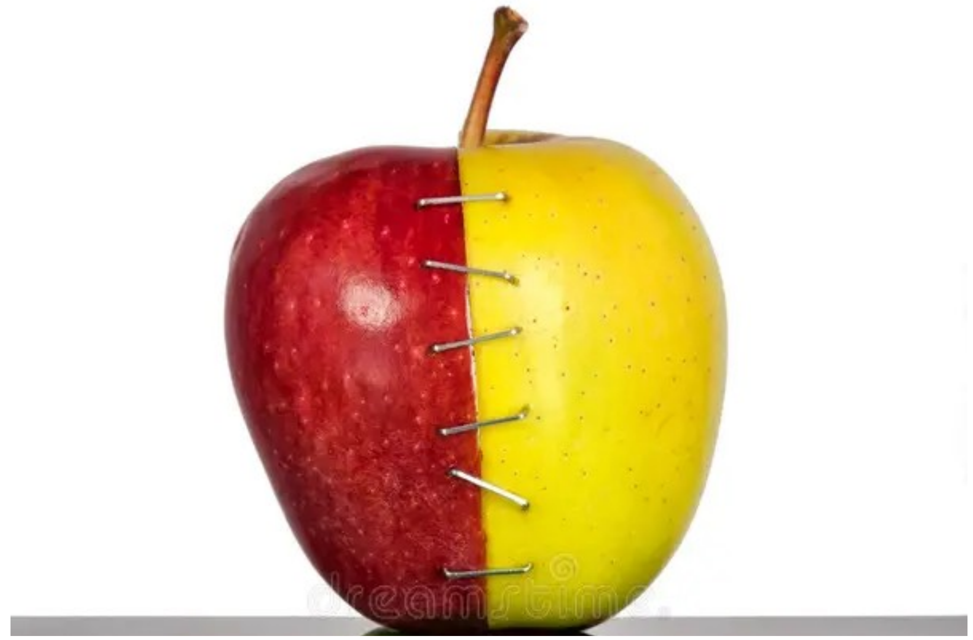

Multi-trace operators in CFTs

Agnese Bissi (ICTP & Uppsala University)

50 + ε years of conformal bootstrap, Pisa

Motivations

- Main aim: understand CFT data, using conformal symmetry and consistency conditions of CFTs.

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- One way to make progress analytically is to consider situation in which it makes sense to chose a perturbative parameter, and study the structure of such expansion.
- In this talk I will focus on **large N** perturbation theory .
- This particular example is mostly interesting due to the connection with theories of gravity in curved space-time.

Large N

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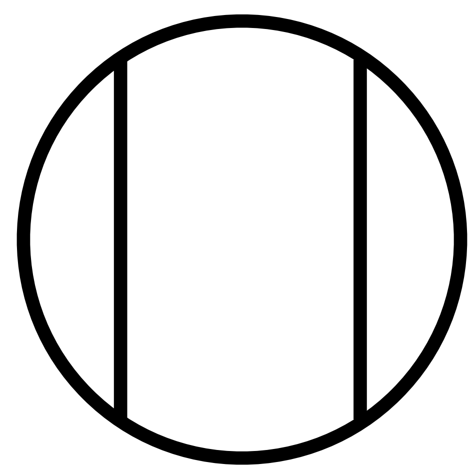
- In the OPE decomposition of generalised free field theories, it is apparent the presence of double trace operators.



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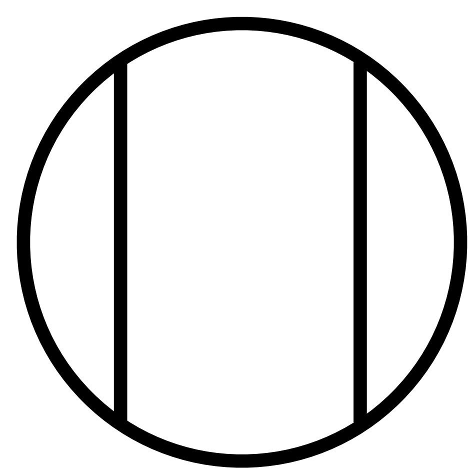

- Generically multi-trace operators correspond to multi-particle states in AdS.

Approach I

Study four point functions of single trace operators $\mathcal{O}$ at large N (simplest possible example of $\mathcal{O}^4$)

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4) \rangle$$

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$$\langle \mathcal{O}\mathcal{O}[\mathcal{O}_1\mathcal{O}_2\ldots\mathcal{O}_m]\rangle \sim N^{-m}$$

$[\mathcal{O}_1\mathcal{O}_2\ldots\mathcal{O}_m]$ are m-trace operators, with $\langle [\mathcal{O}_1\mathcal{O}_2\ldots\mathcal{O}_m][\mathcal{O}_1\mathcal{O}_2\ldots\mathcal{O}_m]\rangle = 1$

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Warning: degeneracy among states having the same Δ and ℓ .

Approach II

Study four point functions of higher trace operators
$$\mathcal{O}_{DT} \sim \left[\mathcal{O}\mathcal{O}\right].$$

One example is

$$\langle \mathcal{O}_{DT}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

In this case, triple-trace operators appear already at leading order.

Approach III

Study higher point functions of single traces at large N

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\mathcal{O}(x_5)\rangle$$

This situation is much richer but also harder.

see for instance Harris, Kaviraj, Mann, Quintavalle, Schomerus, 2024

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Supersymmetry helps in constraining the structure of the correlators (protected quantities).

It is also very interesting due to the connection with supergravity amplitudes.

AdS/CFT correspondence

CFT

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4 dimensional $\mathcal{N} = 4$
Super Yang Mills with
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theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$

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$$N \sim g_s^{-1}$$

$$\lambda = g_{YM}^2 N = (\alpha')^{-2}$$

Setup

Weakly coupled regime in the bulk is **supergravity** and corresponds to large central charge and string length to zero.

