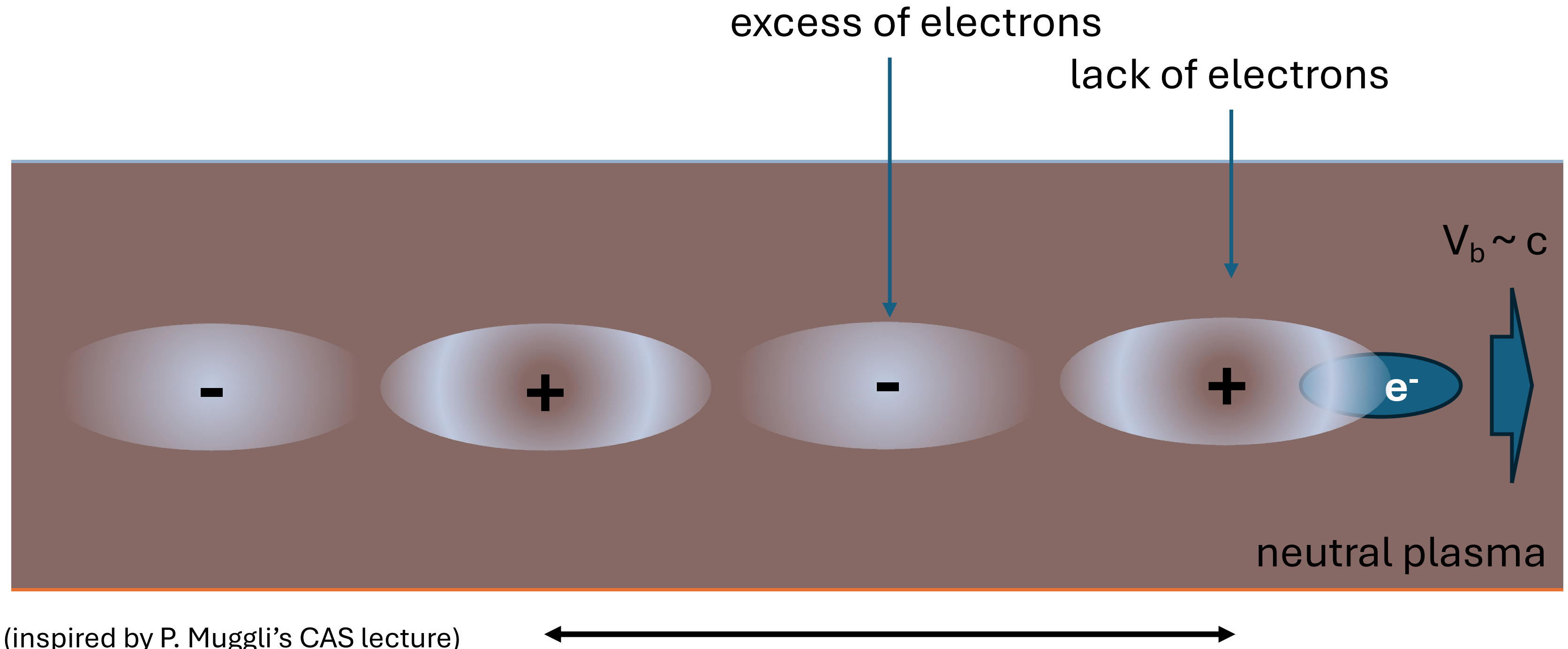

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$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

→ Wakefields ←

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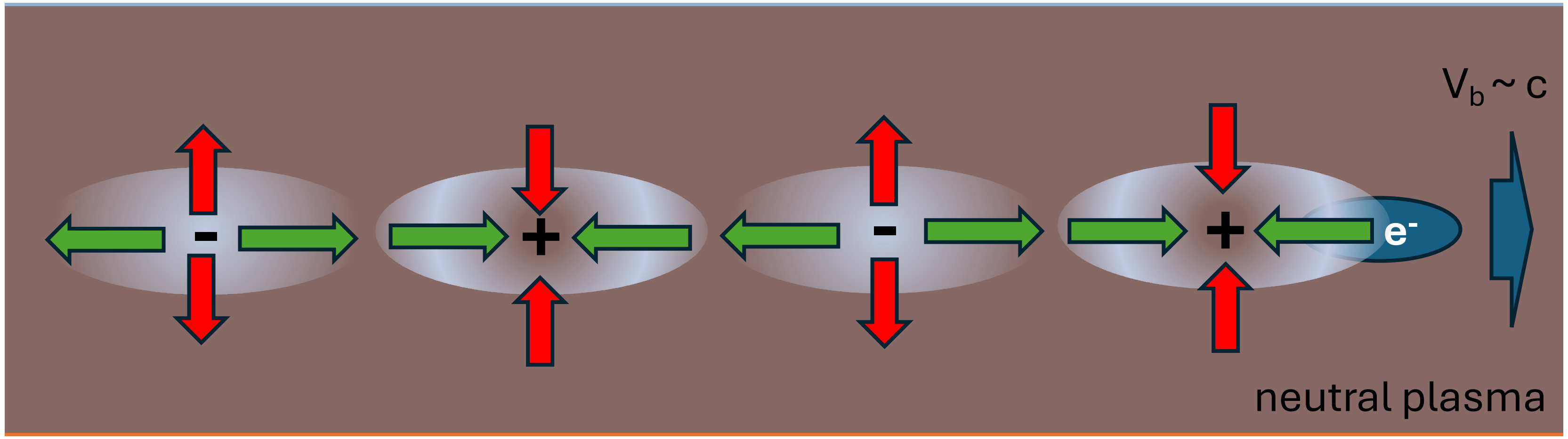

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$$\lambda_{pe} = \frac{2\pi c}{\omega_{pe}}$$

⇄ Longitudinal (accelerating – decelerating) wakefields

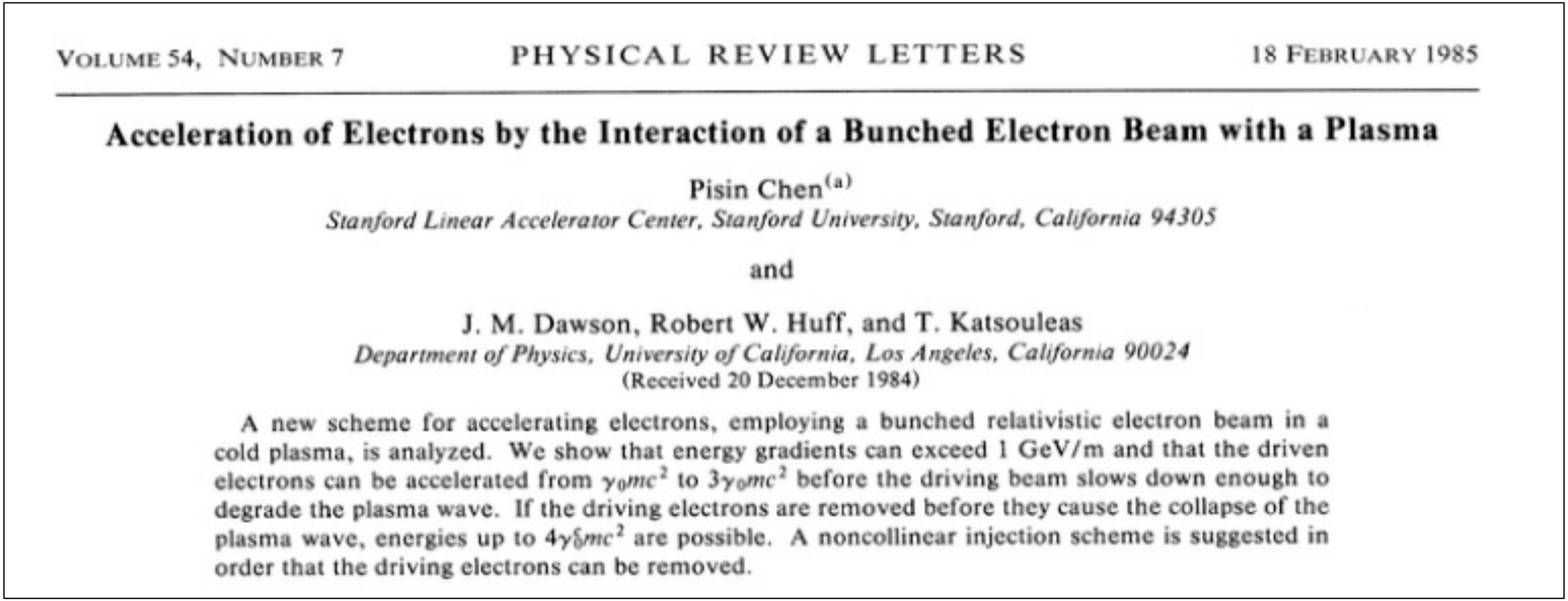
⇅ Transverse (focusing – defocusing) wakefields

$$\boldsymbol{\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$$

→ Wakefields ←

I. Linear Regime

- Discussed for the first time here:



VOLUME 54, NUMBER 7 PHYSICAL REVIEW LETTERS 18 FEBRUARY 1985

Acceleration of Electrons by the Interaction of a Bunched Electron Beam with a Plasma

Pisin Chen[(a)]
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

J. M. Dawson, Robert W. Huff, and T. Katsouleas
Department of Physics, University of California, Los Angeles, California 90024

(Received 20 December 1984)

A new scheme for accelerating electrons, employing a bunched relativistic electron beam in a cold plasma, is analyzed. We show that energy gradients can exceed 1 GeV/m and that the driven electrons can be accelerated from $\gamma_0 mc^2$ to $3\gamma_0 mc^2$ before the driving beam slows down enough to degrade the plasma wave. If the driving electrons are removed before they cause the collapse of the plasma wave, energies up to $4\gamma_0^2 mc^2$ are possible. A noncollinear injection scheme is suggested in order that the driving electrons can be removed.

I. Linear Regime

Basic mechanism:

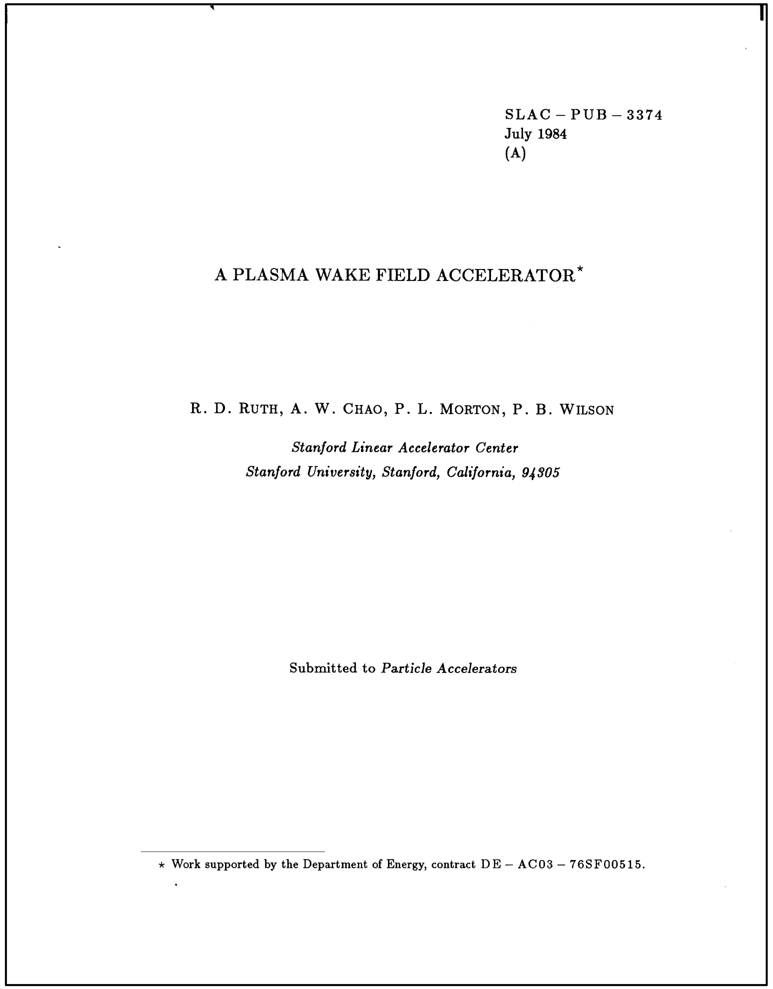
→ Nonrelativistic fluid equations:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0$$

← continuity equation

$$\frac{\partial v}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \frac{e}{m} (\vec{E} + \vec{v} \times \vec{B})$$

← Newton’s law, Lorentz force



SLAC – PUB – 3374
July 1984
(A)

A PLASMA WAKE FIELD ACCELERATOR*

R. D. RUTH, A. W. CHAO, P. L. MORTON, P. B. WILSON

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

Submitted to *Particle Accelerators*

---

* Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

(Detailed didactic explanation)

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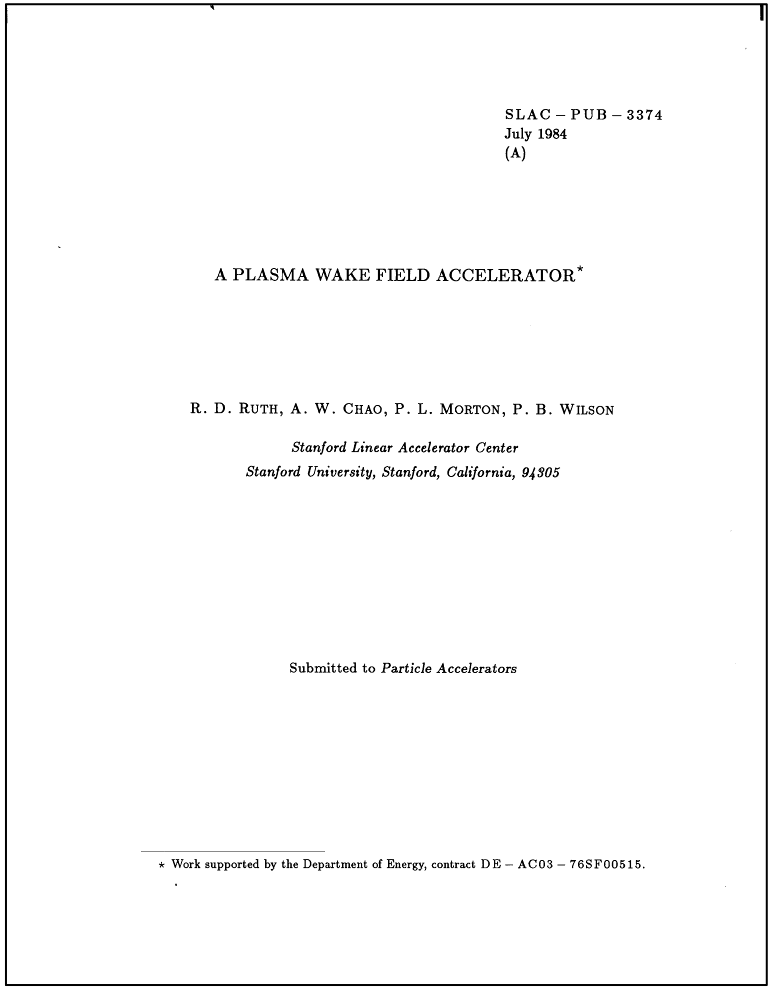
→ Introduce a perturbation **n_1 << n_{pe}** due to a bunch with density n_b<<n_{pe}
→ Linearize
→ …

$$\frac{\partial n_1}{\partial t} + n_{pe}(\nabla \cdot \vec{v}) = 0$$ 🡰 continuity equation

$$\frac{\partial v}{\partial t} = \frac{e}{m}\vec{E}$$ 🡰 Newton’s law, Lorentz force



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July 1984
(A)

A PLASMA WAKE FIELD ACCELERATOR*

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Stanford University, Stanford, California, 94305

Submitted to *Particle Accelerators*

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* Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

(Detailed didactic explanation)

I. Linear Regime

$$\frac{\partial n_1}{\partial t} + n_{pe}(\nabla \cdot \vec{v}) = 0$$ ← continuity equation

$$\frac{\partial v}{\partial t} = \frac{e}{m}\vec{E}$$ ← Newton's law, Lorentz force

+

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = \frac{e}{\varepsilon_0}(n_1 + n_b)$$

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→ combining

→ …

No spatial derivatives → no group velocity → no energy transfer

$$\frac{\partial^2 n_1}{\partial t^2} + \omega_{pe}^2 n_1 = -\omega_{pe}^2 n_b,$$

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$$\frac{\partial^2 n_1}{\partial t^2} + \omega_{pe}^2 n_1 = -\omega_{pe}^2 n_b,$$ **1-D:** $\boldsymbol{n_b = \sigma\delta(z - v_b t)}$

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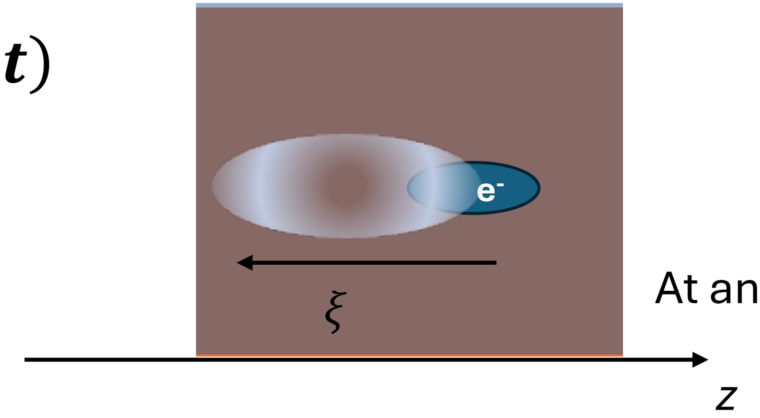
Change of variables:

- z: distance along plasma
- t: time

→ $\xi = v_b t - z$: co-moving frame



At any time *t*

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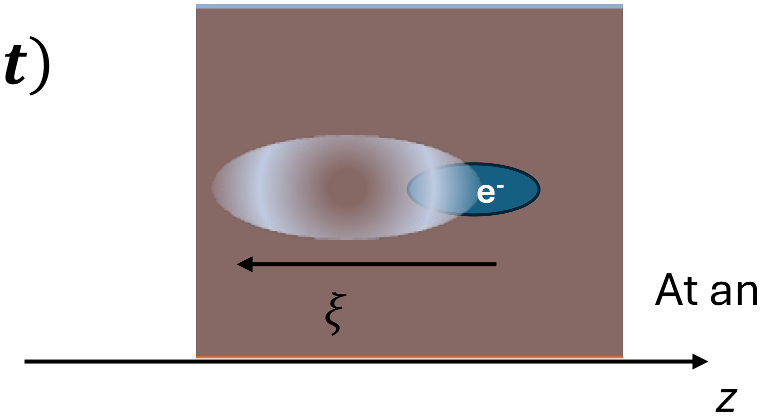

At any time *t*

$$\frac{\partial^2 n_1}{\partial \xi^2} + k^2 n_1 = -k^2 \sigma\delta(\xi),$$ where $k = \frac{\omega_{pe}}{v_b}$ → phase velocity $v_\varphi = v_b$ ➔ **no dephasing**!

I. Linear Regime

$$\frac{\partial^2 n_1}{\partial \xi^2} + k^2 n_1 = -k^2 \sigma \delta(\xi)$$

→ $n_1 = -k\sigma \sin(k\xi)\,, \xi > 0$

→ $n_1 = 0, \qquad\qquad \xi < 0$

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At $\xi = 0$, i.e., within the driving bunch??

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Consider the energy deposited by the bunch per unit length:

$$\frac{\varepsilon_0}{2}\, E^2(peak) = \frac{(e\sigma)^2}{2\, \varepsilon_0}$$

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must be equal to the energy lost by the bunch:

$$E(\xi = 0)e\sigma = \frac{(e\sigma)^2}{2\,\varepsilon_0} \Rightarrow \boldsymbol{E(\xi = 0) = \frac{e\sigma}{2\,\varepsilon_0}}$$

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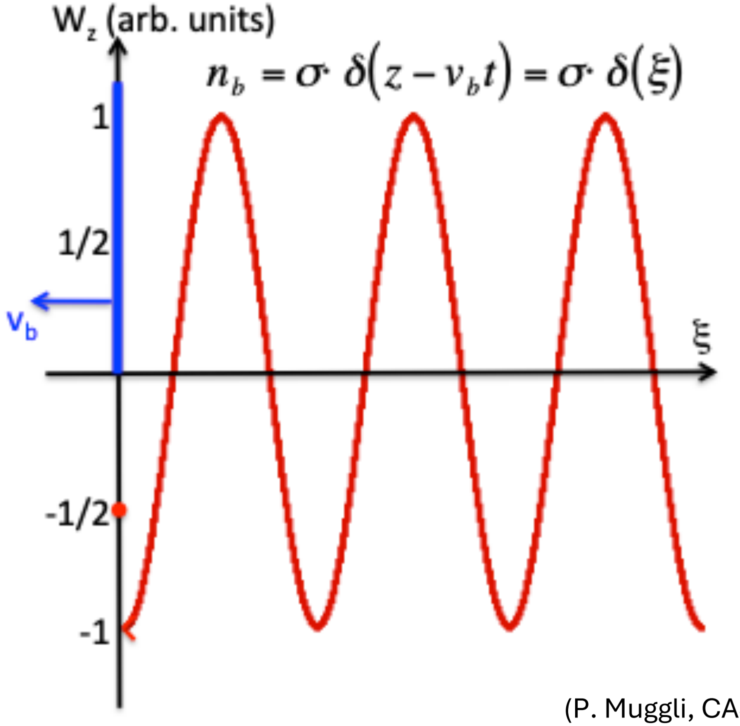

(P. Muggli, CAS 2014)

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The energy gain behind the bunch E₊ can be twice as high as the energy loss E₋!
It satisfies the "fundamental theorem of beam loading": $R = \frac{E_+}{E_-} \le 2$

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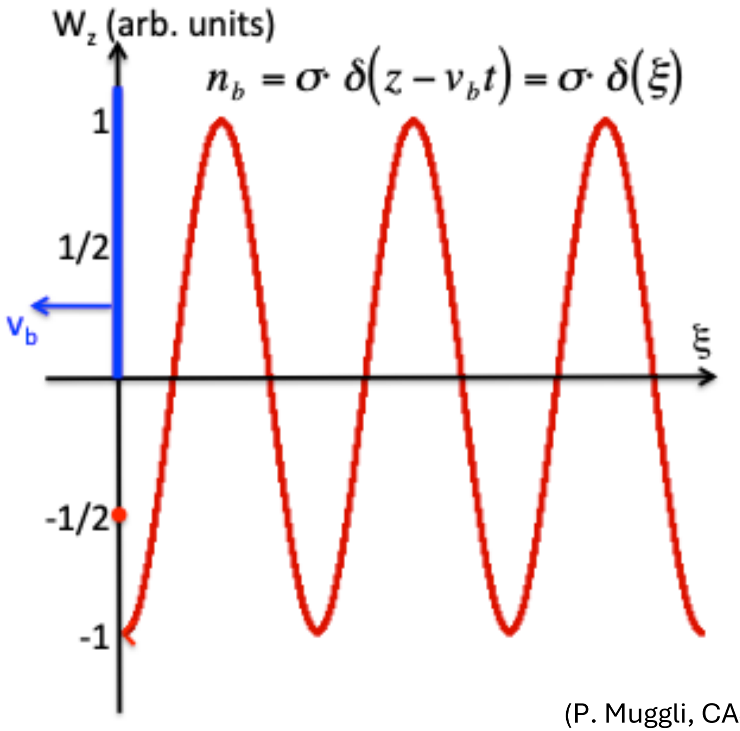

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$$E(\xi = 0)e\sigma = \frac{(e\sigma)^2}{2\,\varepsilon_0} \Rightarrow \boldsymbol{E(\xi = 0) = \frac{e\sigma}{2\,\varepsilon_0}}$$

Energy conservation is satisfied:

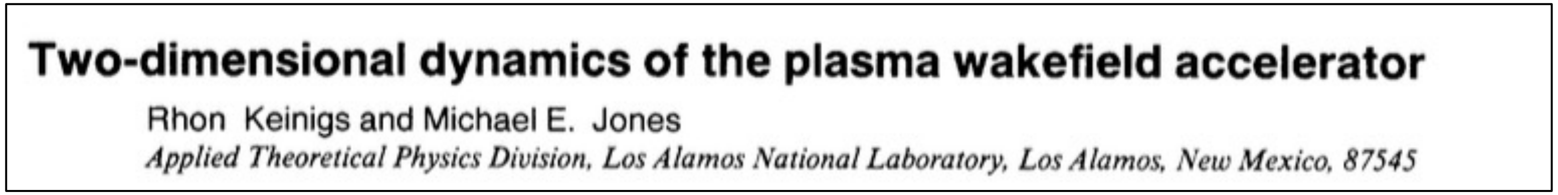
The energy gain behind the bunch E_+ can be twice as high as the energy loss E_-! It satisfies the "fundamental theorem of beam loading": $R = \frac{E_+}{E_-} \leq 2$

➔

Energy balance:
$Q_D E_- \geq Q_W E_+$

I. Linear Regime

If you are interested into the full derivation and much deeper considerations:



Two-dimensional dynamics of the plasma wakefield accelerator

Rhon Keinigs and Michael E. Jones

Applied Theoretical Physics Division, Los Alamos National Laboratory, Los Alamos, New Mexico, 87545

Let’s switch to “real” 2-D world:

$$n_b(\xi, r) = n_{b0}\, n_{b\perp}(r) n_{b\parallel}(\xi)$$

→ Apply Green’s function to the solution obtained before:

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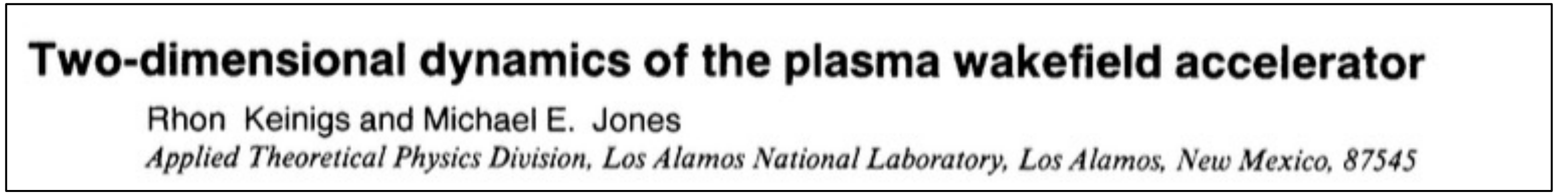

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Longitudinal wakefields

Convolution

Bunch distribution

Green’s function

Radial dependency

$$R(r) = k_{pe}^2 K_0(k_{pe}r)\int_0^r r' n_{b\perp}(r') I_0(k_{pe}r')dr'$$
$$+k_{pe}^2 I_0(k_{pe}r)\int_r^{\infty} r' n_{b\perp}(r') K_0(k_{pe}r')dr'$$

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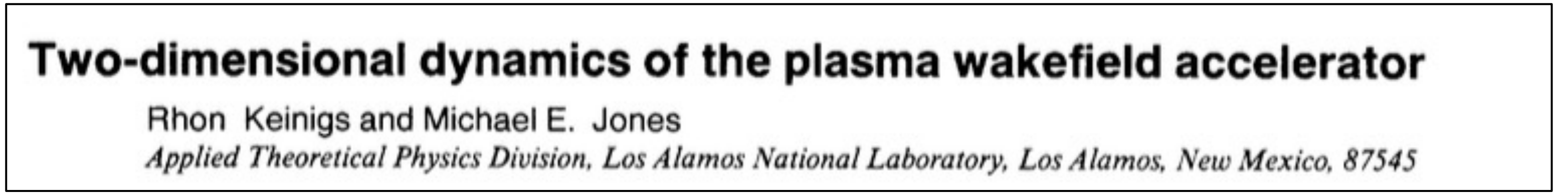

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$$W_\perp(\xi, r) = \frac{-n_{b0} q}{\varepsilon_0 k_{pe}} \int_{-\infty}^{\xi} n_{b||}(\xi') sin(k_{pe}(\xi - \xi')) d\xi' \cdot \frac{dR(r)}{dr}$$

Transverse wakefields

$$W_\perp = e(E_r + v_b \times B_\theta)$$

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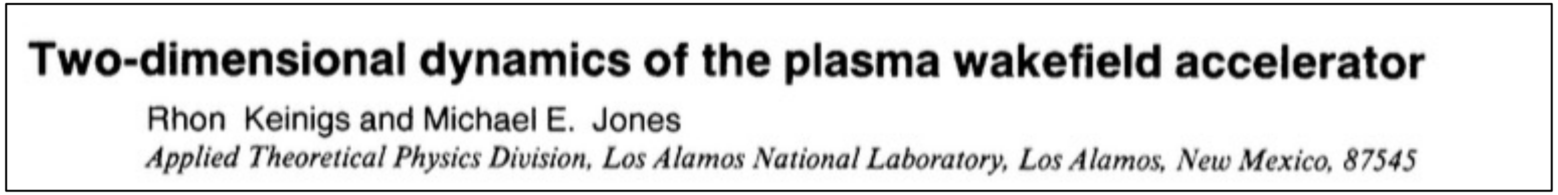

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The relevant beam parameter is the bunch DENSITY, NOT CHARGE!!

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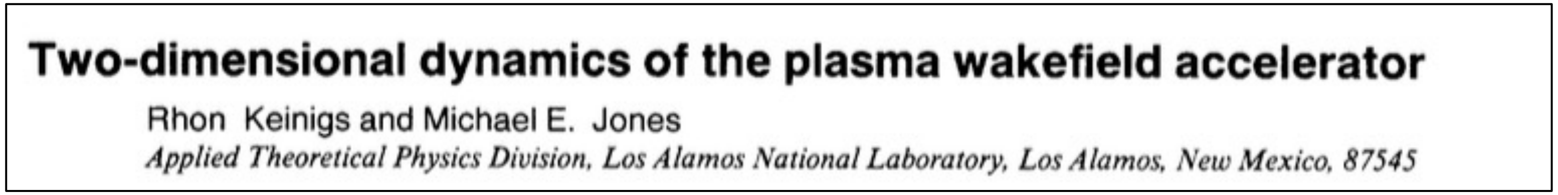

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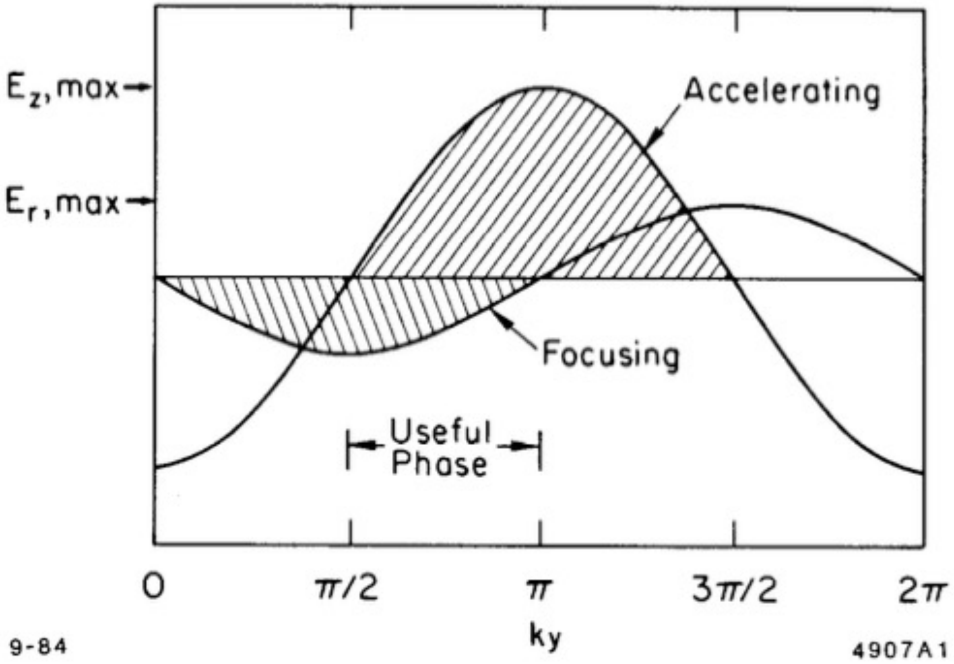
Wakefields are sinusoidal, $\frac{\pi}{2}$ out of phase wrt each other



(Ruth et al.)

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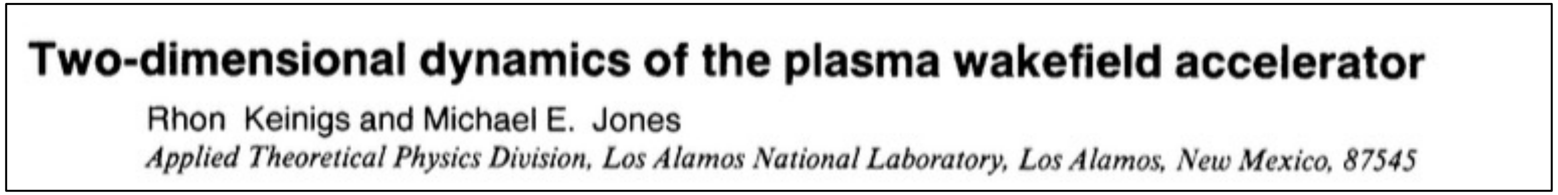

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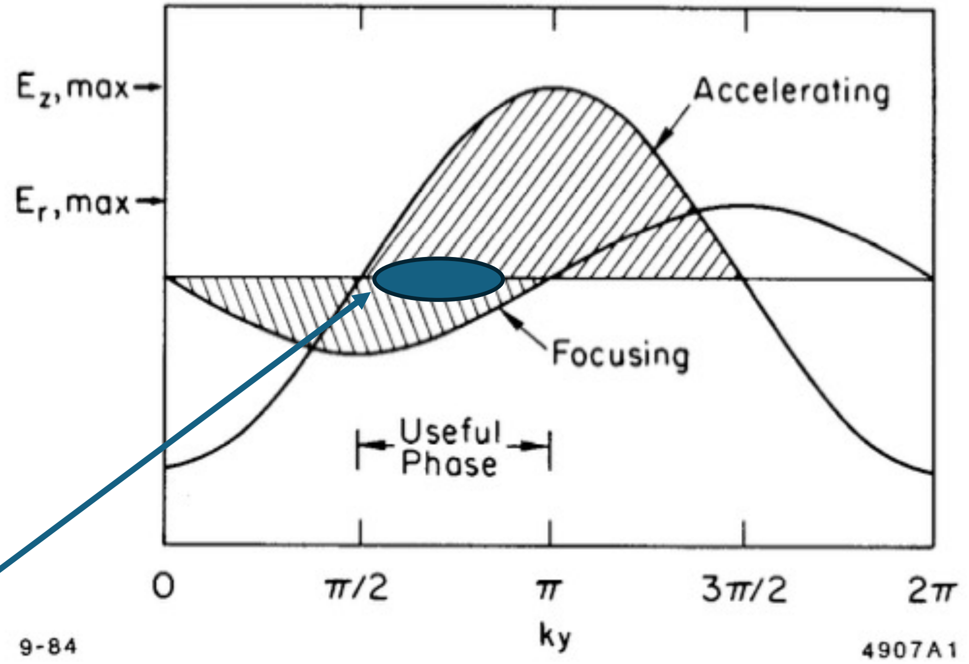
Wakefields are sinusoidal, $\frac{\pi}{2}$ out of phase wrt each other

Have to give up on the maximum accelerating gradient to have stable beam



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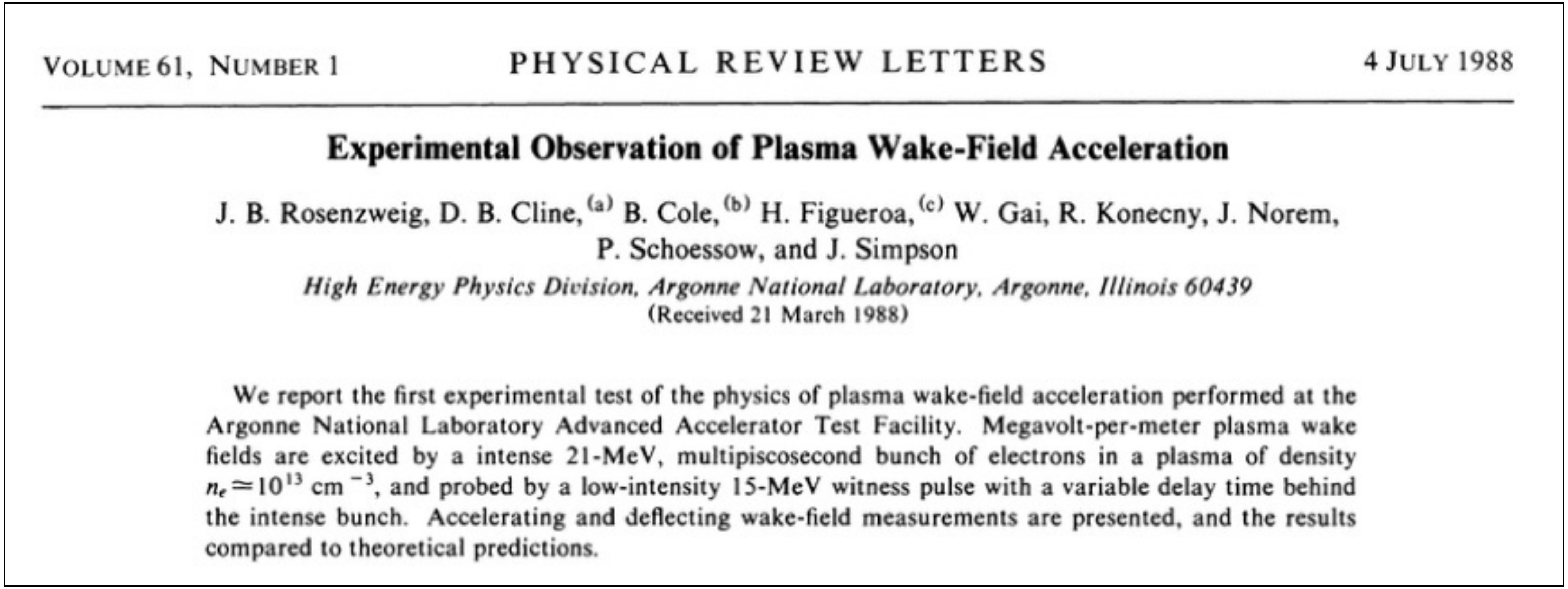
I. Linear Regime - Results

First experimental demonstration: 1988, Argonne National Laboratory (US)



VOLUME 61, NUMBER 1 PHYSICAL REVIEW LETTERS 4 JULY 1988

Experimental Observation of Plasma Wake-Field Acceleration

J. B. Rosenzweig, D. B. Cline,(a) B. Cole,(b) H. Figueroa,(c) W. Gai, R. Konecny, J. Norem,
P. Schoessow, and J. Simpson
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

(Received 21 March 1988)

We report the first experimental test of the physics of plasma wake-field acceleration performed at the Argonne National Laboratory Advanced Accelerator Test Facility. Megavolt-per-meter plasma wake fields are excited by a intense 21-MeV, multipiscosecond bunch of electrons in a plasma of density $n_e \simeq 10^{13}$ cm $^{-3}$, and probed by a low-intensity 15-MeV witness pulse with a variable delay time behind the intense bunch. Accelerating and deflecting wake-field measurements are presented, and the results compared to theoretical predictions.

