A New Twist on Spin

(in progress, with D. Baumann,
G. Mathys, F. Rost)

<u>Goal</u>: Kinematic variables that make <u>all</u> constraints from conformal symmetry manifest:

* SO(1,4) + conservation *

Main Result:

These variables exist!

$p_\mu^2 = 0$ — $\not{P}_{AB} = \lambda^a{}_A \lambda_{aB}$

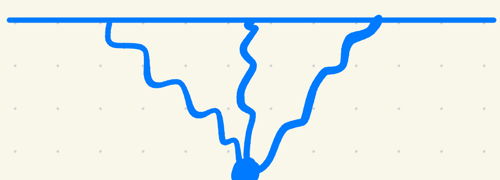
NULL CONE $\to$ SPINOR HELICITY VARS. $\to$

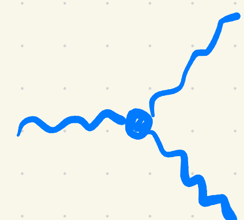
$\to$ "TWISTORS" $Z_A = \pi_a \lambda^a{}_A$

$$J^{ab}(P) = \int_{\mathrm{Ker}\,\not{P}} DZ\, f(Z)\, \pi^a \pi^b$$

Beautiful connection to Flat Space Amplitudes!

$$\langle T T T \rangle = \int [DZ] (\ldots) \times M_{flat}$$





Properly interpreted...

All ingredients already in literature

Chiodaroli, Gunaydin, Johansson, Roiban '22

Ward '89

Neiman '14

Adamo, Skinner, Williams '17

Osborn, Petkou '93

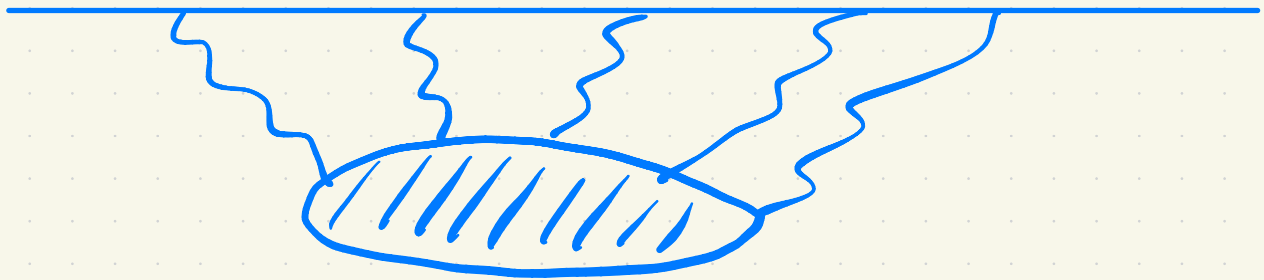
Costa, Penedones, Poland, Rychkov '11

We just mixed them in a new way

Motivation:

* Cosmology
* Success story in flat space

Cosmology

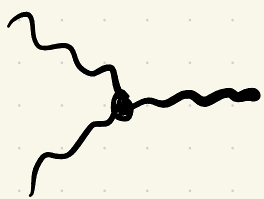


What is (tree-level) cosmological correlator of $\begin{cases} \text{Y.M.} \\ \text{G.R.} \end{cases}$ at n- points, tree-level?

Flat Space

Right variables: Spinor-Helicity

$$p_\mu^2 = 0 \implies (p\cdot\vec{\sigma})_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$$



$$1^+ 2^+ 3^+ : \left([12][23][31]\right)^S$$

$$1^+ 2^+ 3^- : \left(\frac{[12]^3}{[23][31]}\right)^S$$

"INEVITABLE"

Current Approaches

* Embedding Space — $SO(1,4)$ manifest; Conservation imposed

* Momentum Space — $SO(1,4)$ not manifest; Conservation easy

* Mellin Space — Conservation not obvious

First Idea

Embedding Space Rays are NULL MOMENTA

$$P_\mu^2 = 0 \longrightarrow P = \lambda\lambda$$

2D CFT: $$P_{\alpha\dot\alpha} = \lambda_\alpha \tilde\lambda_{\dot\alpha}$$

Works for any Δ. What's special about J/T?

