Colloquium:
the quest for the physical foundations of music

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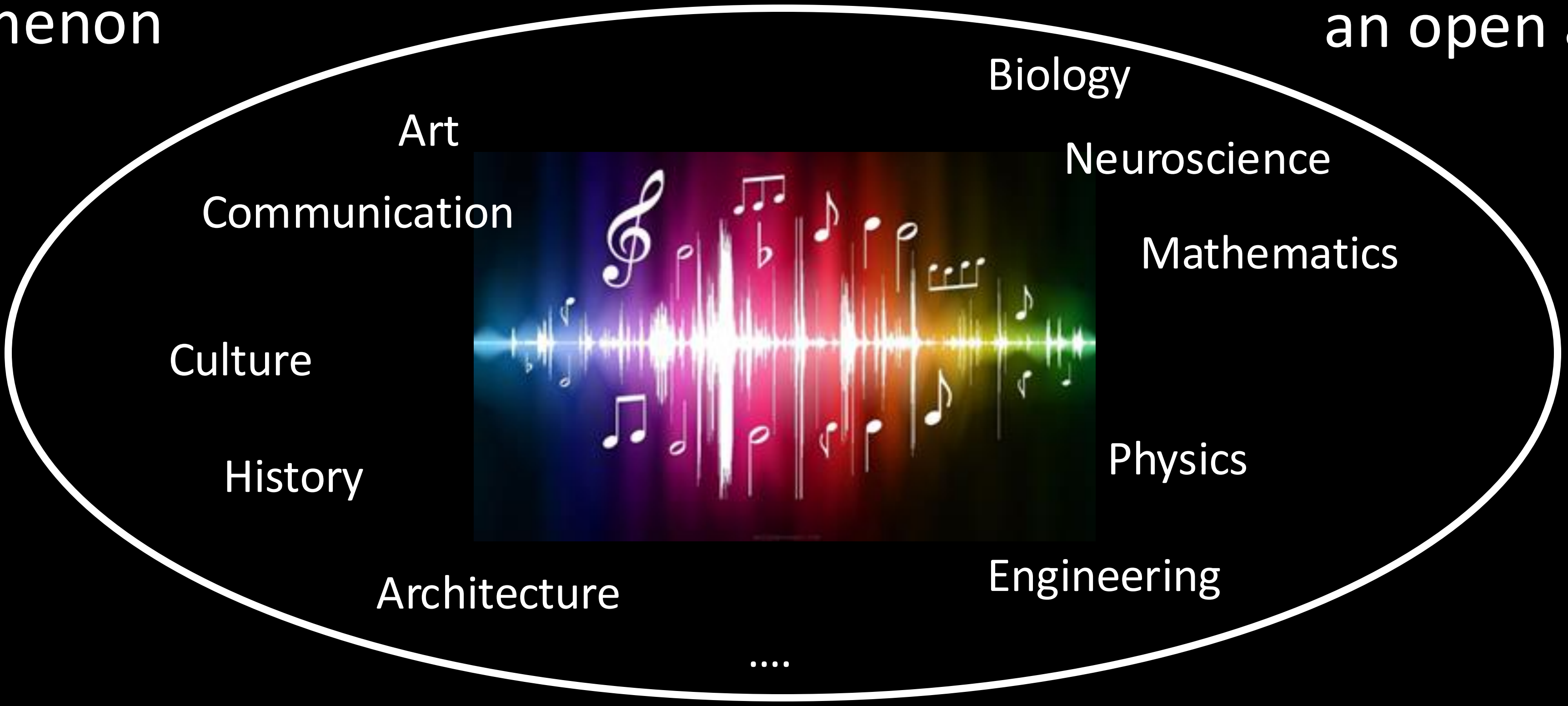
Music is a complex interdisciplinary phenomenon

Science of music is an open and active field



Dyad's consonance and dissonance: combining the compactness and roughness approaches
I Masina, G Lo Presti, D Stanzial, The European Physical Journal Plus 137 (11) (2022) 1254

Triad's consonance and dissonance: a detailed analysis of compactness models
I Masina, The European Physical Journal Plus 138 (7) (2023) 606

Triad's consonance and dissonance: combining roughness and compactness models
I Masina, G Lo Presti, The European Physical Journal Plus 139 (1) (2024) 79

Online lectures of CERN Academic Training:
The Physics of Music; from Pythagoras to Microtones
https://indico.cern.ch/event/1172806/

COLLOQUIUM CONTENT

→The Detector: Our yet unknown hearing system

→ The Sources of musical «tones»

→ Psychoacoustic perceptions for simultaneous tones

→ Consonance and Dissonance (C&D) as an «observable quantity»

→ C&D and Musical Practice: a flash review on scale’s evolution

→ Modeling C&D: Literature review of past models

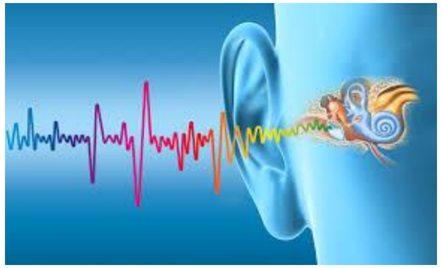
→ Modeling C&D: Our models and related analysis

→ Conclusions

OUR DETECTOR: THE EAR

The ear (nearly) functions as a Fourier analysis device

Ohm's law of hearing: the perception of the tone of a sound is a function of the frequencies and amplitudes of the harmonics and not of the phase relationships between them



https://www.odyo.ca/anatomy-of-the-ear-and-the-hearing-system-works/

Signal associated to pressure waves in air, time domain $s(t)$ → $S(f)$ (S)FT in frequency domain

Fourier Analysis

The Fourier Transform $S(f)$ of a time-dependent function (signal) $s(t)$ is defined as:

$$S(f) := \int_{-\infty}^{+\infty} s(t) e^{-i2\pi f t} \, dt$$

Typically, the ***Power Spectral Density*** $|S(f)|^2$ is analyzed, which relates to the *autocorrelation*

To capture transients, time-frequency analysis through the *Short-Time* Fourier Transform is common:

$$S_{st}(t, f) = \int_{-\infty}^{+\infty} s(\tau) w(\tau - t) e^{-i2\pi f \tau} \, d\tau$$

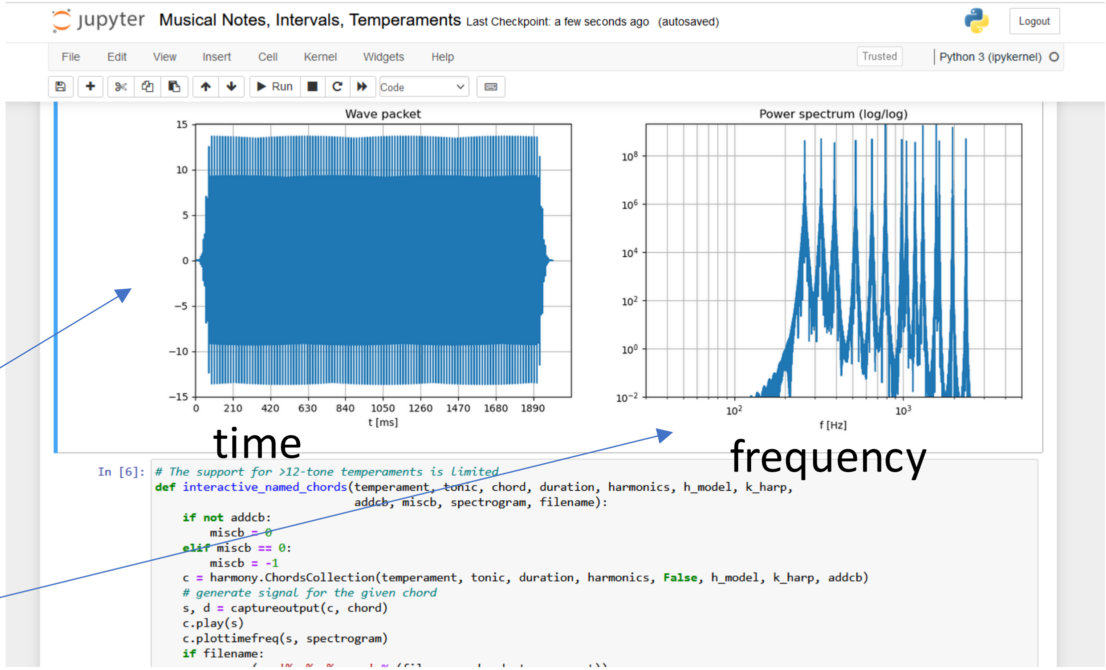
where $w(t)$ is a *windowing* function (typically a Gaussian). For human ear, its width is about 0.05 s.

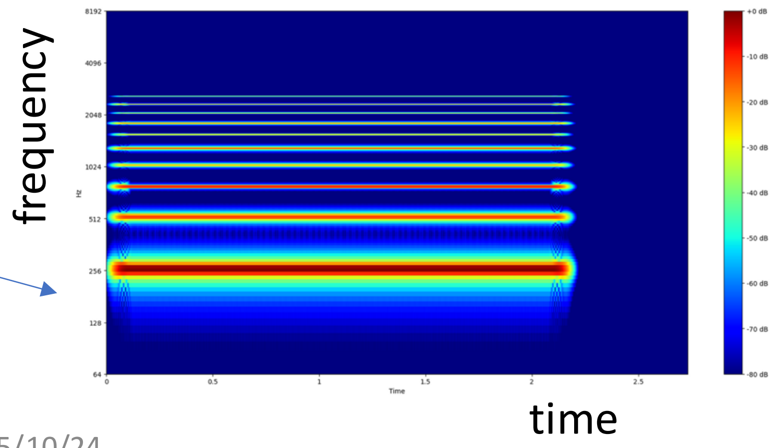
The ***Spectrogram*** is defined as $|S_{st}(t, f)|^2$. Easy to produce with your smartphone!

OUR PYTHON TOOLBOX TO TEST AND BENCHMARK

We* have developed a set of iPython notebooks to explore sounds and their acoustic effects

- Based on popular Data Science libraries (numpy, `matplotlib`)
- As shown, we display
  - Time-domain waveform
  - Power spectral density
  - Spectrogram



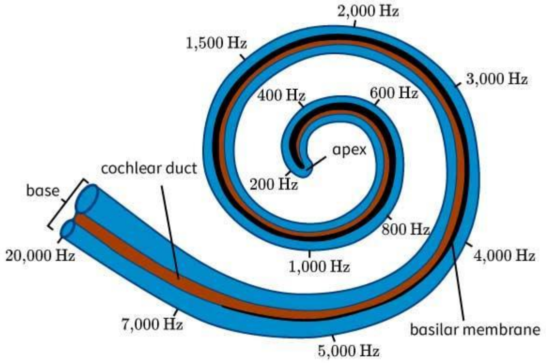


*credits: G.Lo Presti (CERN)

OUR DETECTOR: THE EAR

Ohm's law is consistent with the *place theory of hearing*, which correlates the observed pitch with the position along the basilar membrane of the inner ear that is stimulated by the corresponding frequency.
An electrical signal is produced (ions) and transmitted to the brain.

https://en.wikipedia.org/wiki/File:Journey_of_Sound_to_the_Brain.ogv



However, this process is far from being fully understood…

As an example:
→ the time-domain waveform is also actually relevant (not just the spectrogram)
→ non-linearities might also be relevant and represent a current subject of research,
see e.g.
J.N. Oppenheim and M.O. Magnasco,
Human Time-Frequency Acuity Beats the Fourier Uncertainty Principle
PRL 110, 044301 (2013)

MUSICAL TONES

The hearing system can establish:

1. the PITCH of a sound if its harmonics (or partials) are «harmonic»

fundamental frequency of the tone

$$f_n = n f_1$$

where n=1,2,3,...

The (whole) sound $\{f_1\}$ is then said to be a *(musical) tone*

2. the TIMBRE of a tone, due to its harmonic's amplitudes

harmonic's amplitudes

$$s(t) = \sum_{n=1,2,\dots} A_n \sin(2\pi n f_1 t) \quad \Rightarrow \quad S(f) = \sum_{n=1,2,\dots} A_n \, \delta(f - n f_1)$$

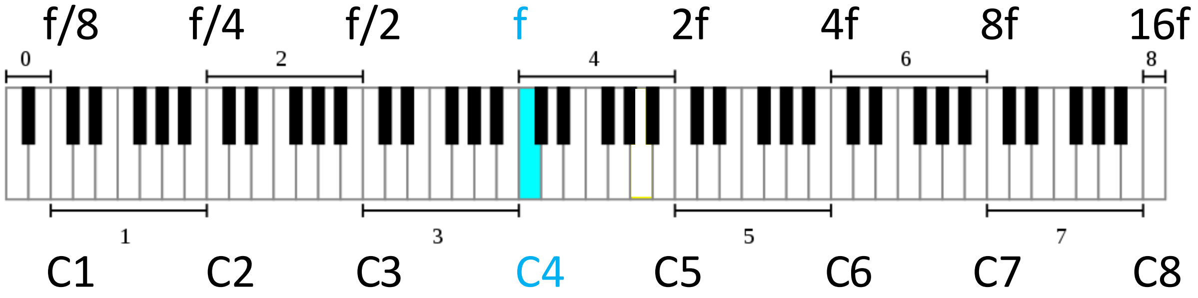
MUSICAL TONES

3

OCTAVE EQUIVALENCE (universal through cultures, yet not fully understood):

tone {f} is perceived to be equivalent to tones {2f}, {4f}, {8f}, …

C4: f=262 Hz



In the audible range (20 Hz - 20 kHz), all C's are C's

4

DISCRIMINATION LIMEN (or *Just Noticeable Difference*)

the frequency difference which makes two sounds perceived as *different pitches* when heard separately (in sequence) is 3-4 Hz from C1 up to C5, then it increses

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OUR SOURCES

1) Human Voice

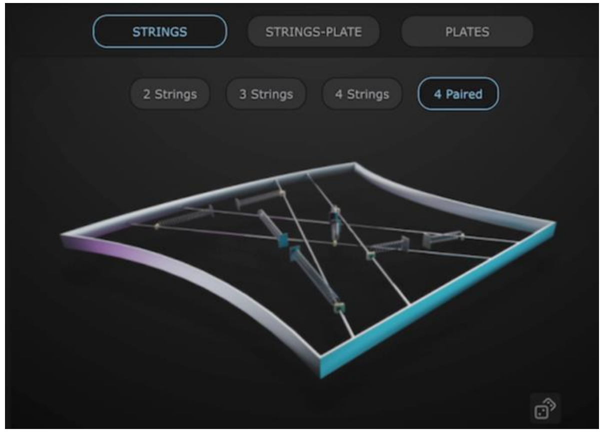
2) Acoustics instruments:
strings (plucked, striked, bowed),
air columns (flutes, …),
…

3) Digital instruments

Sound synthesis

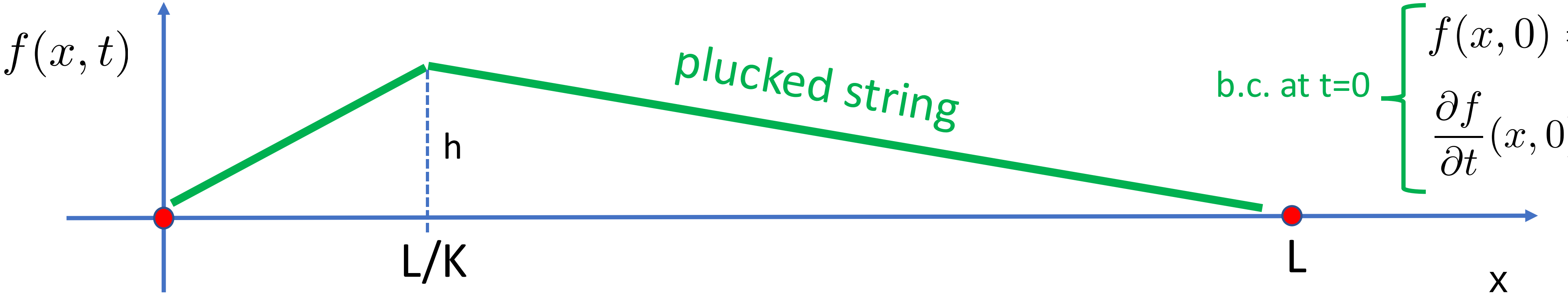
As a «close» example,
Edinbourg-Bologna Group

Physical Audio® Modus
by M. Ducceschi (UniBo)
https://physicalaudio.co.uk/



In the following, a self-made example: plucked string, from theory to simulation

The vibrating string with fixed endpoints + damping



b.c. at t=0
$$\begin{cases} f(x,0) = \text{triangle} \\ \dfrac{\partial f}{\partial t}(x,0) = 0 \end{cases}$$

The ear detects the (S)FT of the pressure wave generated in air, that is of a signal that can be written as

$$s(t) = \sum_{n=1,2,\dots} (\cancel{C_n^s} + C_n^c) \sin(2\pi f_n t) \times e^{-n\frac{\Gamma}{2}t}$$

$$C_n^c = \underbrace{\frac{2hK^2}{\pi^2(K-1)}}_{C_1^c} \frac{1}{n^2} \sin\frac{n\pi}{K}$$

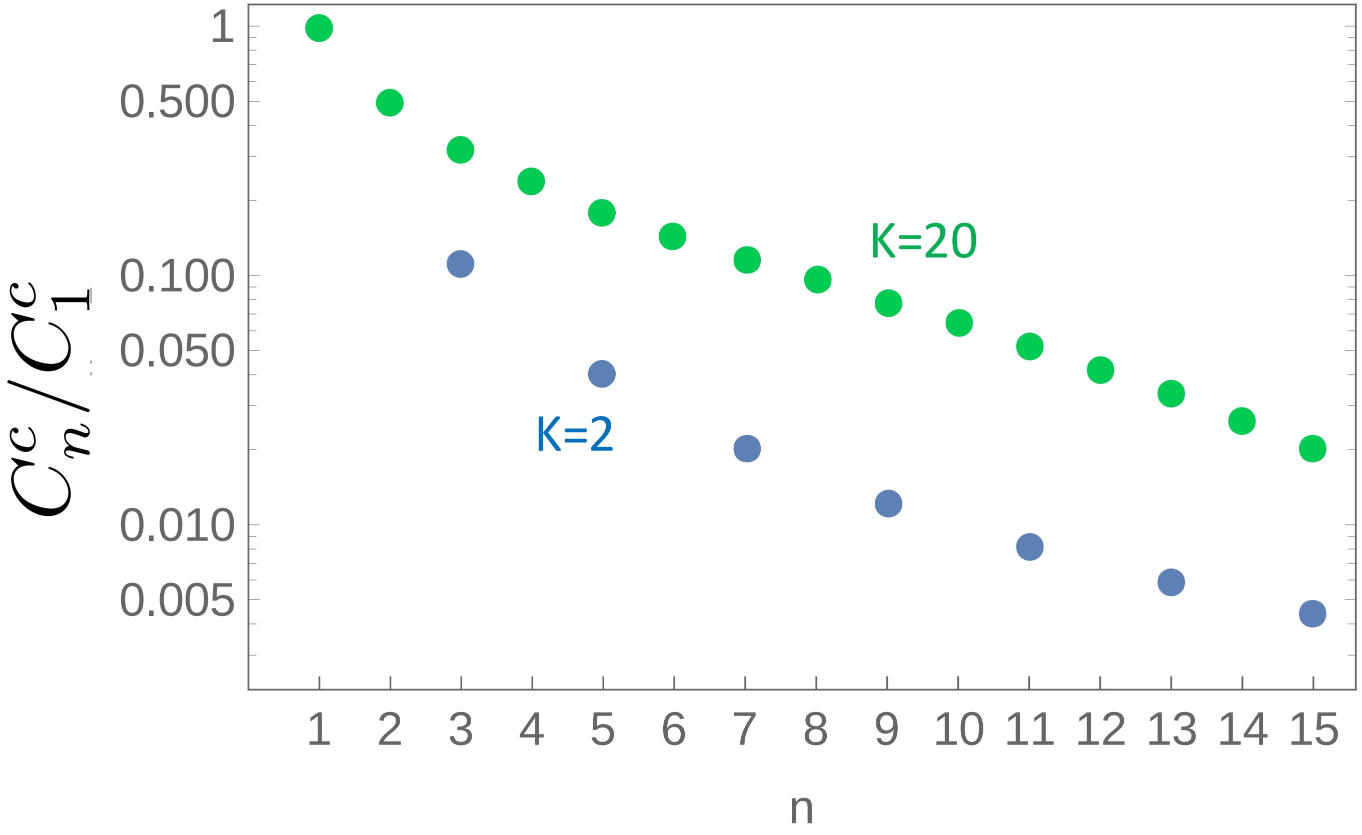
damping factor:

the higher is the harmonic the sooner it vanishes

Take e.g. Γ=O(1) s^{-1}

Plucked string: harp (K=2) vs guitar (K=20)

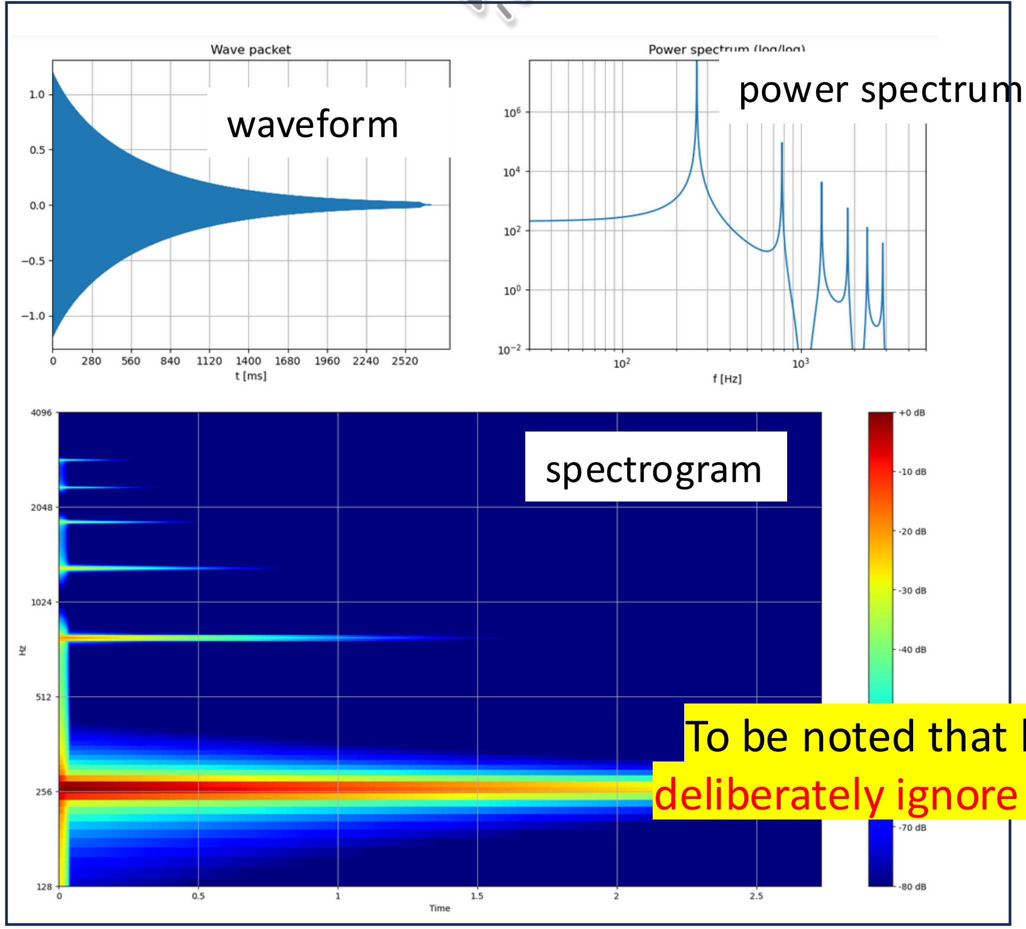
Different timbre is expected depending on where you pluck the string