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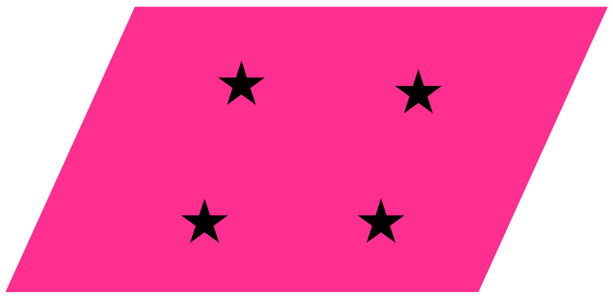

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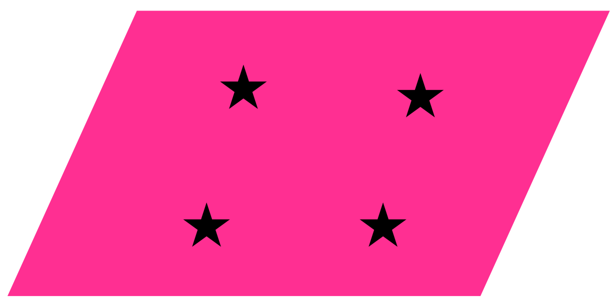

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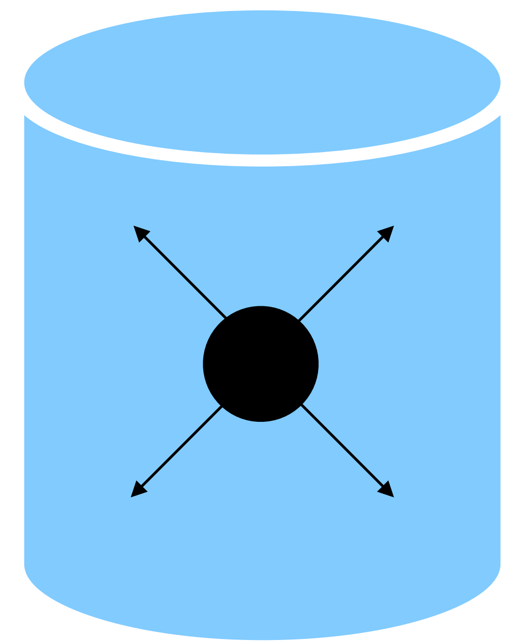

$\mathrm{AdS}_{d+1} \times \mathrm{S}^q$

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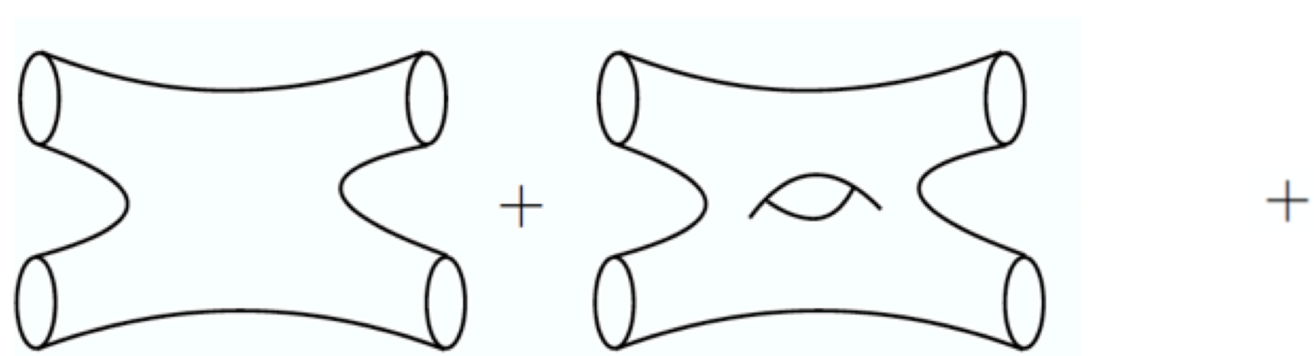

$$A^{(0)}(\alpha', s, t, u) \; + \; g_s^{-2} A^{(1)}(\alpha', s, t, u) \; +$$

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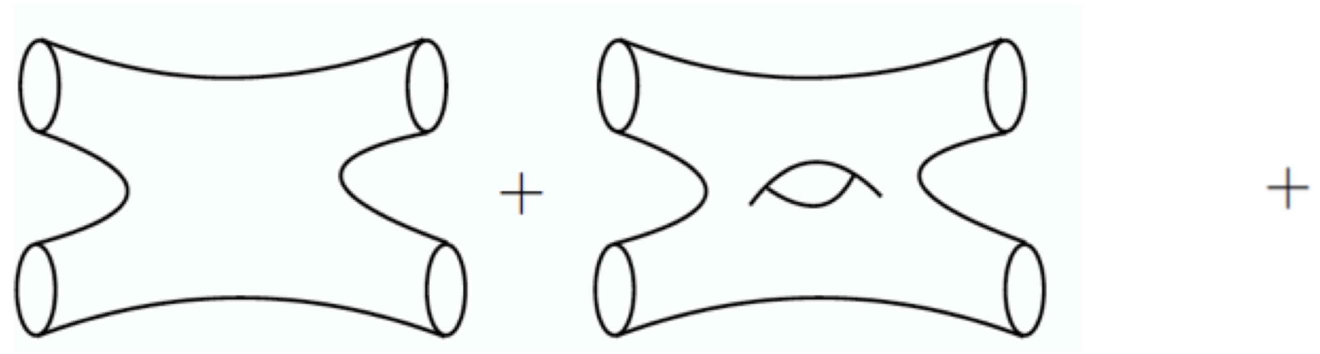

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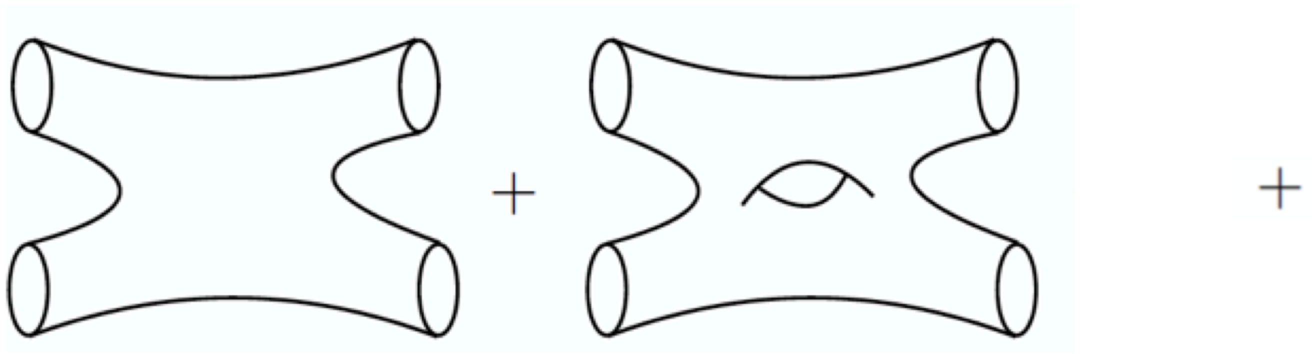

