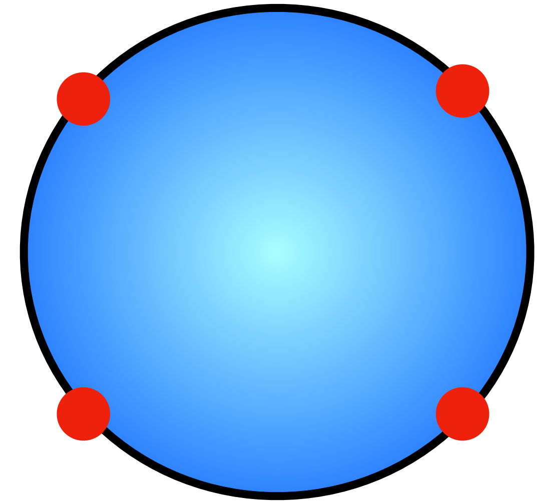
Taming Mass Gap with Anti-de Sitter Space

Lorenzo Di Pietro
(Università di Trieste)

18/02/2024, Pisa

50+ε Years of Conformal Bootstrap

QFT in AdS



- IR regulator with symmetries

- Radius L : probe of the theory at different scales

- Infinite volume: symmetry breaking, phase transitions possible (unlike on sphere)

- Asymptotic observables: correlation functions in nonlocal CFT

Setup: **<u>Dimensional Transmutation in AdS</u>**

- UV classical theory: massless degrees of freedom, weakly interacting $g \ll 1$;
- IR: mass gap $\Lambda \sim \mu e^{-\frac{1}{\beta_0 g^2(\mu)}}$;

Setup: **Dimensional Transmutation in AdS**

- UV classical theory: massless degrees of freedom, weakly interacting $g \ll 1$;
- IR: mass gap $\Lambda \sim \mu e^{-\frac{1}{\beta_0 g^2(\mu)}}$;
- Varying the AdS radius L we can interpolate;
- In some cases, a sharp transition between the two regimes must occur, detectable from the bCFT.

Opportunity: studying the bCFT as a way to detect/prove the mass gap in the bulk

Outline:

- Brief review of the example of Yang-Mills in AdS_4 [Aharony, Berkooz, Tong, Yankielowicz]
- [Copetti, DP, Ji, Komatsu]: we study 2d examples

→ O(N) sigma model

→ O(N) Gross-Neveu model

Additional computational handle: **Large N**

Outline:

- Brief review of the example of Yang-Mills in AdS_4 [Aharony, Berkooz, Tong, Yankielowicz]
- [Copetti, DP, Ji, Komatsu]: we study 2d examples

→ O(N) sigma model

→ O(N) Gross-Neveu model

Additional computational handle: **Large N**

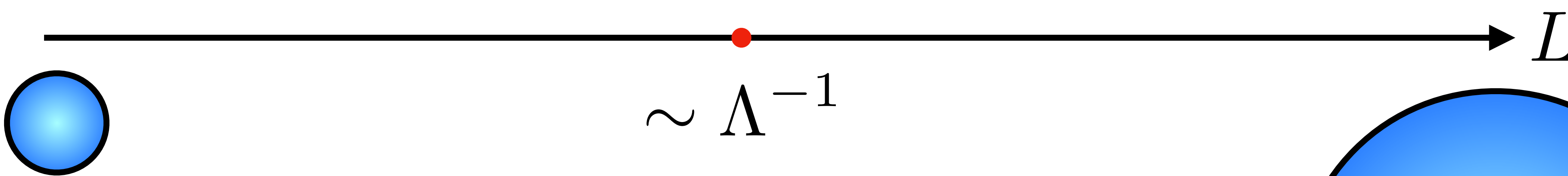
One more 50th birthday to celebrate:

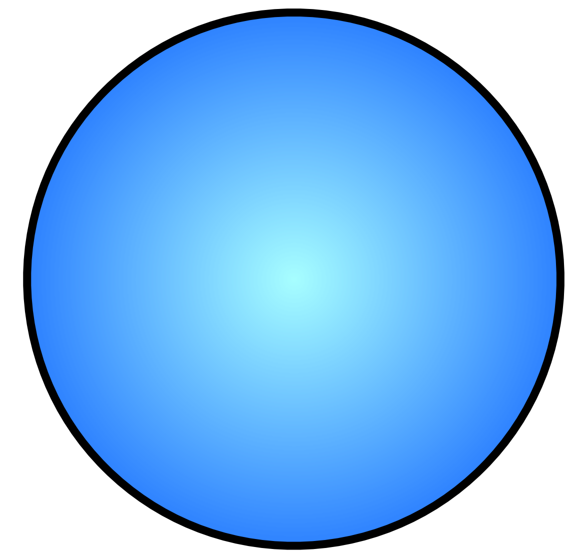
- ’t Hooft: large N gauge theory and mesons in QCD_2
- Coleman, Jackiw, Politzer: O(N) model at large N
- Gross, Neveu

all came out in 1974

Yang-Mills in AdS$_4$

[Aharony, Berkooz, Tong, Yankielowicz]





$$ds^2_{\text{AdS}} = L^2 \frac{dz^2 + d\vec{x}^2}{z^2}$$

- $L \ll \Lambda^{-1}$: with **Dirichlet** boundary condition

$$A^a_\mu \underset{z \to 0}{\sim} z J^a_\mu + \dots$$

➡ bulk gauge symmetry = boundary global symmetry

- $L \gg \Lambda^{-1}$: no coloured asymptotic states, mass gap

➡ $\Delta \sim L\Lambda \gg 1 \,,$ ~~J^a_μ~~

Why a sharp transition?

