Positivity Bounds on Massive Vectors

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$50 + \varepsilon$ Years of Conformal Bootstrap

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Plan of the presentation

1. Introduction and motivations
2. Setup
3. Results
4. Discussion and future directions

Introduction and motivations

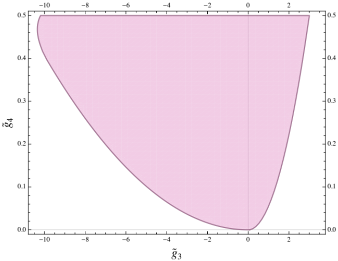
When is an effective field theory (EFT) UV completable?

Impose basic assumptions to derive dispersion relations on EFT coefficients:

$$g_n \sim \int_{M^2}^{\infty} ds \, \frac{\text{Im}\, A(s,t)}{s^{1+n}}.$$

Impose positivity in the UV to get bound [Caron-Huot, Van Duong 2020]

$$\# < \frac{g_n}{g_m} < \#.$$



Final result: 2d plot for $\frac{g_n}{g_m}$ vs $\frac{g_p}{g_m}$.

Our work:

- spinning massive particles (spin-1)
  $\mathcal{L} = -\frac{1}{4}F^2 - \frac{1}{2}m^2 A^2 + \dots$.

Introduction and motivations

Assumptions/properties:

- unitarity
- causality (analyticity)
- crossing symmetry, partial wave expansion
- asymptotic behavior at $|s| \to \infty$
- weak coupling in the low-energy (loop suppression)

How can we use this?

- photons [Vichi et al. 2021], gravitons [DSD et al. 2022]
- large N QCD (mesons, glueballs?) [Albert,Rastelli 2021]
- massive gravity [Cheung,Remmen 2016,Riva et. al 2023]
- etc.

Setup

- 17 amplitudes $A^I(s,t)$, $I = \{++++\}, \{+0-0\}, \dots$
- They can be written as "structure × function"

$$A^I(s,t) = \sum_J {E^I}_J(s,t) F_J(s,t).$$

- ${E^I}_1 = (\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot \epsilon_4), \ \dots, \ {E^I}_{17} = (\epsilon_1 \cdot p_4)(\epsilon_2 \cdot p_3)(\epsilon_3 \cdot p_2)(\epsilon_4 \cdot p_1).$
- F_J have crossing properties:

$$F_J(u,t) = C^{su}_{JK} F_K(s,t), \quad F_J(t,s) = C^{st}_{JK} F_K(s,t).$$

- Regge boundedness:

$$\lim_{|s|\to\infty} \frac{A^I(s,t)}{s^2} = 0.$$

- Partial wave decomposition:

$$\mathrm{Im}\, A^I(s,t) = \sum_\ell 16\pi(2\ell+1)\sqrt{\frac{s}{s-4m^2}}\, d_I^{(\ell)}(\theta)\rho^I_\ell(s).$$

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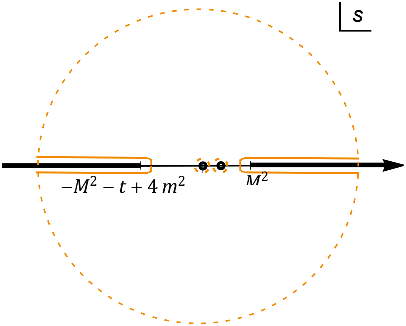


Contour integral:

$$\oint_\infty \frac{ds}{2\pi i}\frac{A^I(s,t)}{s^{k+1}} = 0, \qquad k \geqslant 2.$$

After manipulations and $(\partial_t)^m|_{t=0}\dots$ Dispersion relation:

$$g_n = \sum_\ell \int_{M^2}^\infty ds\, \mathbf{K}^J(s)\rho_\ell^J(s).$$

$\rho_\ell^J(s)$ positive definite $\implies$ optimization problem.

Setup

Spin and mass are complicated. Recall that $A^I = E^I{}_J F_J < s^2$.

