Introduction to Plasma Physics

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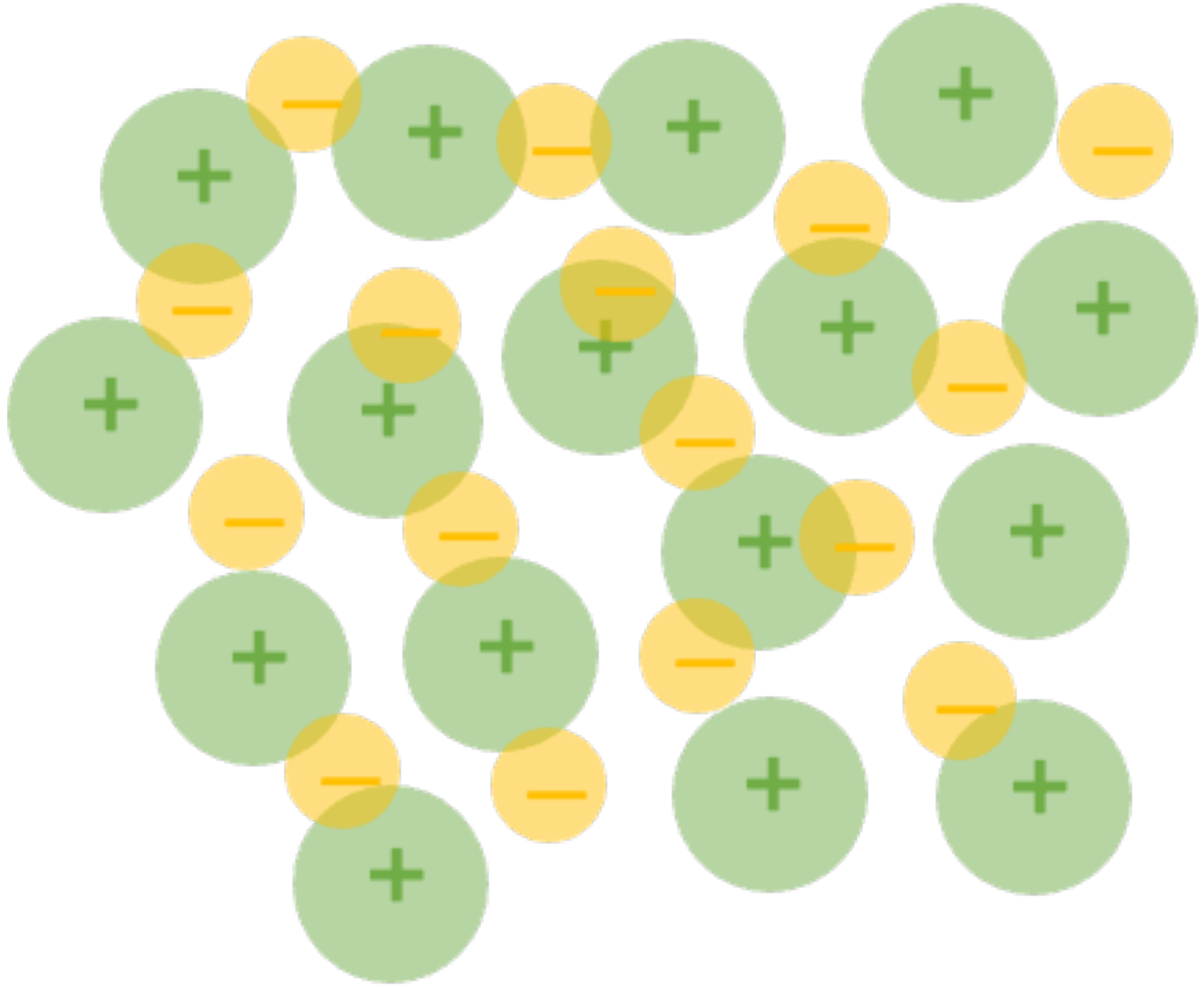

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European Research Council EIC Pathfinder
Open under grant agreement No 101047223.

Acknowledgments

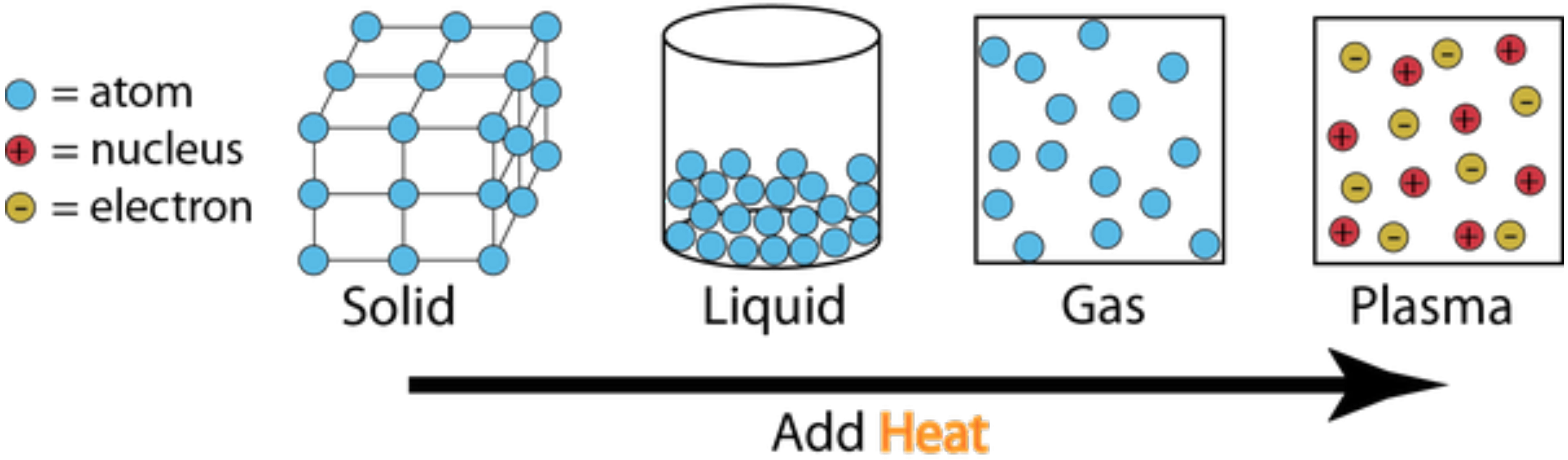
Lecture based on:

- Francis F. Chen, *Introduction to Plasma Physics and Controlled Fusion*, Springer Cham, DOI 10.1007/978-3-319-22309-4, third edition, 2016
- Paul Gibbon, *Introduction to Plasma Physics*, arXiv:2007.04783v1, CAS - CERN Accelerator School: High Gradient Wakefield Accelerators, 11-22 March 2019, Sesimbra, Portugal
- Laurent Gremillet, *Introduction à l'interaction laser-plasma relativiste, 2012*

Contents

What is a plasma?

For general public:



What is a plasma?

Simple definition of plasma

Quasi-neutral gas of charged particles showing *collective behaviour*

Quasi-neutrality: particle density of electrons n_e, and ions, n_i with charge state Z are locally balanced

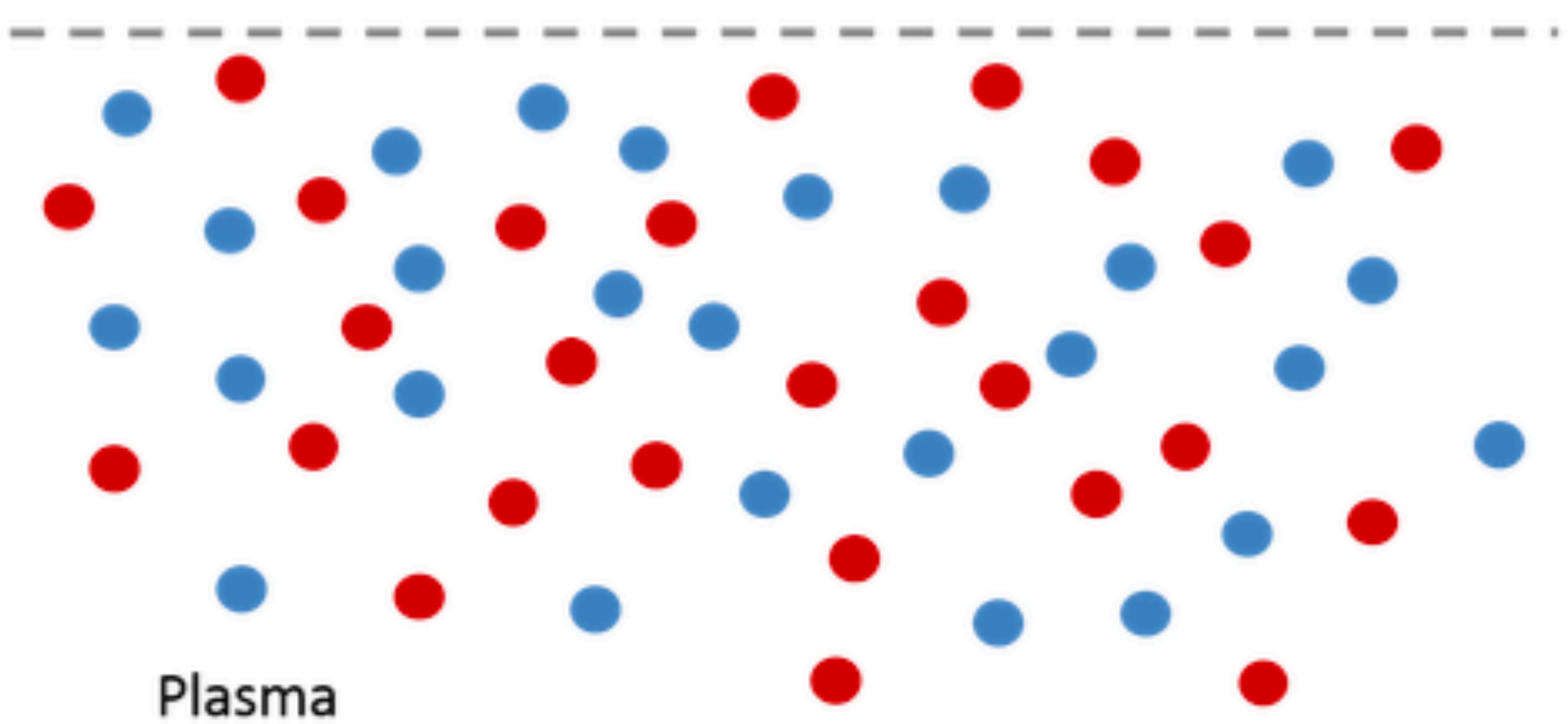
$$n_e \simeq Zn_i$$

Collective behaviour: local disturbances from equilibrium can have strong influence on remote regions

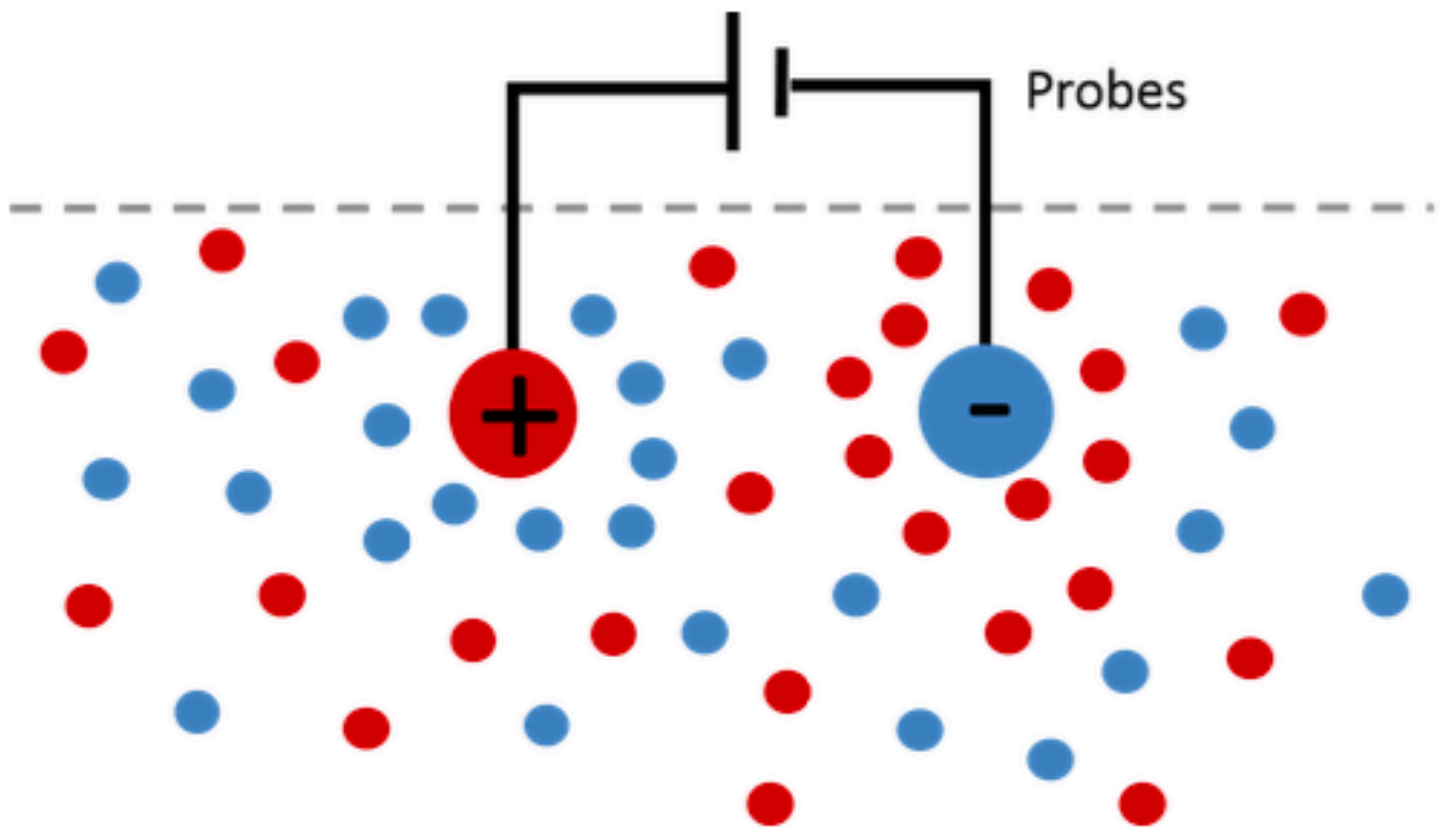
$$\vec{\nabla} \cdot \vec{E} = \frac{Zn_i - n_e}{\epsilon_0}$$

Debye shielding

Equilibrium plasma



Perturbed plasma



Debye length

Thermal equilibrium $(T_e = T_i)$ $\quad \frac{1}{2}m_e\langle v_e^2\rangle = \frac{1}{2}m_i\langle v_i^2\rangle = \frac{3}{2}k_BT_e$

For H_2 plasma $\frac{\langle v_i\rangle}{\langle v_e\rangle} = \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \simeq \frac{1}{43}$, ions are stationary compared to e^-

$\Rightarrow n_i \simeq n_0$, and thus $n_e = n_0 \exp(e\phi/k_BT_e)$ (Boltzmann distribution)

Recall Gauss law (1D) $\nabla^2\phi = \partial_x^2\phi = -\frac{\rho}{\epsilon_0} = -\frac{e}{\epsilon_0}(n_i - n_e)$

After some maths, we can solve $\phi(r) = \frac{1}{4\pi\epsilon_0}\frac{e^{-r/\lambda_D}}{r}$, $\lambda_D = \left(\frac{\epsilon_0 k_B T_e}{e^2 n_e}\right)^{1/2}$

Particles per Debye sphere $N_D = n_e\frac{4\pi}{3}\lambda_D^3$, *plasma parameter* $g = \frac{1}{n_e}\lambda_D^3$

Where are plasmas?