$L \ll \Lambda^{-1}$: small perturbation of the free limit.

bCFT: Mean-field theory of J^a_μ + small corrections

→ No candidate scalar operator that can continuously recombine:

$$\partial^\mu J^a_\mu = \mathcal{O}^a$$

→ scalar, adjoint, $\Delta = 3$

→ J^a_μ must exist for a finite range of L

Natural expectation: $L_{\text{crit}} \sim \Lambda^{-1}$

Possibilities for the transition:

Possibilities for the transition:

(1) **Higgs:** a bulk charged scalar condenses

bCFT: an adjoint scalar operator hits $\Delta = 3$ and recombines $\partial^\mu J^a_\mu = \mathcal{O}^a$

Possibilities for the transition:

(1) **Higgs:** a bulk charged scalar condenses

bCFT: an adjoint scalar operator hits $\Delta = 3$ and recombines $\partial^\mu J^a_\mu = \mathcal{O}^a$

(2) **Decoupling:** gluon states become null and are removed from the spectrum

bCFT: $\langle J_\mu(x) J_\nu(0) \rangle = C_J \frac{\delta_{\mu\nu} - 2\frac{x_\mu x_\nu}{x^2}}{(x^2)^2} \;, \quad C_J \underset{L \to L_{\rm crit}}{\longrightarrow} 0$

Possibilities for the transition:

(1) **Higgs:** a bulk charged scalar condenses

bCFT: an adjoint scalar operator hits $\Delta = 3$ and recombines $\partial^\mu J^a_\mu = \mathcal{O}^a$

(2) **Decoupling:** gluon states become null and are removed from the spectrum

bCFT: $\langle J_\mu(x) J_\nu(0) \rangle = C_J \frac{\delta_{\mu\nu} - 2\frac{x_\mu x_\nu}{x^2}}{(x^2)^2}$, $C_J \underset{L \to L_{\rm crit}}{\longrightarrow} 0$

(3) **Tachyon:** instability towards generation of a VEV, a weakly-coupled scalar ϕ with $M^2 < M^2_{\rm BF}$

bCFT: $\Delta(\mathcal{O}_\phi) = \frac{3}{2}$, two bCFTs related by $\mathcal{O}_\phi^2$ merge

We propose a generalization of (3), without need of weak coupling, which is realized in 2d toy models:

(4) **Marginality:**

bCFT: a singlet scalar operator $\mathcal{O}$ hits $\Delta = 3$

The bulk YM coupling induces running of the associated marginal coupling:

$$\int_{\mathrm{AdS}_4} \frac{1}{2g^2}\,\mathrm{tr}[F^2] + \int_{\partial} y\,\mathcal{O} \;\; , \quad \beta_y \sim A\left(\frac{1}{g^2} - \frac{1}{g^2_{\mathrm{crit}}}\right) - B\,y^2$$

[Antunes, Lauria, van Rees]

Merger and annihilation for $g^2 > g^2_{\mathrm{crit}}$

[Gorbenko, Rychkov, Zan]

A **bootstrap approach** to the mass gap in YM?

Family of (non-local) CFT_3s with G_{YM} global symmetry

$$\langle J_\mu(x) J_\nu(0) \rangle = C_J \frac{\delta_{\mu\nu} - 2\frac{x_\mu x_\nu}{x^2}}{(x^2)^2} \,, \qquad C_J \sim \frac{1}{g^2(L)}$$

$C_J \to \infty$: approaches mean-field theory of J^a_μ

Known perturbative data: $1/C_J$ corrections to the double-trace data [Caron-Huot, Li]

Can it be found studying $\langle JJJJ \rangle$?

[He, Rong, Su, Vichi]

AdS$_4$: deconfinement-confinement transition

Analogous phenomenon in AdS$_2$?

AdS$_4$: deconfinement-confinement transition

Analogous phenomenon in AdS$_2$?

“**Goldstone-gapped transition**”:

- Bulk theory with continuous global symmetry, $\nabla^M J^a_M = 0$
- Small L: bc that breaks the symmetry

➡ $J^a_z \underset{z\to 0}{\sim} t^a$, t^a is a marginal operator of the bCFT

“conformal manifold” of bCFTs related by bulk symmetry, AdS version of Goldstone’s theorem.

- Large L: Coleman-Mermin-Wagner

➡ this bc must have disappeared

“**Goldstone-gapped transition**”:

- Bulk theory with continuous global symmetry, $\nabla^M J^a_M = 0$
- Small L: bc that breaks the symmetry

➡ $J^a_z \underset{z\to 0}{\sim} t^a$, t^a is a marginal operator of the bCFT

“conformal manifold” of bCFTs related by bulk symmetry, AdS version of Goldstone’s theorem.