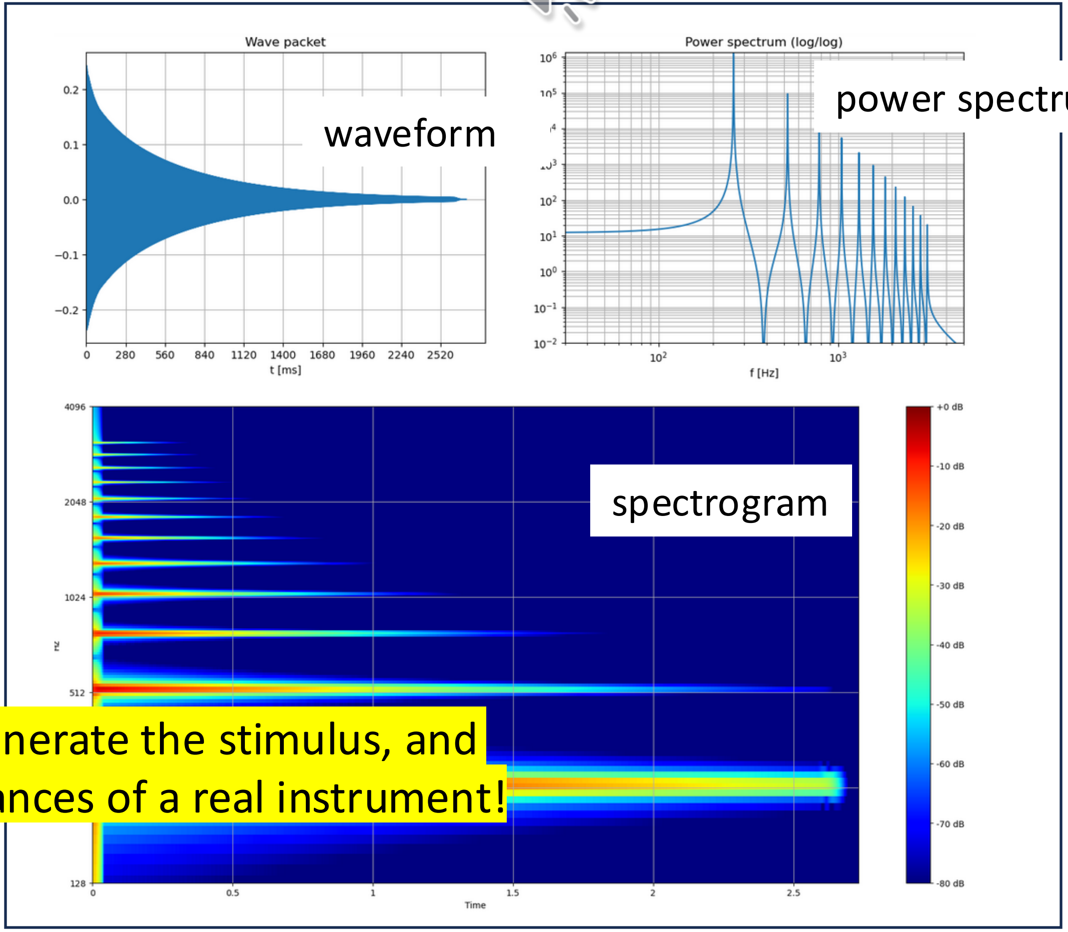
Plucked string: harp (K=2) vs guitar (K=20)

We take f=C_4 , Γ=1.5 s^{-1}, and generate a waveform with numpy

K=2 (Harp)



K=20 (Guitar)



To be noted that here we generate the stimulus, and deliberately ignore the resonances of a real instrument!

COLLOQUIUM CONTENT

COMBINING FREQUENCIES

Now let's combine two (pure) tones with frequency f_1 and f_2, and explore the auditory effects when hearing them together:

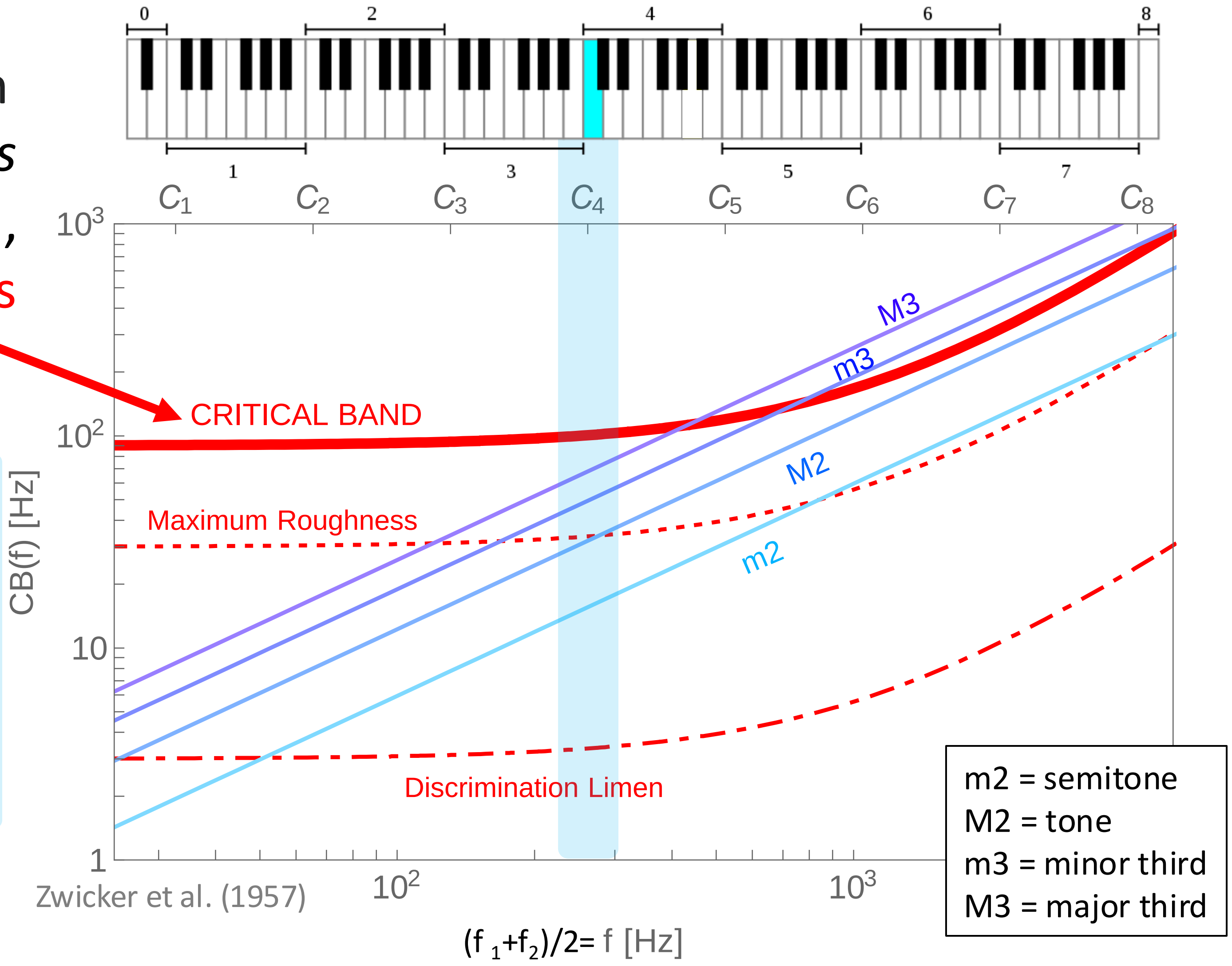
- "very close" frequencies: *primary beatings* (waveform modulation amplitude from 0 to max)
- "close" frequencies within Critical Bandwidth: significant *roughness*
- "far away" frequencies: combined sound relatively less rough
- "simple ratios" (when $f_2/ f_1 = k$ = fraction made with small integers*): peaks of *consonance*
- "close to simple ratios": *secondary beatings* (modulation amplitude from min>0 to max)

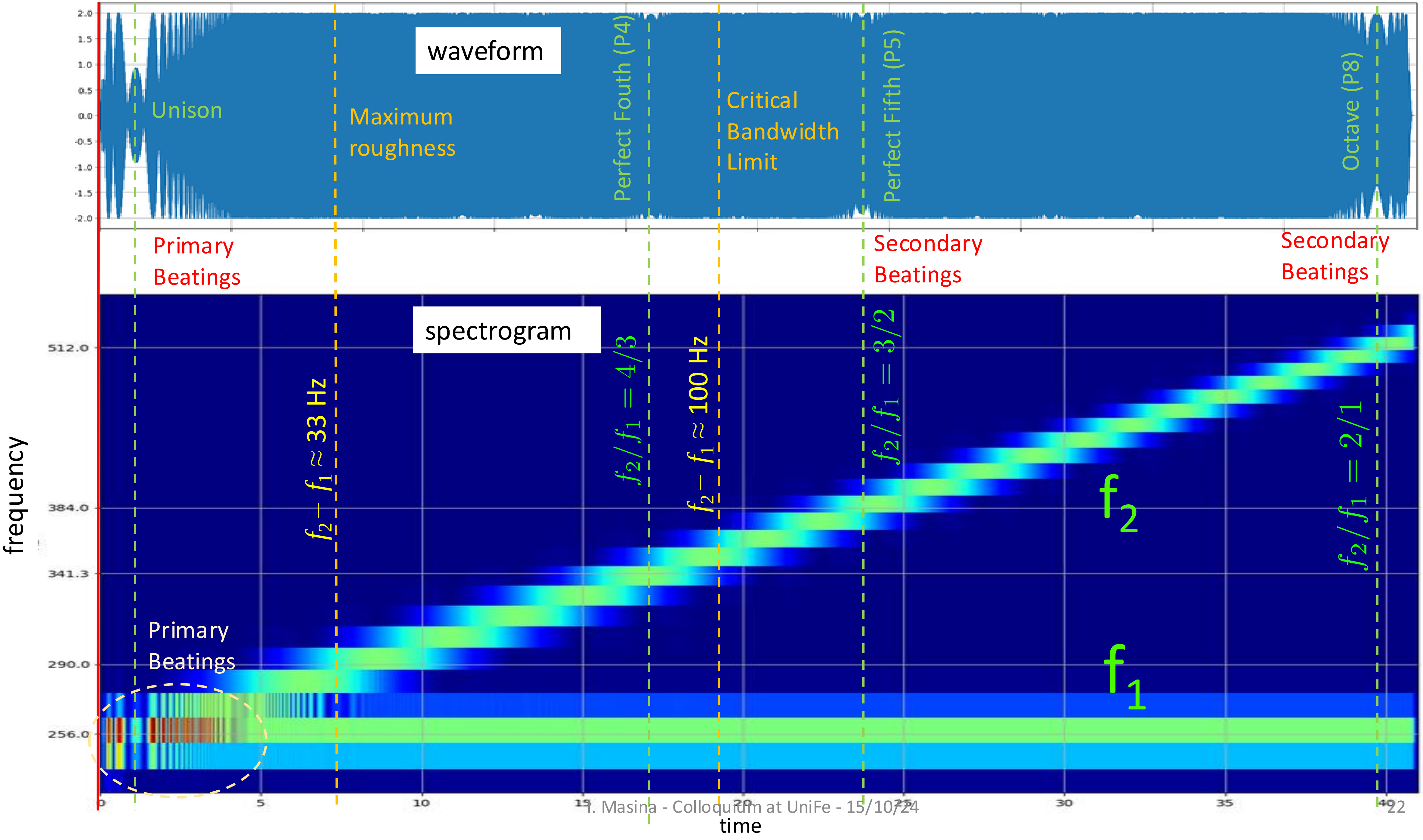
Critical Bandwidth (and Discrimination Limen)

the frequency band, $\Delta f = f_2 - f_1$, within which a second sound (f_2) *interferes* in the perception of the first (f_1), giving roughness

For frequences around C_4 (262 Hz), CB is about 100 Hz:

(primary) beatings are heard until Δf is about 12Hz, roughness between 12Hz and 100Hz, with a maximum at about 33Hz.

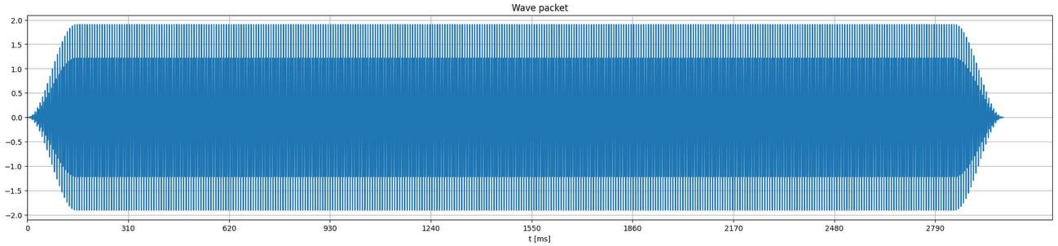



waveform
Unison
Maximum roughness
Perfect Fouth (P4)
Critical Bandwidth Limit
Perfect Fifth (P5)
Octave (P8)
Primary Beatings
Secondary Beatings
Secondary Beatings
spectrogram
$f_2 - f_1 \approx 33$ Hz
$f_2 / f_1 = 4/3$
$f_2 - f_1 \approx 100$ Hz
$f_2 / f_1 = 3/2$
$f_2 / f_1 = 2/1$
f_2
f_1
Primary Beatings
frequency
time

Secondary beatings of P5: $f_2 = 3/2\, f_1$, with $f_1 = C_4$

PURE TONES

Perfect Fifth:

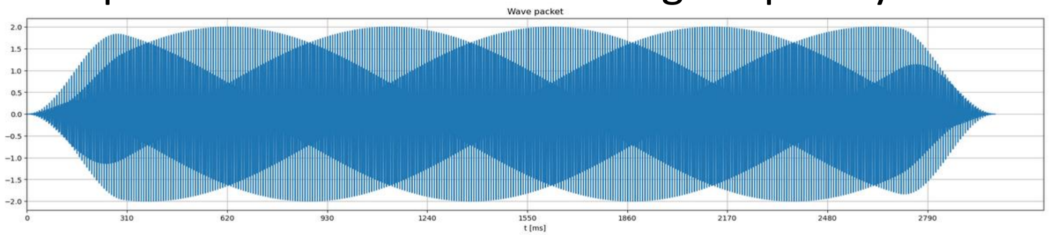


amplitude modulation at beating frequency of 2 Hz

Mistuned Fifth:

$f_2 = 3/2\, f_1 + 1\ \text{Hz} \approx 3/2\, f_1$

→ secondary beatings

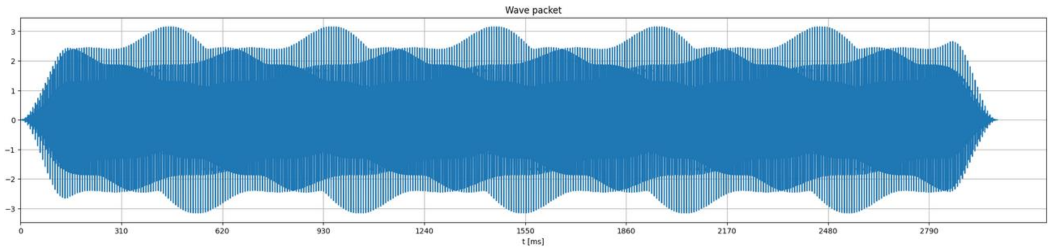


The hear is not sensitive to a difference of 1 Hz. However, the hear is extremely sensitive to beatings: this is why we "tune" musical instruments based on beatings!

COMPLEX TONES (n=5)

Mistuned Fifth:

→ higher harmonics reinforce beatings



Fundamental Bass

Let's now assume:

$$\frac{f_2}{f_1} = \frac{m}{n} \quad m, n \in \mathbb{N} \text{ and coprime}$$

→ We can define a new frequency f_0 , whose inverse represents the period of the waveform

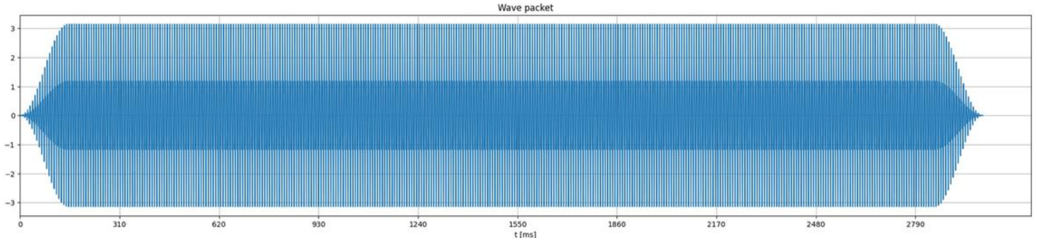
$$f_0 = \frac{f_2}{m} = \frac{f_1}{n}$$

For *m, n* small, it is known in music as the Fundamental Bass, Common Bass or Missing Fundamental.
It is VIRTUAL: not present in the frequency spectrum, but...

Fundamental Bass for P5: $f_2 = 3/2\, f_1$, with $f_1 = C_4$

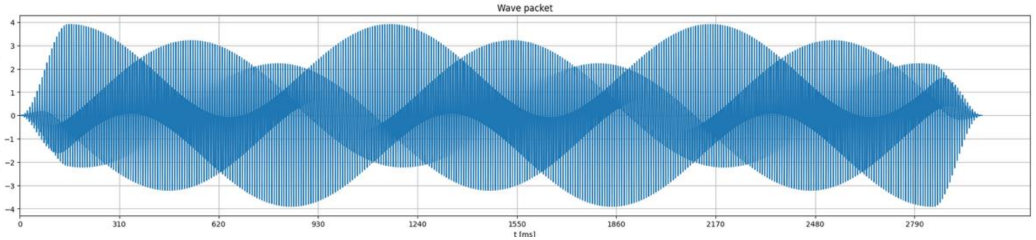
PURE TONES

Perfect Fifth + f_0:
($f_0 = f_1/2 = C_3$)



Perfect Fifth + mistuned $f = f_0 + 1$ Hz:

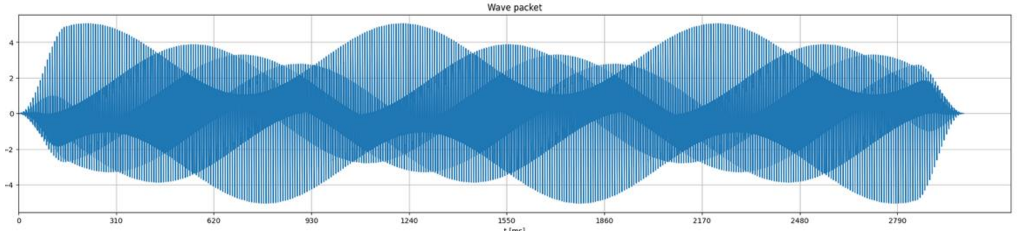
→ (tertiary) Beatings generated (as if f_0 were physically present



COMPLEX TONES (n=5)

Perfect Fifth + (pure) mistuned $f = f_0 + 1$ Hz

→ Beatings reinforced (out of tune piano)



COLLOQUIUM CONTENT

A CONSONANCE TEST

Several scientists and musical theorists did a psychoacoustic test to formalize C/D
In the past, limited to one octave and using a specific musical instrument. E.g.,

- Foderà (early 1800)
- Schwartz (early 1900)
- Bowling Purves and Gill (2018), *Vocal similarity predicts the relative attraction of musical chords* Proceedings of the National Academy of Sciences 115(1):201713206

We tried it ourselves, with some variants:

→ we built a harmonic timbre that **does not resemble** too much a familiar instrument
slow exponential decay + partials with 1/n amplitude

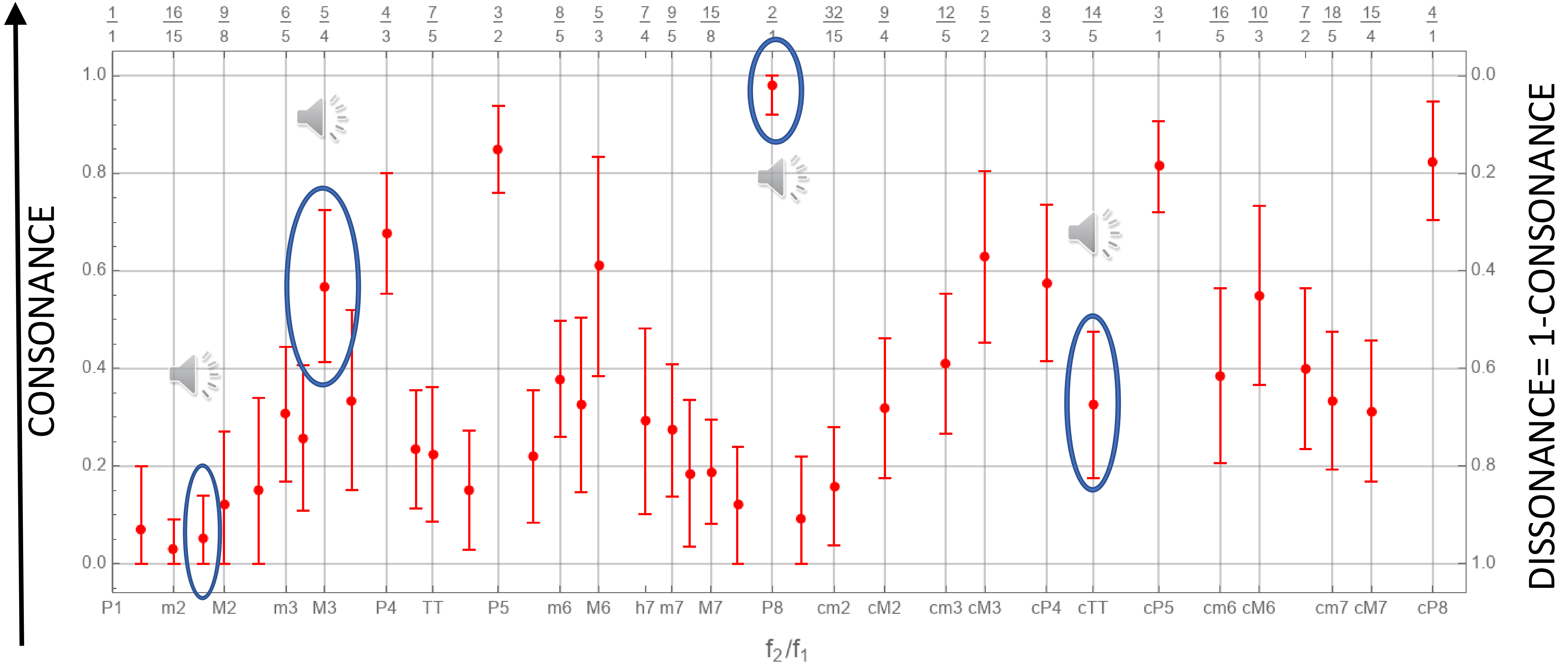
→ We chose **38** ratios, with f_1 = C_4 and f_2 running up to 4 f_1 = **2 octaves**

We let volunteers hear several dyads and provide a degree of perceived consonance within a scale 1 to 5: 1 = very dissonant, 3 = neither diss. nor cons., 5 = very consonant

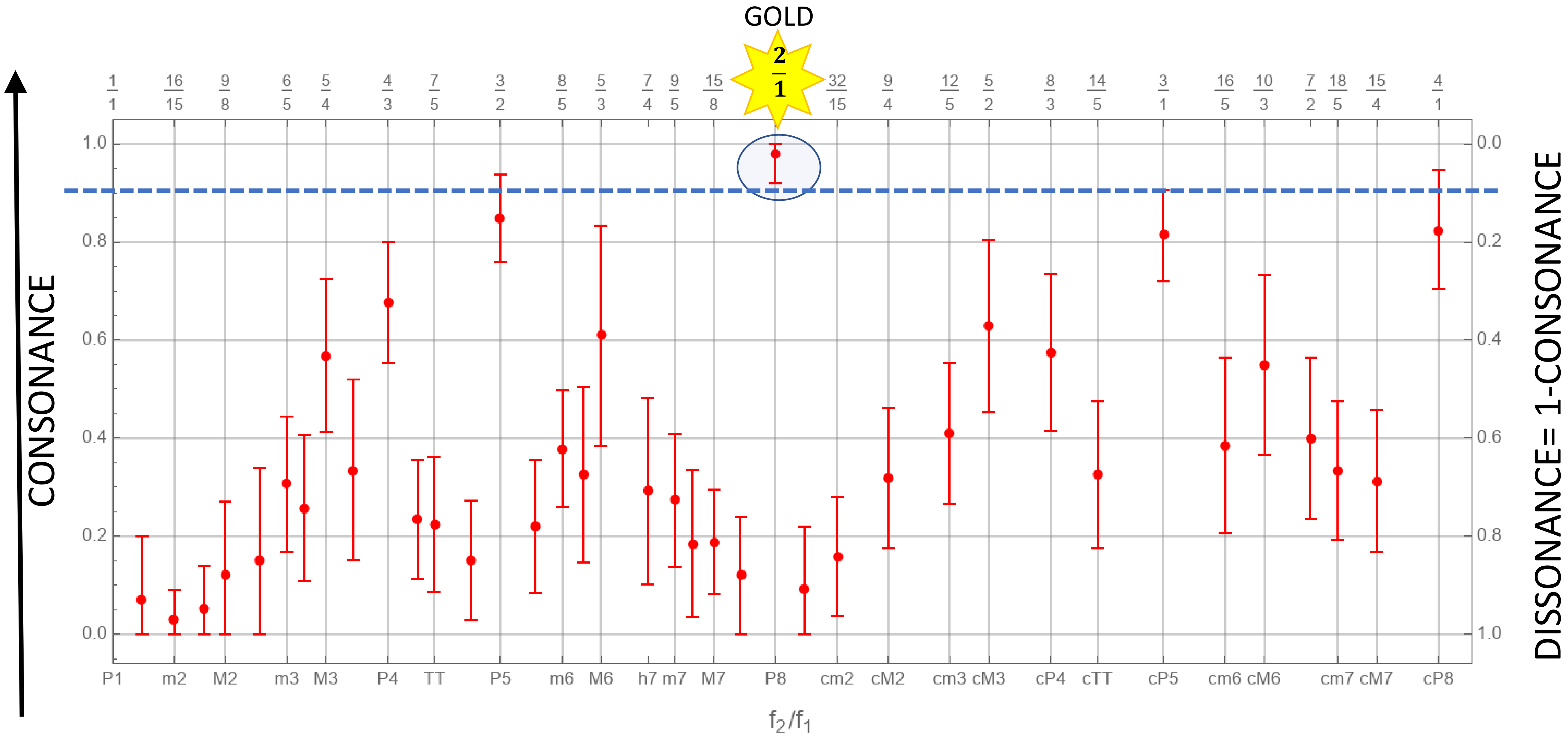
Test Results

[IM, GLP, DS, EPJP 2022]

Results from 20 individuals. Error bars represent one standard deviation.

