Bootstrapping gauge theories (QCD)

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LPENS CNRS

based on 2309.12402 and *to appear* with Martin Kruczenski

Low energy physics of asymptotically free gauge theory

asymptotically free gauge theory $SU(N_c)$

chiral symmetry breaking and confinement

N_f massive quarks $m_q \ll \Lambda_{\rm QCD}$ fundamental representation of gauge group

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$$\mathcal{L} = i \sum_{j}^{N_f} \bar{q}_j \not{D} q_j - \sum_{j}^{N_f} m_q \bar{q}_j q_j - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \text{gauge fixing} + \text{ghost}$$

gauge theory parameters: N_c N_f m_q Λ_{QCD}

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gauge theory parameters: $N_c \quad N_f \quad m_q \quad \Lambda_{\mathrm{QCD}}$

What is the low energy physics?

Physics of Goldstone bosons

chiral symmetry breaking

explicitly — *dynamically*

quark mass $m_q \bar{q}_j q_j$

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

condensate $\langle \bar{q}_j q_j \rangle$

(approximate) Goldstone bosons dominate the low energy physics

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dynamically — *condensate* $\langle \bar{q}_j q_j \rangle$

(approximate) Goldstone bosons dominate the low energy physics

e.g. $N_f = 2$ pions $\pi_0 = \pi^3 \quad \pi_\pm = \frac{1}{\sqrt{2}}(\pi^1 \pm i\pi^2)$

very low energy effective Lagrangian (lowest order):

$$\mathcal{L} = \frac{f_\pi^2}{4}\left\{\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + m_\pi^2 \mathrm{Tr}\left(U + U^\dagger\right)\right\} \qquad U = e^{i\frac{\vec{\tau}\cdot\vec{\pi}}{f_\pi}}$$

$$\mathcal{L}_2^{2\pi} = \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}^2 \qquad \mathcal{L}_2^{4\pi} = \frac{1}{6f_\pi^2}\Big((\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 - \vec{\pi}^2(\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi})\Big) + \frac{m_\pi^2}{24f_\pi^2}(\vec{\pi}^2)^2 \quad \ldots$$

The problem of strongly coupled physics

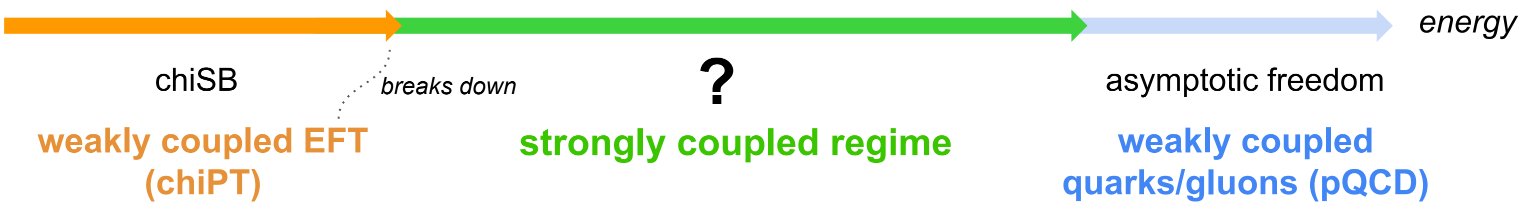
chiSB

weakly coupled EFT (chiPT)

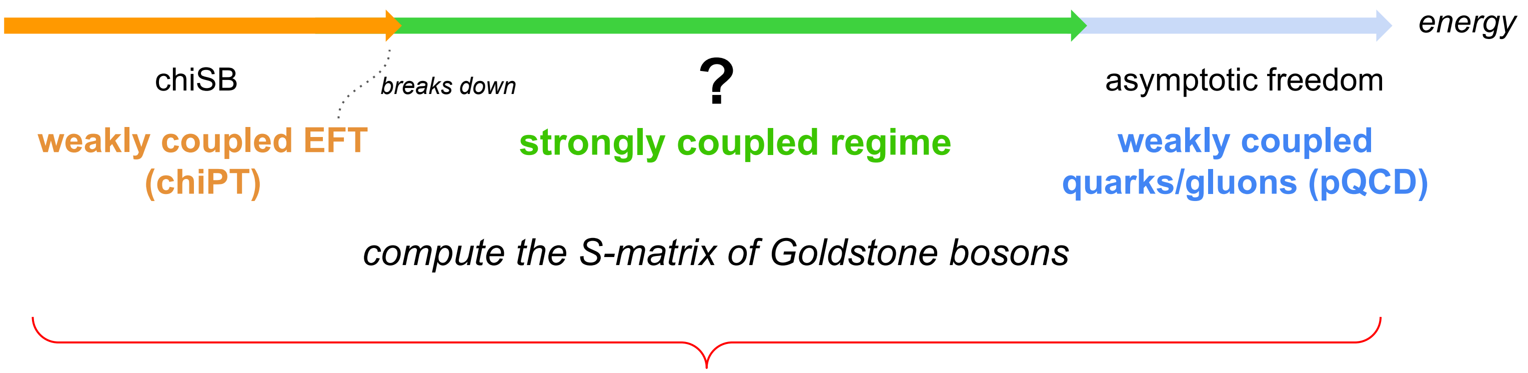
asymptotic freedom

weakly coupled quarks/gluons (pQCD)

The problem of strongly coupled physics



The problem of strongly coupled physics



gauge theory bootstrap

The problem of strongly coupled physics

energy

chiSB · *breaks down* · ? · asymptotic freedom

weakly coupled EFT (chiPT) · **strongly coupled regime** · **weakly coupled quarks/gluons (pQCD)**

compute the S-matrix of Goldstone bosons

gauge theory bootstrap

rules of the game: assume — *chiral symmetry breaking* & *confinement*

input — gauge theory parameters – define the theory

as few as possible (universal) low energy parameters

Gauge theory bootstrap

- **Pure S-matrix bootstrap:**

symmetry, analyticity, crossing, unitarity

Gauge theory bootstrap

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- **Chiral symmetry breaking:**

general very low energy behavior

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- **Form factor bootstrap + SVZ + asymptotics:**

gauge theory information

Gauge theory bootstrap

- **Pure S-matrix bootstrap:** ← $SU(N_f)_V$

symmetry, analyticity, crossing, unitarity

- **Chiral symmetry breaking:** ← $f_\pi \quad m_\pi$

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gauge theory information

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general very low energy behavior ← $f_\pi \quad m_\pi$

- **Form factor bootstrap + SVZ + asymptotics:**

gauge theory information ← $N_c \quad m_q \quad \Lambda_{\text{QCD}}$

Numerical test of the method: $N_f = 2 \quad N_c = 3$

can be compared with experimental data

for general gauge theories — compare with lattice data

partial waves

$$f_\ell^I(s)$$

form factors

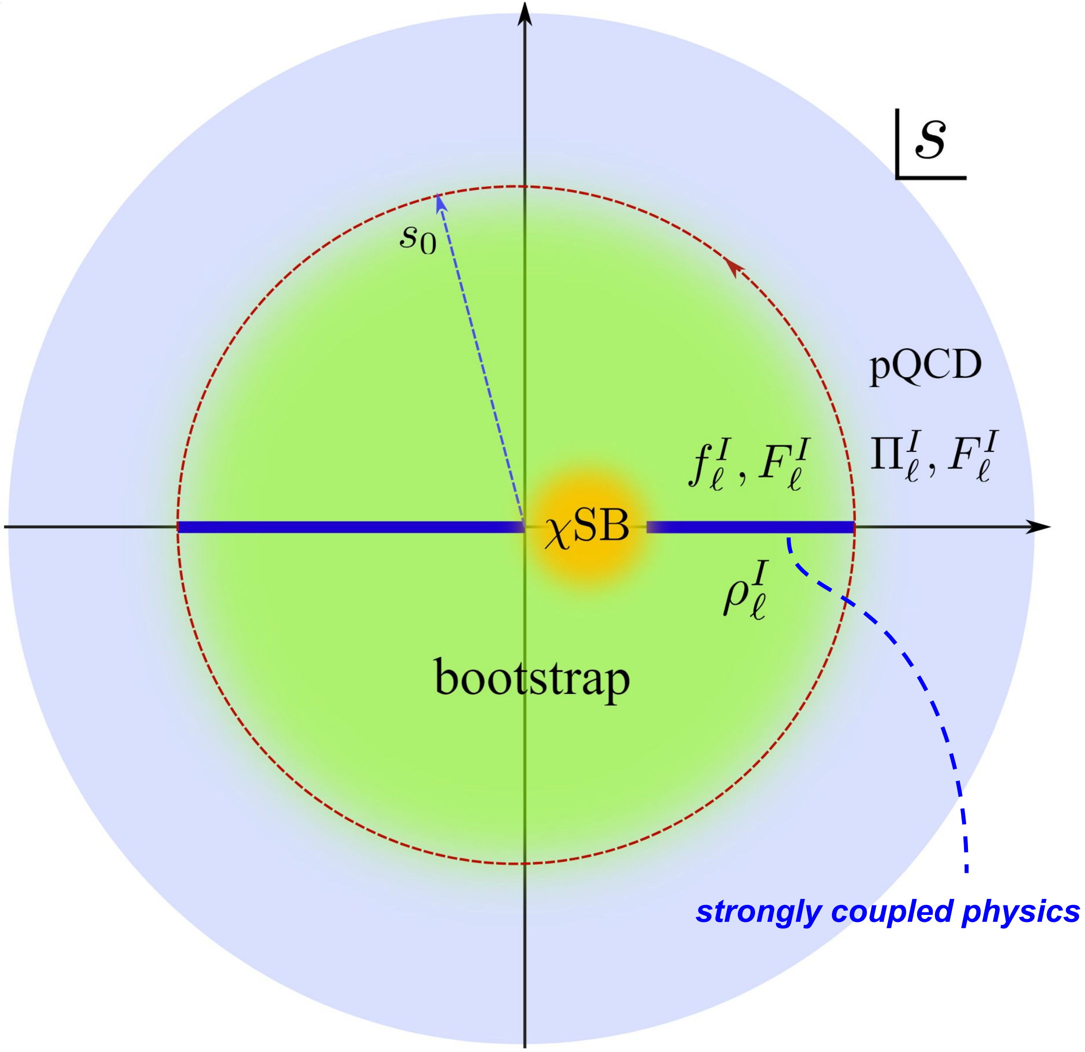
$$F_\ell^I(s)$$

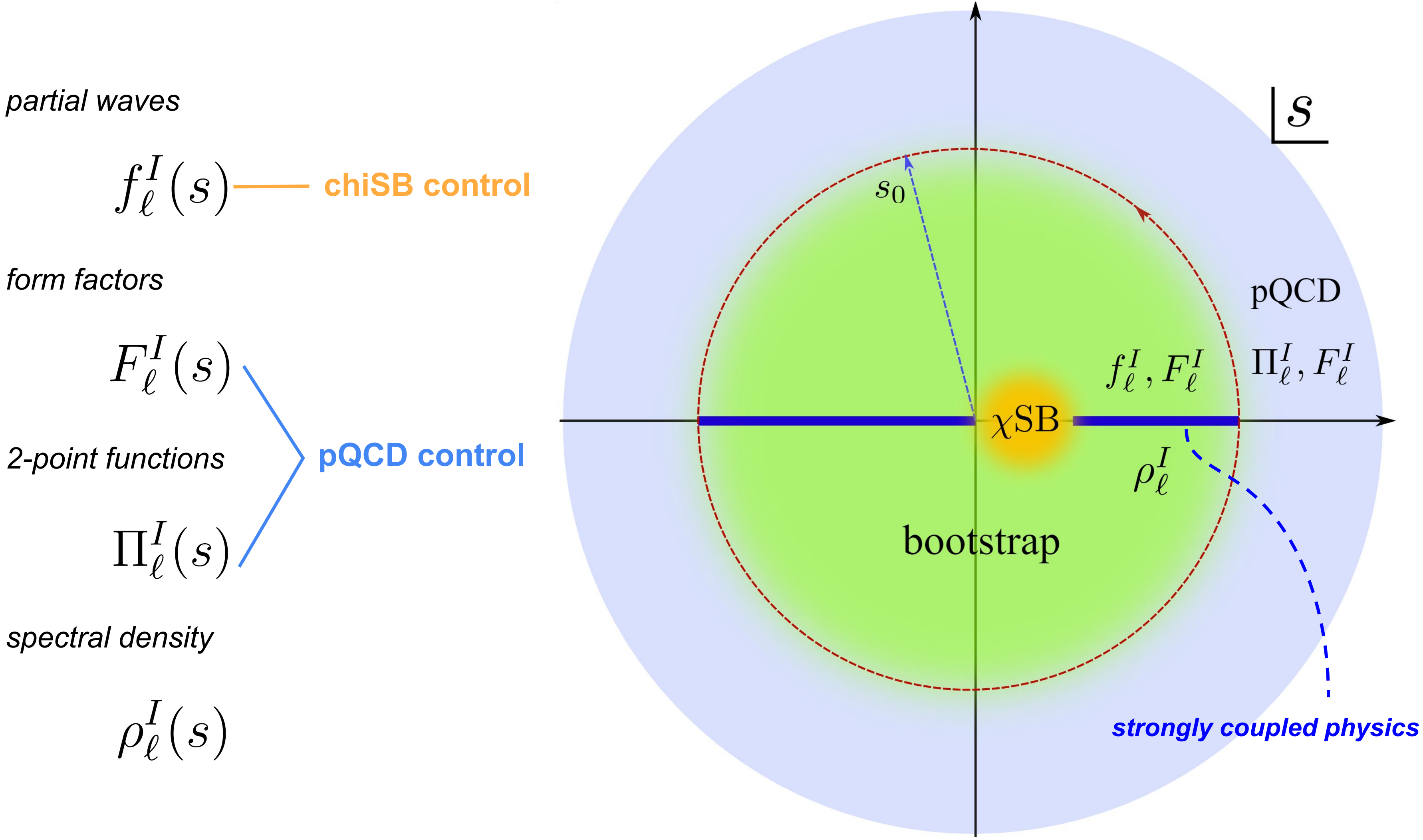
2-point functions

$$\Pi_\ell^I(s)$$

spectral density

$$\rho_\ell^I(s)$$





partial waves
$f_\ell^I(s)$
chiSB control
form factors
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2-point functions
pQCD control
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s
s_0
pQCD
Π_ℓ^I, F_ℓ^I
f_ℓ^I, F_ℓ^I
χSB
ρ_ℓ^I
bootstrap
strongly coupled physics

Gauge theory bootstrap

- **Pure S-matrix bootstrap:**

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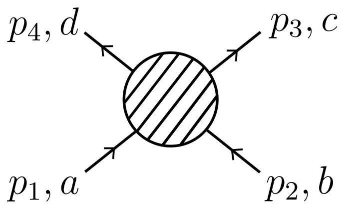
$$SU(N_f)_V$$

Pure S-matrix bootstrap

modern S-matrix bootstrap: [Paulos, Penedones, Toledo, van Rees, Vieira, 2016 & 2017]



constrain amplitudes using generic consistency conditions

2-to-2 pion scattering: $\pi_a(p_1) + \pi_b(p_2) \to \pi_c(p_3) + \pi_d(p_4)$

$$s = (p_1 + p_2)^2$$
$$t = (p_1 - p_3)^2$$
$$u = (p_1 - p_4)^2$$

$$\langle p_1, a; p_2, b|\mathbf{T}|p_3, c; p_4, d\rangle = A(s,t,u)\delta_{ab}\delta_{cd} + A(t,s,u)\delta_{ac}\delta_{bd} + A(u,t,s)\delta_{ad}\delta_{bc}$$

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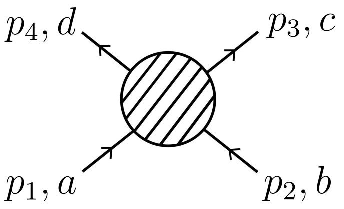

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crossing $A(s,t,u) = A(s,u,t)$

analyticity cuts $s,t,u > 4$

$$m_\pi = 1$$

$$A(s,t,u) = \frac{1}{\pi^2}\int_4^\infty dx \int_4^\infty dy \left[\frac{\rho_1(x,y)}{(x-s)(y-t)} + \frac{\rho_1(x,y)}{(x-s)(y-u)} + \frac{\rho_2(x,y)}{(x-t)(y-u)} \right]$$ +subtraction terms

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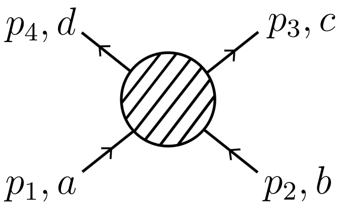

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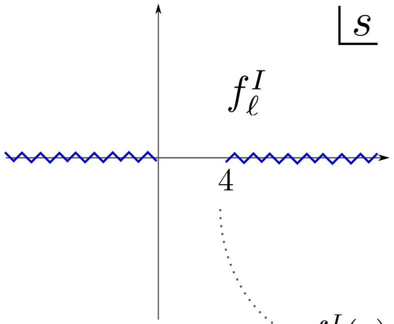
parameters: $\{\rho_{\alpha=1,2}(x,y), \dots\}$ numerics: discretize $\{\rho_{\alpha,ij}, \dots\}$ bootstrap variables

Pure S-matrix bootstrap



$$
\begin{aligned}
T^{I=0}(s,t,u) &= 3A(s,t,u) + A(t,s,u) + A(u,t,s) \\
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\end{aligned}
$$

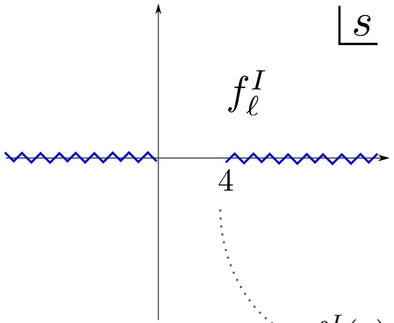
$SU(2)_V$ isospin

symmetry

⇓

$$f_\ell^I(s) = \frac{1}{4}\int_{-1}^{1} d\cos\theta \, P_\ell(\cos\theta) T^I(s,t)$$

Pure S-matrix bootstrap



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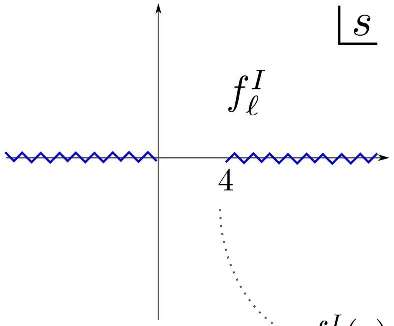
analytic function of s