- Large L: Coleman-Mermin-Wagner

➡ this bc must have disappeared

Objective: understand this transitions in 2d examples that are under control thanks to large N

O(N) sigma model in AdS_2

$$S = \int \tfrac{1}{2}(\partial\phi^i)^2 \quad , \quad i = 1, \ldots, N \quad , \quad (\phi^i)^2 = \frac{N}{g^2}$$

with Hubbard-Stratonovich auxiliary field σ :

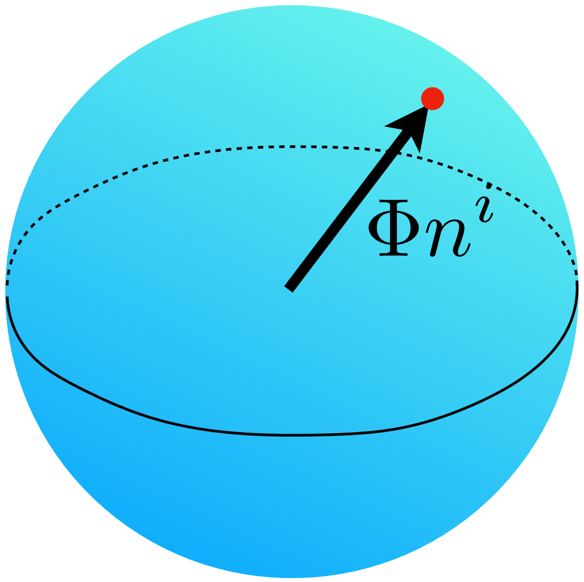
$$S = \int \left[\tfrac{1}{2}(\partial\phi^i)^2 + \sigma\left((\phi^i)^2 - \frac{N}{g^2}\right)\right] + N\mathrm{tr}\log(-\Box + 2\sigma)$$

O(N) sigma model in AdS$_2$

$$S = \int \tfrac{1}{2}(\partial\phi^i)^2 \;, \quad i = 1, \ldots, N \;, \quad (\phi^i)^2 = \frac{N}{g^2}$$

with Hubbard-Stratonovich auxiliary field σ :

$$S = \int \left[\tfrac{1}{2}(\partial\phi^i)^2 + \sigma\left((\phi^i)^2 - \frac{N}{g^2}\right)\right] + N \operatorname{tr}\log(-\Box + 2\sigma)$$



$$\sigma = \Sigma + \frac{1}{\sqrt{N}}\delta\sigma \;,$$

$$\phi^i = (\sqrt{N}\Phi + \rho)n^i + \pi^i \;,$$

$$(n^i)^2 = 1 \;, \; n^i\pi^i = 0$$

Vacuum equations: $\begin{cases} \Sigma\,\Phi = 0 \\ \Phi^2 - \frac{1}{g^2} + \operatorname{tr}\left[\frac{1}{-\Box+2\Sigma}\right] = 0 \end{cases}$

Vacuum equations: $\begin{cases} \Sigma\,\Phi = 0 \\ \Phi^2 - \frac{1}{g^2} + \mathrm{tr}\left[\frac{1}{-\Box+2\Sigma}\right] = 0 \end{cases}$

$$-\frac{1}{g^2_{\mathrm{reg}(\mu)}} + \frac{1}{2\pi}\log\mu + f(\Sigma) = \frac{1}{2\pi}\log\Lambda + f(\Sigma)$$

Vacuum equations: $$\begin{cases} \Sigma\,\Phi = 0 \\ \Phi^2 - \frac{1}{g^2} + \mathrm{tr}\left[\frac{1}{-\Box+2\Sigma}\right] = 0 \end{cases}$$

$$-\frac{1}{g^2_{\mathrm{reg}(\mu)}} + \frac{1}{2\pi}\log\mu + f(\Sigma) = \frac{1}{2\pi}\log\Lambda + f(\Sigma)$$

- Flat space: $$f(\Sigma) \underset{\Sigma\to 0}{\sim} -\frac{1}{2\pi}\log\Sigma$$

➡ no SSB solution with $\Sigma = 0\ ,\ \Phi \neq 0$

Vacuum equations: $\begin{cases} \Sigma\,\Phi = 0 \\ \Phi^2 - \frac{1}{g^2} + \mathrm{tr}\left[\frac{1}{-\Box + 2\Sigma}\right] = 0 \end{cases}$

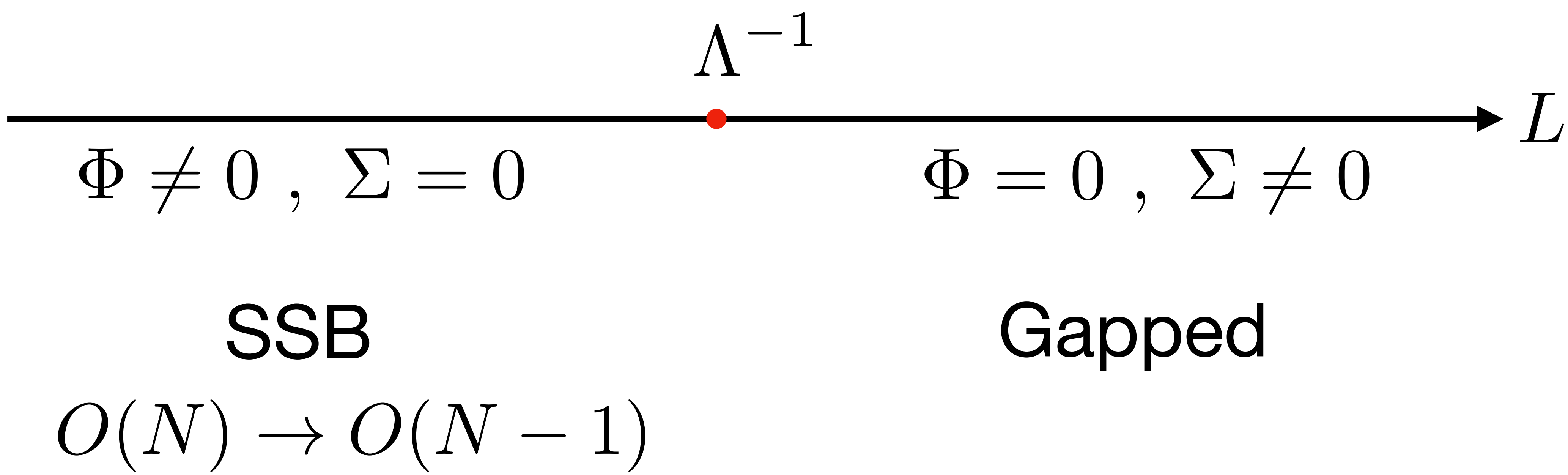
$$-\frac{1}{g^2_{\mathrm{reg}(\mu)}} + \frac{1}{2\pi}\log\mu + f(\Sigma) = \frac{1}{2\pi}\log\Lambda + f(\Sigma)$$