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1. Cosmos (99% of visible universe):

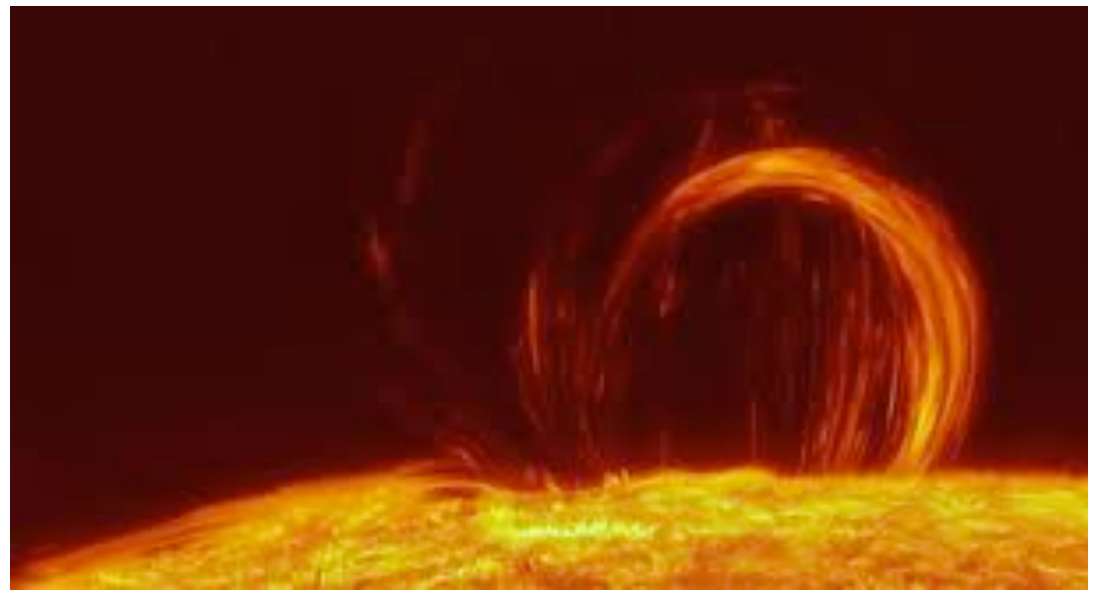
- Interstellar medium
- Stars
- Jets

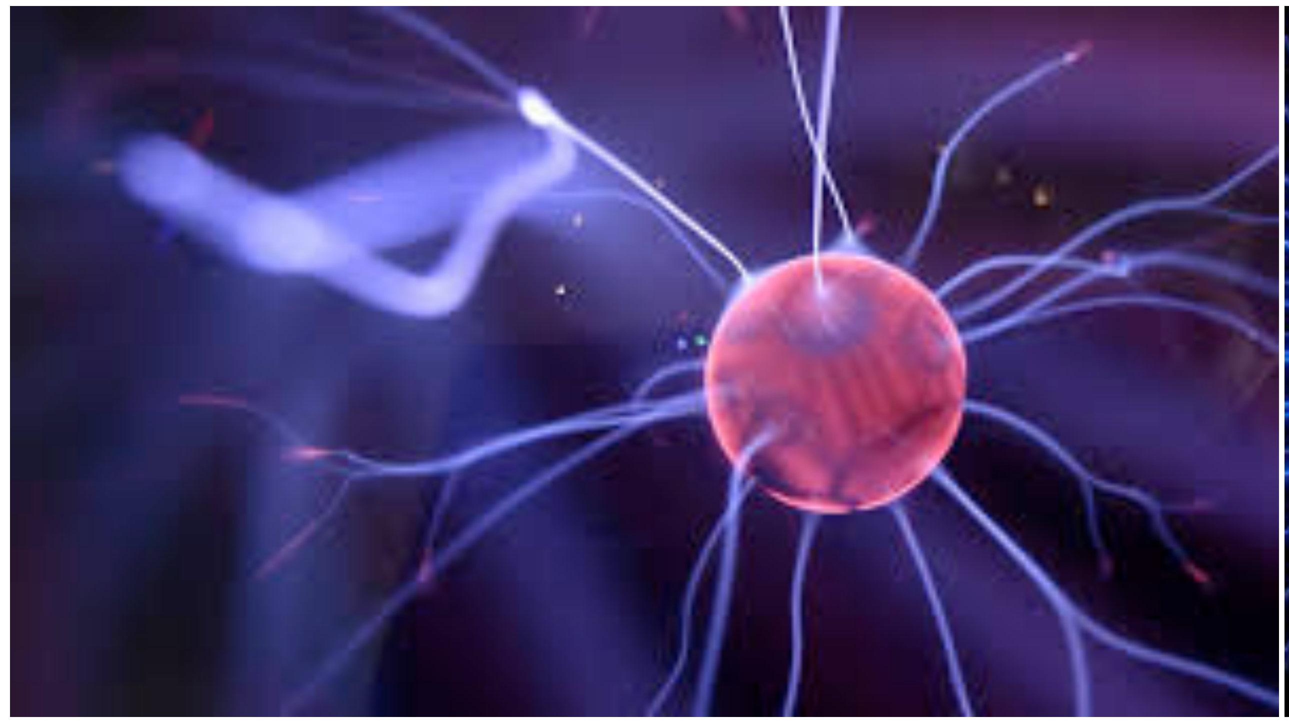
2. Ionosphere:

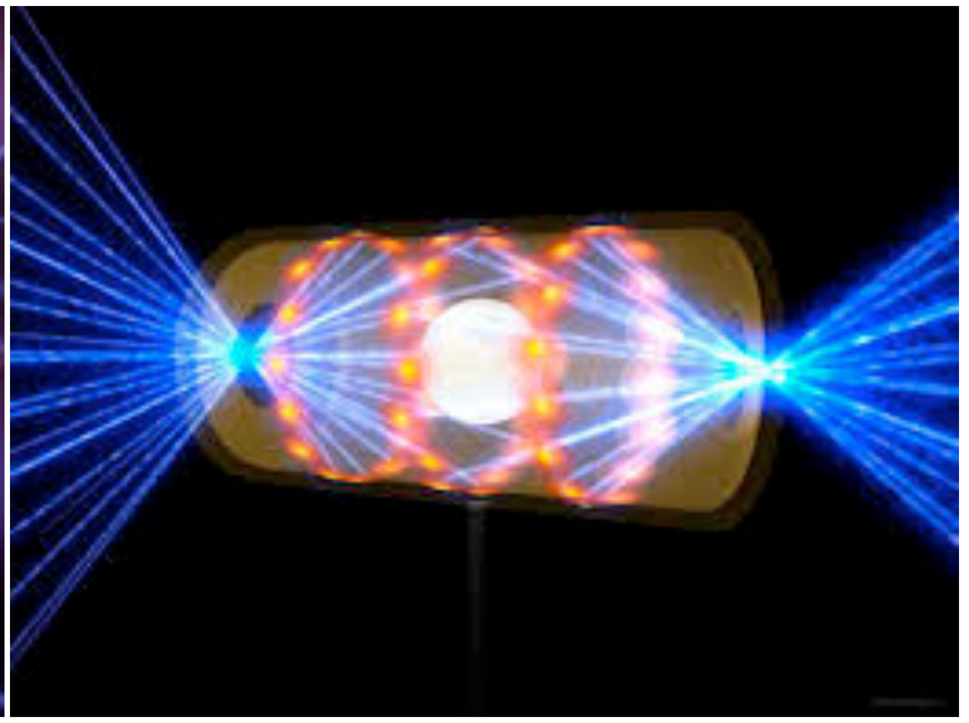
- $\lesssim 50$ km = 10 Earth-radii
- Long-wave radio

3. Earth:

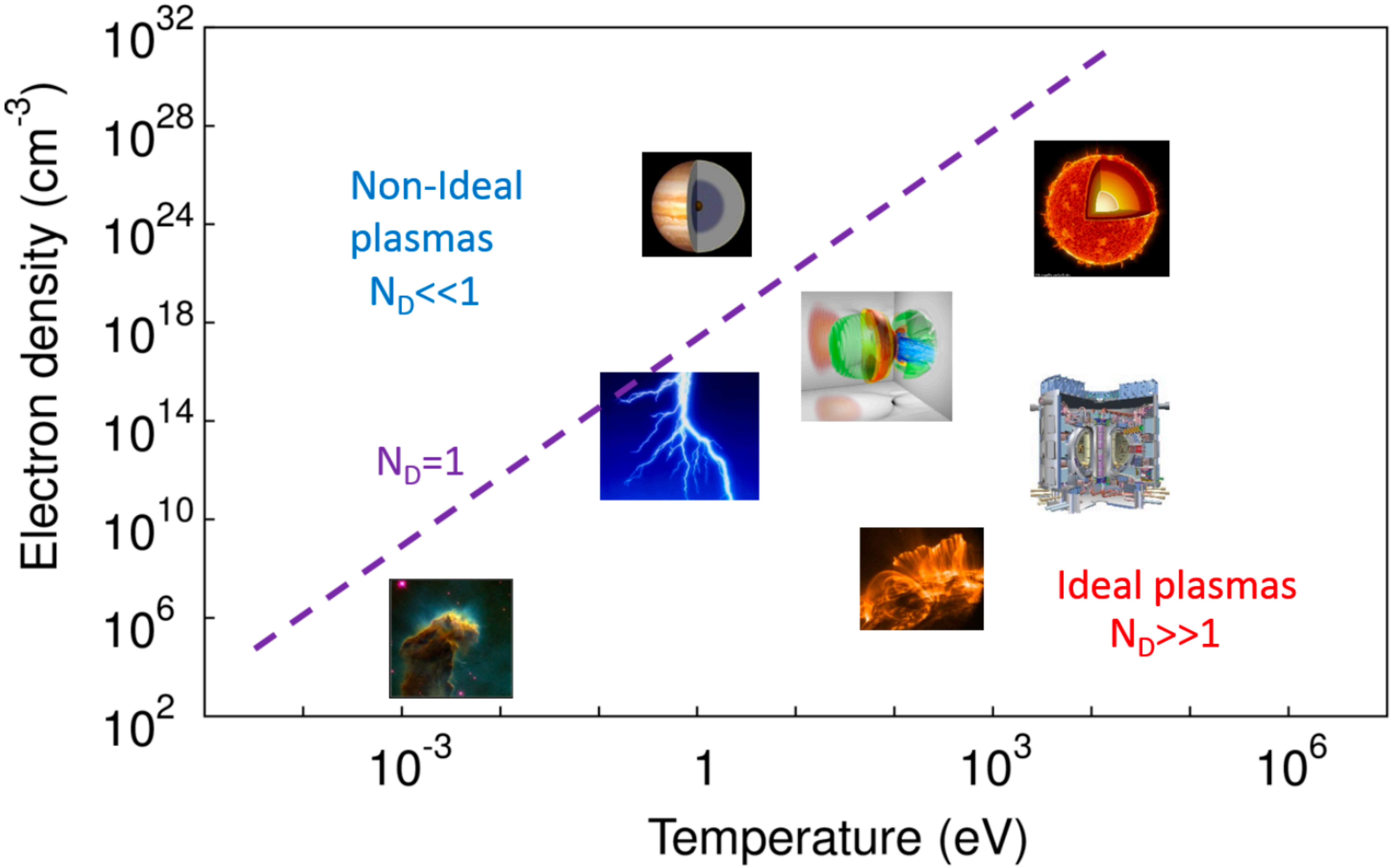
- Fusion devices
- Street Lighting
- Plasma torches
- Discharges - lightning
- Plasma accelerators and radiation sources