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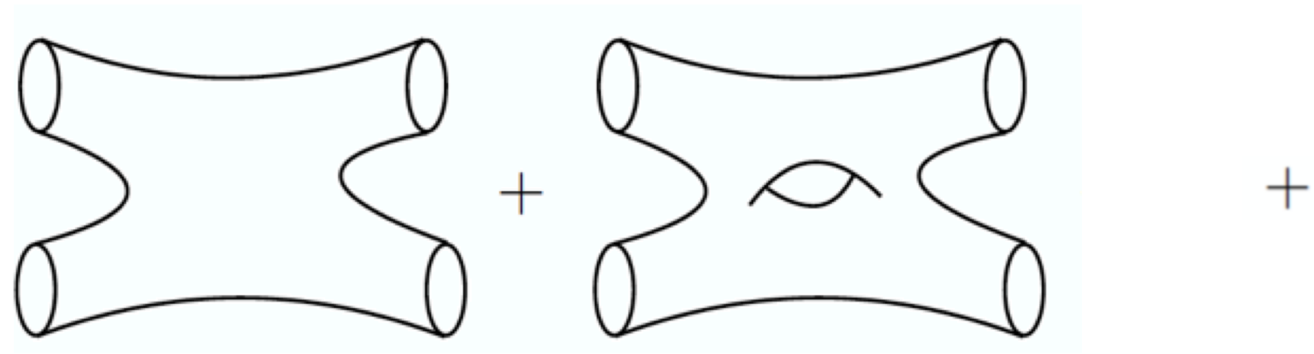

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Bound states of single particle states

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Idea

Understand how to use the symmetries of the CFT (conformal symmetry, super symmetry, integrability....) to construct higher order correlators.

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Aharony, Alday, AB, Perlmutter 2016

Caron Huot 2017

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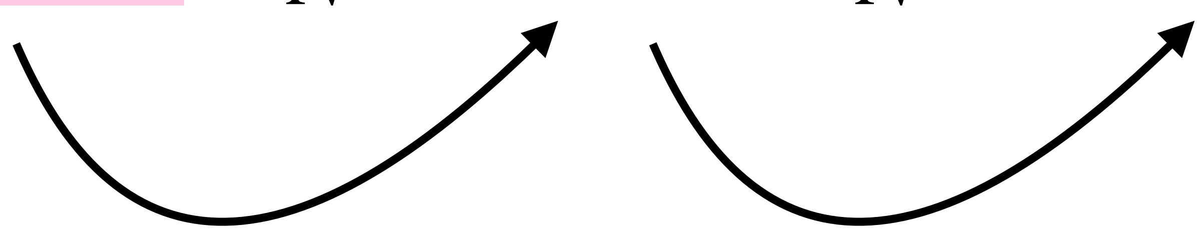

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Method

Let us go back to the correlator, conformal symmetry fixes the four point function as

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$$\sigma = \frac{y_1\cdot y_3\, y_2\cdot y_4}{y_1\cdot y_2\, y_3\cdot y_4} \quad \tau = \frac{y_1\cdot y_4\, y_2\cdot y_3}{y_1\cdot y_2\, y_3\cdot y_4}$$

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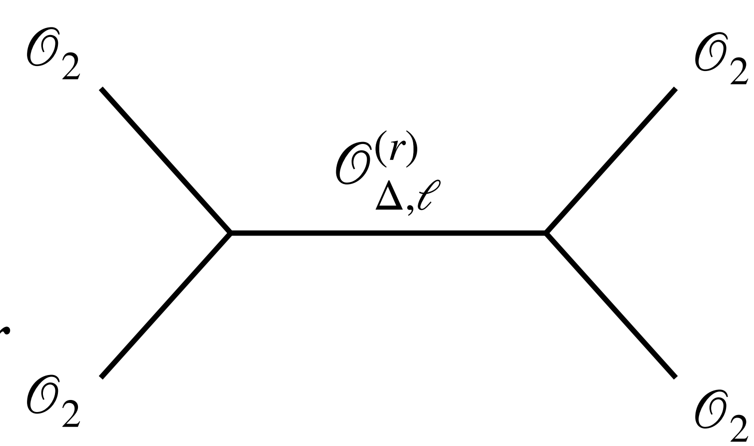

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Supersymmetry

Superconformal Ward Identities let us achieve two goals:

Nirschl, Osborn 2004

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Beem, Rastelli, van Rees 2010

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$$\mathcal{G}(u,v,\sigma,\tau) \to \begin{cases} \mathcal{G}^{short}(u,v) \\ \mathcal{H}(u,v) \end{cases}$$

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$$\mathcal{G}(u,v,\sigma,\tau) \to \mathcal{G}^{short}(u,v) \quad \text{depend on } N$$

$$\mathcal{G}(u,v,\sigma,\tau) \to \mathcal{H}(u,v)$$

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3) the function $\mathscr{H}(u,v)$ is decomposable in terms of superconformal blocks

Comments

1) we are interested in the dynamical part of the correlator $\mathscr{H}(u,v)$

2) when imposing crossing symmetry on the correlator, the two contributions $\mathscr{G}^{short}(u,v)$ and $\mathscr{H}(u,v)$ mix

$$v^2\mathscr{G}^{short}(u,v) - u^2\mathscr{G}^{short}(v,u) + u^2 - v^2 = -\frac{u-v}{c} + v^2\mathscr{H}(u,v) + u^2\mathscr{H}(v,u)$$

3) the function $\mathscr{H}(u,v)$ is decomposable in terms of superconformal blocks

$$\mathscr{H}(u,v) = \sum_{\Delta,\ell} a_{\Delta,\ell}\, g^{s}_{\Delta,\ell}(u,v) = \sum_{\Delta,\ell} a_{\Delta,\ell}\, u^{\frac{\Delta-\ell-4}{2}} g_{\Delta+4,\ell}(u,v)$$

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Inversion Formula

It is possible to write a relation that invert the OPE allowing us to reconstruct the correlator by knowing only its singularities as $v \to 0$ or $\bar{z} \to 1$

Caron Huot 2017

Simmons-Duffin Stanford Witten 2017

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analytic continuation around $\bar{z} \to 1$

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$$c_{\Delta,\ell} \xrightarrow[\Delta \to \Delta_k]{} \frac{a_{\Delta_k,\ell}}{\Delta - \Delta_k}$$

has poles at the dimension of the exchanged operator with residue the square of the three point function