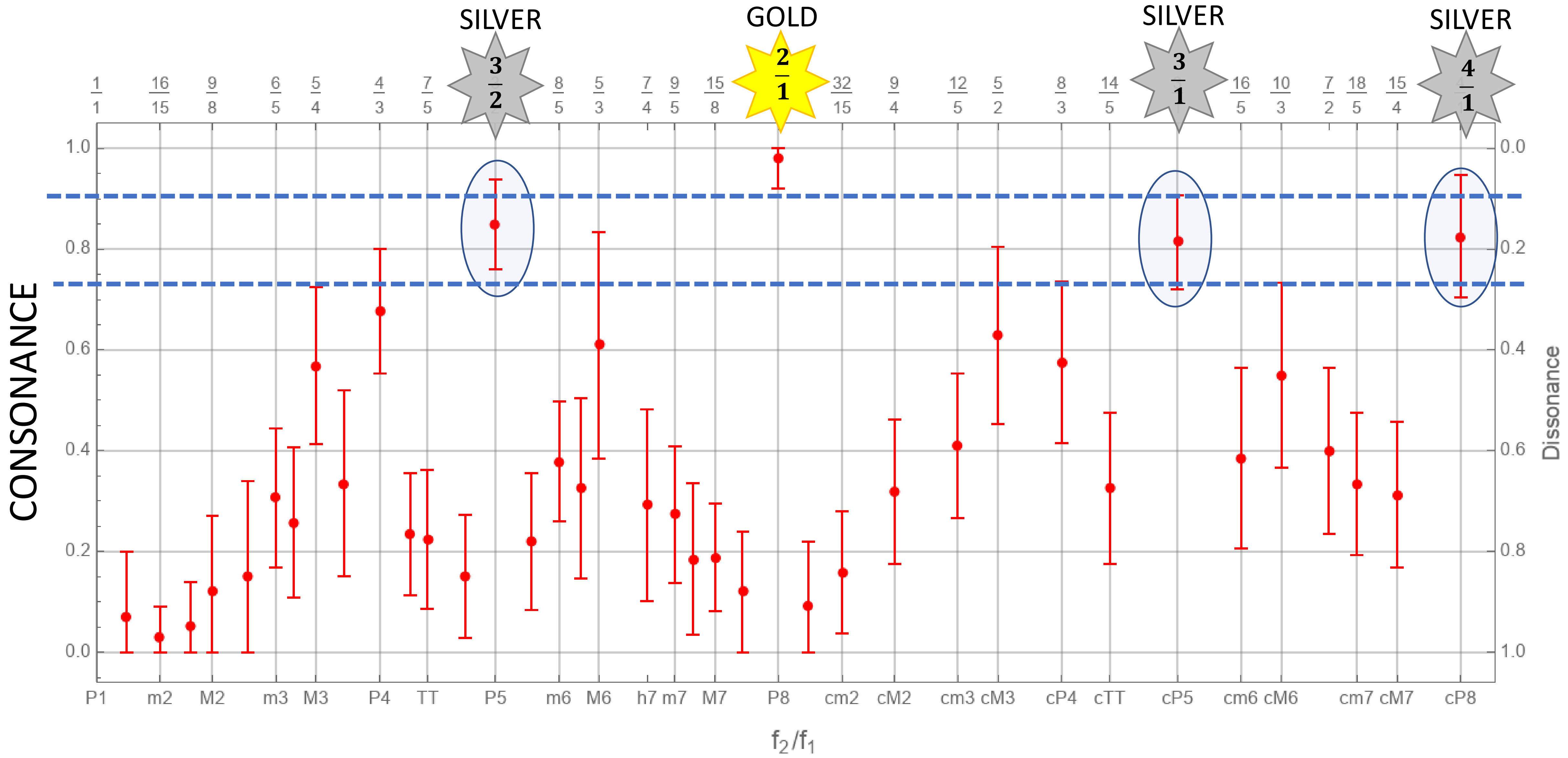


THE WINNERS



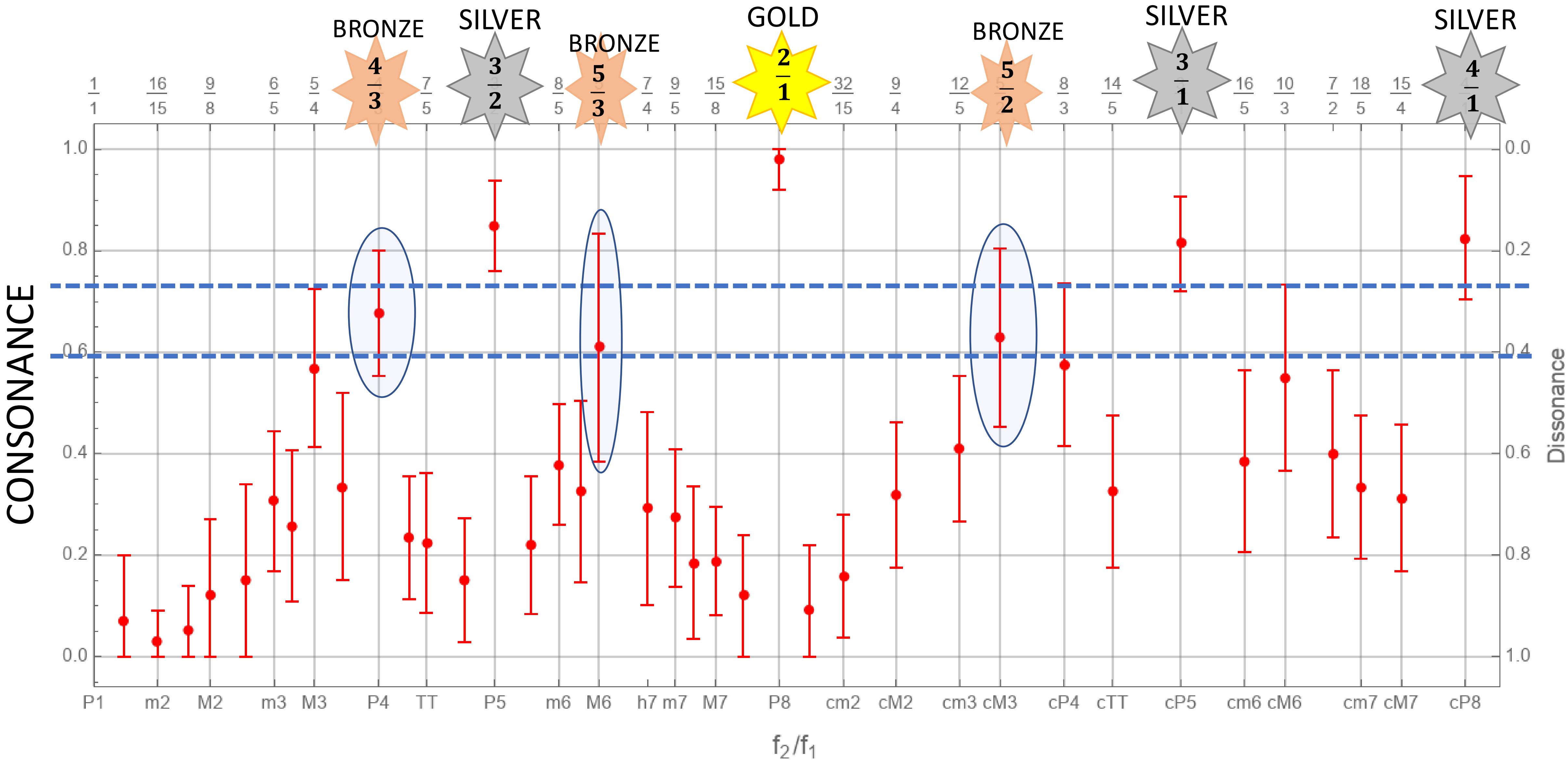
GOLD
2/1
CONSONANCE
DISSONANCE= 1-CONSONANCE
1/1 16/15 9/8 6/5 5/4 4/3 7/5 3/2 8/5 5/3 7/4 9/5 15/8 32/15 9/4 12/5 5/2 8/3 14/5 3/1 16/5 10/3 7/2 18/5 15/4 4/1
P1 m2 M2 m3 M3 P4 TT P5 m6 M6 h7 m7 M7 P8 cm2 cM2 cm3 cM3 cP4 cTT cP5 cm6 cM6 cm7 cM7 cP8
1.0 0.8 0.6 0.4 0.2 0.0
0.0 0.2 0.4 0.6 0.8 1.0
f2/f1

THE WINNERS



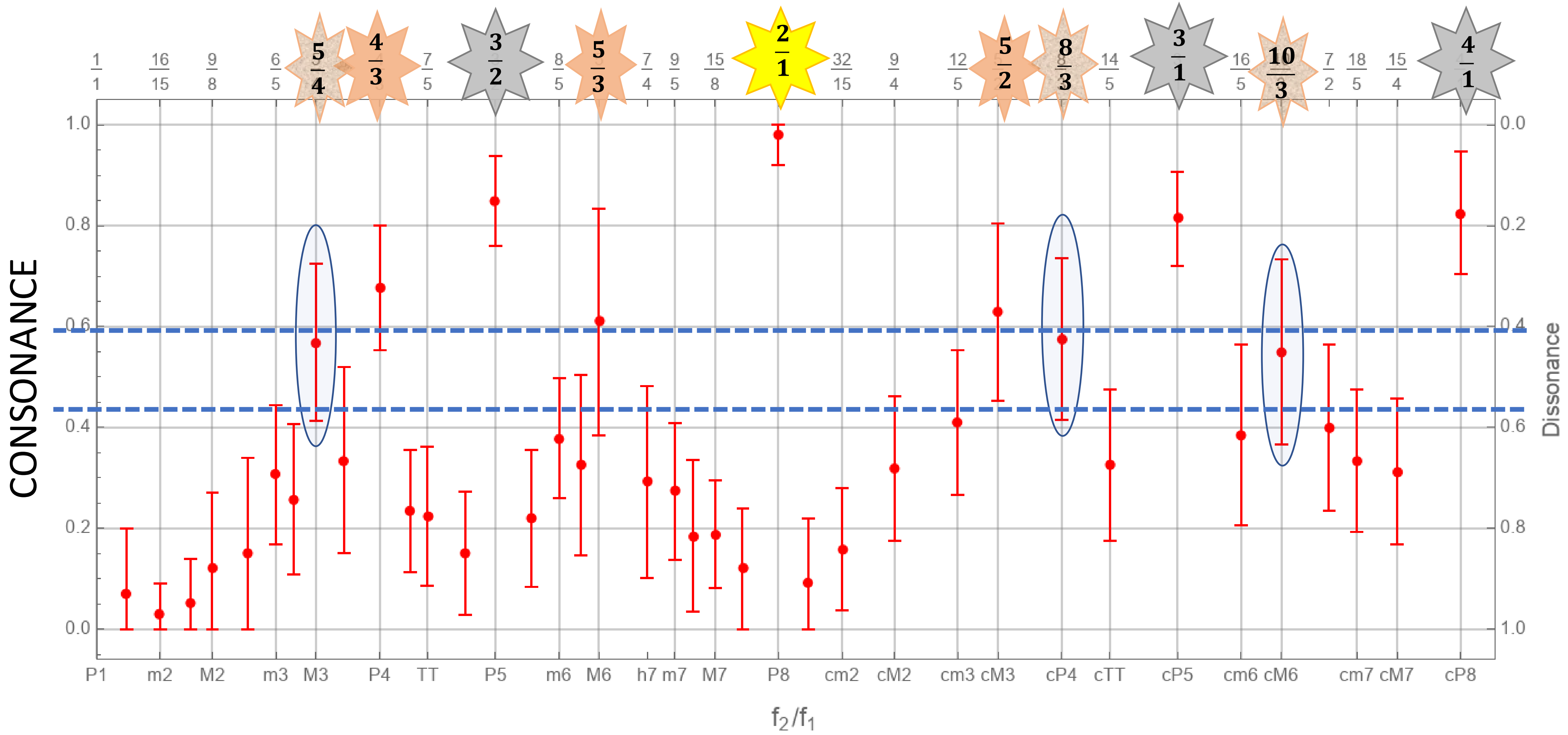
SILVER
GOLD
SILVER
SILVER
3/2
2/1
3/1
4/1
1/1 16/15 9/8 6/5 5/4 4/3 7/5 8/5 5/3 7/4 9/5 15/8 32/15 9/4 12/5 5/2 8/3 14/5 16/5 10/3 7/2 18/5 15/4
CONSONANCE
Dissonance
1.0
0.8
0.6
0.4
0.2
0.0
P1 m2 M2 m3 M3 P4 TT P5 m6 M6 h7 m7 M7 P8 cm2 cM2 cm3 cM3 cP4 cTT cP5 cm6 cM6 cm7 cM7 cP8
f2/f1

THE WINNERS



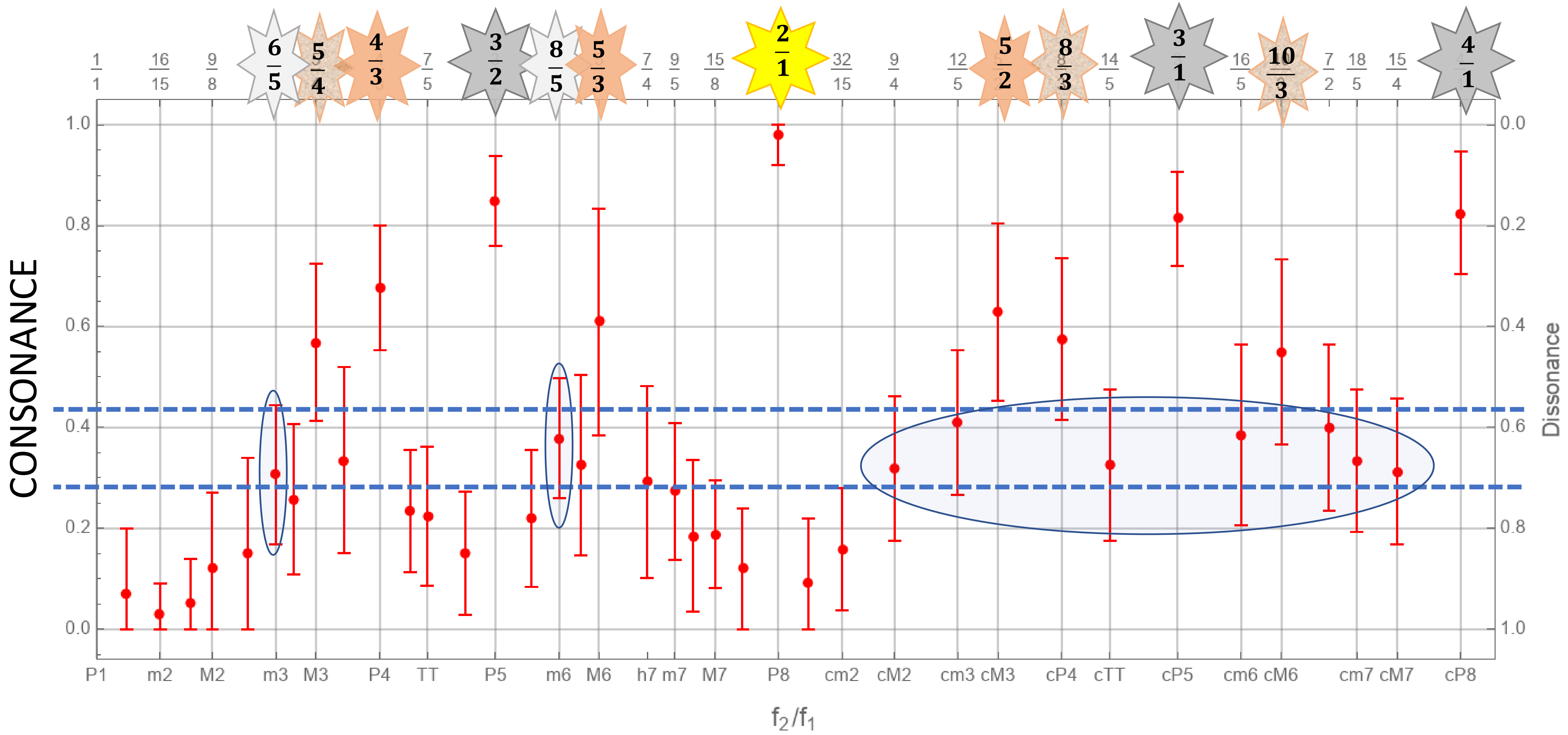
BRONZE
SILVER
BRONZE
GOLD
BRONZE
SILVER
SILVER
CONSONANCE
Dissonance
P1 m2 M2 m3 M3 P4 TT P5 m6 M6 h7 m7 M7 P8 cm2 cM2 cm3 cM3 cP4 cTT cP5 cm6 cM6 cm7 cM7 cP8
f2/f1

THE WOODS



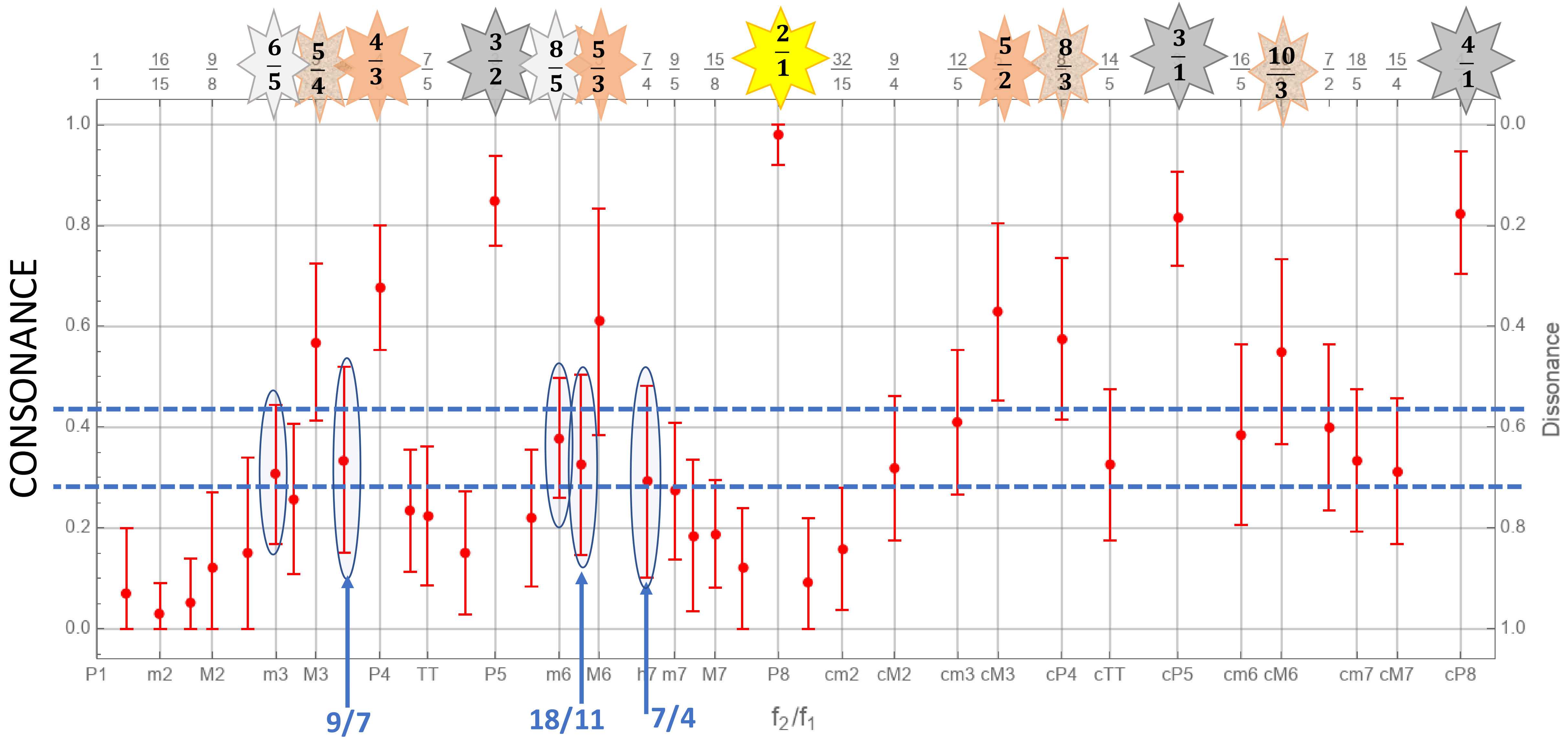
CONSONANCE
Dissonance
f_2/f_1
P1 m2 M2 m3 M3 P4 TT P5 m6 M6 h7 m7 M7 P8 cm2 cM2 cm3 cM3 cP4 cTT cP5 cm6 cM6 cm7 cM7 cP8
1/1 16/15 9/8 6/5 5/4 4/3 7/5 3/2 8/5 5/3 7/4 9/5 15/8 2/1 32/15 9/4 12/5 5/2 8/3 14/5 3/1 16/5 10/3 7/2 18/5 15/4 4/1
0.0 0.2 0.4 0.6 0.8 1.0