- Flat space: $f(\Sigma) \underset{\Sigma\to 0}{\sim} -\frac{1}{2\pi}\log\Sigma$

➡ no SSB solution with $\Sigma = 0\ ,\ \Phi \neq 0$

- AdS: $f(\Sigma) \underset{\Sigma\to 0}{\sim} -\frac{1}{2\pi}\log L$

➡ SSB solution for $L < \Lambda^{-1}$: $\Phi^2 = -\frac{1}{2\pi}\log(L\Lambda)$



So far: bulk analysis of the effective potential.

How is the transition detected from the bCFT?

We look at the **spectrum of $O(N-1)$-singlet scalars.**

We will find: an operator hits marginality at the transition, beyond the SSB bc becomes complex beyond.

Quadratic action for fluctuations:

$$S^{(2)} = \int \frac{1}{2}(\partial\rho)^2 + 2\Phi\delta\sigma\rho + \delta\sigma\left(\frac{1}{-\Box}\right)^2\delta\sigma$$

Expand in a basis of eigenfunctions of the Laplacian:

$$-\Box_{\mathbf{x}}\Omega_\nu(\mathbf{x},\mathbf{y}) = \tfrac{1}{L^2}(\nu^2+\tfrac{1}{4})\Omega_\nu(\mathbf{x},\mathbf{y})\ , \quad \nu\in\mathbb{R}$$

➡ $$\left(\frac{1}{-\Box}\right)^2(\mathbf{x},\mathbf{y}) = \int_{-\infty}^{+\infty} d\nu\, B(\nu)\Omega_\nu(\mathbf{x},\mathbf{y})$$

$$B(\nu) = \tfrac{1}{2\pi}\frac{\psi(i\frac{\nu}{2}+\frac{3}{4})+\psi(-i\frac{\nu}{2}+\frac{3}{4})+\log 4+2\gamma_E}{\nu^2+\frac{1}{4}}$$

[Carmi, DP, Komatsu]

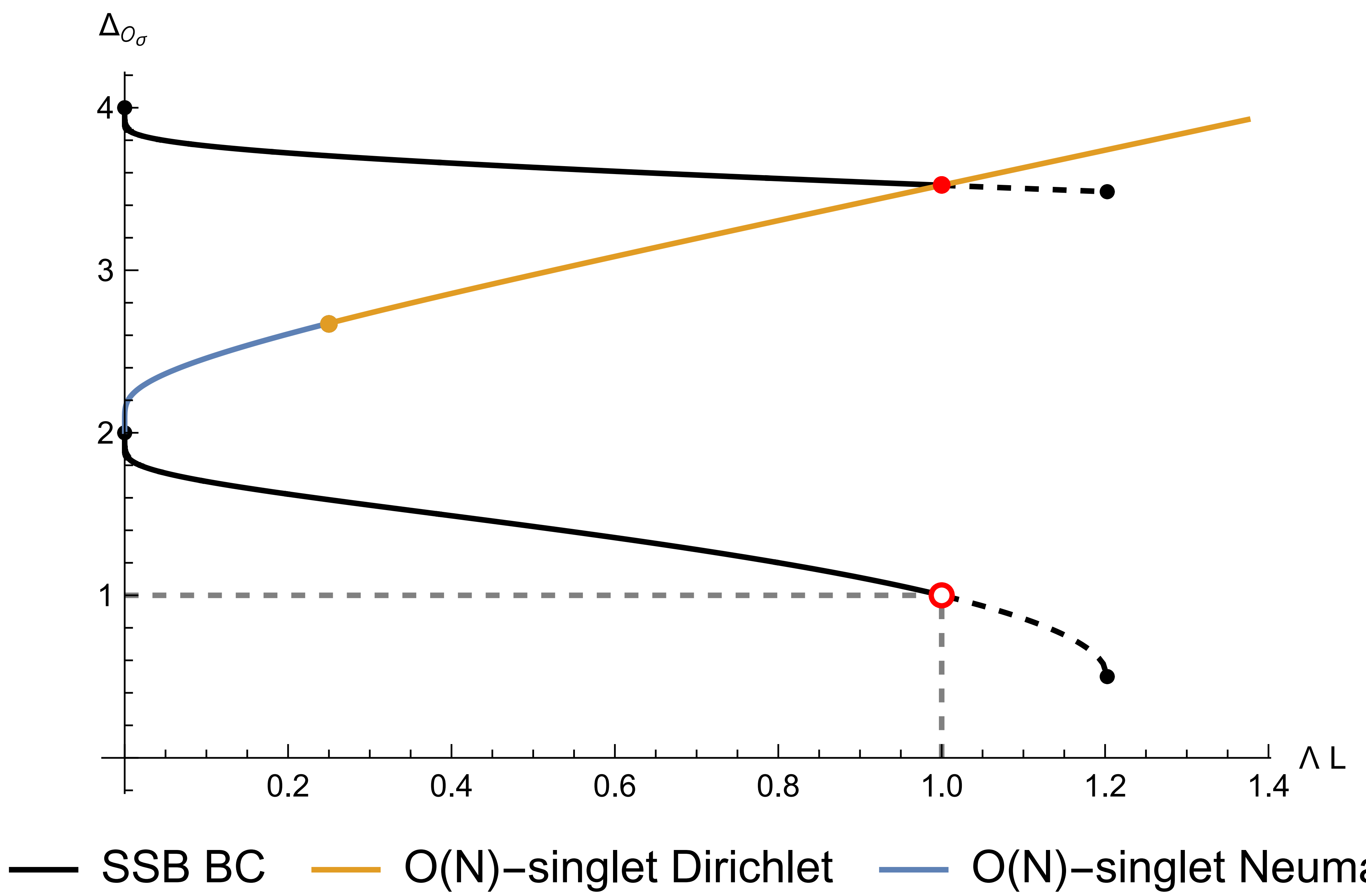
➡ $$\langle\begin{pmatrix}\delta\sigma\\ \rho\end{pmatrix}\begin{pmatrix}\delta\sigma & \rho\end{pmatrix}\rangle(\nu) = \begin{pmatrix}-2B(\nu) & -2\Phi\\ -2\Phi & \nu^2+\frac{1}{4}\end{pmatrix}^{-1}$$