Caron Huot 2017

Simmons-Duffin Stanford Witten 2017

Tree Level

We expand at leading order N^{-2} and we get

D'Hoker, Freedman, Mathur, Matusis, Rastelli 1999

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Alday, Caron Huot 2018

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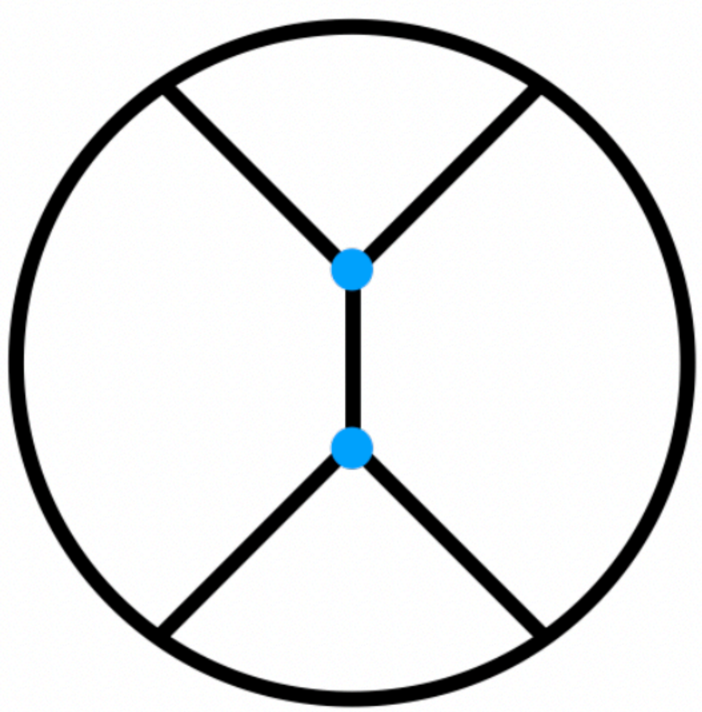

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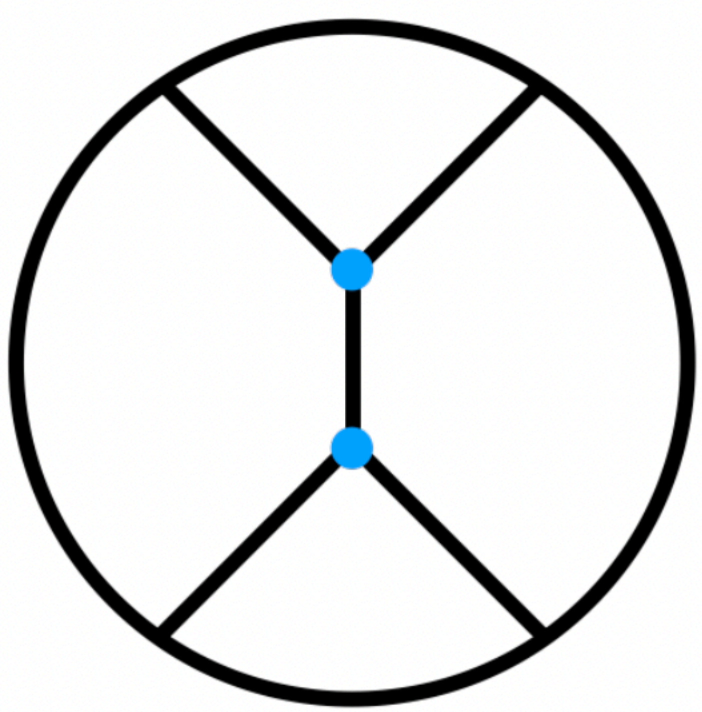

completely fixed by the knowledge of the protected part of the correlator and the leading order data

Schematically

$$\sum_{\Delta,\ell} a_{\Delta,\ell} u^{\frac{\Delta-\ell}{2}} g_{\Delta,\ell}(u,v) = \left(\frac{u}{v}\right)^2 \left(1 + a_{2,0} v g_{2,0}(v,u) + a_{4,0} v^2 g_{4,0}(v,u) + \ldots\right)$$

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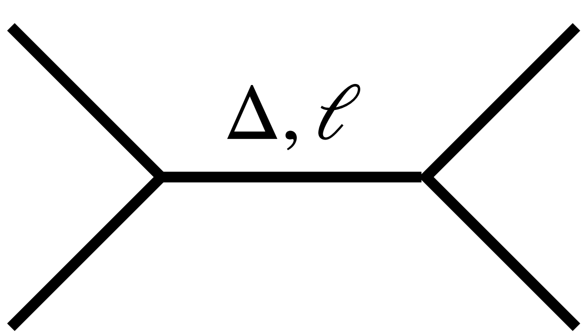

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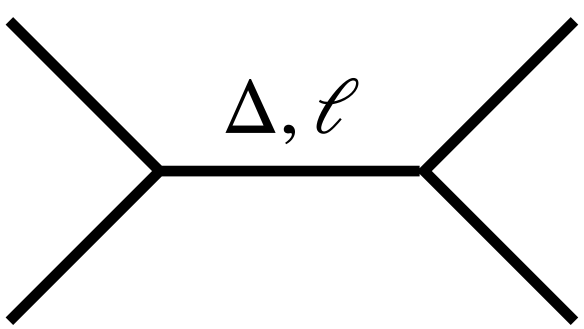

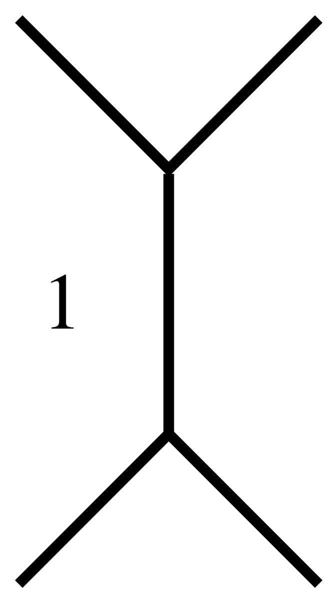

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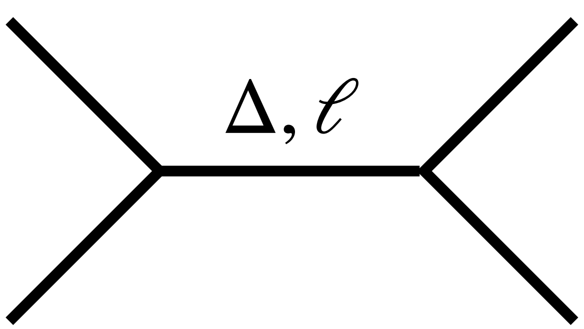