- **Positivity** is easily achieved with A^I, but **crossing** is simpler with F_J.
- A^I mixes the low-energy: observables are **combinations** of EFT coefficients.
- F_J have better Regge behavior than A^I: important for **null constraints**.
- There are **four** positive quantities to use as denominators.

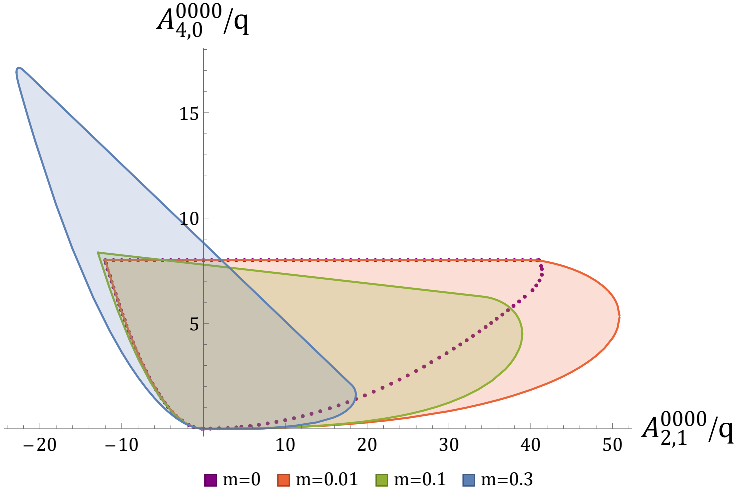
Conclusion

To compute dispersion relations: A^I.
To generate null constraints: F_J.

Results

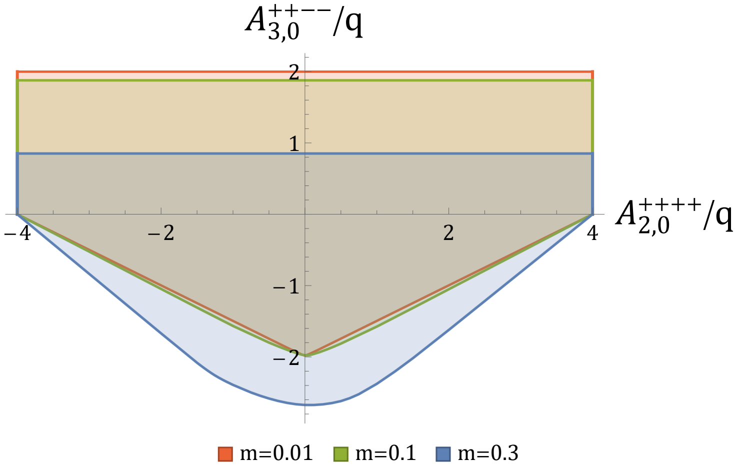
"Scalar" plots

$$A^{\lambda_i}_{k,\ell} \sim \sum_\ell \int_{M^2}^{\infty} ds \, \frac{(\partial_t)^\ell \mathrm{Im}\, A(s,t)}{s^{1+k}}$$



Results

"Photon" plots



Results

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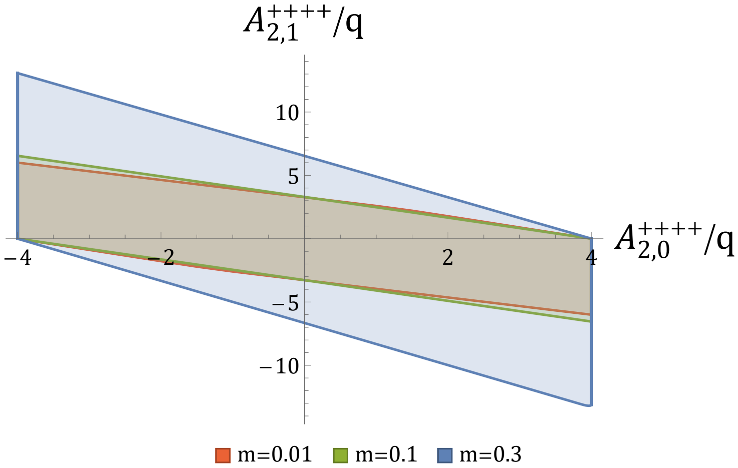