Poles of $\langle \mathcal{O}\mathcal{O}\rangle(\nu)$ give the **bOPE expansion**

→ Spectrum of singlet operators $=$ Poles of $\langle \begin{pmatrix} \delta\sigma \\ \rho \end{pmatrix} \begin{pmatrix} \delta\sigma & \rho \end{pmatrix} \rangle(\nu)$

Poles of $\langle \mathcal{O}\mathcal{O}\rangle(\nu)$ give the **bOPE expansion**

➡ Spectrum of singlet operators $=$ Poles of $\left\langle \begin{pmatrix} \delta\sigma \\ \rho \end{pmatrix} \begin{pmatrix} \delta\sigma & \rho \end{pmatrix} \right\rangle(\nu)$

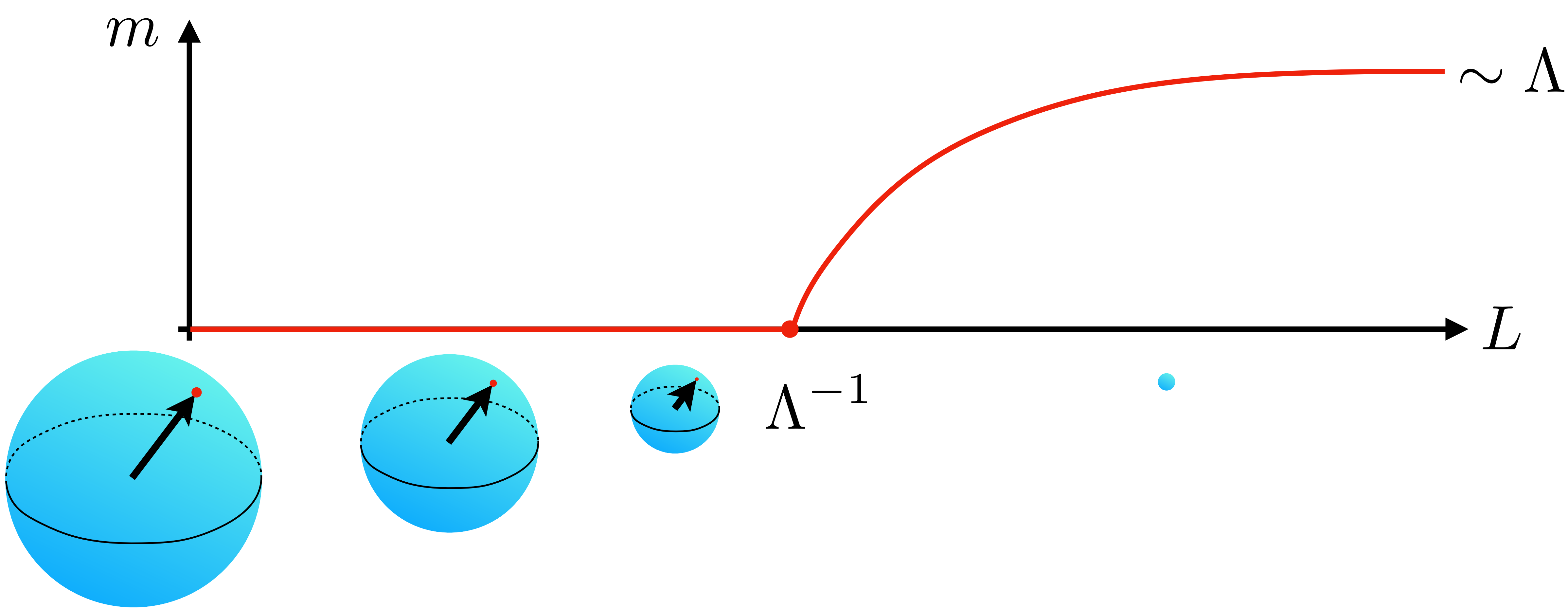


At $L = \Lambda^{-1}$ an operator $\hat{\sigma}$ becomes marginal.

