Factorization for J/ψ leptoproduction at small p_T

Samuel F. Romera
University of the Basque Country (UPV/EHU)

In collaboration with M.G. Echevarria and P. Taels

eman ta zabal zazu
Universidad del País Vasco
Euskal Herriko Unibertsitatea

EHU QC
EHU Quantum Center

Gluon TMDs and Quarkonium

- We are interested in **TMDs**.

- The quark TMDs have been extensively studied by theoretical approaches as well as experimental measurements. On the contrary, **the gluon TMDs are almost unknown** from experimental aspect. ATLAS Collab., https://arxiv.org/abs/2202.00487

- **Quarkonium production is a good tool to extract gluon TMDs**:
  - Heavy quarks are sensitive to the gluon content of hadrons.

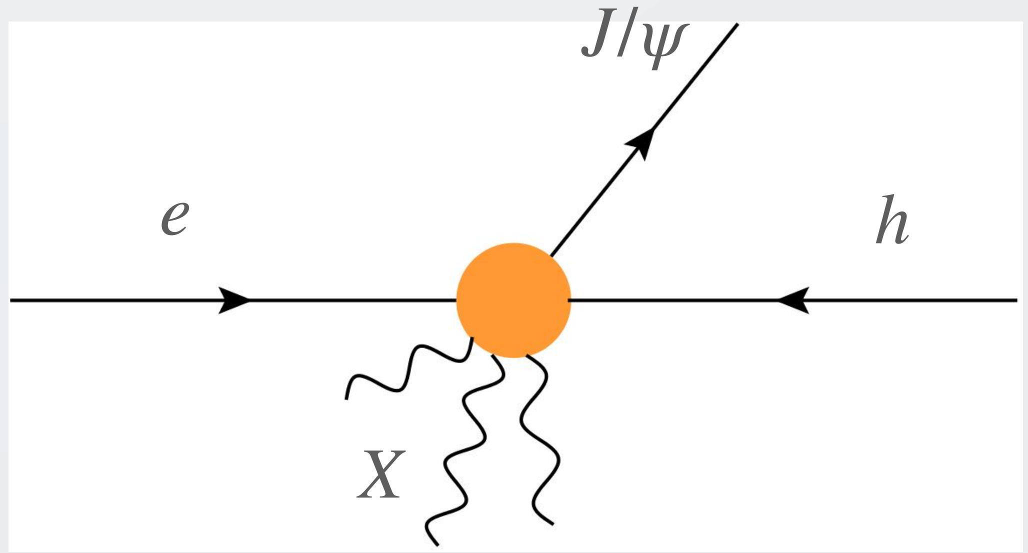
    W. J. den Dunnen, J.-P. Lansberg, C. Pisano, M. Schlegel, https://arxiv.org/abs/1401.7611 *Dominantly produced by g-g fusion*
    J.-P. Lansberg, C. Pisano, F. Scarpa, M. Schlegel, https://arxiv.org/abs/1710.01684 *Gluon TMD fits in Di-J/ψ production*
  - J/ψ is straightforward to detect and there are numerous events.

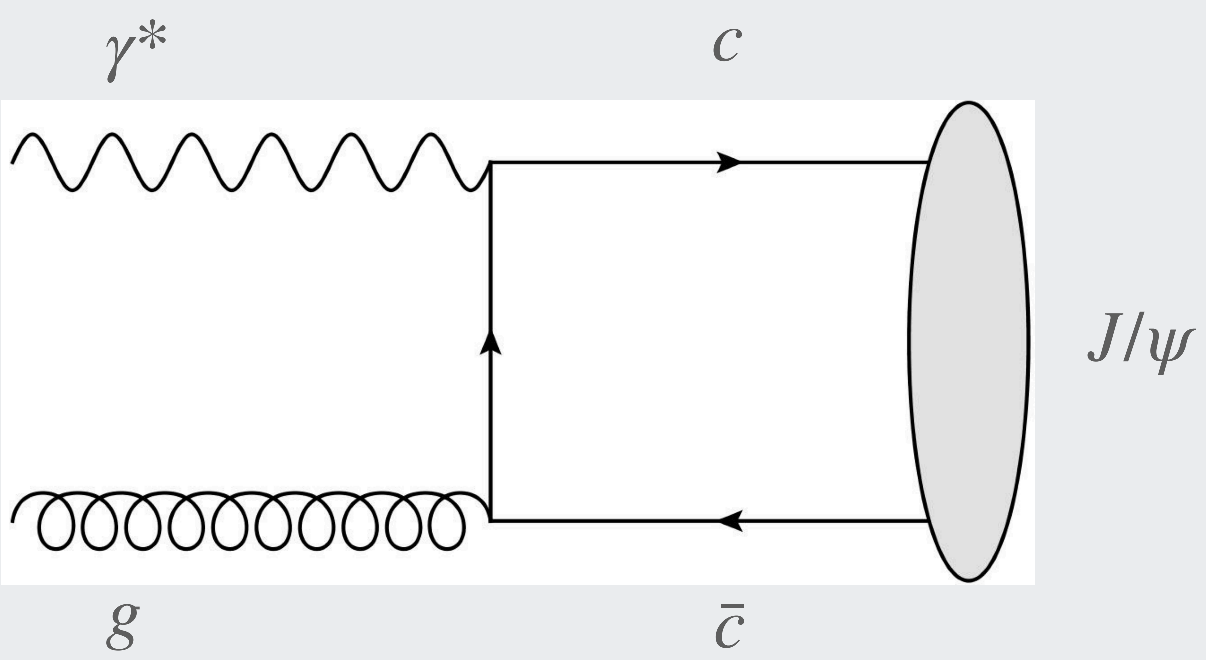
- **Challenges**: no universality of the LDMEs in the NRQCD approach, quarkonium production mechanism?, theoretical framework, phenomenological analysis, etc.

  U. D'Alesio, A. Mukherjee, F. Murgia, C. Pisano and S. Rajesh, https://arxiv.org/abs/2203.03299 *NRQCD for quarkonium production mechanism*
  H.S. Chung, http://arxiv.org/abs/2211.10201v1 *Review on universality*
  and many others

Setup (of the talk)

Leptoproduction, e-h





A. Bacchetta, D. Boer, C. Pisano, P. Taels, arXiv:1809.02056

e+e-, pp, photoproduction…

S. Fleming, A.K. Leibovich, T. Mehen, arXiv:hep-ph/0607121
S. Fleming, A.K. Leibovich, T. Mehen, arXiv:hep-ph/0306139
M.G. Echevarria, arXiv:1907.06494
S. Fleming, Y. Makris, T. Mehen, arXiv:1910.03586

- **Quarkonium production mechanism: NRQCD factorization**

G.T. Bodwin, E. Braaten, G.P. Lepage, arXiv:hep-ph/9407339

NRQCD: m, mv, mv^2 (potential, soft, ultra-soft)

$$d\sigma(\gamma^* + g \to J/\psi + X) = \sum_N d\sigma(\gamma^* + g \to c\bar{c}(N) + X)\left\langle \mathcal{O}_{N\to J/\psi}\right\rangle$$

SDCs LDMEs

$$\mathcal{O}_{N\to J/\psi} = \chi^\dagger \mathcal{K}_N^\dagger \psi \sum_X \left(\left| J/\psi + X \right\rangle + \left\langle J/\psi + X \right| \right) \times \psi^\dagger \mathcal{K}_N \chi$$

$$N = {}^{2S+1}L_J^{[col.]}$$

S: spin
L: angular momentum
J: total angular momentum

$p_T \gg M$ *Collinear factorization + LDMEs*

$p_T \ll M$ *TMD factorization + TMDShFs*

$p_T \sim m_c v$ $\lambda = p_T/M$

Intermediate region

D. Boer, U. D'Alesio, F. Murgia, C. Pisano, S. Rajesh, arXiv: 2004.06740
U. D'Alesio, L. Maxia, F. Murgia, C. Pisano, S. Rajesh, arXiv:2110.07529
D. Boer, J. Bor, L. Maxia, C. Pisano, F. Yuan, arXiv: 2304.09473