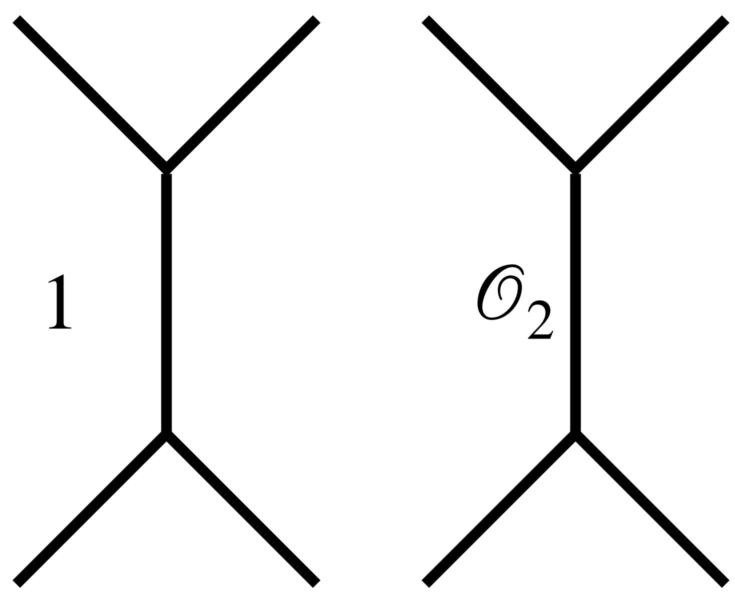

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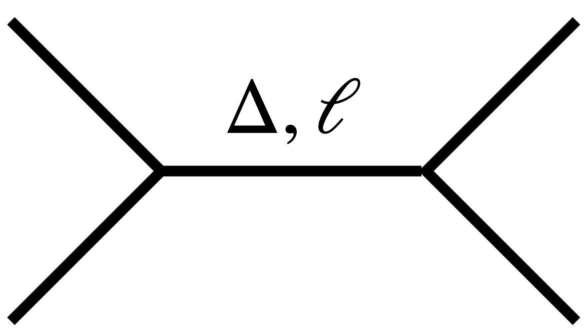
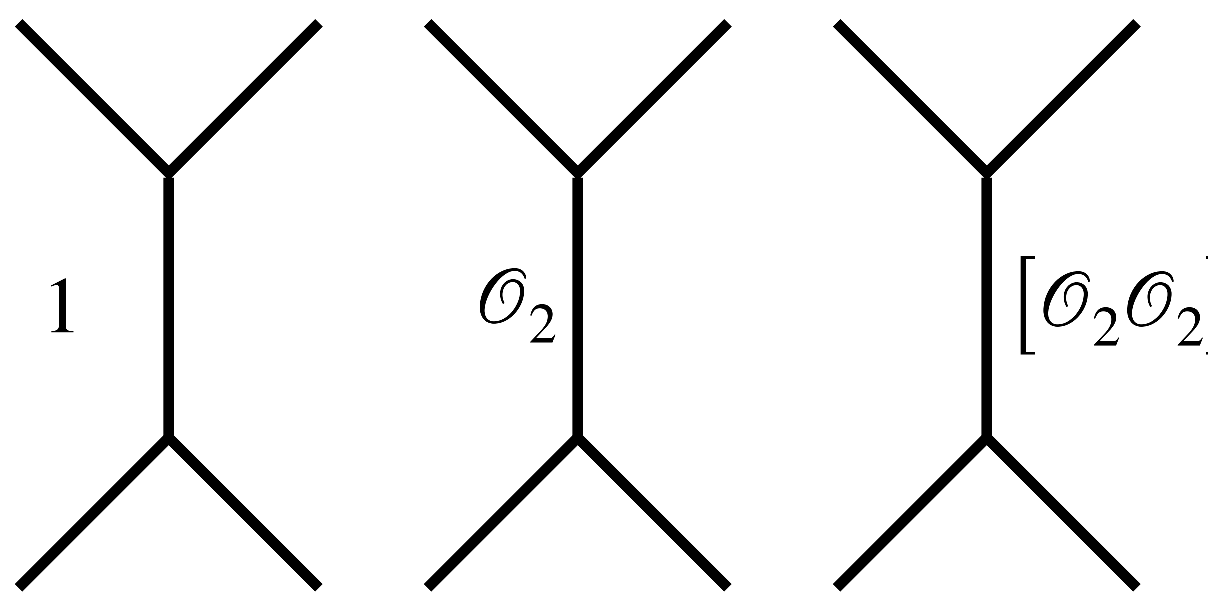
double trace





One Loop

At one loop the situation is different, mainly for two reasons:

Aprile, Drummond, Heslop, Paul 2017
Alday, AB 2017

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completely specified by tree level data!

Aprile, Drummond, Heslop, Paul 2017

Alday, AB 2017

Mixing

Caveat: mixing between different operators with the same bare dimension and quantum numbers.

$$\sum_{n,\ell} a^{(0)}_{n,\ell} \qquad \sum_{n,\ell} a^{(0)}_{n,\ell}\gamma^{(1)}_{n,\ell} \qquad \sum_{n,\ell} a^{(0)}_{n,\ell}\left(\gamma^{(1)}_{n,\ell}\right)^2$$