$f_\ell^I(0 < s < 4)$ *real linear functionals of bootstrap variables*

unphysical region

Pure S-matrix bootstrap



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analytic function of s

physical kinematic region $s > 4$

$$S_\ell^I(s) = 1 + i\pi\sqrt{\frac{s-4}{s}} f_\ell^I(s) = \eta_\ell^I(s)\, e^{2i\delta_\ell^I(s)}$$

phase shift

$f_\ell^I(0 < s < 4)$ *real linear functionals of bootstrap variables*

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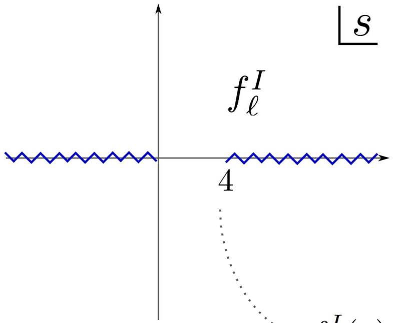

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$f_\ell^I(0 < s < 4)$ *real linear functionals of bootstrap variables*

phase shift

unphysical region

$$|S_\ell^I(s^+)| \le 1,\ s > 4 \quad \forall \ell, I$$

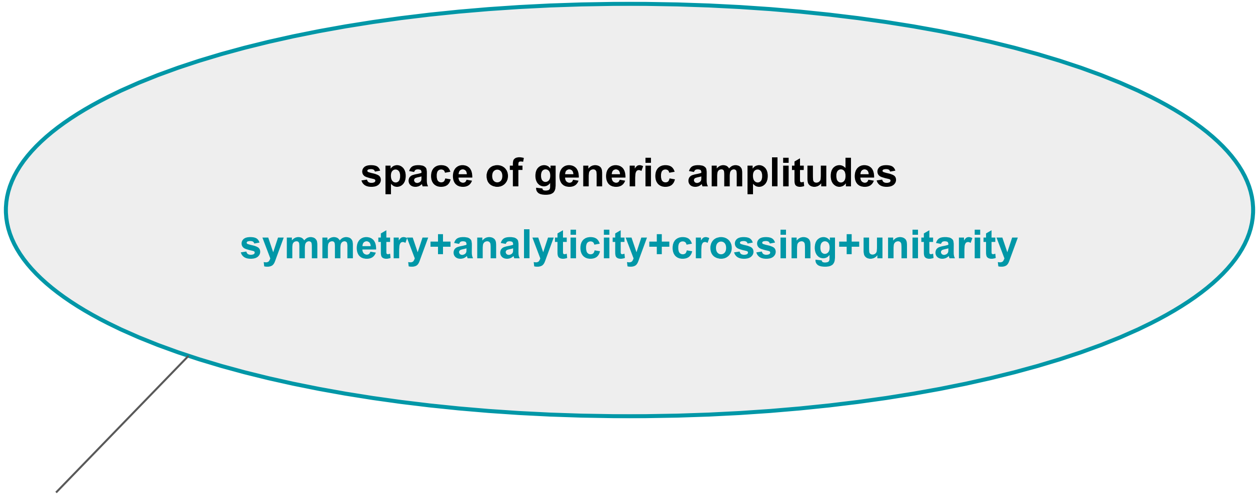
unitarity

positive semidefinite → convex space of amplitudes

$$\begin{pmatrix} 1 & S_\ell^I(s) \\ S_\ell^{I*}(s) & 1 \end{pmatrix} \succeq 0$$

convex optimization

Pure S-matrix bootstrap

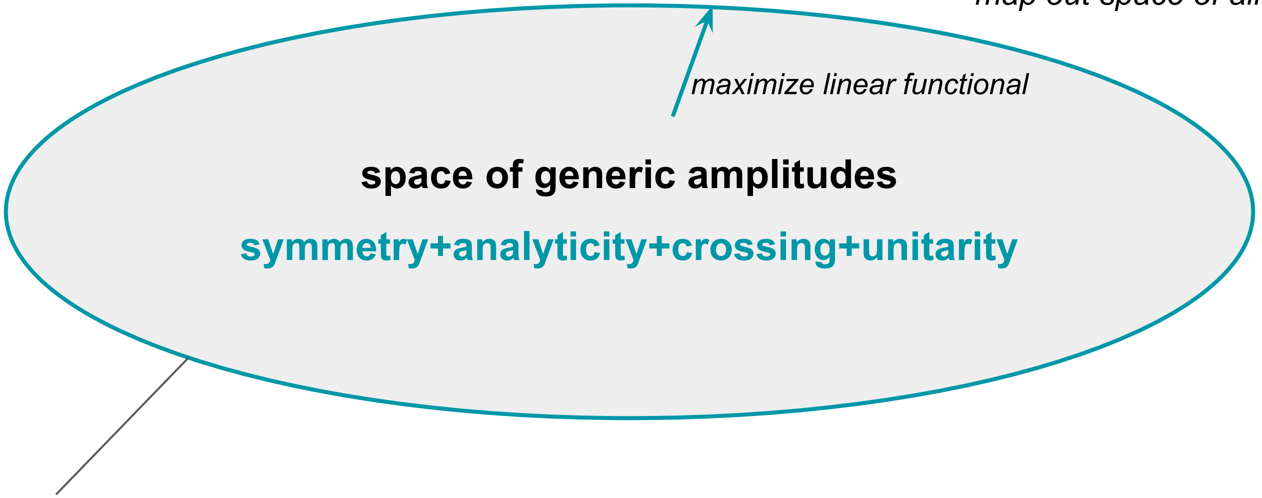


space of parameters under constraints

$$\left\{ \rho_{\alpha=1,2}(x,y), \ldots \right\}$$

Pure S-matrix bootstrap

putting bounds,
map out space of allowed amplitudes



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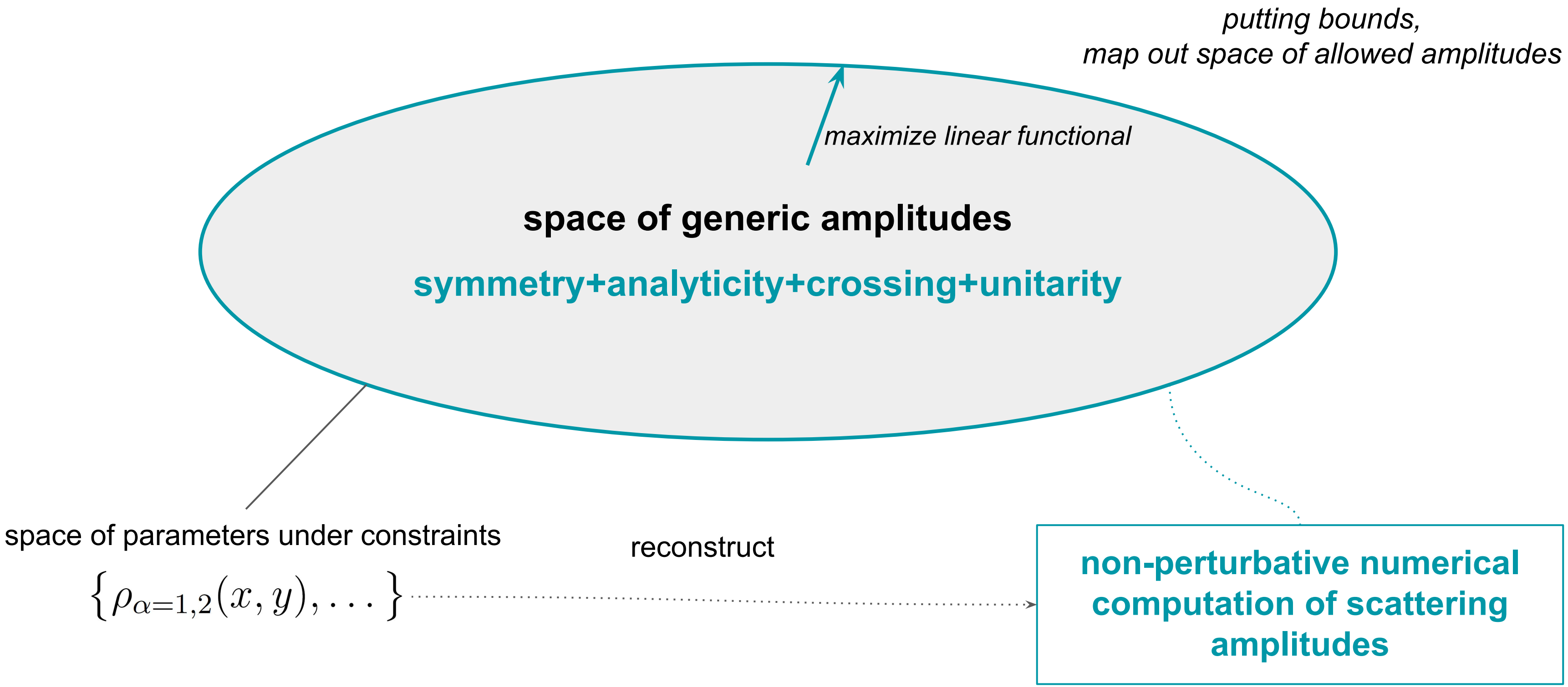
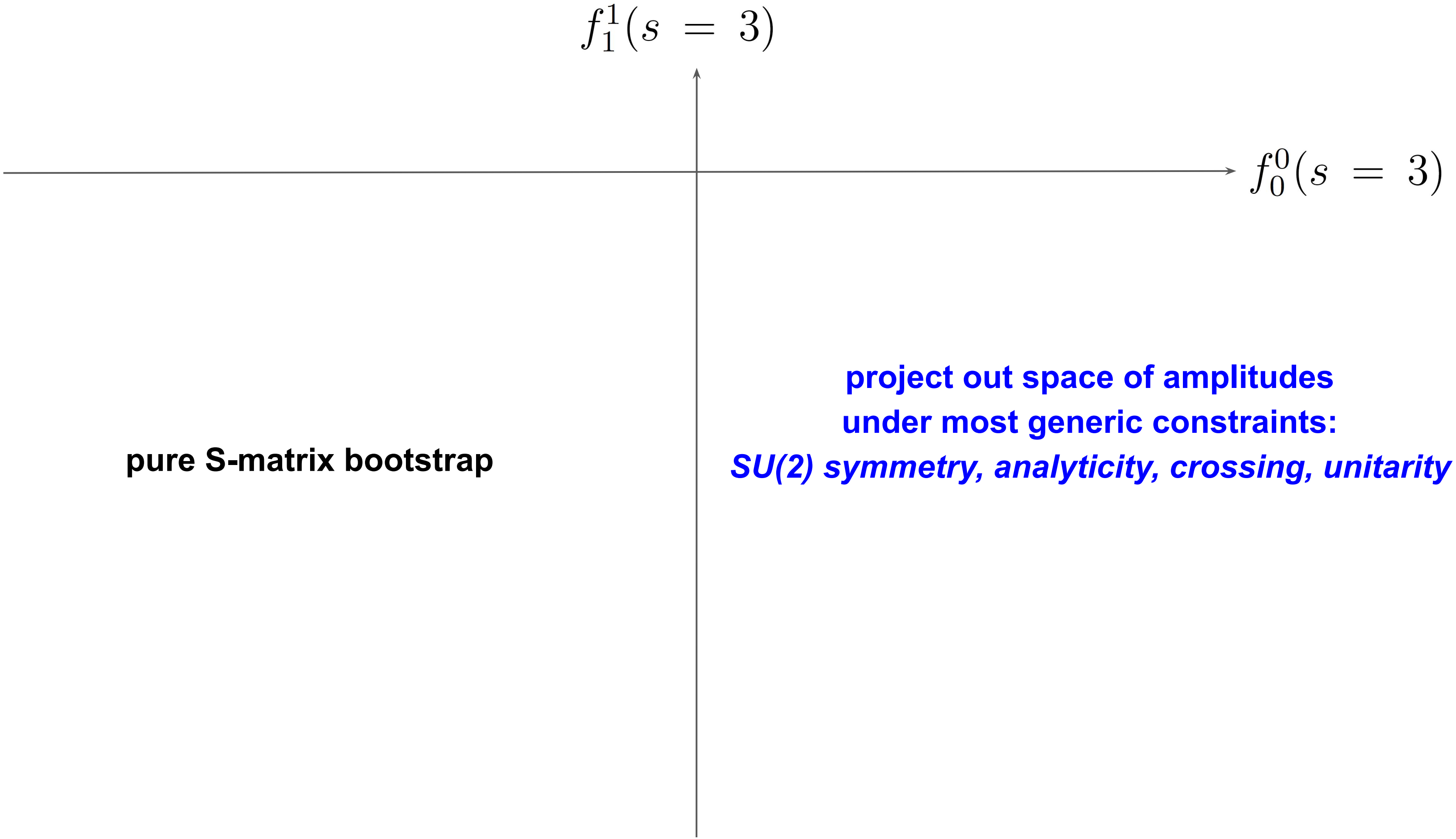
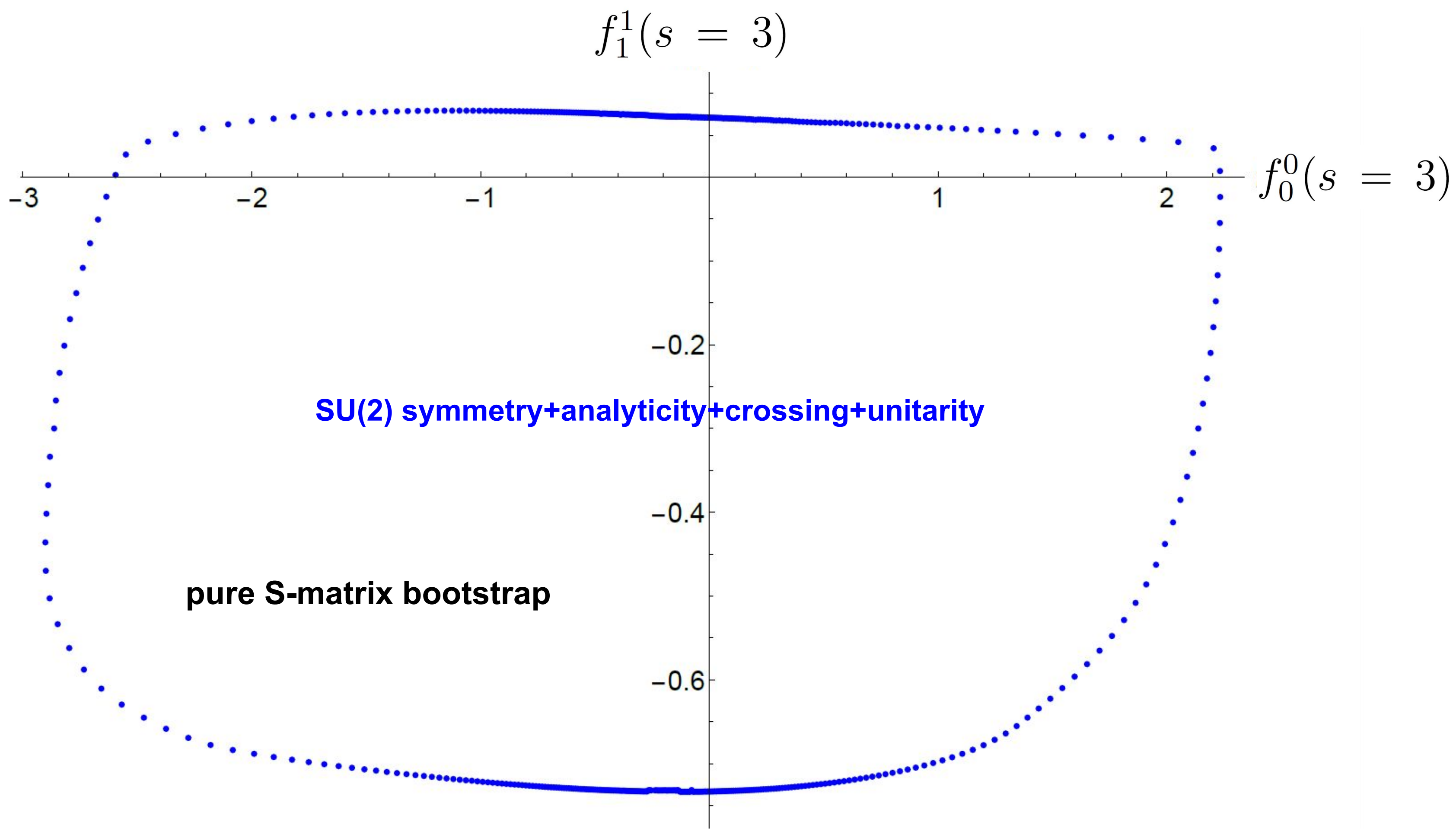



$f_1^1(s = 3)$
$f_0^0(s = 3)$
pure S-matrix bootstrap
project out space of amplitudes
under most generic constraints:
SU(2) symmetry, analyticity, crossing, unitarity



each boundary point: an extremal numerical amplitude

Gauge theory bootstrap

- **Pure S-matrix bootstrap:**

symmetry, analyticity, crossing, unitarity

$$SU(N_f)_V$$

+

- **Chiral symmetry breaking:**

general very low energy behavior

$$f_\pi \quad m_\pi$$

Weakly coupled Goldstone bosons

chiral symmetry breaking: weakly coupled Goldstone bosons at very low energy

interaction: $$\mathcal{L}_2^{4\pi} = \frac{1}{6f_\pi^2}\Big((\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 - \vec{\pi}^2(\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi})\Big) + \frac{m_\pi^2}{24f_\pi^2}(\vec{\pi}^2)^2$$

tree-level amplitude: $$A_{\text{tree}}(s,t,u) = \frac{4}{\pi}\frac{s-m_\pi^2}{32\pi f_\pi^2}$$ *linear in s* [Weinberg, 1966]

good in the unphysical region (very low energy) $0 < s,t,u < 4m_\pi^2$

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to get an idea of the weakly coupled pions: $$\langle\pi^0\pi^0|\mathbf{T}|\pi^0\pi^0\rangle = A(s,t,u) + A(t,s,u) + A(u,t,s) \qquad \lambda = \frac{\pi}{4}T\left(\frac{4}{3},\frac{4}{3},\frac{4}{3}\right)$$

$$\lambda_{\text{tree}} = \frac{m_\pi^2}{32\pi f_\pi^2} \simeq 0.023 \ll \lambda_{\max} \simeq 2.661$$

max coupling by S-matrix bootstrap (ACU)

primal: [PPTvRV, 2017]

dual: [YH, Kruczenski, 2021]