THE PAPERS

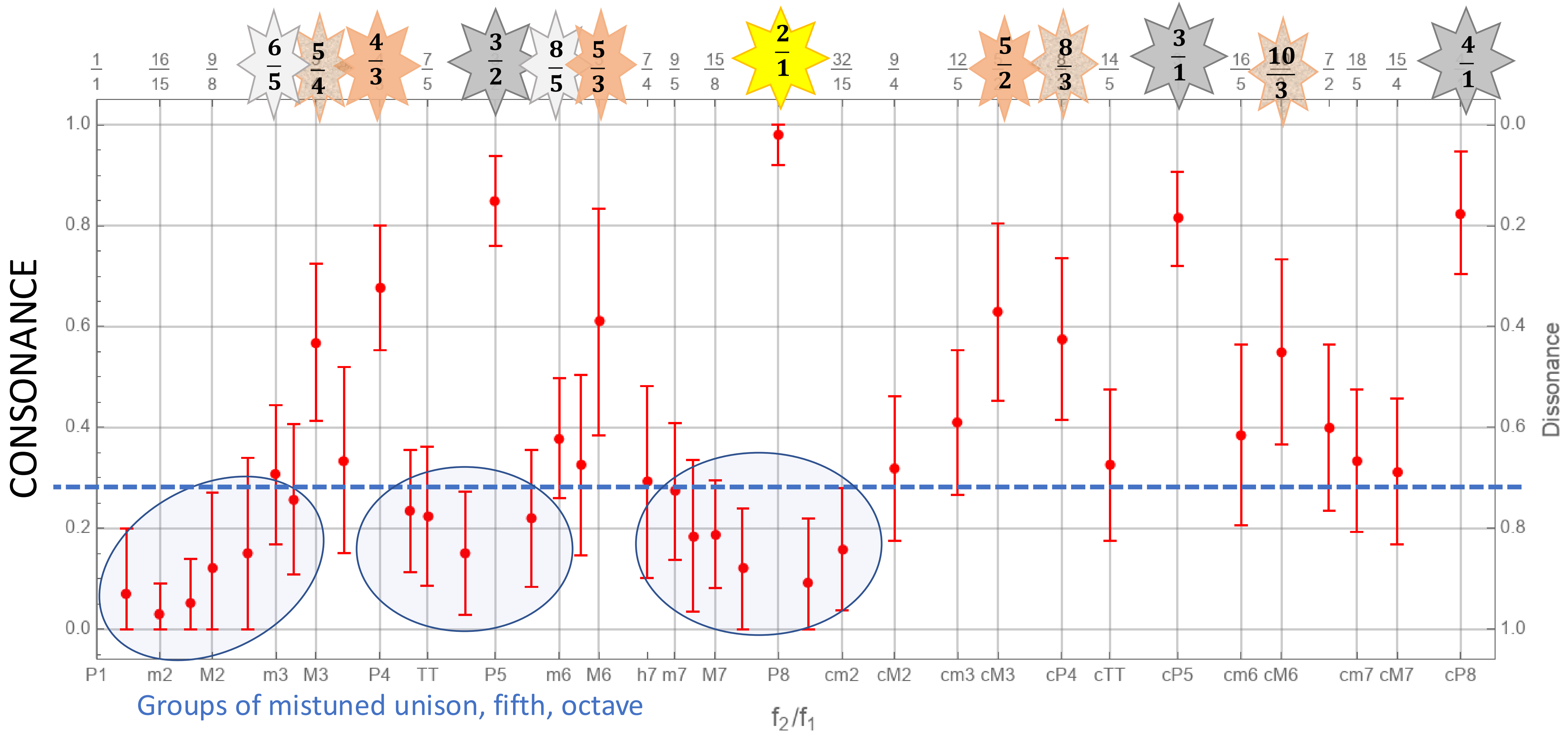


CONSONANCE
Dissonance
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
P1 m2 M2 m3 M3 P4 TT P5 m6 M6 h7 m7 M7 P8 cm2 cM2 cm3 cM3 cP4 cTT cP5 cm6 cM6 cm7 cM7 cP8
f_2/f_1

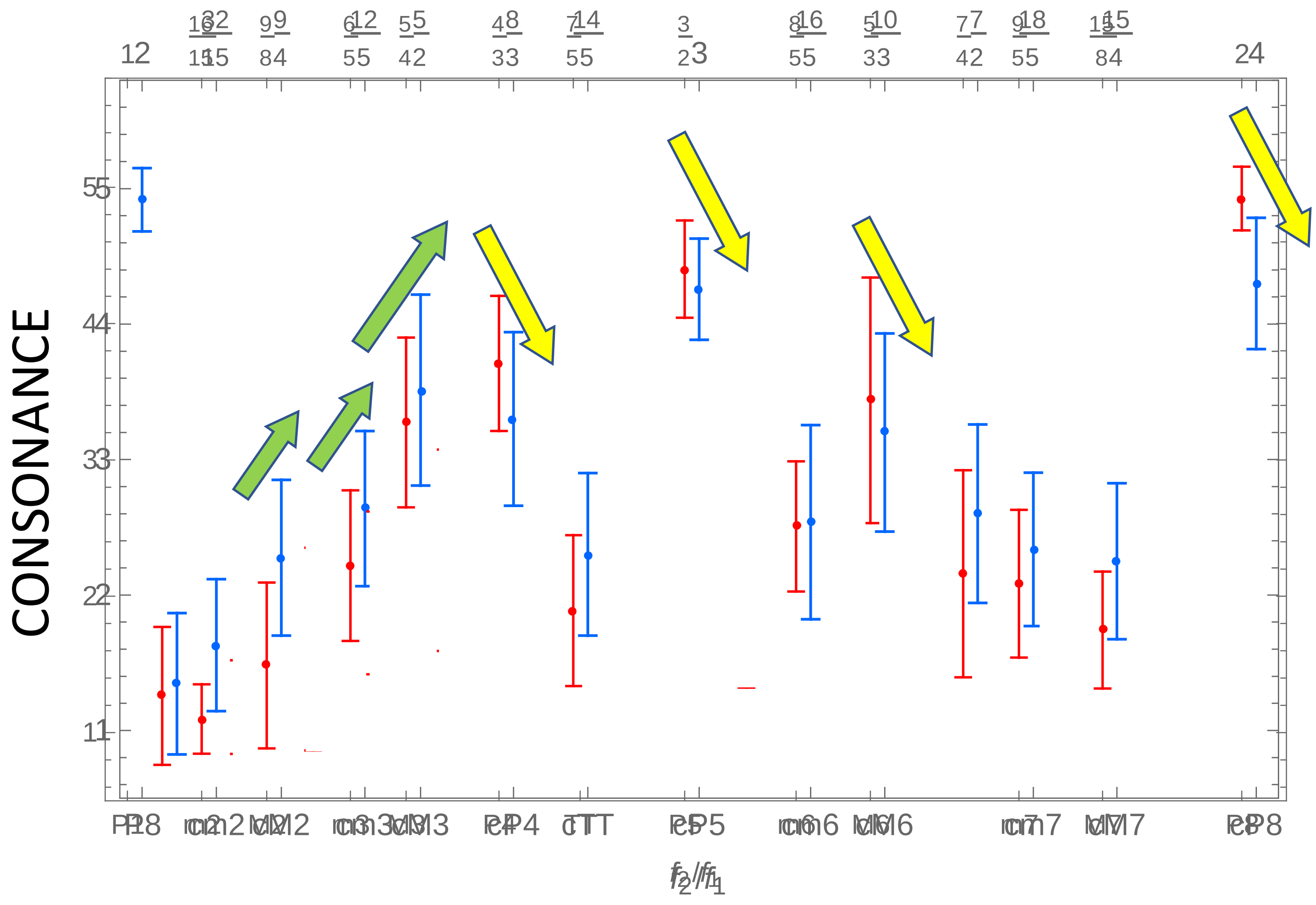
THE INTRUDERS AMONG PAPERS (1st octave)



THE LASTS



Groups of mistuned unison, fifth, octave first vs second octave: quite (not fully) equivalent



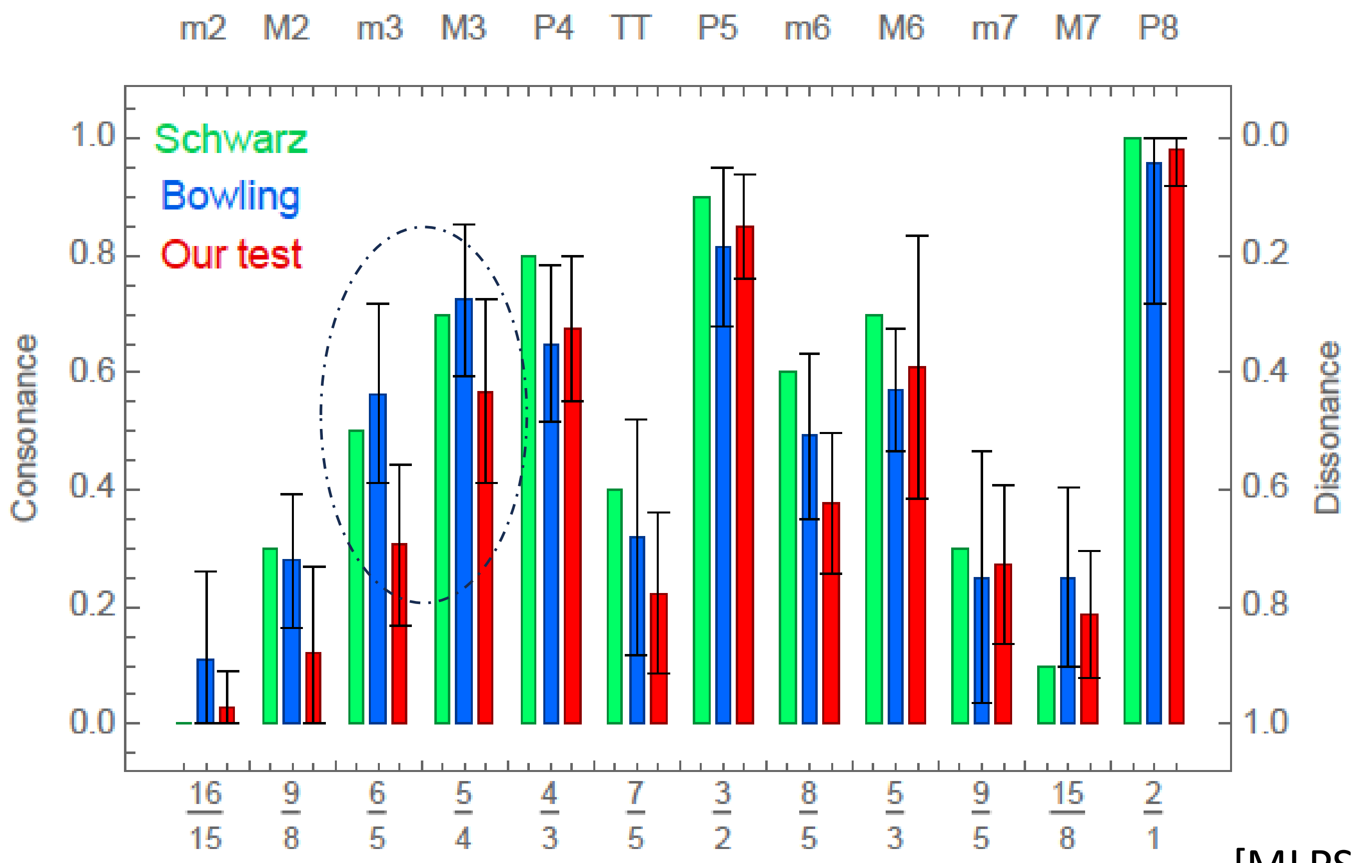
More consonant dyads are worse if compound*, except for cM3 (tenth)

*E.g. Compound octave less severe than mistuned unison

Octave equivalence is NOT fully TRUE!
(as composers know)

COMPARISON WITH OTHER TESTS

Other test restricted to chromatic scale (12) within 1 octave
tipically use piano timbre → cultural effect emphasized (thirds) → that is why we rather used a «neutral» timbre



[MLPS EPJP 2022]

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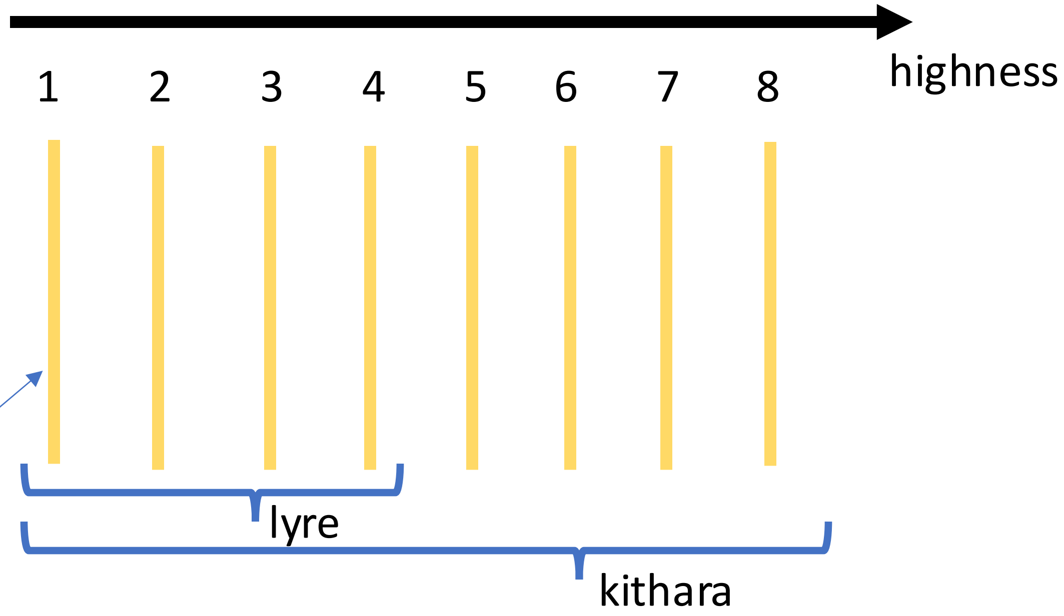
ANCIENT GREEK MUSIC

Consonance was obtained by a suitable tuning of the 8 strings of the kithara (or the 4 of the lyre)
Musical practice was melodic (voice at unison or octave with the instrument)



Take this lower string fixed,
how to tune the other strings? … with the 3 best consonances!

Indeed, only 3 intervals (dyads) were considered to be consonant and were involved in the tuning procedure

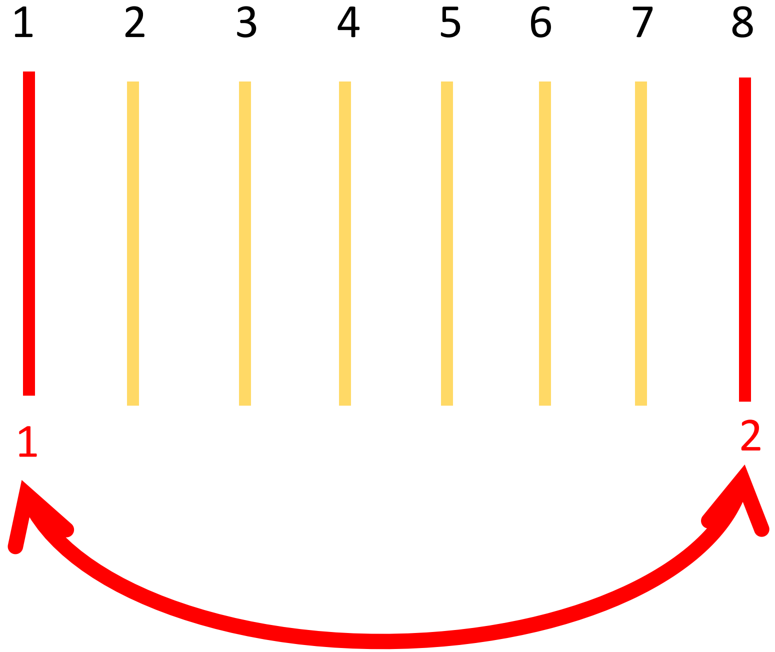
ANCIENT GREEK MUSIC: CONSONANT INTERVALS

$L_A = 2/3\ L$ $L_B = 1/3\ L$

monochord

$$\frac{f_8}{f_1} = \frac{L_A}{L_B} = \frac{2}{1}$$

GOLD



1) διὰ πασῶν = through all → **THE OCTAVE (P8)**

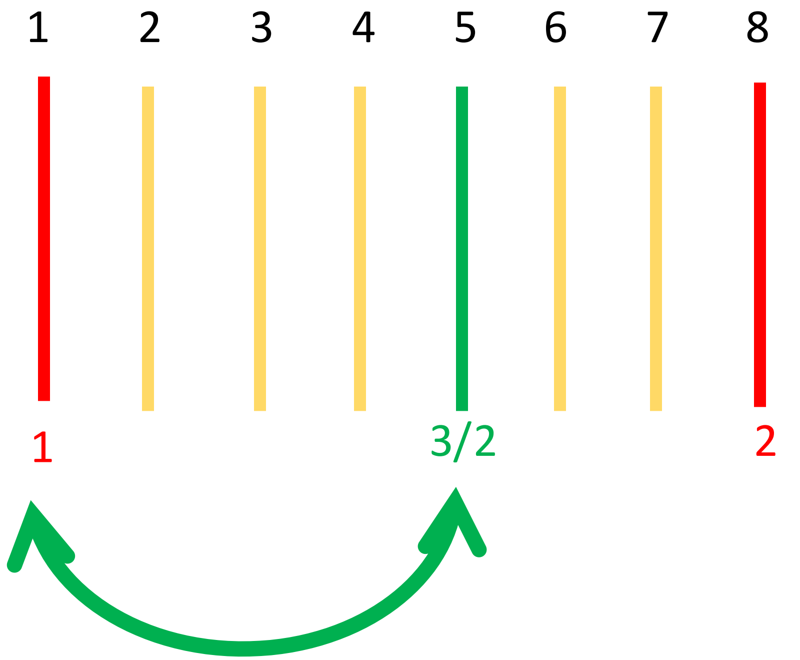
ANCIENT GREEK MUSIC: CONSONANT INTERVALS

$L_A = 3/5\ L$ $L_B = 2/5\ L$

monochord

$$\frac{f_5}{f_1} = \frac{L_A}{L_B} = \frac{3}{2}$$

SILVER



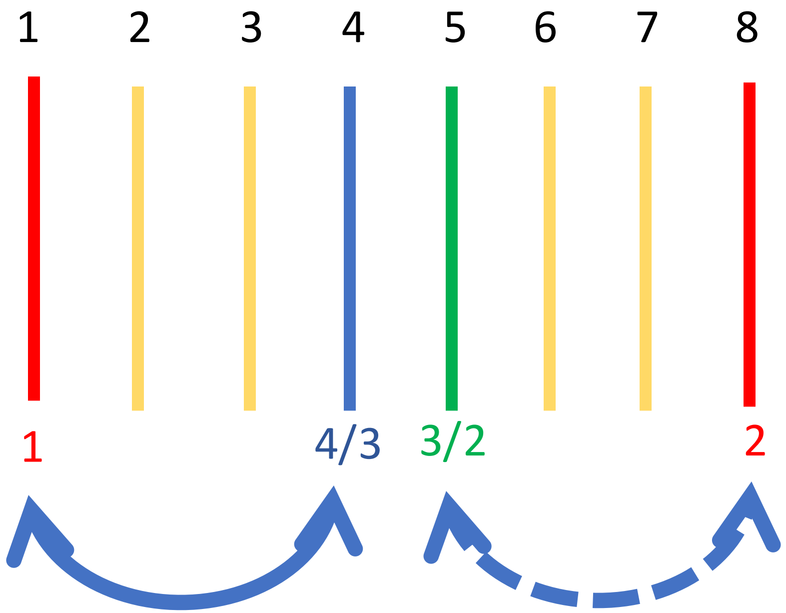
2) διὰ πέντε = through five → **THE FIFTH (P5)**

ANCIENT GREEK MUSIC: CONSONANT INTERVALS

monochord

$$\frac{f_4}{f_1} = \frac{L_A}{L_B} = \frac{4}{3}$$

BRONZE



3) διὰ τεσσάρων = through four → **THE FOURTH (P4)**

ANCIENT GREEK MUSIC: 7 NOTES FOR MANY MODES

Two equal tetrachords, with the two middle strings tuned differently (via fifths and fourths) leading to MODES

tetrachord — Diazeusi (separation) — tetrachord

Modern names	Diatonic mode	1–2	2–3	3–4	4–5	5–6	6–7	7–8
Ionian	Lydian	e	e	l	e	e	e	l
Dorian	Phrygian	e	l	e	e	e	l	e
Phrygian	Dorian	l	e	e	e	l	e	e
etc	etc							

String 1: 1; string 4: 4/3; string 5: 3/2; string 8: 2

Also denoted «Pythagorean (diatonic) scale»

epogdoon (1/8 more)

(take 4th and 5th strings)

$$e = \frac{f_5}{f_4} = \frac{\frac{3}{2}}{\frac{4}{3}} = \frac{9}{8}$$

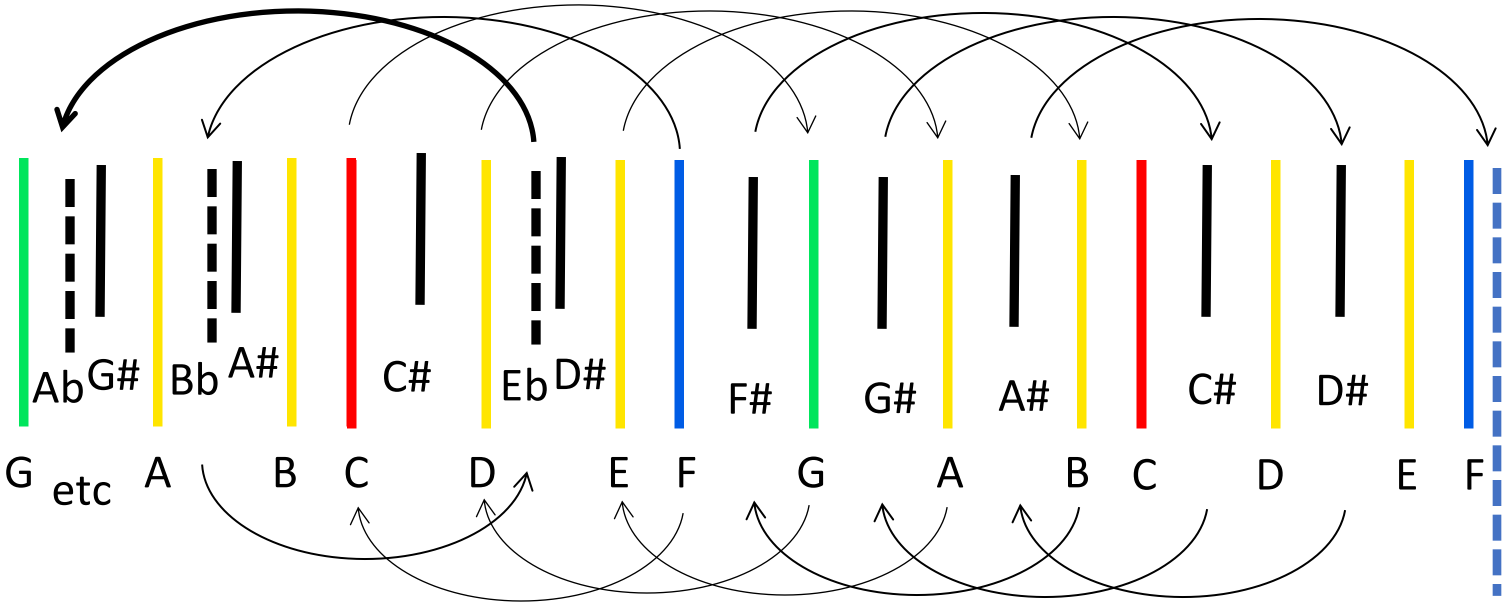
l = limma (residue)