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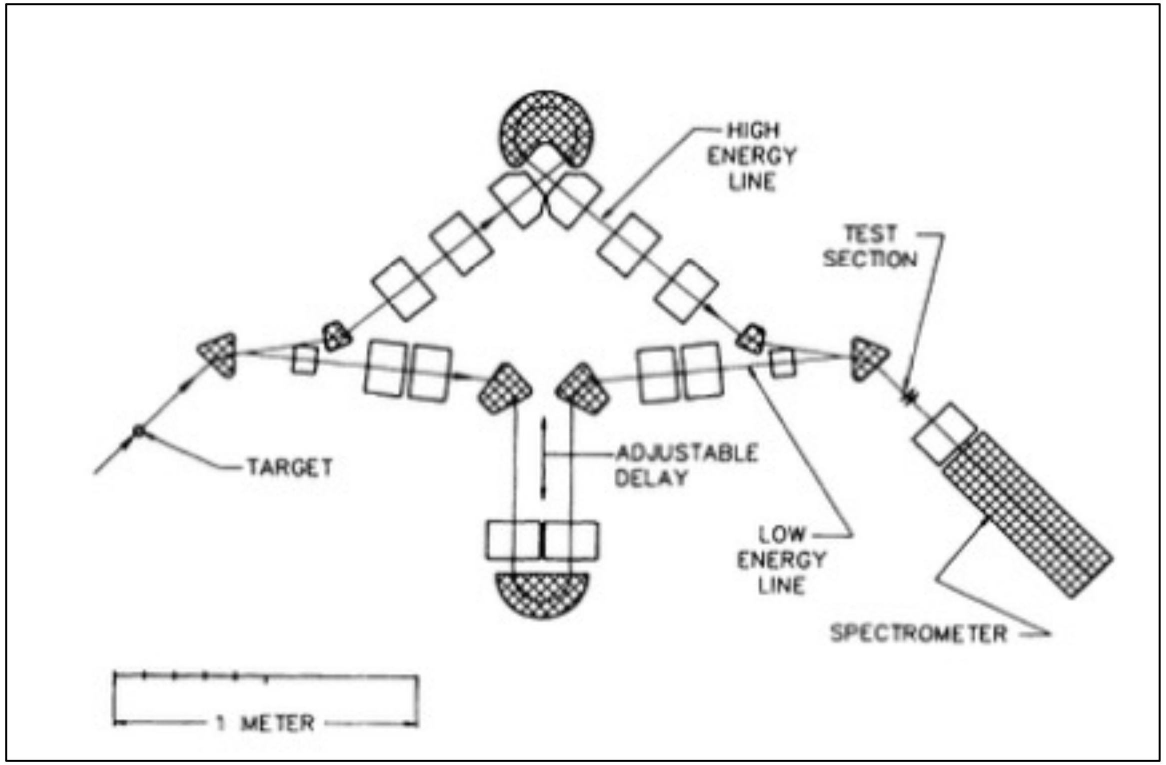

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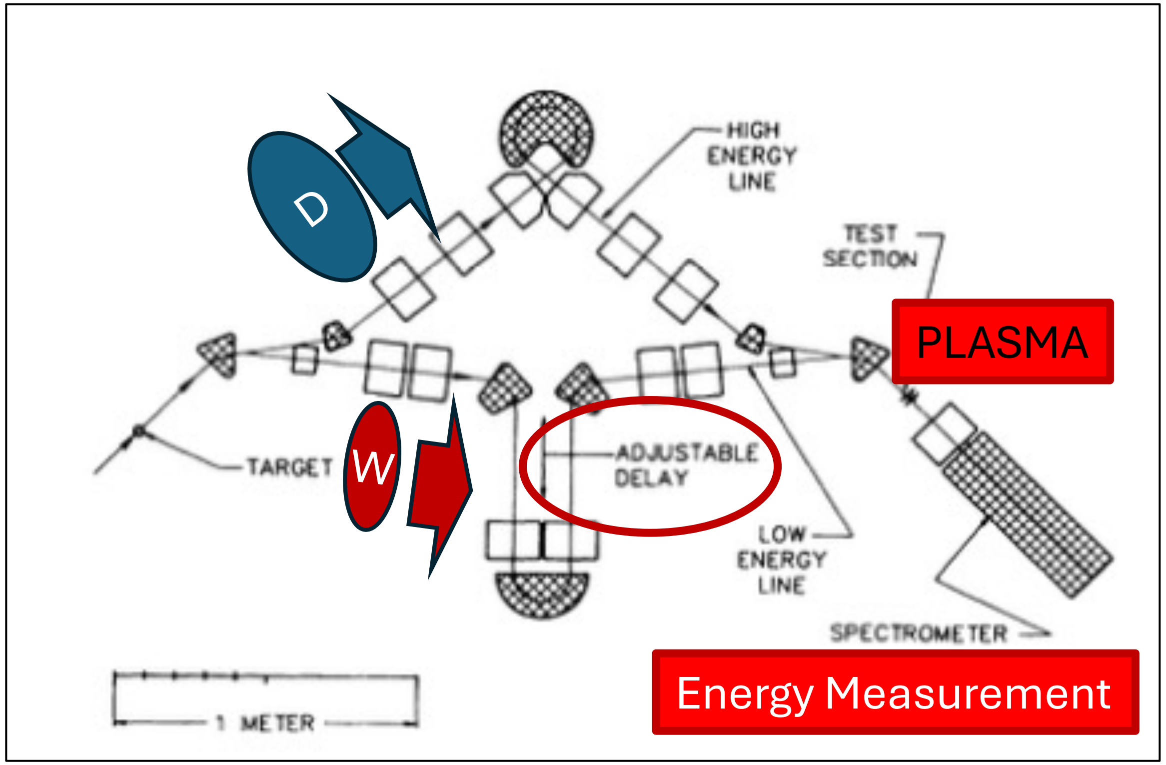

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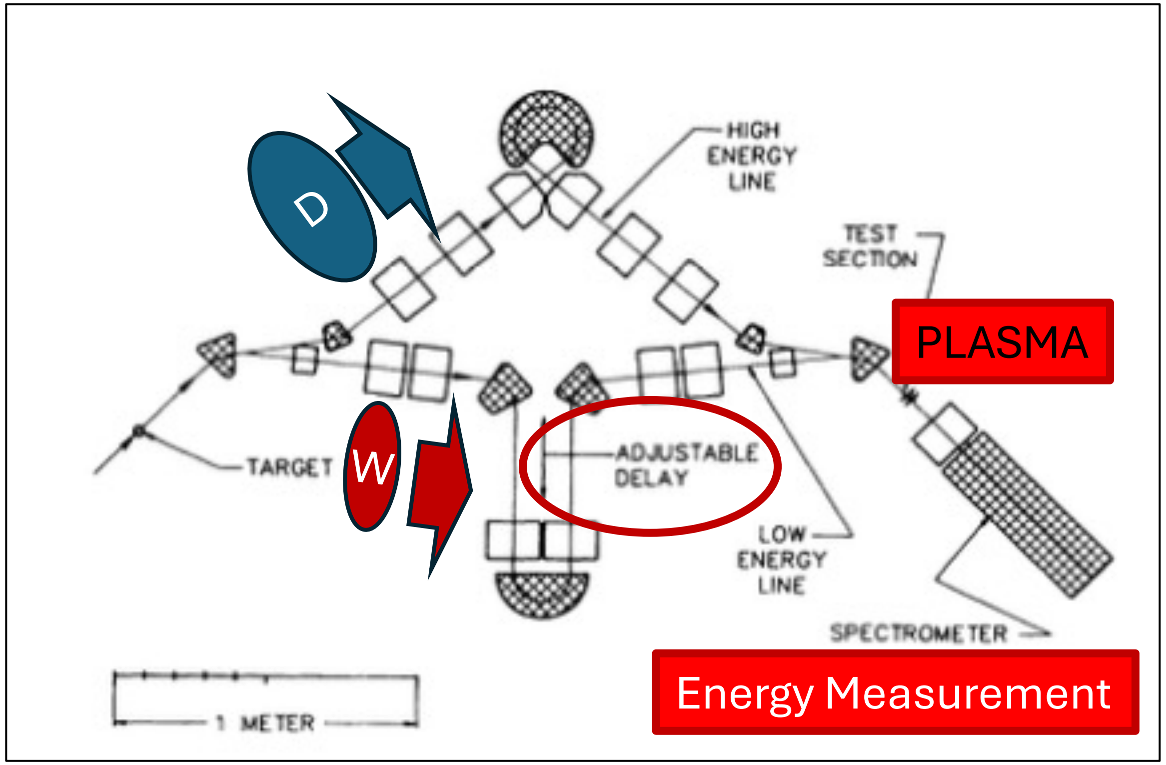

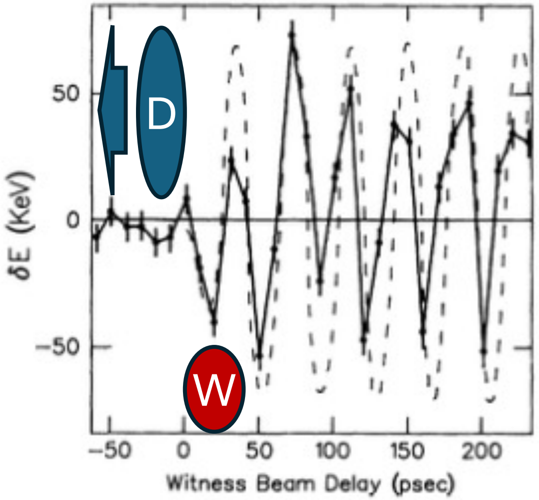

- Measurement of Witness energy as a function of delay
- Sinusoidal
- Q: HOW DO I TEST THE EXPECTATION WITH FREQUENCY?

I. Linear Regime - Results

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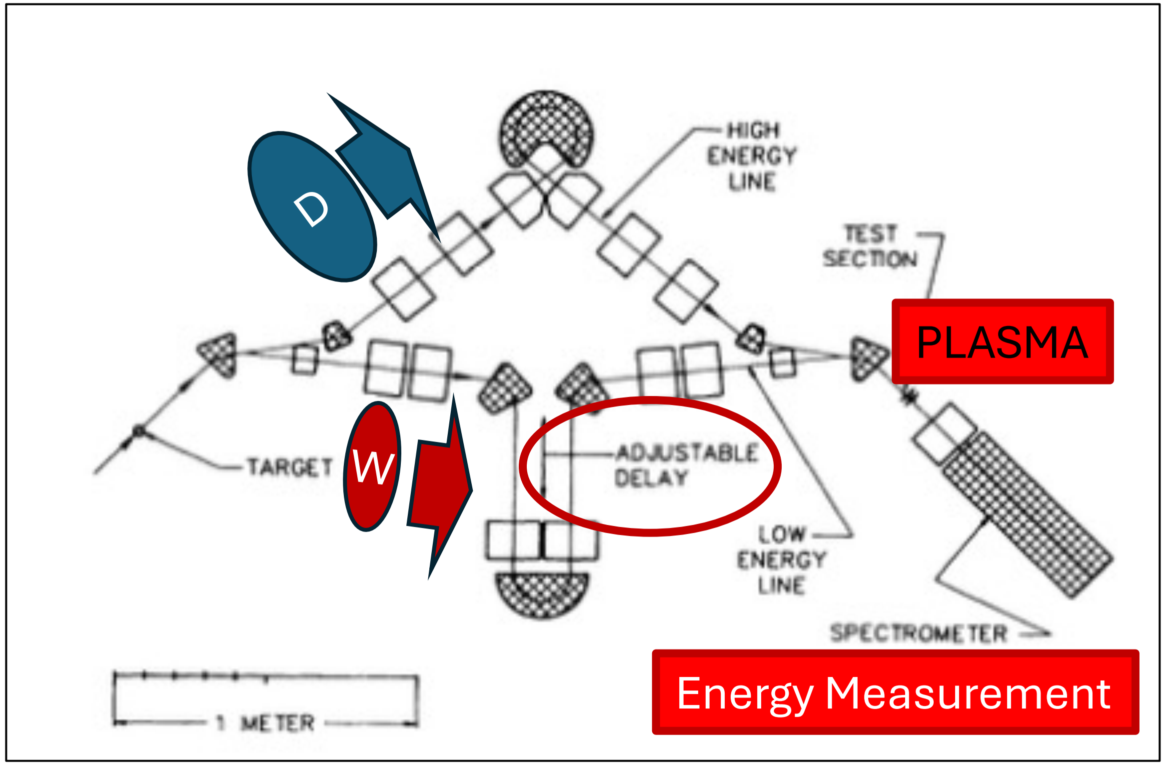

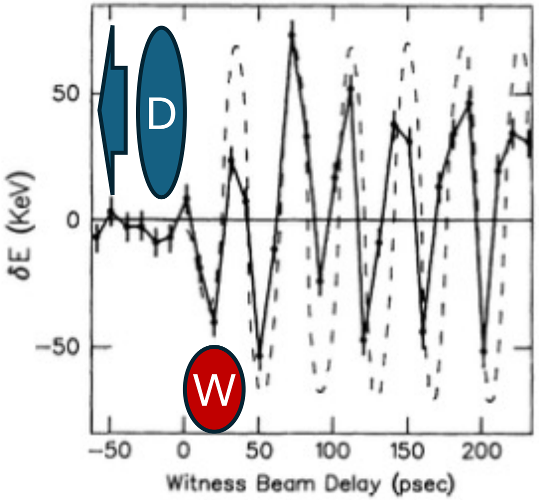

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  → I vary n_{pe}

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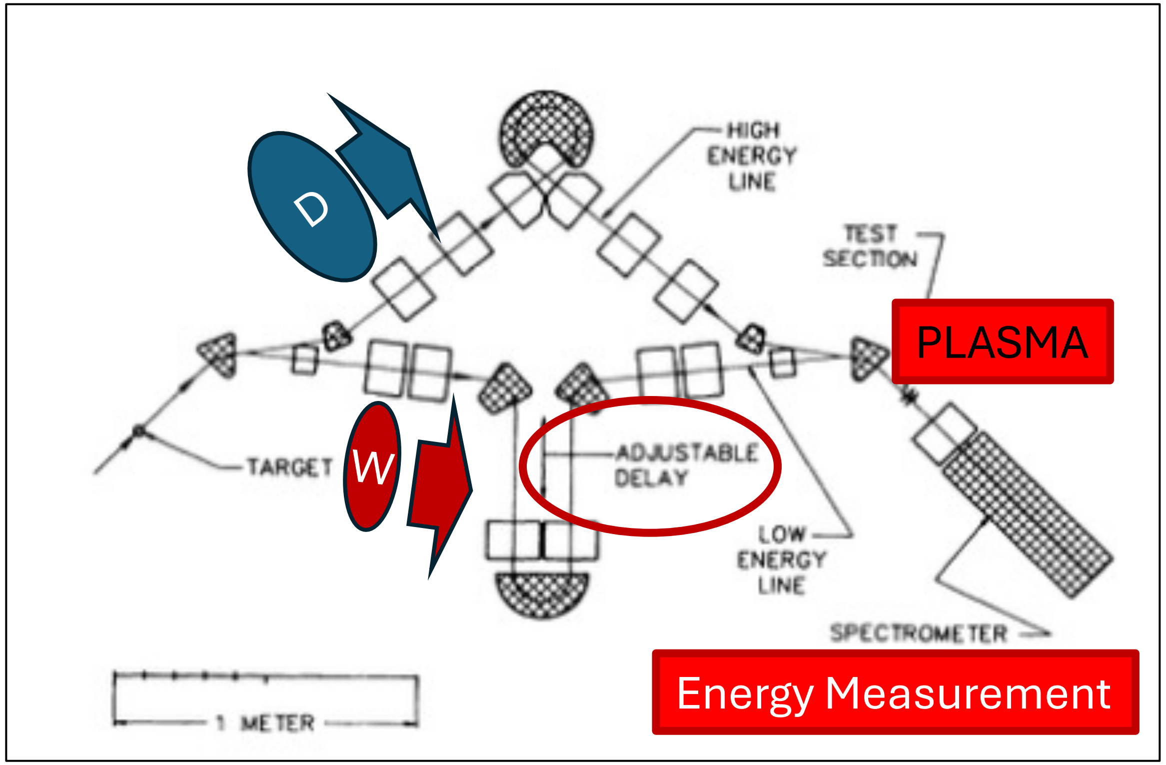
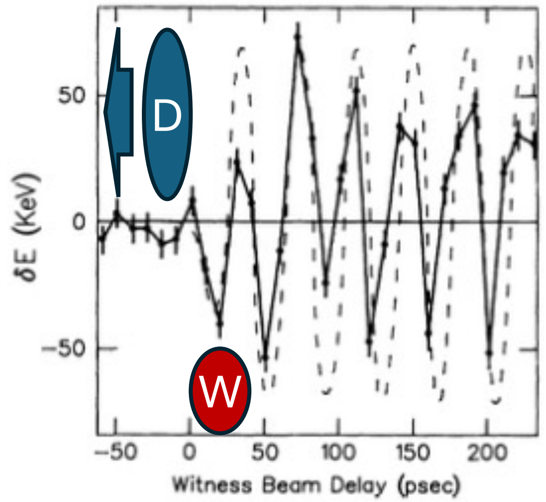
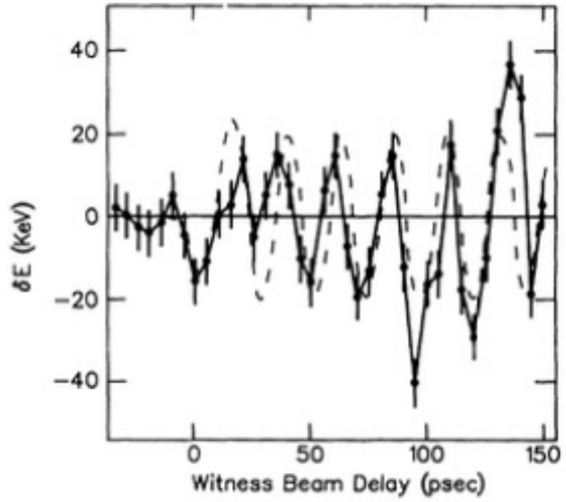


n_{pe} = 8.6 x 10^{12} cm^{-3}



n_{pe} = 2.3 x 10^{13} cm^{-3}



- Measurement of Witness energy as a function of delay
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- Q: HOW DO I TEST THE EXPECTATION WITH FREQUENCY?
  → I vary n_{pe}

- n_b =5.7 x 10^{10} cm^{-3} << n_{pe} ➔ linear regime

I. Linear Regime - Conclusions

A few considerations:

- Bunch requirements:
  - $\sigma_r \leq \frac{c}{\omega_{pe}}$ ➔ avoid instabilities (CFI, see later..)

plasma skin depth:
distance over which plasma
screens electromagnetic fields

$$E_z(\xi, r) = \frac{n_{b0} q}{\varepsilon_0} \int_{-\infty}^{\xi} n_{b||}(\xi') cos(k_{pe}(\xi - \xi')) d\xi' \cdot R(r)$$

$$W_{\perp}(\xi, r) = \frac{-n_{b0} q}{\varepsilon_0 k_{pe}} \int_{-\infty}^{\xi} n_{b||}(\xi') sin(k_{pe}(\xi - \xi')) d\xi' \cdot \frac{dR(r)}{dr}$$

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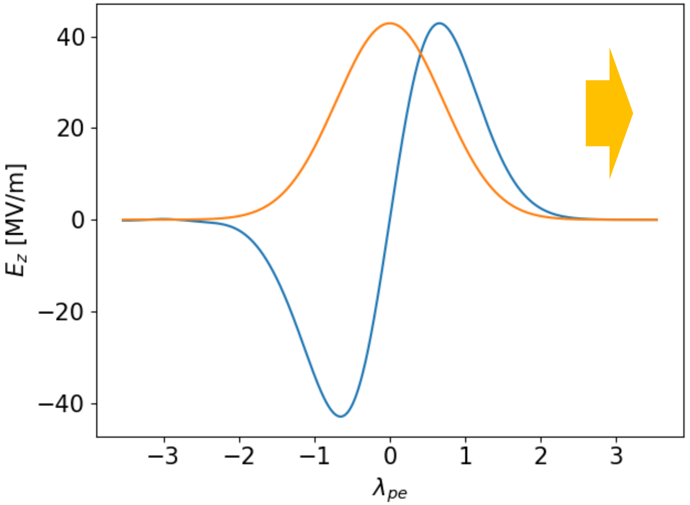
$\sigma_z = \lambda_{pe}$: TOO LONG!

➔ NO WAKEFIELDS BEHIND THE BUNCH



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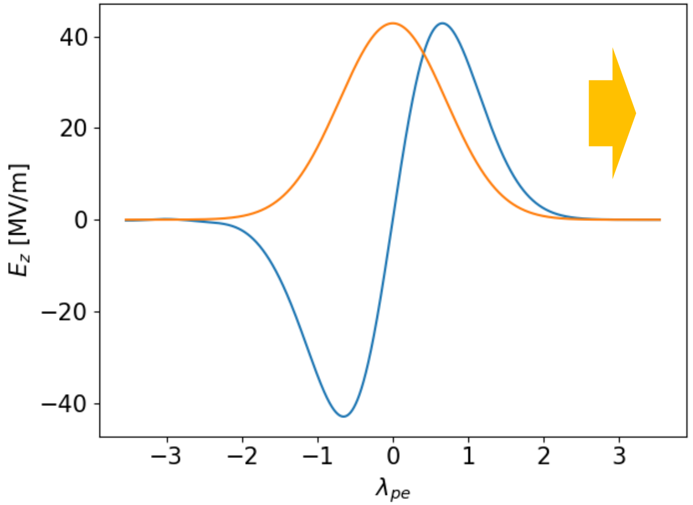
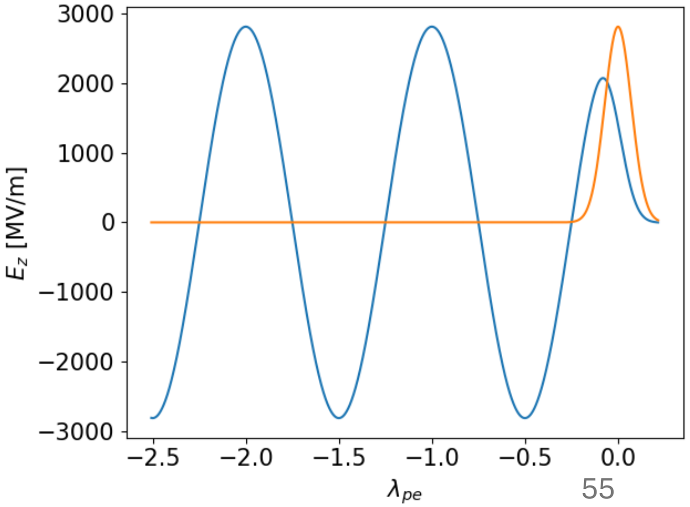
➔ NO WAKEFIELDS BEHIND THE BUNCH



$\sigma_z = \frac{1}{4}\lambda_{pe}$: TOO SHORT!

➔ R<<2: NOT EFFECTIVE!



I. Linear Regime - Conclusions

$$E_z(\xi,r) = \frac{n_{b0}q}{\varepsilon_0}\int_{-\infty}^{\xi} n_{b||}(\xi')cos(k_{pe}(\xi-\xi'))d\xi' \cdot R(r)$$

$$W_\perp(\xi,r) = \frac{-n_{b0}q}{\varepsilon_0 k_{pe}}\int_{-\infty}^{\xi} n_{b||}(\xi')sin(k_{pe}(\xi-\xi'))d\xi' \cdot \frac{dR(r)}{dr}$$

A few considerations:

- Bunch requirements:
  - $\sigma_r \leq \frac{c}{\omega_{pe}}$ ➔ avoid instabilities (CFI, see later..)
  - $\sigma_z = \sqrt{2}\frac{c}{\omega_{pe}}$ ➔ most effective

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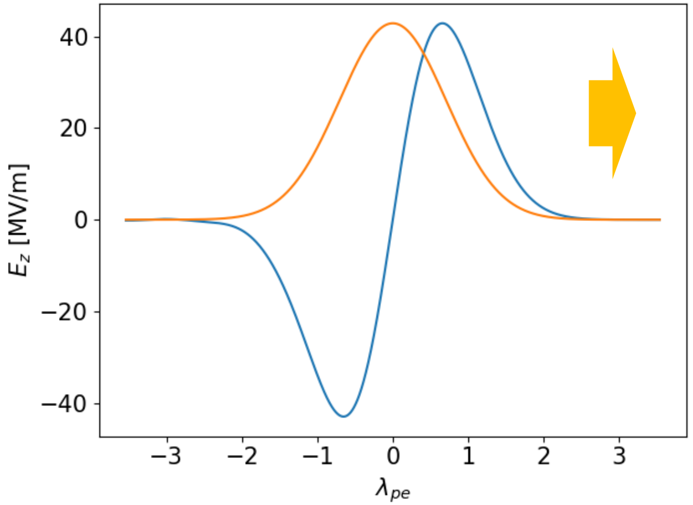
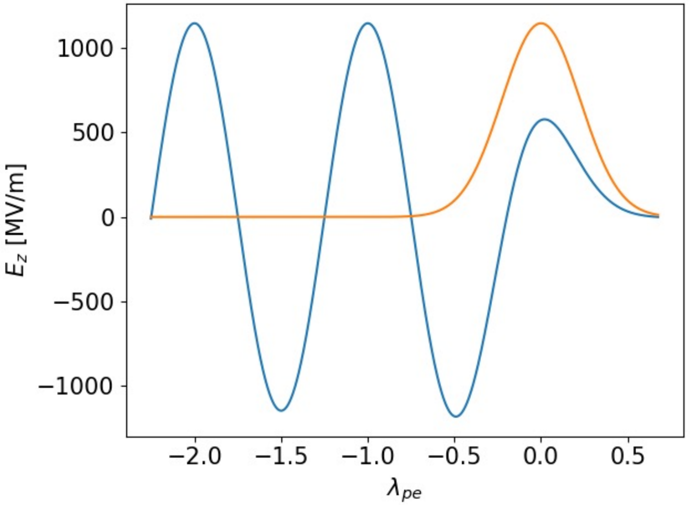


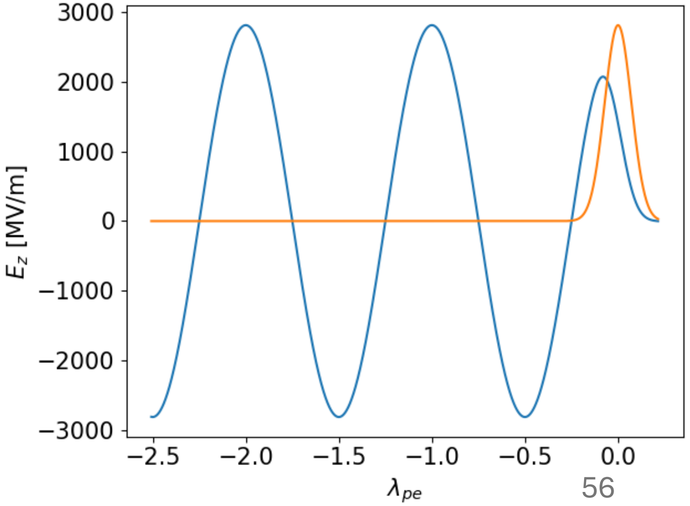
$\sigma_z = \sqrt{2}\frac{c}{\omega_{pe}}$: JUST RIGHT!

➔ R=2: EFFECTIVE!



$\sigma_z = \frac{1}{4}\lambda_{pe}$: TOO SHORT!

➔ R<<2: NOT EFFECTIVE!



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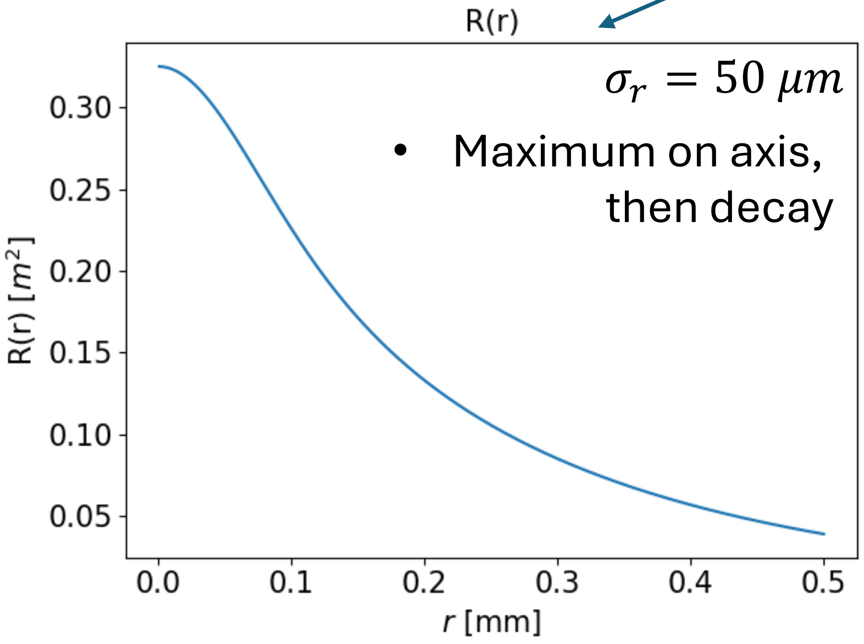
  - Non-uniform E_z along r ➔ energy spread



$\sigma_r = 50\ \mu m$

- Maximum on axis, then decay

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- Radial dependency:
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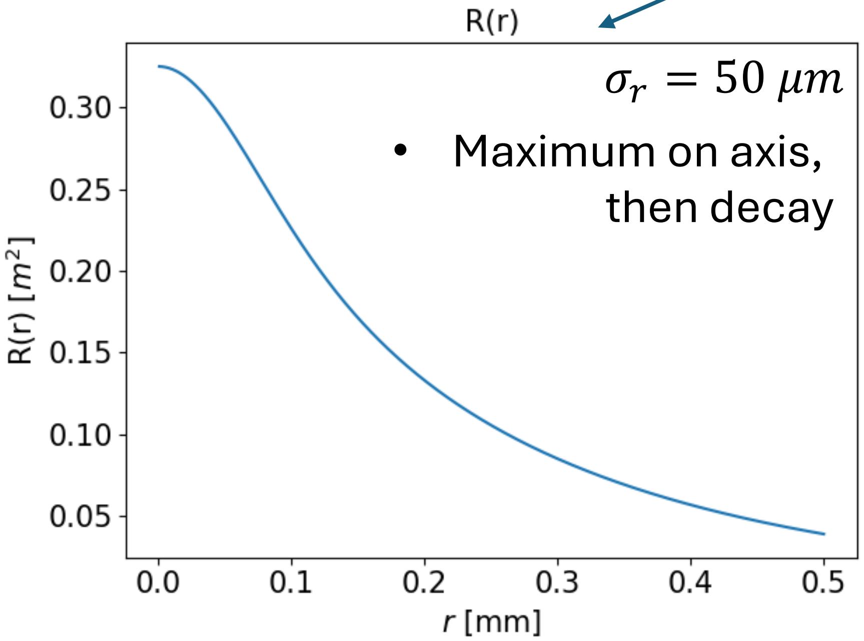
  - Non-linear $W_\perp$ along r ➔ emittance growth



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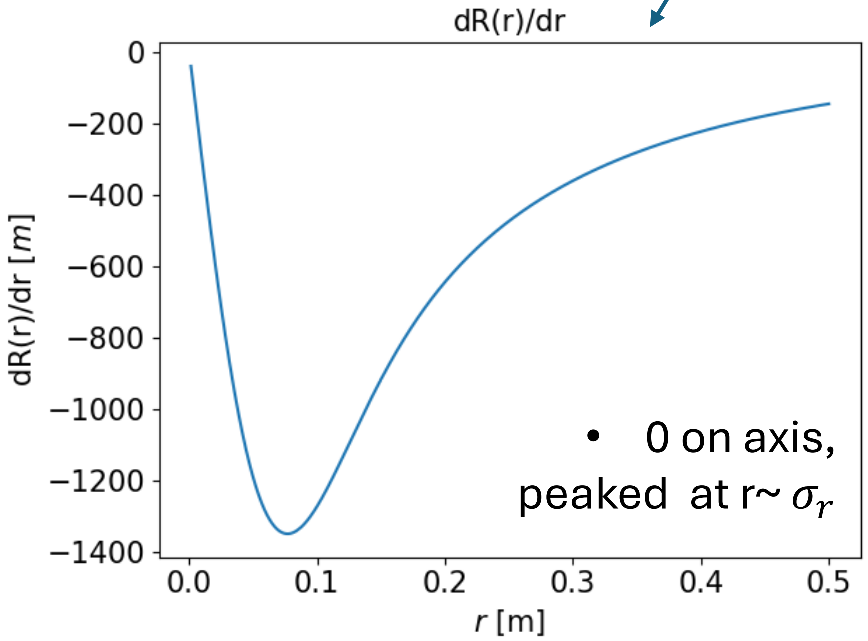

- 0 on axis, peaked at r~ σ_r

I. Linear Regime - Conclusions

Linear Regime:

- **Satisfying analytical solutions**
- Limited accelerating gradient
- **Radial dependencies ➔ Energy spread + Emittance Growth**

→ Switch to non-linear regime

II. Non-linear Regime

- When the electric field of the bunch is strong enough to expel ALL plasma electrons
→ BUBBLE of plasma electrons around a column of pure ions

Requirement:

$$n_b \gg n_{pe}$$

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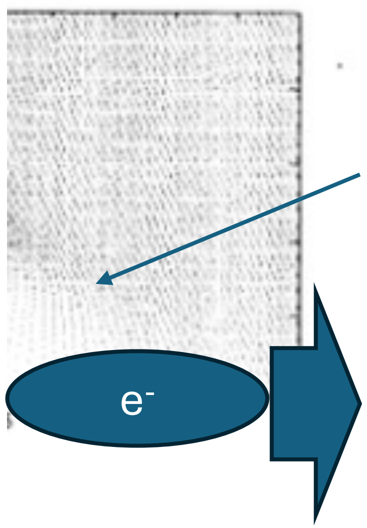

Plasma electrons are pushed out

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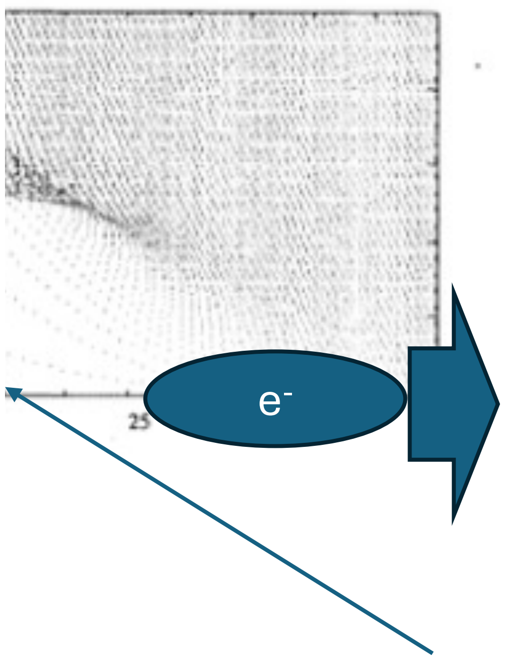

No electrons on axis
(blowout)
→ Strong restoring force from ions

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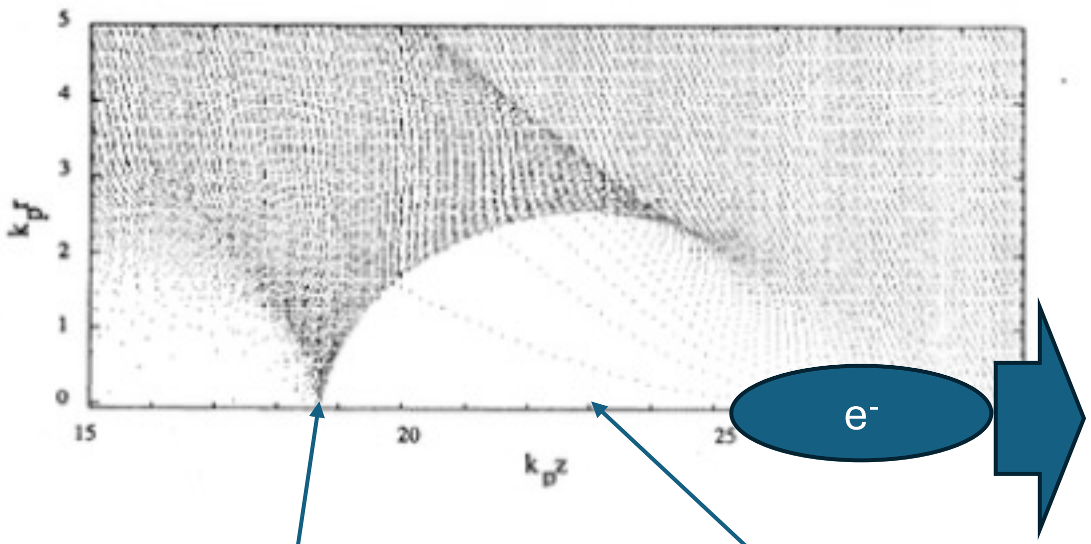

They cross the axis after ~λ_{pe}
→ Singularity
→ Non-linearities

No electrons on axis (blowout)

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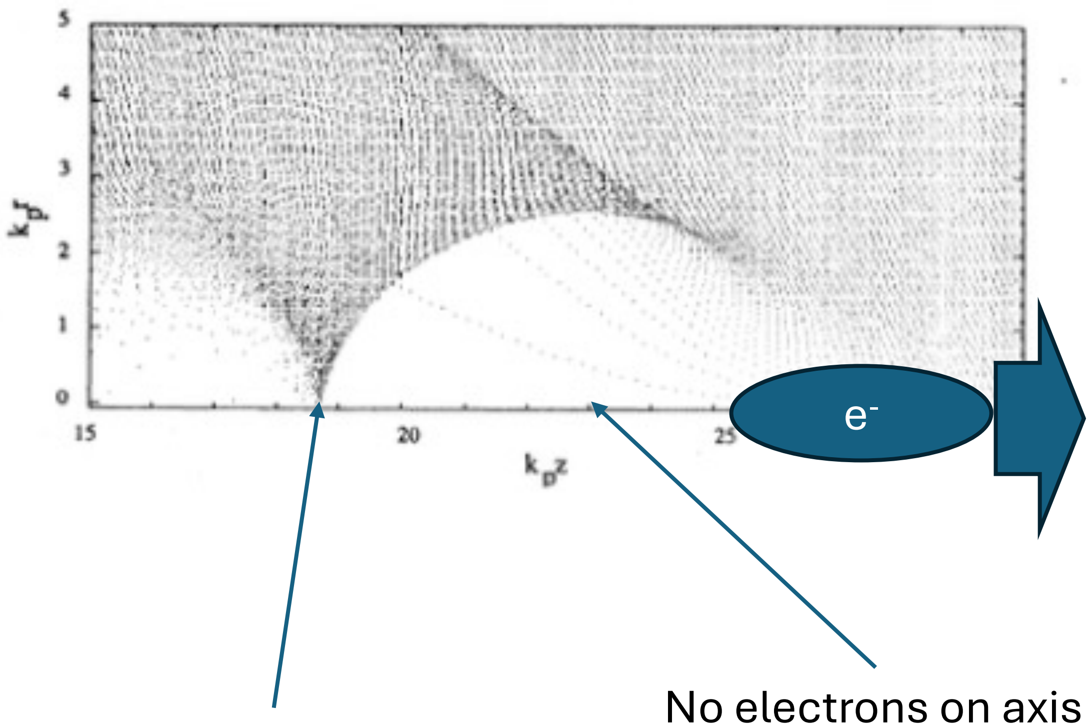

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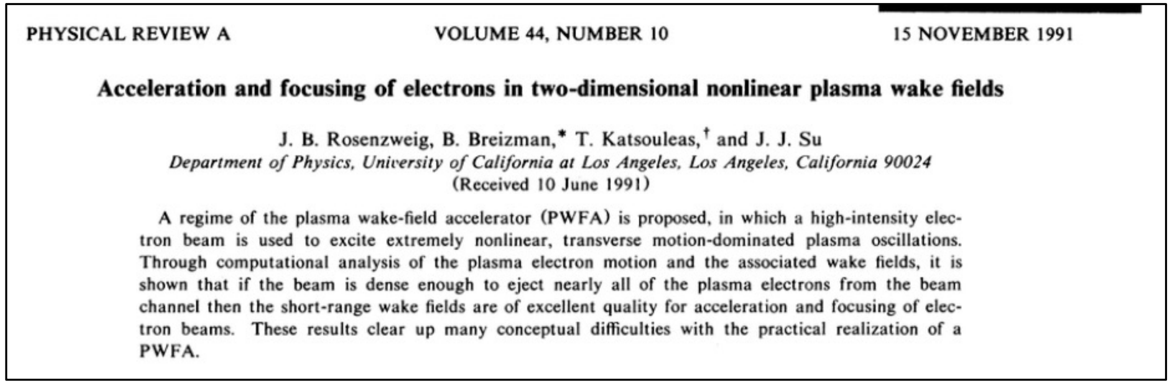

PHYSICAL REVIEW A VOLUME 44, NUMBER 10 15 NOVEMBER 1991

Acceleration and focusing of electrons in two-dimensional nonlinear plasma wake fields

J. B. Rosenzweig, B. Breizman,* T. Katsouleas,† and J. J. Su
Department of Physics, University of California at Los Angeles, Los Angeles, California 90024
(Received 10 June 1991)

A regime of the plasma wake-field accelerator (PWFA) is proposed, in which a high-intensity electron beam is used to excite extremely nonlinear, transverse motion-dominated plasma oscillations. Through computational analysis of the plasma electron motion and the associated wake fields, it is shown that if the beam is dense enough to eject nearly all of the plasma electrons from the beam channel then the short-range wake fields are of excellent quality for acceleration and focusing of electron beams. These results clear up many conceptual difficulties with the practical realization of a PWFA.