Chiral symmetry breaking input

approximate linear behavior at very low energy: input in gauge theory bootstrap

S0: $f^0_{0,\text{tree}}(s) = \frac{2}{\pi}\frac{2s - m_\pi^2}{32\pi f_\pi^2}$ P1: $f^1_{1,\text{tree}}(s) = \frac{2}{\pi}\frac{s - 4m_\pi^2}{96\pi f_\pi^2}$ S2: $f^2_{0,\text{tree}}(s) = \frac{2}{\pi}\frac{2m_\pi^2 - s}{32\pi f_\pi^2}$

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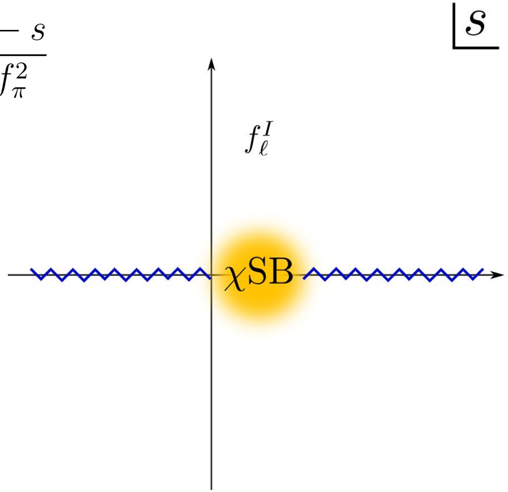

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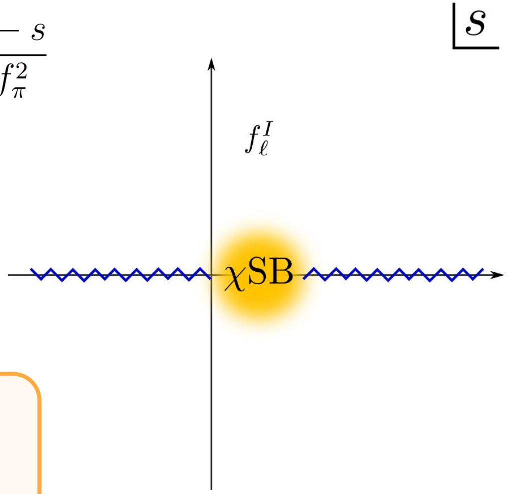
numerically

requires p.w. in the bootstrap match the tree level p.w. in unphysical region

$$f_0^0(s) \simeq f^0_{0,\text{tree}}(s) \qquad f_1^1(s) \simeq f^1_{1,\text{tree}}(s) \qquad f_0^2(s) \simeq f^2_{0,\text{tree}}(s) \qquad 0 < s < 4m_\pi^2$$



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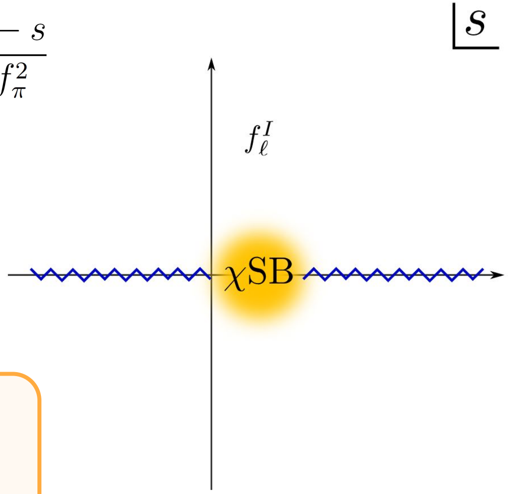

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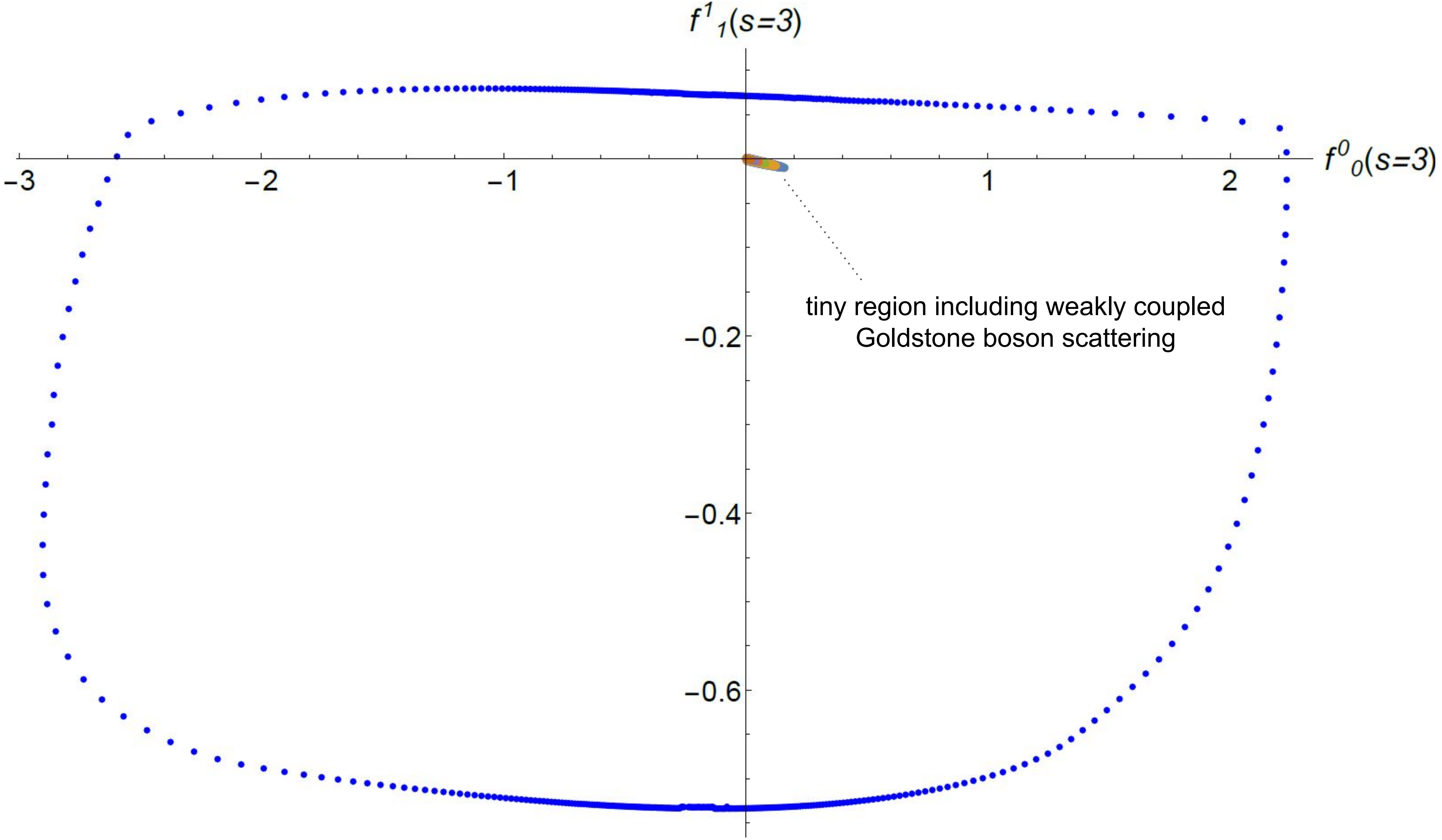
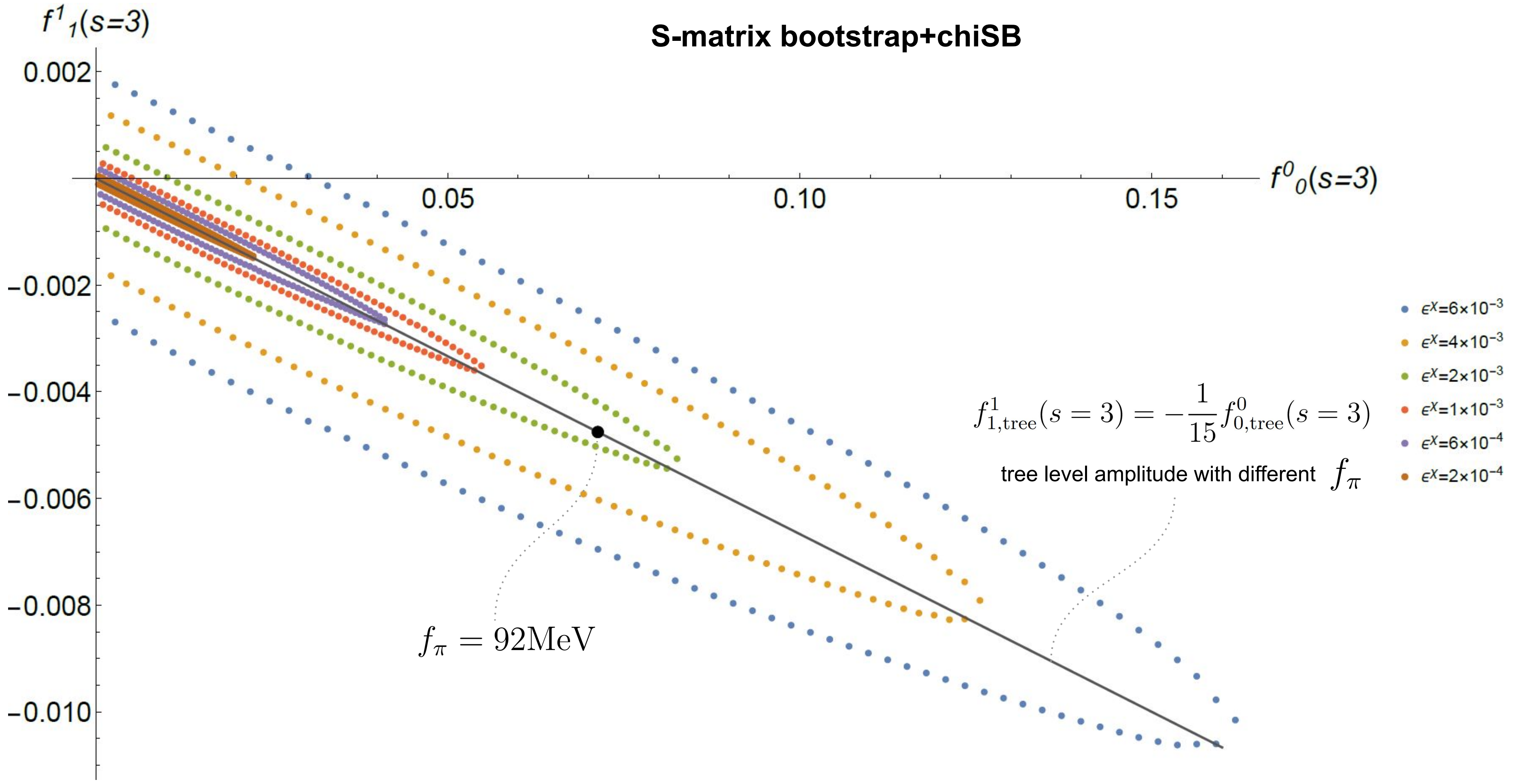
too loose: large deviation from chiSB prediction
too tight: exclude the desired theory

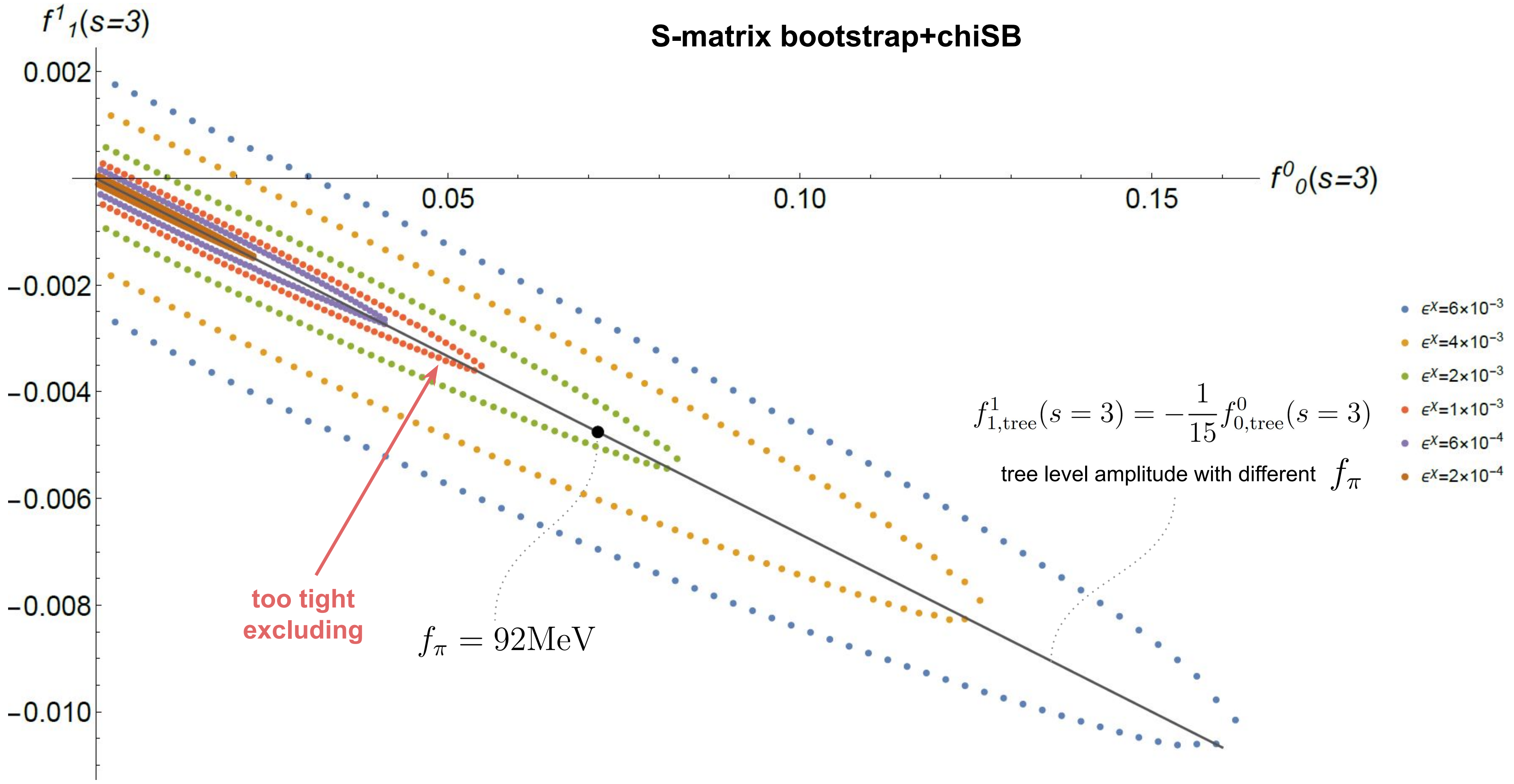
numerics with a series of tolerance

use $f_\pi = 92\text{MeV}$ **to select appropriate tolerance**

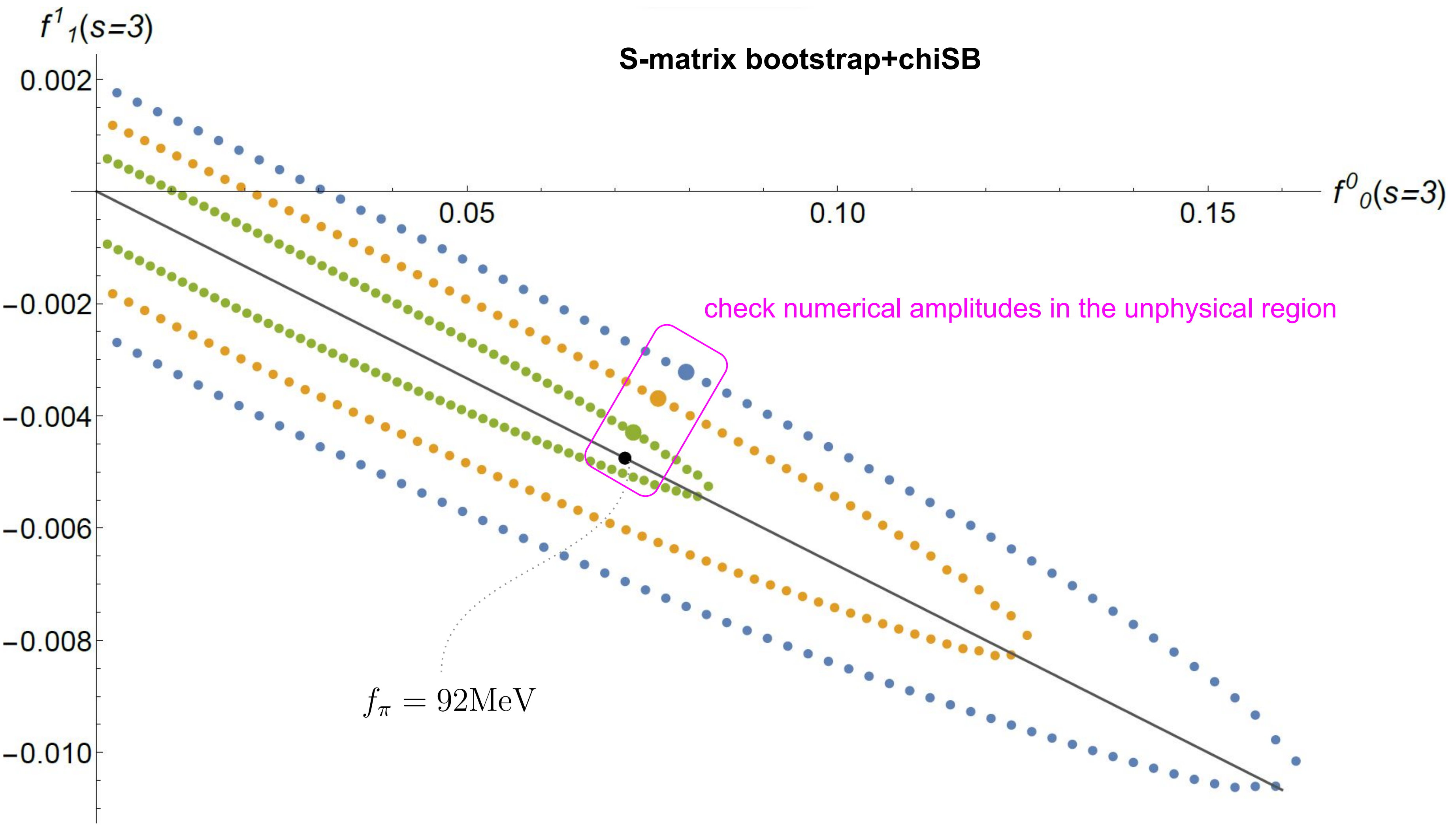

$f^1{}_1(s=3)$
$f^0{}_0(s=3)$
−3
−2
−1
1
2
−0.2
−0.4
−0.6
tiny region including weakly coupled
Goldstone boson scattering