About this talk…

- Factorization:
  - Definition of operators in NRQCD+TMD framework
  - Matching onto QCD
  - TMD shape function (TMDShF)

- LDMEs and TMDShFs at NLO:
  - Calculation at NLO
  - Renormalization group equations
  - Matching onto LDMEs

- Discussion on Hard function

N-operators: scaling

In the equations: $\psi \equiv J/\psi$

$$z = \frac{P \cdot P_\psi}{P \cdot q}$$

J/ψ production in SIDIS: $\ell(k) + N(P_N) \to \ell'(k') + J/\psi(P_\psi) + X(P_X)$

$$W^{\mu\nu} = \sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{iq\cdot\xi} < N | J^{\mu\dagger}(\xi) | J/\psi, X > < J/\psi, X | J^\nu(0) | N >$$

$$J^\nu = \sum_{\underline{\mathcal{O}_N}} \sum_p J_N^{\nu(p)} \qquad N = {}^{2S+1}L_J^{[col.]}$$

N–operators (defined in vNRQCD+SCET framework)

- **Puzzle pieces** for $\gamma^* + g \to c\bar{c}$ (2 power expansions: v y λ):

Gluons: $B^\mu_{n\perp} \sim \lambda$ *In SCET* Heavy quarks: $\psi, \chi \sim \mathrm{v}^{3/2}$ *In vNRQCD*

→ The lowest power operator will scale as $\lambda \cdot \mathrm{v}^3$ → Color octet configuration

- **Power-counting in vNRQCD (v):** $\underbrace{\left\langle {}^3S_1^{[1]} \right\rangle,}_{\mathrm{v}^3} \underbrace{\left\langle {}^1S_0^{[8]} \right\rangle, \left\langle {}^3S_1^{[8]} \right\rangle, \left\langle {}^3P_J^{[8]} \right\rangle}_{\mathrm{v}^7}$ $\left\langle \mathcal{O}({}^{2S+1}L_J) \right\rangle \sim \mathrm{v}^{3+2L+2E+4M}$

Dominant Fock state

$$N = {}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_J^{[8]} \longrightarrow \left\{ \mathcal{O}^\mu_{{}^1S_0^{[8]}}, \mathcal{O}^\mu_{{}^3S_1^{[8]}}, \mathcal{O}^\mu_{{}^3P_0^{[8]}}, \mathcal{O}^\mu_{{}^3P_1^{[8]}}, \mathcal{O}^\mu_{{}^3P_2^{[8]}} \right\}$$

N-operators: definition

- **Definition of operators:**

$$\mathcal{O}^{\mu}_{{}^1S_0^{[8]}} = \Gamma^{\mu\alpha}_{{}^1S_0^{[8]}} \left(\underbrace{\mathcal{S}_v^{cd}\, \psi^\dagger_{\mathbf{p}_c}\, T^d\, \chi_{\mathbf{p}_{\bar c}}}_{\text{pair formation}} \right) \times \left(\underbrace{\mathcal{S}_n^{ce}\, B^e_{n\perp\alpha}}_{\text{incoming gluon}} \right)$$

$$\mathcal{O}^{\mu}_{{}^3S_1^{[8]}} = \Gamma^{\mu\alpha\rho}_{{}^3S_1^{[8]}} \left(\mathcal{S}_v^{cd}\, \psi^\dagger_{\mathbf{p}_c}\, T^d\, \underbrace{\left(\boldsymbol{\Lambda}\cdot\boldsymbol{\sigma}\right)_\rho}_{\text{spin-triplet}}\, \chi_{\mathbf{p}_{\bar c}} \right) \times \left(\mathcal{S}_n^{ce}\, B^e_{n\perp\alpha} \right)$$

* $\langle c\bar{c} |\, \psi^\dagger \chi \,| 0 \rangle = M\, \xi^\dagger \eta$

$M = 2\, m_c$

$$\mathcal{O}^{\mu}_{{}^3P_J^{[8]}} = \Gamma^{\mu\alpha\sigma\rho}_{{}^3P_J^{[8]}} \left(\mathcal{S}_v^{cd}\, \underbrace{\frac{\left(\boldsymbol{\Lambda}\cdot\mathbf{q}\right)_\sigma}{M\,{*}}}_{L=1}\, \psi^\dagger_{\mathbf{p}_c}\, T^d \left(\boldsymbol{\Lambda}\cdot\boldsymbol{\sigma}\right)_\rho \chi_{\mathbf{p}_{\bar c}} \right) \times \left(\mathcal{S}_n^{ce}\, B^e_{n\perp\alpha} \right)$$

Properties of the boost matrix

$$(\Lambda\cdot P)_i = 0\,, \qquad \underbrace{(\Lambda\cdot k_1)_i}_{\text{photon}} = -\underbrace{(\Lambda\cdot k_2)_i}_{\text{gluon}}$$

See also: S. Fleming, Y. Makris, T. Mehen, arXiv:1910.03586

N-operators: definition

▸ Definition of operators:

$$\mathcal{O}^{\mu}_{{}^1S_0^{[8]}} = \Gamma^{\mu\alpha}_{{}^1S_0^{[8]}} \Big(\underbrace{\mathcal{S}_v^{cd}\,\psi^\dagger_{\mathbf{p}_c}\,T^d\,\chi_{\mathbf{p}_{\bar c}}}_{\text{pair formation}}\Big) \times \Big(\underbrace{\mathcal{S}_n^{ce}\,B^e_{n\perp\alpha}}_{\text{incoming gluon}}\Big)$$

$$\mathcal{O}^{\mu}_{{}^3S_1^{[8]}} = \Gamma^{\mu\alpha\rho}_{{}^3S_1^{[8]}} \Big(\mathcal{S}_v^{cd}\,\psi^\dagger_{\mathbf{p}_c}\,T^d\,\underbrace{\left(\boldsymbol{\Lambda}\cdot\boldsymbol{\sigma}\right)_\rho}_{\text{spin-triplet}}\,\chi_{\mathbf{p}_{\bar c}}\Big) \times \Big(\mathcal{S}_n^{ce}\,B^e_{n\perp\alpha}\Big)$$

$$* \, \langle c\bar{c}|\,\psi^\dagger\chi\,|0\rangle = M\,\xi^\dagger\eta$$

$$M = 2\,m_c$$

$$\mathcal{O}^{\mu}_{{}^3P_J^{[8]}} = \Gamma^{\mu\alpha\sigma\rho}_{{}^3P_J^{[8]}} \left(\mathcal{S}_v^{cd}\,\underbrace{\frac{\left(\boldsymbol{\Lambda}\cdot\mathbf{q}\right)_\sigma}{M\,*}}_{\text{L=1}}\,\psi^\dagger_{\mathbf{p}_c}\,T^d\left(\boldsymbol{\Lambda}\cdot\boldsymbol{\sigma}\right)_\rho\chi_{\mathbf{p}_{\bar c}}\right) \times \Big(\mathcal{S}_n^{ce}\,B^e_{n\perp\alpha}\Big)$$

Properties of the boost matrix

$$\left(\Lambda\cdot P\right)_i = 0\,, \qquad \underbrace{\left(\Lambda\cdot k_1\right)_i}_{\text{photon}} = -\underbrace{\left(\Lambda\cdot k_2\right)_i}_{\text{gluon}}$$