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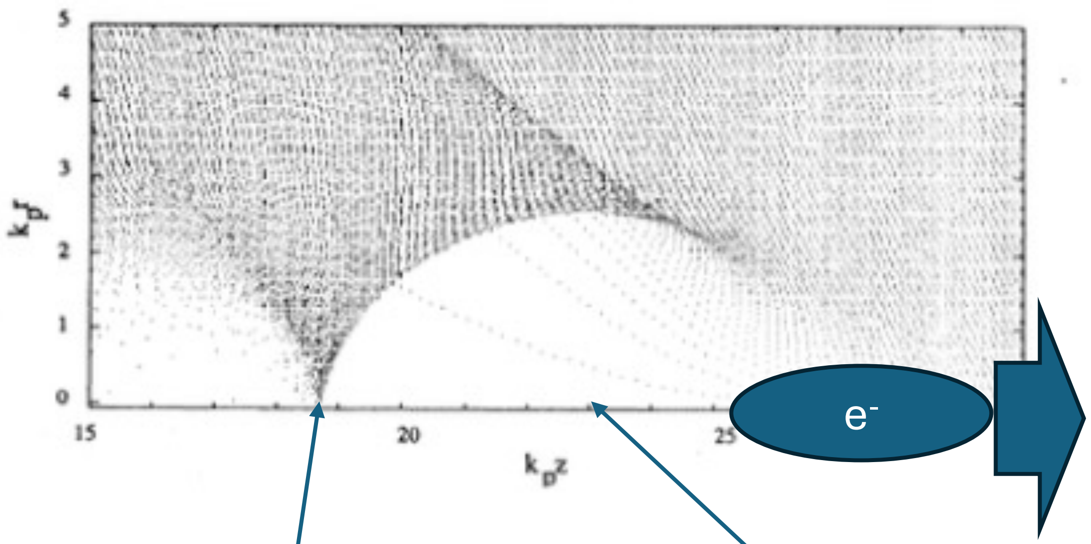

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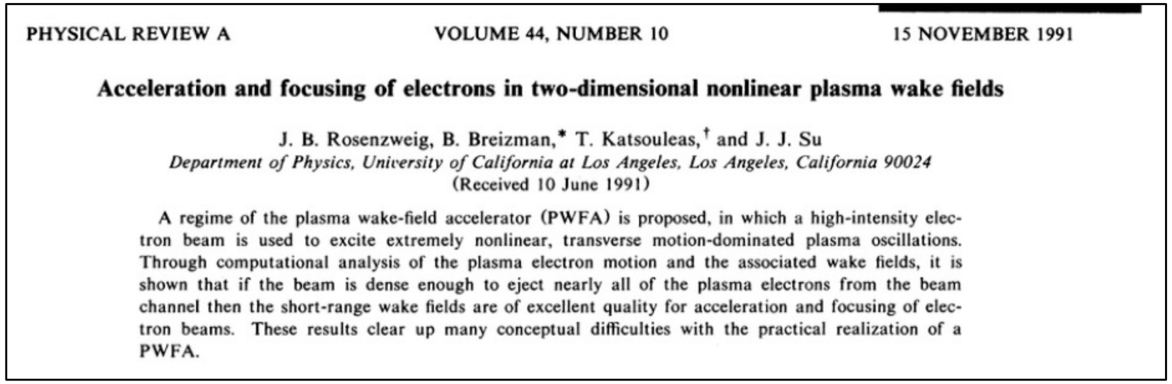

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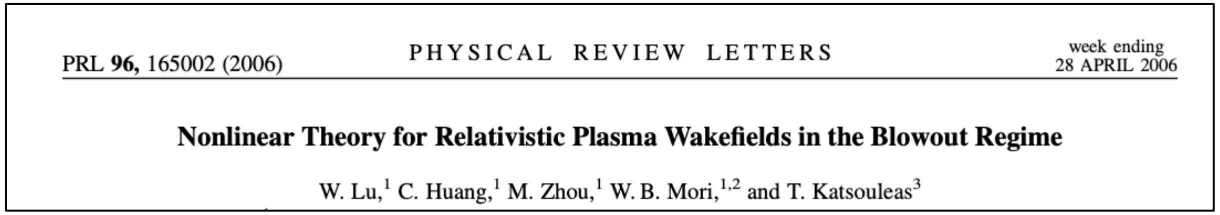
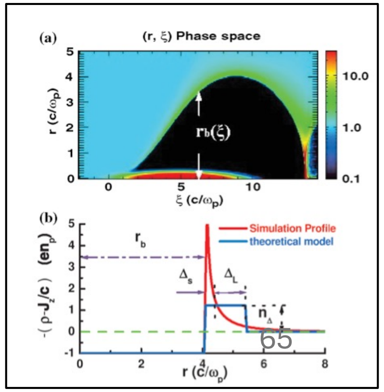


PRL **96,** 165002 (2006) PHYSICAL REVIEW LETTERS week ending 28 APRIL 2006

Nonlinear Theory for Relativistic Plasma Wakefields in the Blowout Regime

W. Lu,[1] C. Huang,[1] M. Zhou,[1] W. B. Mori,[1,2] and T. Katsouleas[3]

For an elegant description of the bubble evolution



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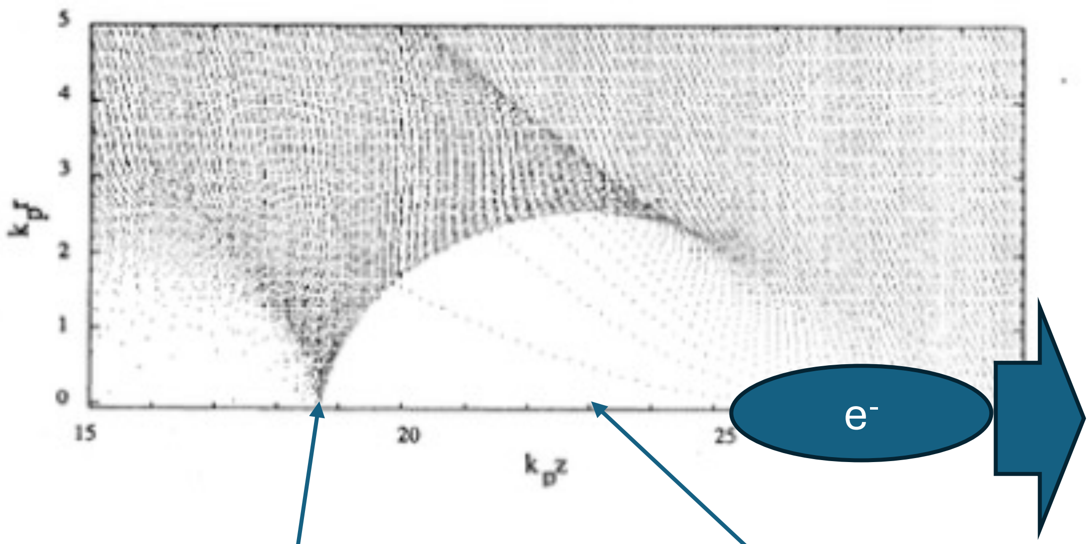

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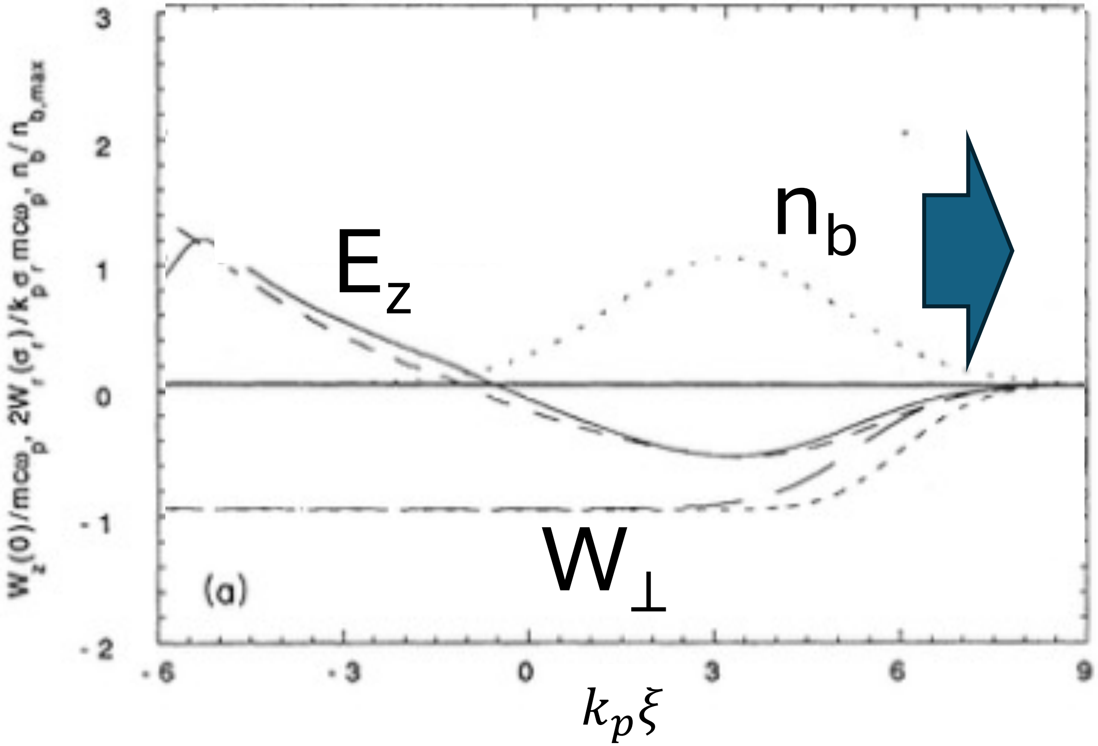
No electrons on axis (blowout)



Along ξ:

- Periodic ''Steepened'' accelerating field
- Uniform focusing field

II. Non-linear Regime

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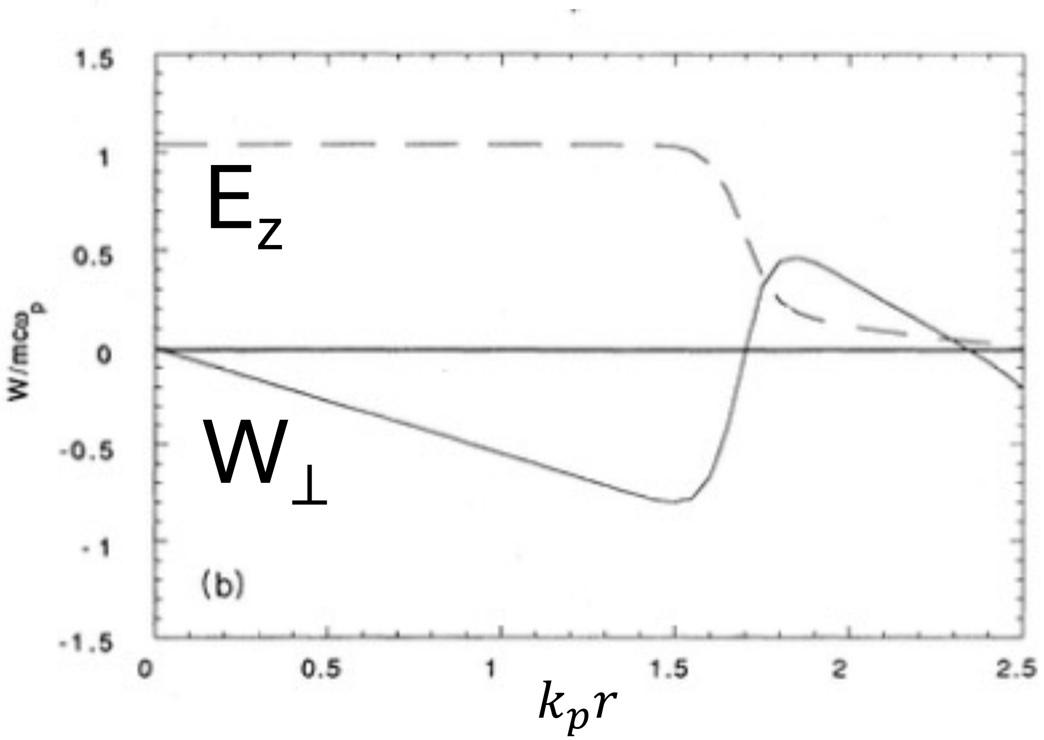
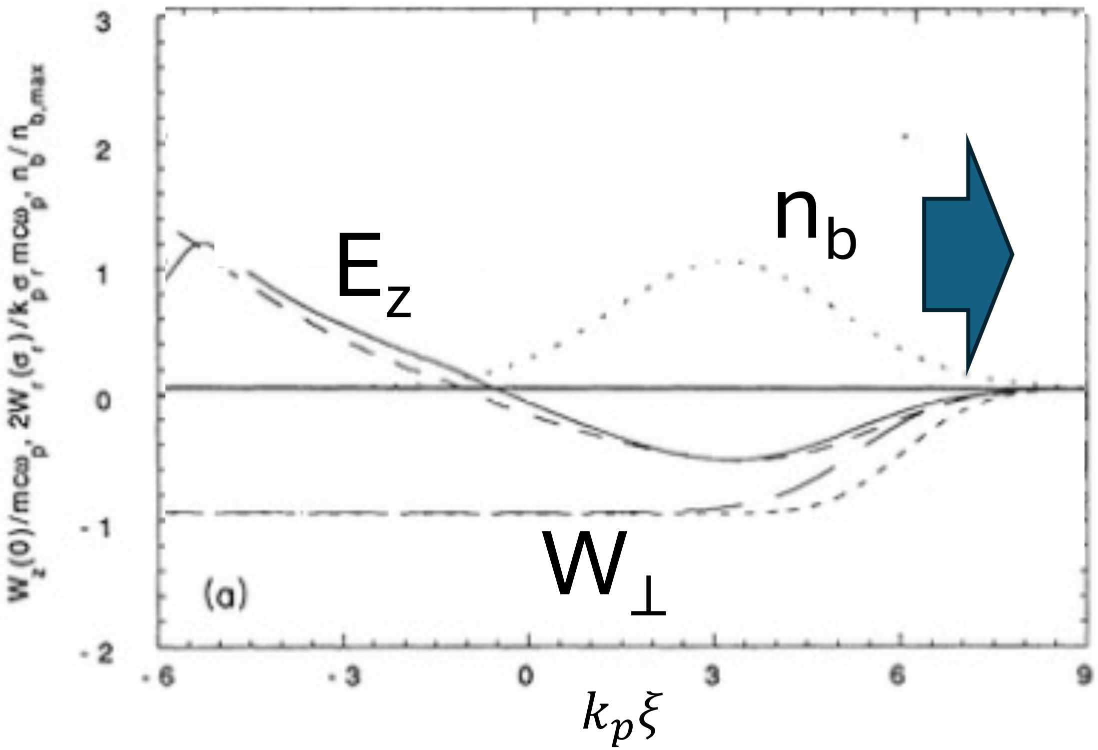




Along r (behind the bunch):

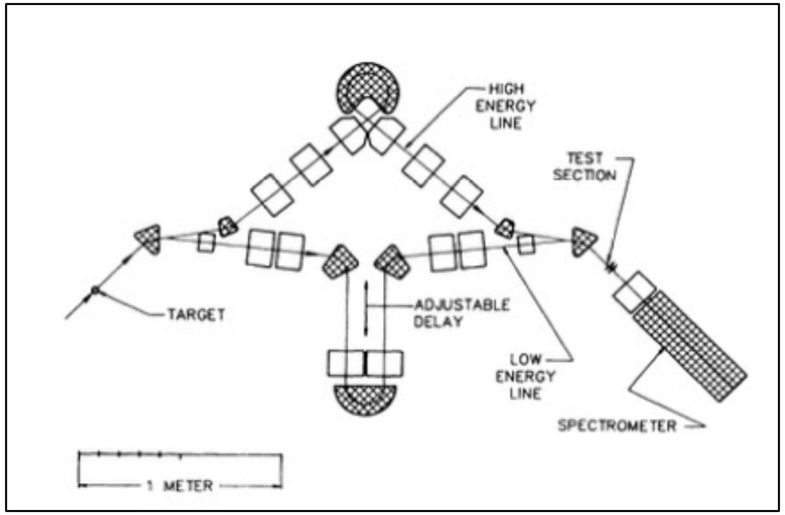
- Uniform accelerating field ➔ uniform acceleration
- Linear focusing force ➔ possible emittance preservation

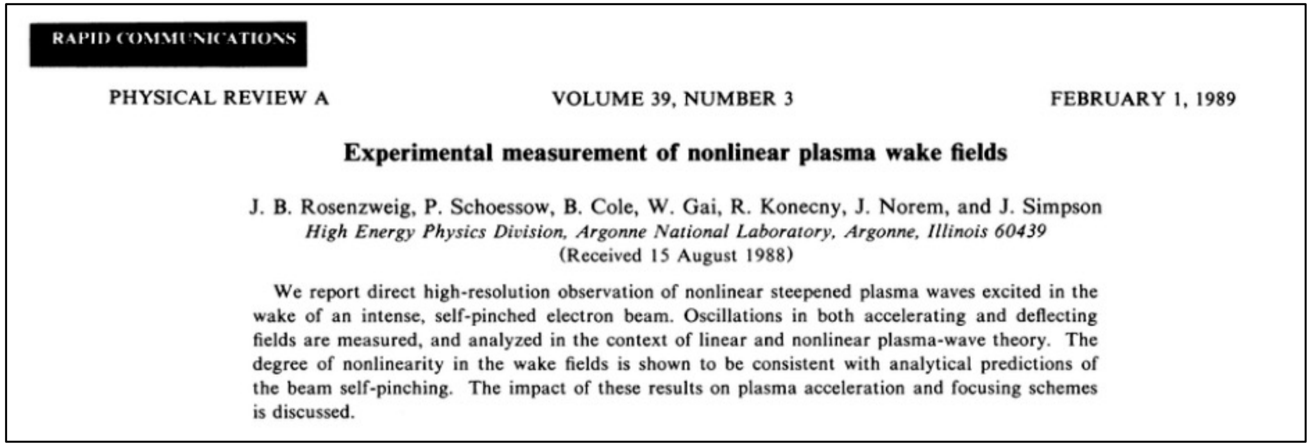
Along ξ:

- Periodic ''Steepened'' accelerating field
- Uniform focusing field

II. Non-linear Regime - Results

- Observation at Argonne:
  same setup, increased n_b





RAPID COMMUNICATIONS

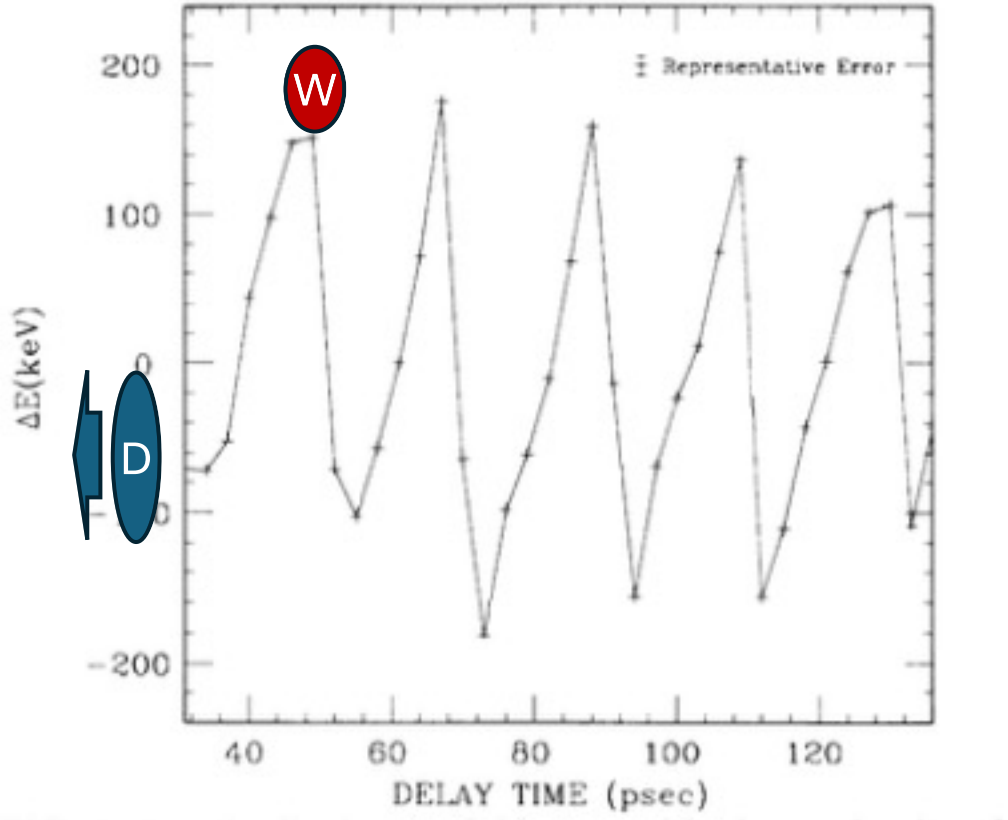
PHYSICAL REVIEW A VOLUME 39, NUMBER 3 FEBRUARY 1, 1989

Experimental measurement of nonlinear plasma wake fields

J. B. Rosenzweig, P. Schoessow, B. Cole, W. Gai, R. Konecny, J. Norem, and J. Simpson
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

(Received 15 August 1988)

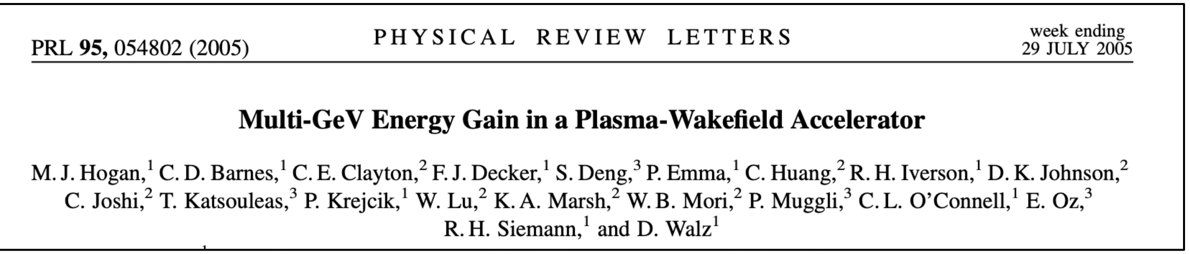
We report direct high-resolution observation of nonlinear steepened plasma waves excited in the wake of an intense, self-pinched electron beam. Oscillations in both accelerating and deflecting fields are measured, and analyzed in the context of linear and nonlinear plasma-wave theory. The degree of nonlinearity in the wake fields is shown to be consistent with analytical predictions of the beam self-pinching. The impact of these results on plasma acceleration and focusing schemes is discussed.



Periodic, saw-tooth accelerating field

II. Non-linear Regime - Results

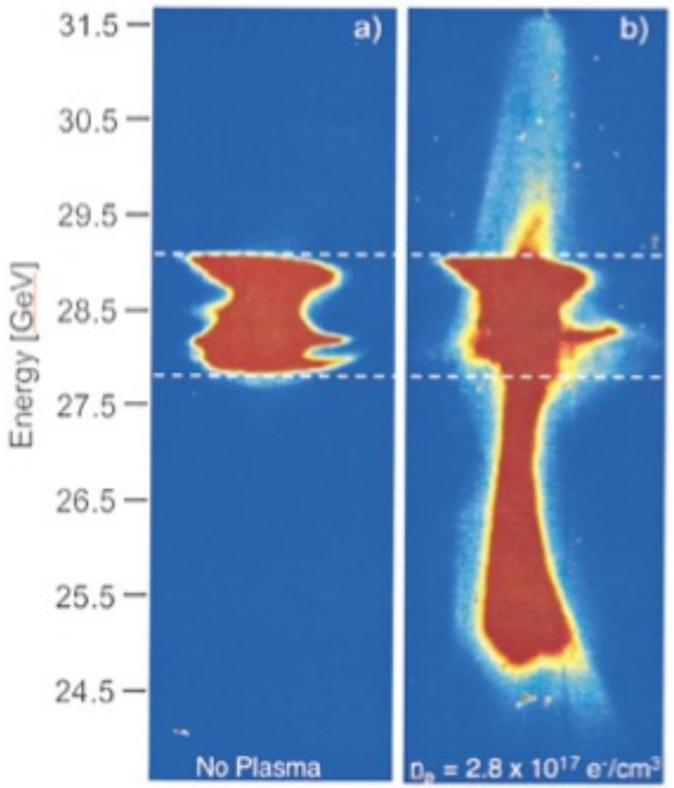
- After this, almost all experiments worked in the non-linear/bubble/blowout regime



PRL **95,** 054802 (2005) PHYSICAL REVIEW LETTERS week ending 29 JULY 2005

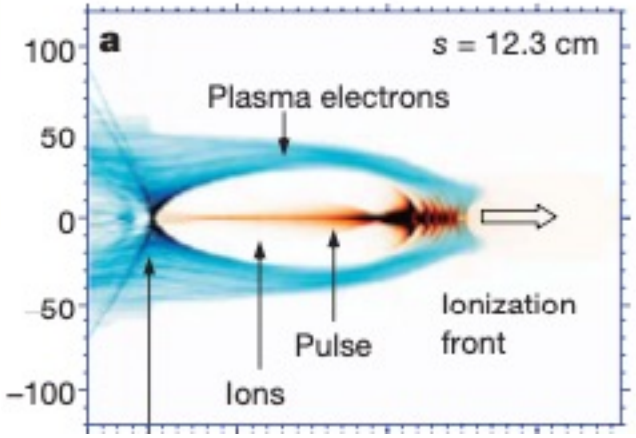
Multi-GeV Energy Gain in a Plasma-Wakefield Accelerator

M. J. Hogan,[1] C. D. Barnes,[1] C. E. Clayton,[2] F. J. Decker,[1] S. Deng,[3] P. Emma,[1] C. Huang,[2] R. H. Iverson,[1] D. K. Johnson,[2] C. Joshi,[2] T. Katsouleas,[3] P. Krejcik,[1] W. Lu,[2] K. A. Marsh,[2] W. B. Mori,[2] P. Muggli,[3] C. L. O'Connell,[1] E. Oz,[3] R. H. Siemann,[1] and D. Walz[1]



~ 2.7 GeV in 10 cm

$n_{pe} = 2.8 \times 10^{17}$ cm^{-3}



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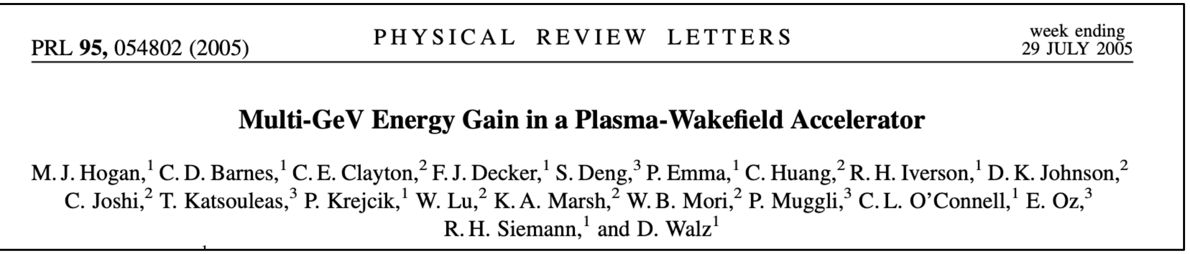

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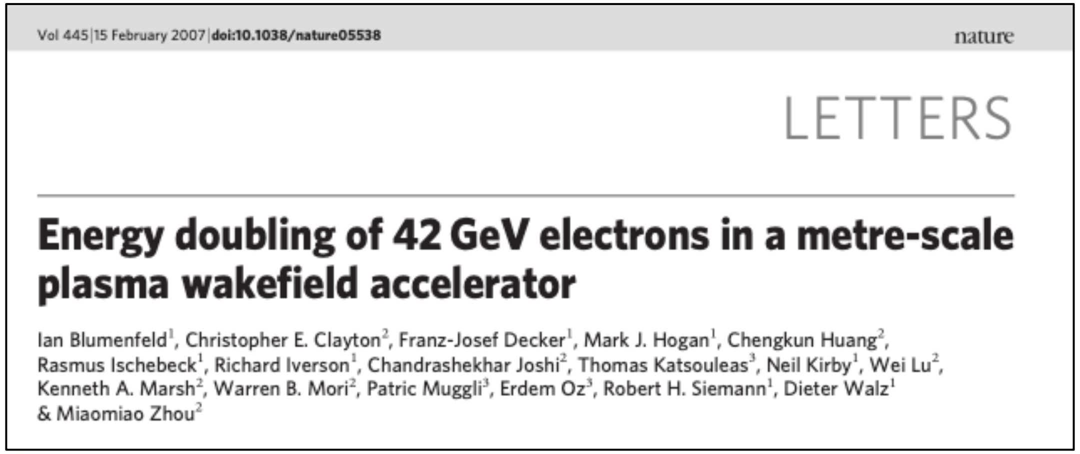
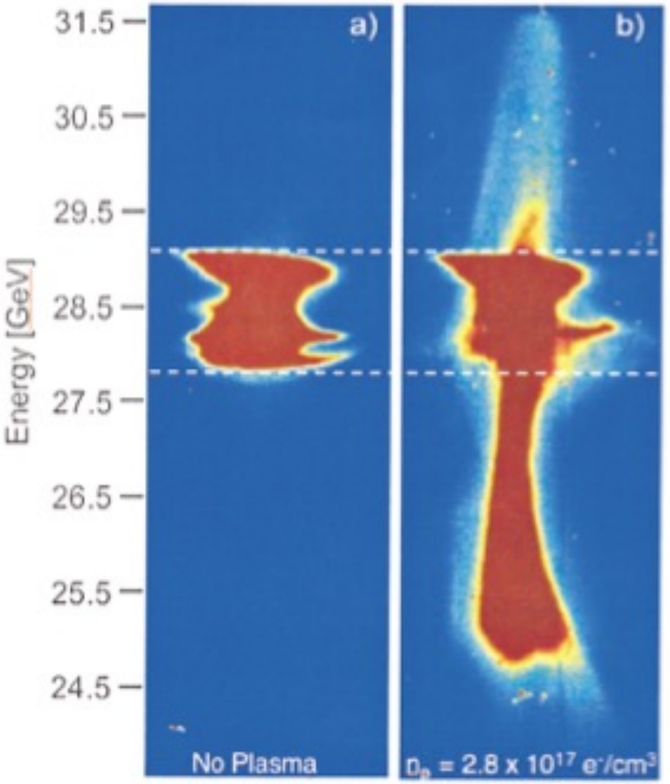


Vol 445|15 February 2007|**doi:10.1038/nature05538** nature

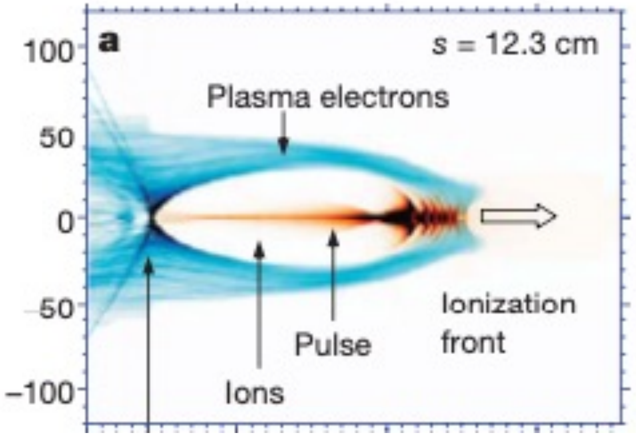
LETTERS

Energy doubling of 42 GeV electrons in a metre-scale plasma wakefield accelerator

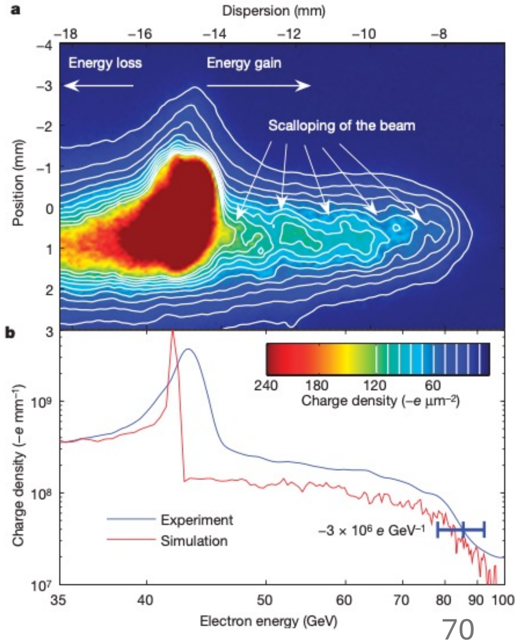
Ian Blumenfeld[1], Christopher E. Clayton[2], Franz-Josef Decker[1], Mark J. Hogan[1], Chengkun Huang[2], Rasmus Ischebeck[1], Richard Iverson[1], Chandrashekhar Joshi[2], Thomas Katsouleas[3], Neil Kirby[1], Wei Lu[2], Kenneth A. Marsh[2], Warren B. Mori[2], Patric Muggli[3], Erdem Oz[3], Robert H. Siemann[1], Dieter Walz[1] & Miaomiao Zhou[2]



~ 2.7 GeV in 10 cm
$n_{pe} = 2.8 \times 10^{17}$ cm^{-3}



~ 42 GeV in 85 cm
$n_{pe} = 2.8 \times 10^{17}$ cm^{-3}



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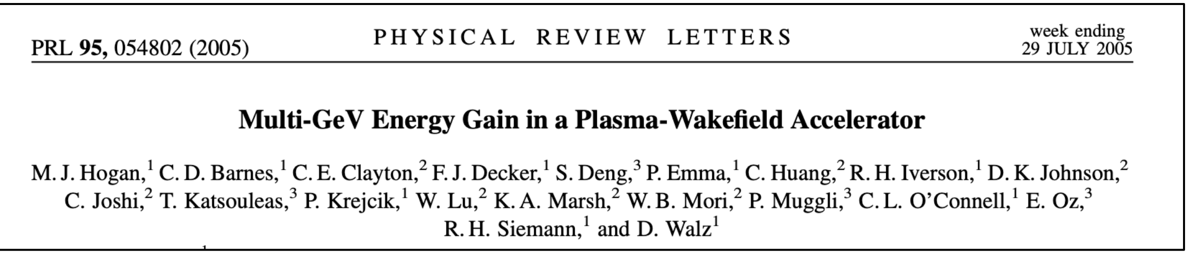

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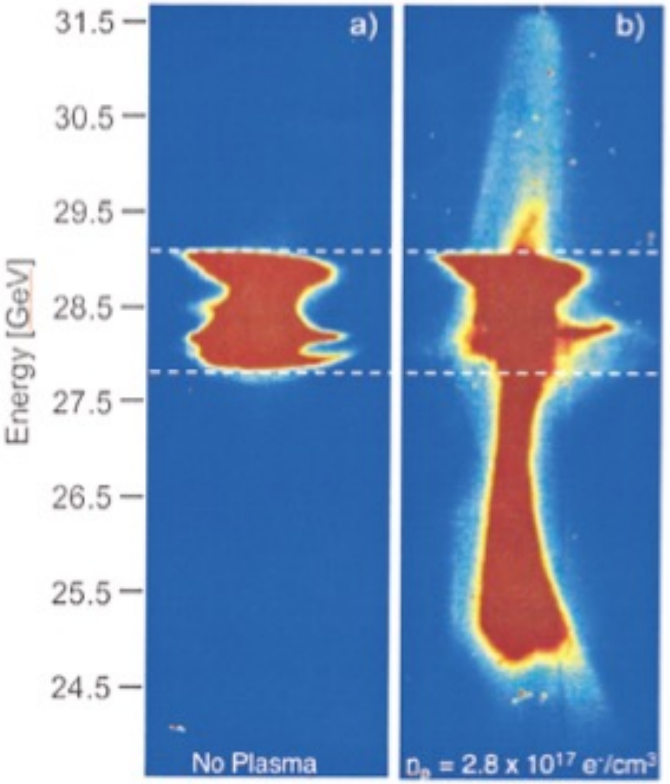

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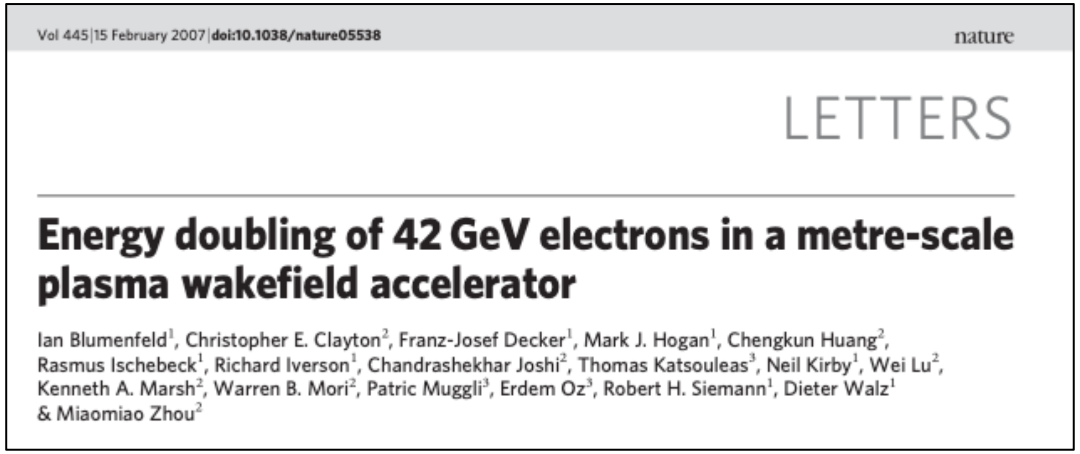

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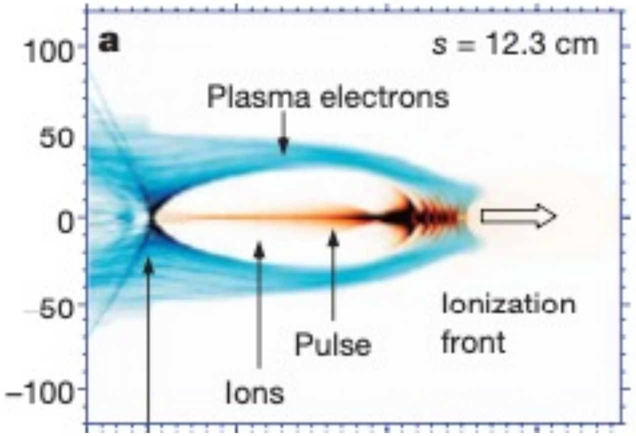
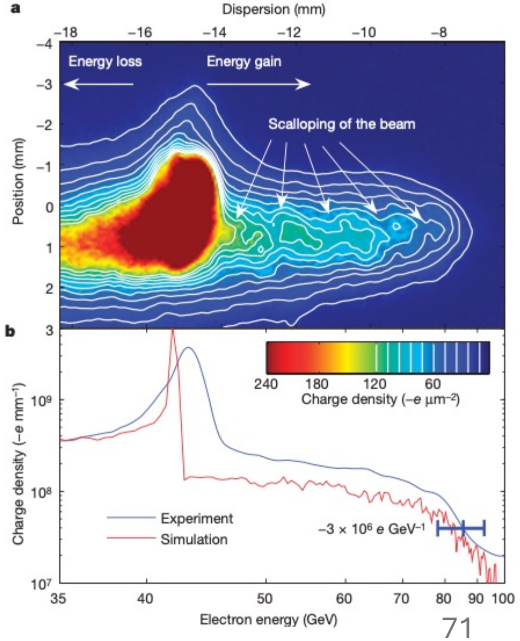




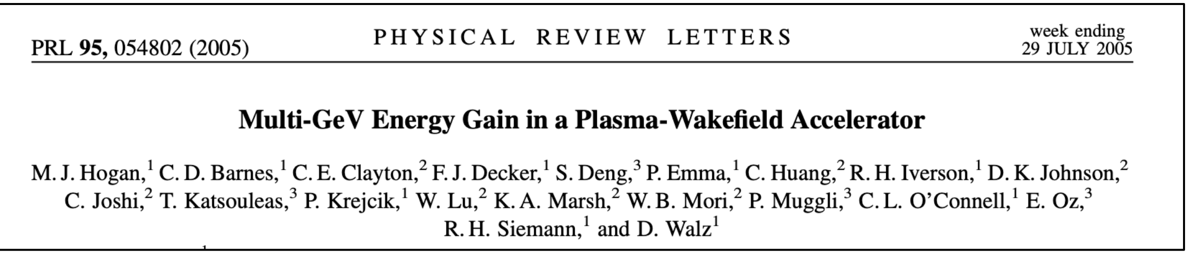
~ 42 GeV in 85 cm

$n_{pe} = 2.8 \times 10^{17}\ cm^{-3}$

BUT: What is ''wrong'' with these results from an accelerator point of view?