Alday, Zhou 2019 2020

Aprile, Drummond, Heslop, Paul 2017 2018 2019

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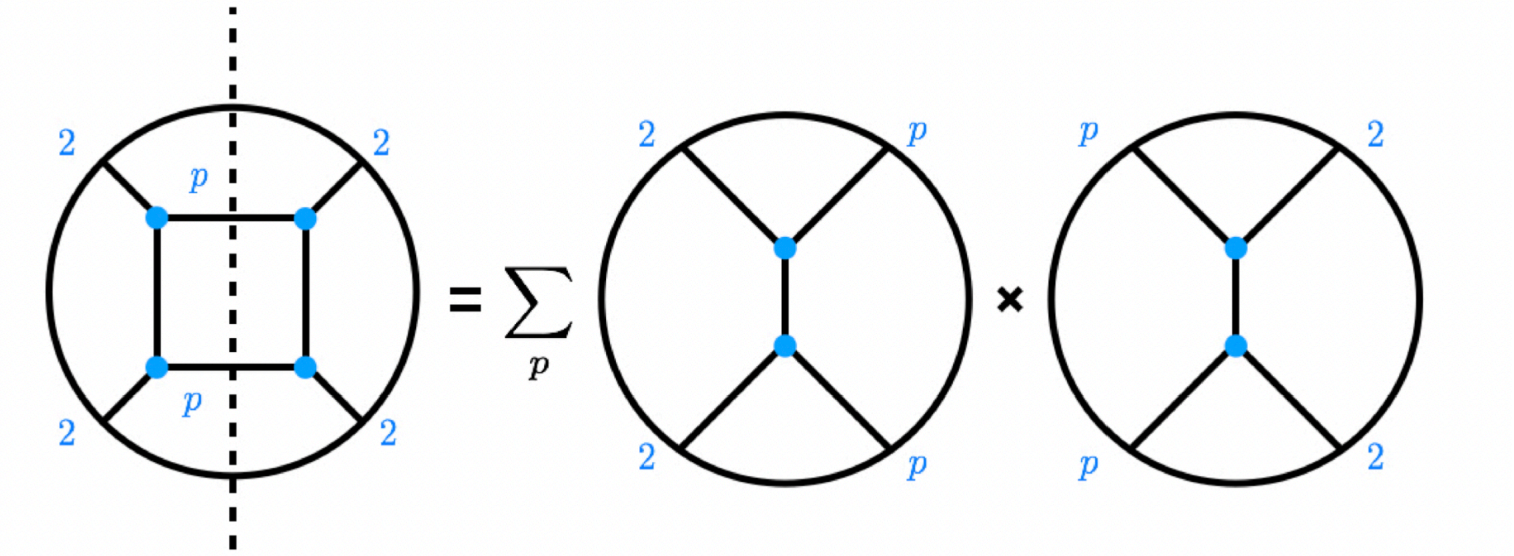

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1) At higher orders, there are higher trace operators that start contributing to the double discontinuity.

2) There are further mixing problems to take into account and it becomes unfeasible.

How to approach higher trace

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1) Quarter-BPS intrinsically double trace

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How to approach higher trace

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3) Higher loops

Quarter BPS operators

AB, G. Fardelli, A. Manenti 2022

$$\mathcal{O}_{pq} \sim \mathrm{Tr}\left(\varphi^{M_1}\ldots\right)\mathrm{Tr}\left(\ldots\varphi^{M_\Delta}\right)P_{M_1\ldots M_\Delta} + \frac{1}{N}\left(\text{single trace}\right)$$

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AB, G. Fardelli, A. Manenti 2022

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projector in the $[p, q, p]_R$

Quarter BPS operators

AB, G. Fardelli, A. Manenti 2022

$$\mathcal{O}_{pq} \sim \mathrm{Tr}\left(\varphi^{M_1}\ldots\right)\mathrm{Tr}\left(\ldots\varphi^{M_\Delta}\right) P_{M_1\ldots M_\Delta} + \frac{1}{N}\,\text{(single trace)}$$

projector in the $[p, q, p]_R$

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Proliferation of $SU(4)_R$ tensor structure in the OPEs.

For instance $\mathcal{O}_2 \times \mathcal{O}_2$ has 6 structure, $\mathcal{O}_{02} \times \mathcal{O}_2$ has 10 structures, $\mathcal{O}_{02} \times \mathcal{O}_{02}$ has 42 structures

How to deal with them?

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Use null polarization vectors and use their invariants to group structures.

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Detect protected multiplets!

Four point functions

$$\langle \mathcal{O}_{p_1 q_1} \mathcal{O}_{p_2 q_2} \mathcal{O}_{p_3 q_3} \mathcal{O}_{p_4 q_4} \rangle \sim \sum_k \mathbb{T}_k \mathcal{G}_k(z, \bar{z})$$

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As in the half-BPS case, we can use the inversion formula

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Large degeneracy of states!

Specific case

We studied in details mixed correlators involving $(p, q) \to (0,2)$

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Triple trace operators appearing at lower orders in $1/N$

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Double trace half-BPS

AB, G. Fardelli, A. Manenti in progress

Starting from dimension four operators, there are two half-BPS operators

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$$\mathcal{O}_4^{\mathrm{sp}}(x) = \sqrt{\frac{4(N^2+1)}{(N^2-1)(N^2-4)(N^2-9)}}\left(\mathrm{Tr}\left(\phi^4\right) - \frac{2N^2-3}{N(N^2+1)}\,\mathrm{Tr}\left(\phi^2\right)^2\right)$$

$$\mathcal{O}_4^{\mathrm{dt}}(x) = \sqrt{\frac{2}{N^4-1}}\,\mathrm{Tr}\left(\phi^2\right)^2$$

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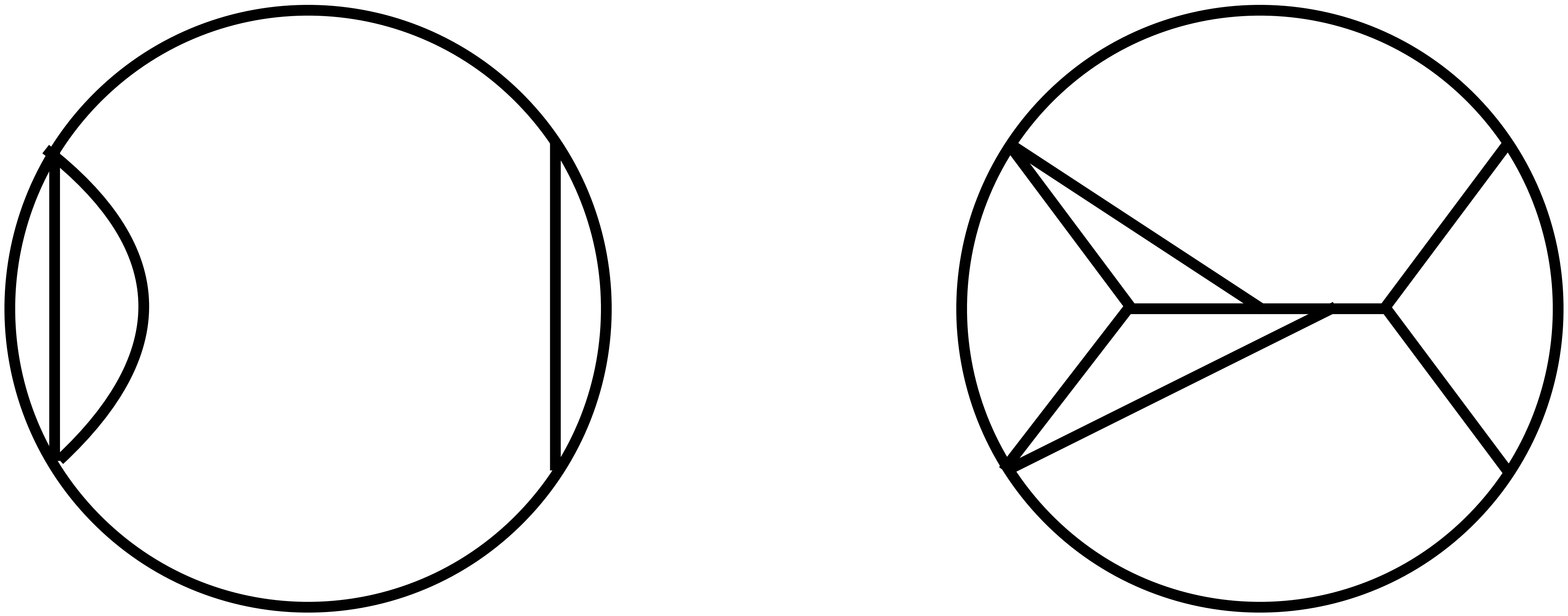
We will be mostly interested in $\mathcal{O}_4^{\mathrm{dt}}$