See also: S. Fleming, Y. Makris, T. Mehen, arXiv:1910.03586

▸ Matching onto QCD (tree level):

$$\Gamma^{\mu\alpha}_{{}^1S_0^{[8]}} = \frac{4\,g_s e}{\mathcal{N}}\,\frac{\epsilon_\perp^{\mu\nu}}{M}$$

$$\Gamma^{\mu\alpha\rho}_{{}^3S_1^{[8]}} = 0$$

Depends on the production mechanism

$$\Gamma^{\mu\nu\sigma\rho}_{{}^3P_J^{[8]}} = \frac{i4\,g_s e}{\mathcal{N}}\,\frac{1}{M}\left\{ g_\perp^{\nu\sigma}\left[g^{\mu\rho}\left(\frac{M^2-Q^2}{M^2+Q^2}\right) - 2\,\bar{n}^\mu\bar{n}^\rho\frac{M^2+Q^2}{P^{+2}} - n^\mu\bar{n}^\rho\right] + g_\perp^{\nu\rho}\left[g^{\mu\sigma} + 2\,\bar{n}^\mu\bar{n}^\sigma\frac{M^2+Q^2}{P^{+2}} - n^\mu\bar{n}^\sigma\left(\frac{M^2-Q^2}{M^2+Q^2}\right)\right] - 4\,g_\perp^{\nu\mu}\,\bar{n}^\sigma\bar{n}^\rho\frac{M^2}{P^{+2}} \right\}$$

→ **There are other results for the matching tensor**

S. Fleming, A. K. Leibovich, T. Mehen, hep-ph/0306139

TMD Shape Function: factorization

In the equations: $\psi \equiv J/\psi$

$P_\psi^\mu = (\omega_g, \omega_q, P_{\psi T})$

$$W^{\mu\nu} = \sum_X \int \sum_N \delta^4(q + P_N - P_\psi - P_X) \langle N \big| J_N^{\dagger\mu}(0) \big| J/\psi, X \rangle \langle J/\psi, X \big| J_N^\nu(0) \big| N \rangle \ .$$

Photon frame

- **Taylor expansion:** $p_g \sim Q_h(1,\lambda^2,\lambda)$, $q \sim Q_h(\lambda^2,1,\lambda)$, $P_\psi \sim Q_h(1,1,\lambda)$

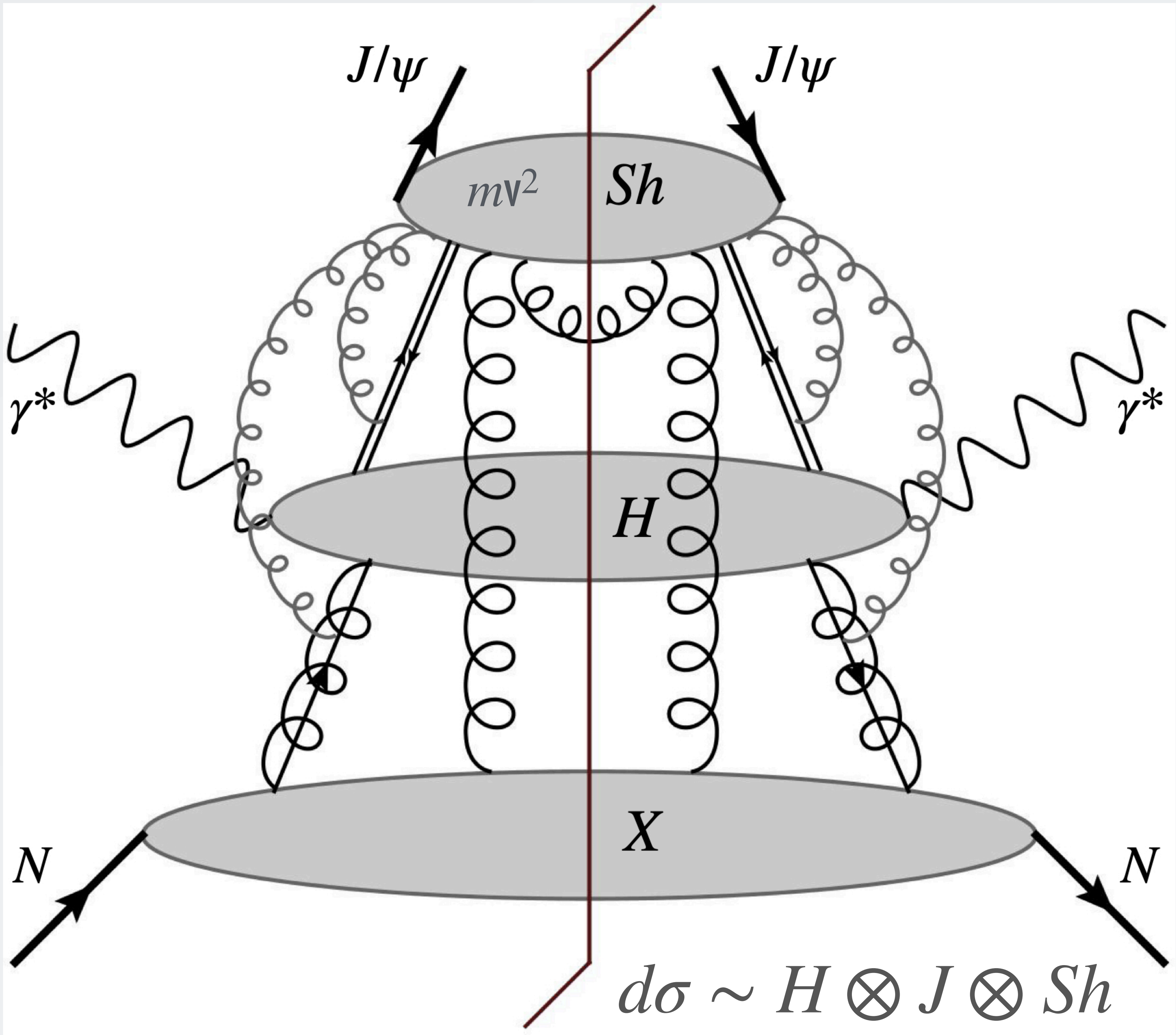
$$\delta^4(q + P_N - P_\psi - P_X) \longrightarrow \delta^2(\mathbf{P}_{\psi\perp} + \mathbf{P}_{X_n\perp} + \mathbf{P}_{X_s\perp}) = \int \frac{d^2\mathbf{b}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot(\mathbf{P}_{\psi\perp}+\mathbf{P}_{X_n\perp}+\mathbf{P}_{X_s\perp})}$$

- **Hilbert space:** $\big| J/\psi, X \rangle = \big| X_n \big\rangle \otimes \big| X_{\bar{n}} \big\rangle \otimes \big| J/\psi, X_s \big\rangle$

TMDShF encodes soft and non-perturbative effects

$$\mathscr{L} = -g_s \sum_{\mathbf{p}} \psi_{\mathbf{p}}^\dagger \left(\frac{\mathbf{B}_{us} \cdot \mathbf{P}}{m_c} \right) \psi_{\mathbf{p}}(x) + (\psi \to \chi)$$

$$W^{\mu\nu} = \sum_N 2H_N(Q,M) \int \frac{d^2\mathbf{b}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\mathbf{P}_{\psi\perp}} \mathrm{Tr} \left[\Gamma_N^{\dagger\mu} \Gamma_N^\nu \, J_n^{(0)}(\omega_g, \mathbf{b}_\perp) \, S_{N\to J/\psi}^{(0)}(\mathbf{b}_\perp) \right]$$



TMD Shape Function: definition

In the equations: $\psi \equiv J/\psi$

$$P^\mu_\psi = (\omega_g, \omega_q, P_{\psi T})$$

$$W^{\mu\nu} = \sum_N 2H_N(Q,M) \int \frac{d^2\mathbf{b}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \mathbf{P}_{\psi\perp}} \mathrm{Tr}\left[\Gamma^{\dagger\mu}_N \Gamma^\nu_N J^{(0)}_n(\omega_g, \mathbf{b}_\perp)\, S^{(0)}_{N\to J/\psi}(\mathbf{b}_\perp)\right]$$