(take 3th and 4th strings for lydian)

$$l = \frac{f_4}{f_3} = \frac{\frac{4}{3}}{e^2} = \frac{256}{243}$$

PYTHAGOREAN CHROMATIC SCALE

Start from F. Up: Descending 4ths, ascending 5ths → 5ths circle do not close F’ different from F
Down: Descending 5ths, ascending 4ths → impossible to make # e b coincide

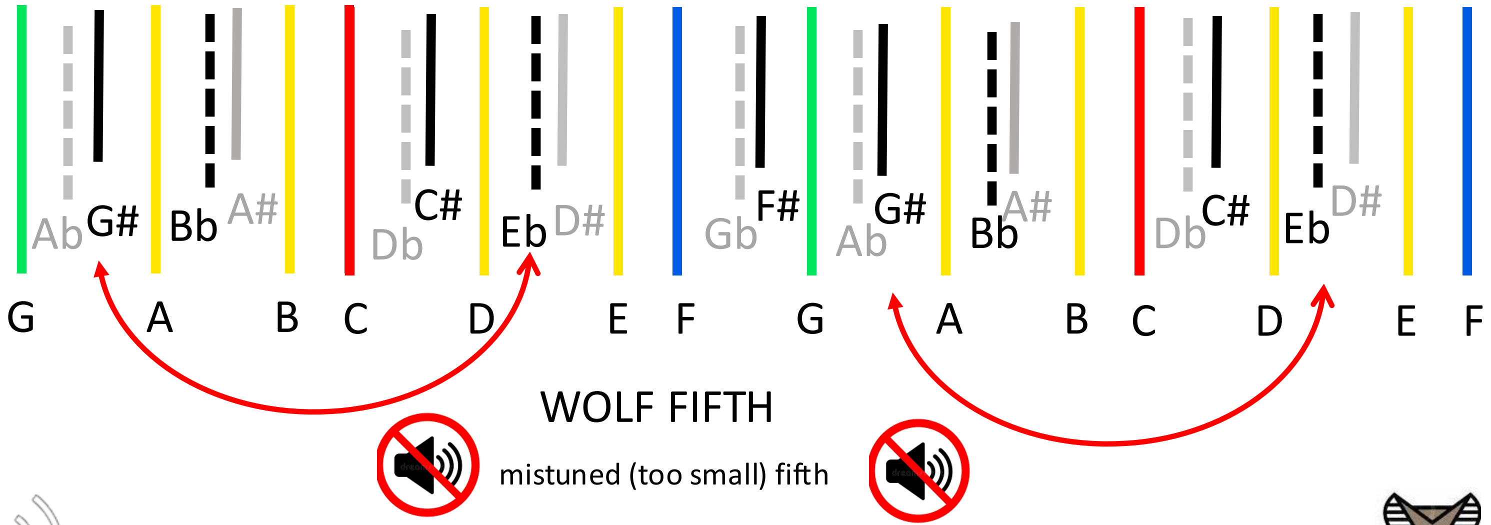


E#=F’ > F by Pythagorean comma

PYTHAGOREAN CHROMATIC SCALE

SOLUTION: Select 12 and do not play the wolf fifth

Example of a specific choice for the blacks:
take 2 descending (Bb Eb, not Ab Db Gb) and 3 ascending (F# C# G#, not D# A#)



P5

OTHER SOLUTION: Take more than 12

12 NOTES FOR MANY SCALES

Antiquity — Pythagorean tuning
Fifths Circle not closed
→ WOLF

1500 — Natural scale (Th, Zarlino)

31-TET (Th, 1550)
($S^{31}=2$)

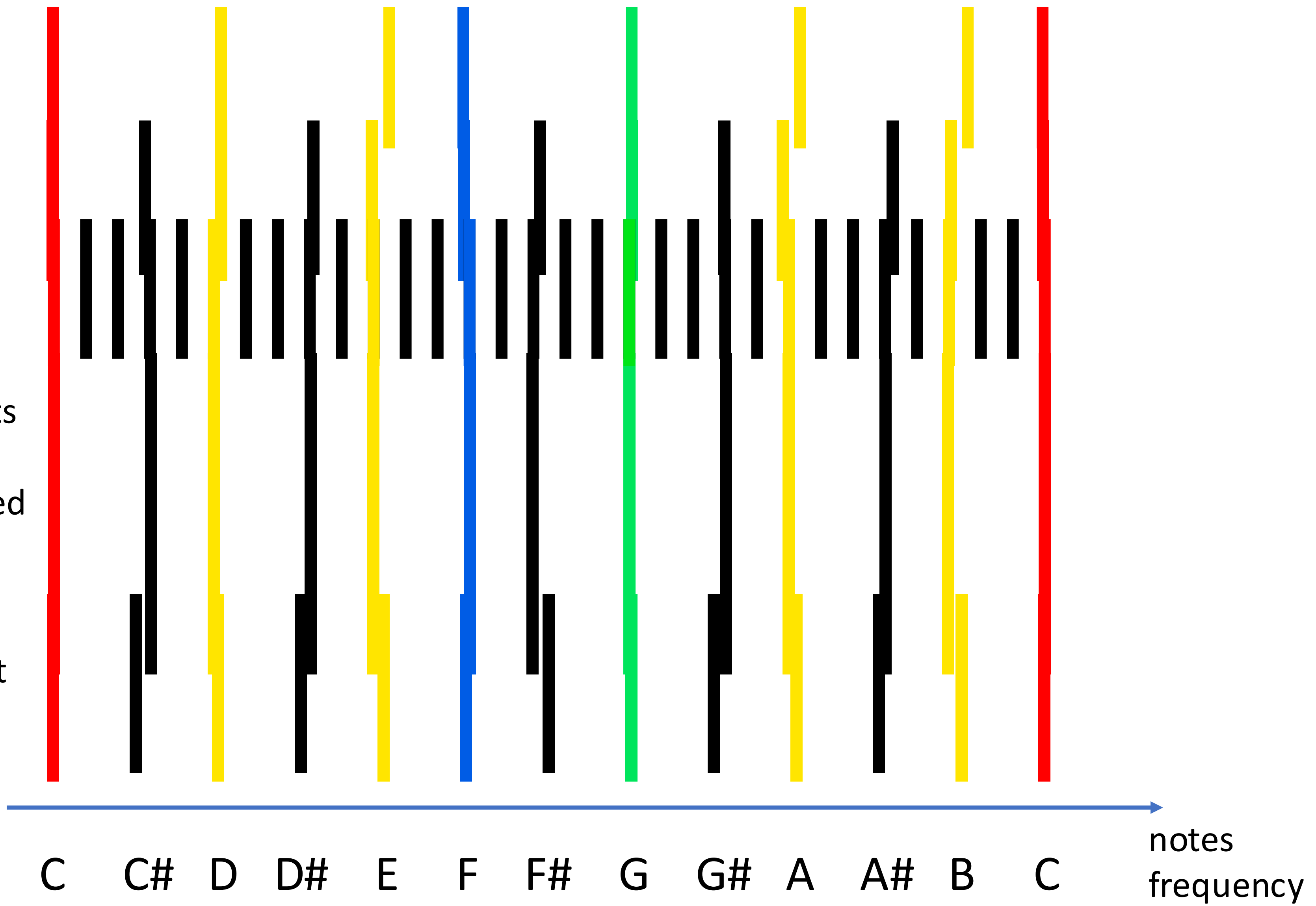
1600

Meantone Temperaments
e.g. Quarter Comma
→ 5ths and 3rds tempered

1700

1800 — 12-TET Equal Temperament
($S^{12}=2$)

time



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APPROACHES TO CONSONANCE AND DISSONANCE

An incomplete list is the following:

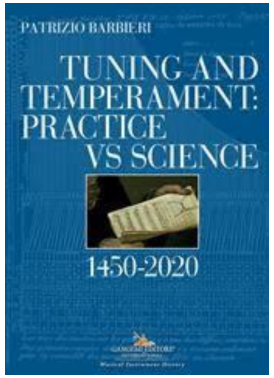
- From Pythagora's School to Zarlino
- Benedetti (recently Tenney)
- Galileo
- Eulero
- Rameau Tartini etc
- Foderà
- Helmholtz (1863)
- Frova
- Plomp Levelt (1965)/Hutkinson Knopoff (1978)
- Sethares (1993)
- Gill Purves (2008)
- Dillon (2013)
- Milne (2013)/Harrison Pearce (2018)
- Stolzenburg (2015)
- …

NUMERICAL

COINCIDENCE THEORIES

ROUGHNESS THEORIES

HARMONICITY THEORIES



See excellent book by P. Barbieri

See review by:
P. Harrison M. Pearce.
Simultaneous Consonance in Music Perception and Composition
Psychol. Rev. 2020 Mar; 127(2): 216–244.

NUMERICAL APPROACH

Antiquity (modes) since Middle Ages (gregorian): only 3 consonant intervals P8 (gold), P5 (silver), P4 (bronze)

→ Pythagorean School:

They are associated to ratios (2/1, 3/2, 4/3) involving only integers from 1 to 4

→ Post-diction: those numbers have a «mystic» role **(*tetraktis)***

Renaissance (counterpoint and polphony): also 6M (bronze) and 3M (woods) are imperfect consonances

→ Zarlino (*Istitutioni harmoniche*,1558):
include 5 and 6 among good numbers (senario) → just (or natural) scale

Even more recently: Mathematical formulas (without underlying physical argument) that reproduce the ordering of the degree of consonance «popularly» accepted have been proposed, e.g. by Euler, Frova, Stolzenburg,…

COINCIDENCE THEORIES

more "coincident" (i.e. in-phase) vibrations of the dyad's notes = more consonance

A **physical theory** known as that of "**coincidences**» [based on arguments from Greek and Roman antiquity] was developed by renaissance theorists.

1) The Venetian mathematician and physicist **Giovanni Battista Benedetti** (1530-1590) expands the coincidence theory by suggesting the (first) indicator, providing reasonable rankings for dyads

$$\frac{f_2}{f_1} = \frac{m}{n} \quad \Rightarrow \quad I^B = m\,n$$

Benedetti's dissonance indicator

Reasonable rankings are obtained:
P8, P5, P4, M6, M3, m3, …

2) In *Discorsi intorno a due nuove scienze* (1638) Galileo interprets consonance as the ear «preference» for two sounds of commensurable frequencies with simple ratios

nr of pulses of the higher string that are sychronous with the lower string

$$\Rightarrow \quad I^G = \frac{1}{m}$$

Galileii's consonance indicator

Reasonable ranking:
P8, P5, P4, M6 and M3, m3, m6, …

CRITICISM: 1) for non in-phase pulses the perception is the same
2) non-continuous indicator: how to deal with mistuned fractions close (within 3Hz) to simple ratios?

HARMONICITY THEORIES

more harmonics coincide = more consonance

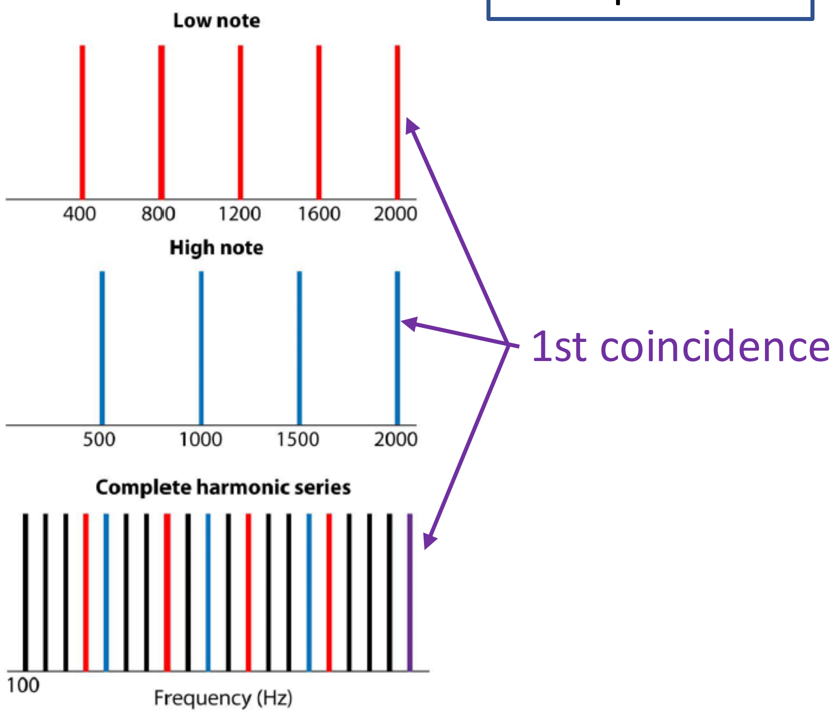
(historically argued during 18th-19th: Rameau, Esteve, Tartini, Pizzati,…)

Representative example: the «percentage similarity» is an indicator by Gill Purves (2008)

$$C^H_{GP} = \frac{m+n-1}{mn}$$

= number of harmonics in common between f_0-f_1 (m) and f_0-f_2 (n) over total number of harmonics of f_0 , up to first coincidence

Example: M3



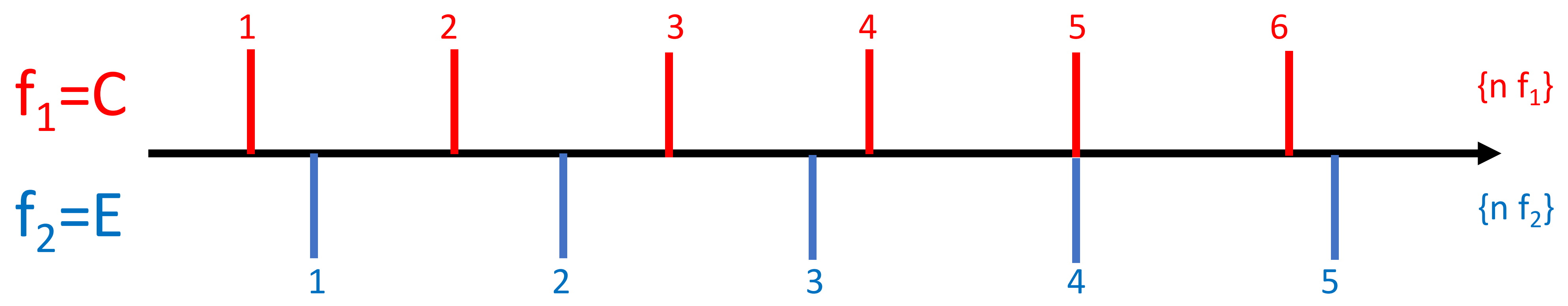
CRITICISM: not continuos

ROUGHNESS THEORIES

more harmonics are beating = more dissonance

(historically developed from fall 19th-20th: Helmholtz 1863, Plomp Levelt 1965, …)

Example: M3



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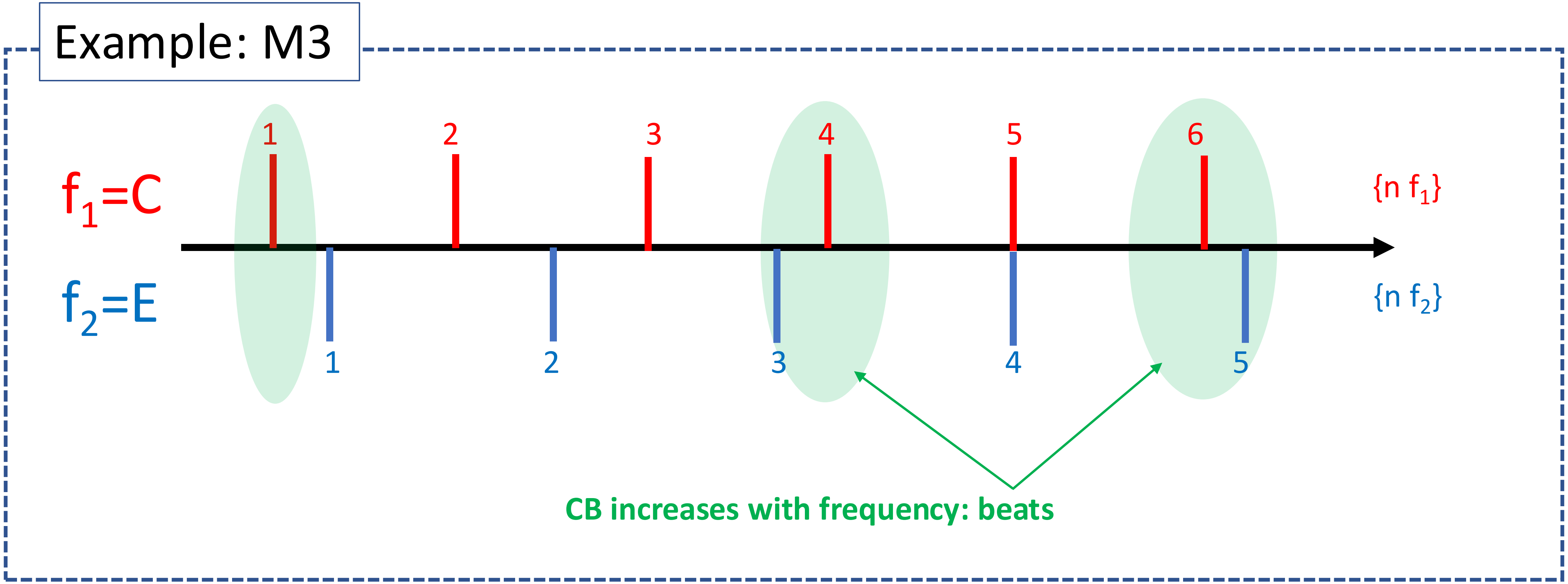

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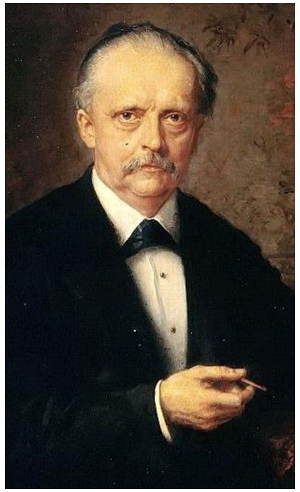
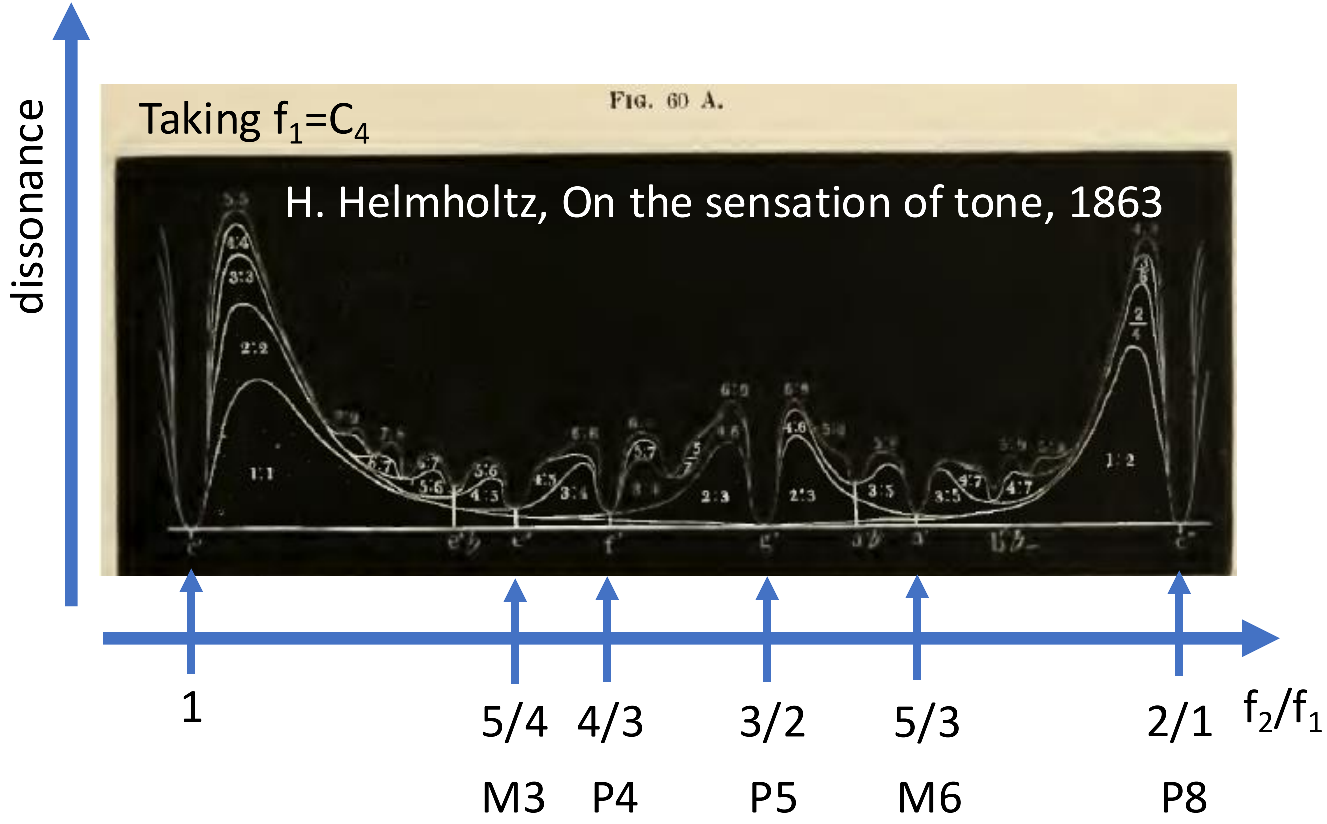




Naturally continuous!
Enthousiastic reaction:
it superseeded
coincidence and
harmonicity theories

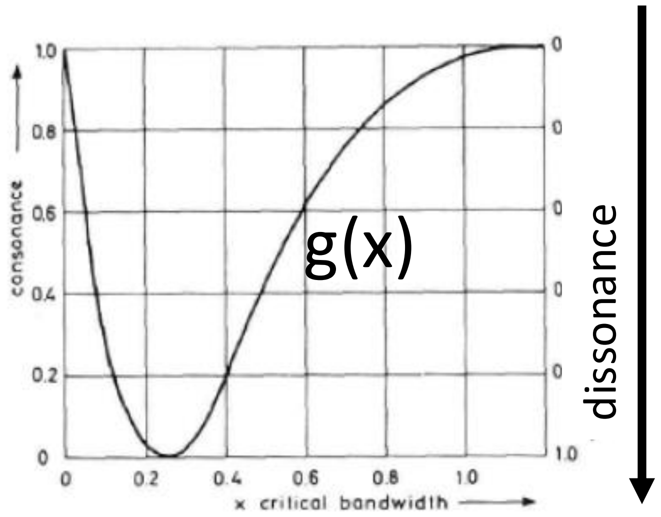
more harmonics are beating = more dissonance

[Plomp Levelt 1965]