Plasma classification

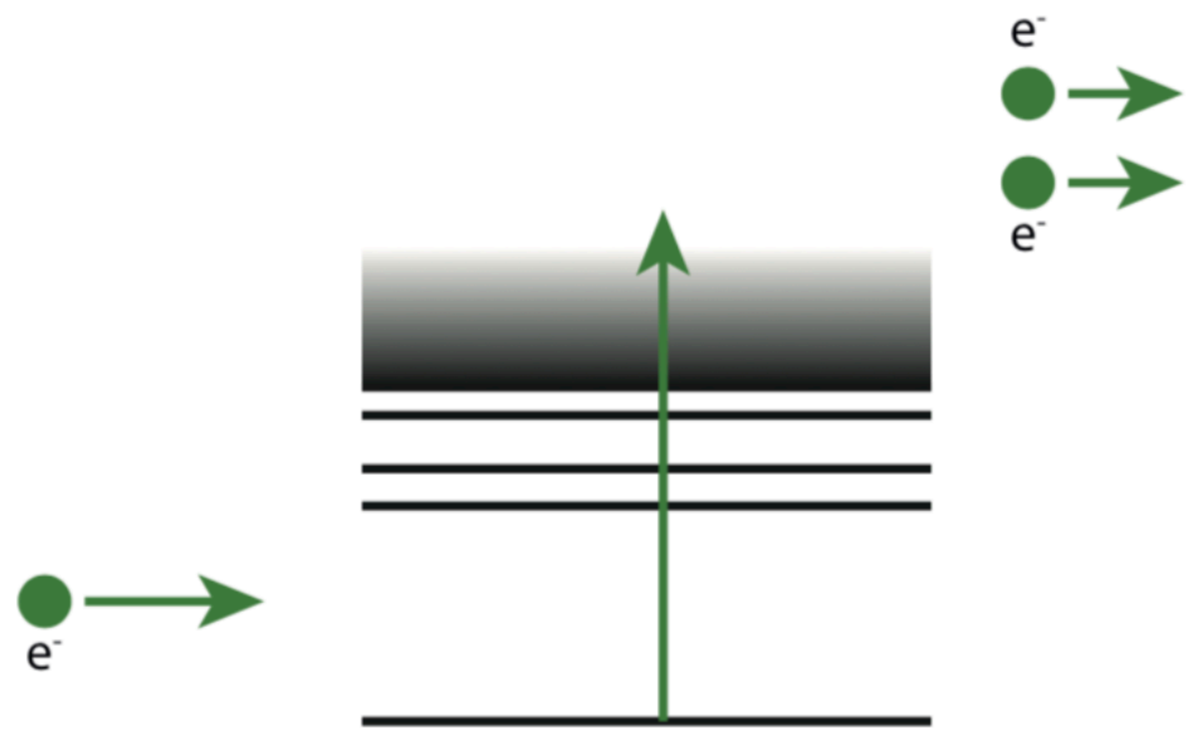


Contents

How to create a plasma

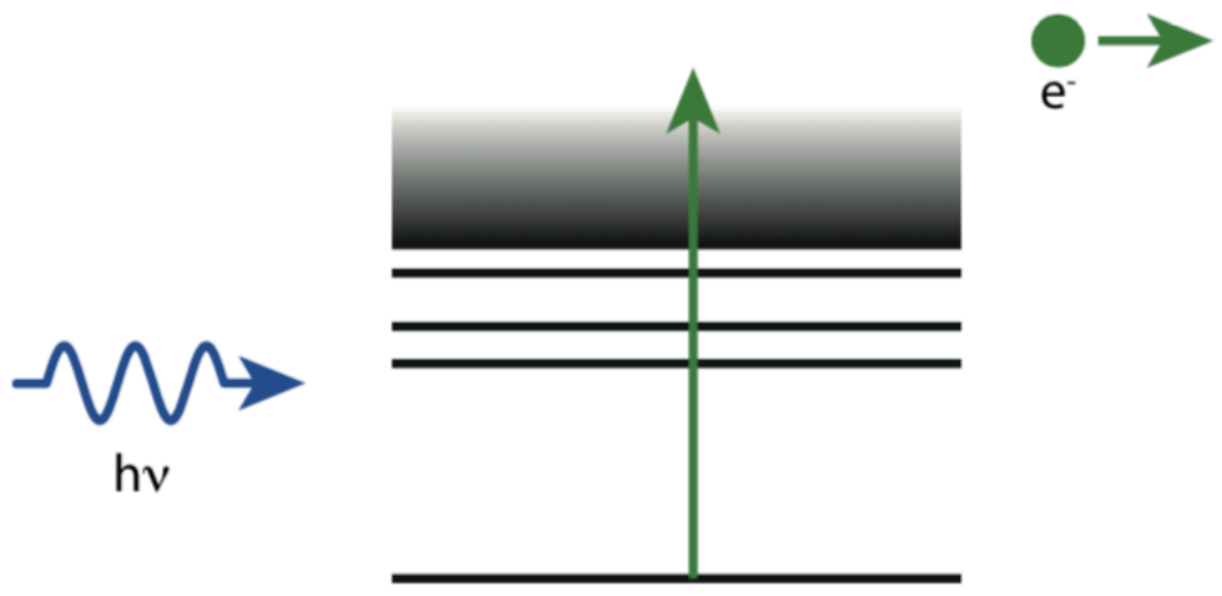
Impact ionisation

$$A^{Z+} + e^- \rightarrow A^{(Z+1)+} + e^{-\prime} + e^{-\prime\prime}$$



Photon ionisation

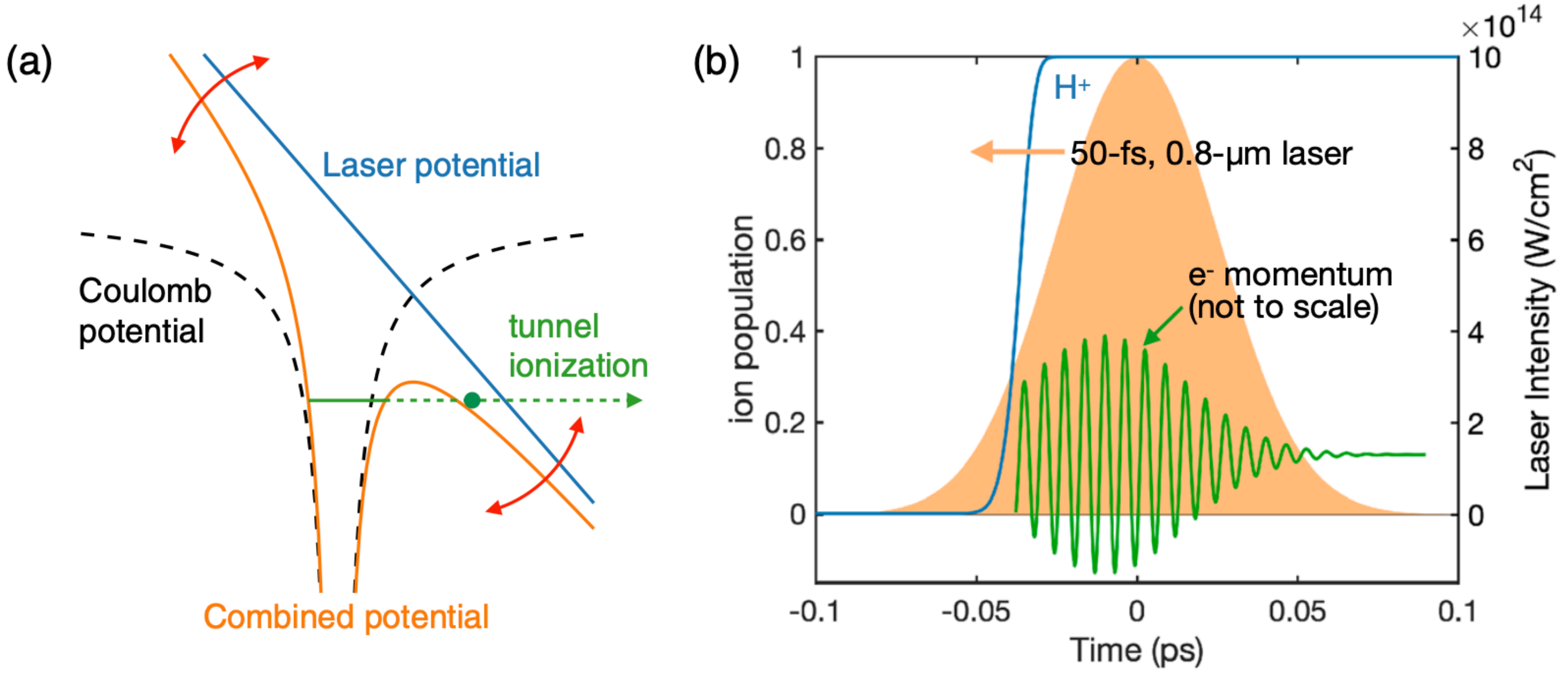
$$A^{Z+} + \hbar\omega \rightarrow A^{(Z+1)+} + e^-$$



Field ionisation - Laser

Electric field (*classical*) experience by the Hydrogen e^-:

$$E_a = \frac{e}{4\pi\epsilon_0 a_B^2} \simeq 5.1 \times 10^9\,\mathrm{V/m} \rightarrow I_a \simeq 3.5 \times 10^{16}\,\mathrm{W/cm^2}$$



Intense electric field bends Coulomb potential and leads to tunnel ionisation*

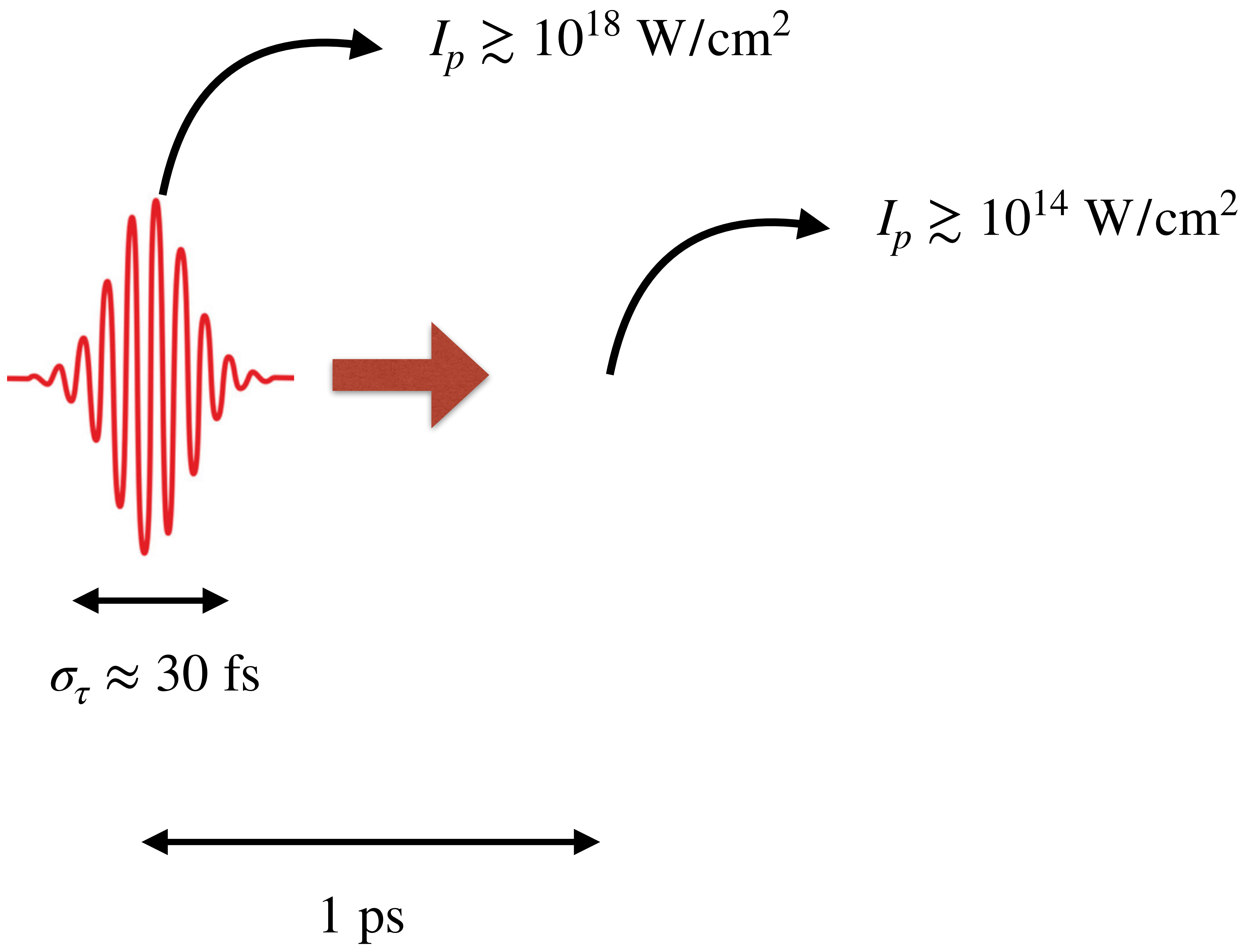
Multiphoton effects: ionisation threshold $\sim 10^{14}\,\mathrm{W/cm^2}$

Field ionisation also applies to discharge plasma sources and relativistic particle beam ionisation

*C. Zhang *et al.*, Review of Modern Plasma Physics, Volume 7, article number 34, (2023)