II. Non-linear Regime - Results

- After this, almost all experiments worked in the non-linear/bubble/blowout regime



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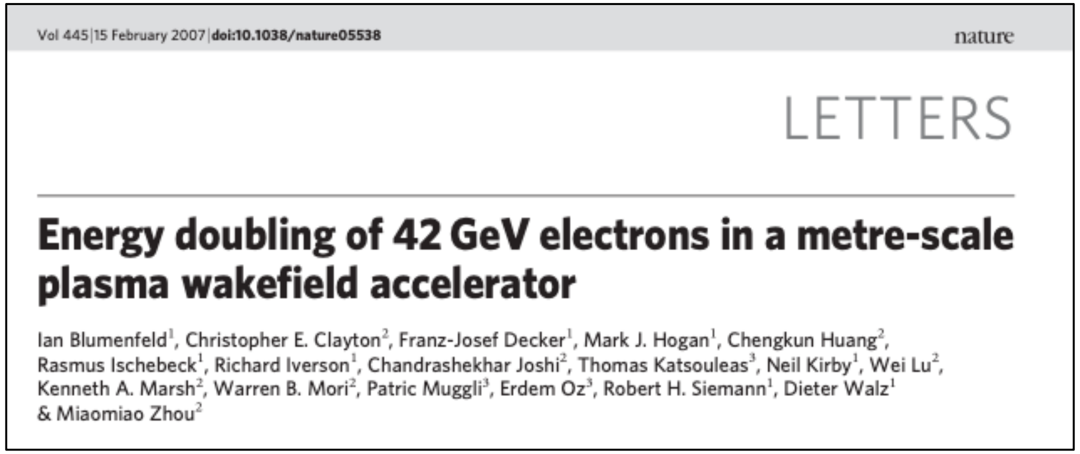

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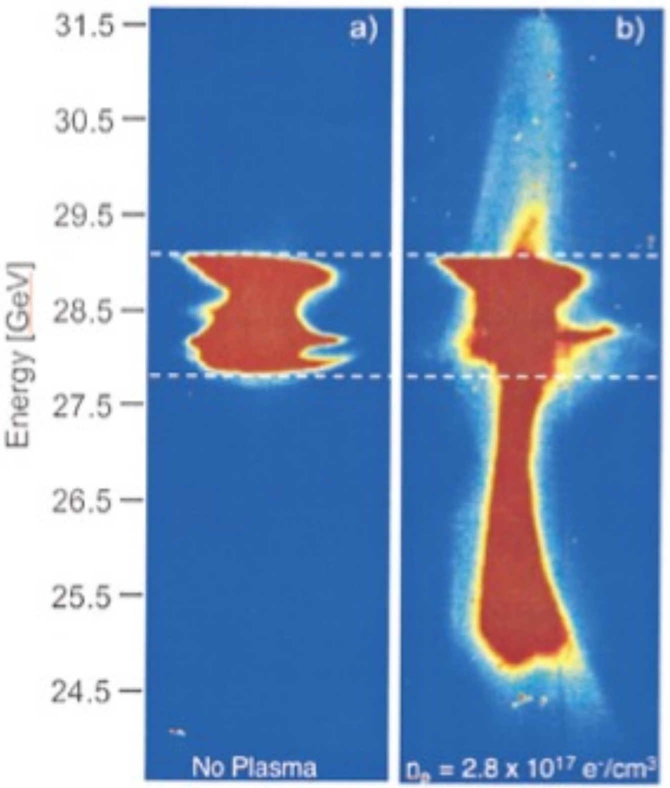
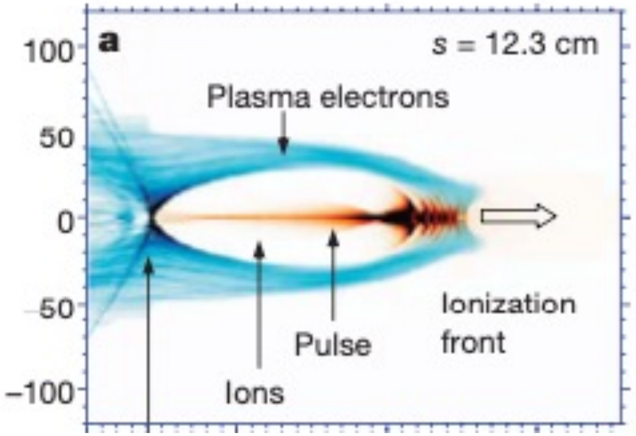
LETTERS

Energy doubling of 42 GeV electrons in a metre-scale plasma wakefield accelerator

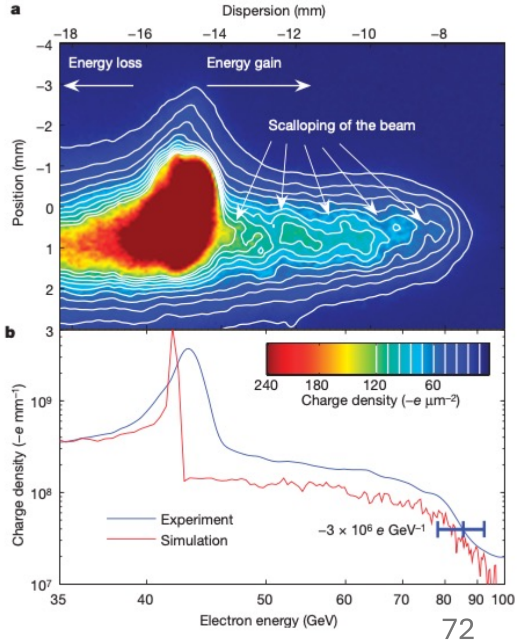
Ian Blumenfeld[1], Christopher E. Clayton[2], Franz-Josef Decker[1], Mark J. Hogan[1], Chengkun Huang[2], Rasmus Ischebeck[1], Richard Iverson[1], Chandrashekhar Joshi[2], Thomas Katsouleas[3], Neil Kirby[1], Wei Lu[2], Kenneth A. Marsh[2], Warren B. Mori[2], Patric Muggli[3], Erdem Oz[3], Robert H. Siemann[1], Dieter Walz[1] & Miaomiao Zhou[2]



~ 2.7 GeV in 10 cm
n_{pe} = 2.8 x 10^{17} cm^{-3}



~ 42 GeV in 85 cm
n_{pe} = 2.8 x 10^{17} cm^{-3}



BUT: What is ''wrong'' with these results from an accelerator point of view?
''Only'' acceleration of electrons, not bunches

II. Non-linear Regime - Results

- After this, almost all experiments worked in the non-linear/bubble/blowout regime
  → transition to bunch acceleration
  → not only quantity (energy) but also quality (charge, energy spread, emittance)

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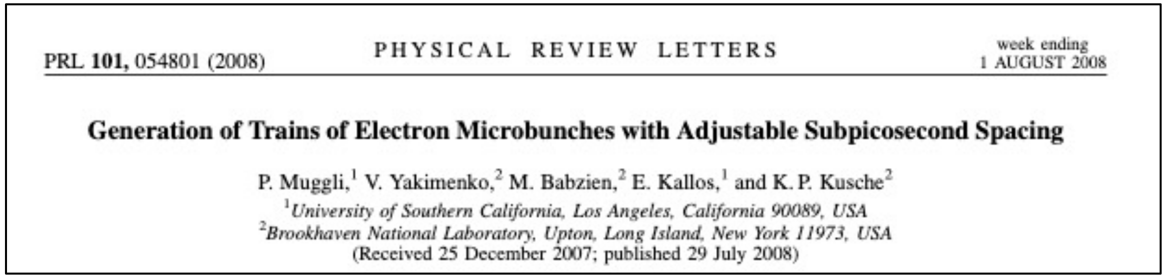
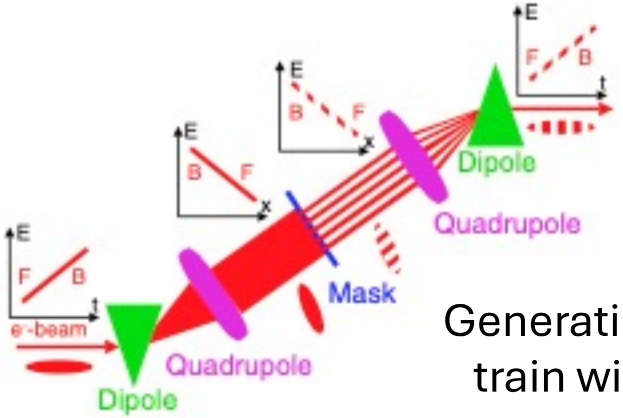


PRL **101,** 054801 (2008) PHYSICAL REVIEW LETTERS week ending 1 AUGUST 2008

Generation of Trains of Electron Microbunches with Adjustable Subpicosecond Spacing

P. Muggli,[1] V. Yakimenko,[2] M. Babzien,[2] E. Kallos,[1] and K. P. Kusche[2]
[1]*University of Southern California, Los Angeles, California 90089, USA*
[2]*Brookhaven National Laboratory, Upton, Long Island, New York 11973, USA*
(Received 25 December 2007; published 29 July 2008)



Generation of bunch train with mask in dispersive section (and chirped bunch)

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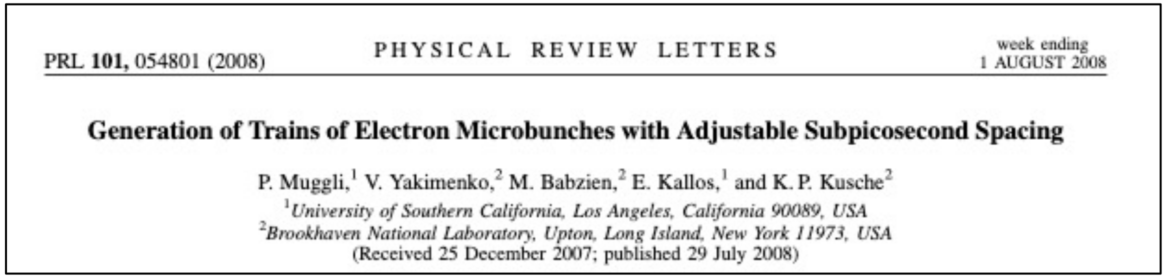

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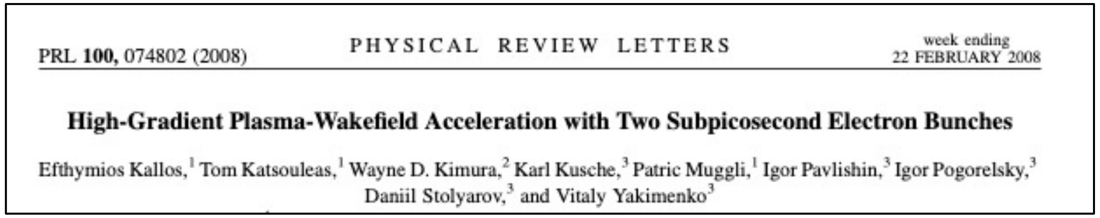
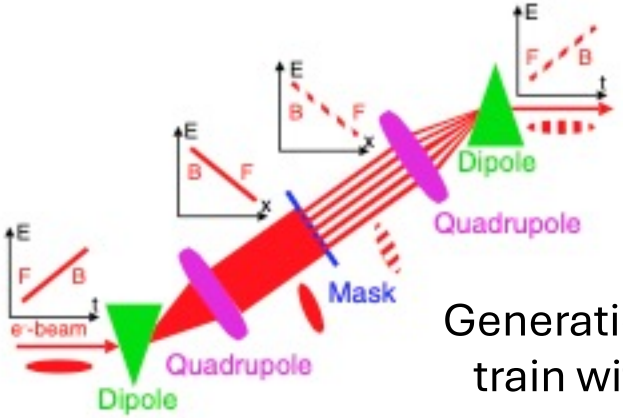


PRL **100,** 074802 (2008) PHYSICAL REVIEW LETTERS week ending 22 FEBRUARY 2008

High-Gradient Plasma-Wakefield Acceleration with Two Subpicosecond Electron Bunches

Efthymios Kallos,[1] Tom Katsouleas,[1] Wayne D. Kimura,[2] Karl Kusche,[3] Patric Muggli,[1] Igor Pavlishin,[3] Igor Pogorelsky,[3] Daniil Stolyarov,[3] and Vitaly Yakimenko[3]

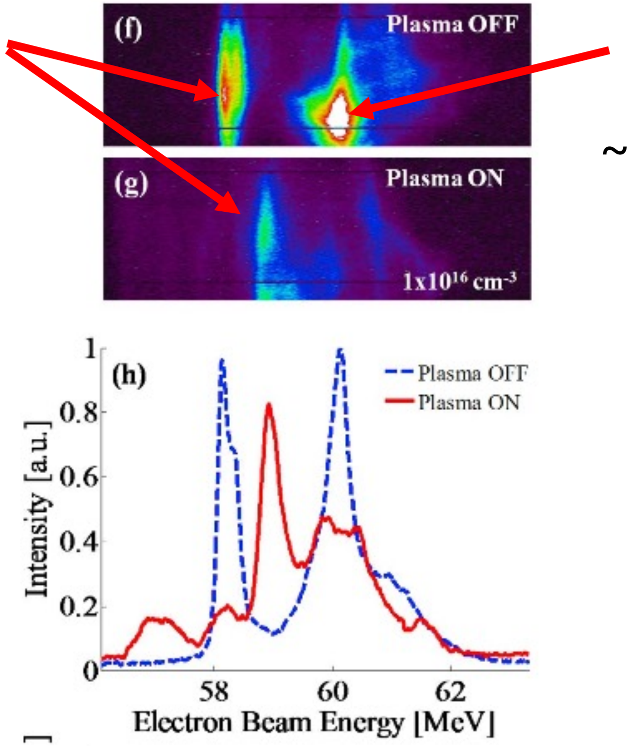


Generation of bunch train with mask in dispersive section (and chirped bunch)

then

WITNESS

DRIVER



~ 0.9 MeV in 6 mm

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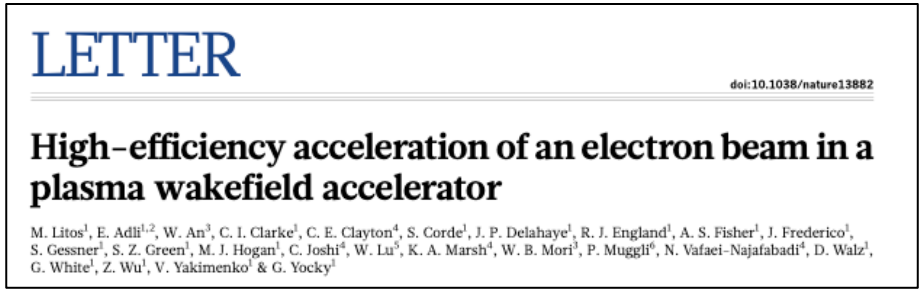
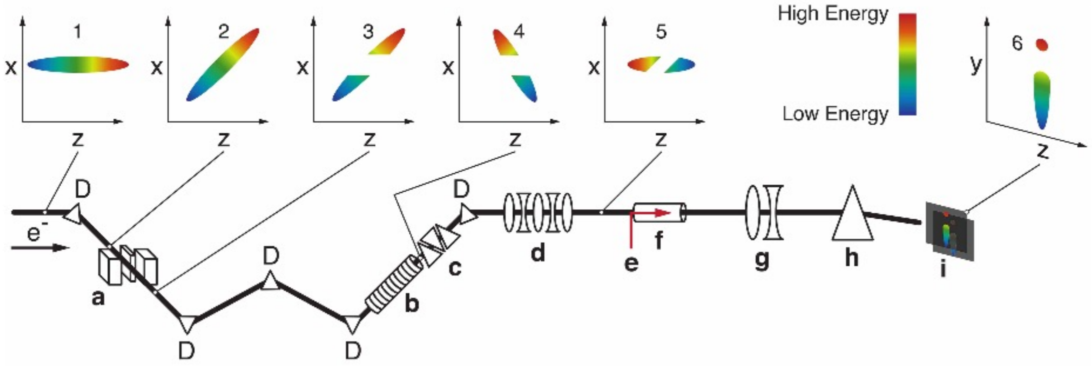


LETTER

doi:10.1038/nature13882

High-efficiency acceleration of an electron beam in a plasma wakefield accelerator

M. Litos[1], E. Adli[1,2], W. An[3], C. I. Clarke[1], C. E. Clayton[4], S. Corde[1], J. P. Delahaye[1], R. J. England[1], A. S. Fisher[1], J. Frederico[1], S. Gessner[1], S. Z. Green[1], M. J. Hogan[1], C. Joshi[4], W. Lu[5], K. A. Marsh[4], W. B. Mori[3], P. Muggli[6], N. Vafaei-Najafabadi[4], D. Walz[1], G. White[1], Z. Wu[1], V. Yakimenko[1] & G. Yocky[1]



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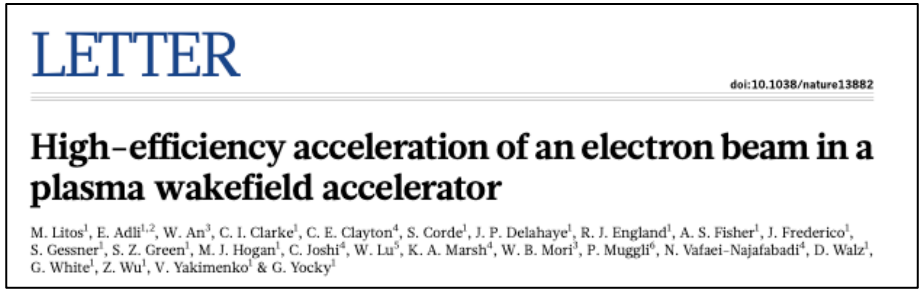

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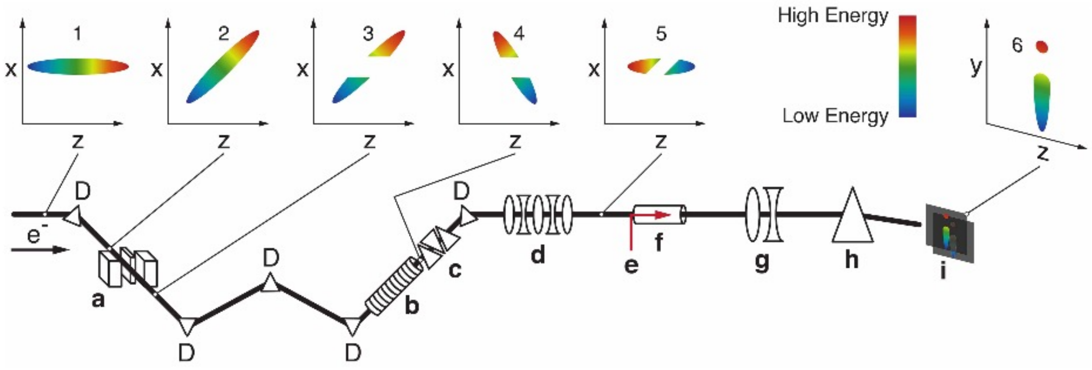

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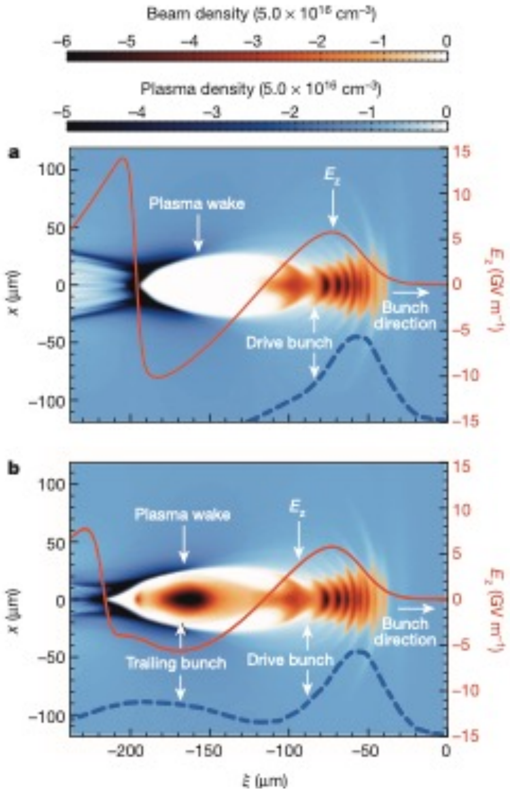

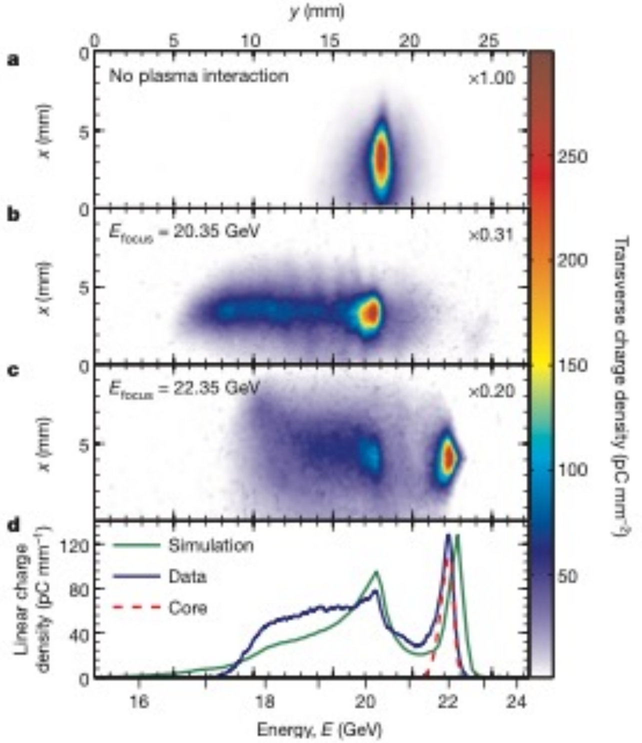

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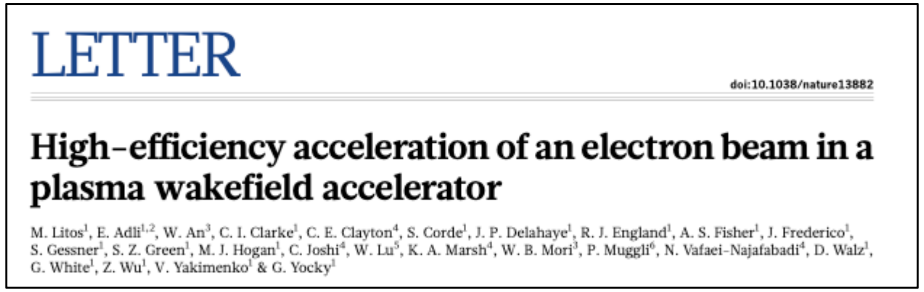

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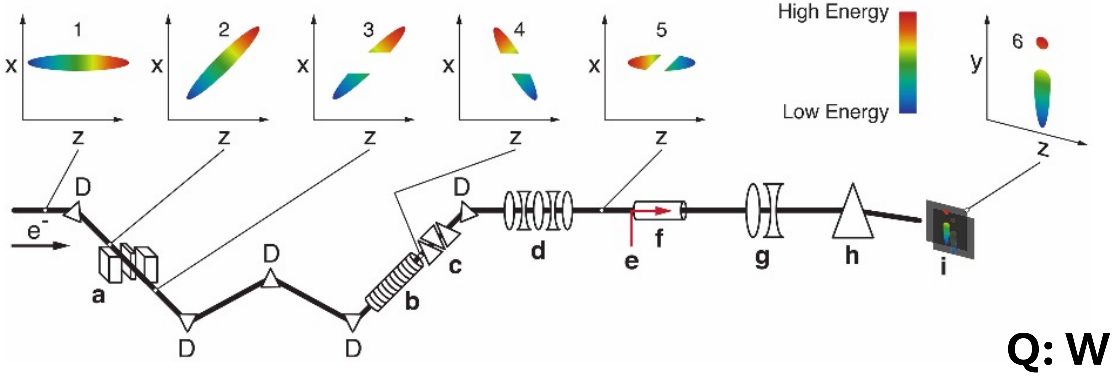

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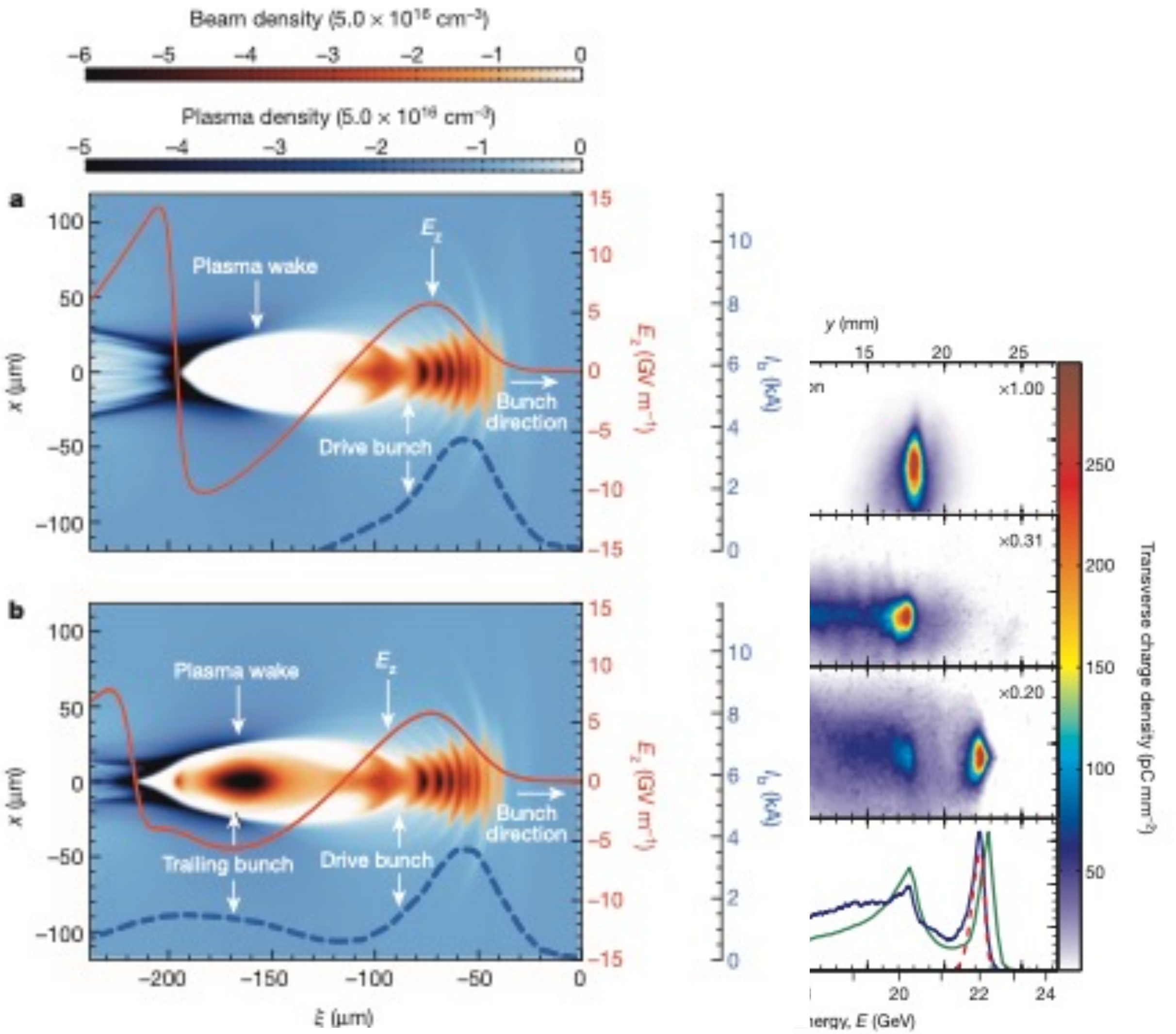
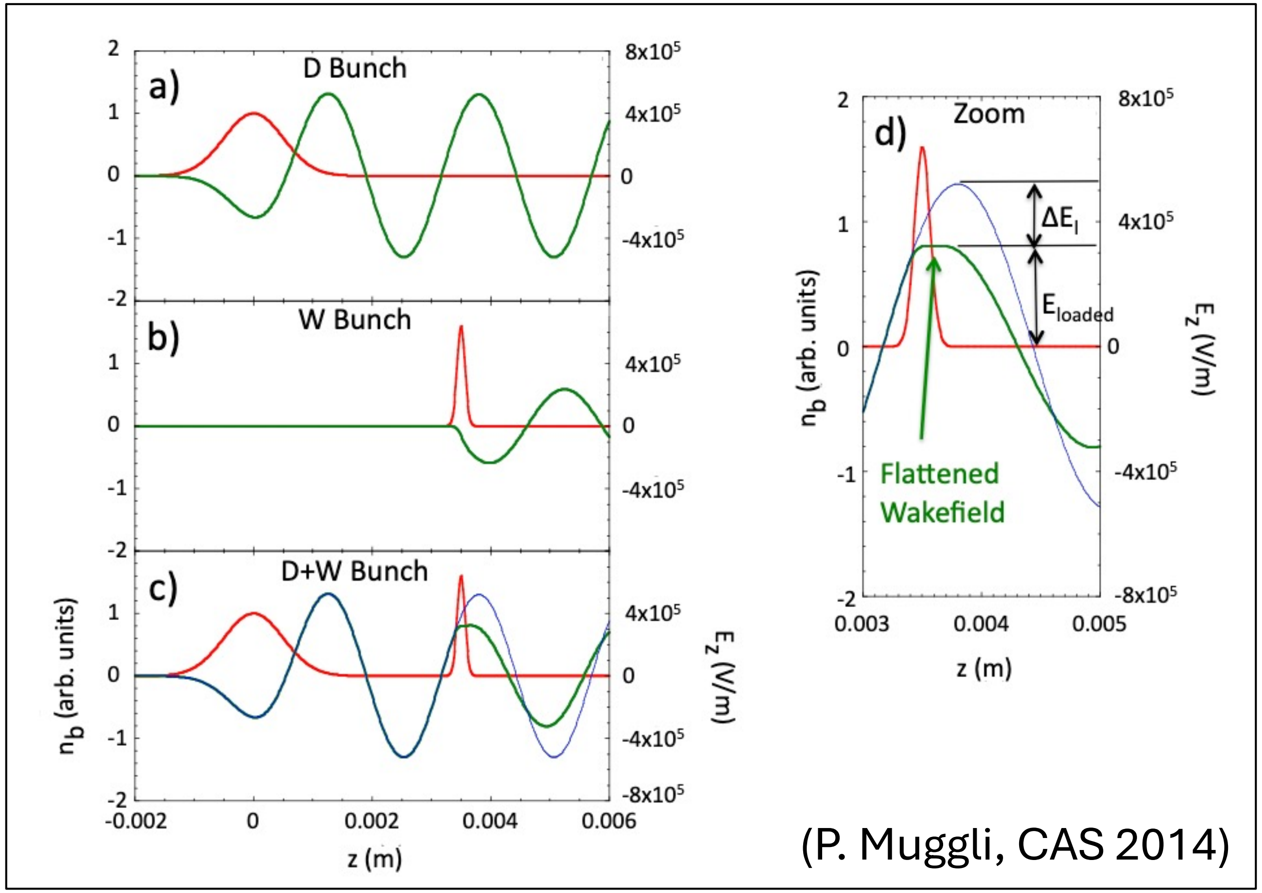


**Q: WHY IS THE ACCELERATING FIELD DIFFERENT WITH AND WITHOUT WITNESS BUNCH?
IS IT GOOD OR BAD?**

II. Non-linear Regime – Beam Loading

➔ BEAM LOADING:
The presence of the witness bunch affects the wakefields

Linear regime



(P. Muggli, CAS 2014)

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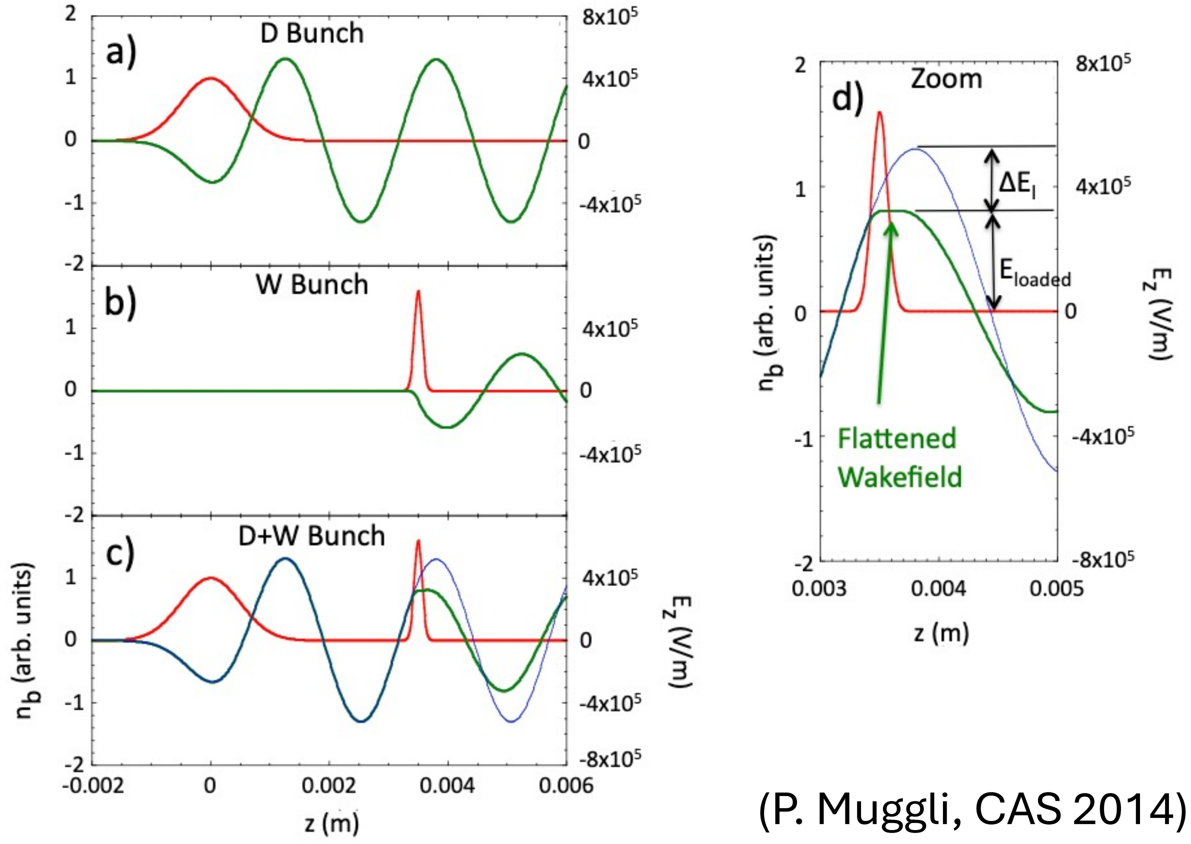