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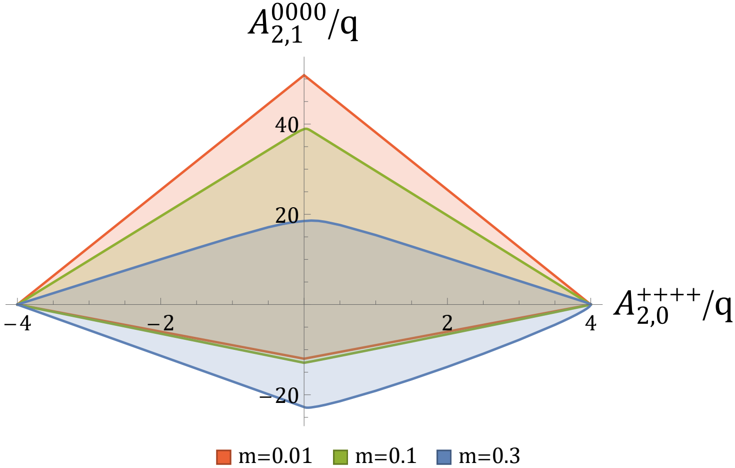
"Mixed" plots



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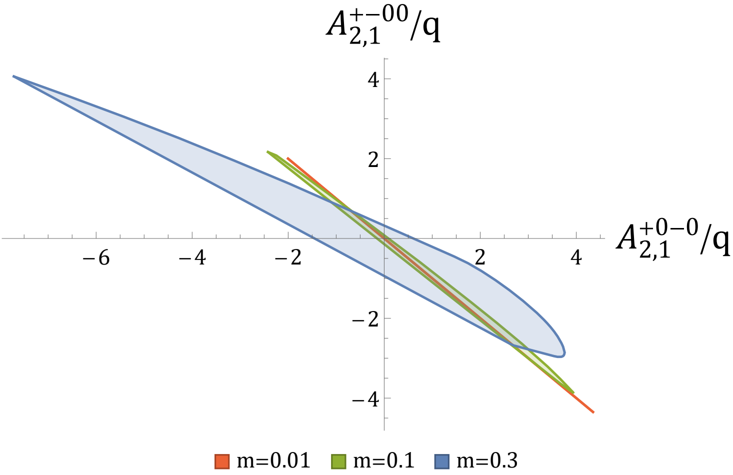


Results

We can populate the plots with simple UV completions. These arise from integrating out scalars and vectors of mass M at tree-level.

For example:

- real massive scalar ϕ

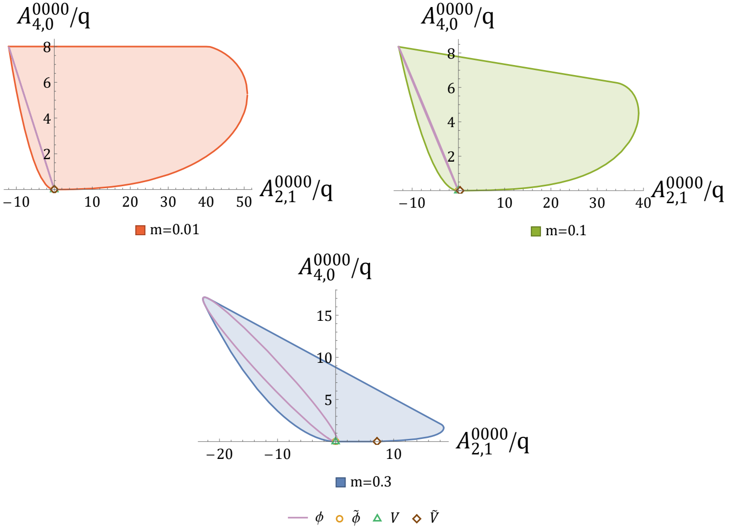
$$\mathcal{L} \supset \lambda_\phi^{(1)} F^{\mu\nu} F_{\mu\nu} \phi + \lambda_\phi^{(2)} A^\mu A_\mu \phi.$$