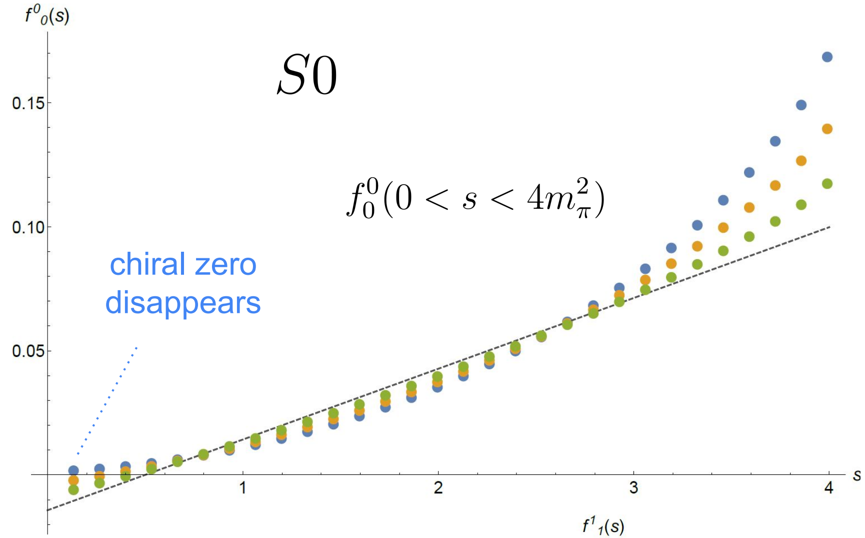
S-matrix bootstrap+chiSB
$f^1{}_1(s=3)$
$f^0{}_0(s=3)$
0.002
−0.002
−0.004
−0.006
−0.008
−0.010
0.05
0.10
0.15
$\epsilon^X=6\times10^{-3}$
$\epsilon^X=4\times10^{-3}$
$\epsilon^X=2\times10^{-3}$
$\epsilon^X=1\times10^{-3}$
$\epsilon^X=6\times10^{-4}$
$\epsilon^X=2\times10^{-4}$
$f^1_{1,\text{tree}}(s=3) = -\frac{1}{15} f^0_{0,\text{tree}}(s=3)$
tree level amplitude with different f_π
$f_\pi = 92\text{MeV}$

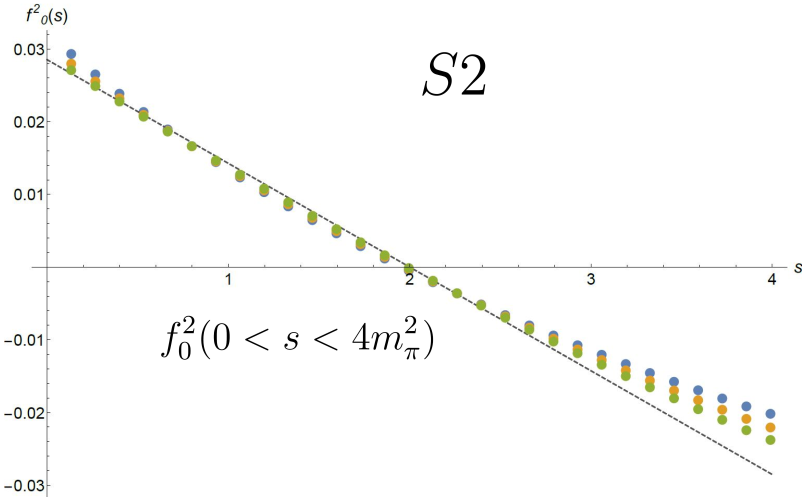


S-matrix bootstrap+chiSB
$f^1{}_1(s=3)$
$f^0{}_0(s=3)$
0.002
−0.002
−0.004
−0.006
−0.008
−0.010
0.05
0.10
0.15
$f^1_{1,\text{tree}}(s=3) = -\frac{1}{15} f^0_{0,\text{tree}}(s=3)$
tree level amplitude with different f_π
$\epsilon^X=6\times10^{-3}$
$\epsilon^X=4\times10^{-3}$
$\epsilon^X=2\times10^{-3}$
$\epsilon^X=1\times10^{-3}$
$\epsilon^X=6\times10^{-4}$
$\epsilon^X=2\times10^{-4}$
too tight excluding
$f_\pi = 92\text{MeV}$



S-matrix bootstrap+chiSB
$f^1_1(s=3)$
$f^0_0(s=3)$
0.002
−0.002
−0.004
−0.006
−0.008
−0.010
0.05
0.10
0.15
check numerical amplitudes in the unphysical region
$f_\pi = 92\mathrm{MeV}$

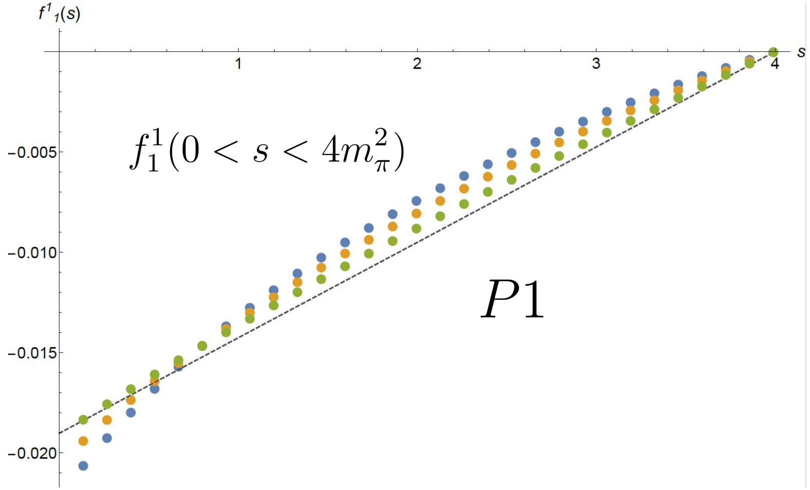




dashed lines: tree-level p.w.

$f^0_{0,\mathrm{tree}}(s)$ $f^1_{1,\mathrm{tree}}(s)$ $f^2_{0,\mathrm{tree}}(s)$

$f_\pi = 92\mathrm{MeV}$

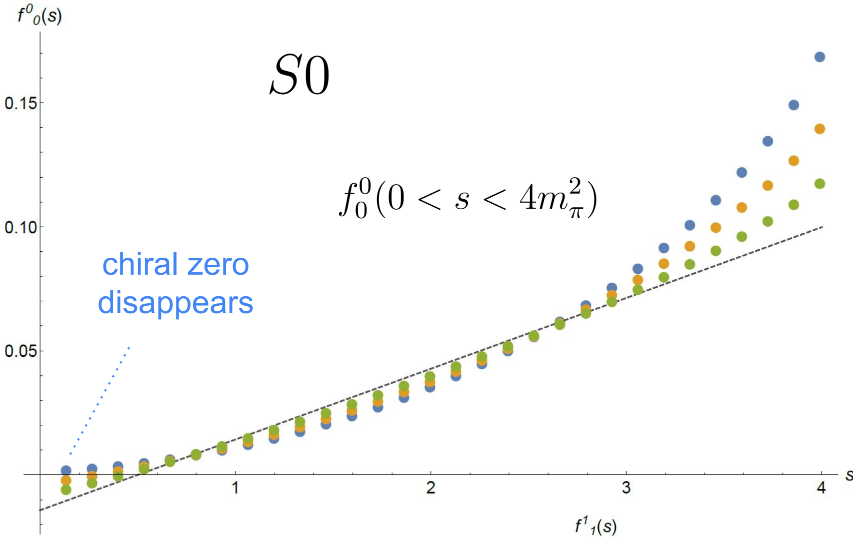


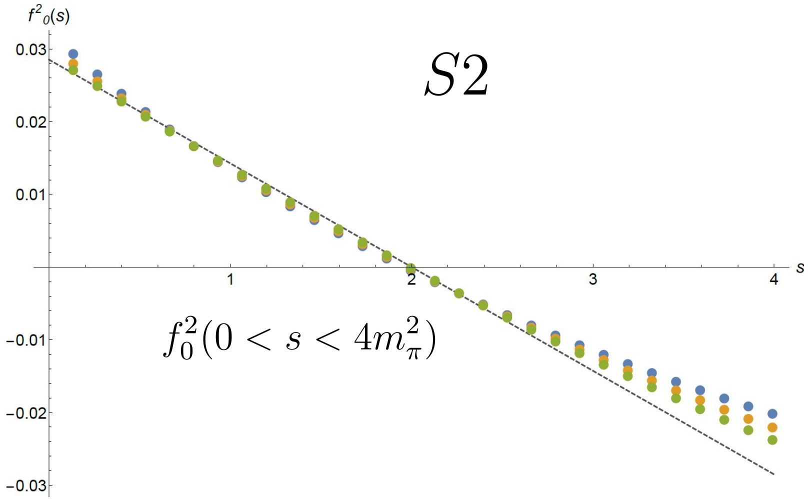
$\epsilon^\chi = 6\times10^{-3}$

$\epsilon^\chi = 4\times10^{-3}$

$\epsilon^\chi = 2\times10^{-3}$

$0 < s < 4m_\pi^2$

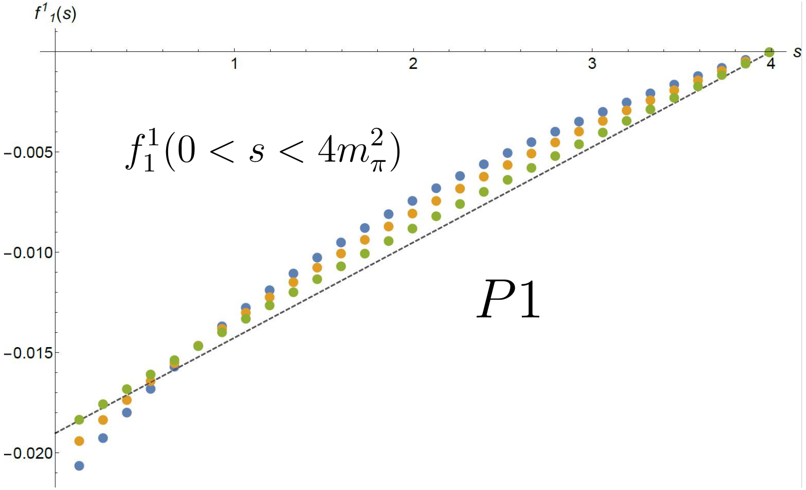




dashed lines: tree-level p.w.

$f^0_{0,\text{tree}}(s) \quad f^1_{1,\text{tree}}(s) \quad f^2_{0,\text{tree}}(s)$

$f_\pi = 92\text{MeV}$



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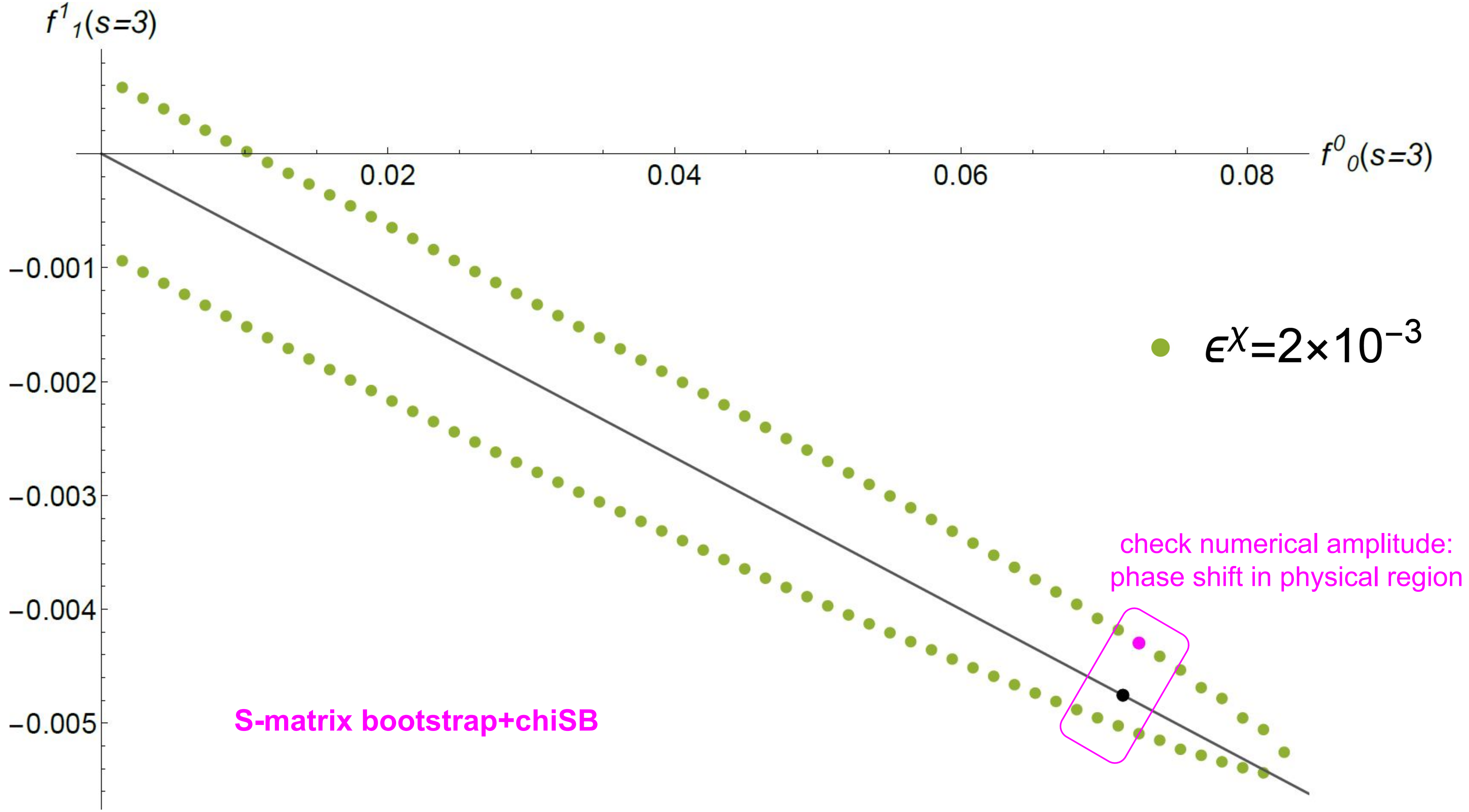
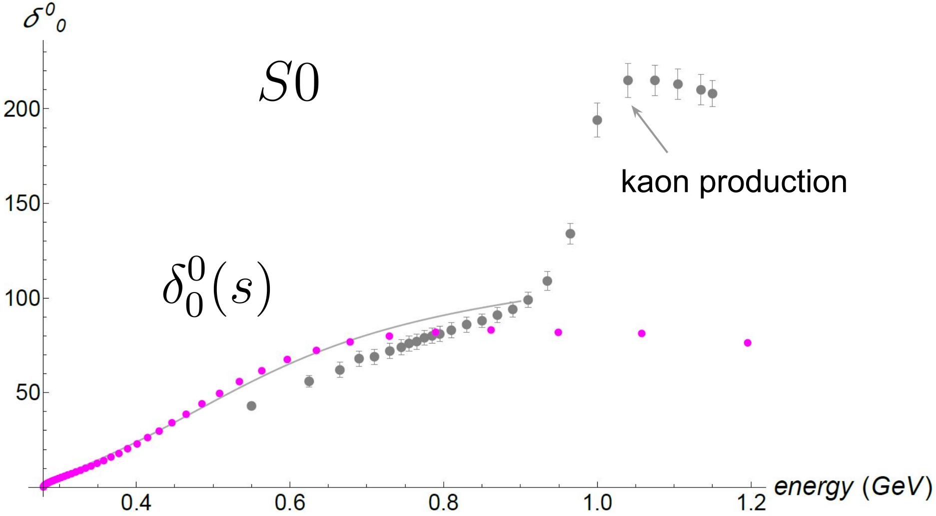
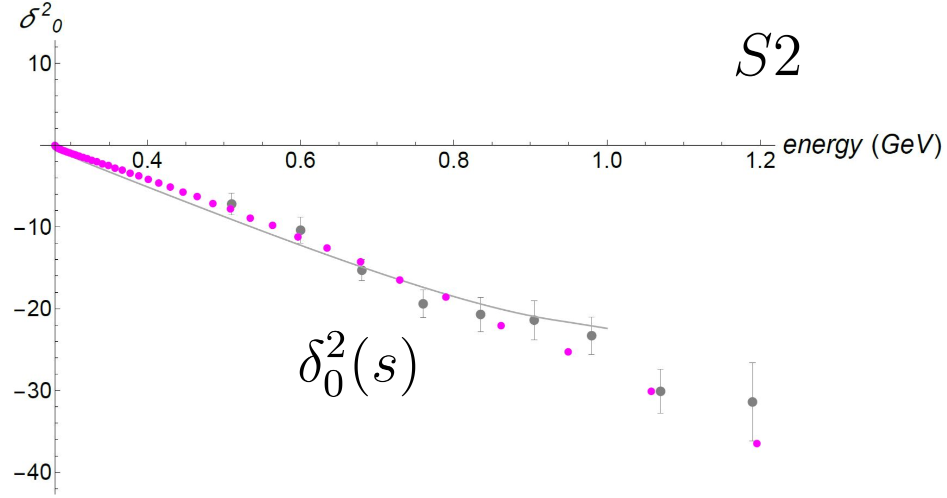