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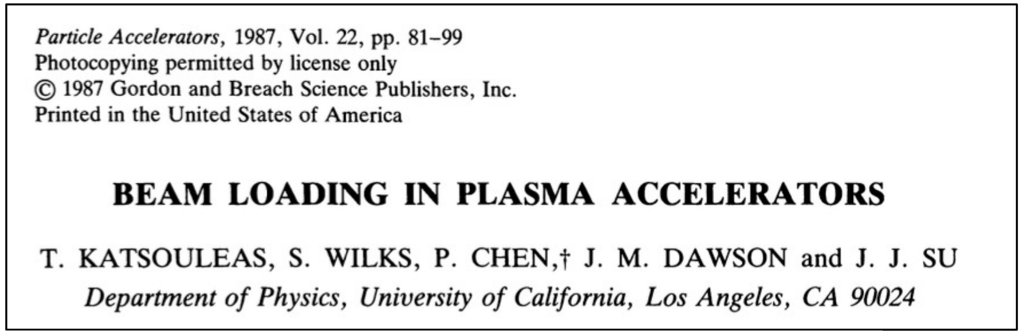

Particle Accelerators, 1987, Vol. 22, pp. 81–99
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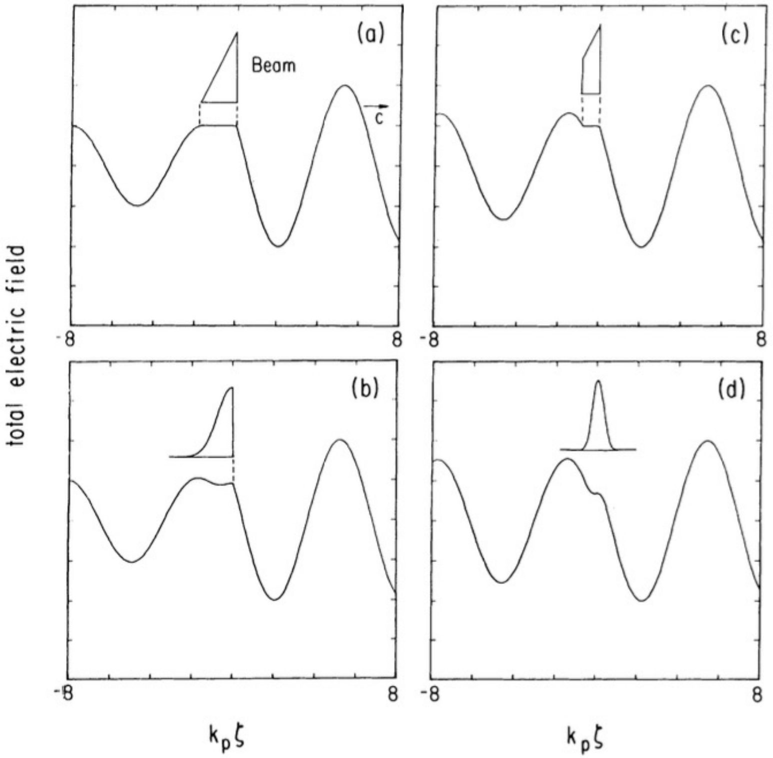
BEAM LOADING IN PLASMA ACCELERATORS

T. KATSOULEAS, S. WILKS, P. CHEN,† J. M. DAWSON and J. J. SU
Department of Physics, University of California, Los Angeles, CA 90024



- Triangular/trapezoidal shape gives constant field
- Short Gaussian placed at the right phase can work

II. Non-linear Regime – Beam Loading

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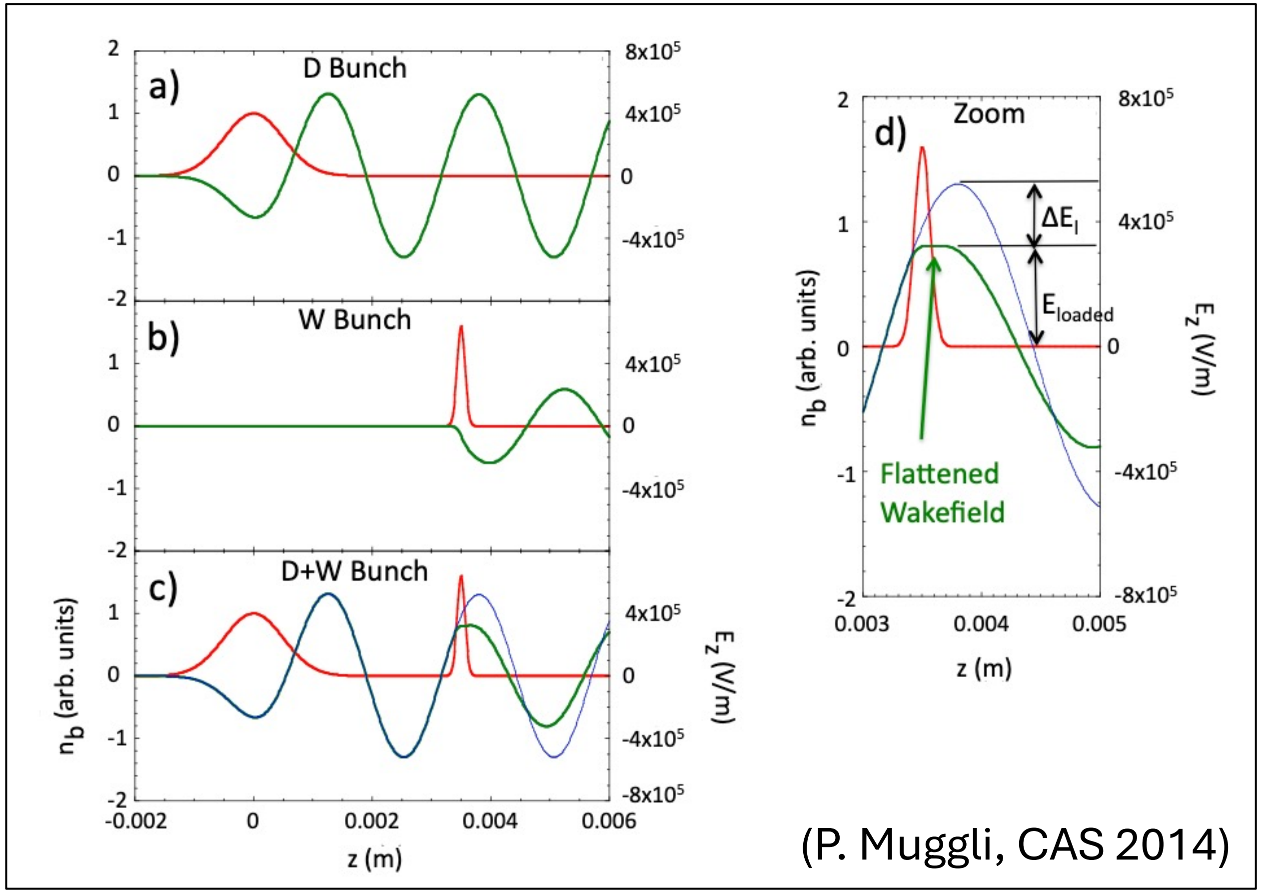
The presence of the witness bunch affects the wakefields

Linear regime



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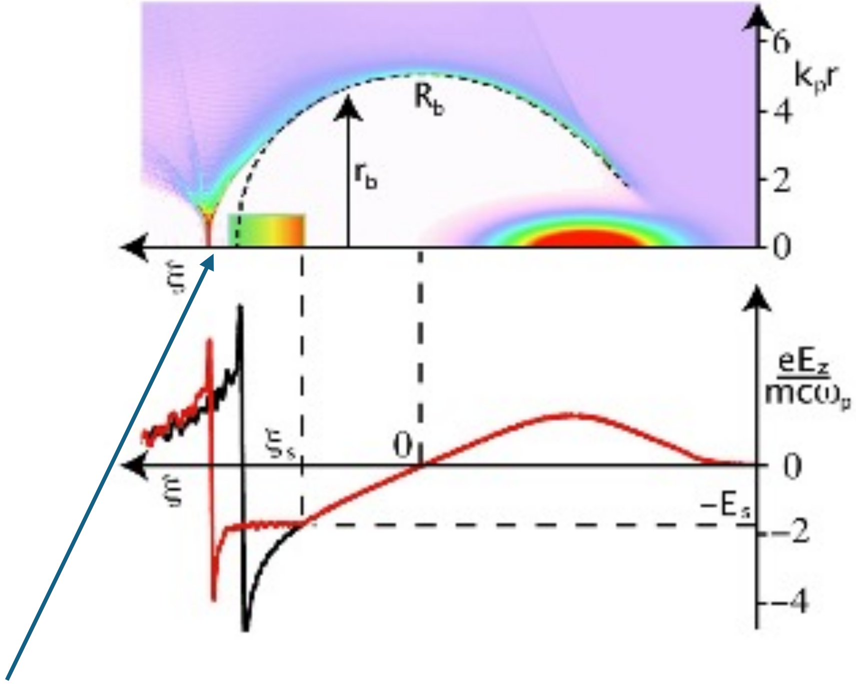
Non-linear regime

PRL **101**, 145002 (2008) PHYSICAL REVIEW LETTERS week ending 3 OCTOBER 2008

Beam Loading in the Nonlinear Regime of Plasma-Based Acceleration

M. Tzoufras,[1] W. Lu,[1] F. S. Tsung,[2] C. Huang,[2] W. B. Mori,[1,2] T. Katsouleas,[3] J. Vieira,[4] R. A. Fonseca,[4] and L. O. Silva[4]



The bubble closes later

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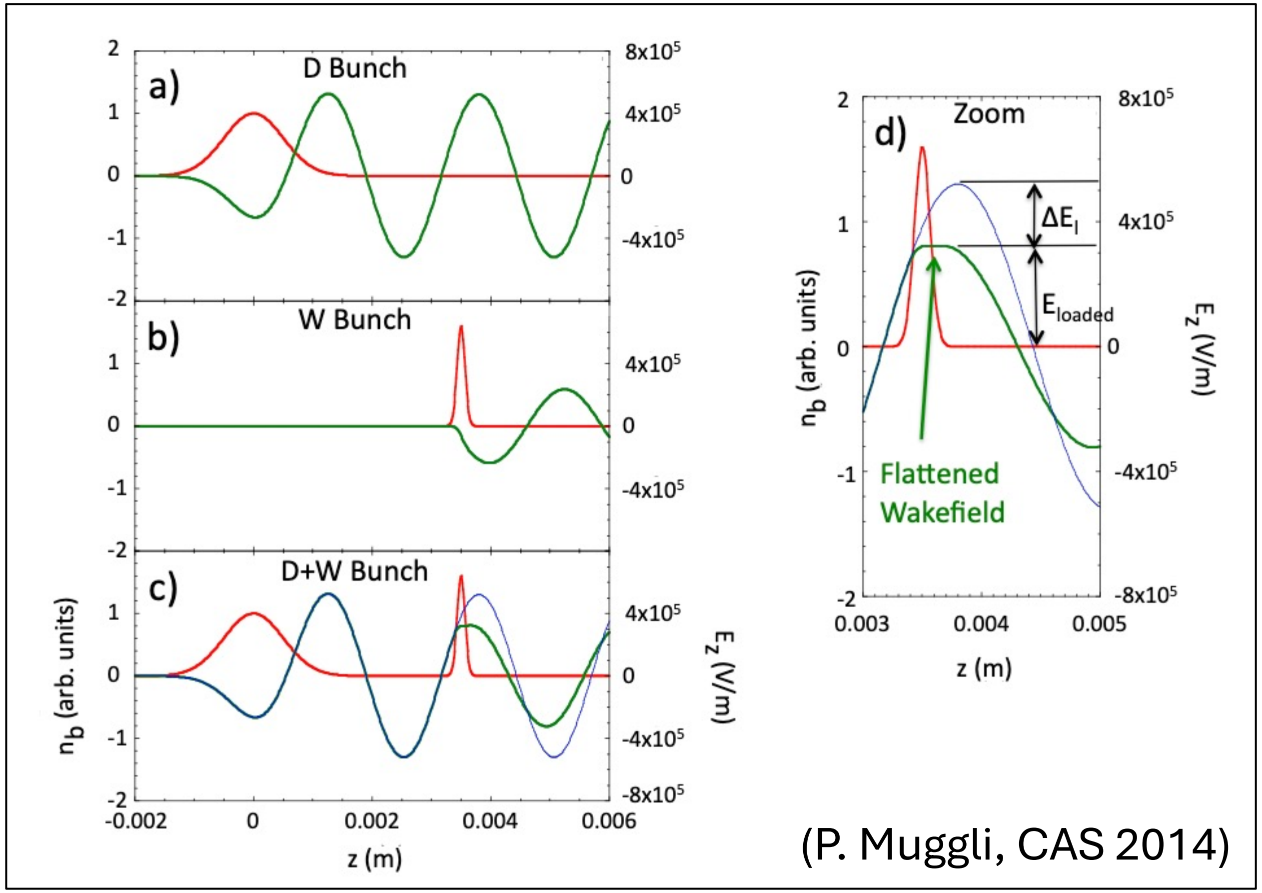

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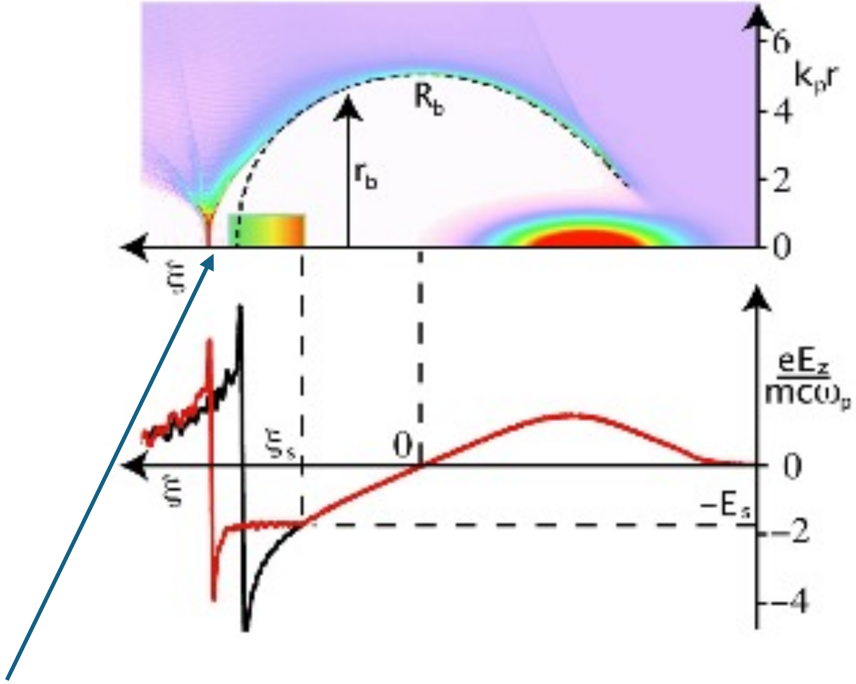
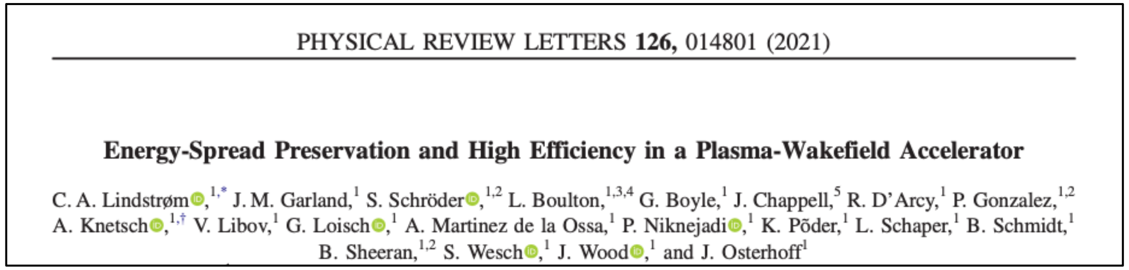


The bubble closes later

The point is: compromise on accelerating gradient ➔ smaller energy spread

II. Non-linear Regime – Beam Loading

➔Experimental Demonstration:



PHYSICAL REVIEW LETTERS **126,** 014801 (2021)

Energy-Spread Preservation and High Efficiency in a Plasma-Wakefield Accelerator

C. A. Lindstrøm,[1,*] J. M. Garland,[1] S. Schröder,[1,2] L. Boulton,[1,3,4] G. Boyle,[1] J. Chappell,[5] R. D'Arcy,[1] P. Gonzalez,[1,2] A. Knetsch,[1,†] V. Libov,[1] G. Loisch,[1] A. Martinez de la Ossa,[1] P. Niknejadi,[1] K. Põder,[1] L. Schaper,[1] B. Schmidt,[1] B. Sheeran,[1,2] S. Wesch,[1] J. Wood,[1] and J. Osterhoff[1]

nature physics

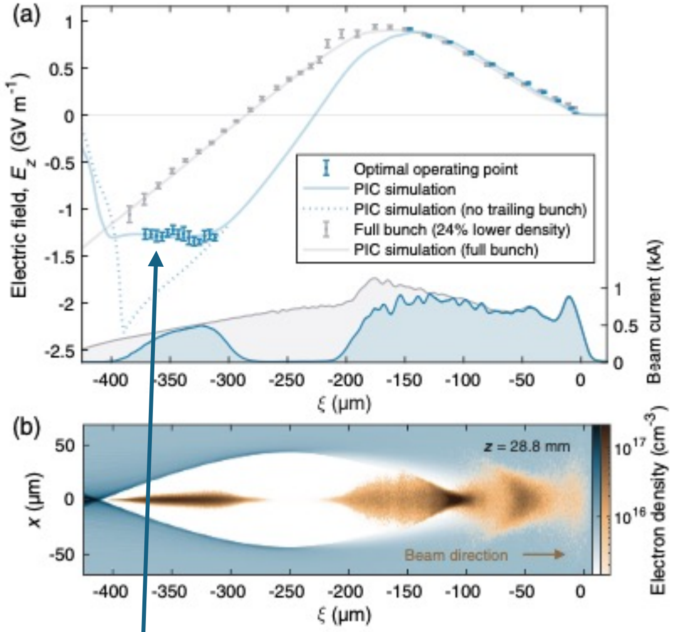
LETTERS

https://doi.org/10.1038/s41567-020-01116-9

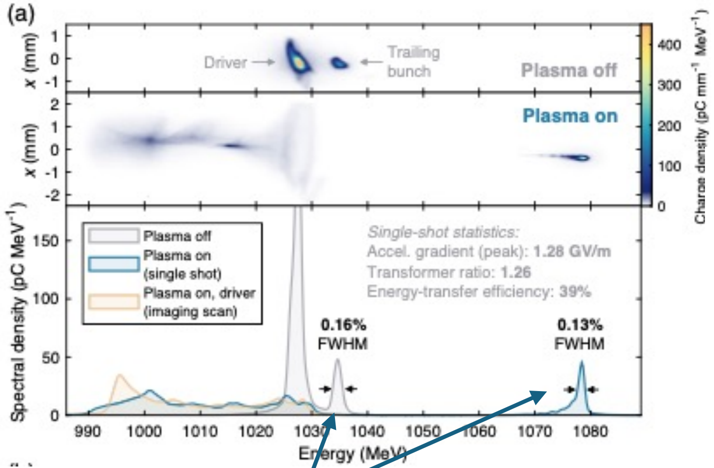
Check for updates

Energy spread minimization in a beam-driven plasma wakefield accelerator

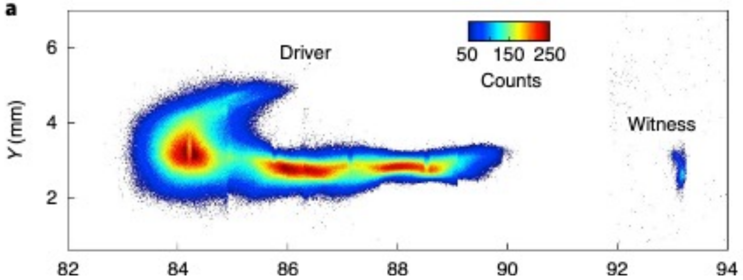
R. Pompili[1✉], D. Alesini[1], M. P. Anania[1], M. Behtouei[1], M. Bellaveglia[1], A. Biagioni[1], F. G. Bisesto[1], M. Cesarini[1,2], E. Chiadroni[1], A. Cianchi[3], G. Costa[1], M. Croia[1], A. Del Dotto[1], D. Di Giovenale[1], M. Diomede[1], F. Dipace[1], M. Ferrario[1], A. Giribono[1], V. Lollo[1], L. Magnisi[1], M. Marongiu[1], A. Mostacci[2], L. Piersanti[1], G. Di Pirro[1], S. Romeo[1], A. R. Rossi[4], J. Scifo[1], V. Shpakov[1], C. Vaccarezza[1], F. Villa[1] and A. Zigler[1,5]

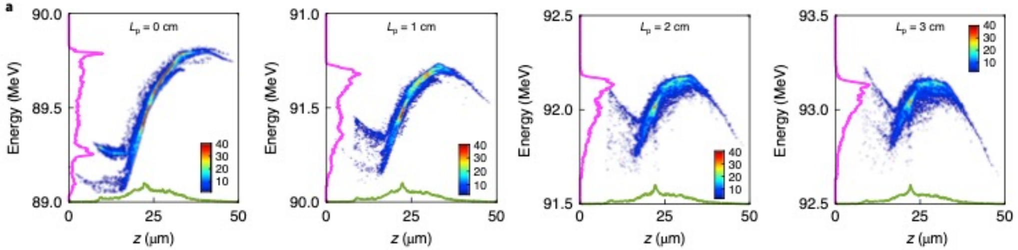


Flattened, loaded wake



~GV/m acceleration
Preserved relative energy spread

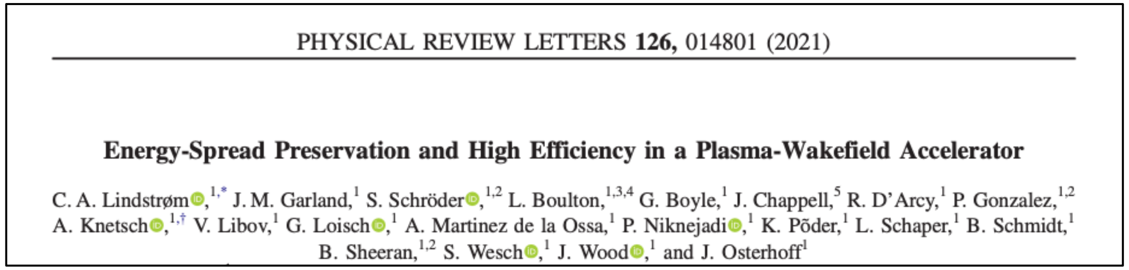




Combination of beam loading and initial chirp to obtain final small energy spread

II. Non-linear Regime – Beam Loading

➔Experimental Demonstration:



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nature physics

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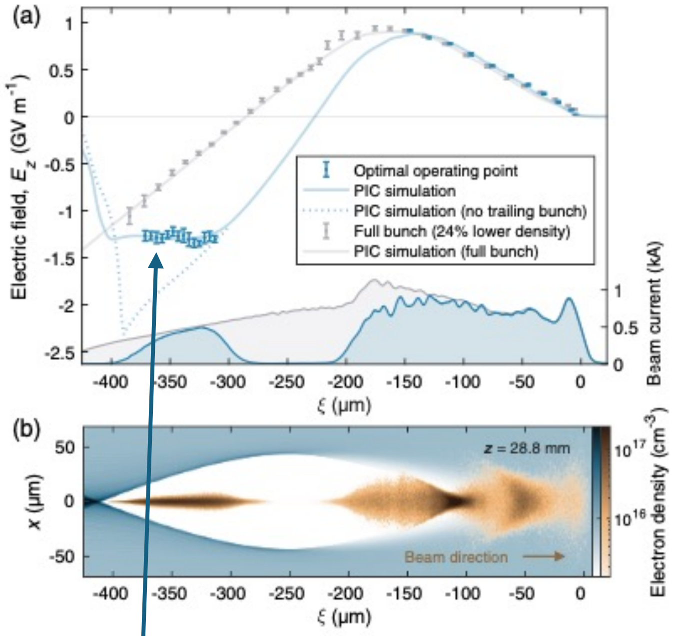
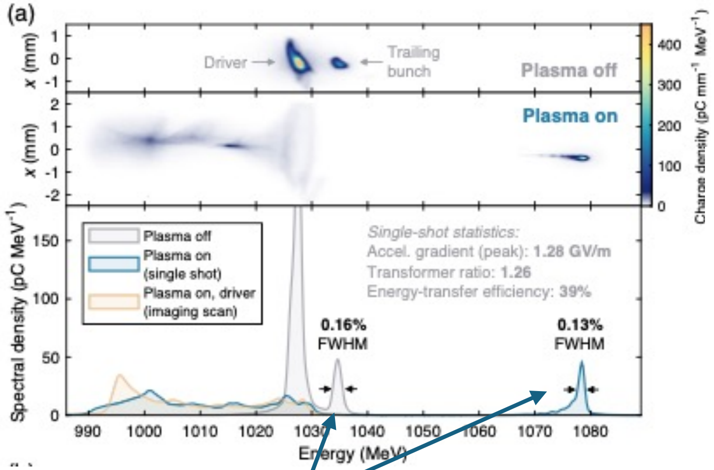
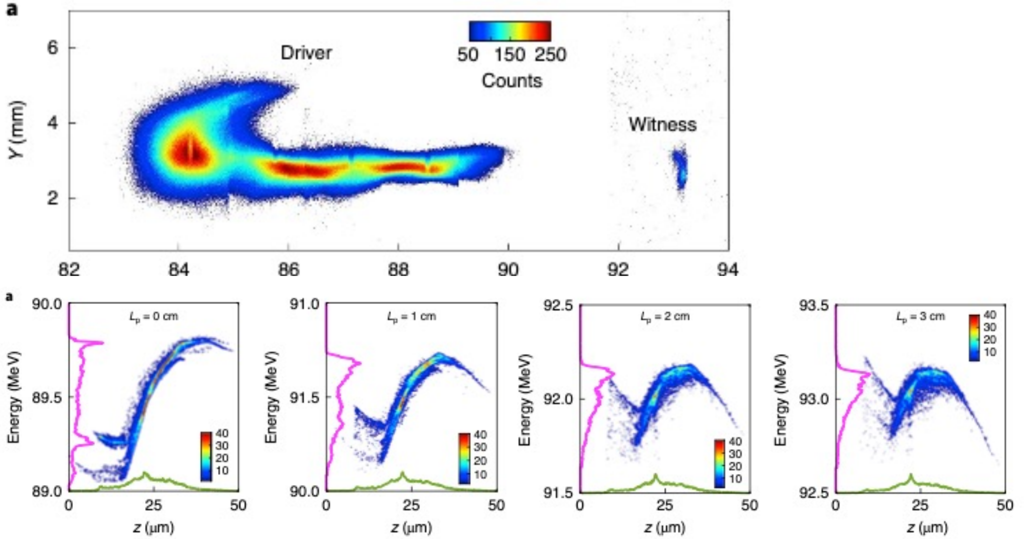




~GV/m acceleration
Preserved relative energy spread



Flattened, loaded wake

Transformer ratio $R = \frac{E_+}{E_-}$ still limited to 2

→ Non-trivial solutions: active reasearch topic

Combination of beam loading and initial chirp to obtain final small energy spread

II. Non-linear Regime – Focusing

➔Ion column provides linear focusing force

Radial electric field: $E_r(r) = \frac{en_{pe}}{2\varepsilon_0}\, r$ (Gauss’ law on cylinder of ions)

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→ Plug it in envelope equation: $\sigma_r''(z) + \sigma_r(z) \underbrace{\left(K - \frac{\epsilon_g^2}{\sigma_r^4(z)}\right)}_{=0:} = 0$

Matching condition ⟶ Equilibrium between focusing force and emittance

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Matching condition $\longrightarrow$ Equilibrium between focusing force and emittance

- $\beta = \frac{\sigma^2(0)}{\epsilon_g} = \sqrt{\frac{2\varepsilon_0 m_e c^2 \gamma}{n_{pe} e^2}}$
- Injection at waist: $\sigma'(z = 0) = 0$

Note: the matching condition is given by β!
→ Stronger focusing required for higher density

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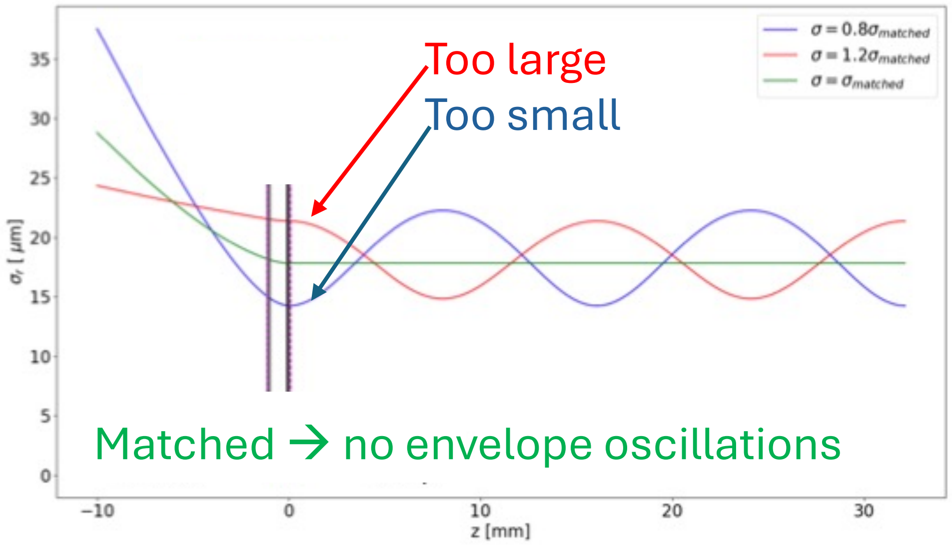


Too large

Too small

Matched → no envelope oscillations

(L. Verra *et al* 2020 *J. Phys.: Conf. Ser.* **1596** 012007)

II. Non-linear Regime – Focusing

➔Ion column provides linear focusing force

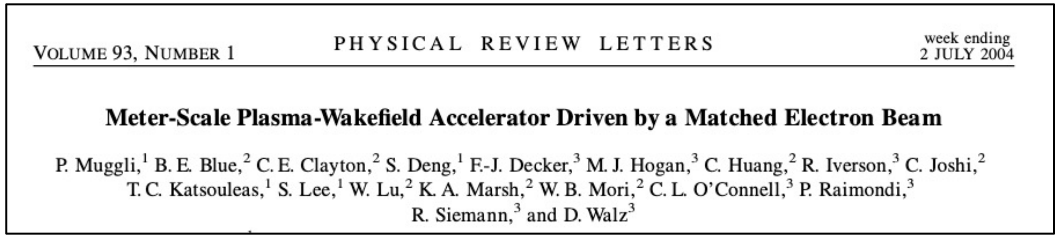
Radial electric field: $E_r(r) = \frac{en_{pe}}{2\varepsilon_0}\, r$ (Gauss' law on cylinder of ions)

→ Plug it in envelope equation: $\sigma_r''(z) + \sigma_r(z)\underbrace{\left(K - \frac{\epsilon_g^2}{\sigma_r^4(z)}\right)}_{=0:} = 0$

Matching condition ⟶ Equilibrium between focusing force and emittance

- $\beta = \frac{\sigma^2(0)}{\epsilon_g} = \sqrt{\frac{2\varepsilon_0 m_e c^2\gamma}{n_{pe}e^2}}$
- Injection at waist: $\sigma'(z=0) = 0$

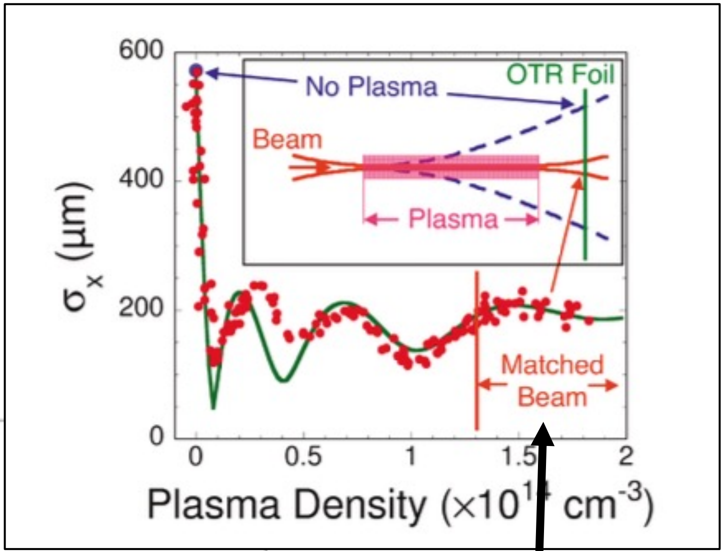
Note: the matching condition is given by β!
→ Stronger focusing required for higher density

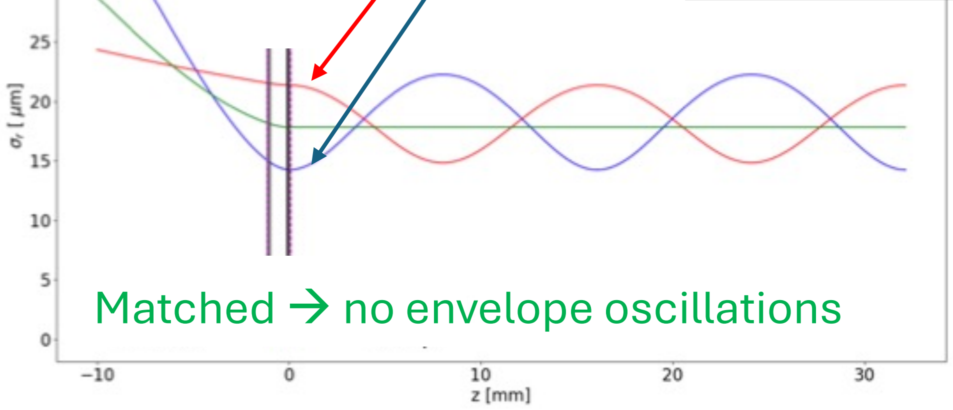


VOLUME 93, NUMBER 1 PHYSICAL REVIEW LETTERS week ending 2 JULY 2004

Meter-Scale Plasma-Wakefield Accelerator Driven by a Matched Electron Beam

P. Muggli,[1] B. E. Blue,[2] C. E. Clayton,[2] S. Deng,[1] F.-J. Decker,[3] M. J. Hogan,[3] C. Huang,[2] R. Iverson,[3] C. Joshi,[2] T. C. Katsouleas,[1] S. Lee,[1] W. Lu,[2] K. A. Marsh,[2] W. B. Mori,[2] C. L. O'Connell,[3] P. Raimondi,[3] R. Siemann,[3] and D. Walz[3]





Too large
Too small

Matched → no envelope oscillations

(L. Verra *et al* 2020 *J. Phys.: Conf. Ser.* **1596** 012007)

Beam leaves the plasma at waist
→ matched
➔constant size

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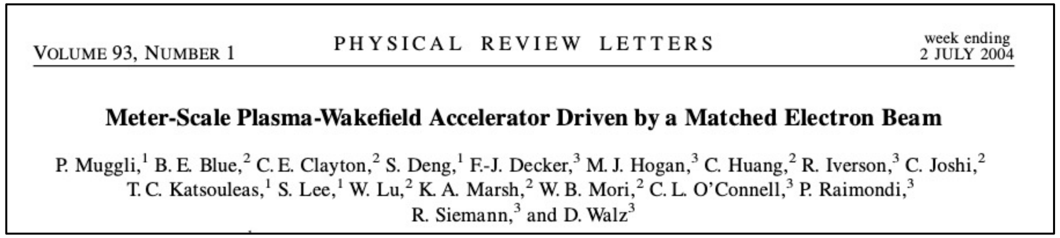
Matching condition →

Equilibrium between focusing force and emittance

Q: WHY IS IMPORTANT TO MATCH THE BEAM ENVELOPE?