Plasmas for particle accelerators

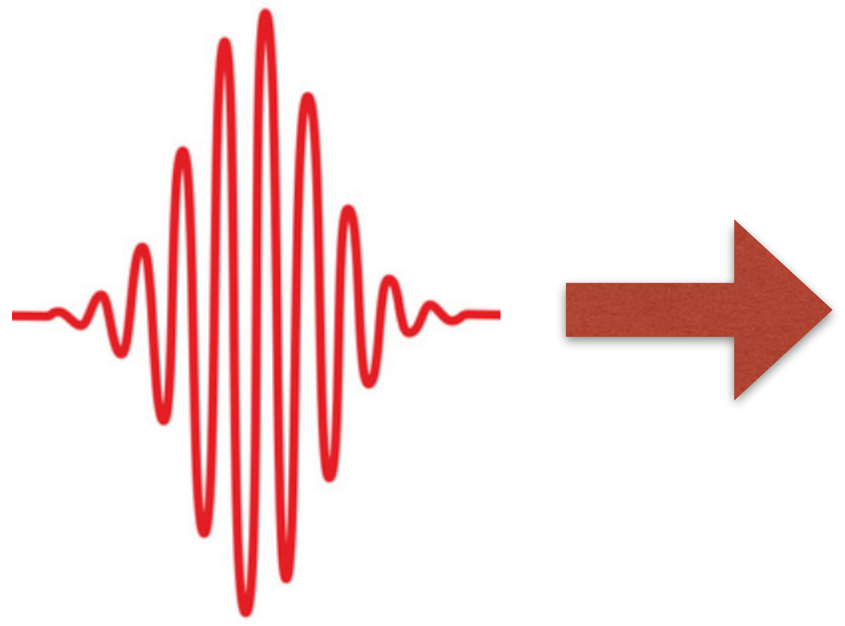
High-power laser

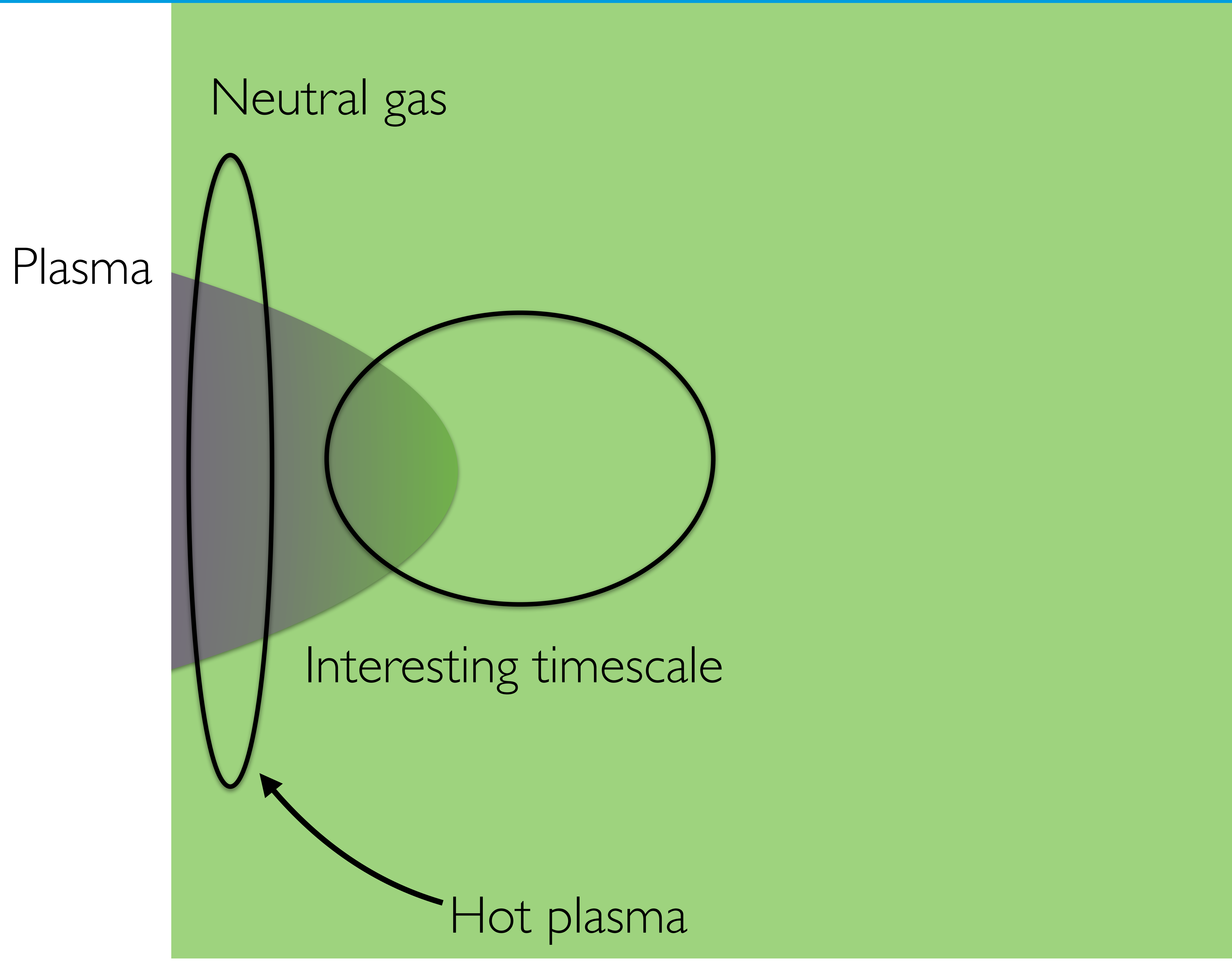


Neutral gas

Plasmas for particle accelerators

High-power laser

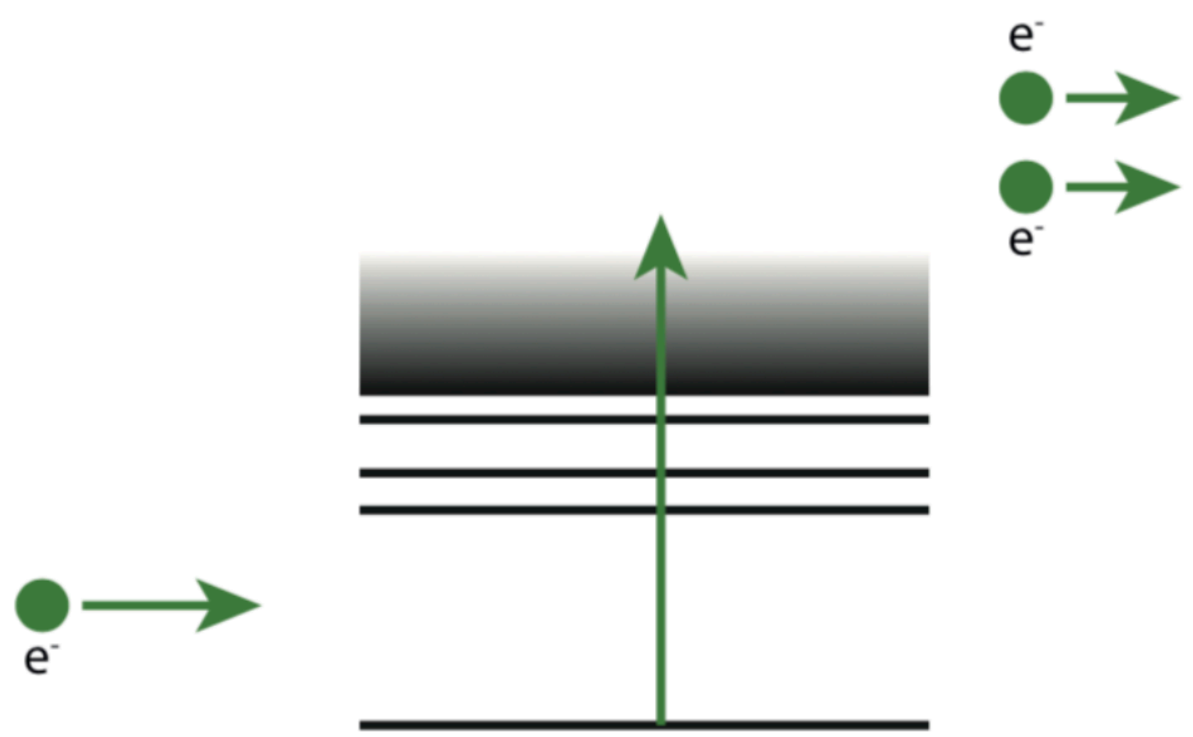


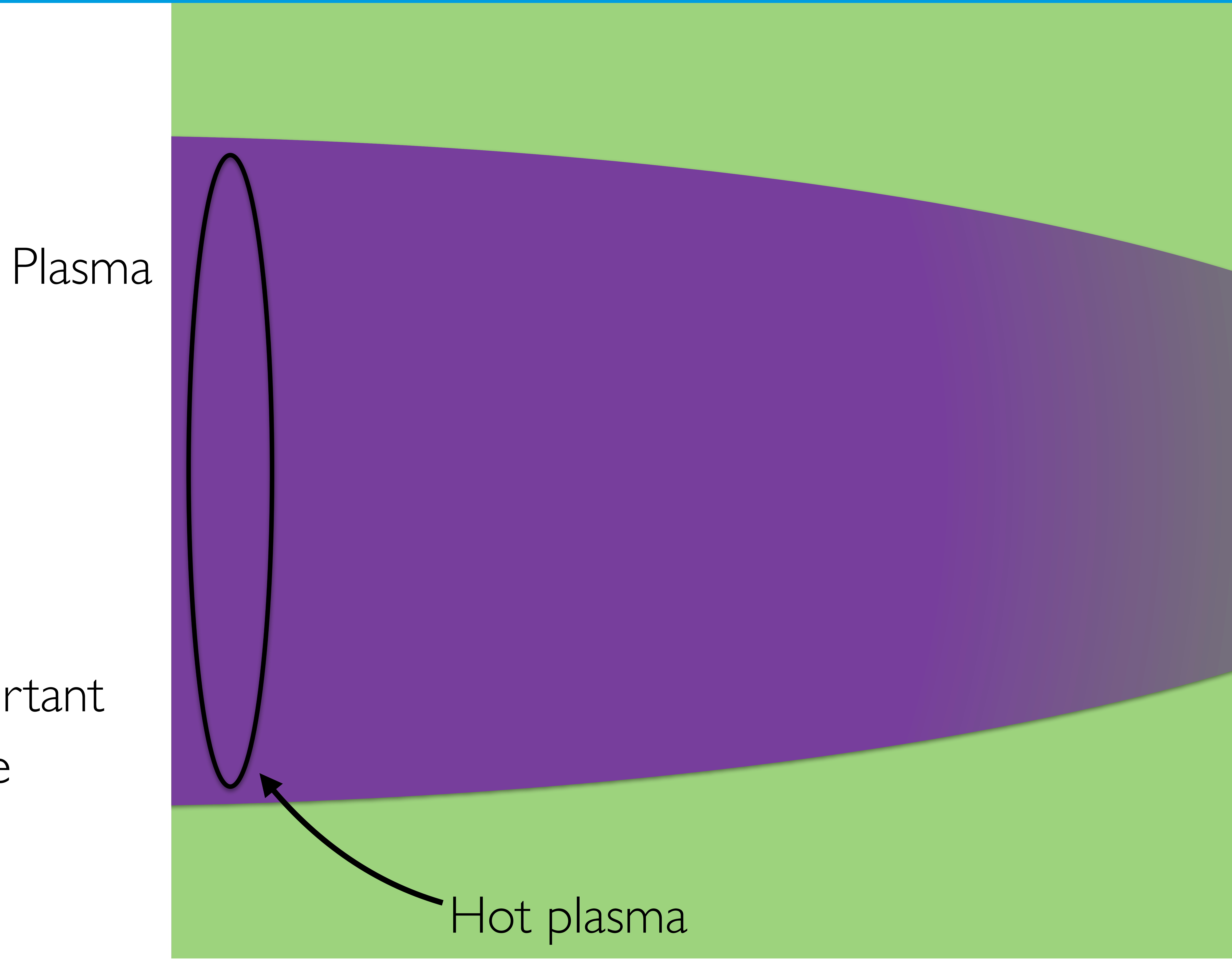


Plasmas for particle accelerators

Impact ionisation

$$A^{Z+} + e^- \rightarrow A^{(Z+1)+} + e^{-\prime} + e^{-\prime\prime}$$





Impact ionisation is going to play an important role in the long timescale evolution of the plasma: cooling down and recovery

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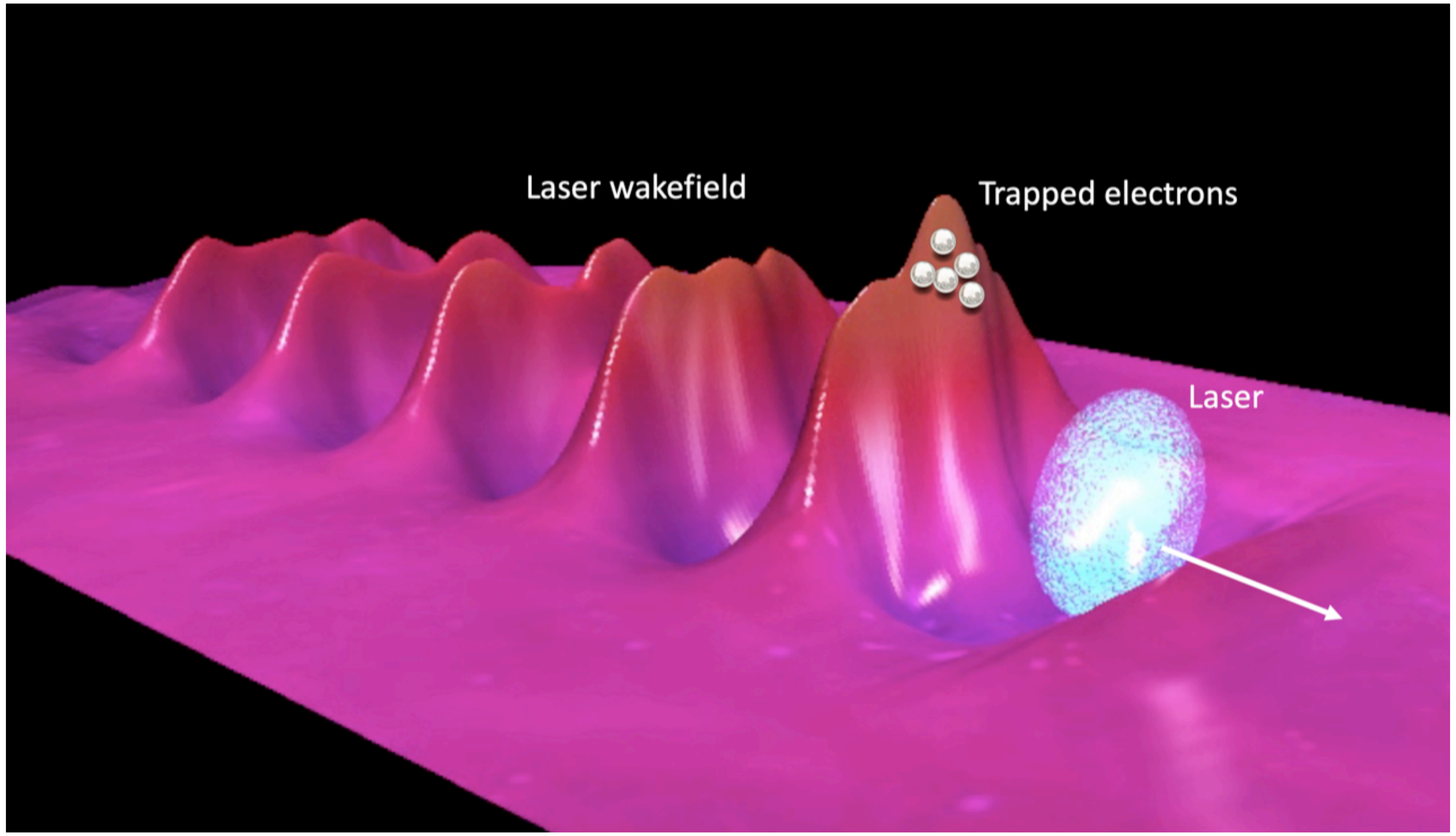
What is a plasma?

How to create a plasma?

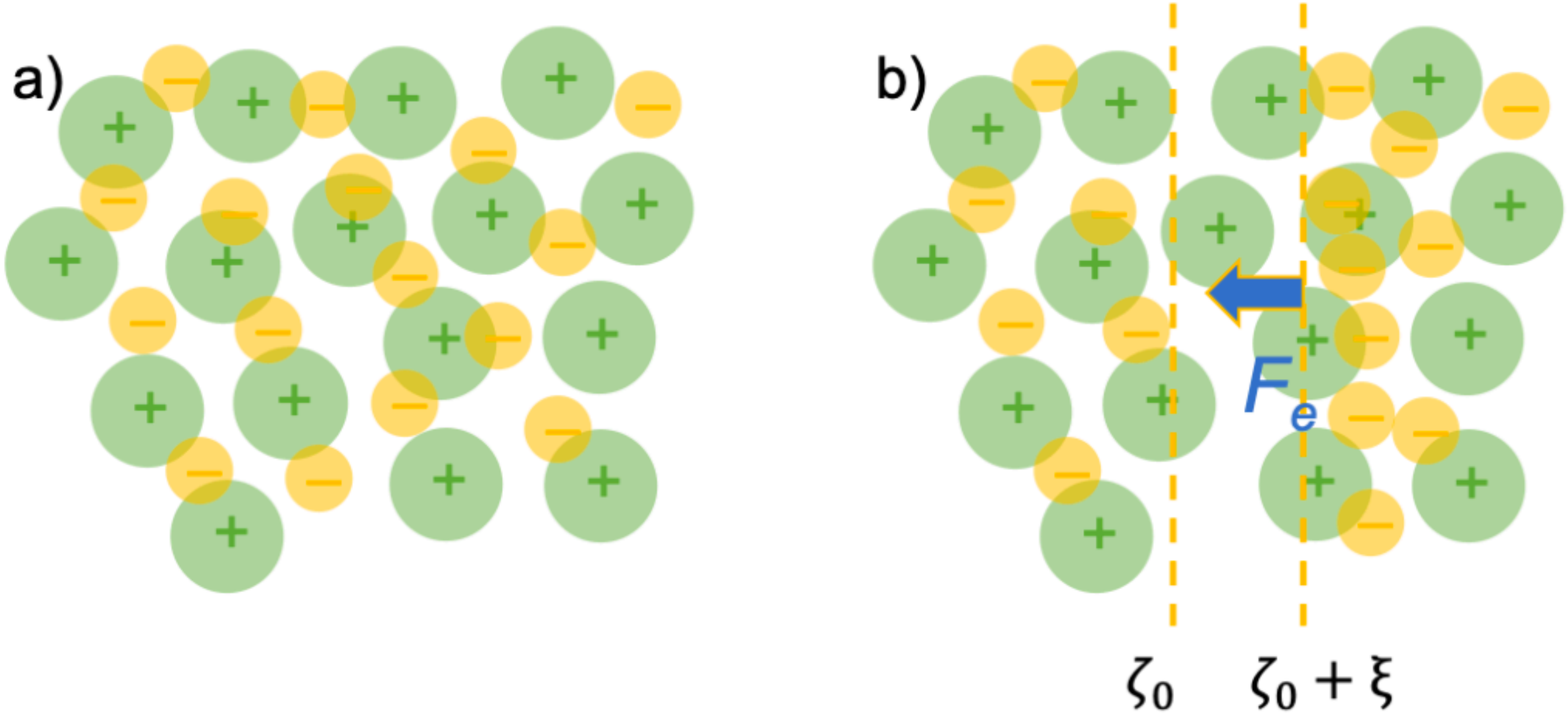
Waves and plasmas

Beam-plasma instabilities

Plasma waves



Plasma waves: plasma frequency

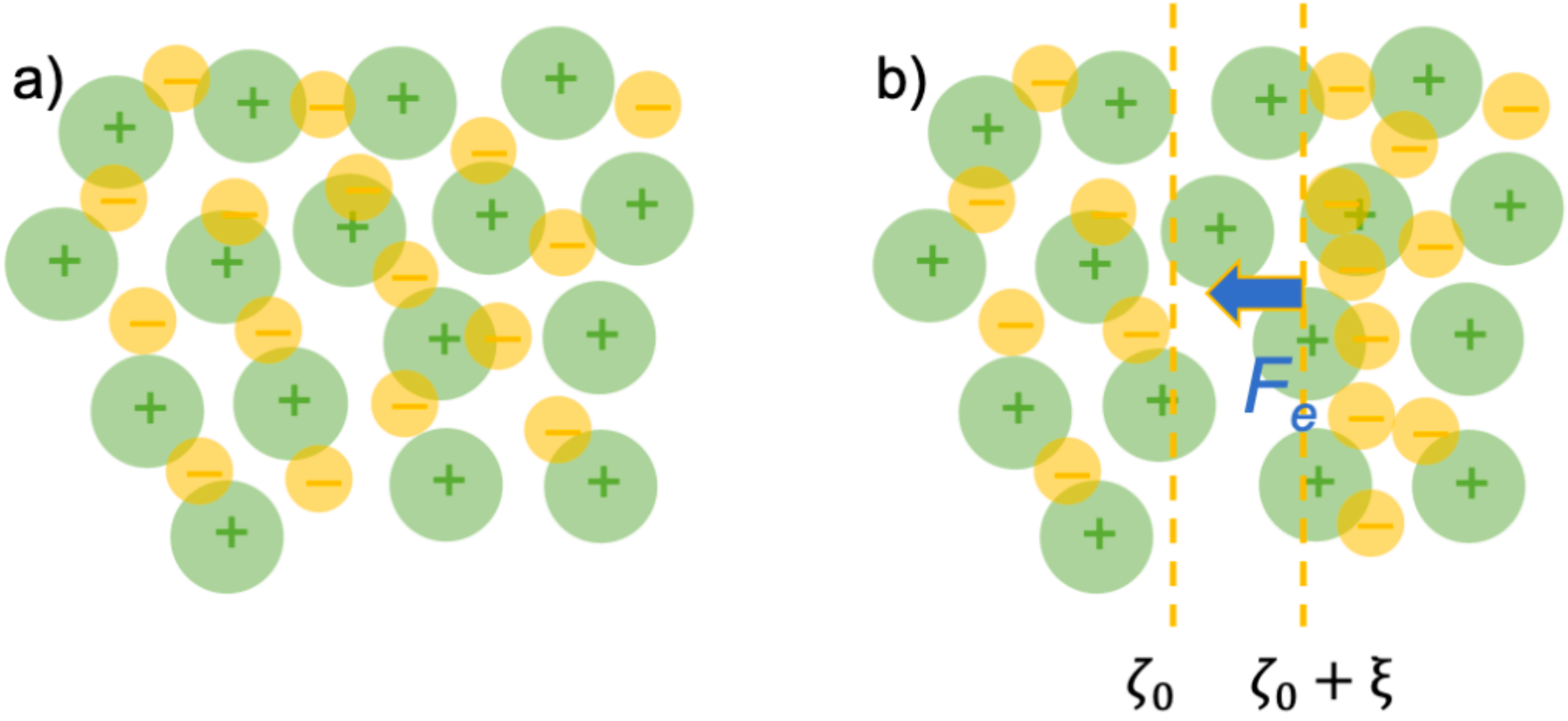


$$E(\xi_0 + \xi) = \frac{n_0 e}{\epsilon_0}\xi$$

$$\partial_t^2 \xi = -\frac{e}{m_e}E_0 = -\frac{e^2 n_0}{m_e \epsilon_0}\xi$$

$$\omega_p = \sqrt{\frac{e^2 n_0}{m_e \epsilon_0}}$$

Plasma waves: wave breaking field



$$\omega_p = \sqrt{\frac{e^2 n_0}{m_e \epsilon_0}} \rightarrow k_p = \frac{\omega_p}{c}$$

$$E(\xi_0 + k_p^{-1}) = \frac{n_0 e}{\epsilon_0} k_p^{-1} = \frac{m_e c \omega_p}{e}$$

$$E_{\mathrm{WB}}(n_p = 10^{18}\ \mathrm{cm}^{-3}) \approx 10^{11}\ \mathrm{V/m}$$