$J(\lambda); T(\lambda)$ HOLOMORPHICITY

First Idea

3D CFT

Embedding Space Rays are NULL MOMENTA

$$P_\mu^2 = 0$$

Symplectic 4x4 matrix

$$(P.\Gamma) = P_{AB} \quad A = 1-4 \qquad \eta_{\mu\nu} \longrightarrow \Omega_{AB}$$

$$\det P_{AB} = (P_\mu^2)^2 \xrightarrow{\text{rank } 2} P_{AB} = \lambda_{aA}\lambda^a_B$$

$a = 1,2$ little group

$A = 1-4$ $SO(1,4)$ index

$$P_{AB} = \lambda_{aA}\lambda^a{}_B$$

Constraint: $\Omega^{AB}\lambda^a{}_A\lambda^b{}_B \equiv$

$$\equiv \langle \lambda^a \lambda^b \rangle = 0$$

Very useful even for generic Δ.

Holomorphicity more confusing.

Idea needed...

Second Idea

(in hindsight...)

λ_{aA}'s span Kernel of P_{AB}, 2D space $\lambda^a \cdot P = 0$

Take $\quad Z_A \equiv \pi_a \lambda^a{}_A$, some linear combination of λ's

HOLOMORPHICITY: $f(Z)$ only

$$J^{ab}(\lambda) = \int \langle \pi d\pi \rangle \pi^a \pi^b f(Z)$$

$$T^{abcd}(\lambda) = \int \langle \pi d\pi \rangle \pi^a \ldots \pi^d f(Z)$$

CONSERVED!

Rules

* SPACETIME: $Z^i_A Z^j_B \Omega^{AB}$ only +
  + Scaling

* CONSERVATION: $f(Z)$'s only
  + $\int (\ )$ invariant under
  $\pi \longrightarrow c\pi$

Pins $\Delta = S + 1$.

Helicity (?)

Instead of (π^a), more invariant way

$$J^+ \equiv \zeta_a \zeta_b J^{ab} = \int_{\mathrm{Ker}P} DZ\,(\sigma^* \cdot Z)^2 f^+(Z)$$

$(\mathrm{Ker}P)^*$

$$J^- = \int DZ\,\left(\sigma^* \cdot \frac{\partial}{\partial Z}\right) f^-(Z)$$

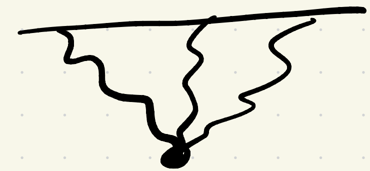
Ansatz

$$\langle JJJ\rangle = \begin{cases} \int [DZ_m]\, dc_{mn} \left(\sigma_m^* \cdot Z_m\right)^2 \mathrm{Exp}\left[i\, c_{mn} Z_m \cdot Z_n\right] \\ \qquad \times\, f_{+++}\left(c_{12}, c_{23}, c_{31}\right) \\ \int [DZ_m]\, dc_{mn} \left(\sigma_1^* \cdot Z_1\right)^2 \left(\sigma_2^* \cdot Z_2\right)^2 \left(\sigma_3^* \cdot \frac{\partial}{\partial Z_3}\right)^2 \times \\ \times\, \mathrm{Exp}\left[i\, c_{mn} Z_m \cdot Z_n\right] \times f_{++-}\left(c_{12}, c_{23}, c_{31}\right) \end{cases}$$

Counting Rules are the same as for flat space Amplitudes!!

$$[mn] \to c_{mn}$$

$$f_{+++} = (c_{12}\, c_{23}\, c_{31})^S$$



F^3/R^3

YM/GR

$$f_{++-} = \left(\frac{c_{12}^{\,3}}{c_{23}\, c_{31}}\right)^S$$

Easy to unpack: δ-function + Wick contractions

Reproduces famous results.

Now + Future

* Helicity + Parity breaking
* 4-pts (Contact, Exchange)
* Recursion Relations
* 4-particle test

P_μ

λ^a_A

$\mathcal{L}_A$