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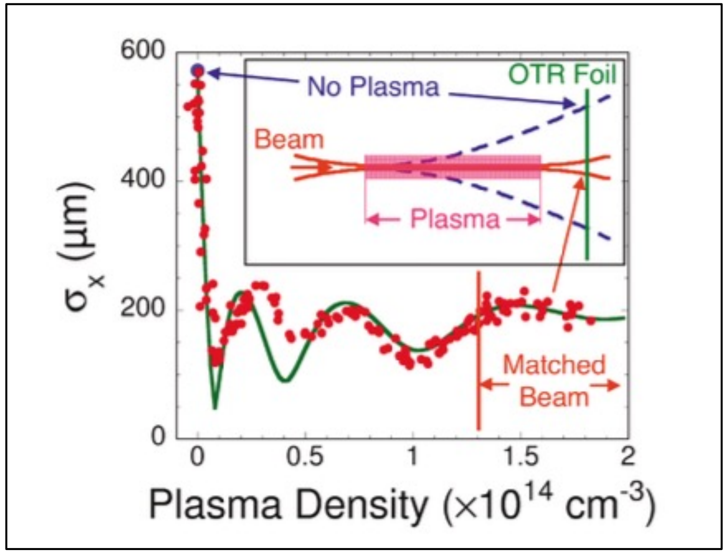

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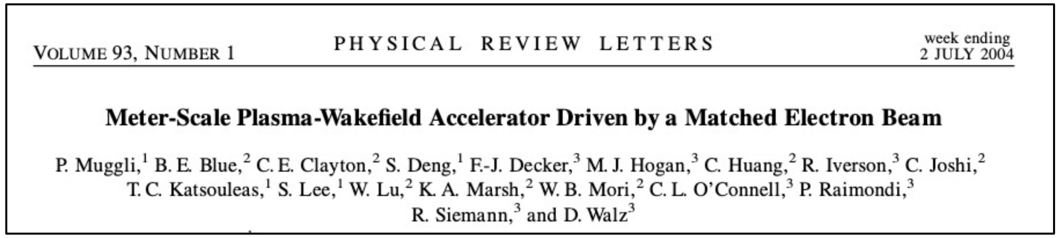

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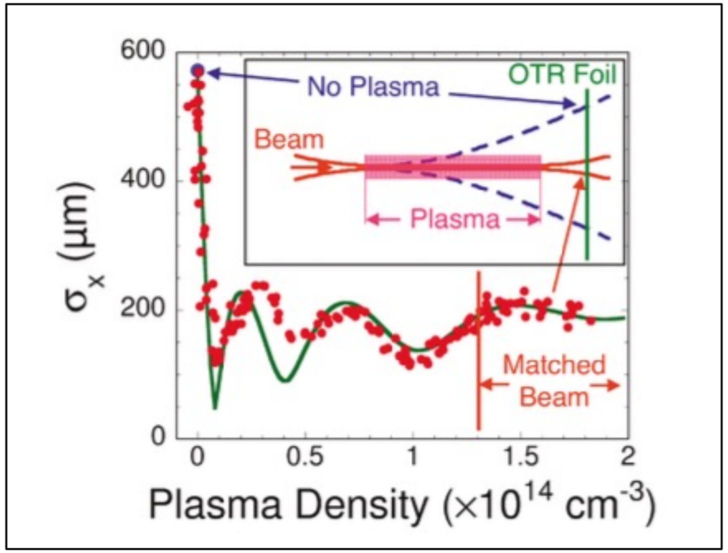

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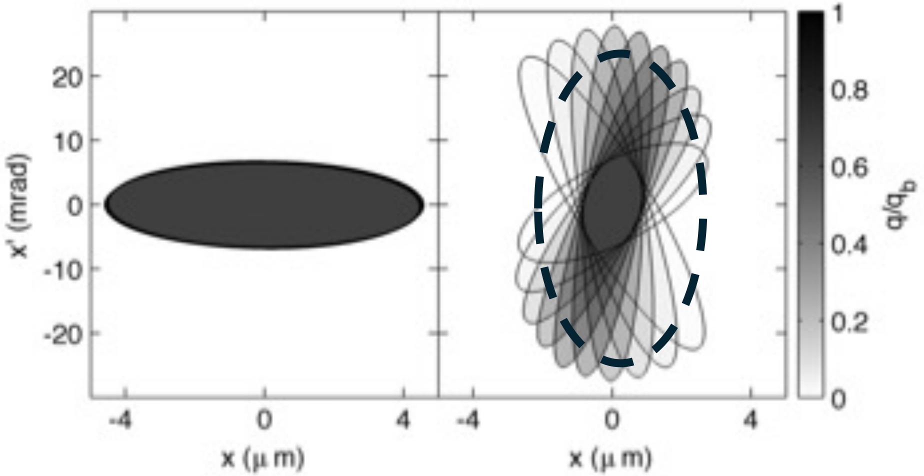


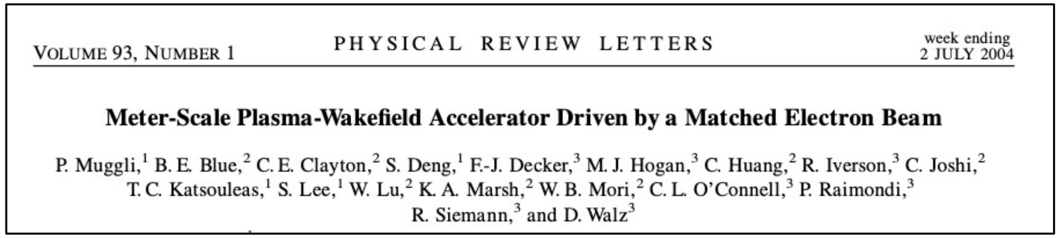
- Finite energy spread:
  → different energy slices rotate at different rates in transverse phase space
  → slice emittance preserved (linear focusing)
  → projected (i.e., overall) emittance grows!

(C.A. Lindstrøm and M. Thévenet 2022 JINST 17 P05016)

II. Non-linear Regime – Focusing

➔Ion column provides linear focusing force

Radial electric field: $E_r(r) = \frac{e n_{pe}}{2\varepsilon_0} r$ (Gauss' law on cylinder of ions)



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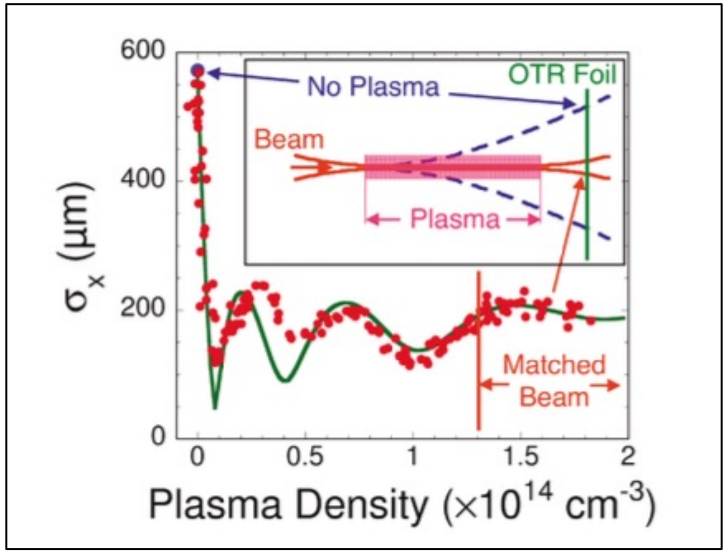

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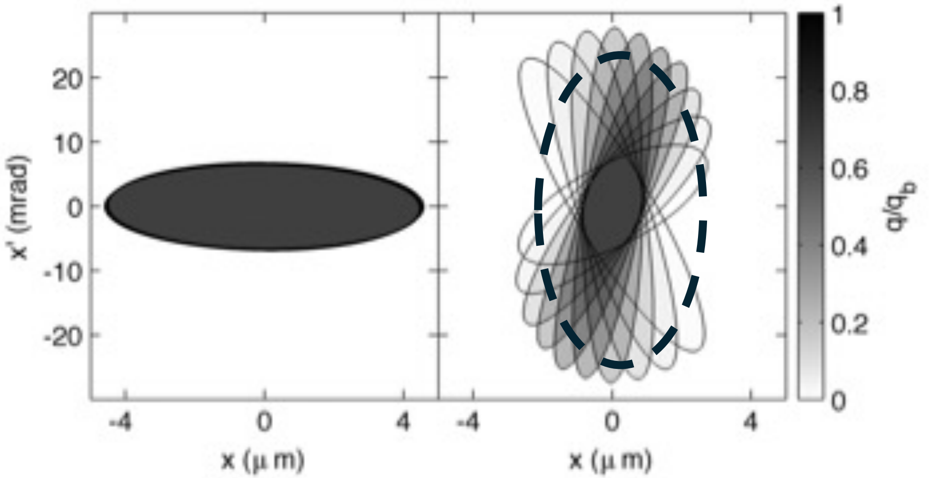


- Finite energy spread:

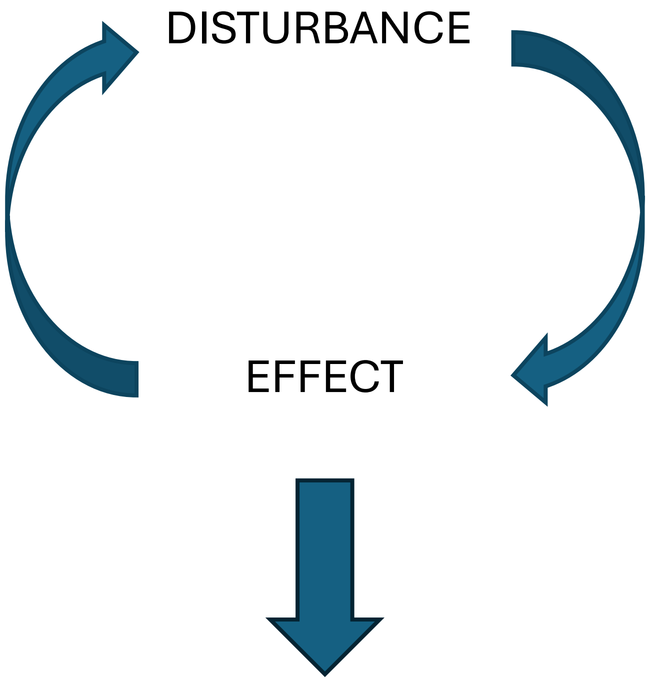
→ ... se phase space

Active Research Topic

(C.A. Lindstrøm and M. Thévenet 2022 JINST 17 P05016)

III. Beam-Plasma Instabilities

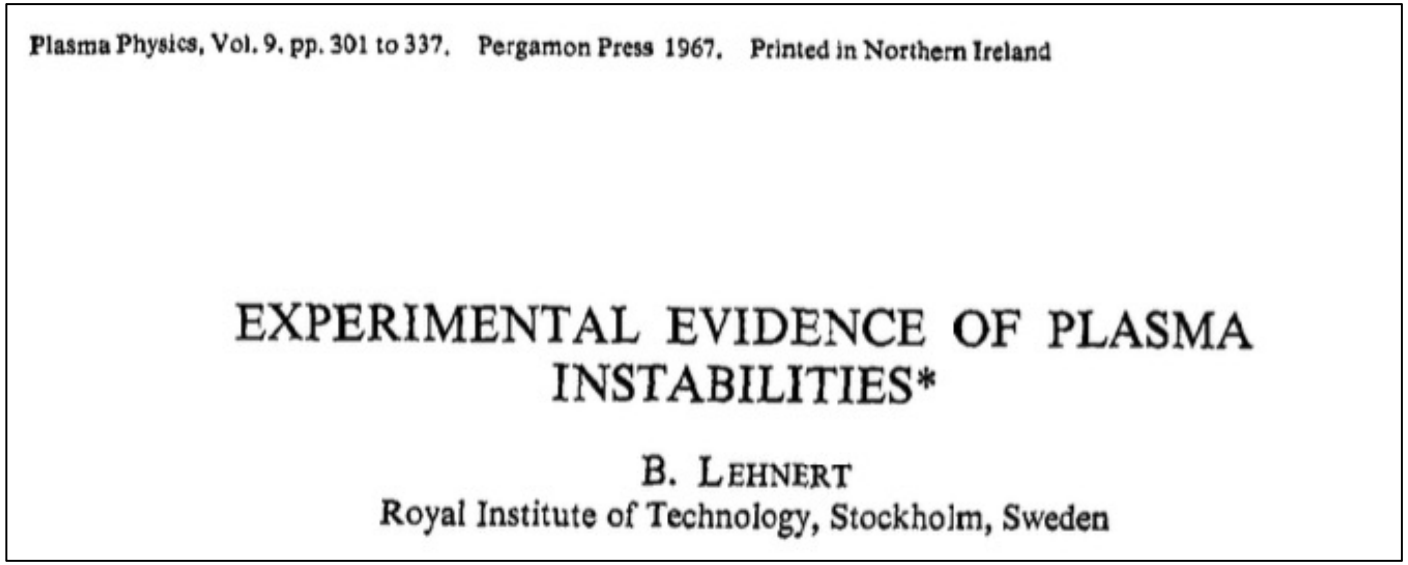
- When the bunch is long and/or wide enough, beam-plasma instabilities may arise
- Need a positive feedback loop for the instability to grow



DISTURBANCE

EFFECT

Until saturation is reached

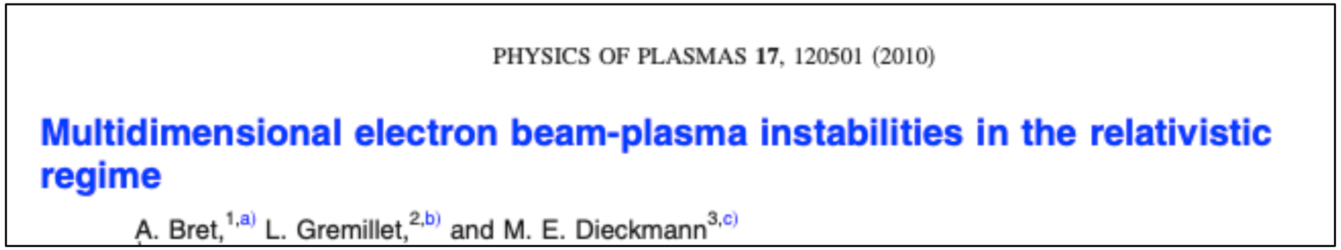


Plasma Physics, Vol. 9, pp. 301 to 337. Pergamon Press 1967. Printed in Northern Ireland

EXPERIMENTAL EVIDENCE OF PLASMA INSTABILITIES*

B. LEHNERT

Royal Institute of Technology, Stockholm, Sweden



PHYSICS OF PLASMAS **17**, 120501 (2010)

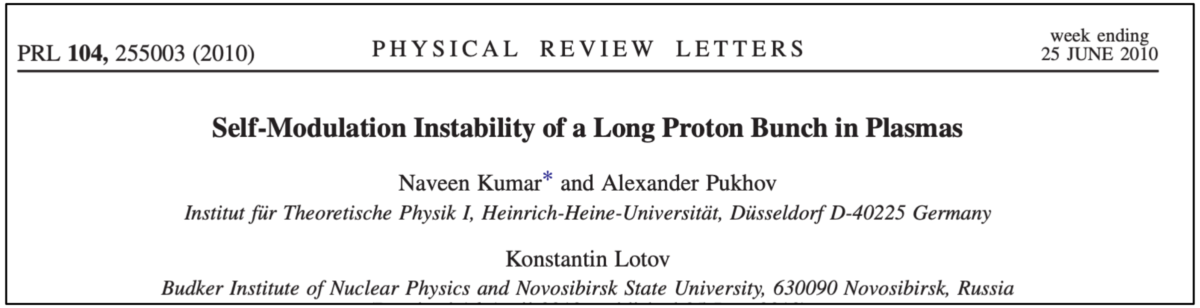
Multidimensional electron beam-plasma instabilities in the relativistic regime

A. Bret,[1,a)] L. Gremillet,[2,b)] and M. E. Dieckmann[3,c)]

III. Beam-Plasma Instabilities – SMI

- Long bunch case: $\sigma_z \gg \lambda_{pe}$

→ Wakefields act on the bunch itself



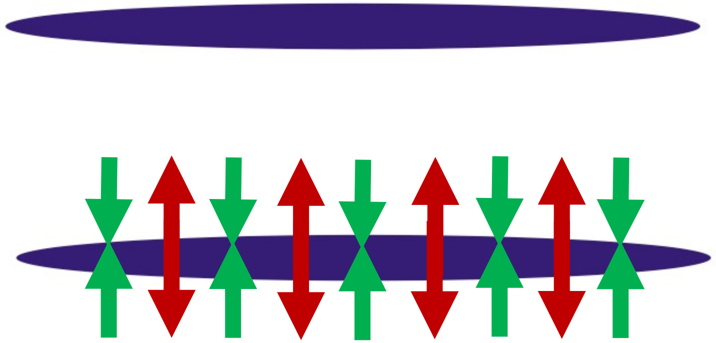
PRL **104,** 255003 (2010) PHYSICAL REVIEW LETTERS week ending 25 JUNE 2010

Self-Modulation Instability of a Long Proton Bunch in Plasmas

Naveen Kumar* and Alexander Pukhov
Institut für Theoretische Physik I, Heinrich-Heine-Universität, Düsseldorf D-40225 Germany

Konstantin Lotov
Budker Institute of Nuclear Physics and Novosibirsk State University, 630090 Novosibirsk, Russia

Growth mechanism



Initial transverse wakefields

Periodic focusing/defocusing fields

Radial bunch and plasma density modulation

Stronger wakefields

Full modulation

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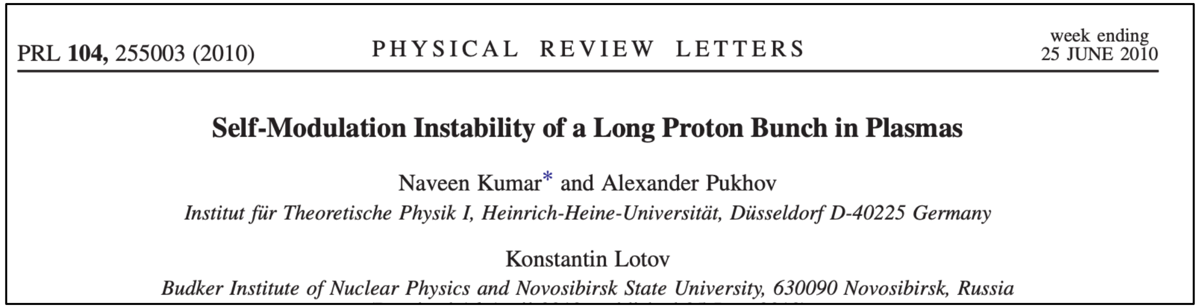

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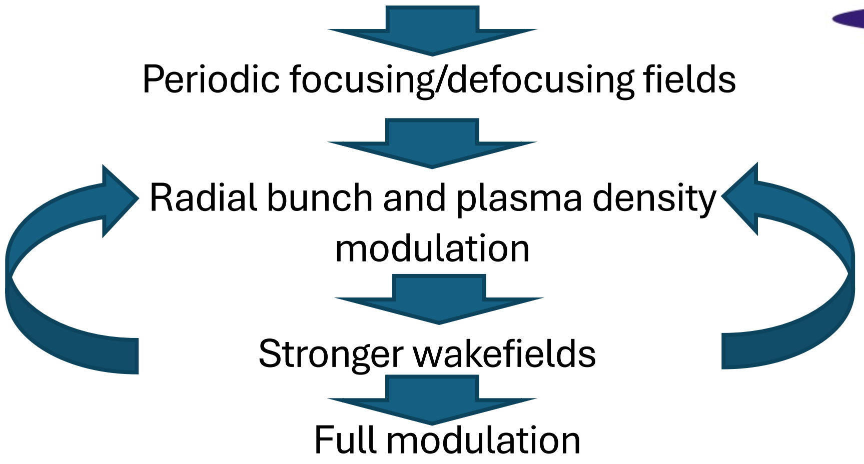

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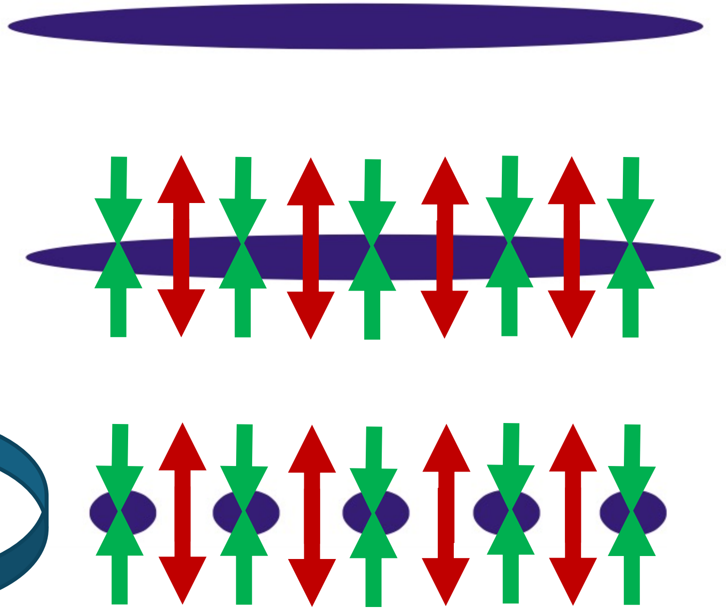

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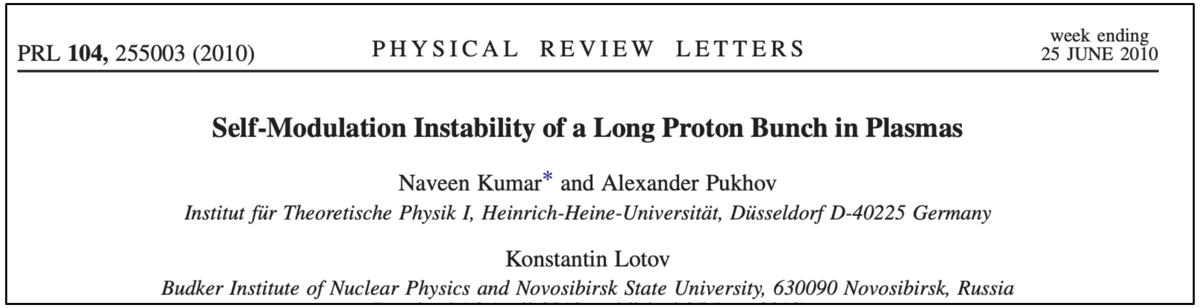

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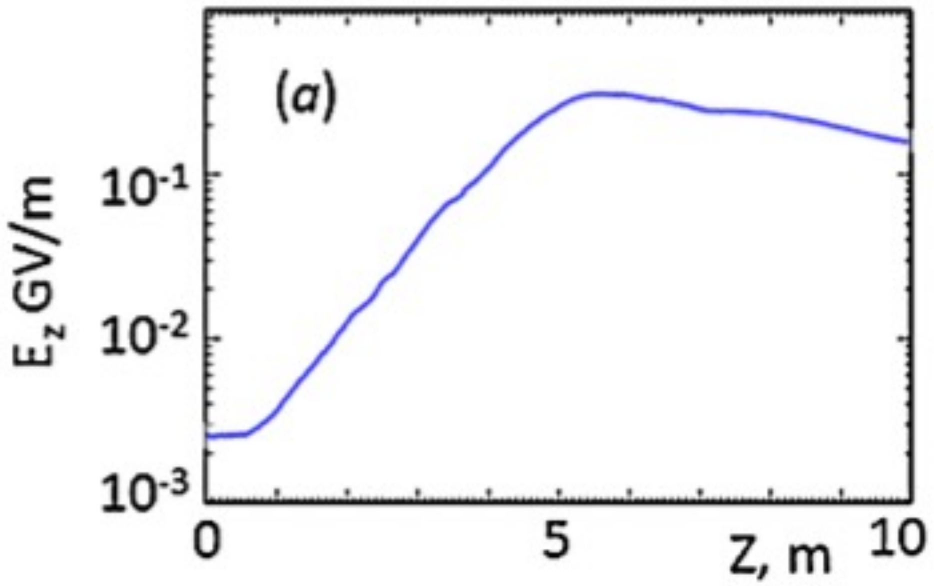
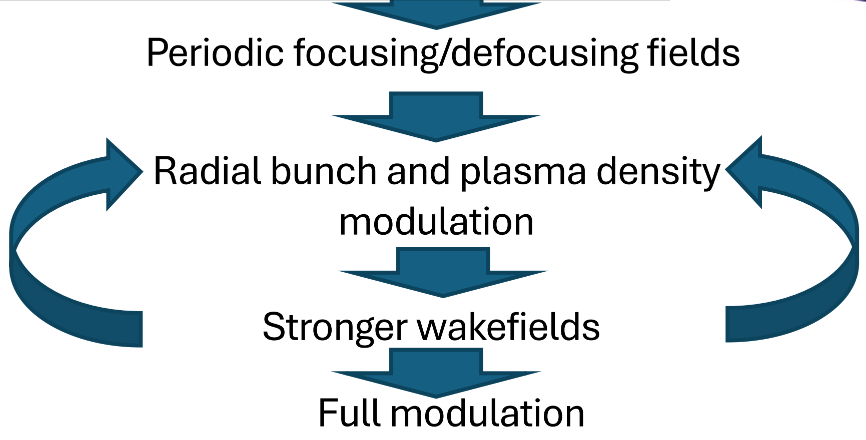
PRL **107,** 145003 (2011) PHYSICAL REVIEW LETTERS week ending 30 SEPTEMBER 2011

Phase Velocity and Particle Injection in a Self-Modulated Proton-Driven Plasma Wakefield Accelerator

A. Pukhov,[1] N. Kumar,[1] T. Tückmantel,[1] A. Upadhyay,[1] K. Lotov,[2] P. Muggli,[3] V. Khudik,[4] C. Siemon,[4] and G. Shvets[4]





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Radial bunch and plasma density modulation

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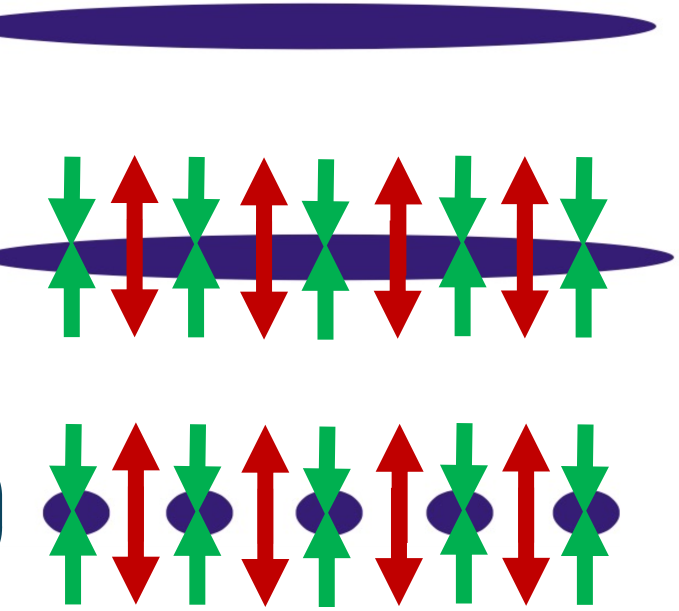
Full modulation



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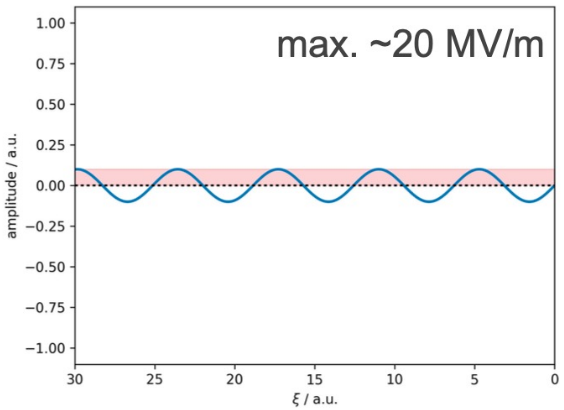
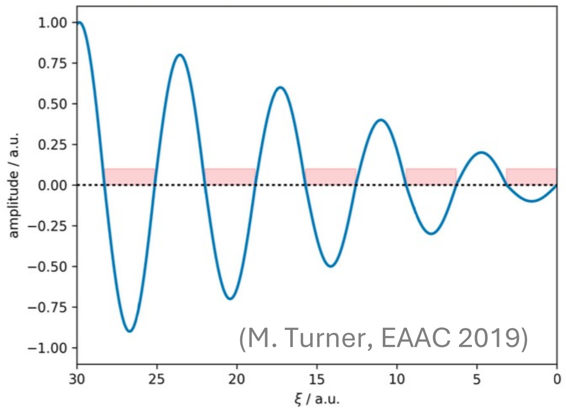
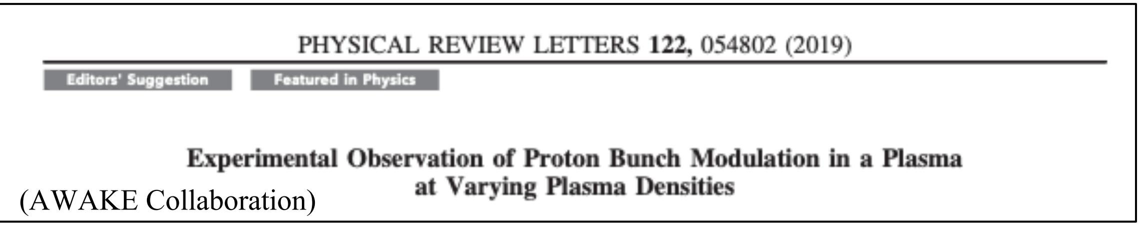
→ Resonant wakefield excitation
➔ interesting for high-gradient acceleration (AWAKE: see Edda Gschwendtner's talk)





(M. Turner, EAAC 2019)

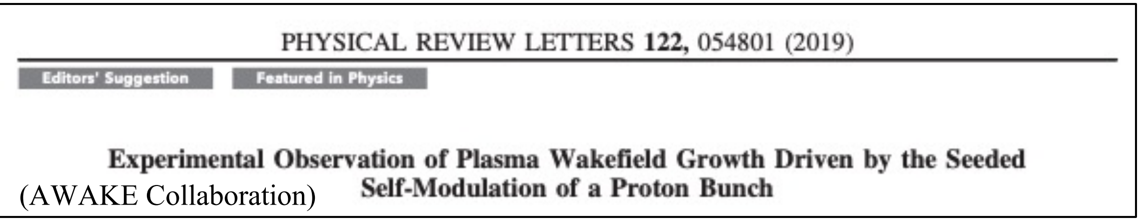


PHYSICAL REVIEW LETTERS **122,** 054802 (2019)

Editors' Suggestion Featured in Physics

Experimental Observation of Proton Bunch Modulation in a Plasma at Varying Plasma Densities

(AWAKE Collaboration)



PHYSICAL REVIEW LETTERS **122,** 054801 (2019)

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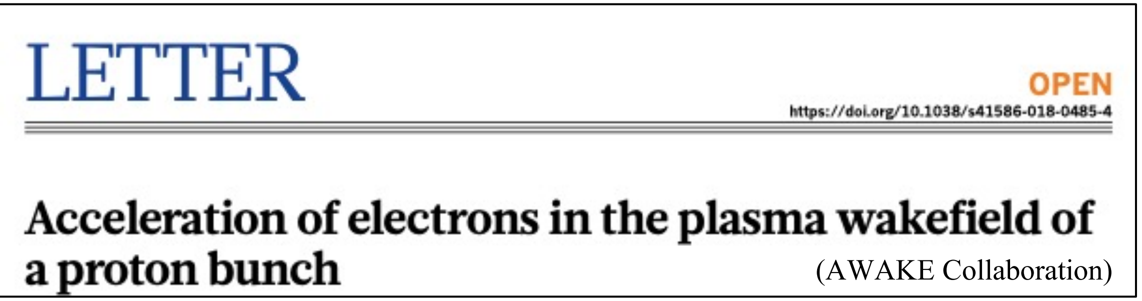


LETTER

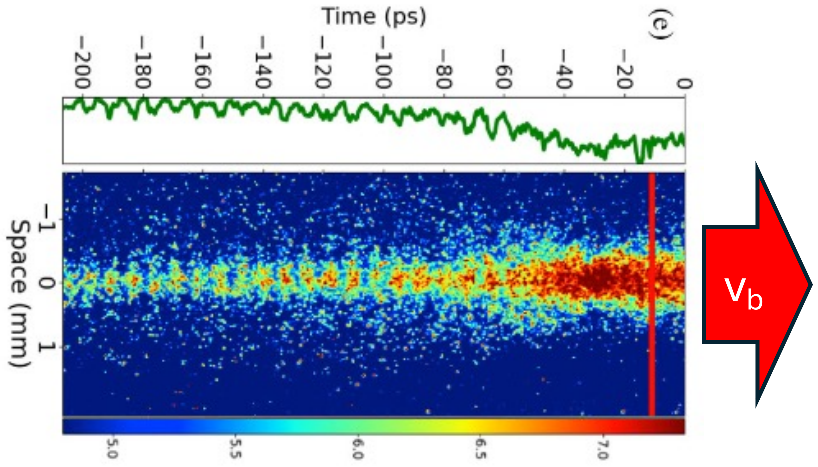
OPEN

https://doi.org/10.1038/s41586-018-0485-4

Acceleration of electrons in the plasma wakefield of a proton bunch

(AWAKE Collaboration)

SMI demonstration



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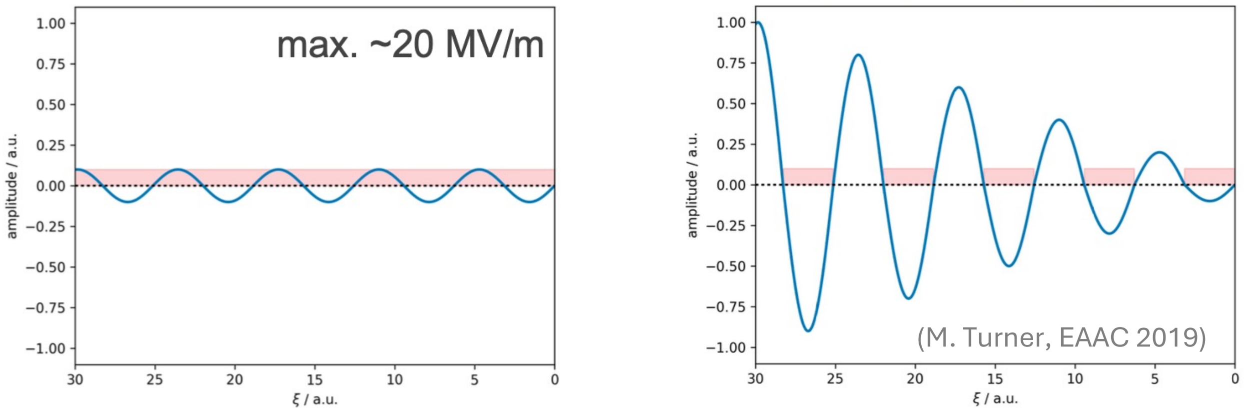

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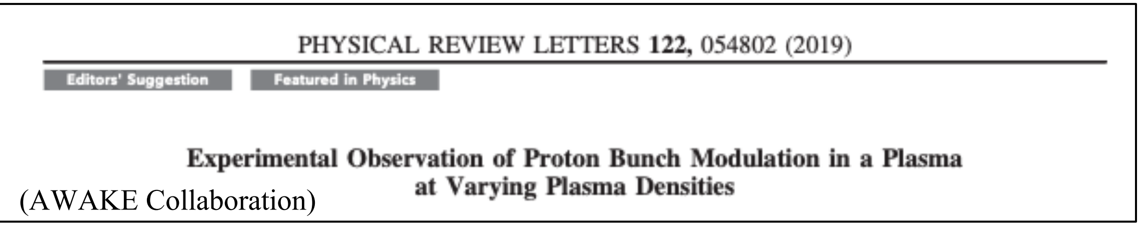

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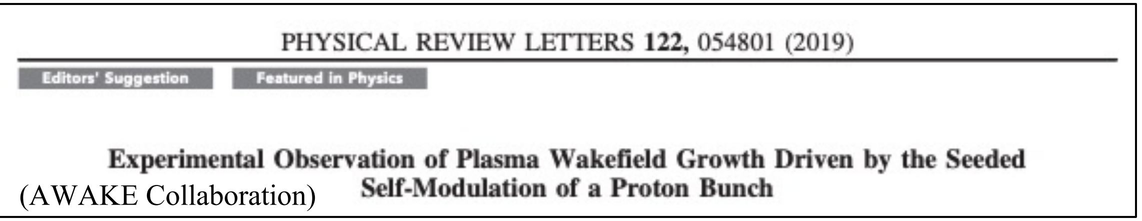

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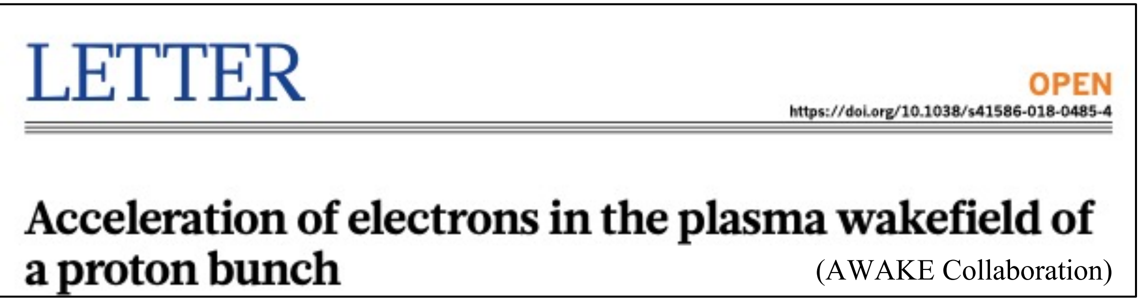


LETTER

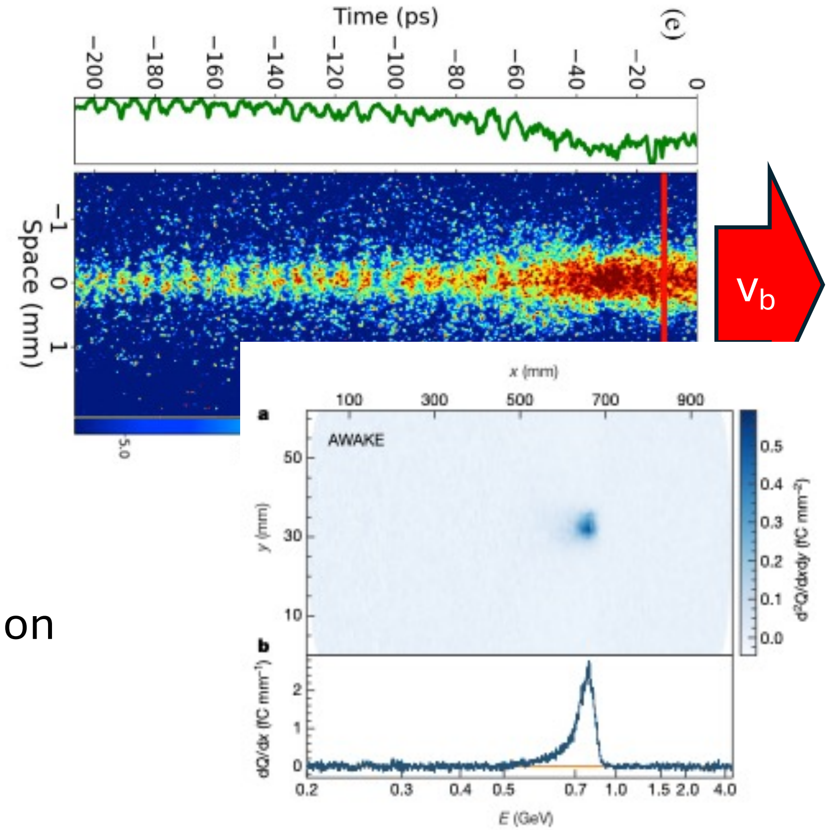
OPEN

https://doi.org/10.1038/s41586-018-0485-4

Acceleration of electrons in the plasma wakefield of a proton bunch

(AWAKE Collaboration)