Waves and plasmas

Theoretical approaches

- First principles N-body dynamics
- Phase-space method - Vlasov-Boltzman equation
- Two fluid equations
- Magnetohydrodynamics

Two fluid (ions and e^-) equations

System variables: (s specie, e for e^-, i for ions)

- Density $n_s(\vec{r}, t)$
- Velocity $\vec{v}_s(\vec{r}, t)$

Fluid equations:

- Continuity: $\partial_t n_s + \vec{\nabla}(n_s \vec{v_s}) = 0$
- Eq. of motion: $m_s d_t \vec{v}_s = q_s(\vec{E} + \vec{v}_s \times \vec{B})$

Maxwell's equations

Assumptions

Stationary ions: $\langle v_i \rangle / \langle v_e \rangle = \left(m_e/m_i\right)^{\frac{1}{2}} \simeq 1/43$

Collision-less plasma

Cold plasma: $P_e = n_0 k_B T_e \simeq 0$

Non-relativistic electrons: $\gamma_e = 1$

Linearisation of equations

In order to simplify the model, we can assume an initially equilibrium plasma of density n_0 and velocity $v_0 = 0$, to which we add a **small** perturbation $n_1(\vec{r}, t), \vec{v_1}(\vec{r}, t), \vec{E_1}(\vec{r}, t), \ldots$

Continuity eq.: $\partial_t n_1 + n_0 \vec{\nabla} \vec{v_1} = 0$

Eq. of motion: $m_e \partial_t \vec{v_1} = q_e \vec{E_1}$

Plasma oscillations

Plasma oscillations

Continuity eq.: $\partial_t n_1 + n_0 \vec{\nabla} \vec{v_1} = 0$

Eq. of motion: $m_e \partial_t \vec{v_1} = q_e \vec{E_1}$

Poisson eq.: $\epsilon_0 \vec{\nabla} \vec{E_1} = q_e n_1$

Oscillatory solutions: $X_1(\vec{r}, t) \propto e^{i(kx-\omega t)}$

Formal substitutions $\partial_t \to -i\omega,\ \vec{\nabla} \to ik$

$$-i\omega n_1 = -n_0 ik v_1$$

$$-i\omega m_e v_1 = -eE_1$$

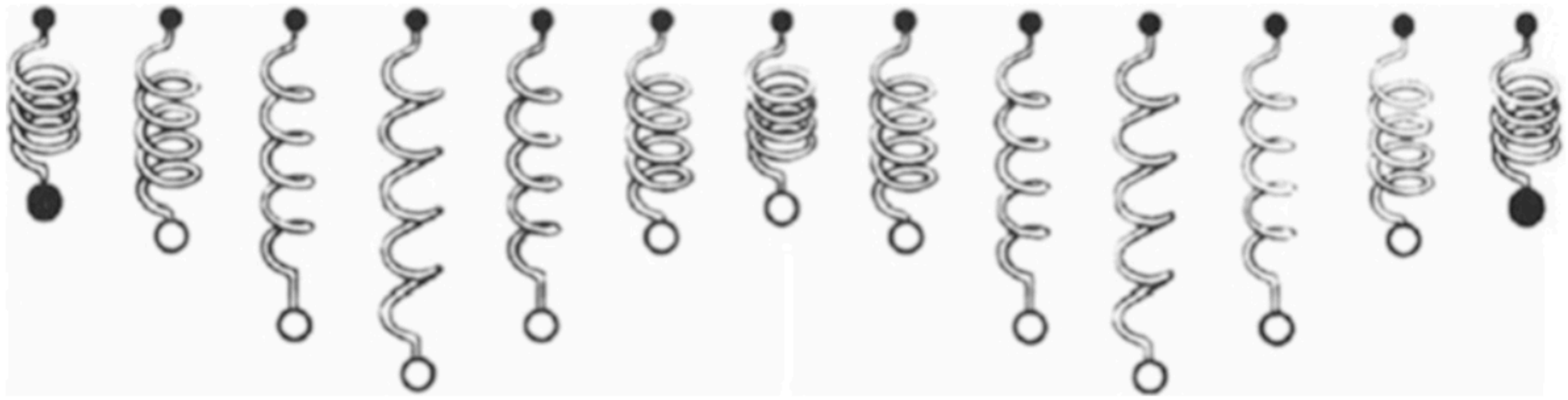
$$ik\epsilon_0 E_1 = -en_1$$

Plasma frequency

$$-i\omega m_e v_1 = -i\frac{n_0 e^2}{\epsilon_0 \omega} v_1 \Rightarrow \omega_p^2 = \omega^2 = \frac{n_0 e^2}{m_e \epsilon_0} \quad \forall k$$

Group velocity $\partial\omega/\partial k = 0$, no propagation

Individual harmonics oscillators



Plasma oscillations with temperature

Plasma oscillations

Continuity eq.: $\partial_t n_1 + n_0 \vec{\nabla} \vec{v_1} = 0$

Eq. of m.: $m_e n_0 \partial_t \vec{v_1} = n_0 q_e \vec{E_1} - 3k_B T_e \overrightarrow{\nabla n_1}$

Poisson eq.: $\epsilon_0 \vec{\nabla} \vec{E_1} = q_e n_1$

Oscillatory solutions: $X_1(\vec{r}, t) \propto e^{i(kx - \omega t)}$

Formal substitutions $\partial_t \rightarrow -i\omega,\ \vec{\nabla} \rightarrow ik$

$$-i\omega n_1 = -n_0 ik v_1$$

$$-i\omega m_e n_0 v_1 = -en_0 E_1 - 3KT_e ikn_1$$

$$ik\epsilon_0 E_1 = -en_1$$

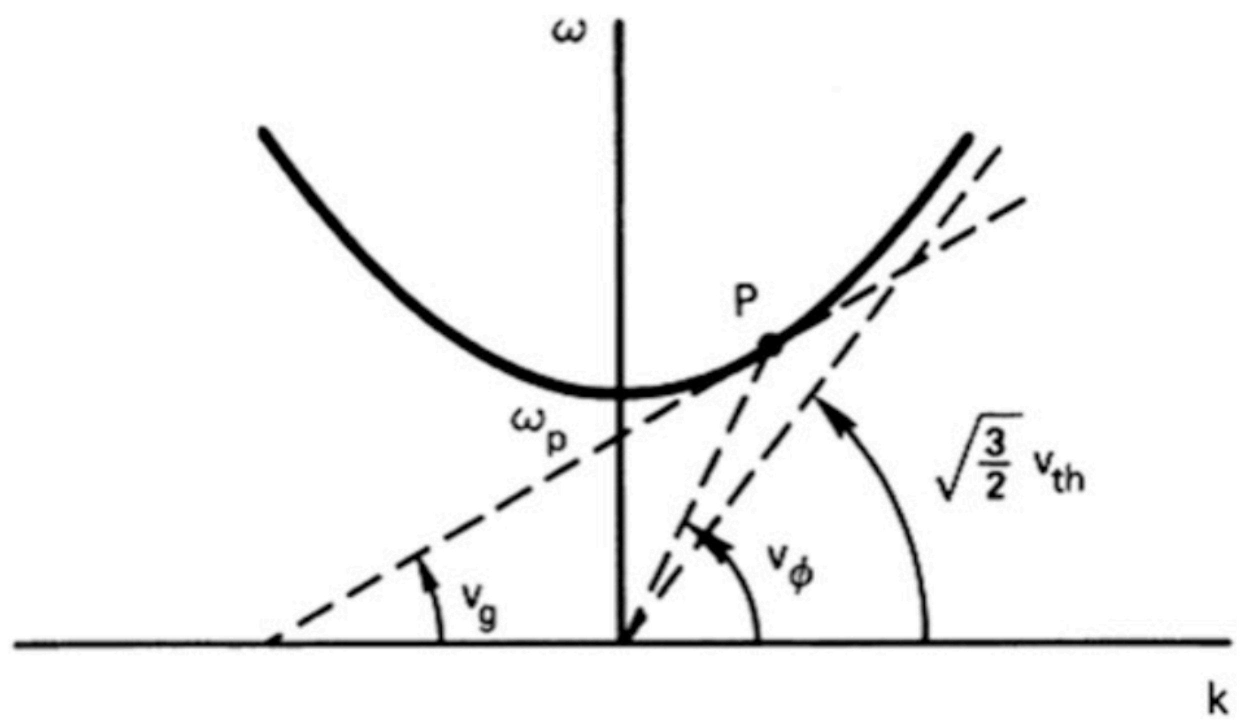
Dispersion relation

$$-i\omega m_e n_0 v_1 = \left[\frac{-n_0 e^2}{ik\epsilon_0} + 3KT_e ik\right]\frac{n_0 ik}{i\omega} v_1 \Rightarrow \omega^2 = \omega_p^2 + \frac{3}{2}k^2 v_{th}^2$$

Group velocity $\partial\omega/\partial k = \frac{3}{2}\frac{v_{th}^2}{v_\phi}$, $v_\phi = \omega/k =$ phase velocity

1D Langmuir waves

$$v_{th}^2 = 2k_B T_e/m_e$$



Cold plasma approximation

$$v_{th} \ll v_g, v_\phi$$

Electromagnetic waves in plasmas

In vacuum

From Maxwell equations

$$\vec{\nabla} \times \vec{E}_1 = -\partial_t \vec{B}_1 \qquad c^2 \vec{\nabla} \times \vec{B}_1 = \partial_t \vec{E}_1$$

Solving for plane waves, $k \cdot E_1 = k \cdot B_1 = 0$

$$\omega^2 = k^2 c^2$$

In plasma

$$\vec{\nabla} \times \vec{E}_1 = -\partial_t \vec{B}_1$$

$$c^2 \vec{\nabla} \times \vec{B}_1 = \frac{\vec{j}_1}{\epsilon_0} + \partial_t \vec{E}_1$$

$\vec{j}_1$ = electronic plasma current

Dispersion relation of EM waves in plasmas

$$c^2 \vec{\nabla} \times \partial_t \vec{B}_1 = \frac{\partial_t \vec{j}_1}{\epsilon_0} + \partial_t^2 \vec{E}_1$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E}_1 = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}_1) - \nabla^2 \vec{E}_1 = \vec{\nabla} \times (-\partial_t \vec{B}_1)$$

$$\downarrow$$

$$-c^2 k^2 \vec{E}_1 = -i\omega \vec{j}_1 / \epsilon_0 - \omega^2 \vec{E}_1$$

Linearised equation of motion:

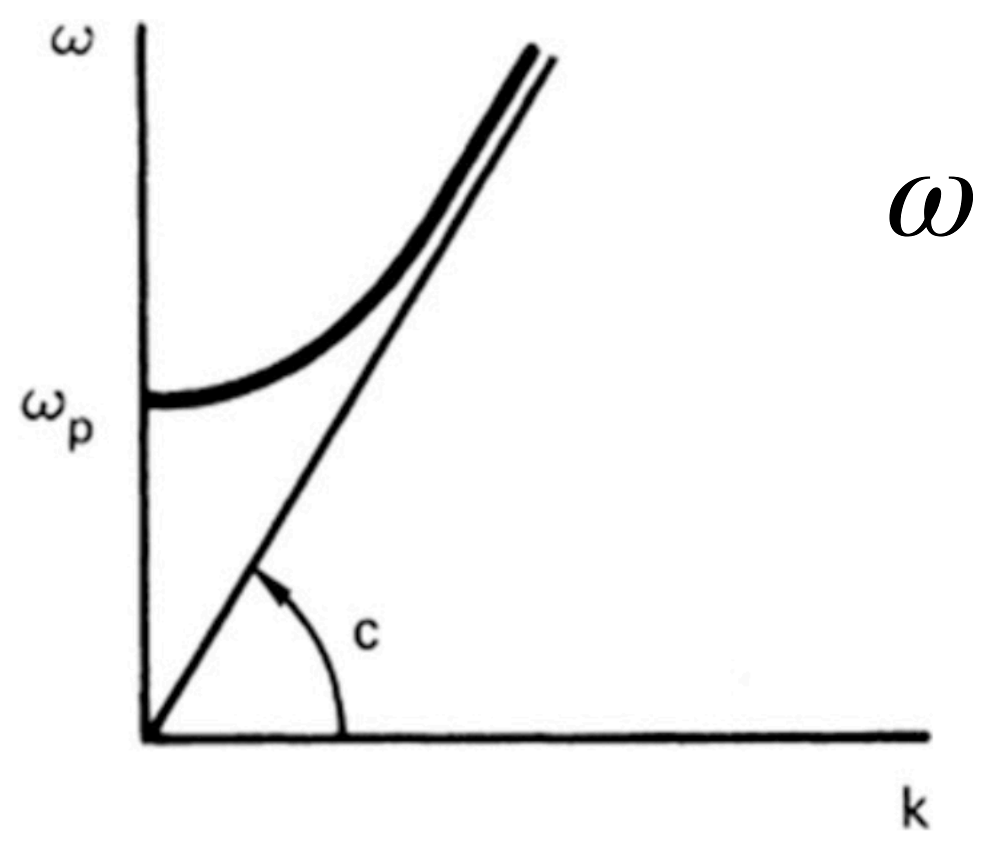
$$m_e \partial_t \vec{v}_1 = q_e \vec{E}_1 \rightarrow -i\omega m_e \vec{j}_1 = n_0 e \vec{E}_1$$