For $L > \Lambda^{-1}$ the SSB Dirichlet bc becomes complex:

$$b^2_{\delta\sigma\,\hat{\sigma}} \underset{\Phi\to 0}{\sim} \frac{144}{\pi^2}\Phi^2 \ , \quad \Phi^2 = -\frac{1}{2\pi}\log(L\Lambda)$$

The SSB Dirichlet continuously connects at $L = \Lambda^{-1}$ with the symmetry-preserving Dirichlet condition $\Phi = 0$ which persists for $L \to \infty$



Gross-Neveu model in AdS$_2$

$$S = \int \bar{\psi}_i \nabla\!\!\!/ \psi^i - \frac{g}{N}(\bar{\psi}_i \psi^i)^2 \,,$$

$$\psi^i = \begin{pmatrix} \chi_L^i \\ \chi_R^i \end{pmatrix}$$

$$i = 1, \ldots, N$$

with Hubbard-Stratonovich auxiliary field σ:

$$S = \int \bar{\psi}_i \nabla\!\!\!/ \psi^i + \sigma \bar{\psi}_i \psi^i - \frac{N^2}{2g}\sigma^2 - N \operatorname{tr} \log (\nabla\!\!\!/ + 2\sigma)$$

Gross-Neveu model in AdS$_2$

$$S = \int \bar{\psi}_i \nabla\!\!\!/ \psi^i - \frac{g}{N}(\bar{\psi}_i \psi^i)^2 \,, \qquad \psi^i = \begin{pmatrix} \chi_L^i \\ \chi_R^i \end{pmatrix} \quad i = 1, \ldots, N$$

with Hubbard-Stratonovich auxiliary field σ:

$$S = \int \bar{\psi}_i \nabla\!\!\!/ \psi^i + \sigma \bar{\psi}_i \psi^i - \frac{N^2}{2g}\sigma^2 - N \operatorname{tr} \log(\nabla\!\!\!/ + 2\sigma)$$

$$\sigma = \Sigma + \frac{1}{\sqrt{N}}\delta\sigma$$

Vacuum equation: $$\frac{\Sigma}{g} + \operatorname{tr}\left[\frac{1}{\nabla\!\!\!/ + 2\Sigma}\right] = 0$$

Symmetry: $$O(2N)_V \times (\mathbb{Z}_2)_A \xrightarrow[\Sigma \neq 0]{} O(2N)_V$$

$$\chi_L^i \ , \ \chi_R^i \longrightarrow \varphi_{L,R}^a \ , \quad a = 1, \ldots, 2N$$

$$\bar{\psi}_i \psi^i = \chi^*_{R\,i}\, \chi_L^i + c.c. = \varphi_R^a \varphi_L^a$$

$$O(2N)_V \quad : \qquad \varphi^a_{R,L} \to M^a{}_b \varphi^b_{R,L}$$

$$(\mathbb{Z}_2)_A \quad : \qquad \chi_L^i \ , \ \chi_R^i \to \chi_L^i \ , -\chi_R^i$$

$$\chi_L^i\ ,\ \chi_R^i \longrightarrow \varphi_{L,R}^a\ ,\quad a=1,\ldots,2N$$

$$\bar{\psi}_i\psi^i = \chi^*_{R\,i}\,\chi^i_L + c.c. = \varphi^a_R\varphi^a_L$$

$$O(2N)_V \ : \qquad \varphi^a_{R,L} \to M^a{}_b\varphi^b_{R,L}$$

$$(\mathbb{Z}_2)_A \ : \qquad \chi_L^i\ ,\ \chi_R^i \to \chi_L^i\ ,-\chi_R^i$$

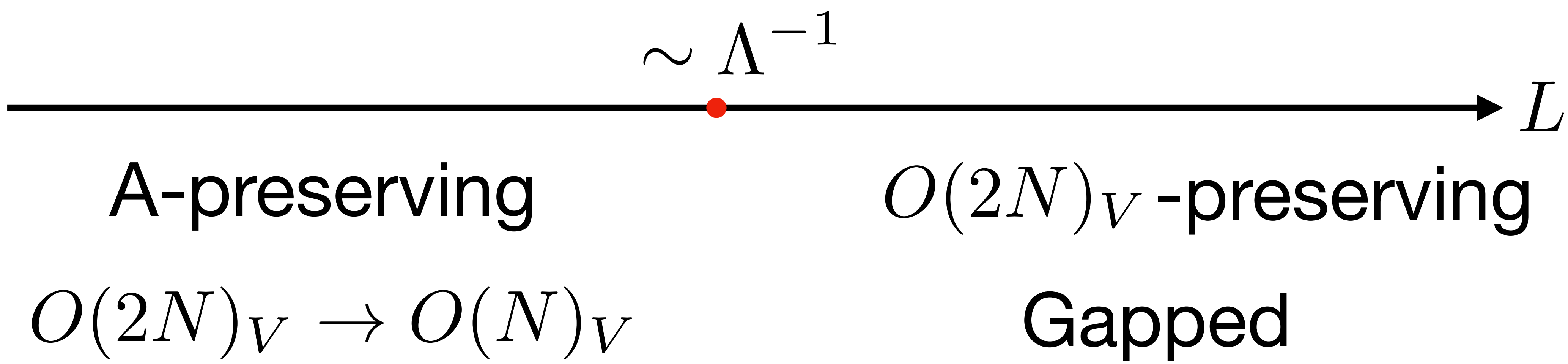
B.c. with “Goldstones”: A-preserving, V-breaking