Injection and acceleration



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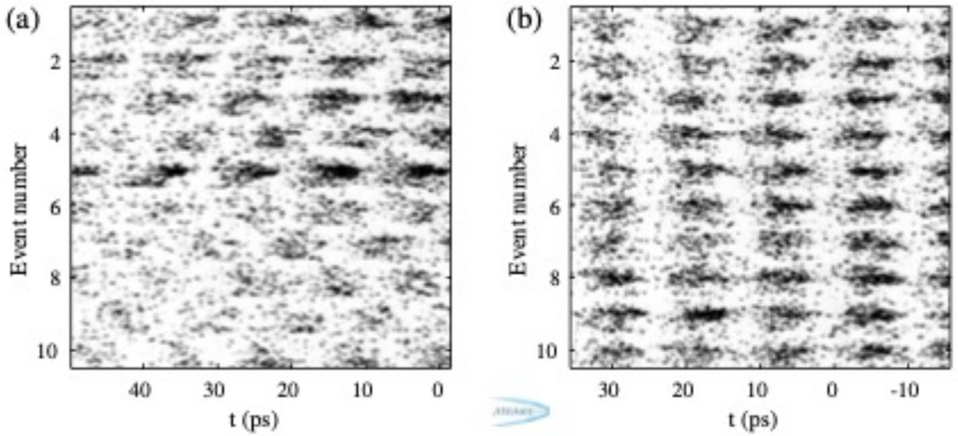
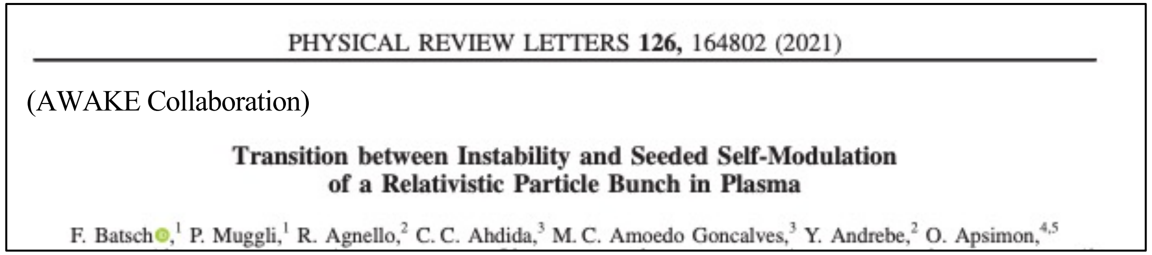
  (AWAKE: see Edda Gschwendtner's talk)

→ but CONTROL is needed!
→ SEEDING: DRIVING **INITIAL WAKEFIELDS** LARGE ENOUGH

NO SEEDING
→ INSTABILITY

SEEDING
→ REPRODUCIBLE




PHYSICAL REVIEW LETTERS **126**, 164802 (2021)

(AWAKE Collaboration)

Transition between Instability and Seeded Self-Modulation of a Relativistic Particle Bunch in Plasma

F. Batsch,[1] P. Muggli,[1] R. Agnello,[2] C. C. Ahdida,[3] M. C. Amoedo Goncalves,[3] Y. Andrebe,[2] O. Apsimon,[4,5]

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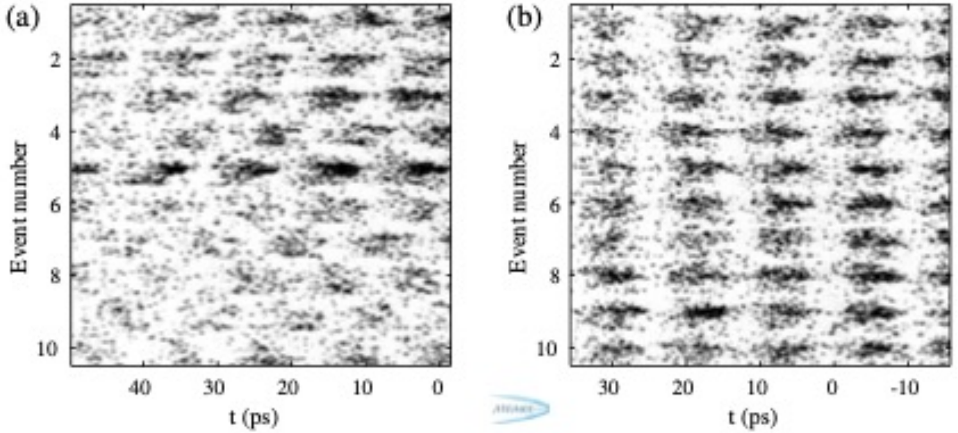
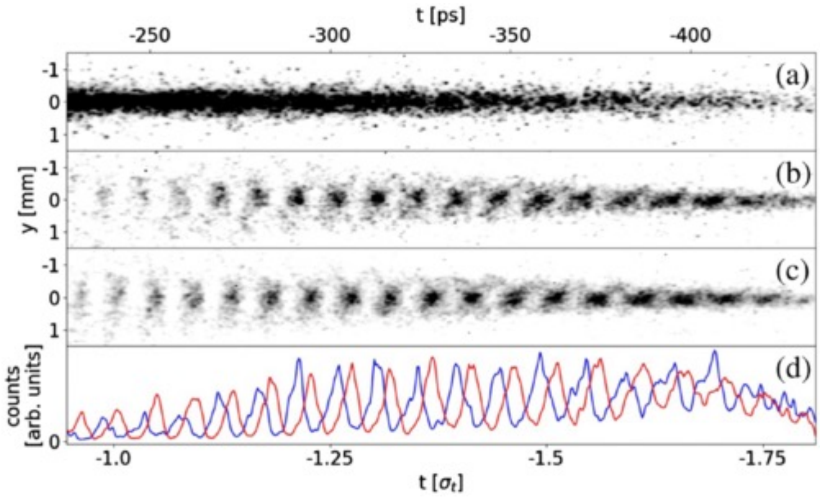
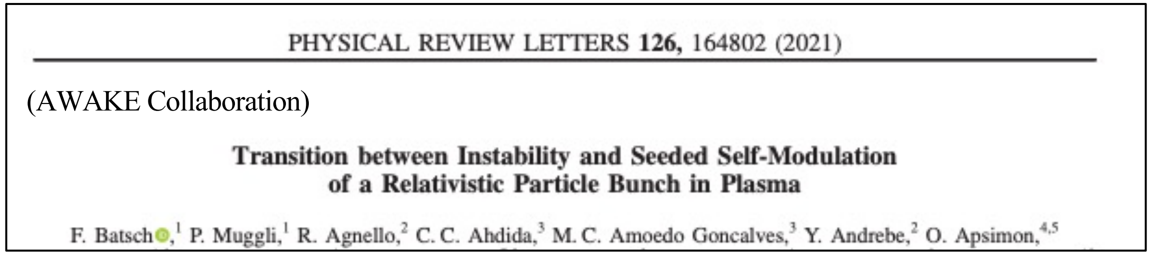
→ REPRODUCIBLE





Seeding with wakefields driven by preceding electron bunch
→ Phase defined by seed bunch



PHYSICAL REVIEW LETTERS **126,** 164802 (2021)

(AWAKE Collaboration)

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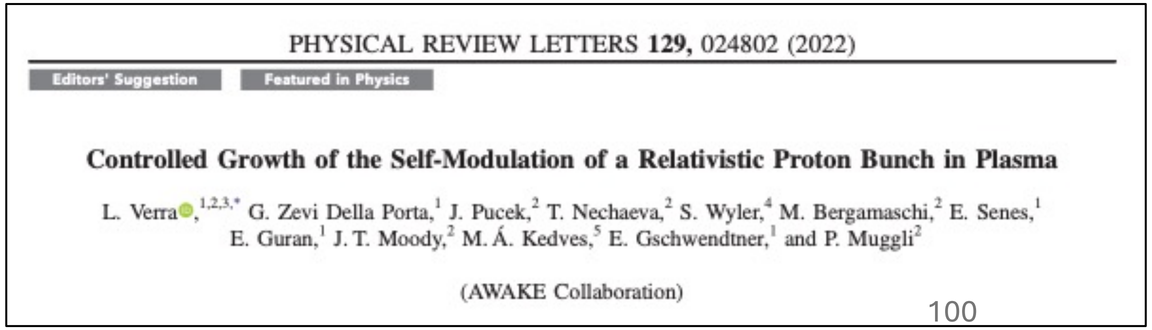

PHYSICAL REVIEW LETTERS **129,** 024802 (2022)

Editors' Suggestion Featured in Physics

Controlled Growth of the Self-Modulation of a Relativistic Proton Bunch in Plasma

L. Verra,[1,2,3,*] G. Zevi Della Porta,[1] J. Pucek,[2] T. Nechaeva,[2] S. Wyler,[4] M. Bergamaschi,[2] E. Senes,[1] E. Guran,[1] J. T. Moody,[2] M. Á. Kedves,[5] E. Gschwendtner,[1] and P. Muggli[2]

(AWAKE Collaboration)

III. Beam-Plasma Instabilities – HOSING

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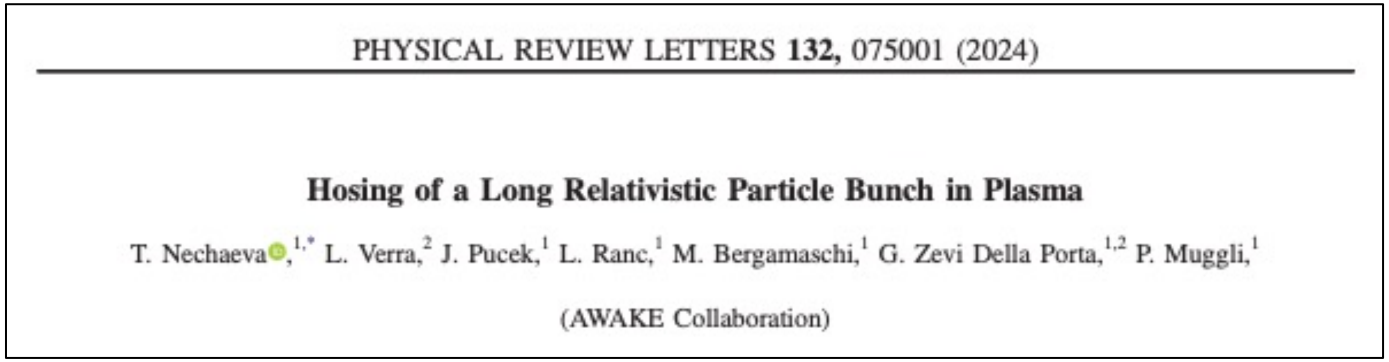
→ Wakefields act on the bunch itself
→ Resonant wakefield excitation



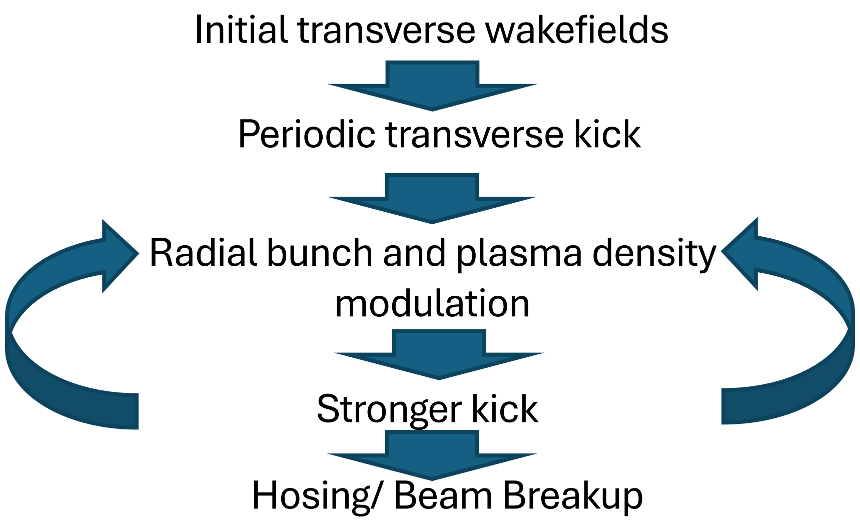
PHYSICAL REVIEW LETTERS **132,** 075001 (2024)

Hosing of a Long Relativistic Particle Bunch in Plasma

T. Nechaeva,[1,*] L. Verra,[2] J. Pucek,[1] L. Ranc,[1] M. Bergamaschi,[1] G. Zevi Della Porta,[1,2] P. Muggli,[1]

(AWAKE Collaboration)

Growth mechanism



Initial transverse wakefields

Periodic transverse kick

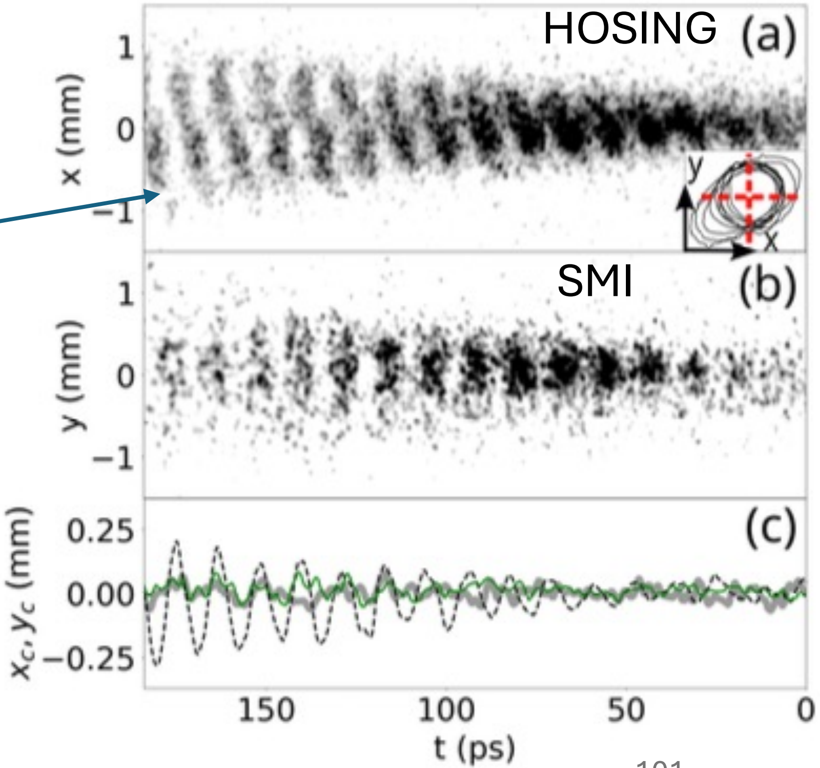
Radial bunch and plasma density modulation

Stronger kick

Hosing/ Beam Breakup

Asymmetric version of SMI

UNWANTED for an accelerator, but important to understand



Active Research Topic

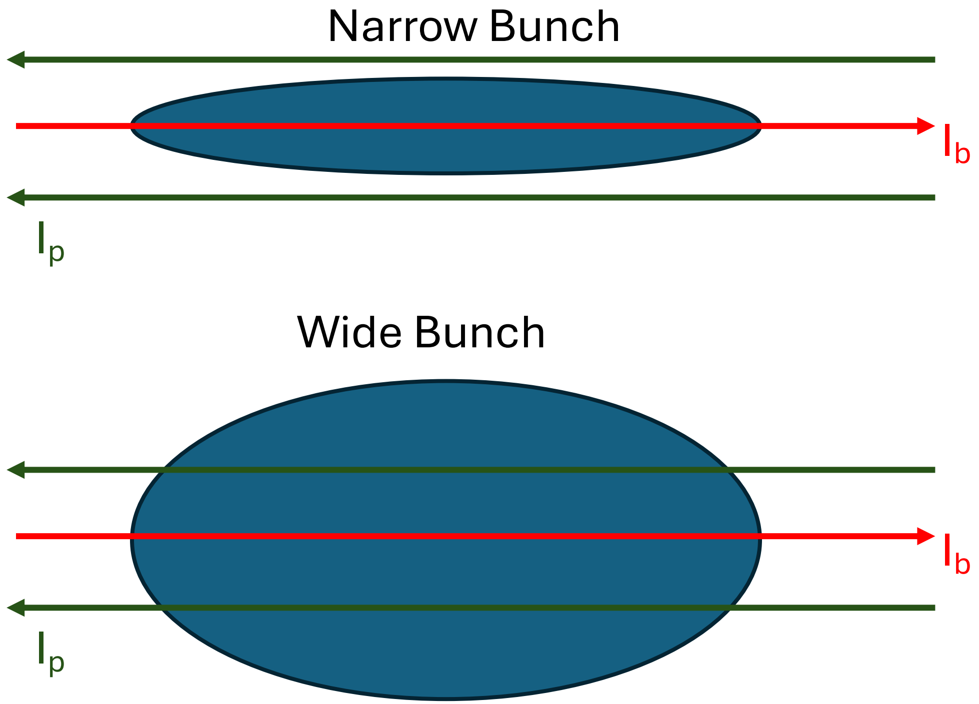
III. Beam-Plasma Instabilities – CFI

- WIDE bunch case: $\sigma_r \gg \lambda_{pe}$

→ Plasma tends to neutralize the bunch current

→ Return current flows within the bunch if $\sigma_r \gg \frac{c}{\omega_{pe}}$ = plasma skin depth

→ Opposite currents tend to repel each other → formation of filaments



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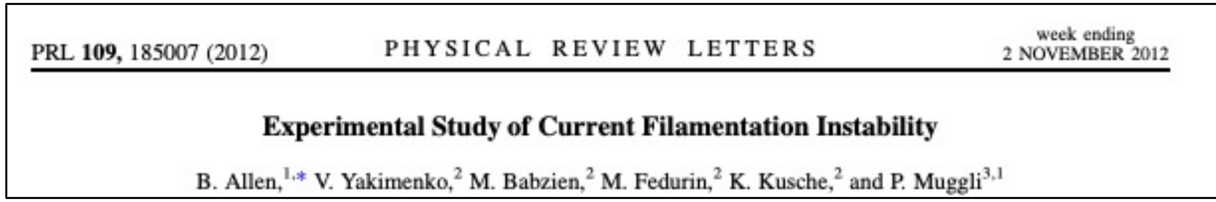
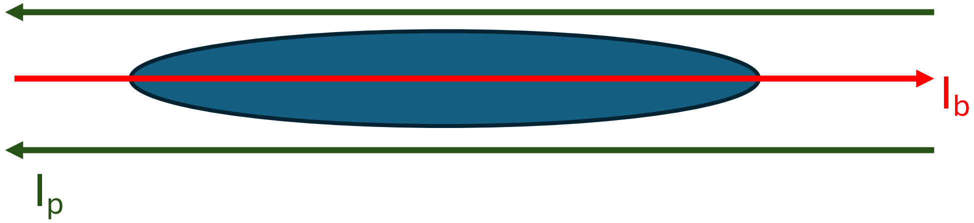


PRL 109, 185007 (2012) PHYSICAL REVIEW LETTERS week ending 2 NOVEMBER 2012

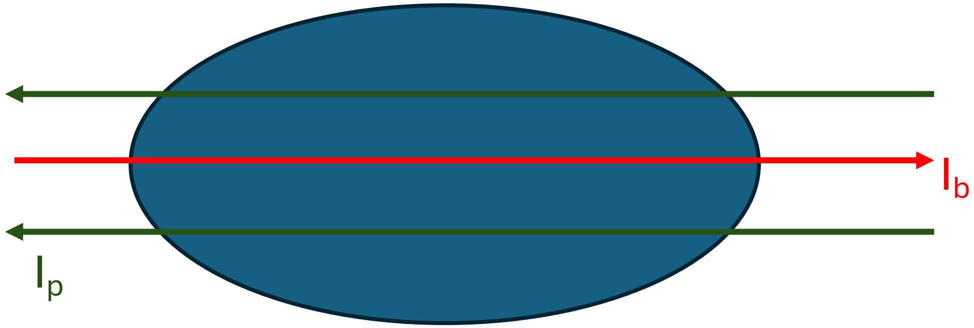
Experimental Study of Current Filamentation Instability

B. Allen,[1,*] V. Yakimenko,[2] M. Babzien,[2] M. Fedurin,[2] K. Kusche,[2] and P. Muggli[3,1]

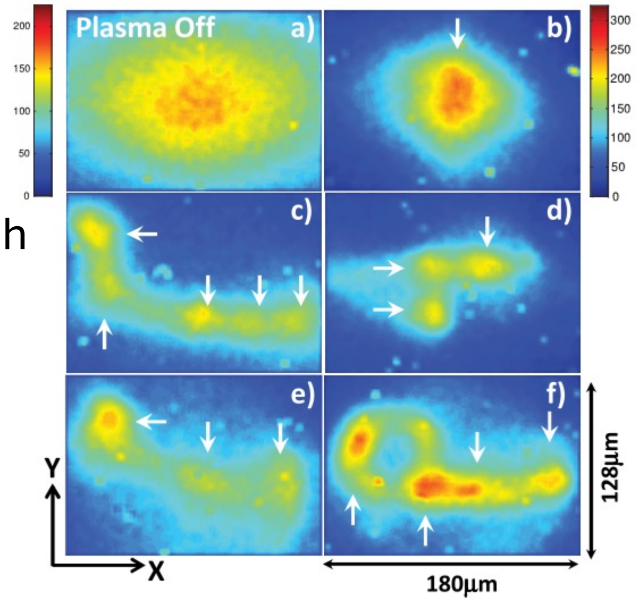
Narrow Bunch



Wide Bunch



Electron bunch



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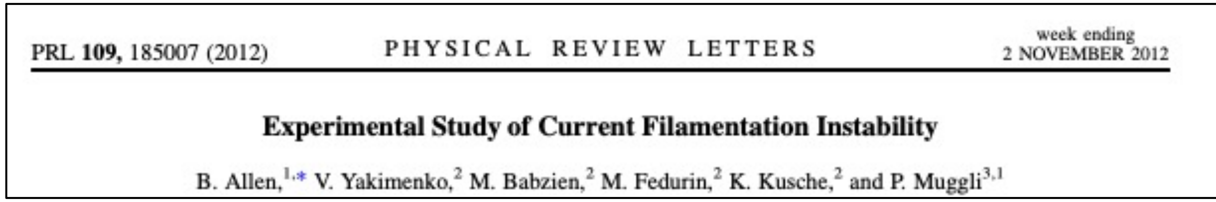

PRL **109,** 185007 (2012) PHYSICAL REVIEW LETTERS week ending 2 NOVEMBER 2012

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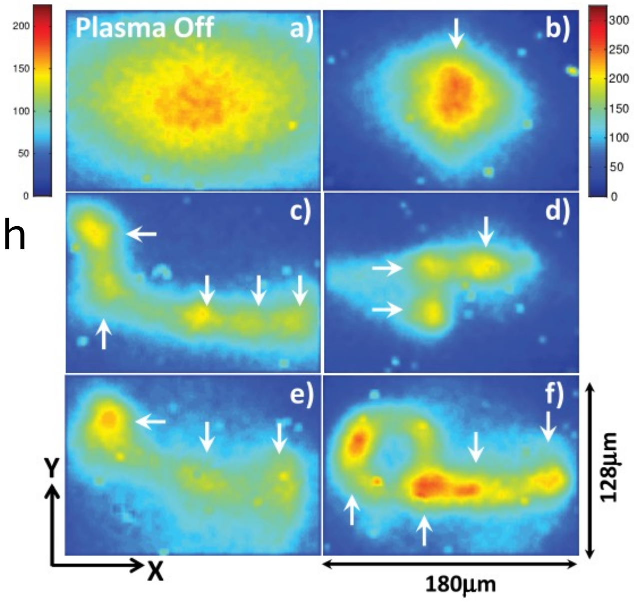

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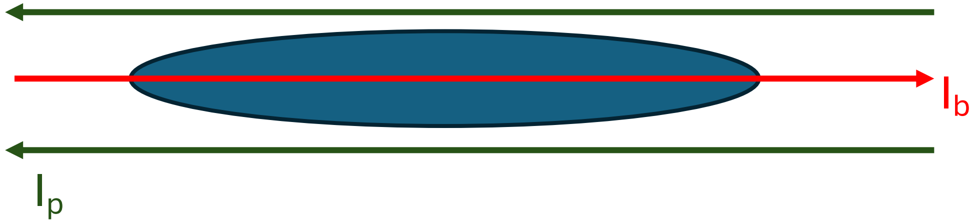
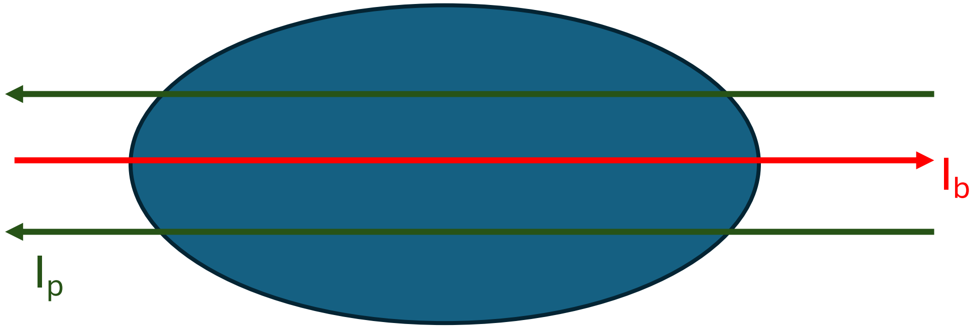
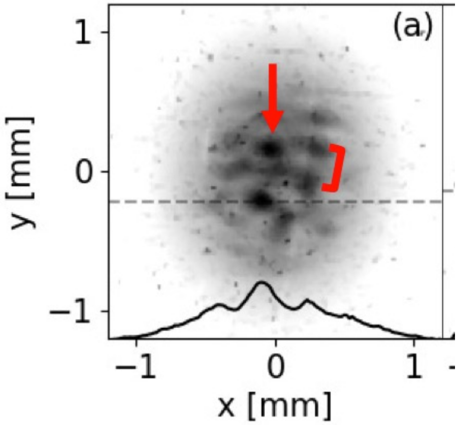
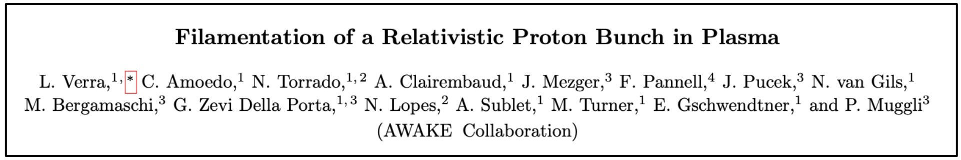


Wide Bunch



Proton bunch





Filamentation of a Relativistic Proton Bunch in Plasma

L. Verra,[1,*] C. Amoedo,[1] N. Torrado,[1,2] A. Clairembaud,[1] J. Mezger,[3] F. Pannell,[4] J. Pucek,[3] N. van Gils,[1]
M. Bergamaschi,[3] G. Zevi Della Porta,[1,3] N. Lopes,[2] A. Sublet,[1] M. Turner,[1] E. Gschwendtner,[1] and P. Muggli[3]
(AWAKE Collaboration)

(Accepted Phys. Rev. E)

Active Research Topic

- UNWANTED for PWFA (upper limit for σ_r)
- Relevant for plasma-astrophysics

Conclusions

- PWFA is an active (and mature) field of research
- Demonstration of very high energy gains
- Quite some work on beam quality
- NOW: working towards real applications

  → Light sources
  → High-energy particle physics

- Your help is needed!!
- All references here: https://istnazfisnucl-my.sharepoint.com/:f:/g/personal/lverra_infn_it/EuyCJFCNnxdDsp99Fh5icw4BmlywN4a2mYmevjnhvxkeCQ?e=tZ8ShE

Thank You For Listening!

Additional Material

II. Non-linear Regime – Transformer Ratio

Transformer ratio $R = \frac{E_+}{E_-}$ still limited to 2

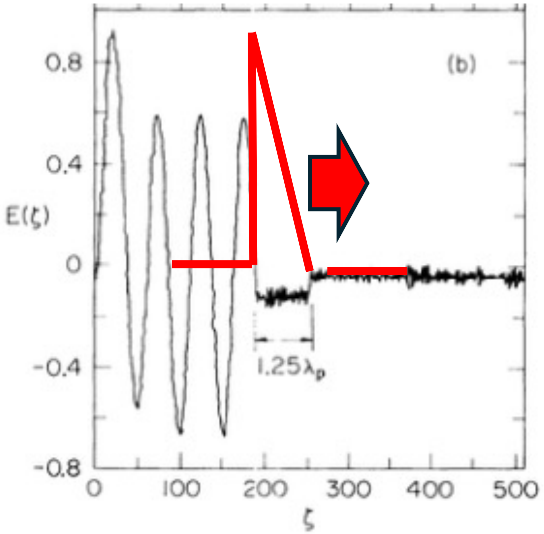
→ How can it be increased?

VOLUME 56, NUMBER 12 PHYSICAL REVIEW LETTERS 24 MARCH 1986

Energy Transfer in the Plasma Wake-Field Accelerator

Pisin Chen,(a) J. J. Su, and J. M. Dawson
Department of Physics, University of California, Los Angeles, California 90024
and
K. L. F. Bane and P. B. Wilson
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 16 September 1985)

The maximum possible transformer ratio for a bunch with given length and total charge corresponds to that charge distribution which causes all particles in the bunch to see the same retarding field



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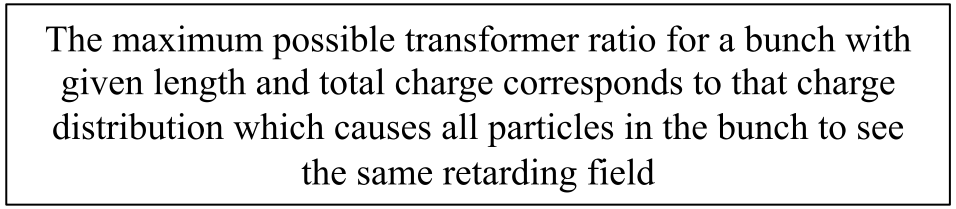

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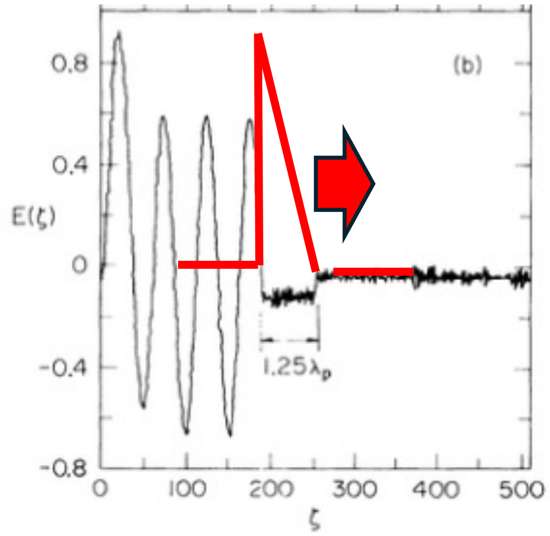
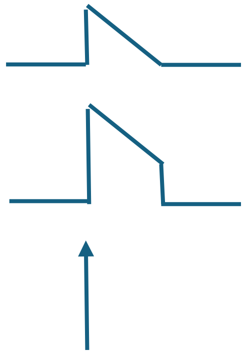


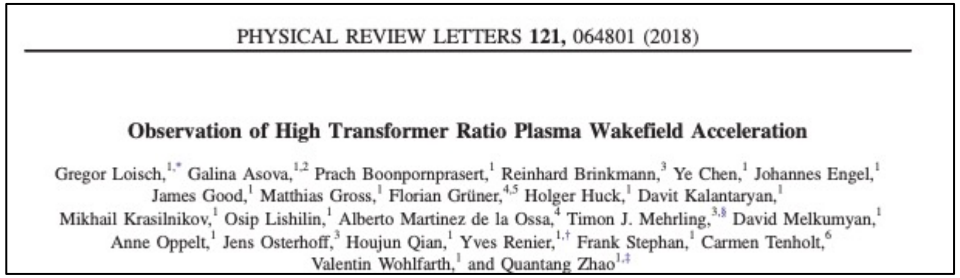
1) Shaping the drive bunch distribution

Triangular / trapezoidal longitudinal distribution



Abrupt down ramp, much shorter than λ_{pe} is key for effective excitation

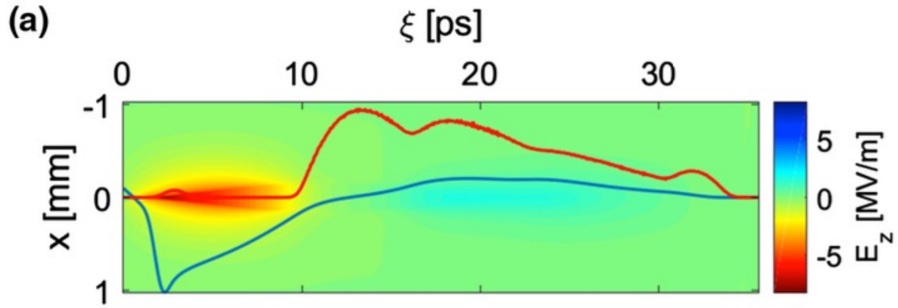
→ Challenging to obtain experimentally

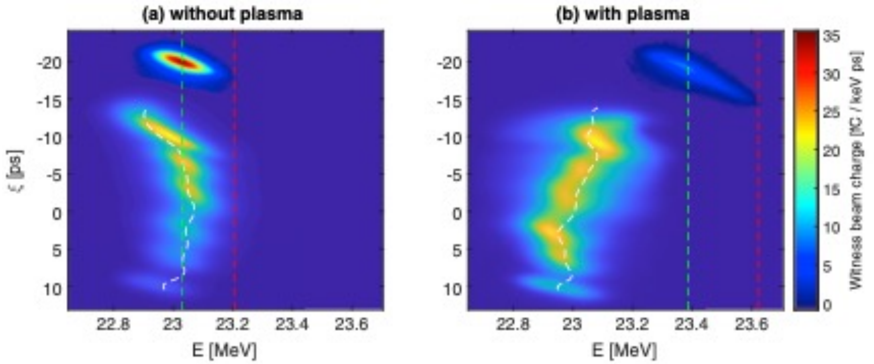


PHYSICAL REVIEW LETTERS **121,** 064801 (2018)

Observation of High Transformer Ratio Plasma Wakefield Acceleration

Gregor Loisch,[1,*] Galina Asova,[1,2] Prach Boonpornprasert,[1] Reinhard Brinkmann,[3] Ye Chen,[1] Johannes Engel,[1] James Good,[1] Matthias Gross,[1] Florian Grüner,[4,5] Holger Huck,[1] Davit Kalantaryan,[1] Mikhail Krasilnikov,[1] Osip Lishilin,[1] Alberto Martinez de la Ossa,[4] Timon J. Mehrling,[3,8] David Melkumyan,[1] Anne Oppelt,[1] Jens Osterhoff,[3] Houjun Qian,[1] Yves Renier,[1,†] Frank Stephan,[1] Carmen Tenholt,[6] Valentin Wohlfarth,[1] and Quantang Zhao[1,‡]





Proof of principle experiment with moderate energy gain

R = 4.6

II. Non-linear Regime – Transformer Ratio

Transformer ratio $R = \frac{E_+}{E_-}$ still limited to 2

→ How can it be increased?

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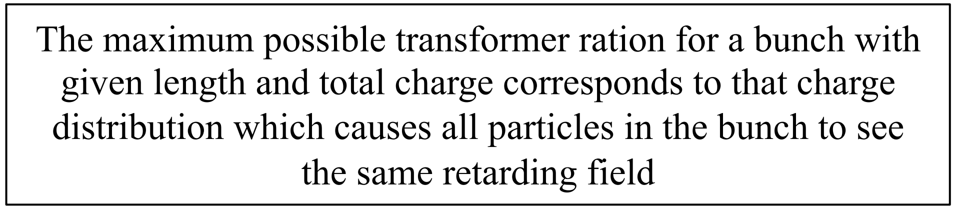

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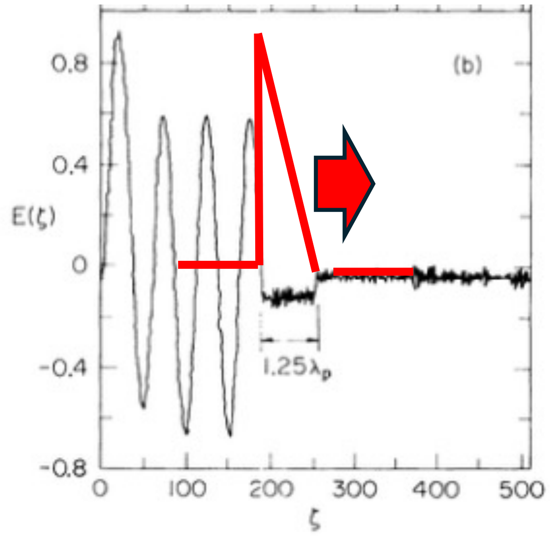
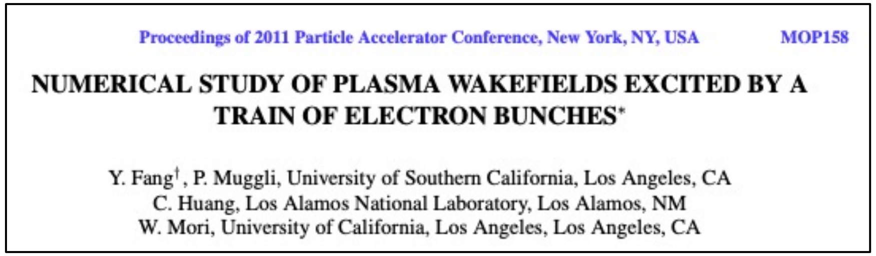


→ Constant decelerating field within each driver

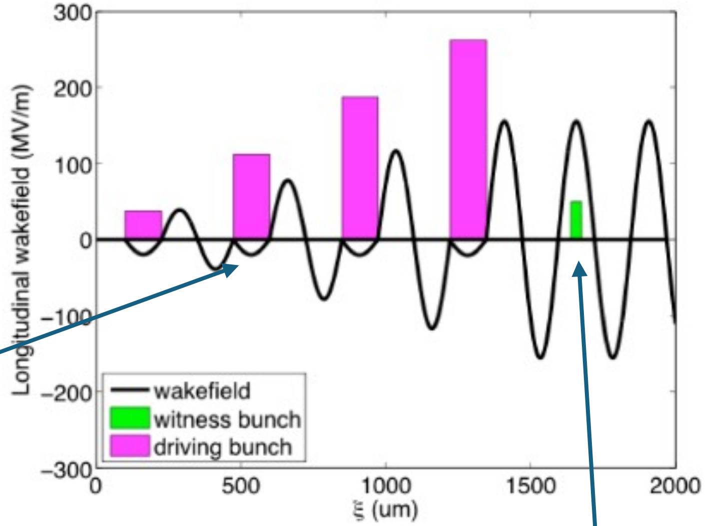
2) Ramped bunch train



Proceedings of 2011 Particle Accelerator Conference, New York, NY, USA MOP158

NUMERICAL STUDY OF PLASMA WAKEFIELDS EXCITED BY A TRAIN OF ELECTRON BUNCHES*

Y. Fang†, P. Muggli, University of Southern California, Los Angeles, CA
C. Huang, Los Alamos National Laboratory, Los Alamos, NM
W. Mori, University of California, Los Angeles, CA



→ Enhanced accelerating field within the witness → **R>>2**

Active Research Topic

IIb. Positrons

→ How do wakefields work with positively charged particles?