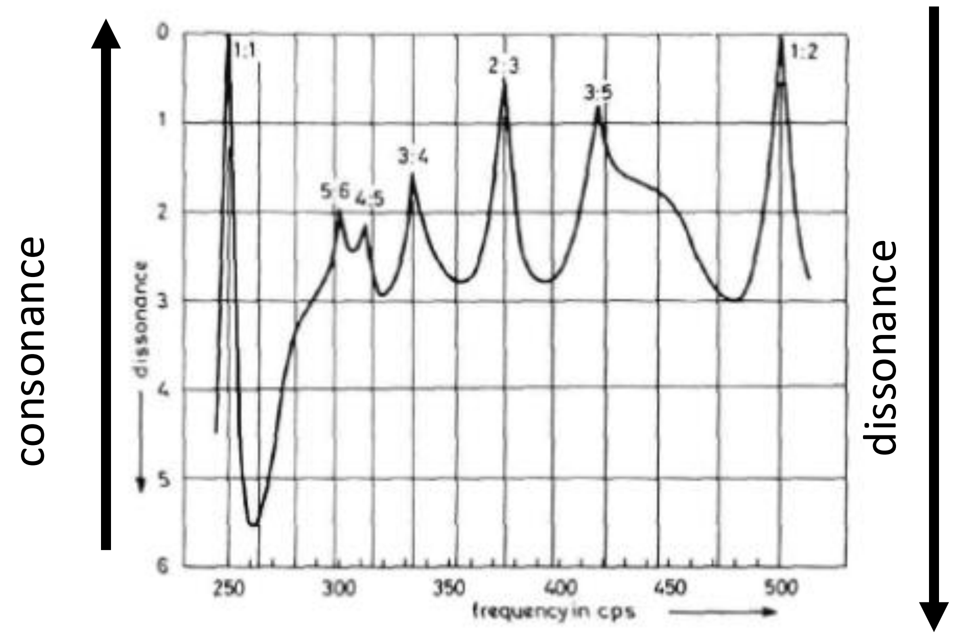
P&L include CB: dissonance indicator with equal weights for n=6 harmonics

$$d(f_1, f_2) = \sum_{f_i, f_j} g\left(\frac{|f_i - f_j|}{CB(\frac{f_i+f_j}{2})}\right)$$

=x



Maximum roughness at about 1/4 of CB



Does not work so well… problems with 6M, 3m, 3M…

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OUR STUDY FOR DYADS [MLPS EPJP 2022]

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THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

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Dyad's consonance and dissonance: combining the compactness and roughness approaches

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Abstract At present, there are two approaches that aim at explaining on physical grounds the psychoacoustic perception of consonance and dissonance for dyads, whose pioneers have been, respectively, Galilei and Helmholtz: One is based on the "compactness" of the waveform of the combined signal, while the other on the absence of "roughness" due to possible beats. We perform a detailed study of each approach and find that none of the associated model versions, not even the more refined ones, is fully satisfactory when faced to perceptual data on dyads. We show that combining the two approaches results instead in a surprisingly successful agreement with perceptual data: This demonstrates that compactness and roughness are both necessary ingredients for a phenomenological description of consonance and dissonance.

simple form
(no prime factors
in common)

Take

$$\frac{f_2}{f_1} = \frac{m}{n} \geq 1$$

and work out C&D «indicators»

OUR STUDY FOR DYADS [MLPS EPJP 2022]

A) Developed many indicators following:

1) «periodicity» approach based on the fundamental bass (upgrade of coincidence theories)

2) harmonicity apprach

3) roughness approach, also including: 1) DL 2) secondary beatings

periodicity and harmonicity indicators provide mathematically similar indicators
→ Compactness indicator

Extended to continuum
by using DL

B) Chi-square Analysis:

1) roughness indicators worse than compactness ones

2) astonishing performance of mixed compactness-roughness theories

THE PERIODICITY APPROACH

shorter period of the «fundamental bass» with respect to the periods of the dyad's tones = more consonance

$$f_0 = \frac{f_1}{n} = \frac{f_2}{m}$$

Basic consonance indicators are (dim analysis!)

$$I_2^P = \frac{f_0}{f_2} = \frac{1}{m} \qquad I_1^P = \frac{f_0}{f_1} = \frac{1}{n}$$

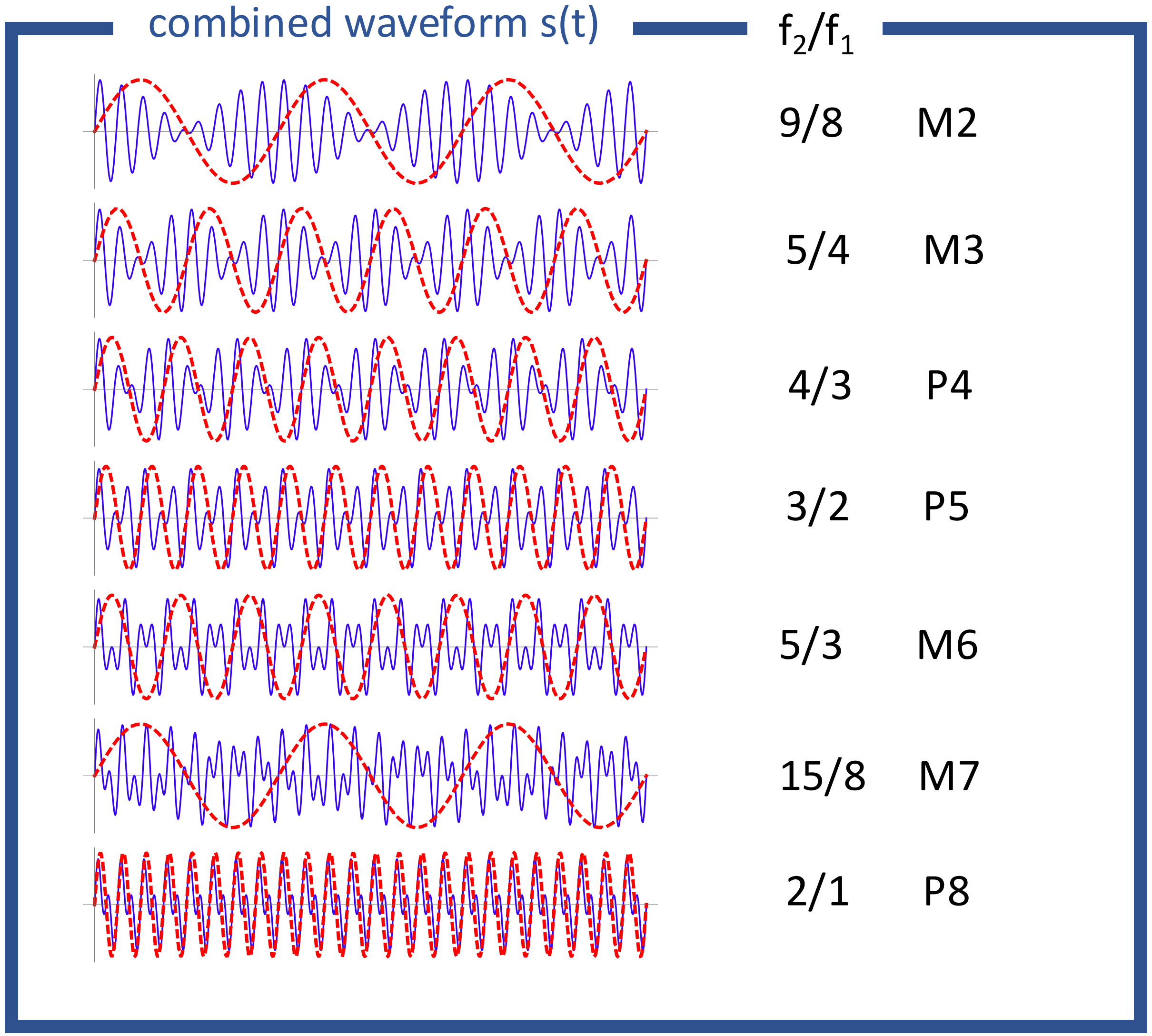
Other indicators are found taking A,G,H means:

$$\begin{cases} f_A = (f_1 + f_2)/2 \\ f_G = \sqrt{f_1 f_2} \\ f_H = f_A^2/f_G \end{cases}$$

$$I_A^P = \frac{f_0}{f_A} = \frac{2}{n+m}$$

$$I_G^P = \frac{f_0}{f_G} = \frac{1}{\sqrt{nm}}$$

$$I_H^P = \frac{f_0}{f_H} = \frac{n+m}{2nm}$$



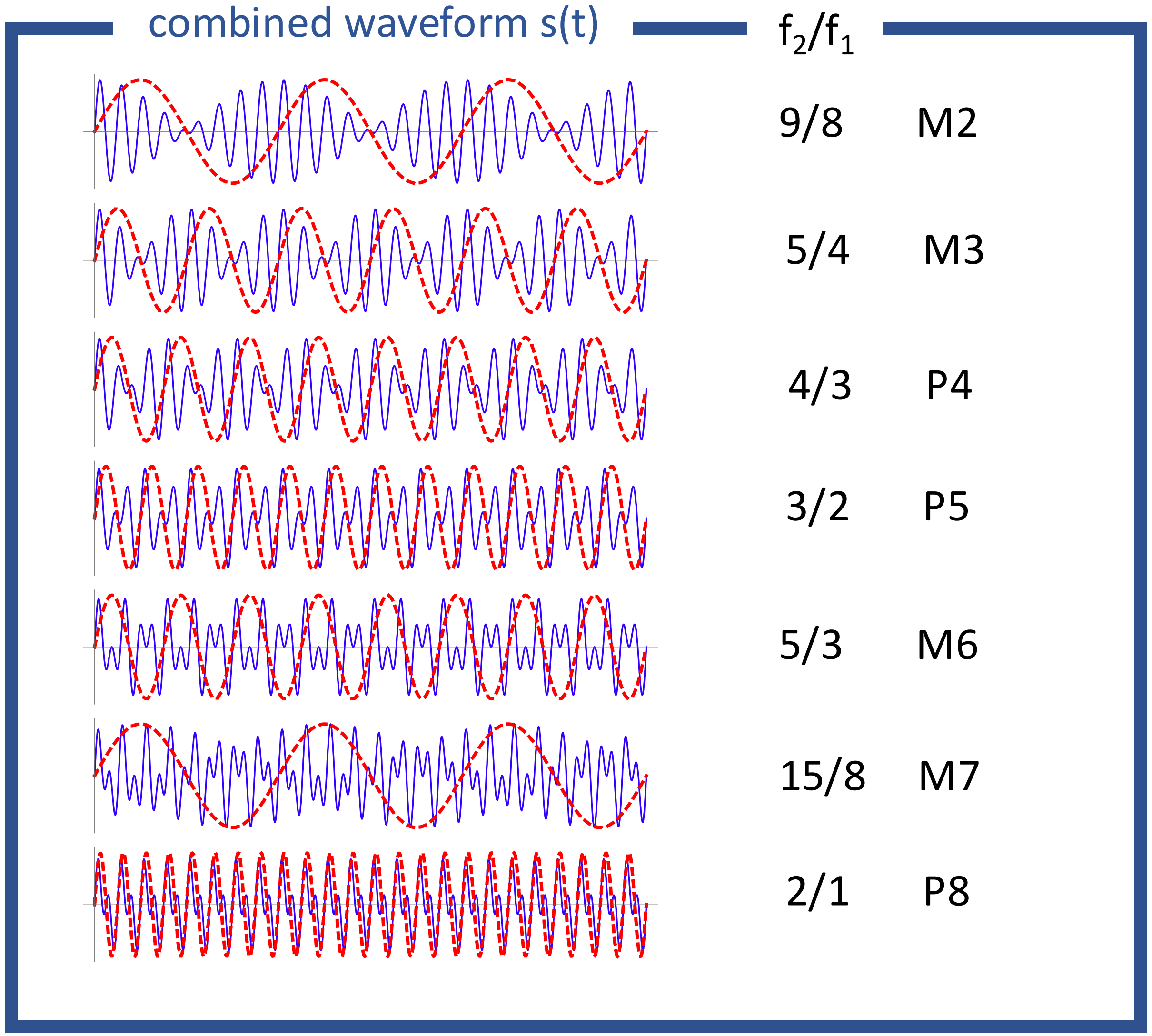
THE PERIODICITY APPROACH

shorter period of the «fundamental bass» with respect to the periods of the dyad's tones = more consonance

Related to Galilei's arguments about «coincidence» theory (16th-18th)

$$I_2^P = \frac{f_0}{f_2} = \frac{1}{m}$$

Number of strikes of the sharper string being in coincidence with strikes of the lower string



THE PERIODICITY APPROACH: CONTINUUM

Problem: discontinuous indicators → Solution: make them continuos by «guassianizing» the peaks with a σ = DL

$$x = f_2/f_1 \qquad C_X^P(x) = \mathrm{Max}_i\, \tilde{I}_X^P(x_i)\, e^{-\frac{(x-x_i)^2}{2\,\sigma(x)^2}}$$

THE PERIODICITY APPROACH: CONTINUUM

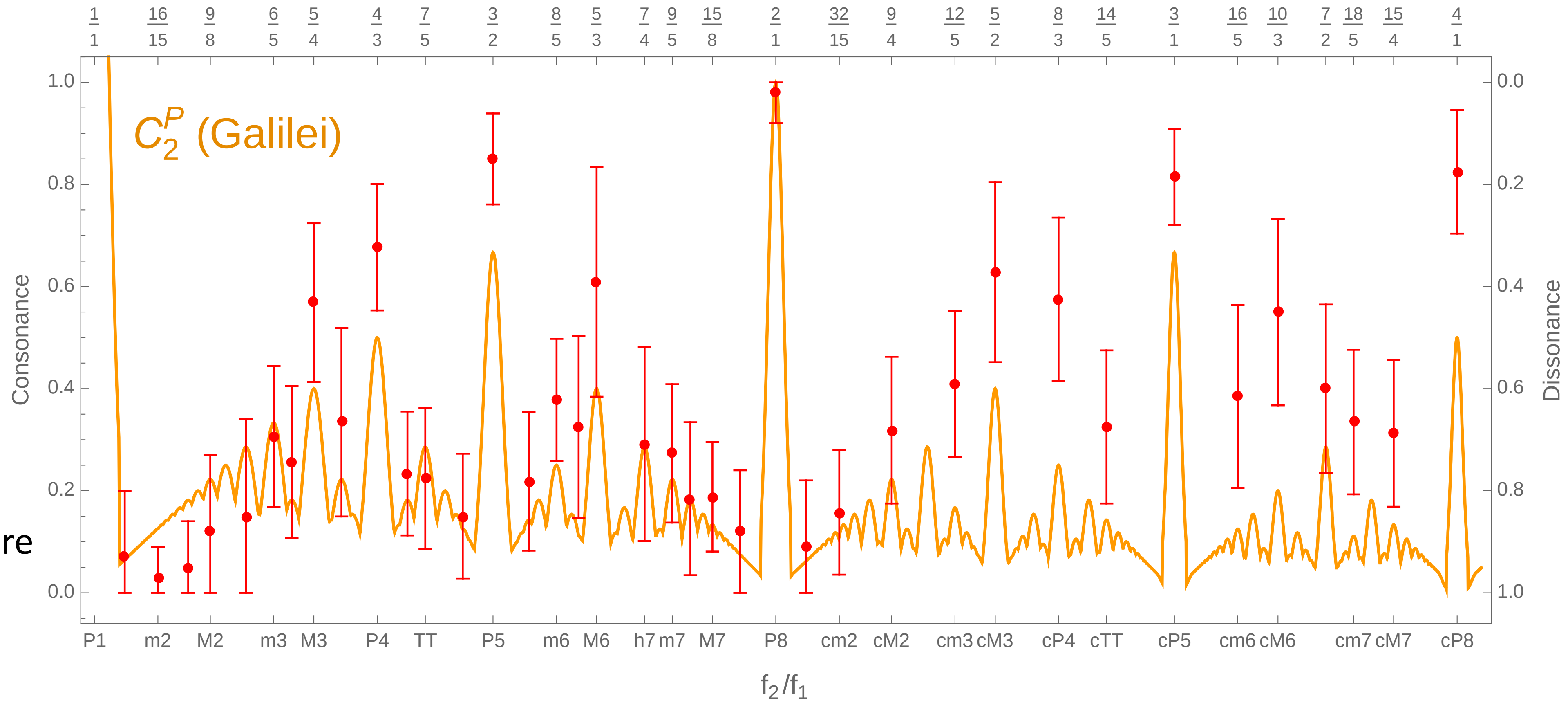
Problem: discontinuous indicators → Solution: make them continuos by «guassianizing» the peaks with a σ = DL

Representative example

$$I_2^P = \frac{f_0}{f_2} = \frac{1}{m}$$

has reduced chi square

$$\tilde{\chi}_G^2 = 1.3$$



Quite good! … but consonance peaks are not reached, especially for 2nd octave

THE HARMONITCITY APPROACH: CONTINUUM

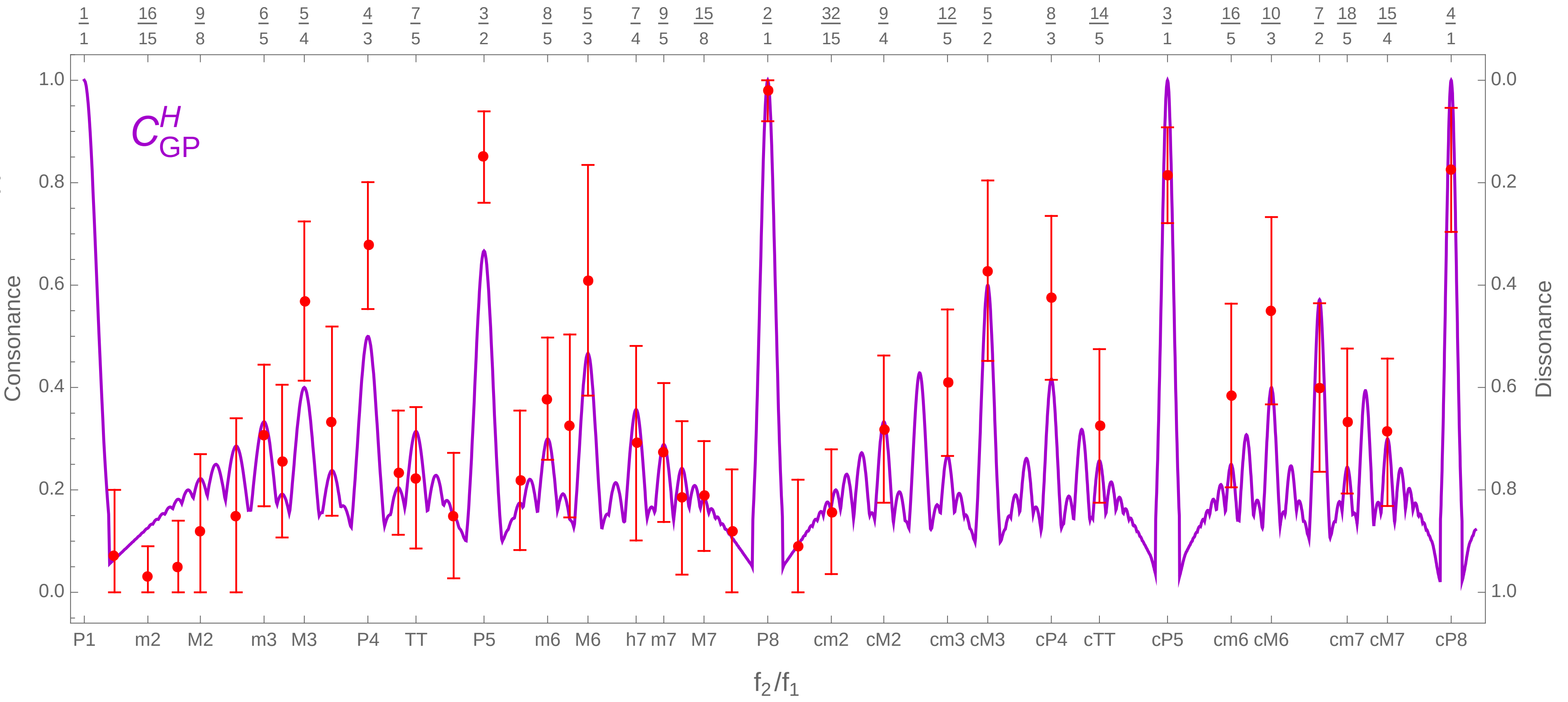
more harmonics coincide = more consonance

Representative example:

$$C^H_{GP} = \frac{m+n-1}{mn}$$

has reduced chi square

$$\tilde{\chi}^2_{GP} = 0.7$$



Good! … but consonance peaks are not reached for 1st octave, even too much for 2nd…

THE ROUGHNESS APPROACH

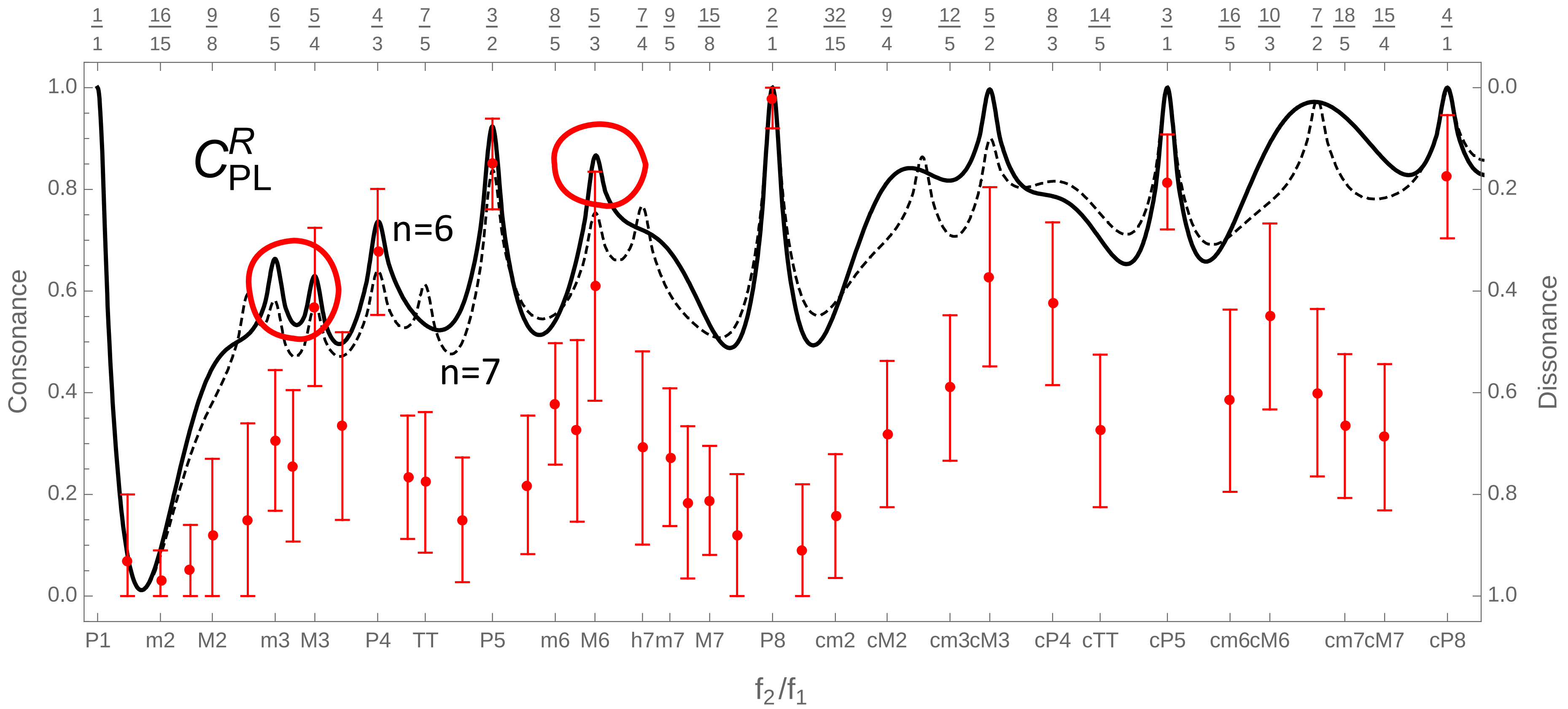
more harmonics are beating = more dissonance