Dispersion relation

$$\omega^2 = \omega_p^2 + c^2 k^2$$

Electromagnetic waves in plasmas

Dispersion relation plot



$$\omega^2 = \omega_p^2 + c^2k^2$$

$$\omega_p^2 = \frac{n_0 e^2}{m_e \epsilon_0}$$

Velocities

$$v_\phi^2 = \frac{\omega^2}{k^2} = c^2 + \frac{\omega_p^2}{k^2} > c^2$$

$$v_g = \frac{\partial \omega}{\partial k} \approx c\left(1 - \frac{\omega_p^2}{2\omega^2}\right) < c$$

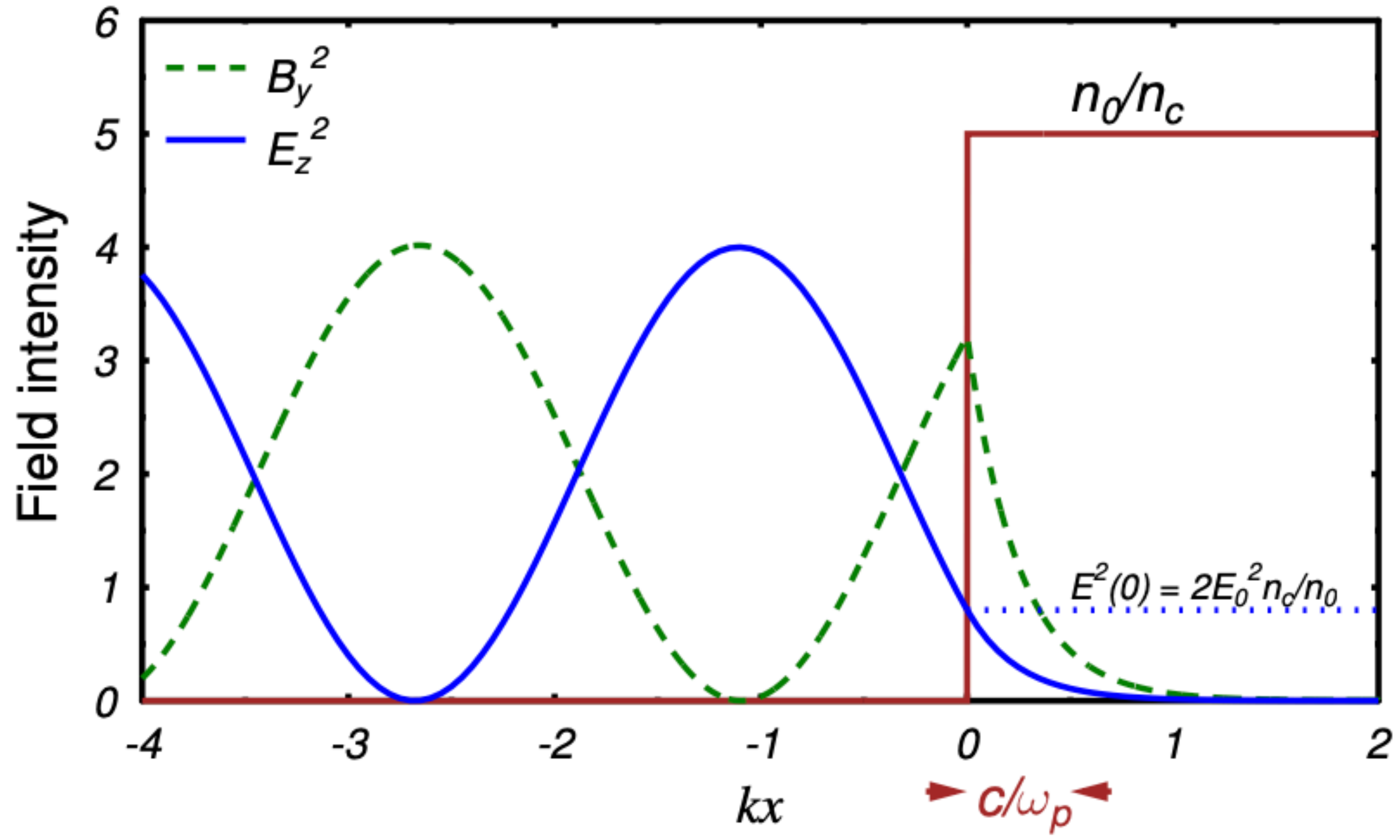
Critical density

Take EM wave of frequency $\omega_0 \gg \omega_p \;\Rightarrow k_{\text{plasma}} < k_{\text{vacuum}}$

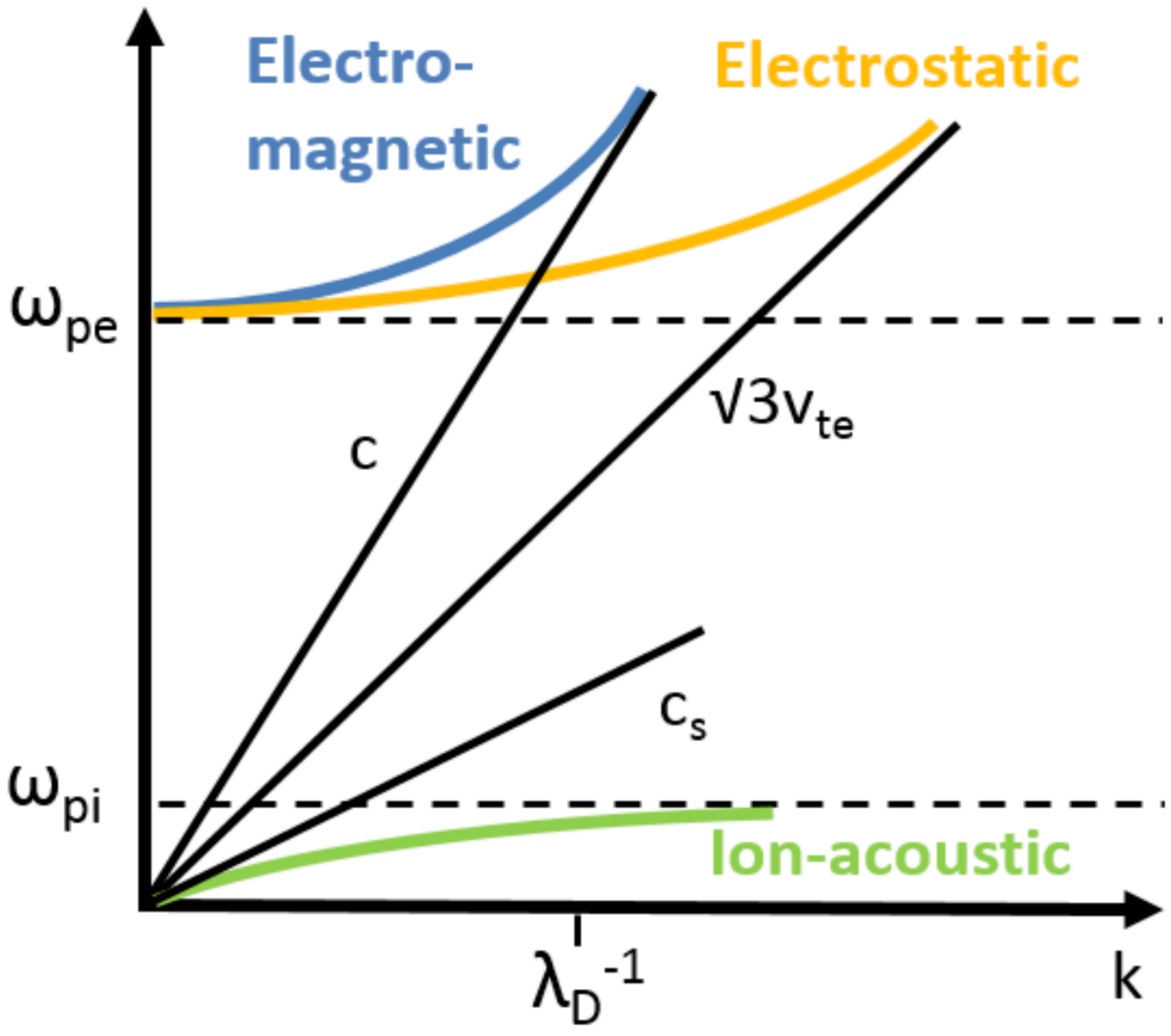
If we increase n_0, as $\omega_p \to \omega,\ \Re(k) \to 0$: no propagation

Define critical density for which EM waves are dumped in plasma

$$n_c = m\epsilon_0\omega^2/e^2$$



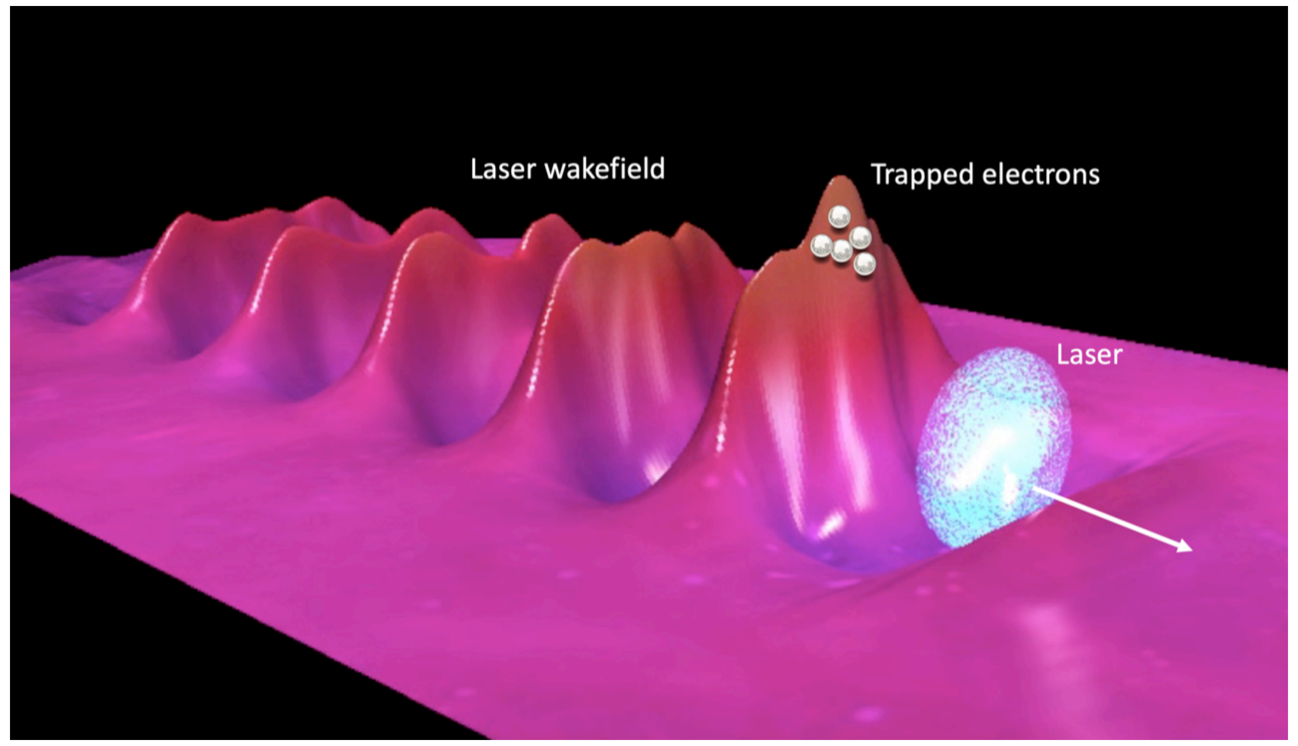
Plasma waves



So far we have introduced two plasma wave-like mechanics:

- Langmuir (electrostatic) waves
  - Too slow to accelerate relativistic particles
- Electromagnetic waves
  - No accelerating field

Plasma wakefield accelerator?



Plasma waves

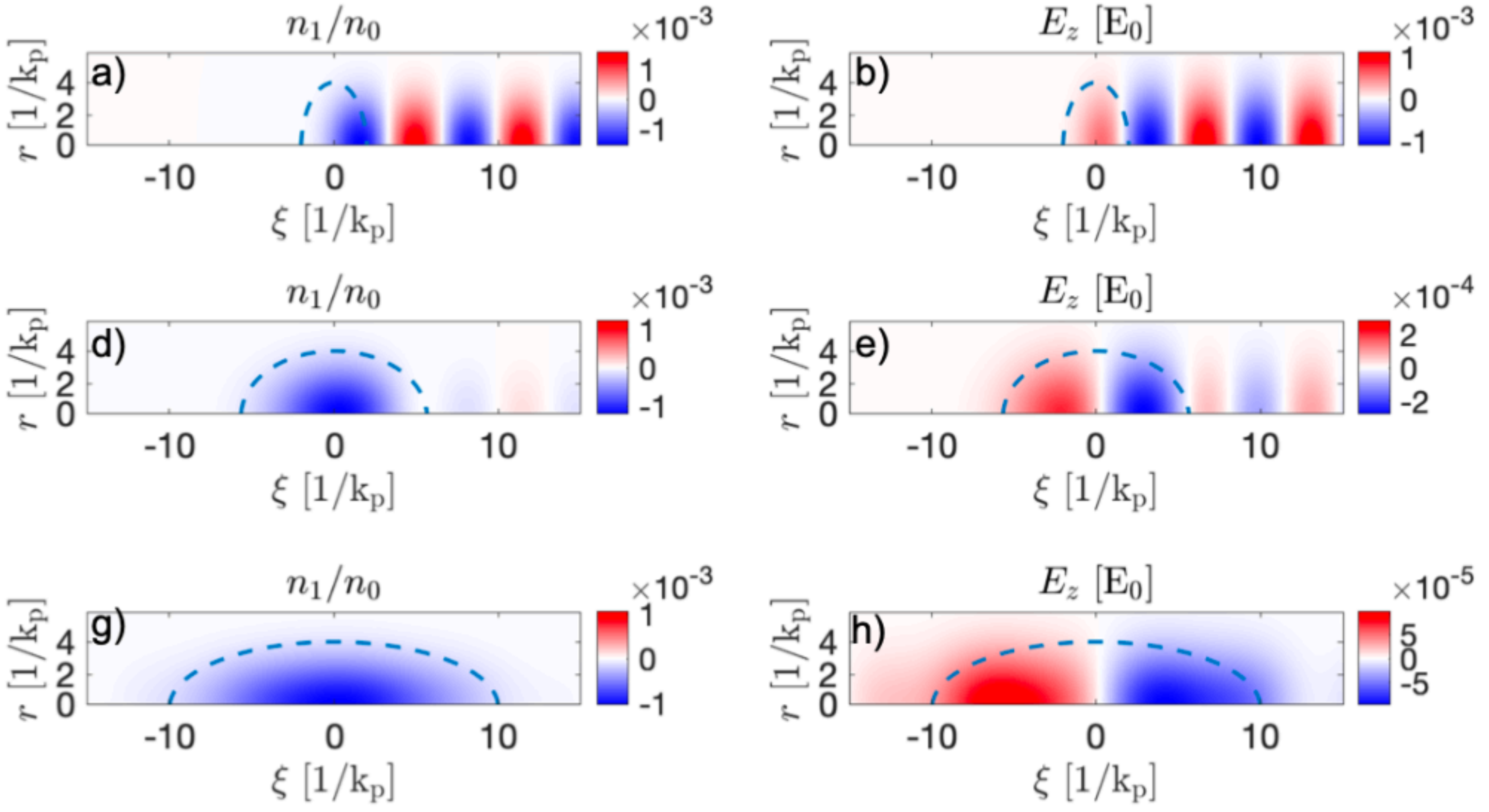
Beam-driven plasma wakefield - PWFA

Same equations (fluid-linear)

Treat beam as perturbative fluid specie

$$\partial_t^2 n_1(\vec{r},t) = -\frac{n_0 e^2}{m_e \epsilon_0}(n_1(\vec{r},t) + n_b(\vec{r},t))$$