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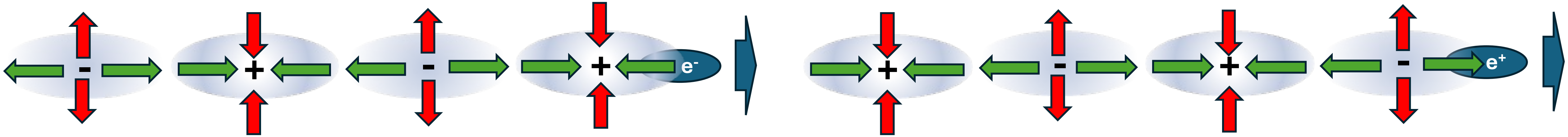
In principle, just a π phase difference



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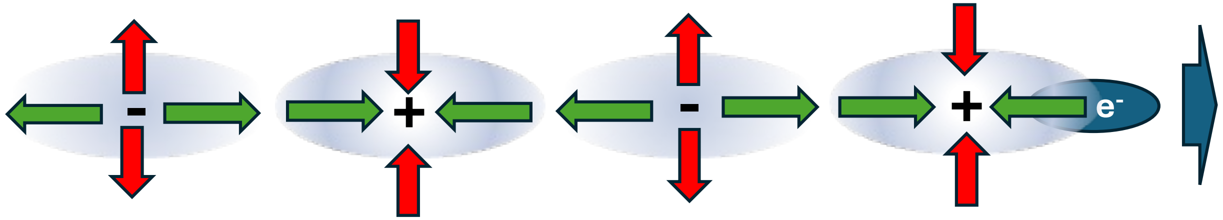

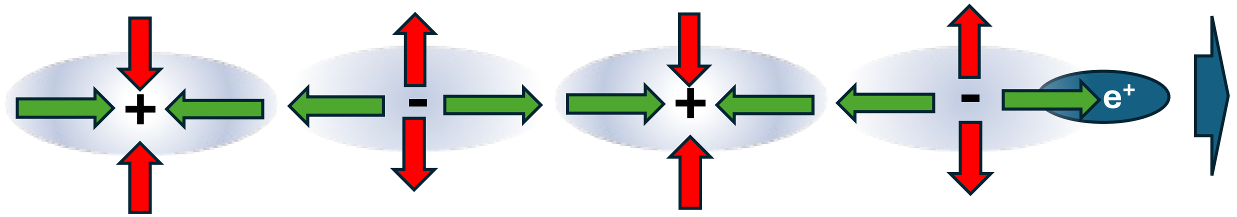

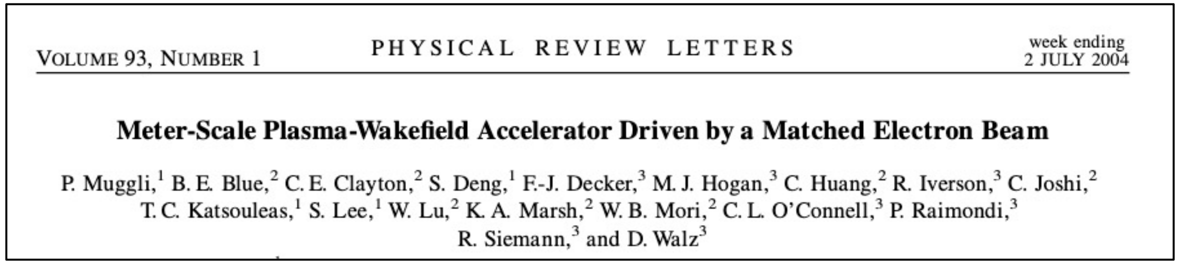

VOLUME 93, NUMBER 1 PHYSICAL REVIEW LETTERS week ending 2 JULY 2004

Meter-Scale Plasma-Wakefield Accelerator Driven by a Matched Electron Beam

P. Muggli,[1] B. E. Blue,[2] C. E. Clayton,[2] S. Deng,[1] F.-J. Decker,[3] M. J. Hogan,[3] C. Huang,[2] R. Iverson,[3] C. Joshi,[2]
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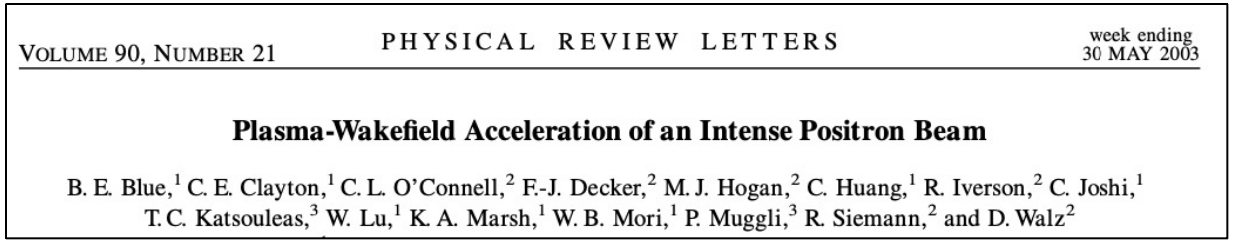

VOLUME 90, NUMBER 21 PHYSICAL REVIEW LETTERS week ending 30 MAY 2003

Plasma-Wakefield Acceleration of an Intense Positron Beam

B. E. Blue,[1] C. E. Clayton,[1] C. L. O'Connell,[2] F.-J. Decker,[2] M. J. Hogan,[2] C. Huang,[1] R. Iverson,[2] C. Joshi,[1]
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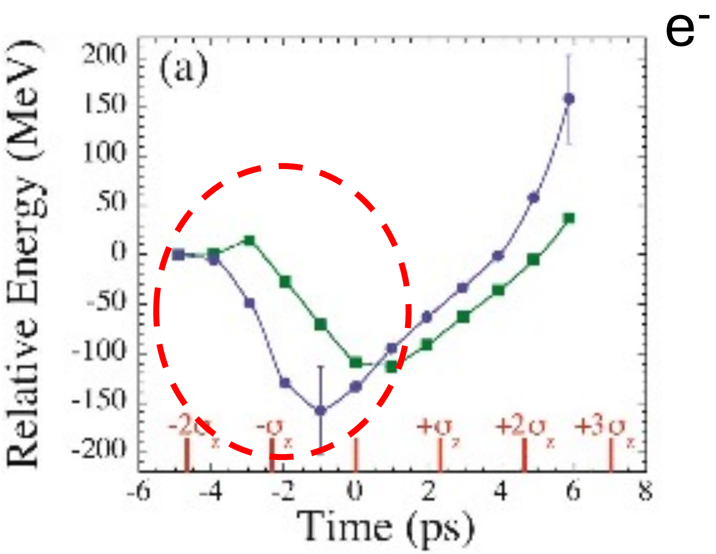
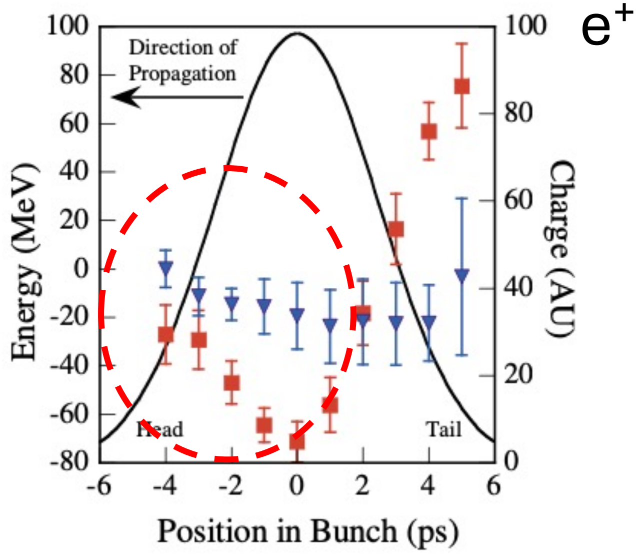
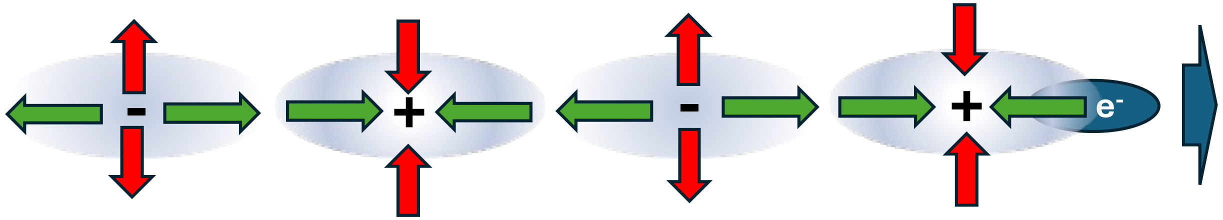


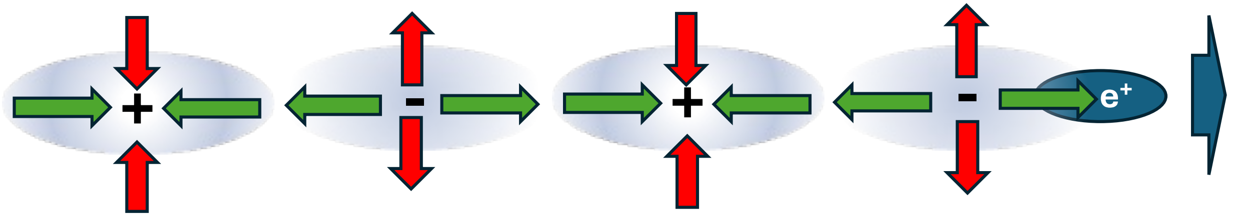
Long Bunches:
Head and center lose energy

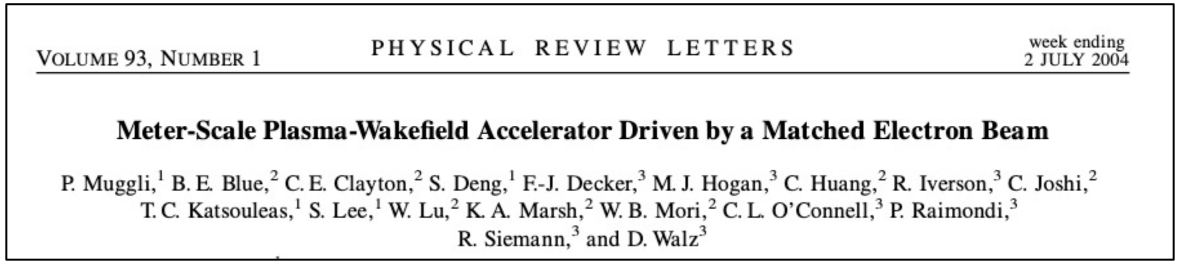


IIb. Positrons

→ How do wakefields work with positively charged particles?
In principle, just a π phase difference (in the linear regime)



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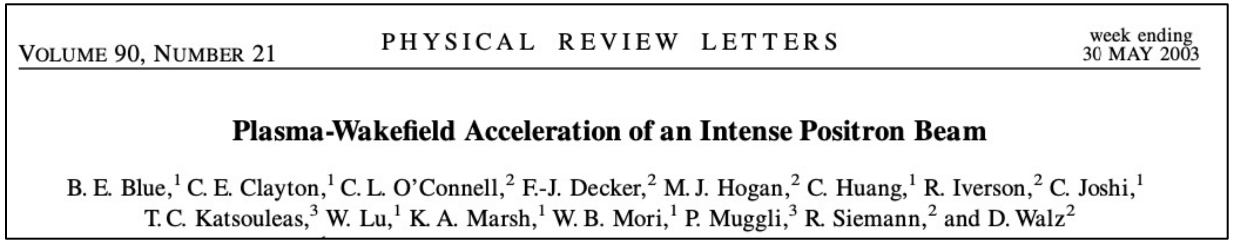
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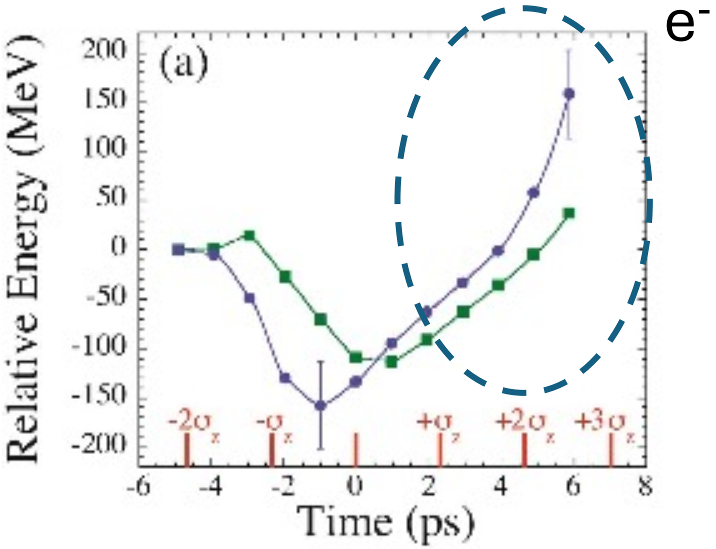
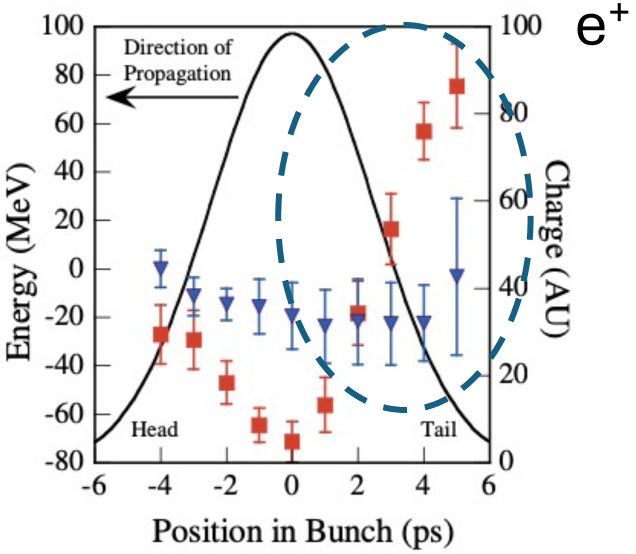


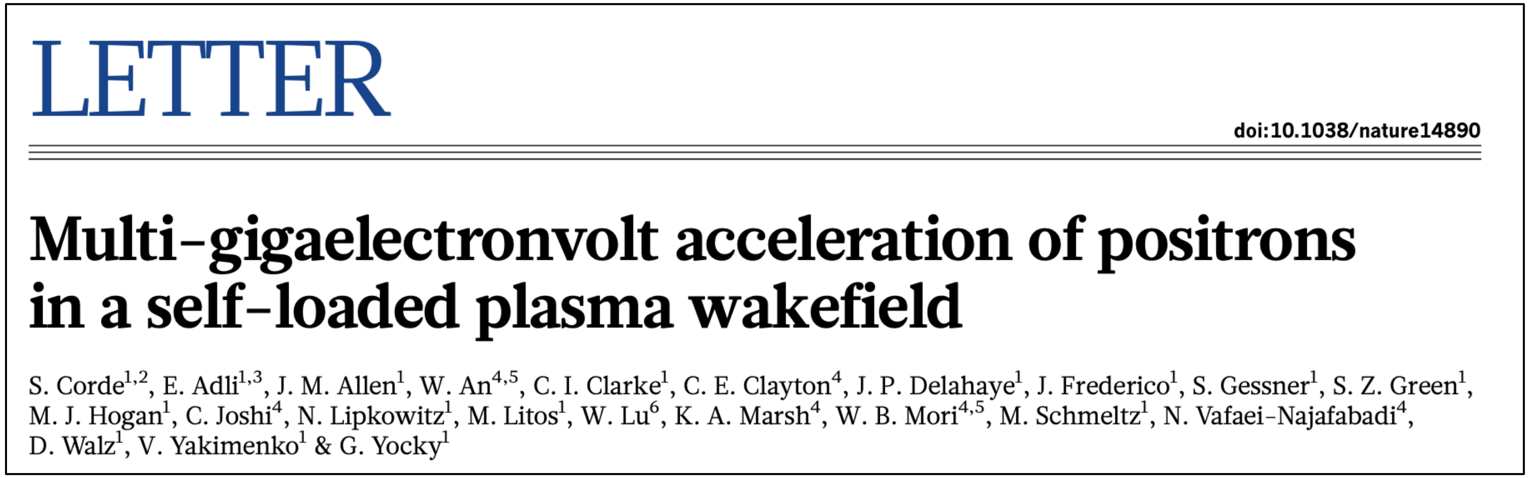
Long Bunches:
Head and center lose energy
Tail gains energy



IIb. Positrons

→ How do wakefields work with positively charged particles?
In principle, just a π phase difference (in the linear regime)

→ Acceleration demonstrated also in the non-linear regime

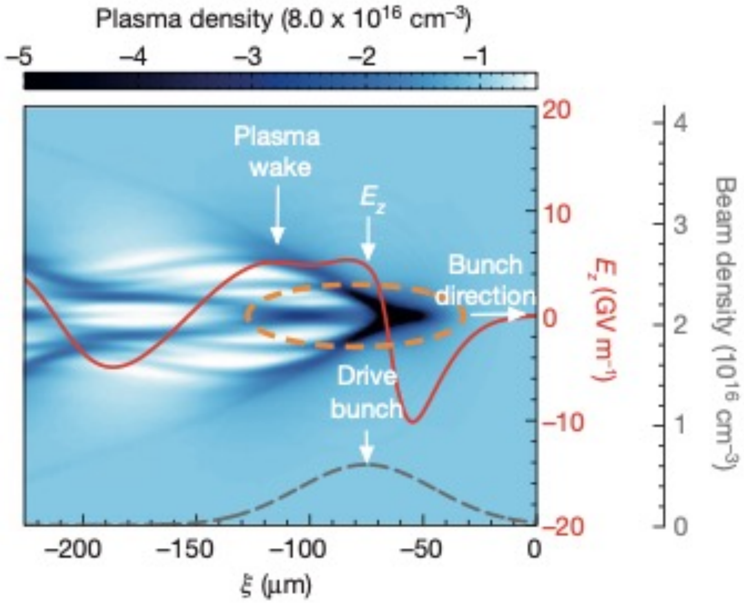


LETTER

doi:10.1038/nature14890

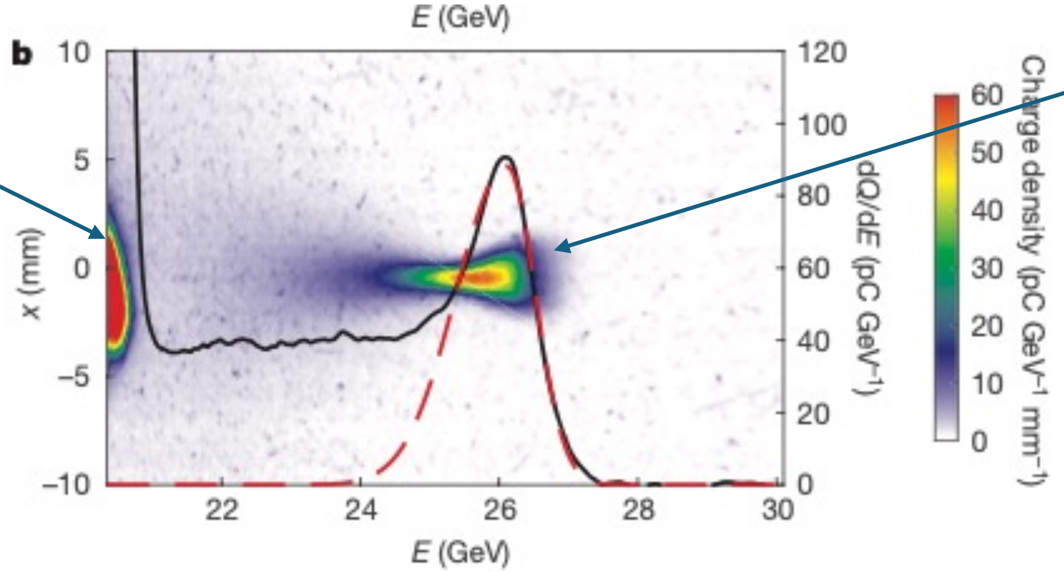
Multi-gigaelectronvolt acceleration of positrons in a self-loaded plasma wakefield

S. Corde[1,2], E. Adli[1,3], J. M. Allen[1], W. An[4,5], C. I. Clarke[1], C. E. Clayton[4], J. P. Delahaye[1], J. Frederico[1], S. Gessner[1], S. Z. Green[1], M. J. Hogan[1], C. Joshi[4], N. Lipkowitz[1], M. Litos[1], W. Lu[6], K. A. Marsh[4], W. B. Mori[4,5], M. Schmeltz[1], N. Vafaei-Najafabadi[4], D. Walz[1], V. Yakimenko[1] & G. Yocky[1]



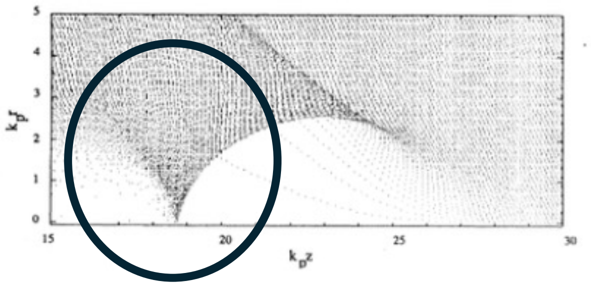
Initial energy

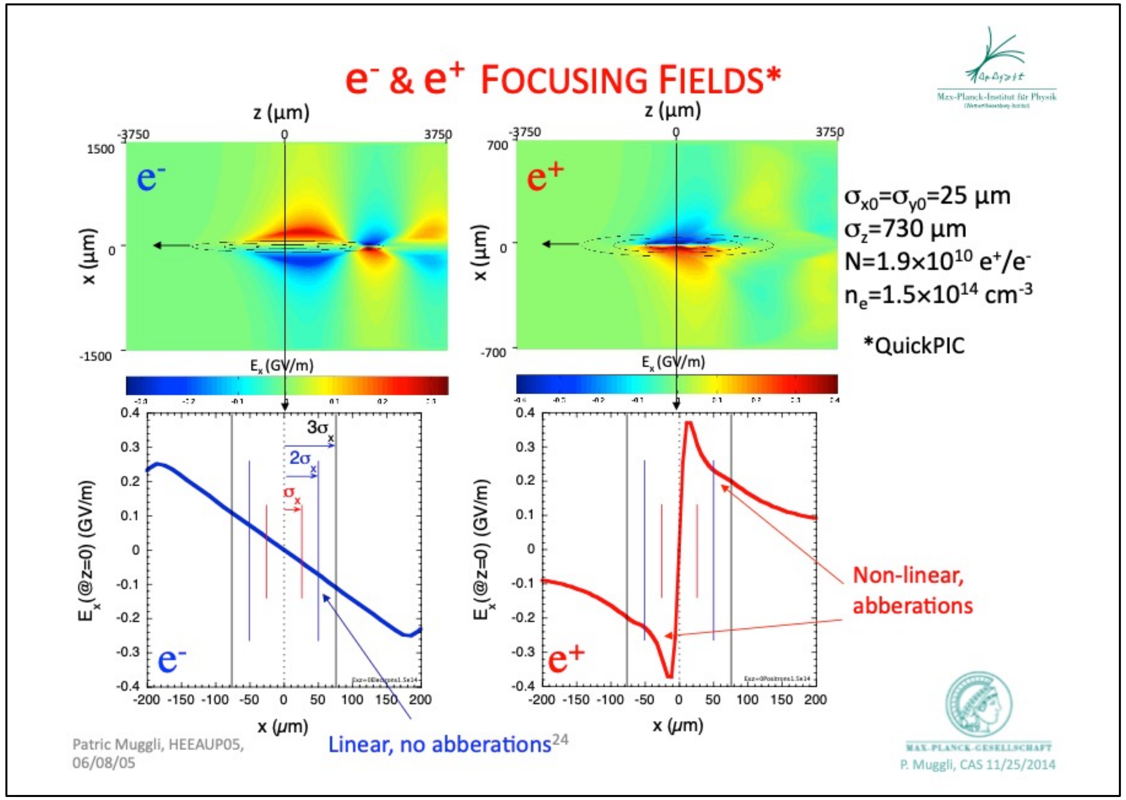
Maximum energy



IIb. Positrons

→ BUT: In the blowout regime, e^+ witness bunches need to be placed in the singularity





e^- & e^+ FOCUSING FIELDS*

$\sigma_{x0}=\sigma_{y0}=25$ µm
$\sigma_z=730$ µm
$N=1.9\times10^{10}$ e^+/e^-
$n_e=1.5\times10^{14}$ cm^{-3}

*QuickPIC

Non-linear, abberations

Linear, no abberations[24]

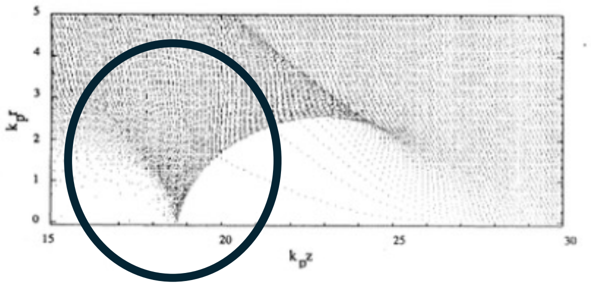
Patric Muggli, HEEAUP05, 06/08/05

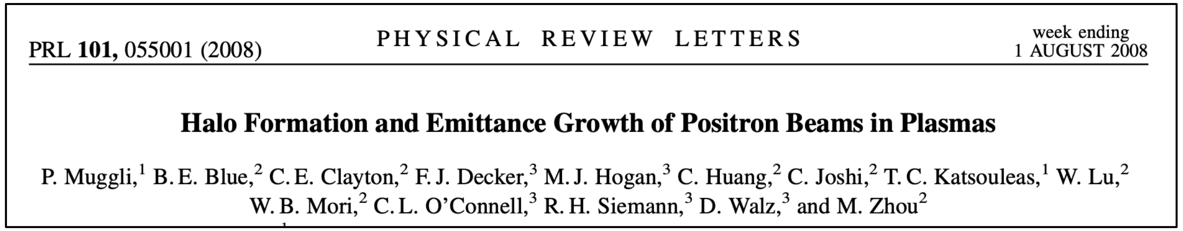
P. Muggli, CAS 11/25/2014

(P. Muggli, CAS 2014)

IIb. Positrons

→ BUT: In the blowout regime, e^+ witness bunches need to be placed in the singularity

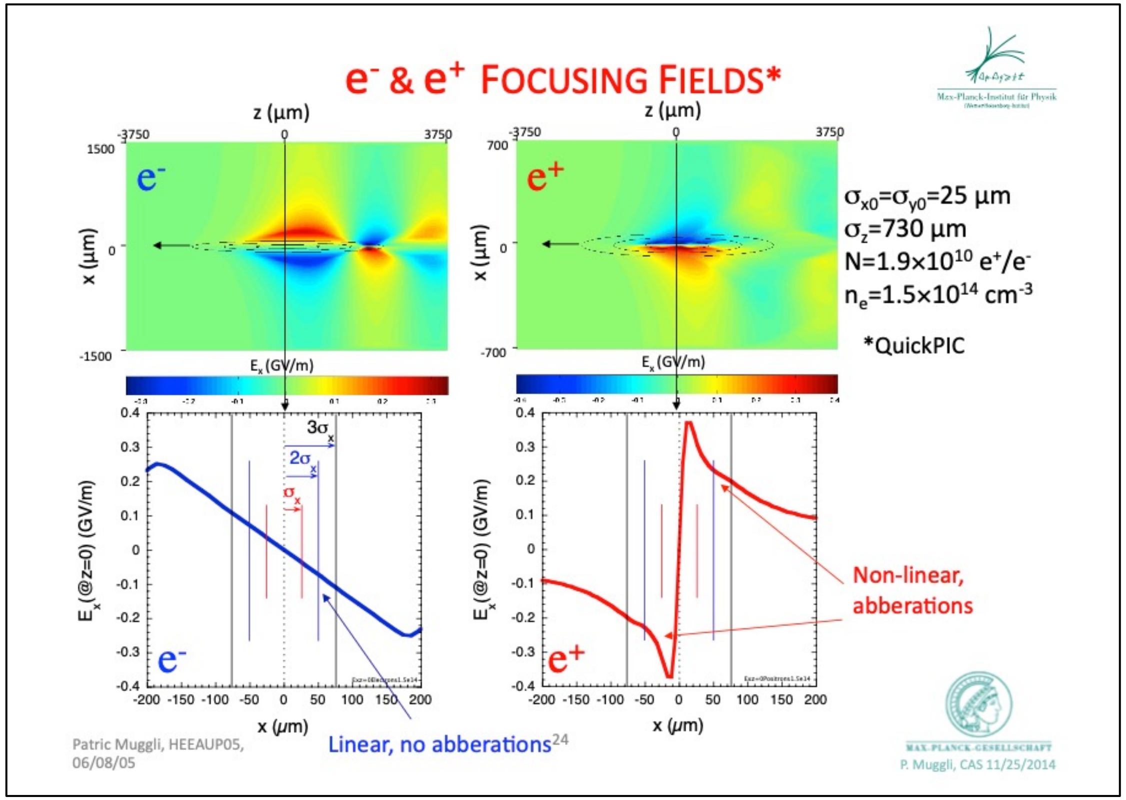




PRL **101,** 055001 (2008) PHYSICAL REVIEW LETTERS week ending 1 AUGUST 2008

Halo Formation and Emittance Growth of Positron Beams in Plasmas

P. Muggli,[1] B. E. Blue,[2] C. E. Clayton,[2] F. J. Decker,[3] M. J. Hogan,[3] C. Huang,[2] C. Joshi,[2] T. C. Katsouleas,[1] W. Lu,[2] W. B. Mori,[2] C. L. O'Connell,[3] R. H. Siemann,[3] D. Walz,[3] and M. Zhou[2]



e^- & e^+ FOCUSING FIELDS*

$\sigma_{x0}=\sigma_{y0}=25$ µm
$\sigma_z=730$ µm
$N=1.9\times10^{10}$ e^+/e^-
$n_e=1.5\times10^{14}$ cm^{-3}

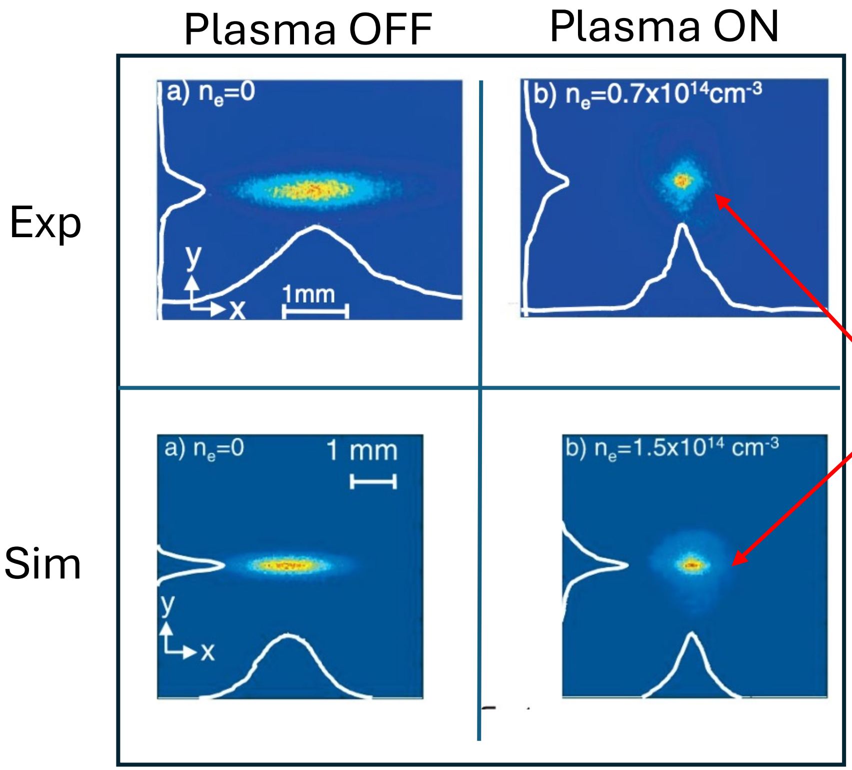
*QuickPIC

Non-linear, abberations

Linear, no abberations[24]

Patric Muggli, HEEAUP05, 06/08/05

P. Muggli, CAS 11/25/2014

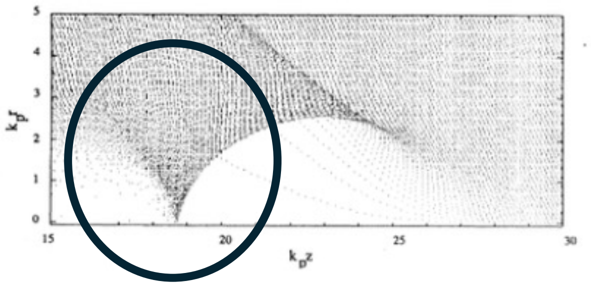


Halo due to aberrations
→ emittance growth

(P. Muggli, CAS 2014)

IIb. Positrons

→ BUT: In the blowout regime, e⁺ witness bunches need to be placed in the singulari




PRL **101,** 055001 (20

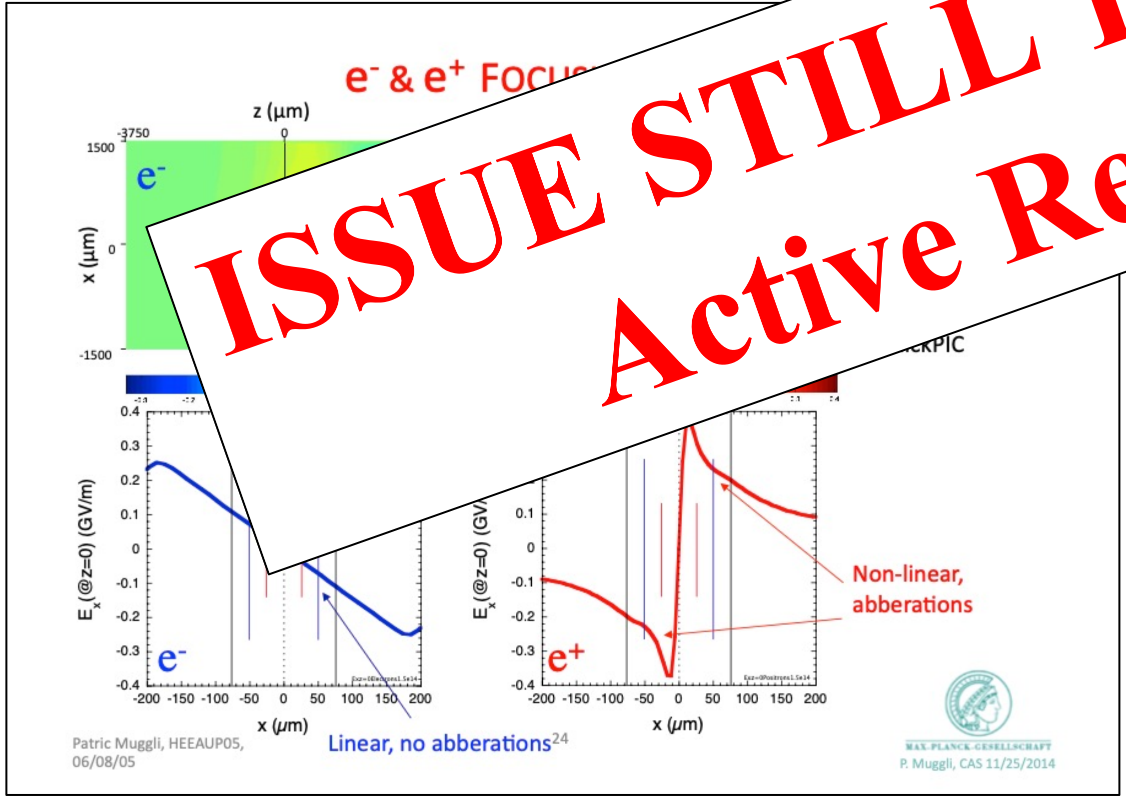
week ending
1 AUGUST 2008

smas

atsouleas,[1] W. Lu,[2]
Zhou[2]

ISSUE STILL TO BE SOLVED!!

Active Research Topic


e⁻ & e⁺ Focus

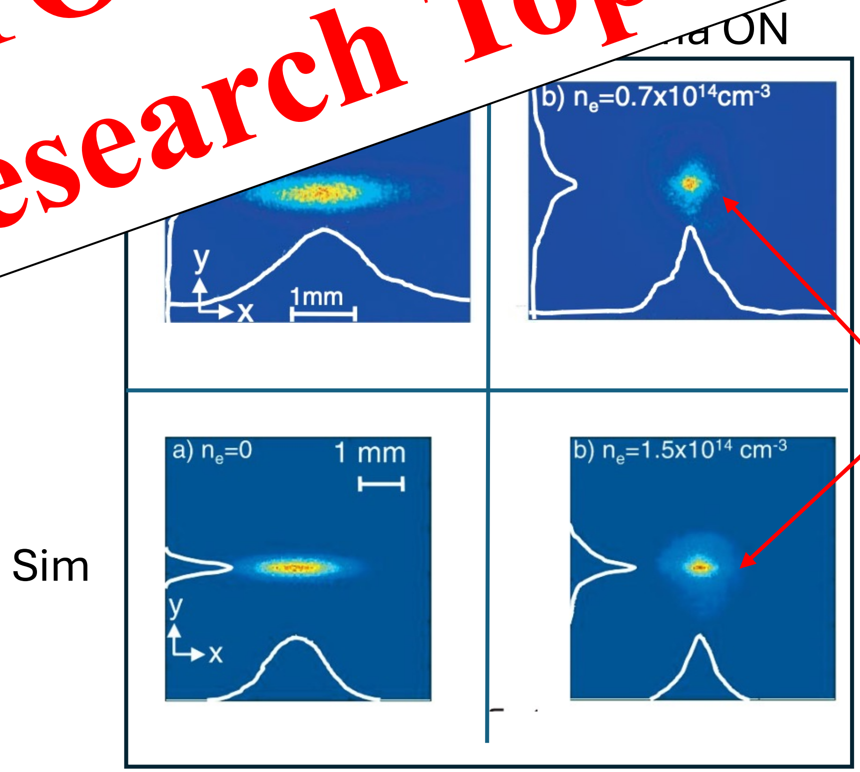
ckPIC

Non-linear, abberations

Linear, no abberations[24]

Patric Muggli, HEEAUP05, 06/08/05

P. Muggli, CAS 11/25/2014



Halo due to aberrations
→ emittance growth

(P. Muggli, CAS 2014)