We found the structure of protected operators and computed the correlator at order $1/c$

Results

$$\langle \mathcal{O}_4^{\mathrm{dt}} \mathcal{O}_4^{\mathrm{dt}} \mathcal{O}_2 \mathcal{O}_2 \rangle$$



Ma, Zhou 2022

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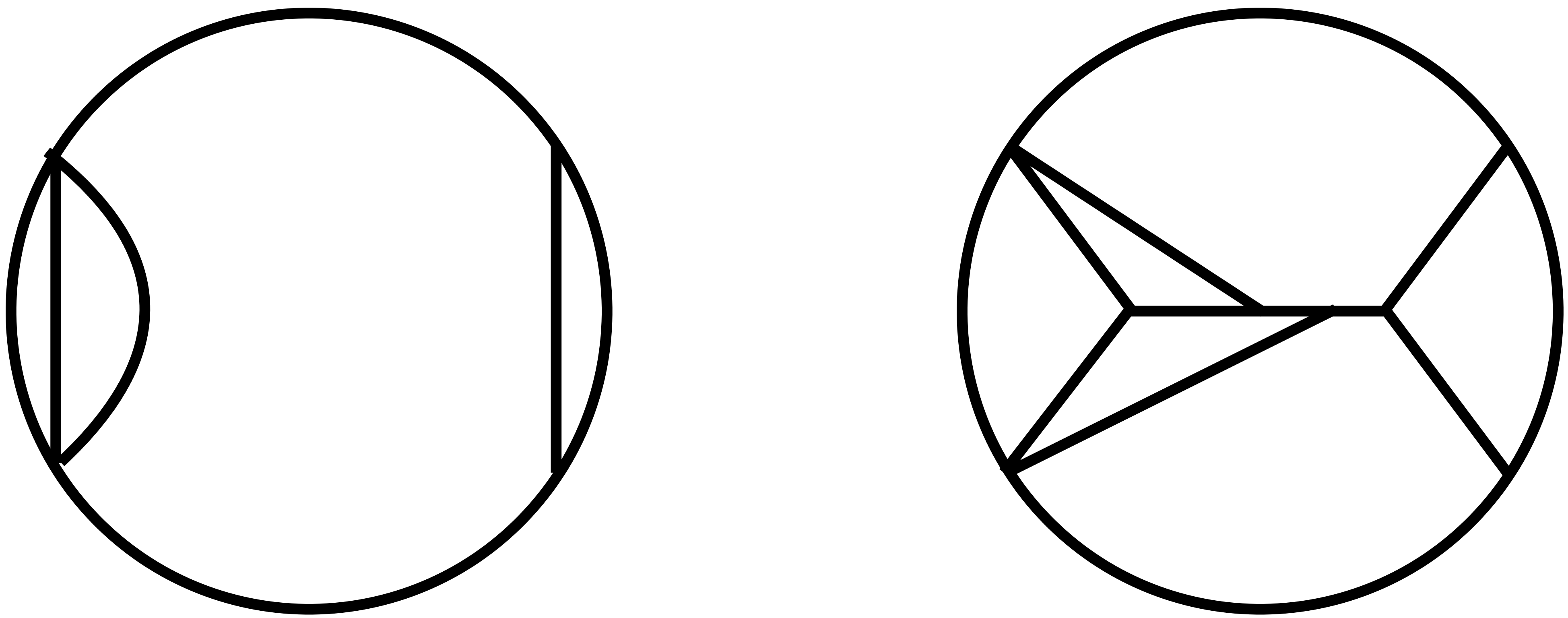

Computed CFT data of non protected, dimension six operators.