$$(\chi_L)^*_i|_\partial = \chi^i_R|_\partial$$

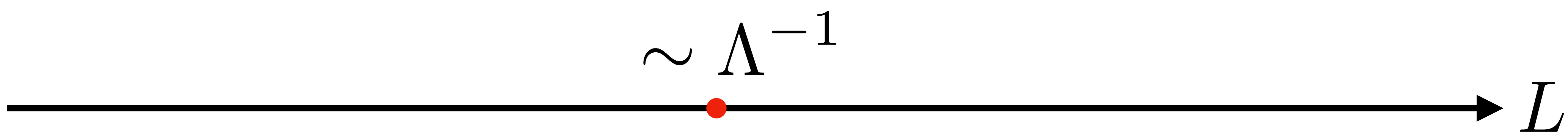
Preserves $\mathbb{Z}_4$ combination of $(\mathbb{Z}_2)_A$ and $O(2N)_V$:

$$\chi_L^i\ ,\ \chi_R^i \to i\chi_L^i\ ,-i\chi_R^i$$

Protects massless phase $\Sigma=0$ and $O(2N)_V \to O(N)_V$



$\sim \Lambda^{-1}$
L
A-preserving
$O(2N)_V$ -preserving
$O(2N)_V \to O(N)_V$
Gapped

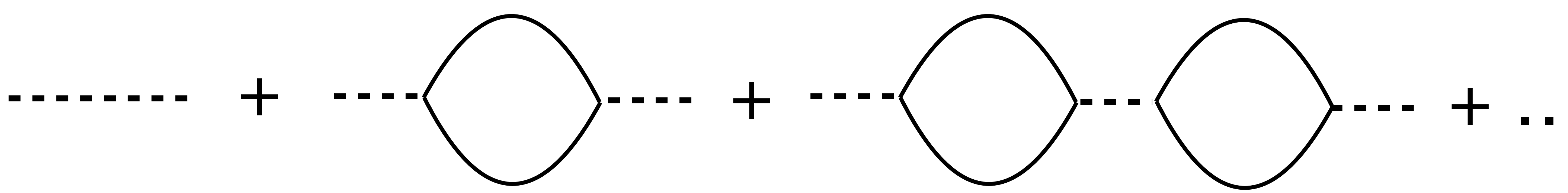


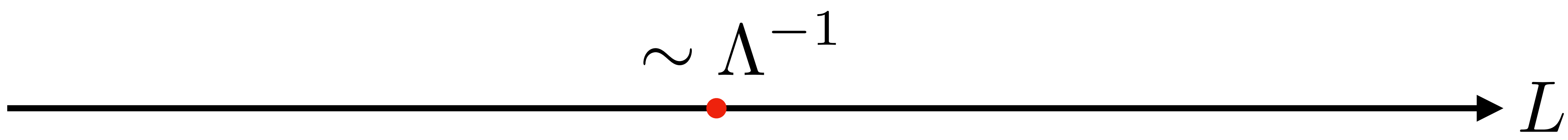
A-preserving | $O(2N)_V$ -preserving

$O(2N)_V \to O(N)_V$ | Gapped

To detect the transition from bCFT:

$$\langle \delta\sigma\delta\sigma \rangle(\nu) = -\frac{1}{g^{-1} - B_A(\nu)}$$



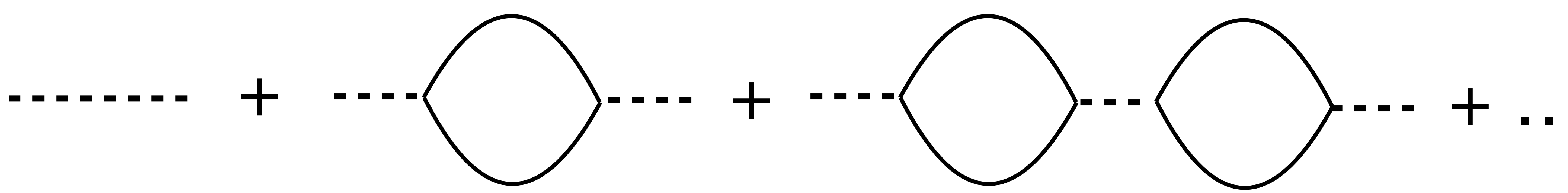


A-preserving — $O(2N)_V$-preserving

$O(2N)_V \to O(N)_V$ — Gapped

To detect the transition from bCFT:

$$\langle \delta\sigma\delta\sigma\rangle(\nu) = -\frac{1}{g^{-1} - B_A(\nu)}$$



$B_A(\nu)$ has a UV divergence: $-\frac{1}{\pi}\left(\frac{1}{\epsilon} - \log(L\mu)\right) + B_A^{\text{finite}}(\nu)$