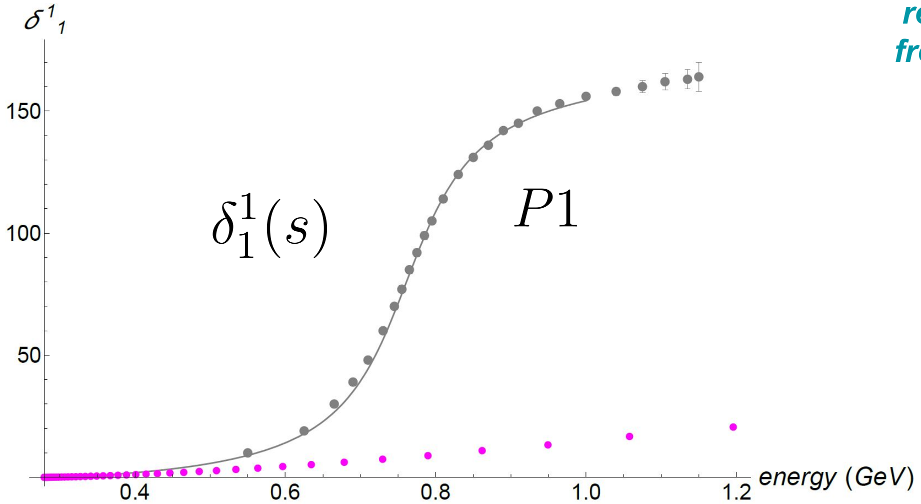
$f^1_1(s=3)$
$f^0_0(s=3)$
0.02
0.04
0.06
0.08
−0.001
−0.002
−0.003
−0.004
−0.005
$\epsilon^\chi=2\times10^{-3}$
check numerical amplitude:
phase shift in physical region
S-matrix bootstrap+chiSB



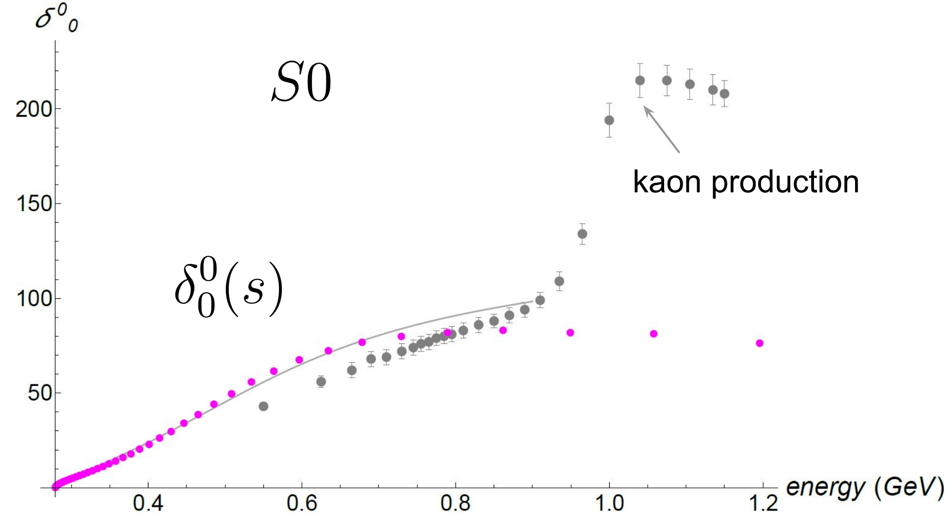


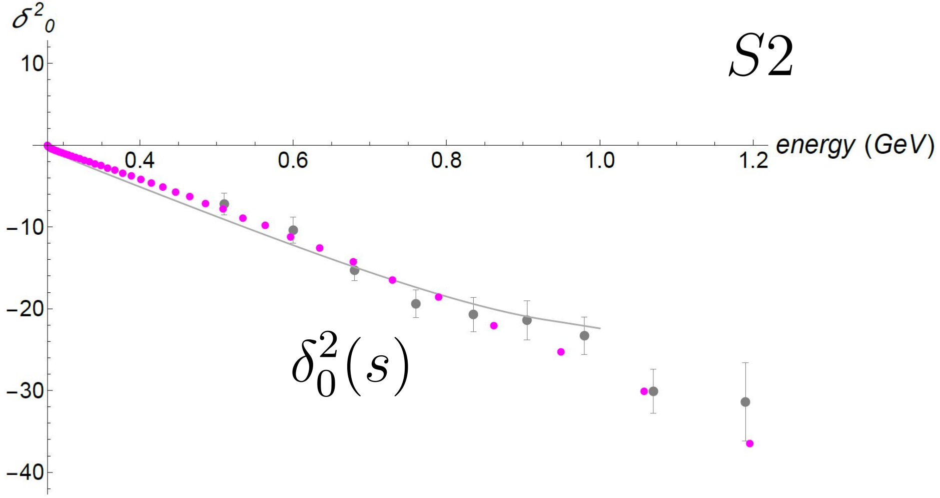
reasonable S0 S2 waves from weakly coupled EFT

phase shift with only chiSB (EFT) input



experimental data (gray dots)
[Protopopescu et al, 1973]
[Losty et al, 1974]
pheno fit (gray line)
[Pelaez, Yndurain, 2005]





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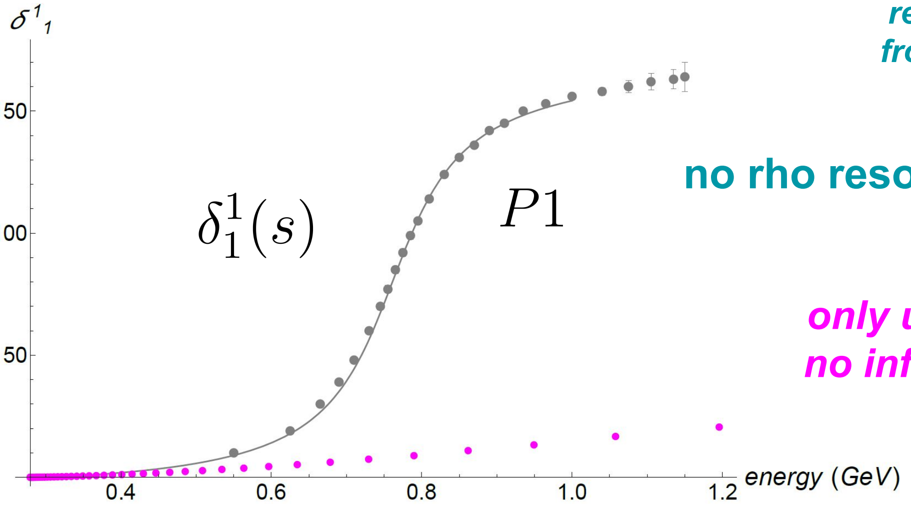

no rho resonance without UV info

makes sense
only used EFT input so far
no information on QCD yet

Gauge theory bootstrap

- **Pure S-matrix bootstrap:**

symmetry, analyticity, crossing, unitarity

$$SU(N_f)_V$$

+

- **Chiral symmetry breaking:**

general very low energy behavior

$$f_\pi \quad m_\pi$$

+

- **Form factor bootstrap + SVZ + asymptotics:**

gauge theory information

$$N_c \quad m_q \quad \Lambda_{\text{QCD}}$$

Form factor bootstrap

[Karateev, Kuhn, Penedones, 2019]

an important development: $|\psi_1\rangle = |p_1, p_2\rangle_{in}\,,\qquad |\psi_2\rangle = |p_1, p_2\rangle_{out}\,,\qquad |\psi_3\rangle = \int dx e^{-i(p_1+p_2)\cdot x}\mathcal{O}(x)|0\rangle$

state created by UV local operator

positive semidefinite matrix $$\langle\psi_a|\psi_b\rangle = \begin{pmatrix} 1 & S & \mathcal{F} \\ S^* & 1 & \mathcal{F}^* \\ \mathcal{F}^* & \mathcal{F} & \rho \end{pmatrix} \succeq 0$$

Form factor bootstrap

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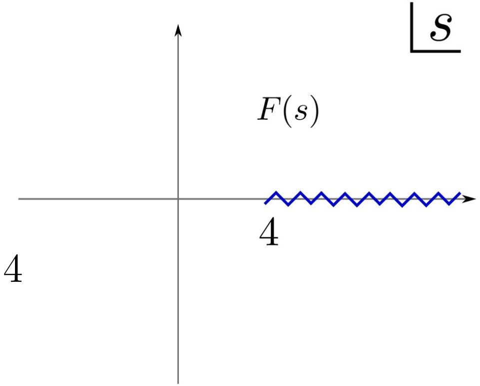
2-particle form factor: ${}_{\text{out}}\langle p_1, p_2|\mathcal{O}(0)|0\rangle = F(s)$ *analytic function of s*

$$F(s) = \frac{1}{\pi}\int_4^\infty dx \frac{\text{Im}F(x)}{x-s}$$ +subtractions

spectral density: $\int \frac{d^4x}{(2\pi)^4} e^{iPx}\langle 0|\mathcal{O}^\dagger(x)\mathcal{O}(0)|0\rangle = \rho(s)$ *supported at $s > 4$*



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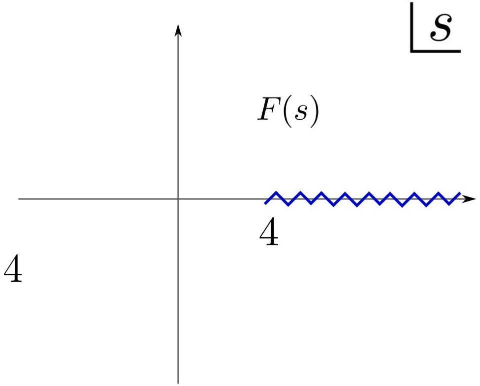


bootstrap variables: $\{\rho_{1,2}(x,y), \ldots, \mathrm{Im}F(x), \rho(x)\}$

allow connection with UV theory

Current correlators from the UV gauge theory

to connect with UV gauge theory

$$\begin{array}{c} \\ \langle \text{in}|_{P',I,\ell} \\ \langle \text{out}|_{P',I,\ell} \\ \langle 0|\mathcal{O}^{\dagger}_{P',I,\ell} \end{array} \begin{array}{c} \begin{array}{ccc} |\text{in}\rangle_{P,I,\ell} & |\text{out}\rangle_{P,I,\ell} & \mathcal{O}_{P,I,\ell}|0\rangle \end{array} \\ \begin{pmatrix} 1 & S^I_\ell(s) & \mathcal{F}^I_\ell \\ S^{I*}_\ell(s) & 1 & \mathcal{F}^{I*}_\ell \\ \mathcal{F}^{I*}_\ell & \mathcal{F}^I_\ell & \rho^I_\ell(s) \end{pmatrix} \succeq 0 \end{array} \qquad s > 4 \quad \forall \ell, I$$

construct operators from gauge theory with desired quantum numbers

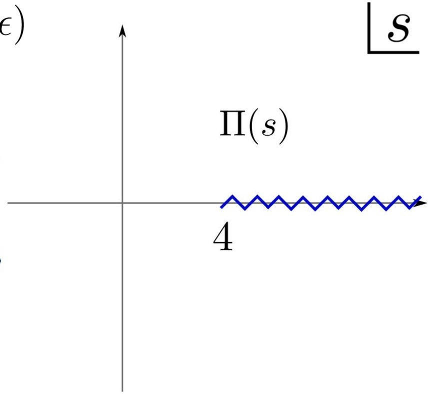
Current correlators from the UV gauge theory

to connect with UV gauge theory

$$\begin{array}{c|ccc} & |\mathrm{in}\rangle_{P,I,\ell} & |\mathrm{out}\rangle_{P,I,\ell} & \mathcal{O}_{P,I,\ell}|0\rangle \\ \hline \langle\mathrm{in}|_{P',I,\ell} & 1 & S_\ell^I(s) & \mathcal{F}_\ell^I \\ \langle\mathrm{out}|_{P',I,\ell} & S_\ell^{I*}(s) & 1 & \mathcal{F}_\ell^{I*} \\ \langle 0|\mathcal{O}^\dagger_{P',I,\ell} & \mathcal{F}_\ell^{I*} & \mathcal{F}_\ell^I & \rho_\ell^I(s) \end{array} \succeq 0 \qquad s > 4 \quad \forall \ell, I$$

construct operators from gauge theory with desired quantum numbers

$$\rho_\ell^I(s) = 2\,\mathrm{Im}\Pi_\ell^I(x + i\epsilon)$$



e.g.

$$S0 \quad : \quad j_S(x) = m_q(\bar{u}u + \bar{d}d) \qquad \Pi_0^0(s) = i\int \frac{d^4x}{(2\pi)^4} e^{iPx} \langle 0|\hat{T}\left\{j_S(x) j_S(0)\right\}|0\rangle$$

$$P1 \quad : \quad j_V^\mu(x) = \frac{1}{2}(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d) \qquad \Pi_1^1(s) = i\int \frac{d^4x}{(2\pi)^4} e^{iPx} \langle 0|\hat{T}\left\{j_\sigma^\dagger(x) j_\sigma(0)\right\}|0\rangle$$

$\ldots$

Current correlators from the UV gauge theory

to connect with UV gauge theory

$$
\begin{array}{c|ccc}
 & |\mathrm{in}\rangle_{P,I,\ell} & |\mathrm{out}\rangle_{P,I,\ell} & \mathcal{O}_{P,I,\ell}|0\rangle \\
\hline
\langle \mathrm{in}|_{P',I,\ell} & 1 & S_\ell^I(s) & \mathcal{F}_\ell^I \\
\langle \mathrm{out}|_{P',I,\ell} & S_\ell^{I*}(s) & 1 & \mathcal{F}_\ell^{I*} \\
\langle 0|\mathcal{O}^\dagger_{P',I,\ell} & \mathcal{F}_\ell^{I*} & \mathcal{F}_\ell^I & \rho_\ell^I(s)
\end{array} \succeq 0 \qquad s > 4 \quad \forall \ell, I
$$

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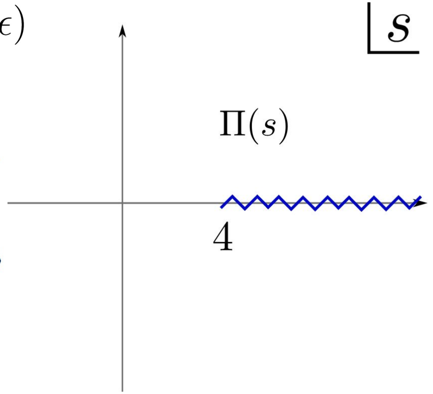

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$\cdots$

large spacelike momenta — asymptotic free region with pQCD computation

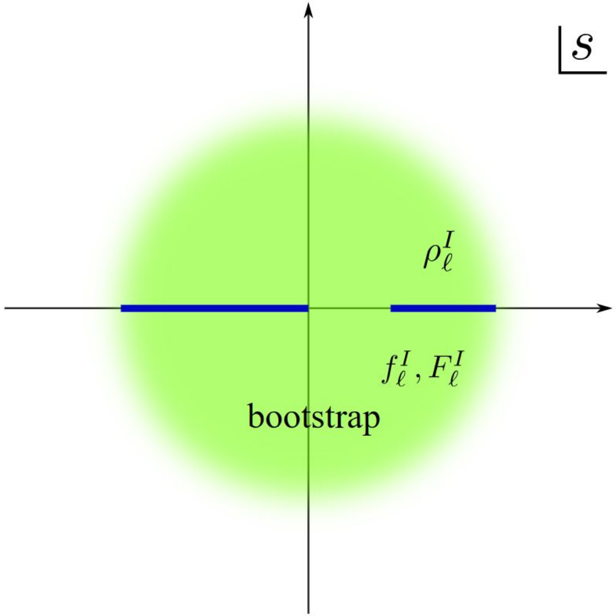
Positive semidefinite matrix – saturation

positive semidefinite $\begin{pmatrix} 1 & S & \mathcal{F} \\ S^* & 1 & \mathcal{F}^* \\ \mathcal{F}^* & \mathcal{F} & \rho \end{pmatrix} \succeq 0 \qquad \forall I,\ \ell,\ s$

iff all its principal minors are non-negative

$$\rho + S^*\mathcal{F}^2 + S(\mathcal{F}^*)^2 - 2|\mathcal{F}|^2 - \rho|S|^2 \geq 0$$

$$\rho \geq 0 \qquad |\mathcal{F}|^2 \leq \rho \qquad |S|^2 \leq 1$$



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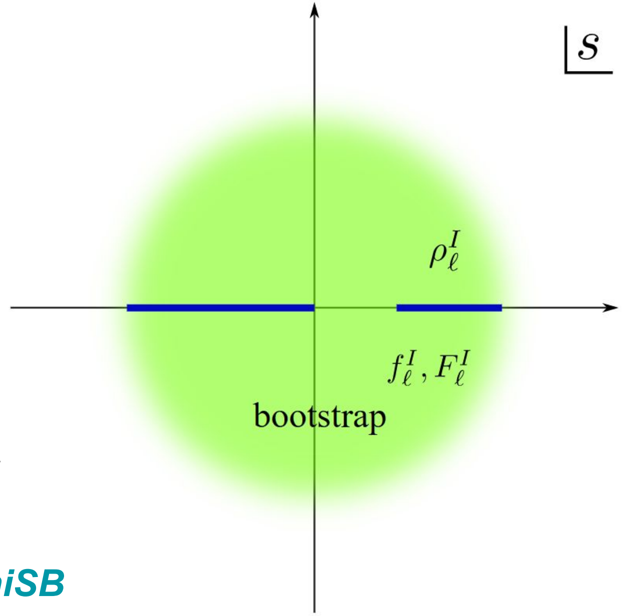
$$\rho \geq 0 \qquad |\mathcal{F}|^2 \leq \rho \qquad |S|^2 \leq 1$$

saturation: $\rho = |\mathcal{F}|^2$

$$|S| = 1 \quad S = \frac{\mathcal{F}}{\mathcal{F}^*}$$

Watson / Muskhelishvili-Omnes



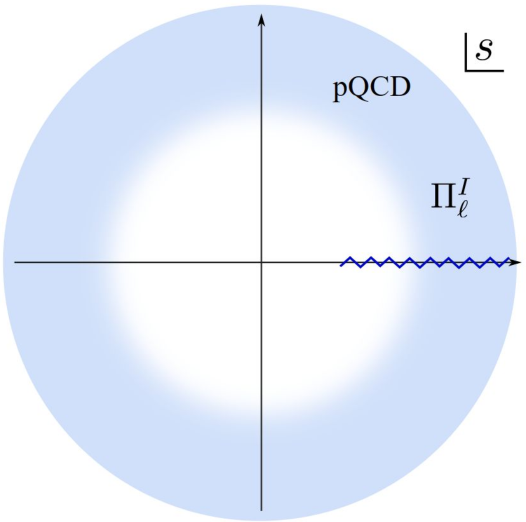
saturation connects quantities controlled by pQCD and chiSB