- **Gluon TMD parton distribution function:**

$$J^{\alpha\alpha'(0)}_n(\omega_g, \mathbf{b}_\perp) = \frac{\theta(\omega_g)}{N_c^2 - 1} \mathrm{tr}_c \left\langle N \right| B^{\alpha'}_{n\perp}(\mathbf{b}_\perp) B^{\alpha}_{n\perp,\omega_g}(0) \left| N \right\rangle$$

- **TMD shape functions:**

$$S^{(0)}_{{}^1S_0^{[8]}\to J/\psi}(\mathbf{b}_\perp) = \frac{1}{N_c^2 - 1} \mathrm{tr}_c \left\langle 0 \right| \left[(\mathcal{S}_v \mathcal{S}_n)^\dagger \chi^\dagger_{\bar{\mathbf{p}}} T^a \psi_{\mathbf{p}} \right](\mathbf{b}_\perp) \mathcal{N}_\psi \left[\mathcal{S}_v \mathcal{S}_n \psi^\dagger_{\mathbf{p}} T^a \chi_{\bar{\mathbf{p}}} \right](0) \left| 0 \right\rangle$$

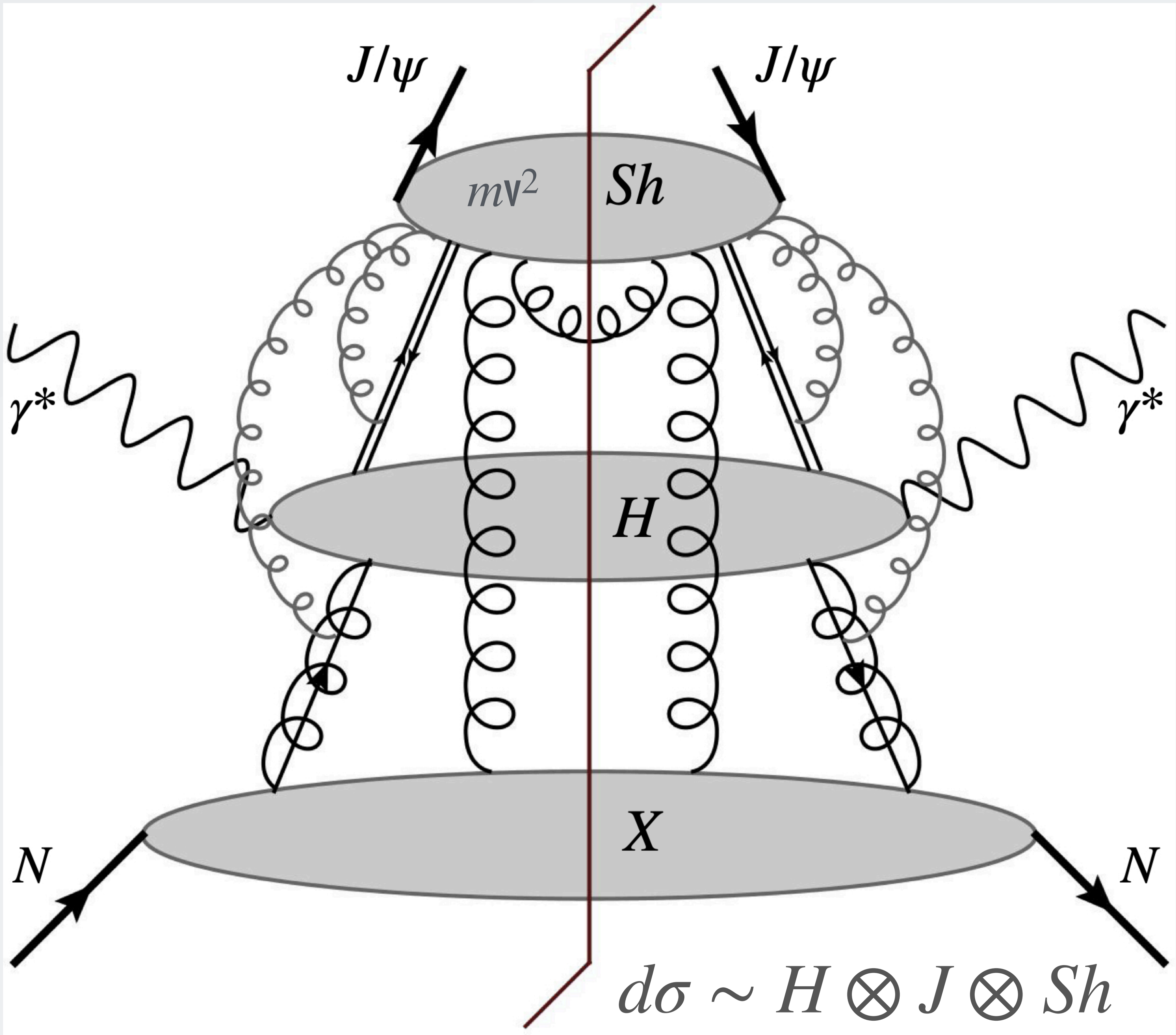
$$S^{\rho\sigma\rho'\sigma'(0)}_{{}^3P_J^{[8]}\to J/\psi}(\mathbf{b}_\perp) = \frac{\Lambda^{\rho'}_{i'} \Lambda^{\sigma'}_{j'} \Lambda^{\rho}_{i} \Lambda^{\sigma}_{j}}{M^2(N_c^2 - 1)}$$

$$\times\, \mathrm{tr}_c \left\langle 0 \right| \left[(\mathcal{S}_v \mathcal{S}_n)^\dagger \chi^\dagger_{\bar{\mathbf{p}}} \sigma^{i'} q^{j'} T^a \psi_{\mathbf{p}} \right](\mathbf{b}_\perp) \mathcal{N}_\psi \left[\mathcal{S}_v \mathcal{S}_n \psi^\dagger_{\mathbf{p}} \sigma^i q^j T^a \chi_{\bar{\mathbf{p}}} \right](0) \left| 0 \right\rangle$$

- **Rapidity renormalization:**

$$G^{\alpha\beta}_{g/P}(x, \mathbf{b}_\perp) = J^{\alpha\beta(0)}_n(x, \mathbf{b}_\perp) \sqrt{S(\mathbf{b}_\perp)} \qquad S_{N\to J/\psi}(\mathbf{b}_\perp) = \frac{S^{(0)}_{N\to J/\psi}(\mathbf{b}_\perp)}{\sqrt{S(\mathbf{b}_\perp)}}$$

Just enough, the shape function only has RDs in the n-collinear sector

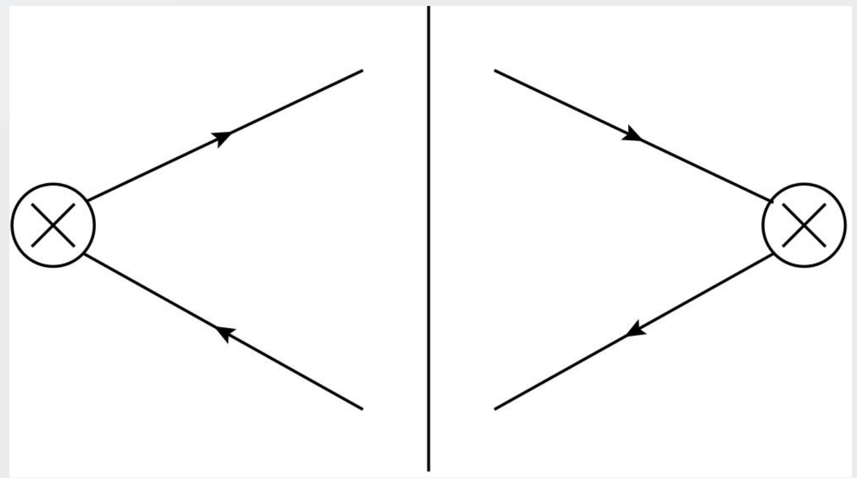


$d\sigma \sim H \otimes J \otimes Sh$

Calculation at NLO

The calculation for the P-wave is analogous

- **Leading order:**



$$\left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}} = M^2\, \eta^\dagger T^a \xi \times \xi^\dagger T^a \eta$$