+ quasi-static approximation…



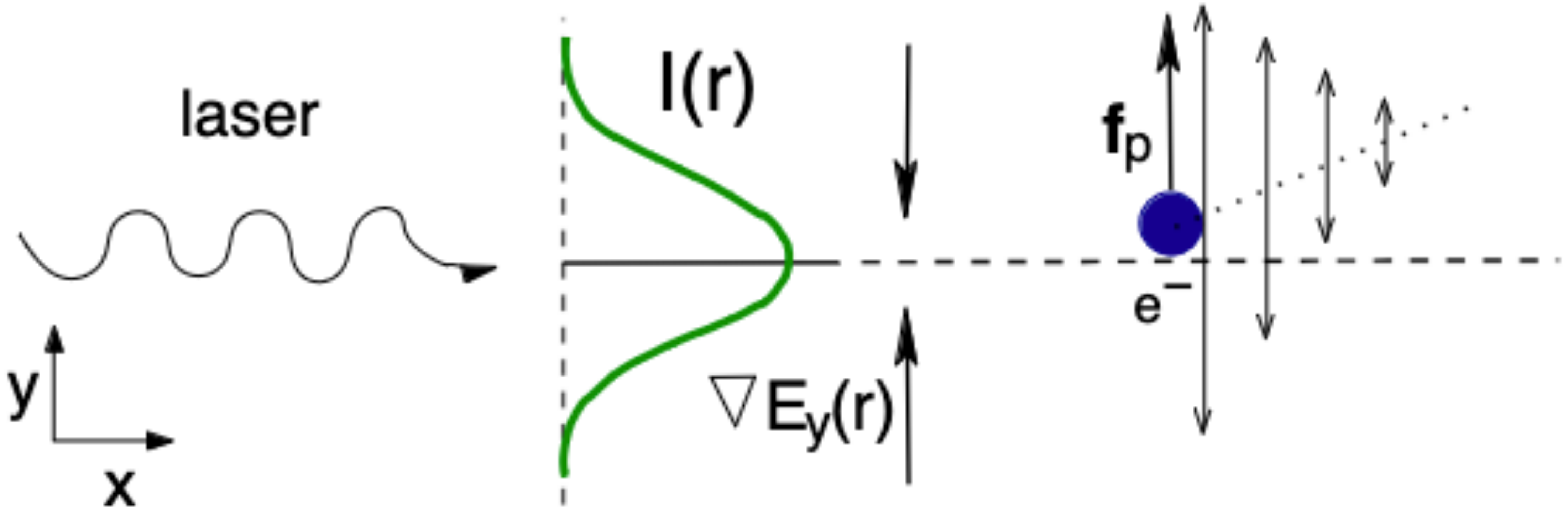
Laser-driven plasma wakefield - LWFA

We have all elements (EM fields + plasma)

Linear wave-e^- interaction: no net energy exchange

We need second order (non-linear) interactions

Ponderomotive force $f_p = -\frac{e^2}{4 m_e \omega^2}\partial_\perp E_y^2$



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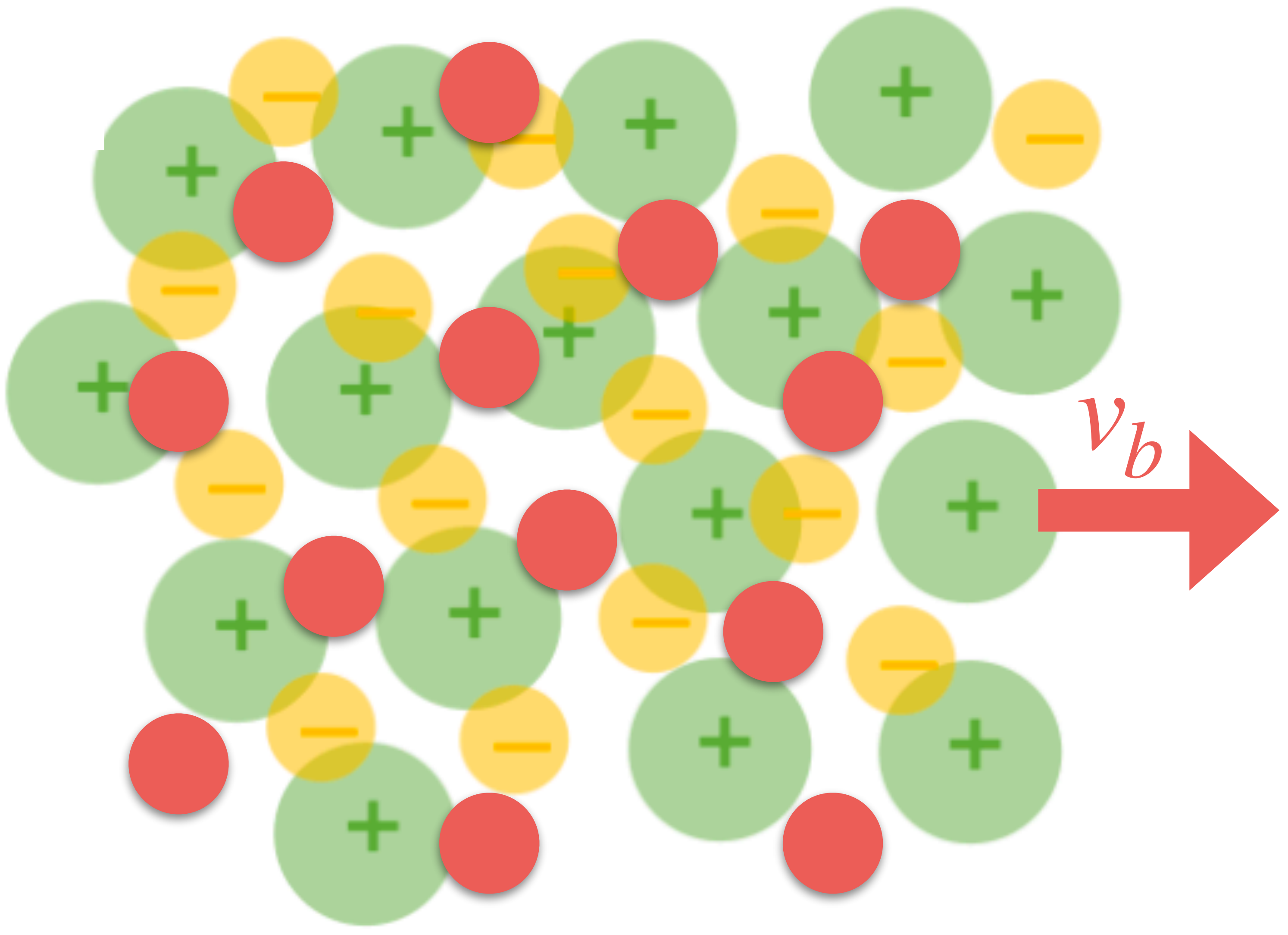
Physical picture



Two electronic fluids model:

Plasma ions (stationary) and electrons

+

Beam electrons

Exponential growth

Initial conditions

Plasma of density n_{p0}

Plasma at rest $v_{p0} = 0$

Beam of density $n_{b0} \ll n_{p0}$

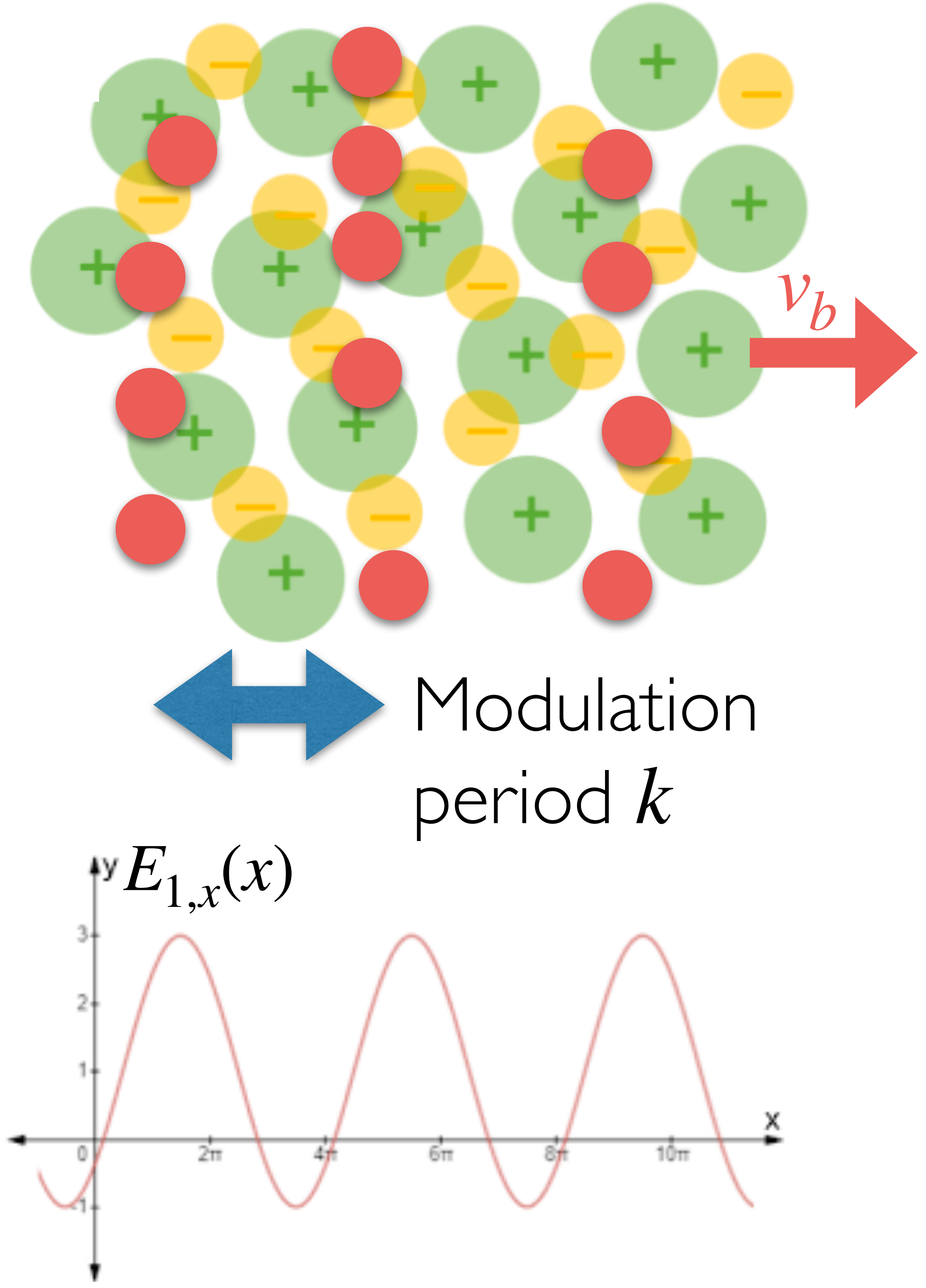
Initial beam velocity v_{0b}

Initial fields $E_0, B_0 = 0$

Note: $B_0 = 0$ means current neutralisation, $j_0 = j_{0b} + j_{0p} = 0$

$n_1(\vec{r}, t), \vec{v_1}(\vec{r}, t), \vec{E_1}(\vec{r}, t), \ldots?$

Periodic Modulation



Beam-plasma instabilities

Main equations for plasma and beam electrons

$$(\partial_t + \mathbf{v_i}\cdot\nabla)\,\mathbf{p_i} = \nabla\phi$$

$$\partial_t n_i + \nabla\cdot(n_i\mathbf{v_i}) = 0$$

2D electrostatic, cold fluid model

1- Momentum equation for **relativistic** beam electrons:

$$(\partial_t + v_{0b}\partial_x)\begin{bmatrix}\gamma_{b0}^3 v_{bx}^{(1)}\\ \gamma_{b0} v_{by}^{(1)}\end{bmatrix} = \begin{bmatrix}\partial_x\phi^{(1)}\\ \partial_y\phi^{(1)}\end{bmatrix}$$

2- Continuity equation for **relativistic** beam electrons:

$$(\partial_t + v_{b0}\partial_x)\,n_b^{(1)} + n_b^{(0)}\left(\partial_x v_{bx}^{(1)} + \partial_y v_{by}^{(1)}\right) = 0\,.$$

3- Momentum eq. + continuity eq. for **rel.** beam electrons:

$$(\partial_t + v_{b0}\partial_x)^2\,n_b^{(1)} = -n_b^{(0)}\left[\gamma_{0b}^{-3}\partial_x^2 + \gamma_{0b}^{-1}\partial_y^2\right]\phi^{(1)}$$

4- Momentum eq. + Continuity eq. for plasma electrons:

$$\partial_t^2 n_p^{(1)} = -n_p^{(0)}\left[\partial_x^2 + \partial_y^2\right]\phi^{(1)}$$

5- Poisson equation

$$\left(\partial_x^2 + \partial_y^2\right)\phi^{(1)} = n_b^{(1)} + n_p^{(1)}$$

6- Put together 3, 4 and 5, oscillatory solutions on k_x

Two stream instability

Dispersion relation

Normalised units

$$1 - \frac{n_{p0}}{\omega^2} + \frac{n_{b0}}{\gamma_b^3}\frac{1}{(\omega - kv_{0b})^2} = 0$$

Dispersion relation

Oscillatory solutions: $X_1(\vec{r}, t) \propto e^{i(kx-\omega t)}$ with $k, \omega \in \Re$

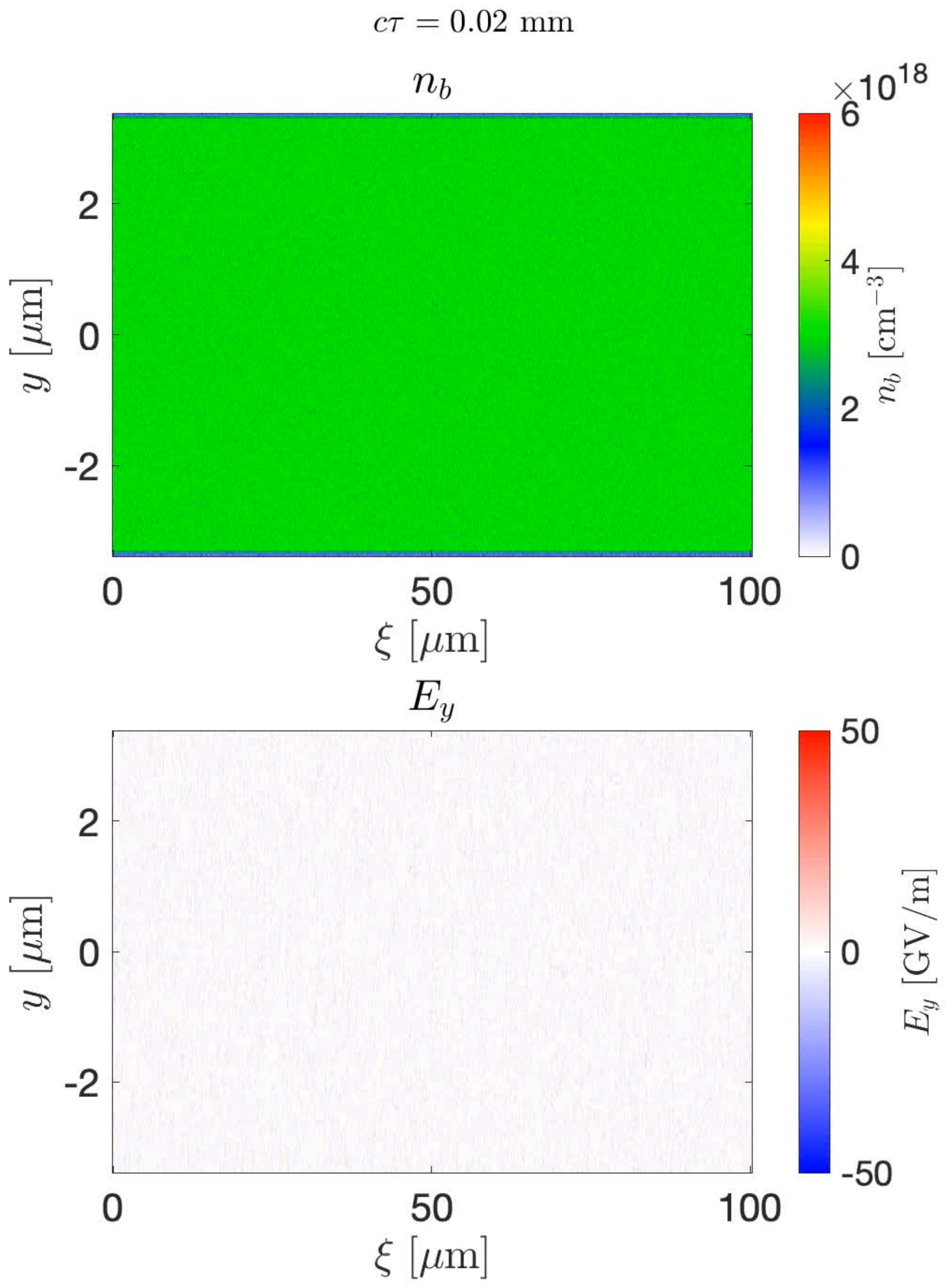
Spatially dumped solution: $X_1(\vec{r}, t) \propto e^{i(kx-\omega t)}$ with $\omega \in \Re, k \in \Im$ with $\mathrm{Im}(k) < 0$