SVZ expansion

[Shifman, Vainshtein, Zakharov, 1979]

OPE: $$T\{j(x)j(0)\} = C_{\mathbb{1}}(x)\ \mathbb{1} + \sum_{\mathcal{O}} C_{\mathcal{O}}(x)\ \mathcal{O}(0)$$

$$\langle 0|T\{j(x)j(0)\}|0\rangle = C_{\mathbb{1}}(x) + C_{\bar{q}q}(x)\ \langle 0|j_S(0)|0\rangle + C_{G^2}(x)\ \langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\,\mu\nu}|0\rangle + \dots$$



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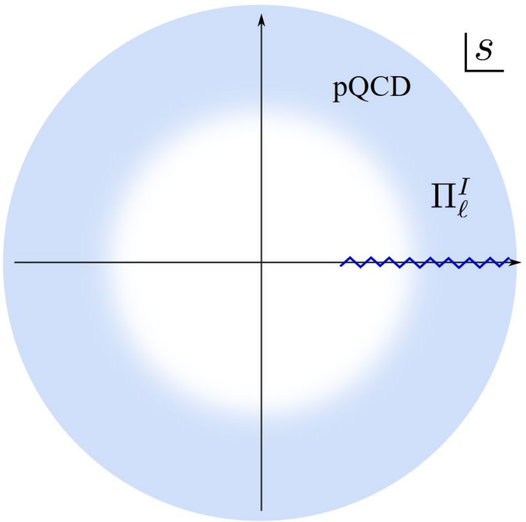
SB vacuum

quark condensate

gluon condensate

pQCD



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SB vacuum

quark condensate

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pQCD

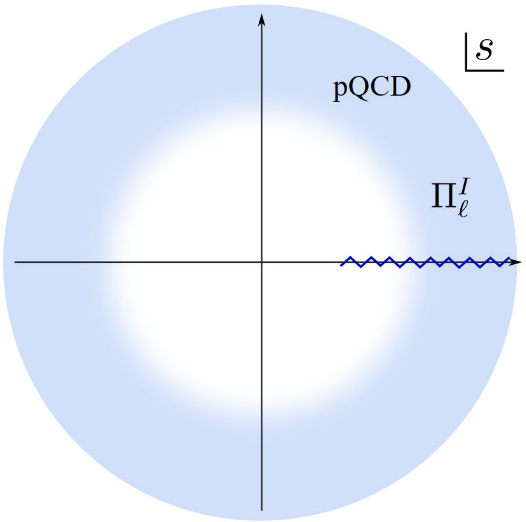
Fourier transform

$$\Pi^0_0(s) = \frac{N_f m_q^2}{(2\pi)^4}\left\{-\frac{3}{8\pi^2}\left(1+\frac{13}{3}\frac{\alpha_s}{\pi}\right)s\ln(-\frac{s}{\mu^2}) - \frac{1}{8s}\langle\frac{\alpha_s}{\pi}G^2\rangle - \frac{3}{2s}\langle j_S\rangle + \ldots\right\}$$

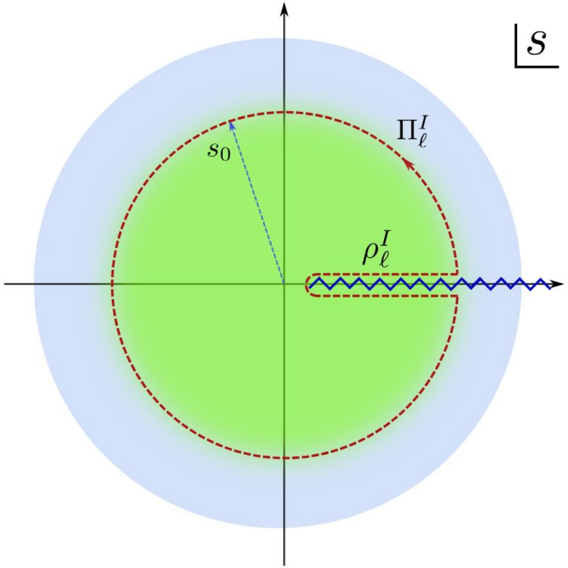
$$\Pi^1_1(s) = \frac{1}{2}\frac{1}{(2\pi)^4}\left\{-\frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)s\ln(-\frac{s}{\mu^2}) + \frac{1}{12s}\langle\frac{\alpha_s}{\pi}G^2\rangle + \frac{1}{s}\langle j_S\rangle + \ldots\right\}$$

$\vdots$

$N_c = 3$



Finite energy sum rule



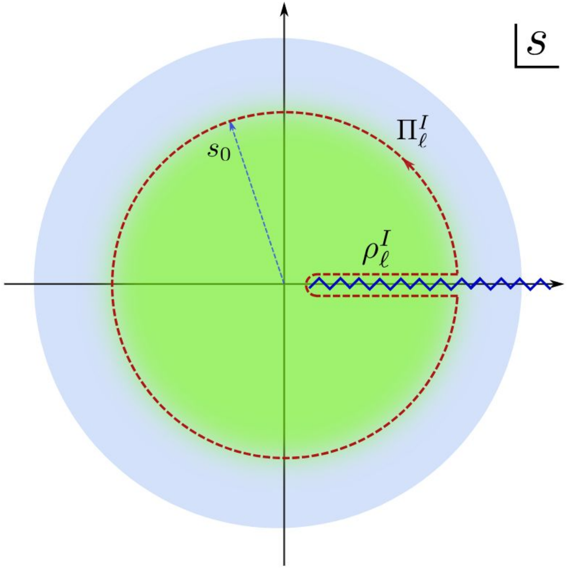
connect pQCD with bootstrap at s0

contour integral $s^n\Pi(s)$ vanishes

SVZ

$$\int_4^{s_0} \rho(x) x^n dx = -s_0^{n+1} \int_0^{2\pi} e^{i(n+1)\varphi}\Pi(s_0 e^{i\varphi}) d\varphi$$

Finite energy sum rule



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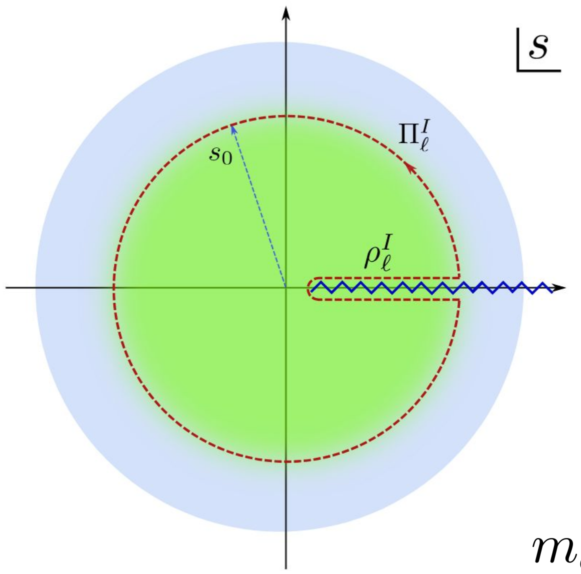
bootstrap variables ← linear constraints **gauge theory information**

Finite energy sum rule



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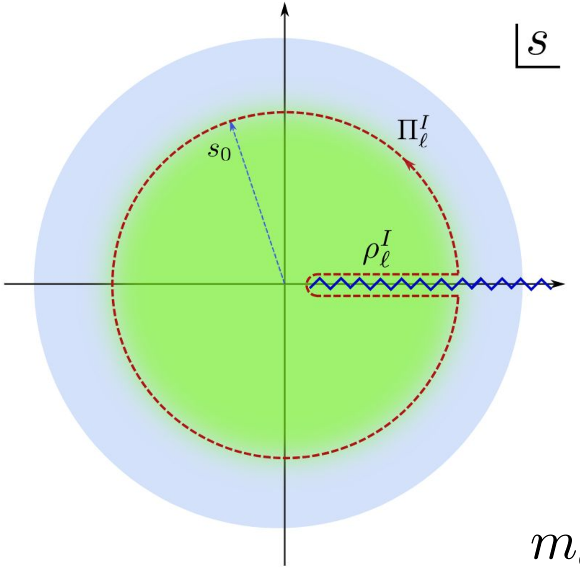
bootstrap variables ← linear constraints — **gauge theory information**

m_q $\quad N_c \quad \Lambda_{\rm QCD}$

$$S0 \quad : \int_4^{s_0}\rho_0^0(x)x^n dx = \frac{s_0^{n+1}N_f m_q^2}{(2\pi)^4}\left\{\frac{3s_0}{4\pi(n+2)}\left(1+\frac{13}{3}\frac{\alpha_s}{\pi}\right)+\delta_n\frac{\pi}{4s_0}\langle\frac{\alpha_s}{\pi}G^2\rangle+\delta_n\frac{3\pi}{s_0}\langle j_S\rangle+\ldots\right\},\quad n\geq 0$$

$$P1 \quad : \int_4^{s_0}\rho_1^1(x)x^n dx = -\frac{s_0^{n+1}}{(2\pi)^4}\frac{1}{2}\left\{-\frac{s_0}{2\pi(n+2)}\left(1+\frac{\alpha_s}{\pi}\right)+\delta_n\frac{\pi}{6s_0}\langle\frac{\alpha_s}{\pi}G^2\rangle+\delta_n\frac{2\pi}{s_0}\langle j_S\rangle+\ldots\right\},\quad n\geq -1$$

Finite energy sum rule



connect pQCD with bootstrap at s0

contour integral $s^n\Pi(s)$ vanishes

SVZ

$$\int_4^{s_0}\rho(x)x^n dx = -s_0^{n+1}\int_0^{2\pi} e^{i(n+1)\varphi}\Pi(s_0 e^{i\varphi})d\varphi$$

bootstrap variables ← linear constraints — **gauge theory information**

m_q N_c $\Lambda_{\rm QCD}$

extra IR parameters suppressed at large s

bootstrap?!?

$$S0\ :\int_4^{s_0}\rho_0^0(x)x^n dx = \frac{s_0^{n+1}N_f m_q^2}{(2\pi)^4}\left\{\frac{3s_0}{4\pi(n+2)}\left(1+\frac{13}{3}\frac{\alpha_s}{\pi}\right)+\delta_n\frac{\pi}{4s_0}\langle\frac{\alpha_s}{\pi}G^2\rangle+\delta_n\frac{3\pi}{s_0}\langle j_S\rangle+\dots\right\},\quad n\geq 0$$

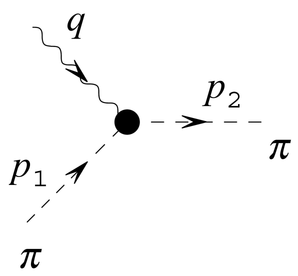
$$P1\ :\int_4^{s_0}\rho_1^1(x)x^n dx = -\frac{s_0^{n+1}}{(2\pi)^4}\frac{1}{2}\left\{-\frac{s_0}{2\pi(n+2)}\left(1+\frac{\alpha_s}{\pi}\right)+\delta_n\frac{\pi}{6s_0}\langle\frac{\alpha_s}{\pi}G^2\rangle+\delta_n\frac{2\pi}{s_0}\langle j_S\rangle+\dots\right\},\quad n\geq -1$$

Asymptotic behavior of form factor from pQCD

perturbative QCD also controls asymptotic behavior of form factors

e.g. electromagnetic FF $\langle \pi(p_2) | J^\mu_{\mathrm{em}}(0) | \pi(p_1) \rangle = (p_1^\mu + p_2^\mu) F_\pi(q^2)$

$$q = (p_2 - p_1)$$



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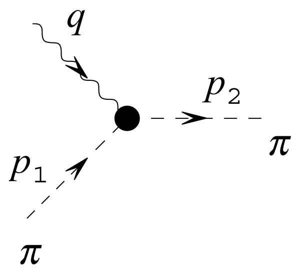

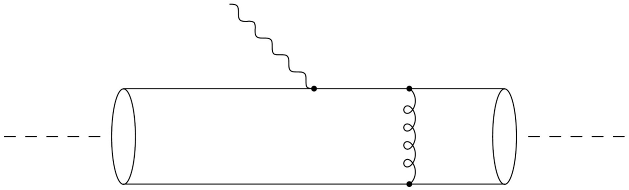

$$F_{\pi}(s) \simeq -\frac{16\pi\alpha_s(s)f_{\pi}^2}{s}$$

[Peter Lepage, Brodsky, 1979]

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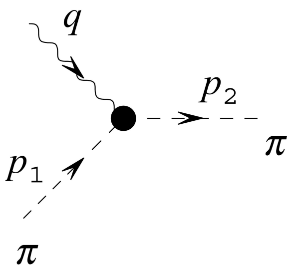

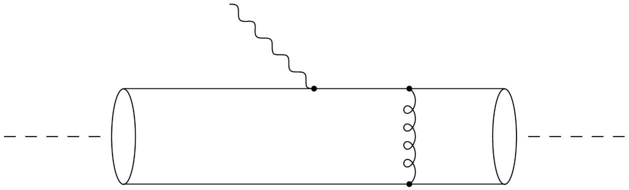

$$F_\pi(s) \simeq -\frac{16\pi\alpha_s(s)f_\pi^2}{s}$$

[Peter Lepage, Brodsky, 1979]

evaluate

in practical numerical implementation
suffices to require smallness above $s = s_0$

$$||\mathcal{F}(s > s_0)|| \lesssim \epsilon$$

Gauge theory parameters: numerical input

$N_f = 2 \qquad N_c = 3$ *for comparison with experiments*

$$s_0 = (1.2\,\mathrm{GeV})^2, \quad \alpha_s \simeq 0.41, \quad m_u \simeq 4\,\mathrm{MeV} \quad m_d \simeq 7.3\,\mathrm{MeV}$$

more recently (to appear):

$$s_0 = (2\,\mathrm{GeV})^2, \quad \alpha_s \simeq 0.31, \quad m_u \simeq 3.6\,\mathrm{MeV} \quad m_d \simeq 6.5\,\mathrm{MeV}$$

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FESR

$$\frac{1}{s_0^{n+2}} \int_4^{s_0} \rho_0^0(x) x^n dx \simeq \frac{6.23 \times 10^{-7}}{n+2}$$

$$\frac{1}{s_0^{n+2}} \int_4^{s_0} \rho_1^1(x) x^n dx \simeq \frac{5.62 \times 10^{-5}}{n+2}$$

$$\frac{1}{s_0^{n+3}} \int_4^{s_0} \rho_2^0(x) x^n dx \simeq \frac{5.13 \times 10^{-5}}{n+3}$$

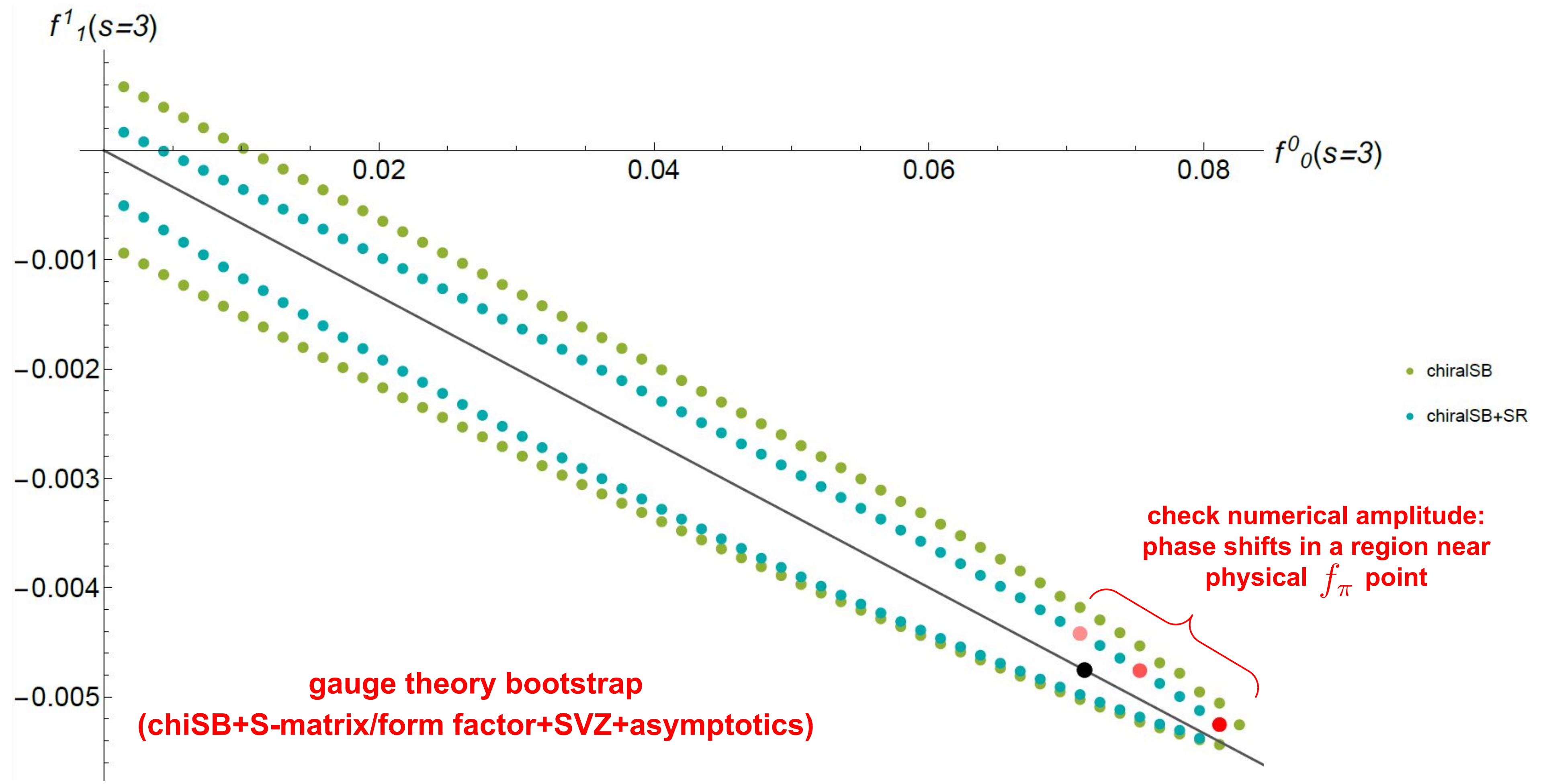
FF asymptotics

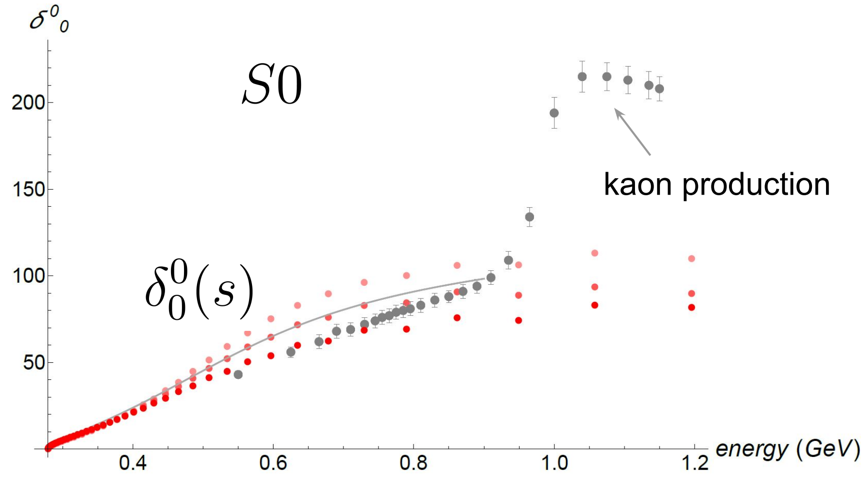
$$||\mathcal{F}_0^0(s > s_0)|| \lesssim 1.7 \times 10^{-8}$$