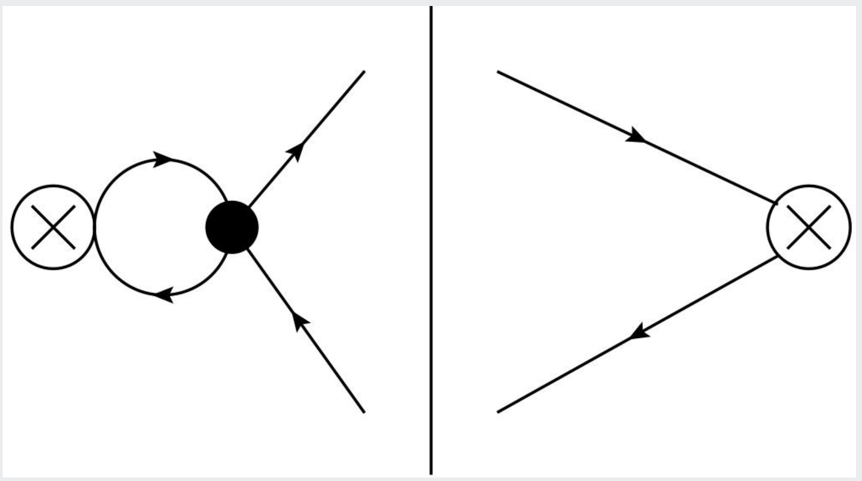
$$S^{\text{LO}}_{{}^1S_0^{[8]} \to J/\psi}(\mathbf{k}_\perp) = \delta^{(2)}(\mathbf{k}_\perp) \left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}}$$

- **LDME:**

$$\left\langle {}^{2S+1}L_J^{[8]} \right\rangle = \frac{4\alpha_s}{3\pi m_c^2} \left(C_F \left\langle {}^{2S+1}L'^{[1]}_{J'} \right\rangle^{\text{LO}} + B_F \left\langle {}^{2S+1}L'^{[8]}_{J'} \right\rangle^{\text{LO}} \right) \left(\frac{1}{\varepsilon_{\text{UV}}} - \frac{1}{\varepsilon_{\text{IR}}} \right)$$

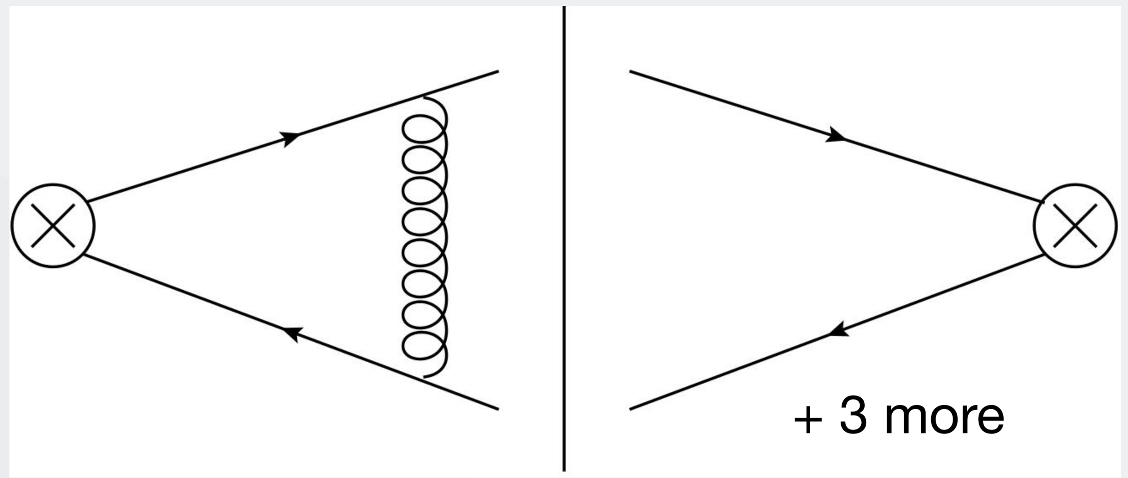
$$\left\langle {}^1S_0^{[8]} \right\rangle = \left(1 + \underbrace{(C_F - C_A/2)\, \frac{\pi\alpha_s}{2\text{v}}}_{\text{Coulomb singularity}} \right) \left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}} + \frac{4\alpha_s}{3\pi m_c^2} \underbrace{\left(C_F \left\langle {}^1P_1^{[1]} \right\rangle^{\text{LO}} + B_F \left\langle {}^1P_1^{[8]} \right\rangle^{\text{LO}} \right)}_{\text{Mixing between channels}} \left(\frac{1}{\varepsilon_{\text{UV}}} - \frac{1}{\varepsilon_{\text{IR}}} \right)$$

$$\mathscr{L}^{\text{Coul.}} = \sum_{\mathbf{p},\mathbf{p}'} \frac{4\pi\alpha_s}{(\mathbf{p}-\mathbf{p}')^2}\, \psi^\dagger_{\mathbf{p}} T^a \psi_{\mathbf{p}'}\, \chi^\dagger_{-\mathbf{p}} \bar{T}^a \chi_{-\mathbf{p}'}$$



$$\mathscr{L}^{\text{Chro.}} = -\, g_s \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \left(\frac{\mathbf{B}_{us} \cdot \mathbf{P}}{m_c} \right) \psi_{\mathbf{p}}(x) + (\psi \to \chi)$$