Temporally growing solutions: $X_1(\vec{r}, t) \propto e^{i(kx-\omega t)}$ with $k \in \Re, \omega \in \Im$ with growth rate $\Gamma = \mathrm{Im}(\omega) > 0$

$$\Gamma_{\mathrm{TSI}} = \frac{\sqrt{3}}{2^{4/3}}\frac{(n_b/n_p)^{1/3}}{\gamma_b}\omega_p$$

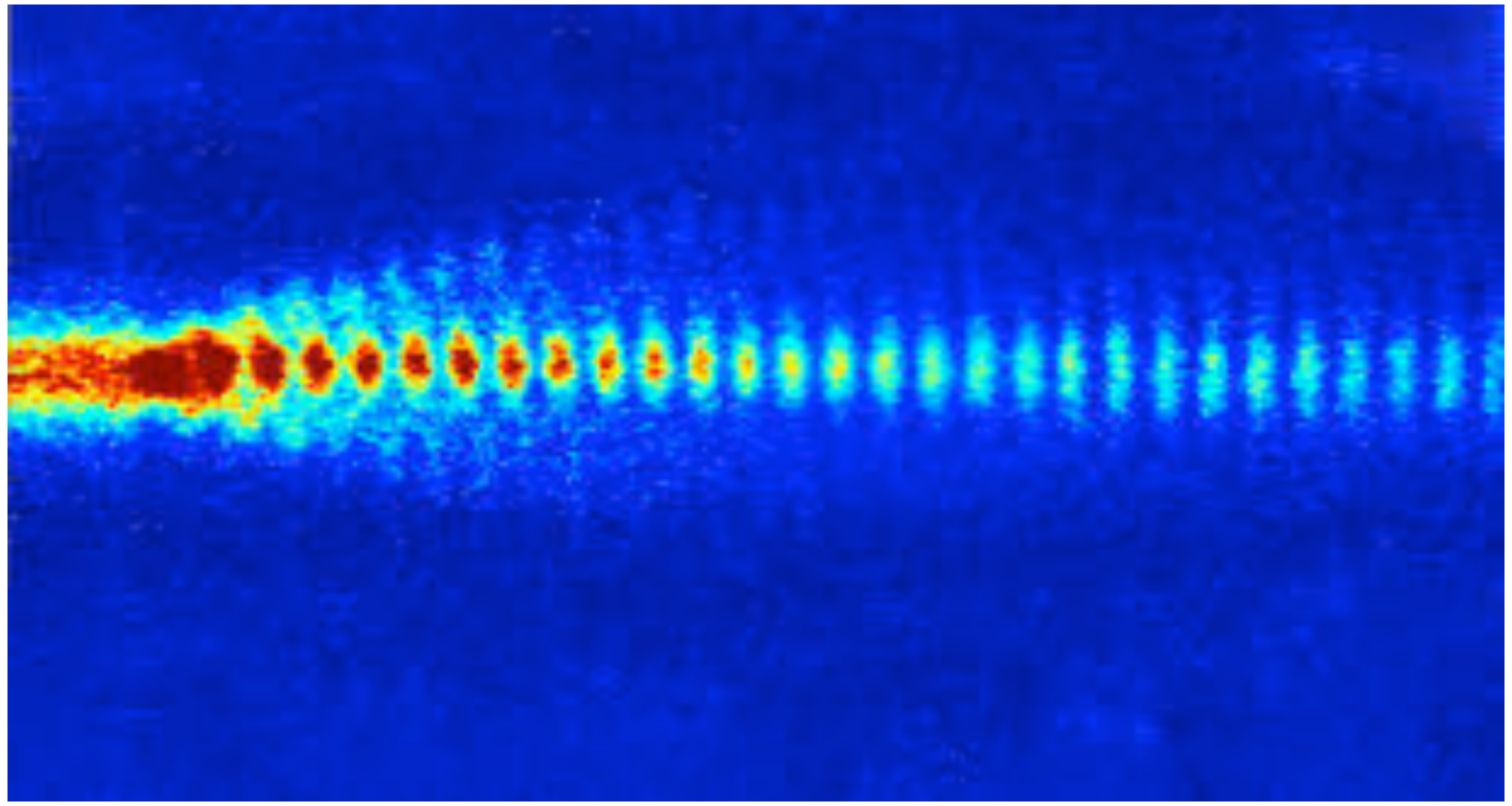
Two stream instability

Simulation movie (PIC)



Beam-plasma instabilities

- Exemplary process of a plasma collective response
- Mechanisms to transform particles' kinetic energy into EM fields
- Interest for astrophysics, but also for applications such as PWFA
- Working principle of AWAKE



Conclusions & Future Work

What is a plasma?

Quasi-neutral gas of charged particles showing *collective behaviour*

99% of visible matter: interest for astrophysics and applications

How to create plasmas?

High Power Lasers

Discharges, X-Rays, particle beams…

Waves and plasmas

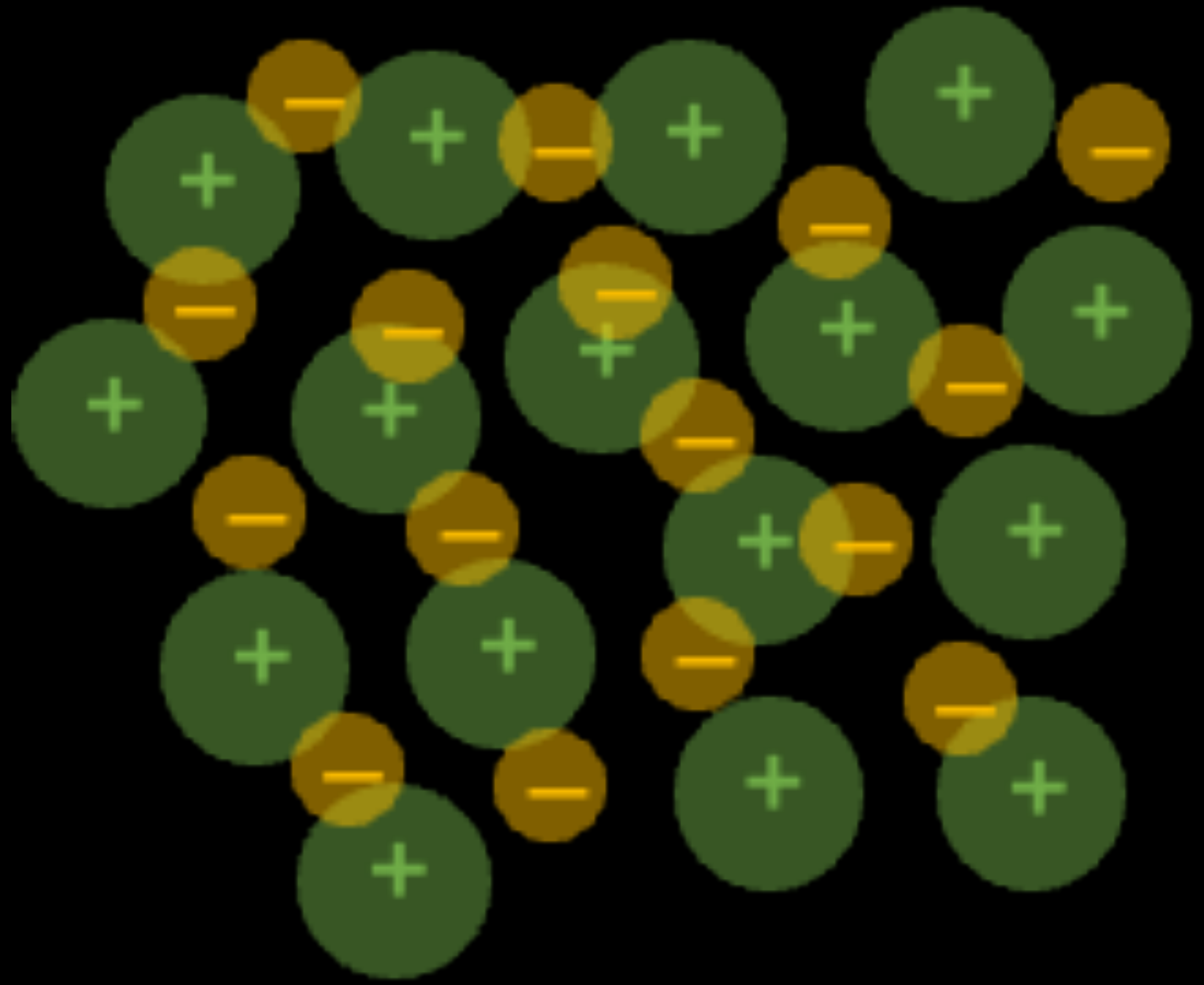
The plasma electronic density sets the oscillatory behaviour of a plasma

EM waves of high amplitude can propagate close to the speed of light: **plasma accelerators!**

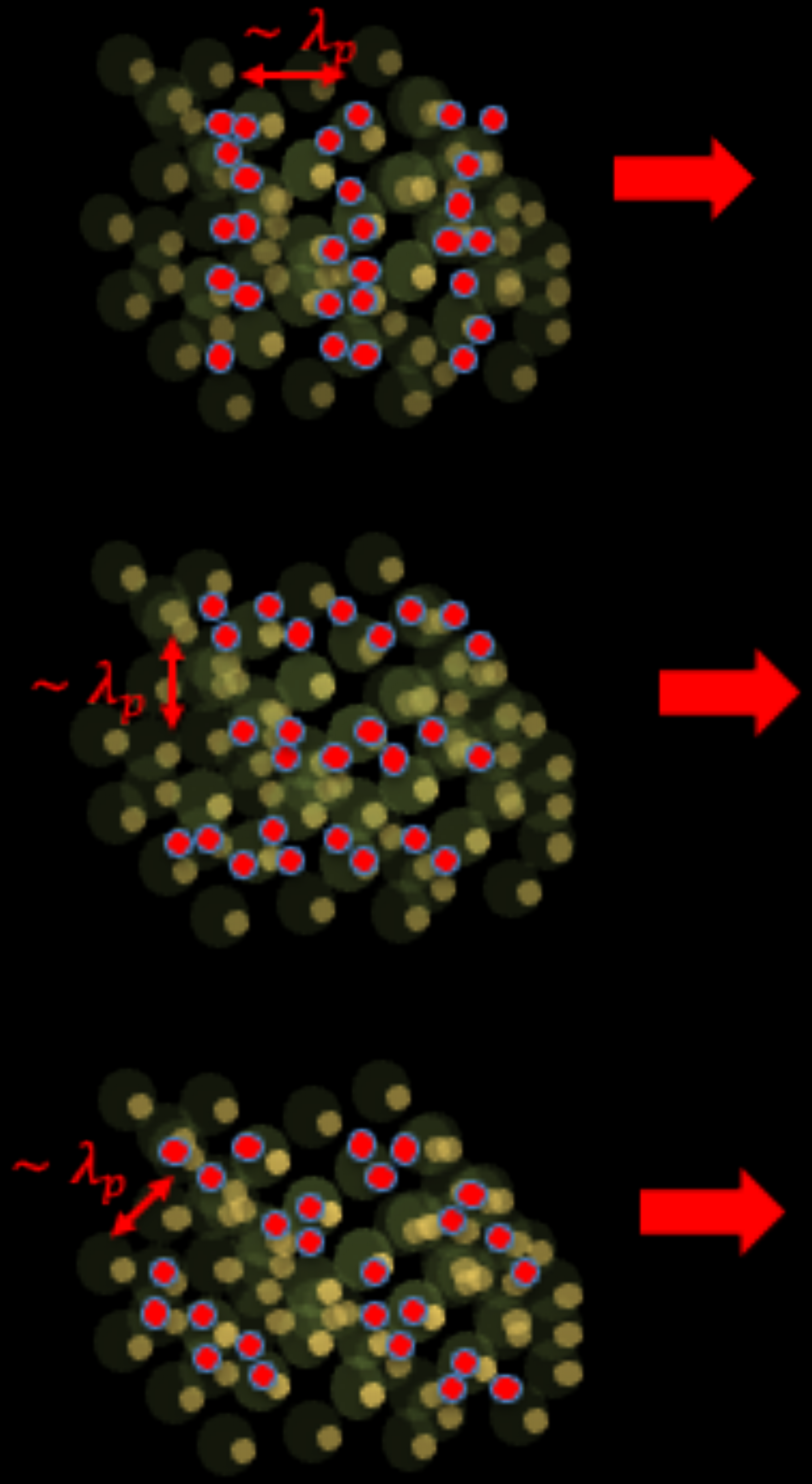
Two stream instabilities

Example of plasma collective response to EM fields

Transformation of particle kinetic energy to EM field energy (radiation)



Thank you



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Ponderomotive force - e^- trajectory derivation

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$$\frac{\partial v_y}{\partial t} = -\frac{e}{m} E_y(r)\,.$$

Slow and fast oscillating e^- trajectories

$$y(t) = y_0(t) + y_1(t)$$

Taylor expansion around slow y_0

$$E_y(y,t) \approx E_0(y_0)\cos\phi + y_1(t)\frac{\partial E_0(y_0)}{\partial y}\cos\phi + \ldots,$$

$$\partial_t^2 y_0 + \partial_t^2 y_1 = \frac{e}{m}\left[E_0(y_0) + y_1(t)\frac{\partial E_0(y_0)}{\partial y}\right]\cos\phi \qquad \partial_t v_{y1} = -\frac{eE_0}{m}\cos\phi \Rightarrow y_1 = -\frac{eE_0}{m\omega^2}\cos\phi$$

Take time averages $\langle \partial_t^2 y_0 \rangle + \langle \partial_t^2 y_1 \rangle = \frac{e}{m}\langle E_0(y_0)\cos\phi\rangle + \frac{e}{m}\langle y_1(t)\frac{\partial E_0(y_0)}{\partial y}\cos\phi\rangle \qquad \langle \partial_t v_{y1}\rangle = -\frac{eE_0}{m}\langle\cos\phi\rangle$

Cancel out

$$\partial_t^2 y_0 = -\frac{e^2}{m\omega^2}E_0\frac{\partial E_0(y)}{\partial y}\cos^2\phi\,. \qquad f_{py} = m\partial_t v_{y0} = -\frac{e^2}{4m\omega^2}\frac{\partial E_0^2}{\partial y}\,.$$