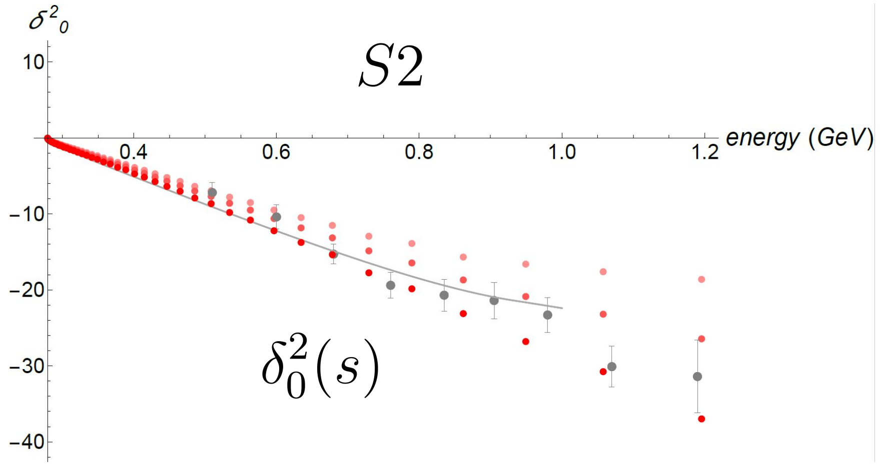
$$||\mathcal{F}_1^1(s > s_0)|| \lesssim 1.7 \times 10^{-6}$$

$$||\mathcal{F}_2^0(s > s_0)|| \lesssim 0.03$$

...

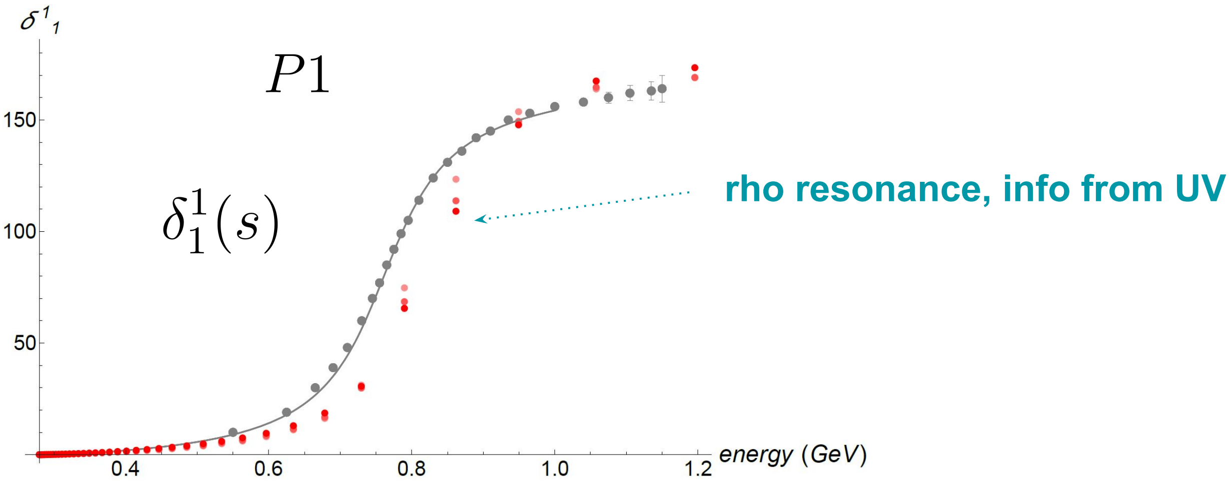

$f^1_1(s=3)$
$f^0_0(s=3)$
0.02
0.04
0.06
0.08
−0.001
−0.002
−0.003
−0.004
−0.005
chiralSB
chiralSB+SR
check numerical amplitude:
phase shifts in a region near
physical f_π point
gauge theory bootstrap
(chiSB+S-matrix/form factor+SVZ+asymptotics)





gauge theory bootstrap

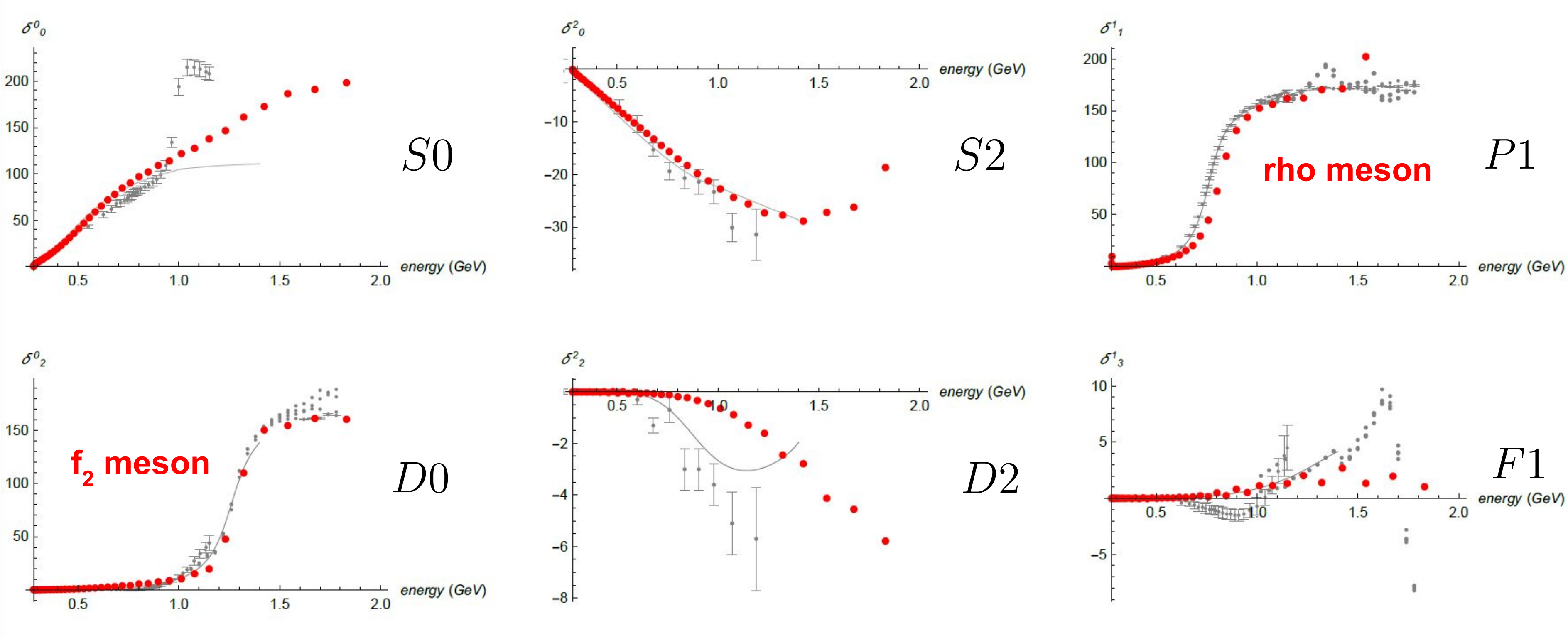
experimental data (gray dots)
[Protopopescu et al, 1973]
[Losty et al, 1974]
pheno fit (gray line)
[Pelaez, Yndurain, 2005]



gauge theory bootstrap

phase shifts up to 2GeV

[YH, Kruczenski, *to appear*]



experiments (gray dots) [Protopopescu et al, 1973][Losty et al, 1974][Hyams et al, 1975] scattering lengths and effective range parameters

$$\mathrm{Re} f_\ell^I(s) \overset{k\to 0}{\simeq} \frac{2m_\pi}{\pi} k^{2\ell}\left(a_\ell^I + b_\ell^I k^2 + \dots\right) \qquad k = \frac{\sqrt{s-4m_\pi^2}}{2}$$

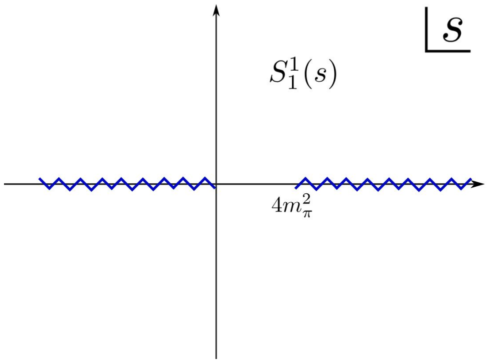
	DFGS	ACGL	CGL	PY	**gauge theory bootstrap**		
$a_0^{(0)}$	0.228 ± 0.012	0.240 ± 0.060	0.220 ± 0.005	0.230 ± 0.010	0.178	0.188	0.201
$a_0^{(2)}$	-0.0382 ± 0.0038	-0.036 ± 0.013	-0.0444 ± 0.0010	-0.0422 ± 0.0022	-0.0362	-0.0388	-0.0425
$b_0^{(0)}$		0.276 ± 0.006	0.280 ± 0.001	0.268 ± 0.010	0.31	0.307	0.297
$b_0^{(2)}$		-0.076 ± 0.002	-0.080 ± 0.001	-0.071 ± 0.004	-0.0629	-0.0681	-0.075

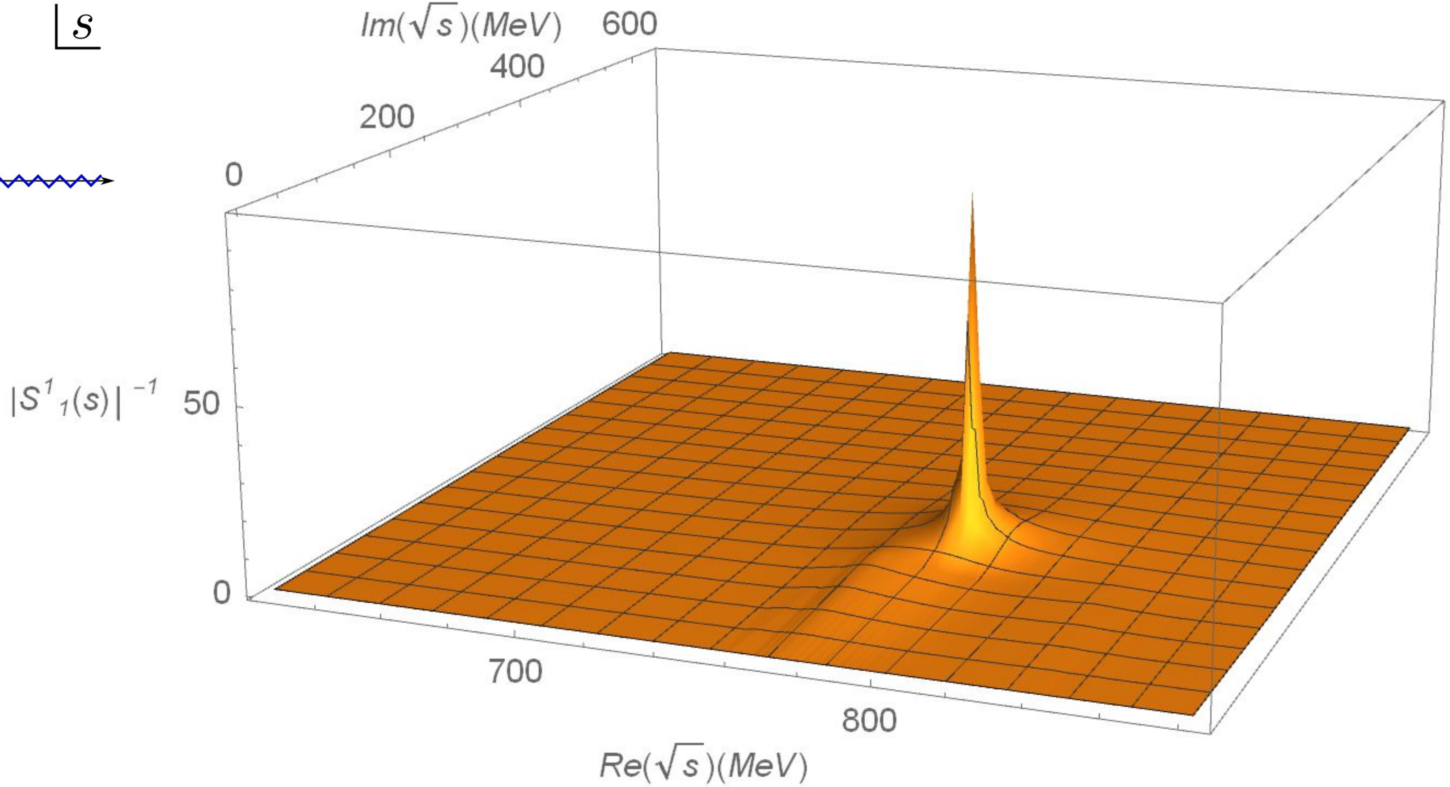
	Nagel	PSGY	CGL	PY			
a_1	38 ± 2	38.5 ± 0.6	37.0 ± 0.13 $[37.9 \pm 0.5]$ [a]	38.1 ± 1.4 $[38.6 \pm 1.2]$ [b] $\times 10^{-3}$	0.0281	0.0304	0.0343

[Nagel et al, 1979][Descotes et al, 2002][Ananthanarayan et al, 2001]
[Colangelo, Gasser, Leutwyler, 2001][Pelaez, Yndurain, 2003]

[YH, Kruczenski, *to appear*]

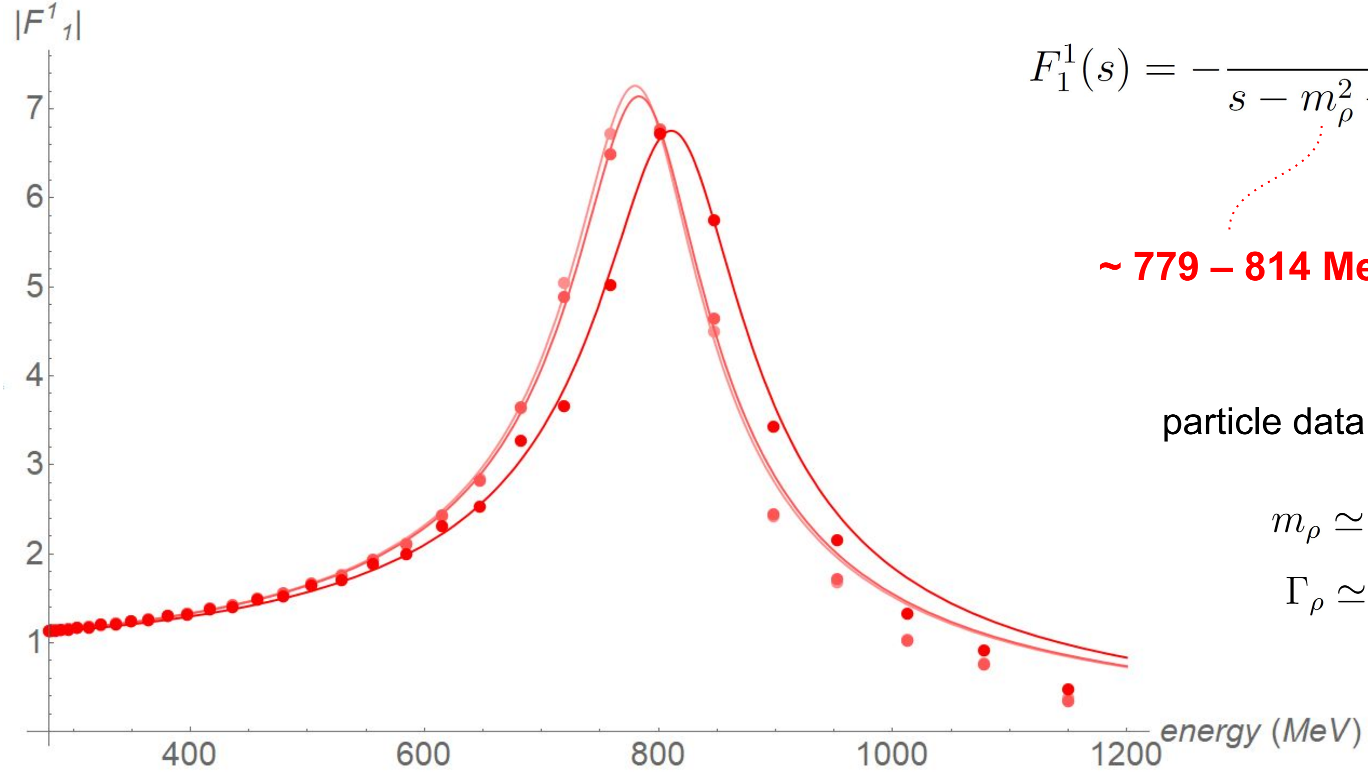
rho meson as pole on the second sheet of $S_1^1(s)$





[YH, Kruczenski, *to appear*]

fit P1 form factor with Breit-Wigner form



$$F_1^1(s) = -\frac{m_\rho^2}{s - m_\rho^2 + i m_\rho \Gamma_\rho \theta(s - 4m_\pi^2)}$$