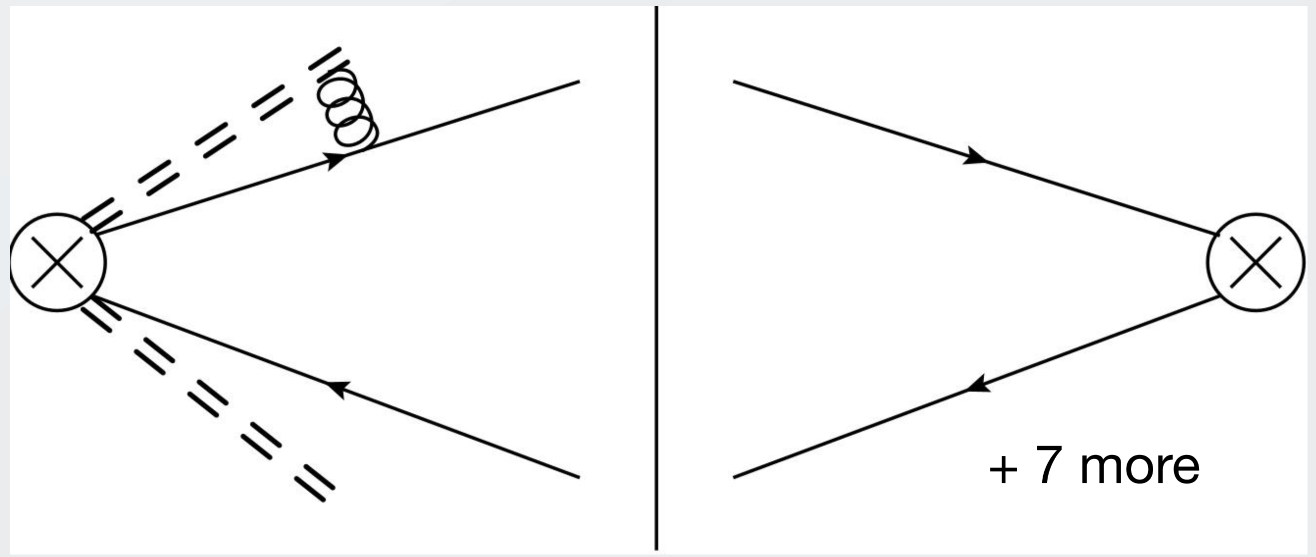
After BPS field redefinition



Calculation at NLO

- **TMDShF:**

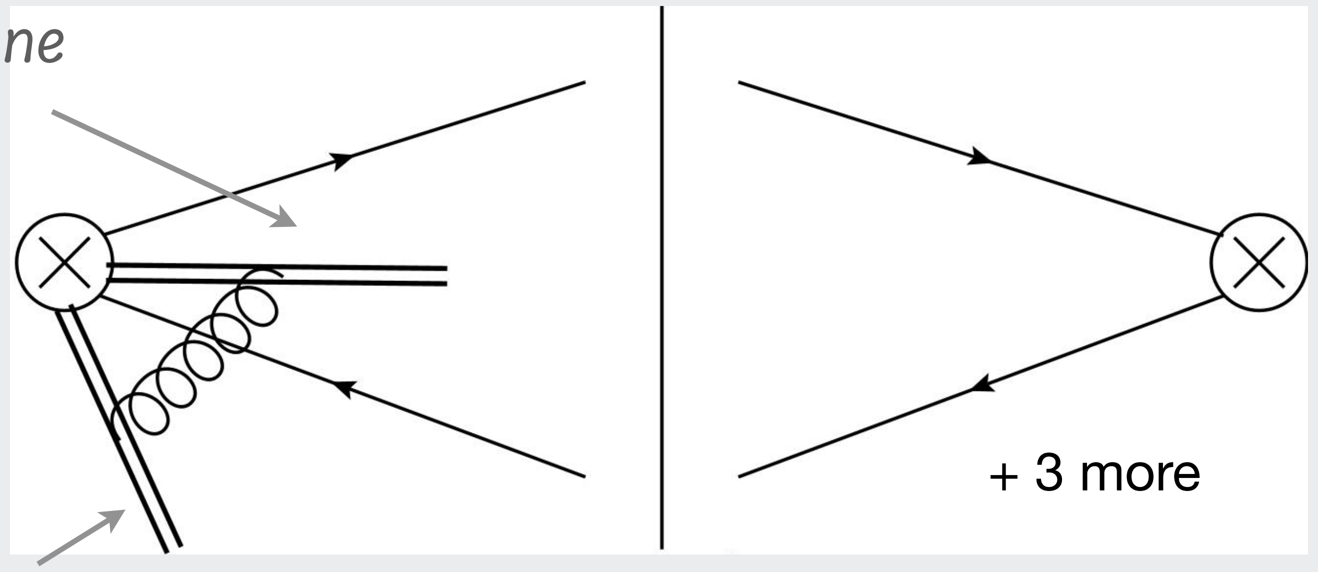
~~I)~~



Vanishes in pairs

quarkonium Wilson line

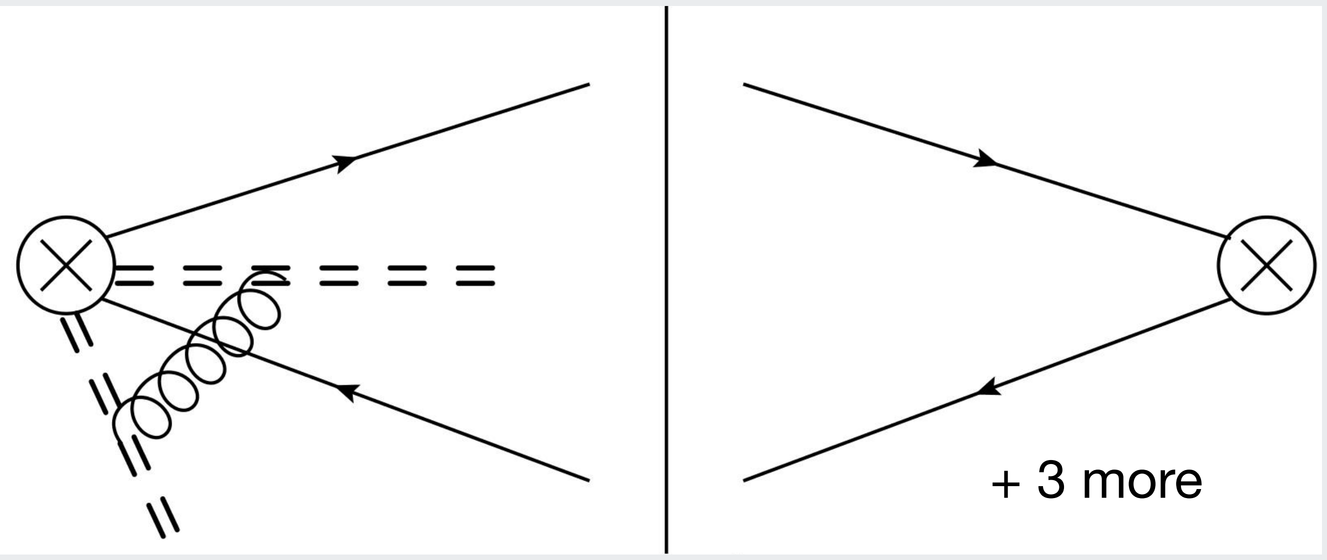
II)



proton Wilson line

Only this set of diagrams survives

~~III)~~



Double-counting

══ *soft*

-------- *usoft*

+ Coulomb + Chromo-electric

The calculation for the P-wave is analogous

δ-regularization

$$S_{{}^1S_0^{[8]}}(\mathbf{k}_\perp;\mu,\delta) = \frac{\alpha_s}{2\pi}\delta^2(\mathbf{k}_\perp)\underbrace{\left((C_F - C_A/2)\frac{\pi^2}{v} - \frac{C_A}{\varepsilon_{\rm IR}}\right)}_{\text{Coulomb}}\left\langle {}^1S_0^{[8]}\right\rangle^{\rm LO}$$

$$-\frac{\alpha_s C_A}{2\pi^2}\pi\delta^2(\mathbf{k}_\perp)\underbrace{\left(\frac{1}{\varepsilon_{\rm UV}^2} - \frac{1}{\varepsilon_{\rm UV}}\ln\frac{\delta^2}{\mu^2} + \frac{1}{2}\ln^2\frac{\delta^2}{\mu^2} + \frac{\pi^2}{4}\right)}_{\text{II}}\left\langle {}^1S_0^{[8]}\right\rangle^{\rm LO}$$

$$-\frac{\alpha_s C_A}{2\pi^2}\underbrace{\frac{1}{\mathbf{k}_\perp^2 - \delta^2}\ln\left(\frac{\delta^2}{\mathbf{k}_\perp^2}\right)}_{\text{II}}\left\langle {}^1S_0^{[8]}\right\rangle^{\rm LO}$$

Half of the soft function

$$+\frac{\alpha_s C_A}{2\pi^2}\left(\underbrace{\frac{\pi}{\varepsilon_{\rm UV}}\delta^2(\mathbf{k}_\perp)}_{\text{II}} - \underbrace{\frac{1}{\mathbf{k}_\perp^2}}_{\text{II}}\right)\left\langle {}^1S_0^{[8]}\right\rangle^{\rm LO}$$

$$+\frac{4\alpha_s}{3\pi^2 m_c^2}\frac{1}{\mathbf{k}_\perp^2}\underbrace{\left(C_F\left\langle {}^1P_1^{[1]}\right\rangle^{\rm LO} + B_F\left\langle {}^1P_1^{[8]}\right\rangle^{\rm LO}\right)}_{\text{Chromo-electric}}$$