Ma, Zhou 2022

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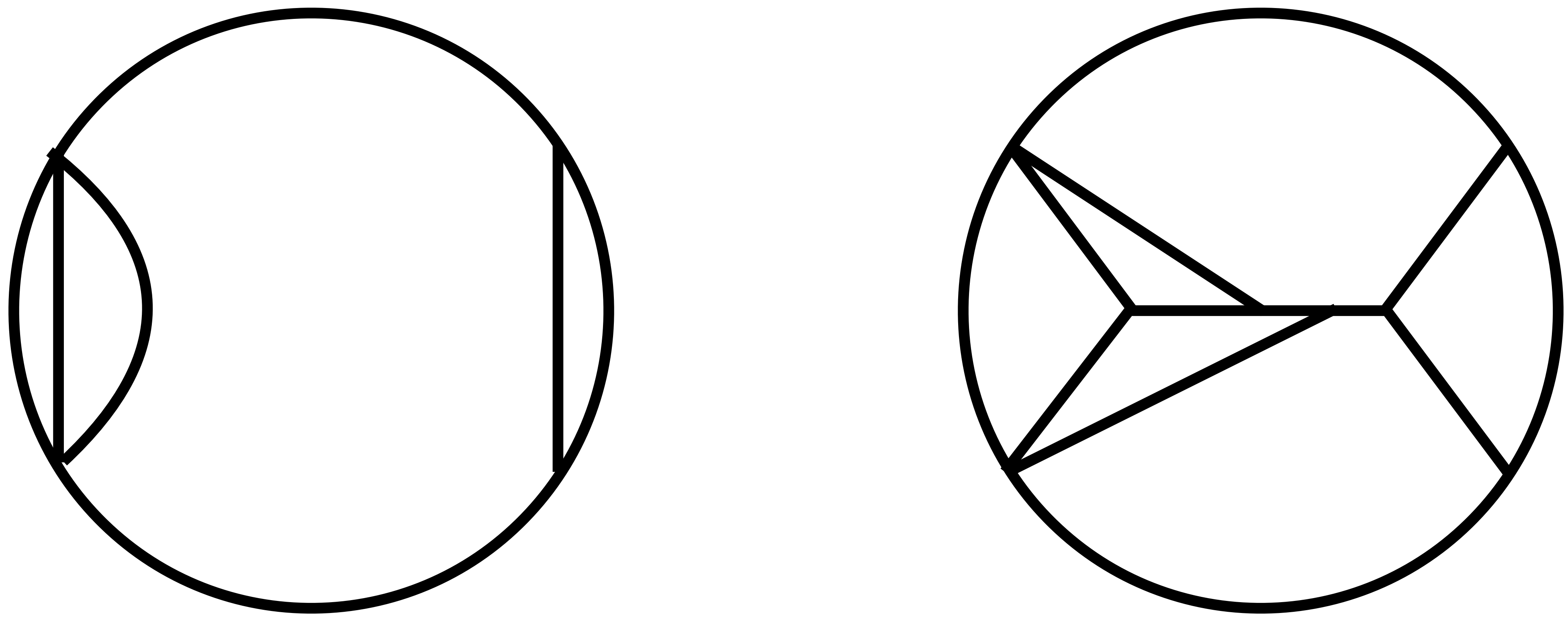

Computed CFT data of non protected, dimension six operators.

While supersymmetry imposes more constraints, there is higher degeneracy.

Open questions

Understand how to systematise these computations

Connect with systematics of AdS Witten diagrams

Use this together with higher point functions

Understand the contribution of higher traces in OPE

Basis of functions?

In AdS_3 it has been shown that it is necessary to include Bloch-Wigner-Ramakrishnan functions.

Ceplak, Giusto, Hughes, Russo 2022

Other results

Drummond, Paul 2022

Huang, Ye Yuan 2021

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- Two loops: OPE reasoning + educated ansatz for the $\mathscr{H}^{(3)}(u,v)$

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checked with flat space

unavoidability of for triple traces

All-loops sugra

Caron-Huot, Trinh 2018
Aprile, Drummond, Heslop, Paul 2018
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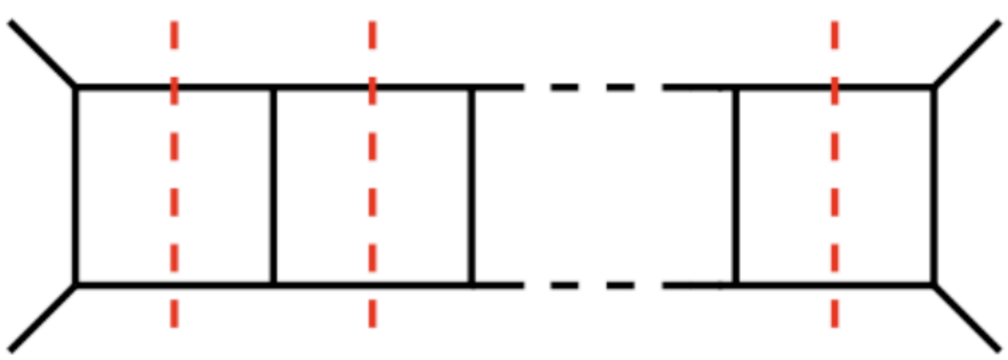

s-channel consecutive cuts

comparison with flat space

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All- loops in ϕ^4

AB, G. Fardelli, M.R. Khansari in progress

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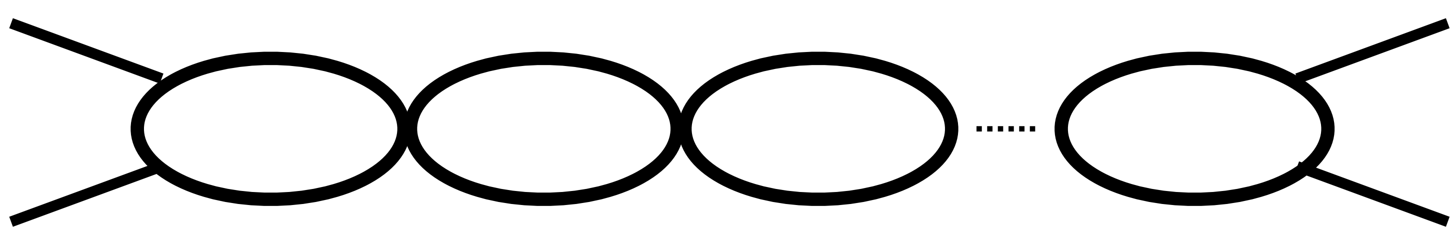

$$\gamma^{(k)} \underset{\ell\to\infty}{\sim} \frac{\log^{k-3}\ell}{\ell^2} + \dots$$

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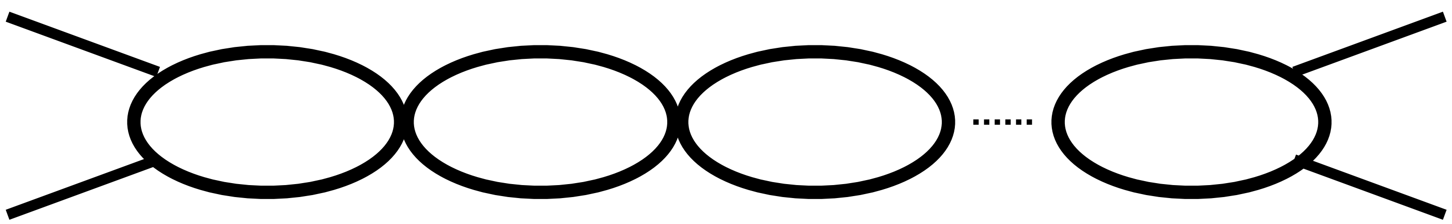

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Can this behaviour constrain higher trace contribution?

AB, G. Fardelli, M.R. Khansari in progress

Conclusions

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Approaches to study double and higher trace operators

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In some cases, there are strong differences between the two

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Use this technology with integrability to include single traces

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Understand how to resum the N expansion

Use this technology with integrability to include single traces