- massive vector V_μ

$$\mathcal{L} \supset \lambda_V V_\mu A_\nu F^{\mu\nu}.$$

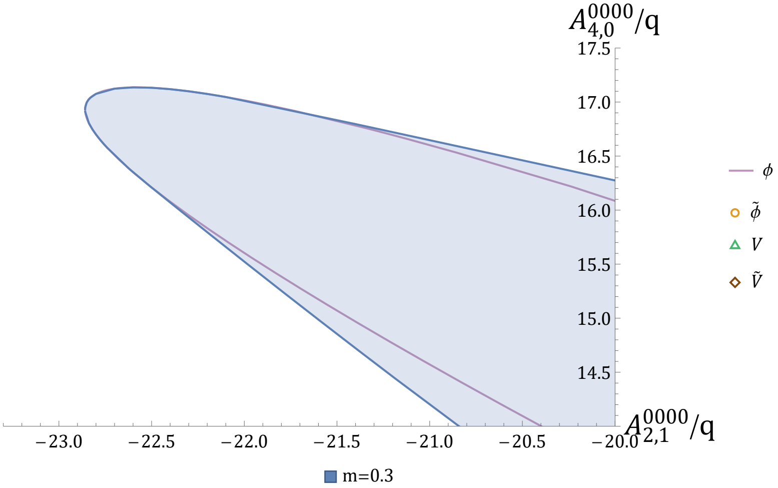
Some of them lie at special points!

"Scalar" plots: UV completions



Results

"Scalar" plots: zooming in



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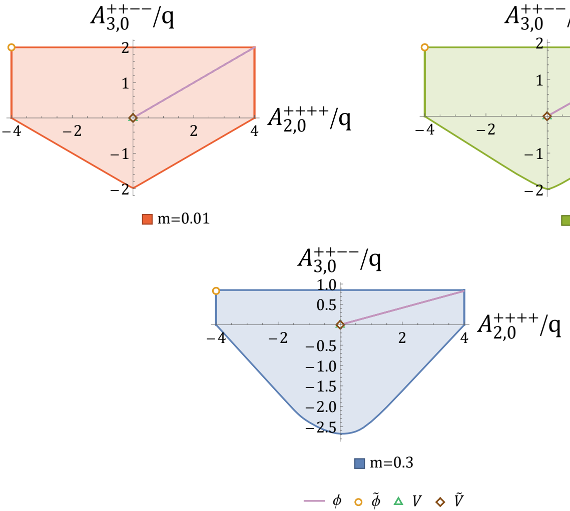

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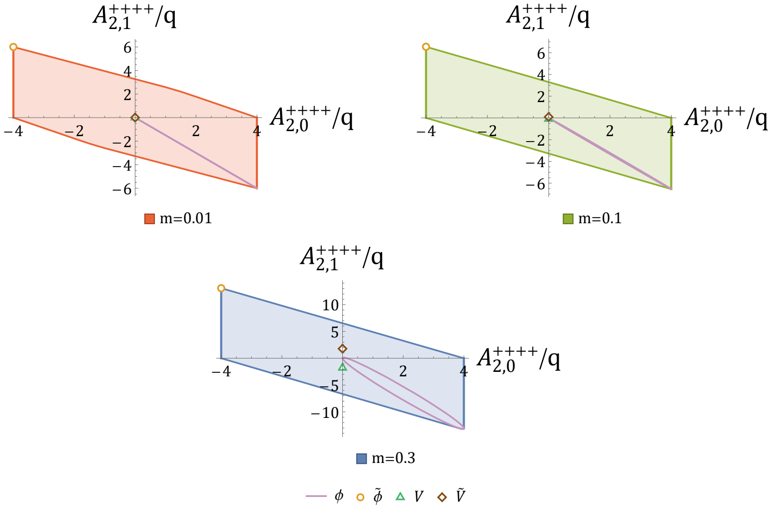