RG evolution: LDME

$$\frac{d}{d\ \ln\mu}\left\langle \mathcal{O}^{n}_{\psi}\right\rangle^{\mu} = \sum_{m} \gamma^{nm}_{\mathcal{O}}\left\langle \mathcal{O}^{m}_{\psi}\right\rangle^{\mu}$$

$$\left\langle \mathcal{O}^{n}_{\psi}\right\rangle^{\mu} = Z^{nm}_{\mathcal{O}}\left\langle \mathcal{O}^{m}_{\psi}\right\rangle^{\mu}$$

$$Z_{{}^1S_0^{[8]}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{4\alpha_s(\mu)}{3\pi m^2}\frac{1}{\varepsilon_{\text{UV}}}\begin{pmatrix} 0 & C_F & B_F \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\gamma_{{}^1S_0^{[8]}} = \frac{8\alpha_s(\mu)}{3\pi m^2}\begin{pmatrix} 0 & C_F & B_F \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\frac{d}{d\ \ln\mu}\left\langle {}^1S_0^{[8]}\right\rangle = \frac{8\alpha_s(\mu)}{3\pi m^2}\left(C_F\left\langle {}^1P_1^{[1]}\right\rangle + B_F\left\langle {}^1P_1^{[8]}\right\rangle\right)$$

$$\left\langle {}^1S_0^{[8]}\right\rangle^{\mu} = \mathcal{U}^{1m}(\mu,\mu_f)\langle \mathcal{O}^m\rangle^{\mu_f}$$

$$\left(\begin{array}{l} \left\langle {}^1S_0^{[8]}\right\rangle^{\mu} = \left\langle {}^1S_0^{[8]}\right\rangle^{\mu_f} - \dfrac{8}{3m^2\beta_0}\ln\left(\dfrac{\alpha_s(\mu)}{\alpha_s(\mu_f)}\right)\left(C_F\left\langle {}^1P_1^{[1]}\right\rangle^{\mu_f} + B_F\left\langle {}^1P_1^{[8]}\right\rangle^{\mu_f}\right) \\ \left\langle {}^3P_J^{[8]}\right\rangle^{\mu} = \left\langle {}^3P_J^{[8]}\right\rangle^{\mu_f} - \dfrac{8}{3m^2\beta_0}\ln\left(\dfrac{\alpha_s(\mu)}{\alpha_s(\mu_f)}\right)\left(C_F\left\langle {}^3D_{J+1}^{[1]}\right\rangle^{\mu_f} + B_F\left\langle {}^3D_{J+1}^{[8]}\right\rangle^{\mu_f}\right) \end{array}\right)$$

Up to NLL

RG evolution: TMDShF

$$L_T = \ln\left(\mu^2 b_T^2 e^{2\gamma_E}/4\pi\right)$$
$$Q_h^4 = \zeta_A\zeta_B$$

$$\frac{d}{d\ln\mu} S_{{}^1S_0^{[8]}\to J/\psi}(b_T;\zeta_B,\mu) = \gamma_{{}^1S_0^{[8]}}(\alpha_s(\mu),\zeta_B,\mu)$$

$$Z_{Sh} = 1 - \frac{\alpha_s C_A}{2\pi}\frac{1}{\varepsilon_{\mathrm{UV}}}\left(1-\ln\frac{\zeta_B}{Q_h^2}\right)$$

$$\gamma_{Sh} \equiv \frac{d}{d\ln\mu}\ln Z_{Sh} = \frac{\alpha_s C_A}{\pi}\left(1-\ln\frac{\zeta_B}{Q_h^2}\right)$$

Consistency result confirmed at NLO

$$\frac{d}{d\ln\zeta_B}\ln S_{{}^1S_0^{[8]}\to J/\psi}(b_T;\mu,\zeta_B) = -\mathcal{D}_g(b_T;\mu)$$

$$\mathcal{D}_g(b_T;\mu) = \frac{\alpha_s C_A}{2\pi}L_T$$

Same as for the TMPDF:

M.G. Echevarria, T. Kasemets, P.J. Mulders, C. Pisano, arXiv:1502.05354
(Higgs production in hadron-hadron collision)

$$\left(\begin{array}{l}
S_{{}^1S_0^{[8]}\to J/\psi}(b_T;\mu_f,\zeta_f) = \exp\left[\int_P\left(\gamma_{{}^1S_0^{[8]}}(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}_g(b_T;\mu)\frac{d\zeta}{\zeta}\right)\right] S_{{}^1S_0^{[8]}\to J/\psi}(b_T;\mu_i,\zeta_i) \\
S_{{}^3P_J^{[8]}\to J/\psi}(b_T;\mu_f,\zeta_f) = \exp\left[\int_P\left(\gamma_{{}^1S_0^{[8]}}(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}_g(b_T;\mu)\frac{d\zeta}{\zeta}\right)\right] S_{{}^3P_J^{[8]}\to J/\psi}(b_T;\mu_i,\zeta_i)
\end{array}\right)$$

Matching onto LDMEs

$$N_{col.} = N_c \text{ or } N_c^2 - 1$$

$$N^S_{pol.} = d - 1$$

$$N^P_{pol.(0)} = 1$$

$$N^P_{pol.(1)} = \frac{(d-1)(d-2)}{2}$$

$$N^P_{pol.(2)} = \frac{(d+1)(d-2)}{2}$$

$$L_T = \ln\left(\mu^2 b_T^2 e^{2\gamma_E}/4\pi\right)$$

- **Operator product expansion:**

$$S_{N\to J/\psi}(b_T;\mu,\zeta_B) = \sum_n C_n^N(b_T;\mu,\zeta_B) \times \frac{\left\langle \mathcal{O}_\psi^n \right\rangle(\mu)}{N_{col.}N_{pol.}} + \mathcal{O}(b_T\Lambda_{QCD})$$

- **Renormalized LDME and TMDShF:**

$$\left\langle {}^1S_0^{[8]} \right\rangle = \left(1 + (C_F - C_A/2)\,\frac{\pi\alpha_s}{2\mathrm{v}}\right)\left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}} - \frac{4\alpha_s}{3\pi m^2}\frac{1}{\varepsilon_{\text{IR}}}\left(C_F\left\langle {}^1P_1^{[1]} \right\rangle^{\text{LO}} + B_F\left\langle {}^1P_1^{[8]} \right\rangle^{\text{LO}}\right)$$

$$S_{{}^1S_0^{[8]}\to J/\psi}(b_T;\mu,\zeta_B) = \left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}} + \frac{\alpha_s}{2\pi}\Bigg[\underset{\textit{Infrared part}}{\underline{\frac{\pi^2}{\mathrm{v}}(C_F - C_A/2)\left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}} - \frac{8}{3m^2}\frac{1}{\varepsilon_{\text{IR}}}\left(C_F\left\langle {}^1P_1^{[1]} \right\rangle^{\text{LO}} + B_F\left\langle {}^1P_1^{[8]} \right\rangle^{\text{LO}}\right)}}$$

$$+ C_A L_T\left(1 - \ln\frac{\zeta_B}{Q^2}\right)\left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}} - \frac{8}{3m^2}L_T\left(C_F\left\langle {}^1P_1^{[1]} \right\rangle^{\text{LO}} + B_F\left\langle {}^1P_1^{[8]} \right\rangle^{\text{LO}}\right)\Bigg]\Bigg\}$$

- **Matching coefficients:**

$$C^S_{{}^1S_0^{[8]}}(b_T;\mu,\zeta_B) = 1 + \frac{\alpha_s C_A}{2\pi}L_T\left(1 - \ln\frac{\zeta_B}{Q_h^2}\right)$$

$$C^S_{{}^1P_1^{[1]}}(b_T;\mu) = -\frac{\alpha_s}{2\pi}\frac{8\,C_F}{3m^2}L_T$$

$$C^S_{{}^1P_1^{[8]}}(b_T;\mu) = -\frac{\alpha_s}{2\pi}\frac{8\,B_F}{3m^2}L_T$$

It is analogous for the P-waves, the matching coefficients will be the same

Check: Hard function

- Virtual contribution of the cross section in **photoproduction** (F. Maltoni, M. L. Mangano, A. Petrelli, hep-ph/9708349):