P&L include CB: dissonance indicator with equal weights for n=6 harmonics (ad hoc, n=7 is a disaster)

Not so good…
problems with 6M,
inversion of m3 & M3, …

Indeed

$$\tilde{\chi}^2_{PL} = 5.9$$



Prediction of too much consonance in general, especially for dissonant dyads…

THE ROUGHNESS APPROACH

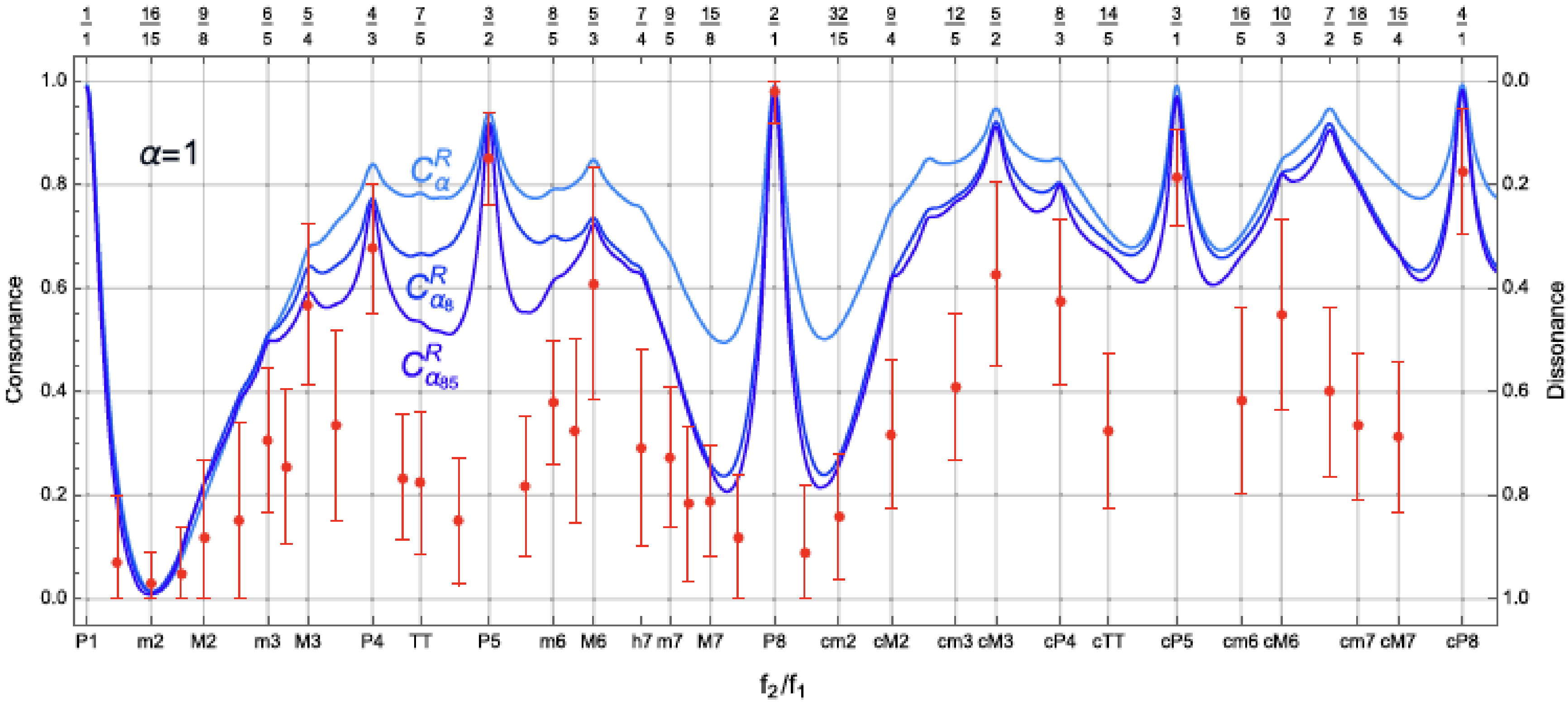
Model can be improved by adding:

1) Proper weights to suppress roughness effect for higher harmonics, like e.g. w_n=1/n   [Hutchinson Knopoff 1978]
2) Modification of g(x) to account for DL
3) Effect of secondary beats of mistuned octave and fifth

Representative example

$$\tilde{\chi}^2_{\alpha_{85}} = 2.9$$

… can't do much better



In any case, too large consonance predicted for dissonant dyads

COMBINING THE COMPACTNESS AND ROUGHNESS APPROACHES

Two approaches do different job: **PRIZES** vs **PENALTIES**

COMBINING THE COMPACTNESS AND ROUGHNESS APPROACHES

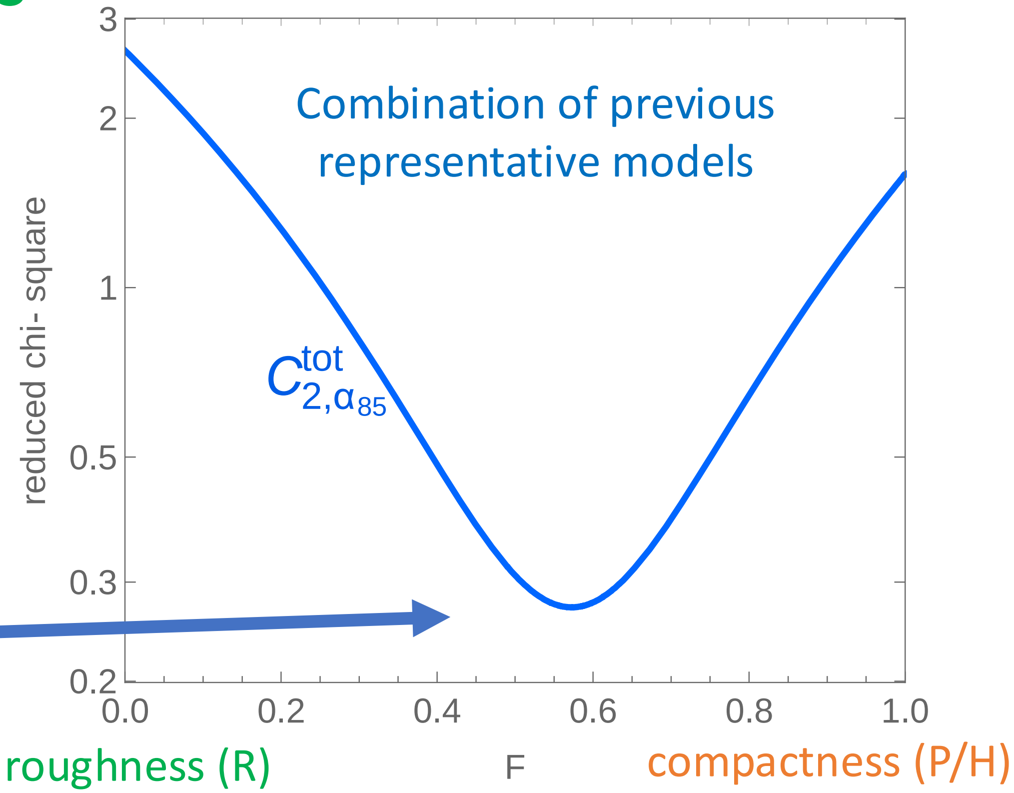
Two approaches do different job: **PRIZES** vs **PENALTIES**

F is fractional contribution of compactness with respect to roughness

$$C^{tot}_{X,Y} = \frac{F\, C^{P/H}_X + (1-F)\, C^R_Y}{N_{X,Y}}$$

normalization

$$\tilde{\chi}^2 \approx 0.3$$

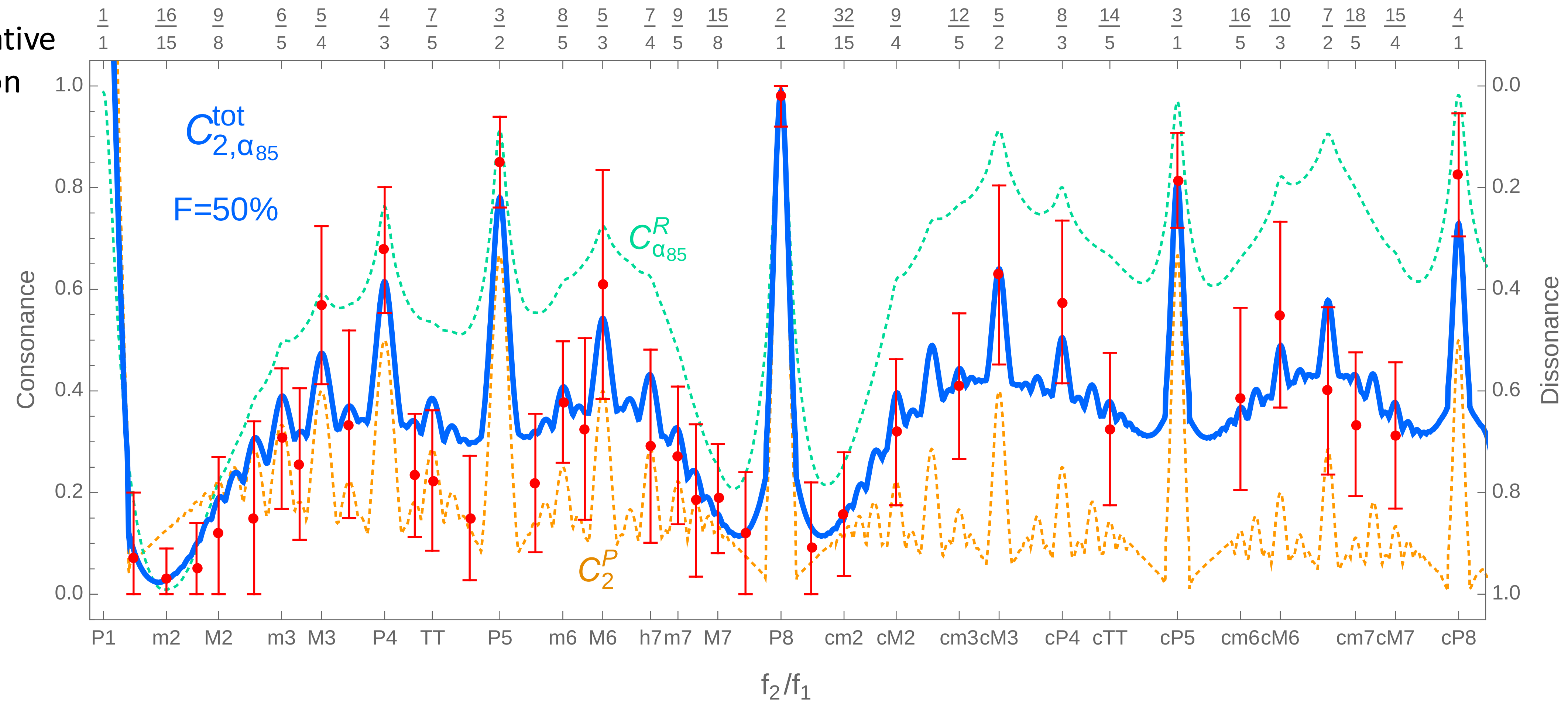


Compactness and roughness are both essential ingredients to explain C&D on physical grounds

chi-square has a minimum for **all possible combinations** of models around F=50%

COMBINING THE COMPACTNESS AND ROUGHNESS APPROACHES

Representative combination

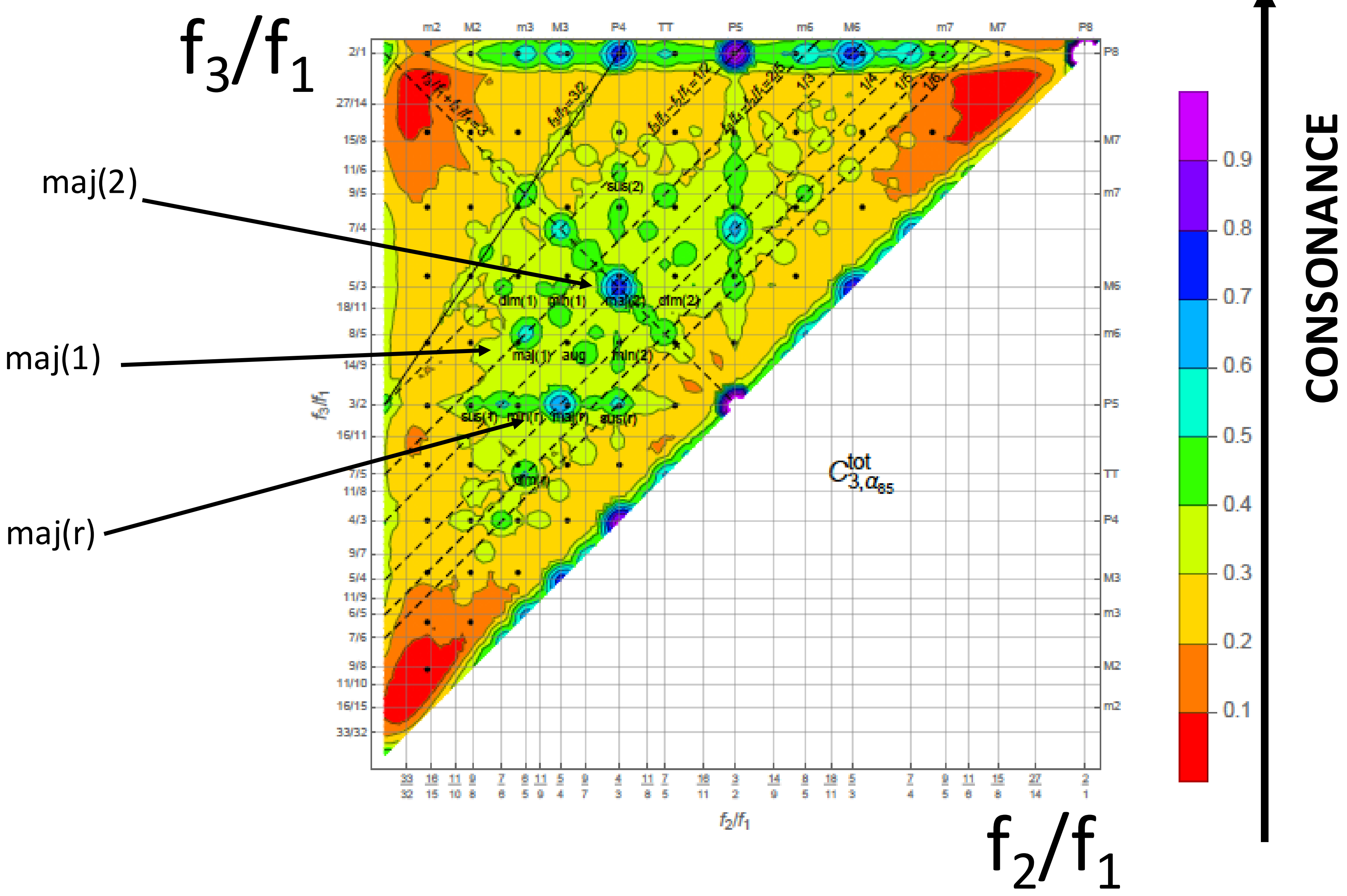


✓ Impressive agreement with data

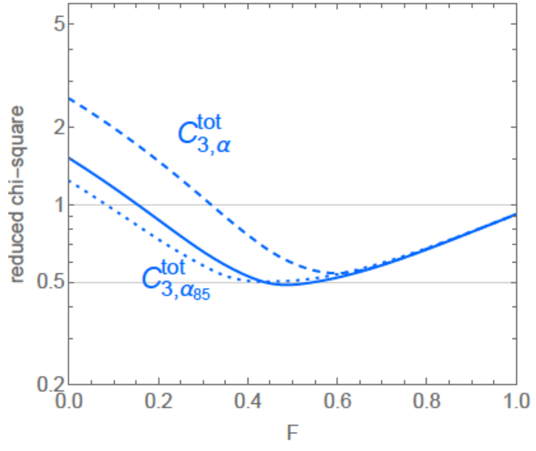
✓ Non trivial that the mean of two models is so much better than each separate model

EXTENSION TO TRIADS GIVES CONSISTENT RESULTS

[IM, EPJP 2023;
IM and GLP, EPJP 2024]



Same conclusions found for also triads, using data on 66 triads from Bowling et al. test

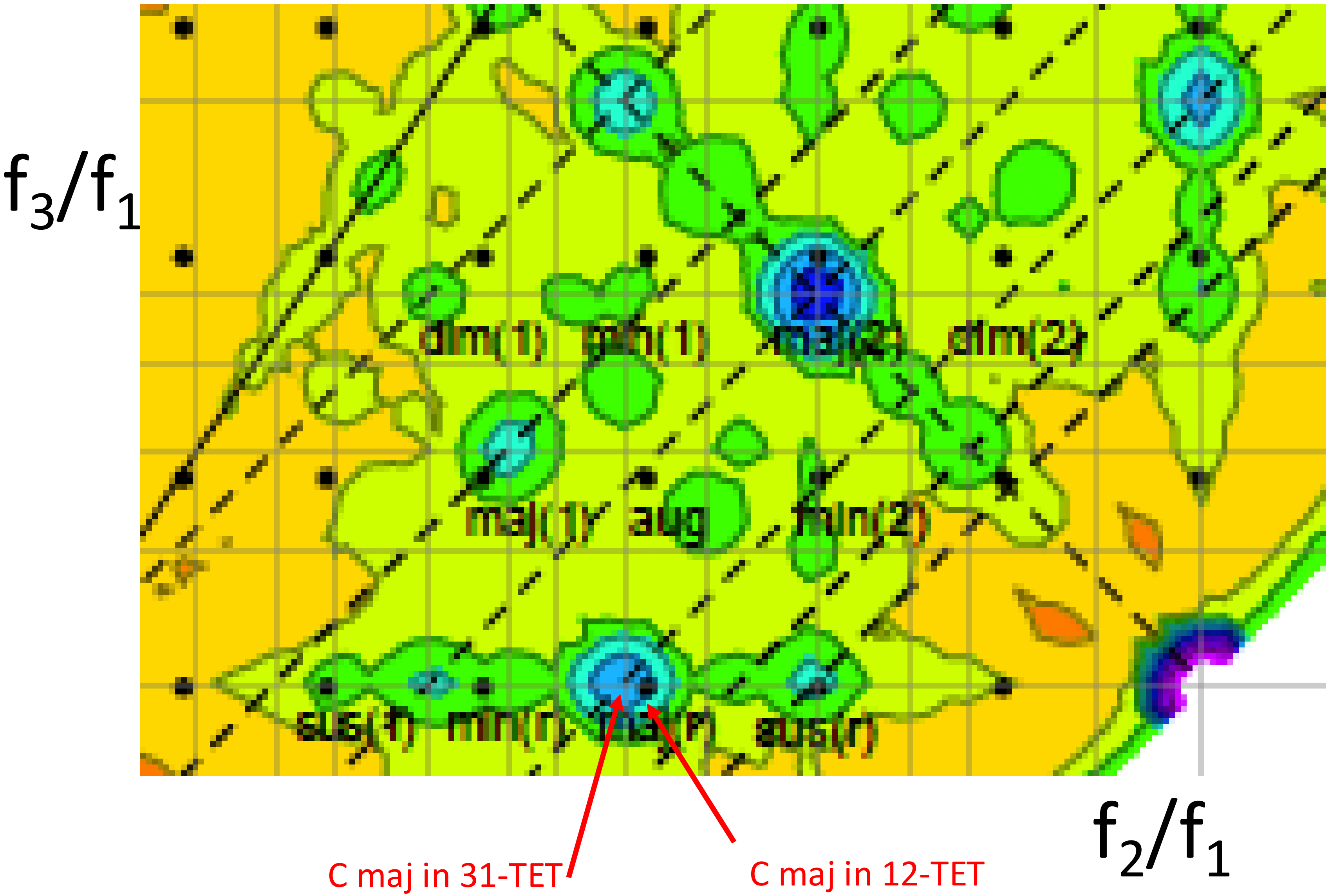


A TRIP TO TRIAD'S LAND

[IM, EPJP 2023;
IM and GLP, EPJP 2024]

C maj in 12-TET: a pipe-like timbre makes those beatings unsatisfactory, in comparison to other tunings. Unequal temperaments typically remain preferred for organs, to limit such beatings

C maj in 31-TET: significantly similar to the Quarter-Comma Meantone temperament

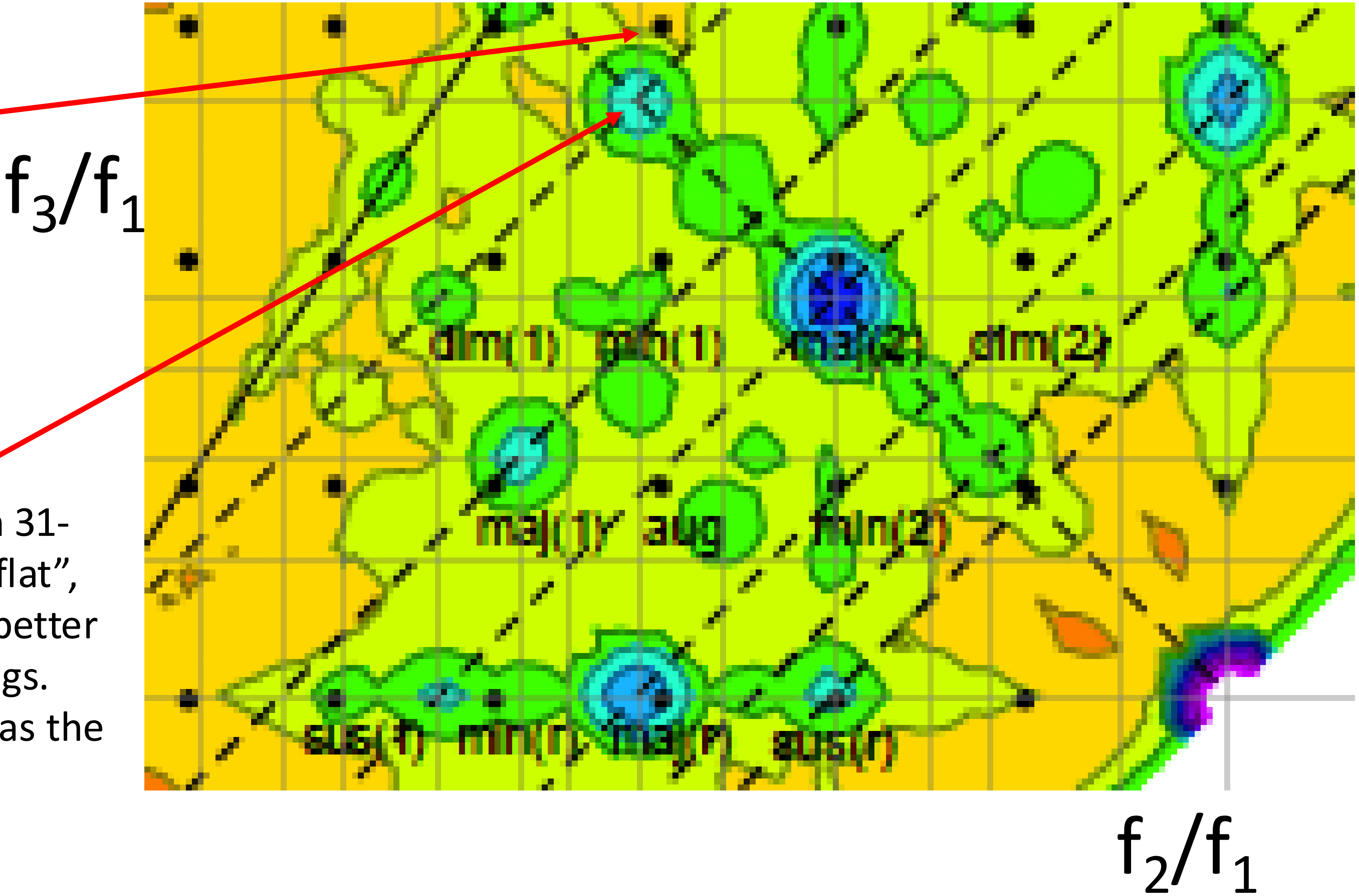


A TRIP TO TRIAD'S LAND

[IM, EPJP 2023;
IM and GLP, EPJP 2024]

C7 chord (no 5) in 12-TET: features beatings; it is a well-known chord to create tension

C7(h7) in 31-TET: the 7th degree in 31-TET can be played with a "B superflat", which approximates the 7:4 ratio better than 12-TET, producing less beatings. As a result, there is less "tension" as the chord is more stable.