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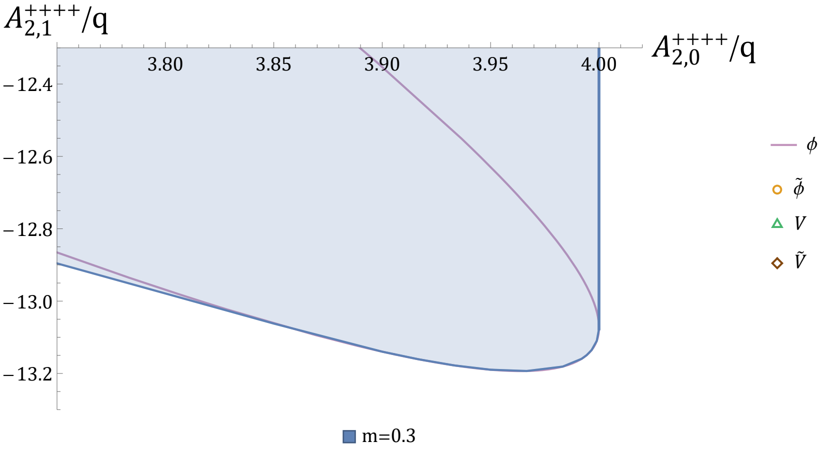

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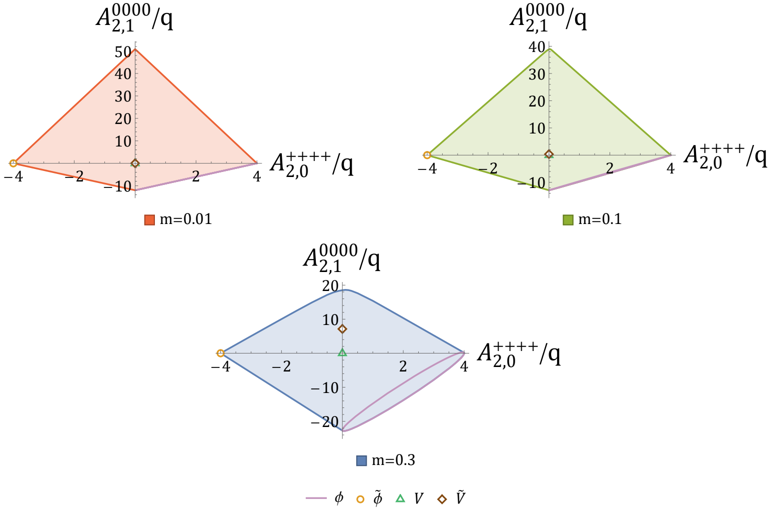

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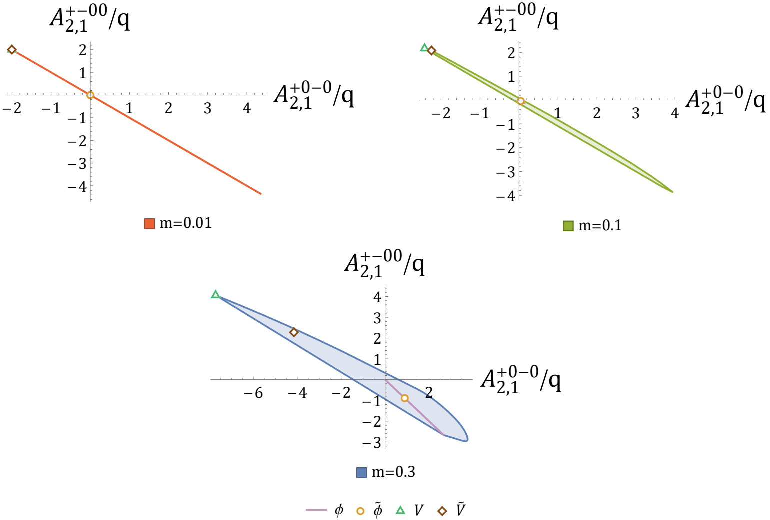

Discussion and future directions

Summary

We can put bounds on EFT coefficients for spinning massive particle scattering.
We obtain regions that are consistent with simple UV completions.

Future directions:

- Non-abelian case
- Massive spin-2
- Glueballs
- ...hopefully many more!

Thanks fo listening!