$Q_h = M_\psi$

$$\sigma_N = \sigma_N^0 \left[1 + \frac{\alpha_s}{2\pi} \left\{ \left[\left(C_F - C_A/2 \right) \frac{\pi^2}{\mathrm{v}} - \frac{C_A}{\varepsilon_{\mathrm{IR}}} \right] - \left[\frac{C_A}{\varepsilon_{\mathrm{IR}}^2} + \frac{1}{\varepsilon_{\mathrm{IR}}} \left(b_0 + C_A \ln \frac{\mu^2}{Q_h^2} \right) \right] - C_A \left(\frac{b_0}{C_A} \ln \frac{\mu^2}{Q_h^2} + \ln \frac{\mu^2}{Q_h^2} + \frac{1}{2} \ln^2 \frac{\mu^2}{Q_h^2} + \frac{\pi^2}{12} \right) + \mathscr{D}_N \right\} \right] .$$

- Virtual contribution of the cross section in **leptoproduction** (M.G. Echevarria, R. Kishore, S.F. Romera, P. Taels, In progress):

$Q_h = f(Q, M_\psi) \gtrsim M_\psi$

$$\sigma_N = \sigma_N^0 \left[1 + \frac{\alpha_s}{2\pi} \left\{ \underbrace{\left[\left(C_F - C_A/2 \right) \frac{\pi^2}{\mathrm{v}} - \frac{C_A}{\varepsilon_{\mathrm{IR}}} \right]}_{\textit{TMDShF}} - \underbrace{\left[\frac{C_A}{\varepsilon_{\mathrm{IR}}^2} + \frac{1}{\varepsilon_{\mathrm{IR}}} \left(b_0 + C_A \ln \frac{\mu^2}{Q_h^2} \right) \right]}_{\textit{Gluon TMDPDF}} + H_N \right\} \right] .$$

TMDShF:

$$S_{{}^1S_0^{[8]} \to J/\psi}(\mathbf{k}_\perp; \mu, \delta^+) = \frac{S(\mathbf{k}_\perp^2; \mu, \delta^+)}{2} \left\langle {}^1S_0^{[8]} \right\rangle^{\mathrm{LO}} + \underbrace{\frac{\alpha_s}{2\pi} \delta^{(2)}(\mathbf{k}_\perp) \left((C_F - C_A/2) \frac{\pi^2}{\mathrm{v}} - \frac{C_A}{\varepsilon_{\mathrm{IR}}} \right) \left\langle {}^1S_0^{[8]} \right\rangle^{\mathrm{LO}}}_{\textit{Virtual contribution}}$$

$$\frac{\alpha_s C_A}{2\pi^2} \left(\frac{\pi}{\varepsilon_{\mathrm{UV}}} \delta^{(2)}(\mathbf{k}_\perp) - \frac{1}{\mathbf{k}_\perp^2} \right) \left\langle {}^1S_0^{[8]} \right\rangle^{\mathrm{LO}} + \frac{4\alpha_s}{3\pi^2 m^2} \frac{1}{\mathbf{k}_\perp^2} \left(C_F \left\langle {}^1P_1^{[1]} \right\rangle^{\mathrm{LO}} + B_F \left\langle {}^1P_1^{[8]} \right\rangle^{\mathrm{LO}} \right)$$

Summary

- In the vNRQCD+SCET framework, the precise definition of operators describing the incoming gluon and the heavy-quark pair within. the previously derived configurations is established.

- While the spin-triplet S-wave emerges as the leading power in the power counting expansion, the heavy-quark pair is not observed in this state.

- At LP in the λ-expansion, we established the factorization of the hadronic tensor into a TMDPDF and a TMDShF for the (u)soft radiation throughout the entire process and the formation of the bound state. Through the calculation, it became apparent that since the TMDShF possesses only one collinear direction, the definition provided is sufficient to eliminate any spurious rapidity regulators.

- We have studied the renormalization group evolution of the LDMEs and the TMDShFs, as well as calculated at next-to-leading order the Wilson matching.

- A concise examination of the process's hard function offers evidence supporting the results of the virtual contribution of the TMDShF and its associated anomalous dimension.

Future: color-singlet contribution, phenomenology, other processes…

Summary

- In the vNRQCD+SCET framework, the precise definition of operators describing the incoming gluon and the heavy-quark pair within. the previously derived configurations is established.

- While the spin-triplet S-wave emerges as the leading power in the power counting expansion, the heavy-quark pair is not observed in this state.

- At LP in the λ-expansion, we established the factorization of the hadronic tensor into a TMDPDF and a TMDShF for the (u)soft radiation throughout the entire process and the formation of the bound state. Through the calculation, it became apparent that since the TMDShF possesses only one collinear direction, the definition provided is sufficient to eliminate any spurious rapidity regulators.

- We have studied the renormalization group evolution of the LDMEs and the TMDShFs, as well as calculated at next-to-leading order the Wilson matching.

- A concise examination of the process's hard function offers evidence supporting the results of the virtual contribution of the TMDShF and its associated anomalous dimension.

Future: color-singlet contribution, phenomenology, other processes…

Thank you!

Backup slides

Matching tensors for J=0,1,2

- Tensorial decomposition:

$$q^i\sigma^j = \frac{\delta^{ij}}{3}\mathbf{q}\cdot\boldsymbol{\sigma} + \frac{e^{ijk}}{2}(\mathbf{q}\times\boldsymbol{\sigma})^k + q^{(i}\sigma^{j)}\ ,$$

- Operators in vNRQCD:

$$\mathcal{O}({}^3P_0^{[8]}) = \frac{1}{3}\psi_{\mathbf{q}}^\dagger\left(\frac{\overleftrightarrow{\mathcal{P}}}{2}\cdot\frac{\boldsymbol{\sigma}}{\sqrt{2}}\right)\sqrt{2}T^a\chi_{-\mathbf{q}} = \frac{1}{3}\psi_{\mathbf{q}}^\dagger\left(\mathbf{q}\cdot\boldsymbol{\sigma}\right)T^a\chi_{-\mathbf{q}}\ ,$$

$$\mathcal{O}^k({}^3P_1^{[8]}) = \frac{1}{2}\psi_{\mathbf{q}}^\dagger\left(\frac{(\overleftrightarrow{\mathcal{P}}\times\boldsymbol{\sigma})^k}{2\sqrt{2}}\right)\sqrt{2}T^a\chi_{-\mathbf{q}} = \frac{1}{2}\psi_{\mathbf{q}}^\dagger\left(\mathbf{q}\times\boldsymbol{\sigma}\right)^k T^a\chi_{-\mathbf{q}}\ ,$$

$$\mathcal{O}^{ij}({}^3P_2^{[8]}) = \psi_{\mathbf{q}}^\dagger\left(\frac{\overleftrightarrow{\mathcal{P}}^{(i}}{2}\frac{\sigma^{j)}}{\sqrt{2}}\right)\sqrt{2}T^a\chi_{-\mathbf{q}} = \psi_{\mathbf{q}}^\dagger\left(q^{(i}\sigma^{j)}\right)T^a\chi_{-\mathbf{q}}\ .$$

- Matching tensors:

$$\Gamma^{\mu\nu}_{{}^3P_0^{[8]}} = \Gamma^{\mu\nu\sigma\rho}_{{}^3P_J^{[8]}}\left(-g_{\sigma\rho} + \frac{P_\sigma P_\rho}{s}\right)\ , \qquad \Gamma^{\mu\nu}_{k,{}^3P_1^{[8]}} = \Gamma^{\mu\nu\sigma\rho}_{{}^3P_J^{[8]}}\epsilon_{\alpha\beta\sigma\rho}\Lambda^\alpha_k\frac{P^\beta}{M}\ , \qquad \Gamma^{\mu\nu}_{ij,{}^3P_2^{[8]}} = \Gamma^{\mu\nu\sigma\rho}_{{}^3P_J^{[8]}}\Lambda_{i\sigma}\Lambda_{j\rho}\ .$$