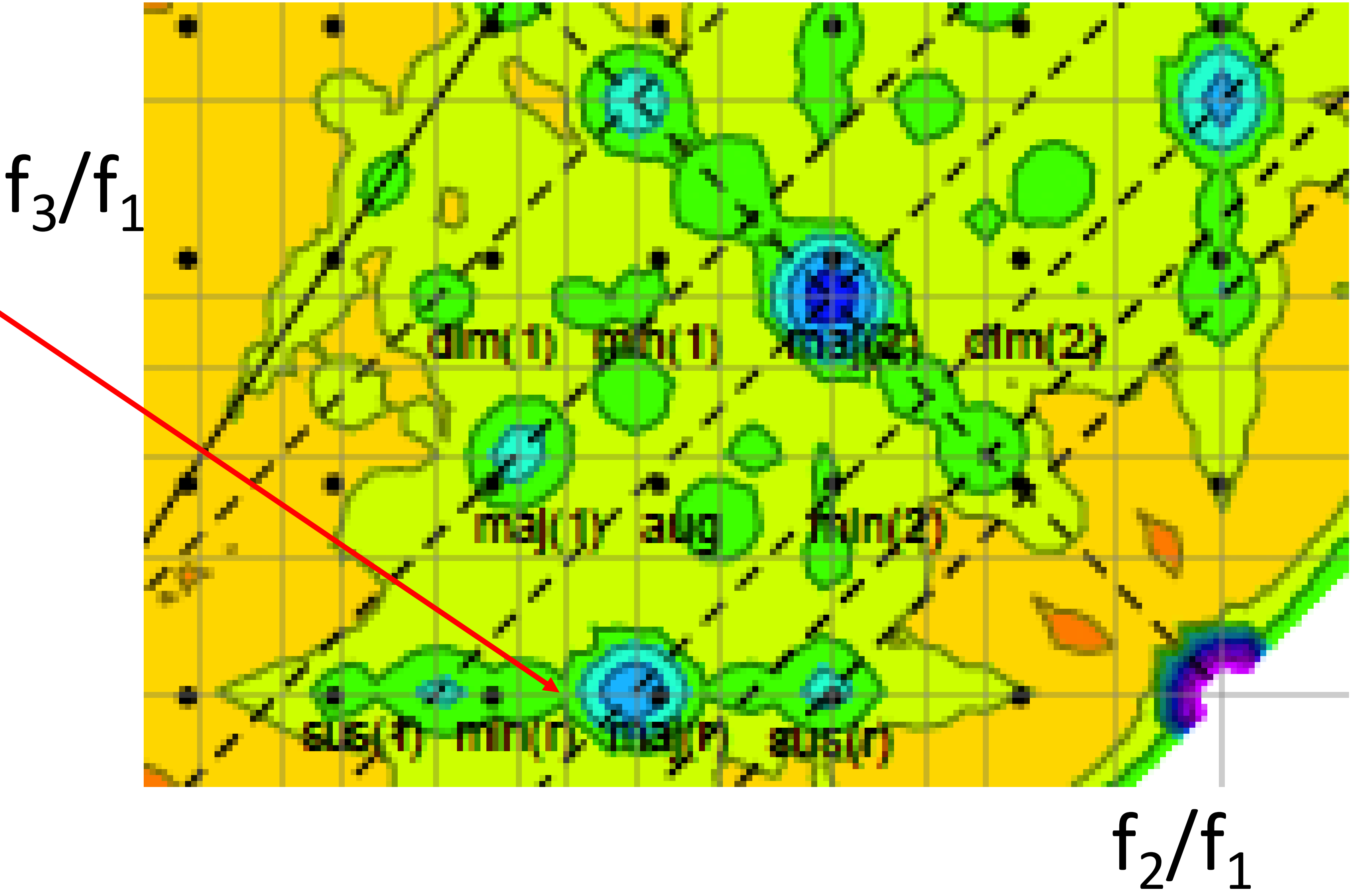


A TRIP TO TRIAD'S LAND

[IM, EPJP 2023;
IM and GLP, EPJP 2024]

C neut (31-TET): with a "neutral" Third (close to 11:9)

More dissonant than a pure minor Third (6:5), significant beatings reinforce the perception of out-of-tune



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CONCLUSIONS

→ Is Music a Science or an Art? Both!

→ Music is objectively founded on physics (via mathematics)
and more generally in science (psyco-acoustics, neuroscience)

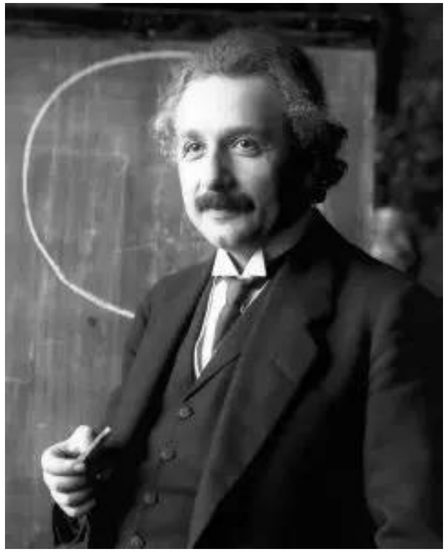
→ Recent advances in C&D → Futher directions: Temperaments (ongoing)

«Music is the pleasure the human mind experiences from counting without being aware that it is counting.» **G. Leibniz**

BACKUP

“Science cannot tell us a word about why music delights us, of why and how an old song can move us to tears.” **E. Schrödinger**, 'Nature and the Greeks' and 'Science and Humanism'

“It would be possible to describe everything scientifically, but it would make no sense; it would be without meaning, as if you described a Beethoven symphony as a variation of wave pressure.” **A. Einstein**



“If I was not a physicist, I would probably be a musician. I often think in music. I live my daydreams in music. I see my life in terms of music. … I cannot tell if I would have done any creative work of importance in music, but I do know that I get most joy in life out of my violin.”
A. Einstein

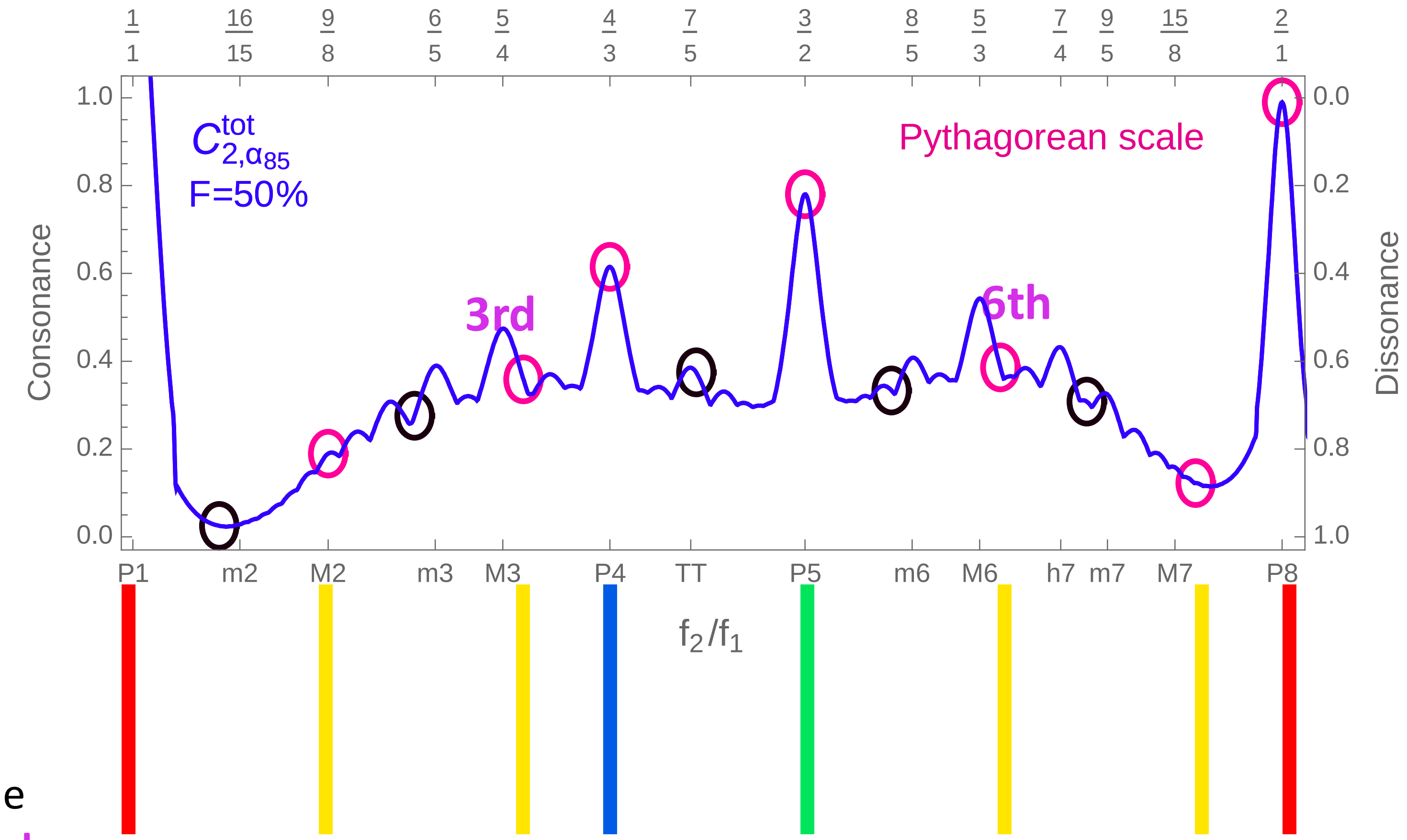
“It occurred to me by intuition, and music was the driving force behind that intuition. My discovery was the result of musical perception.”
A. Einstein, When asked about his theory of relativity

«But the beauty of Einstein’s equations, for example, is just as real to anyone who’s experienced it as the beauty of music. We’ve learned in the 20th century that the equations that work have inner harmony.» **E. Witten**

Pythagorean Tuning

DISADVANTAGE 1:
Thirds and sixths are not at their best

Counterpoint, polyphony and organs led to abandoning Pythagorian scale by upgrading 3rds and 6ths from dissonances to (imperfect) consonances.



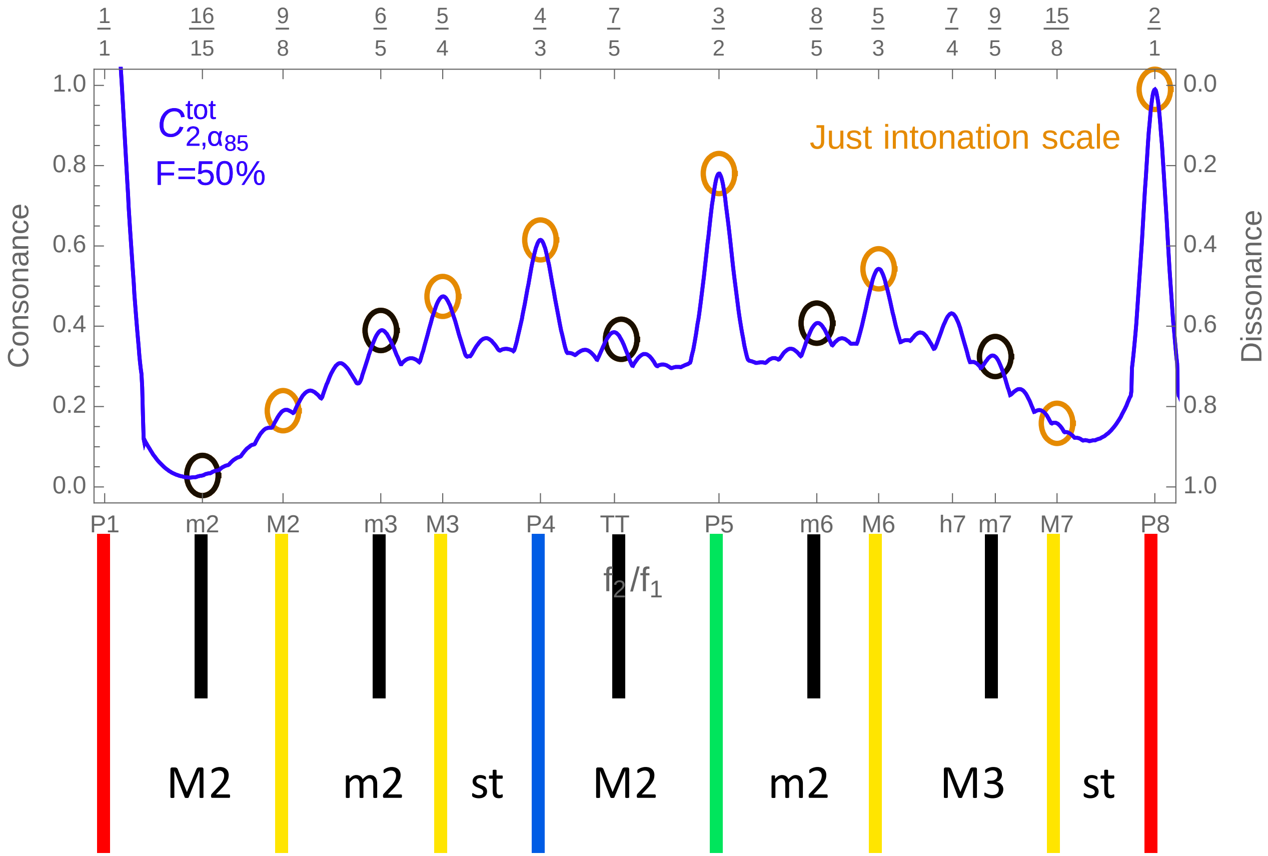
DISADVANTAGE 2:
including chromatic scale (developed in the Middle Ages), the **circle of fifths does not close**

Tempered scales: meantone, equal, etc

Complete Just Intonation Scale

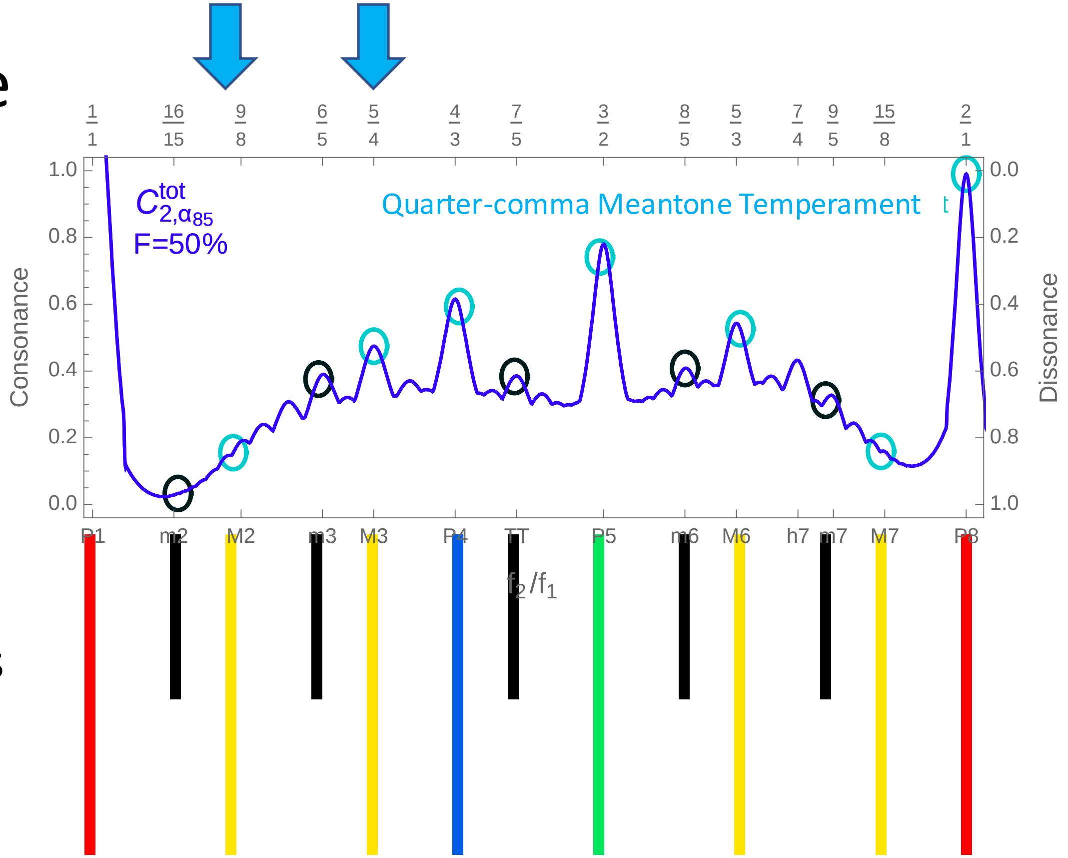
- Advantages
  - All intervals are pure => highest possible consonance
  - Naturally adopted by *a cappella* choirs

- Disadvantages
  - Increased complexity
    - Major tone (tM) and minor tone (tm)
  - Hard to introduce modulations
    - Wolf Fifths and "Wolf Thirds"
    - Key changes during a composition were not common practice…

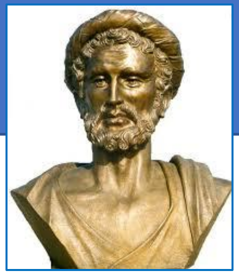


Meantone Temperaments

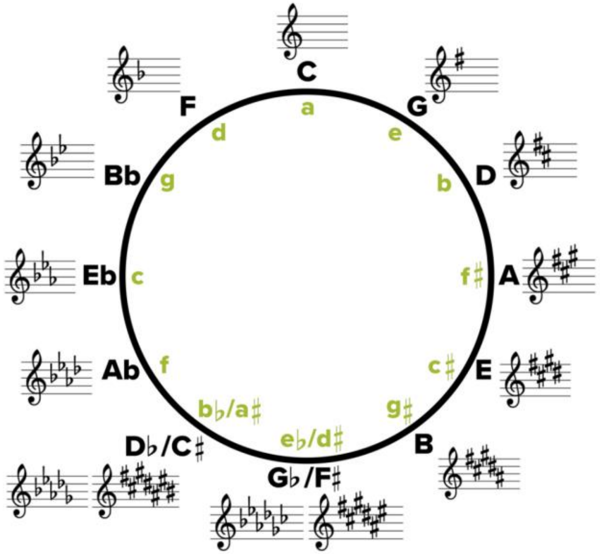
- Principle: keep as many pure Major Thirds as possible, and tune the Fifths accordingly. 4 Fifths = $(3:2)^4$ = 2 Octaves + 1 Major Third + S.C.
  - S.C. = *Syntonic Comma* = 81:80
- Most common: Quarter-Comma Meantone
  - "Mean" Tone: mean of M3 = $\sqrt{5/4}$
  - Why Quarter-Comma? Because it can be obtained by flattening the Fifths by ¼ of a S.C.
- Advantages
  - Tolerable beatings of the Fifths in most tonalities
  - Suitable for Orgues and fixed-tuning instruments
  - Procedures exist to tune instruments based on beatings
- Disadvantages
  - Wolf Fifth still present => "remote" tonalities must be avoided

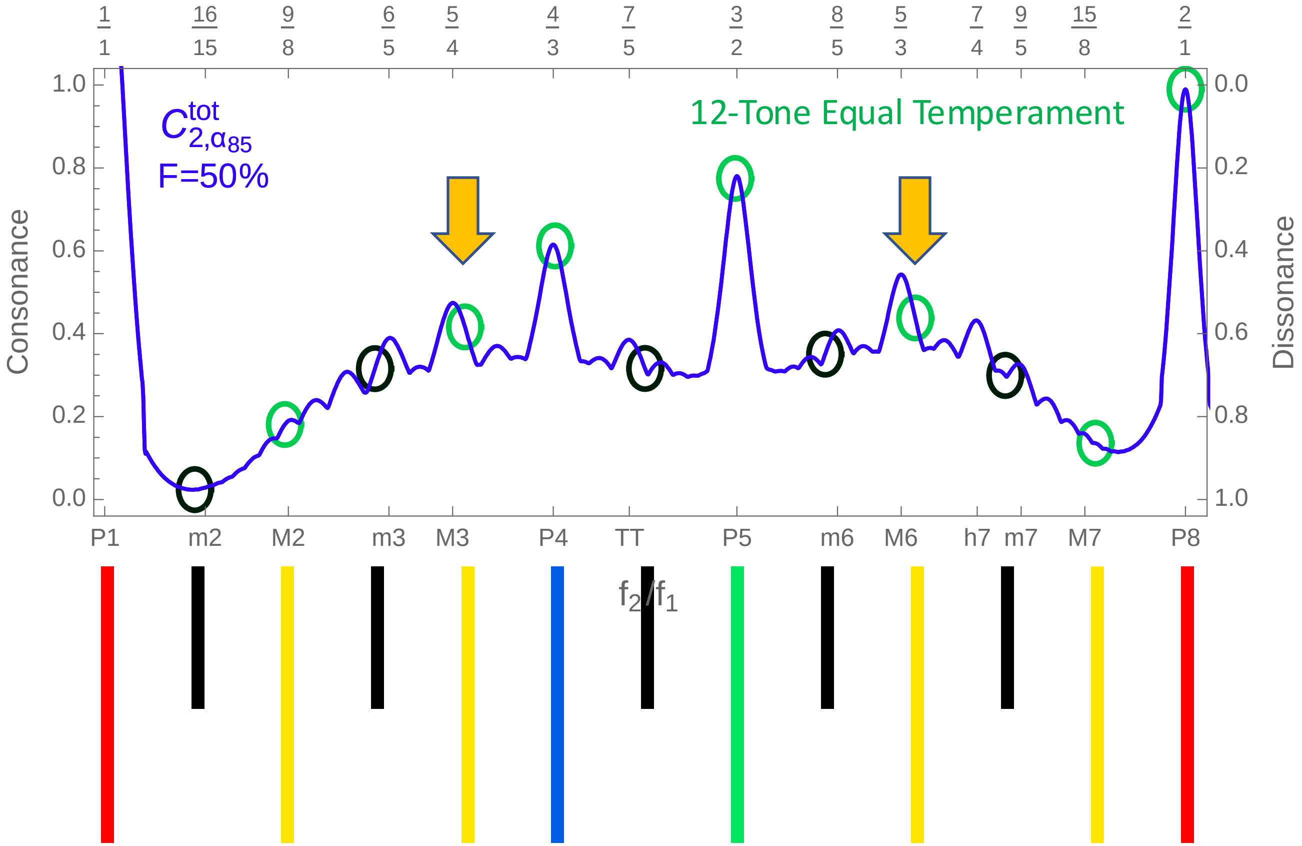


12-TONE EQUALLY TEMPERED SCALE



- Disadvantages
  - Major Thirds and Sixths are not great
  - Minor Thirds are similarly mistuned
- Advantages
  - Excellent approximation of P5
  - Modulations (key changes) allowed for **all tonalities**





31-Tones Equal Temperament

- It was realized that 31-TET is close to Quarter-Comma Meantone
  - Excellent M3
  - Very good P5
- It also matches well some 7-limit ratios, most notably 7:4 (the Harmonic Seventh)