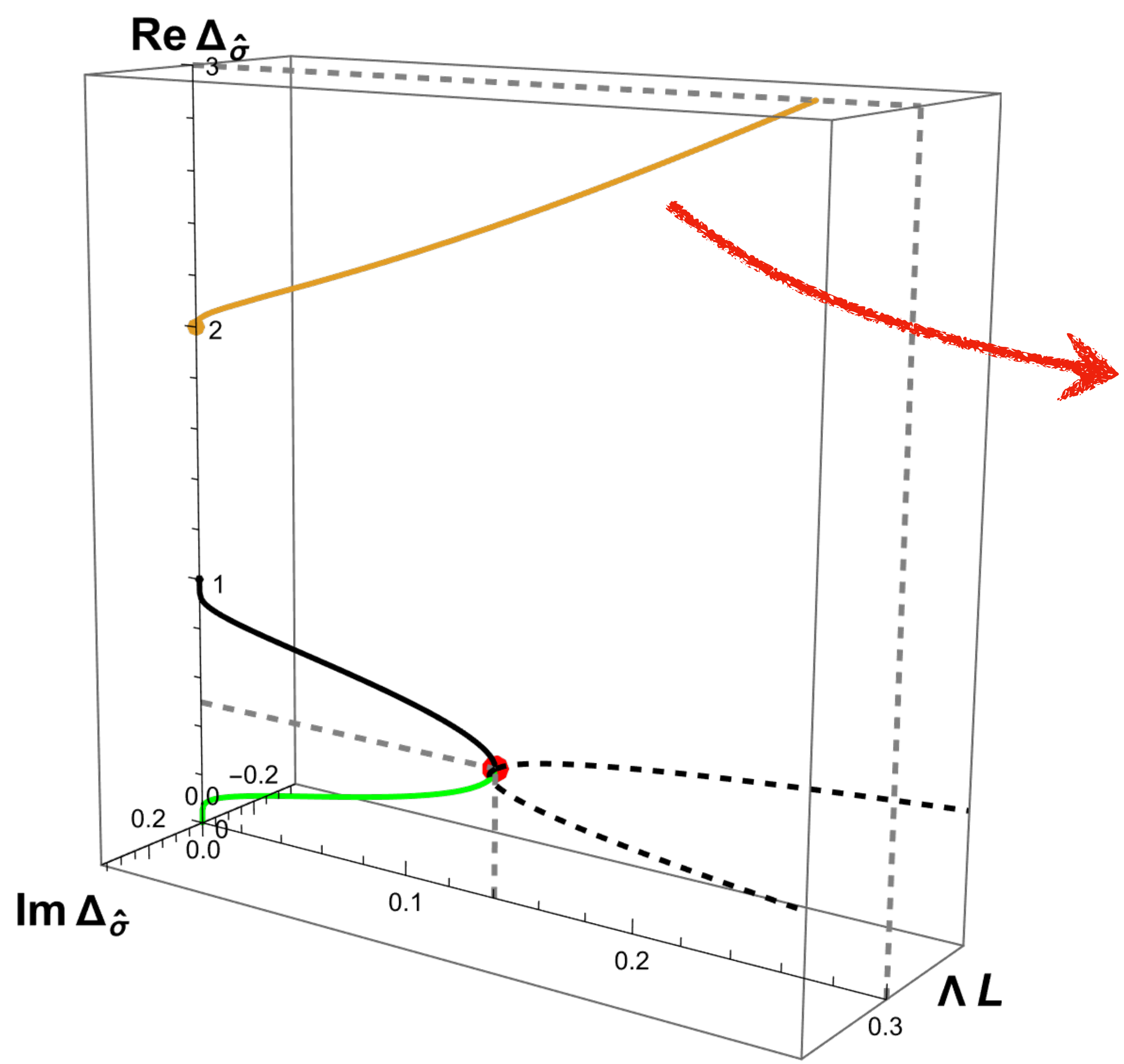
→ g replaced by $\Lambda = \mu e^{-\frac{\pi}{g_{\text{reg}}}}$

The lightest operator $\hat{\sigma}$ in the $\delta\sigma$ bOPE hits $\Delta = 1/2$ at $L = \Lambda^{-1}\frac{e^{-\gamma_E}}{4}$

→ $\hat{\sigma}^2$ singlet marginal operator (large N)

The lightest operator $\hat{\sigma}$ in the $\delta\sigma$ bOPE hits $\Delta = 1/2$ at $L = \Lambda^{-1}\frac{e^{-\gamma_E}}{4}$

➡ $\hat{\sigma}^2$ singlet marginal operator (large N)



discontinuous transition to V-preserving bc

Summary

- Transitions from gapless to gapped phases in AdS
- Solvable examples in 2d point towards merger and annihilation as mechanism for disappearance of gapless b.c.

Future directions

- Can one prove mass gap in O(N) sigma model / Gross-Neveu at small N using 1d bootstrap?
- Other solvable example: $\mathcal{N}=2$ SYM in AdS$_4$
- What could be the (Dirichlet)' that merges with Dirichlet in YM?
- More $1/C_J$ perturbative data

Thank You