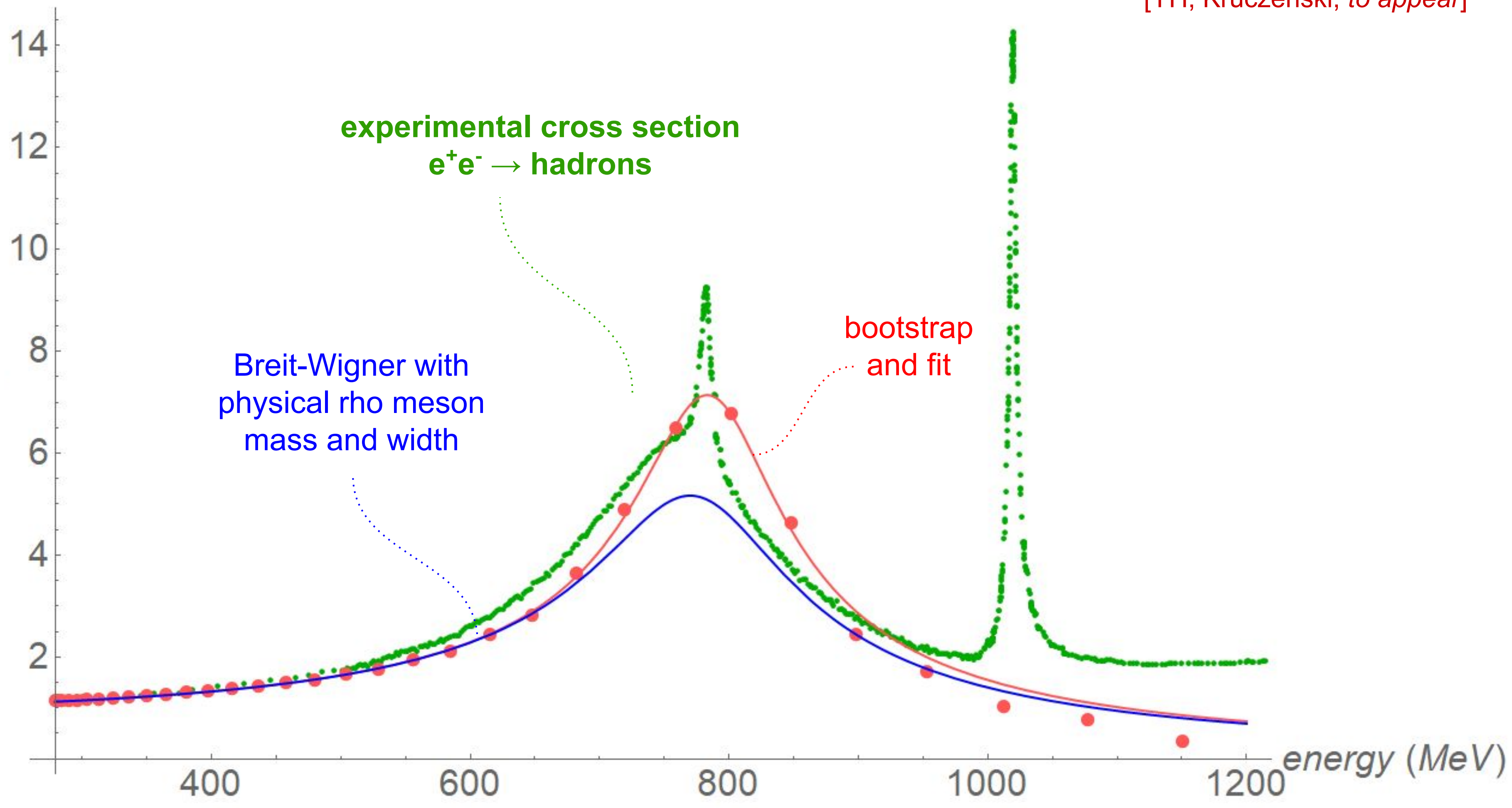
~ 779 – 814 MeV **~ 107 – 128 MeV**

particle data group (T-matrix pole):

$$m_\rho \simeq 761 - 765 \text{ MeV}$$

$$\Gamma_\rho \simeq 142 - 148 \text{ MeV}$$

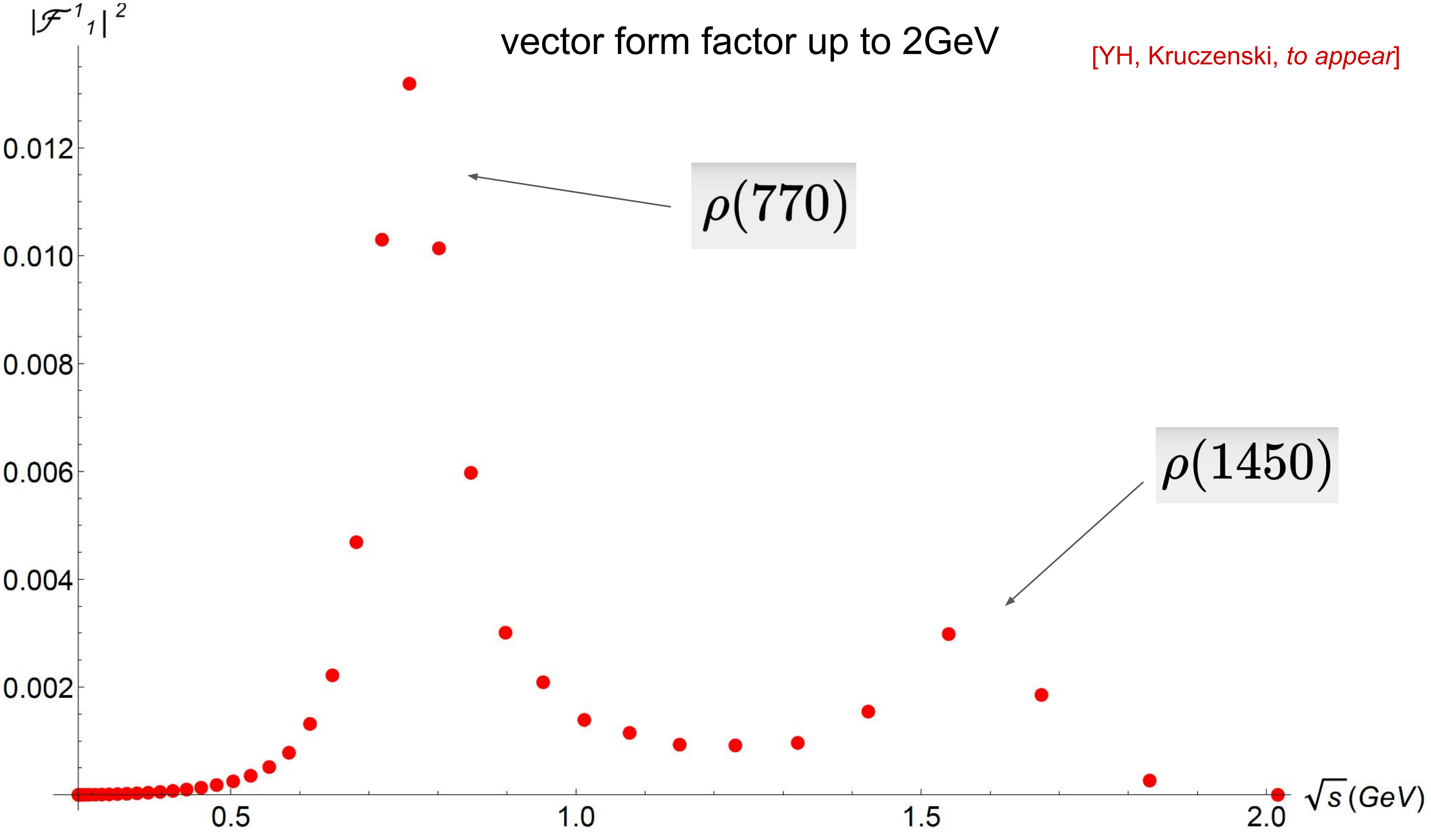
[YH, Kruczenski, *to appear*]



[YH, Kruczenski, *to appear*]
experimental cross section
$e^+e^- \to$ hadrons
bootstrap
and fit
Breit-Wigner with
physical rho meson
mass and width
14
12
10
8
6
4
2
400
600
800
1000
1200
energy (MeV)

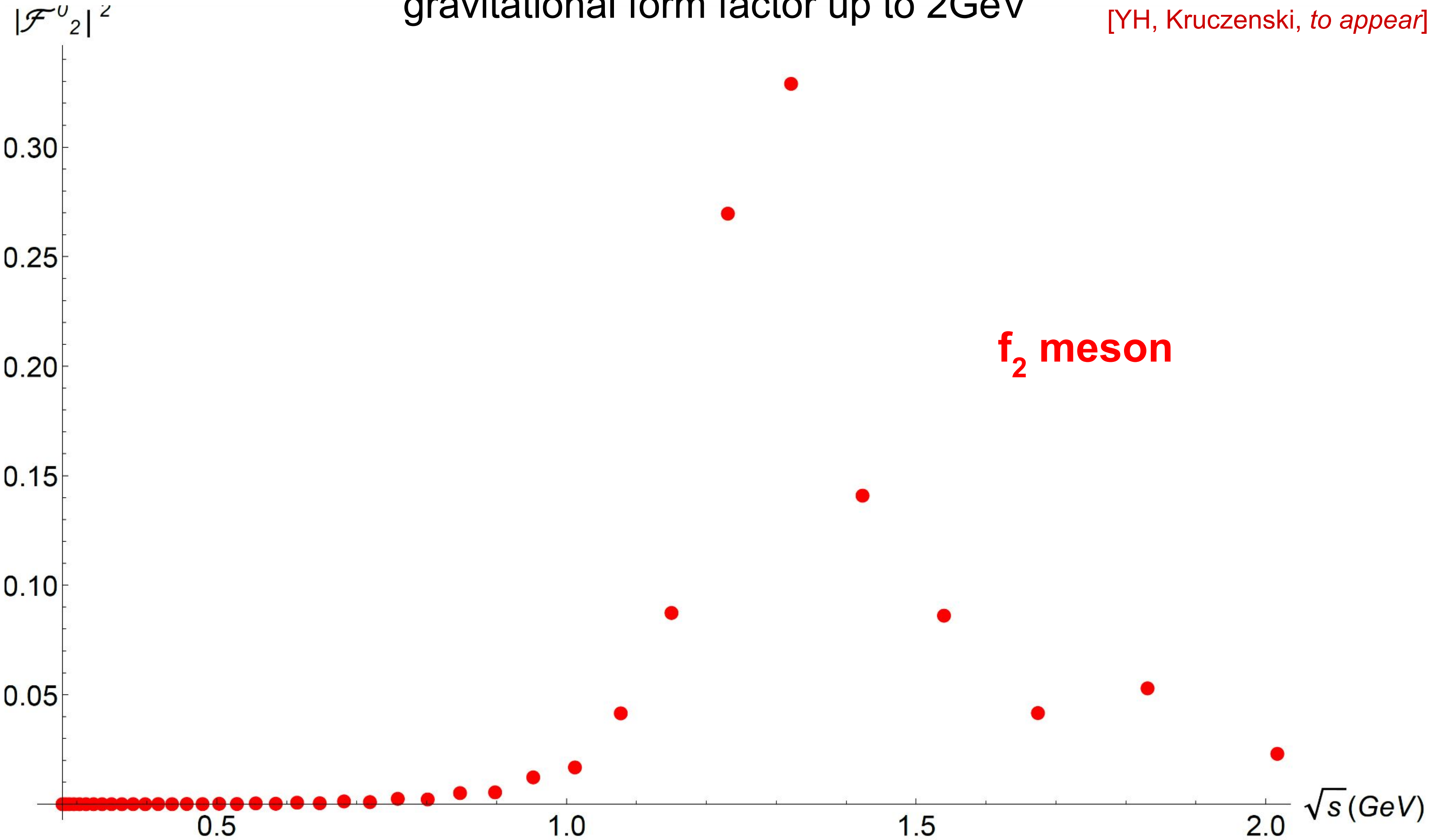
vector form factor up to 2GeV

[YH, Kruczenski, *to appear*]

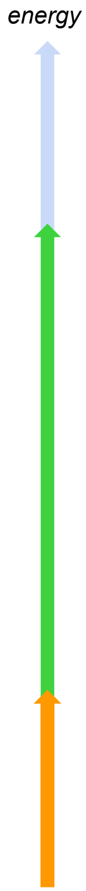


gravitational form factor up to 2GeV

[YH, Kruczenski, *to appear*]



How the gauge theory bootstrap works



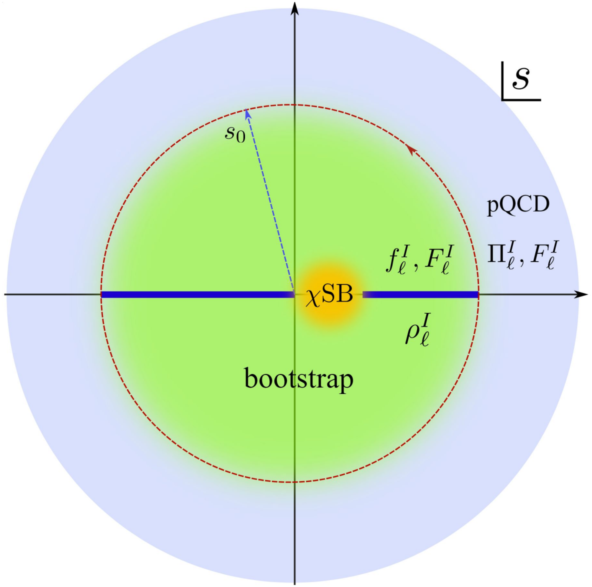
gauge theory info

$$\int_4^{s_0} \rho(x) x^n dx = -s_0^{n+1} \int_0^{2\pi} e^{i(n+1)\varphi} \Pi(s_0 e^{i\varphi}) d\varphi \quad F_\pi(s) \simeq -\frac{16\pi\alpha_s(s) f_\pi^2}{s} \; , \; \ldots$$

pQCD

very low energy behavior $$A(s,t,u) \simeq \frac{4}{\pi} \frac{s - m_\pi^2}{32\pi f_\pi^2}$$

chiSB



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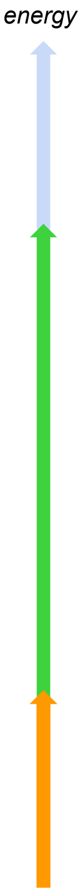

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pQCD

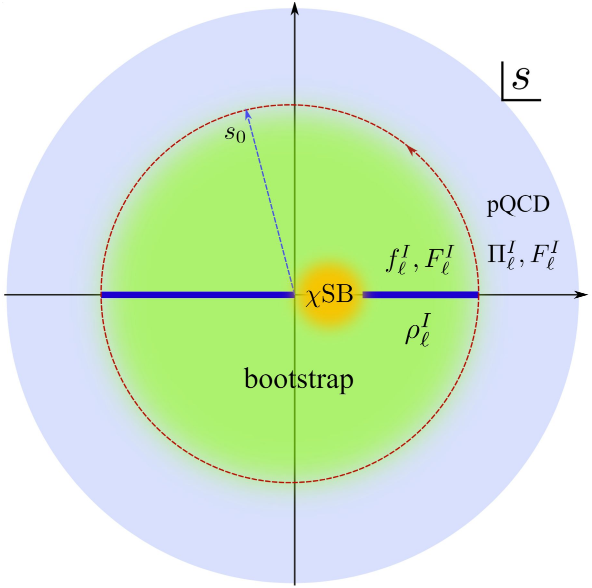
$$\rho(s) = |\mathcal{F}(s)|^2$$

dispersion relation

$$\mathcal{F}(s) = \sqrt{\rho(s)}\,e^{i\alpha(s)} \qquad \ln \mathcal{F}(s) = \frac{1}{2}\ln \rho(s) + i\alpha(s)$$

$$|S(s)| = 1 \qquad e^{2i\alpha(s)} = \frac{\mathcal{F}(s)}{\mathcal{F}^*(s)} = S(s) = e^{2i\delta(s)}$$

very low energy behavior $\quad A(s,t,u) \simeq \frac{4}{\pi}\frac{s-m_\pi^2}{32\pi f_\pi^2}$ **chiSB**



How the gauge theory bootstrap works

energy

gauge theory info

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pQCD

gauge theory bootstrap

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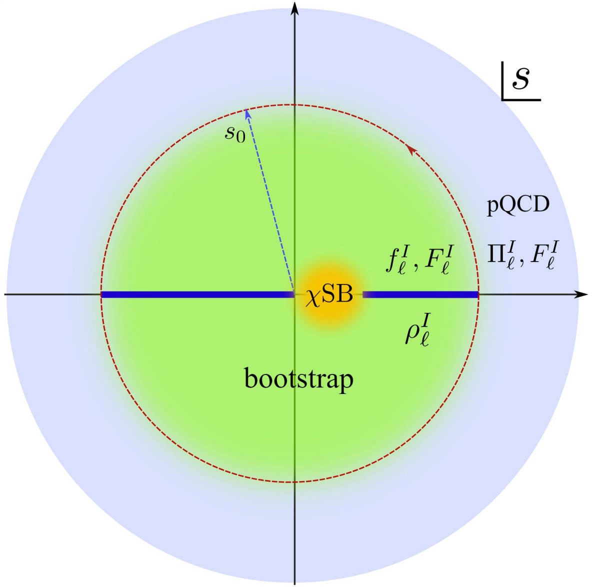
chiSB



Conclusions

- Gauge theory bootstrap:

using only $\underbrace{N_c \; N_f \; m_q \; \Lambda_{\mathrm{QCD}}}_{\text{gauge theory parameters}}$ $\underbrace{f_\pi \quad m_\pi}_{\text{universal low energy parameters}}$

strongly coupled low energy physics of asymptotically free gauge theories

Conclusions

- Gauge theory bootstrap:

using only $\underbrace{N_c\ N_f\ m_q\ \Lambda_{\rm QCD}}_{\textit{gauge theory parameters}}$ $\underbrace{f_\pi\ \ m_\pi}_{\textit{universal low energy parameters}}$

strongly coupled low energy physics of asymptotically free gauge theories

- Numerical test with $N_f = 2\ \ N_c = 3$ find good agreement with experiments

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strongly coupled low energy physics of asymptotically free gauge theories

- Numerical test with $N_f = 2 \;\; N_c = 3$ find good agreement with experiments

- Results suggest: we are on the right track for solving QCD (gauge theories)

Further developments of the framework

Thank you!