Lattice QCD for TMD Physics: Nonperturbative Collins Soper Kernel

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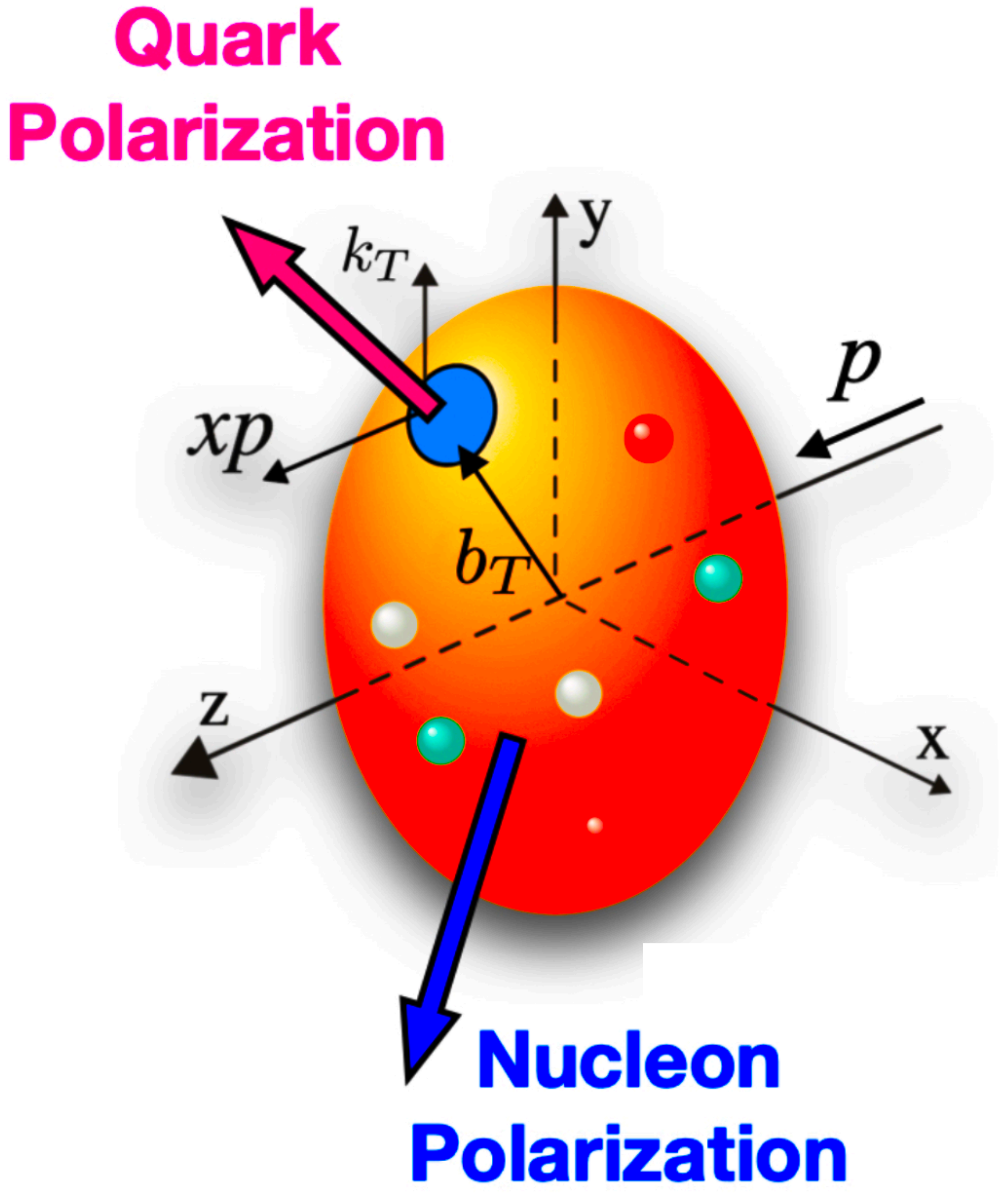
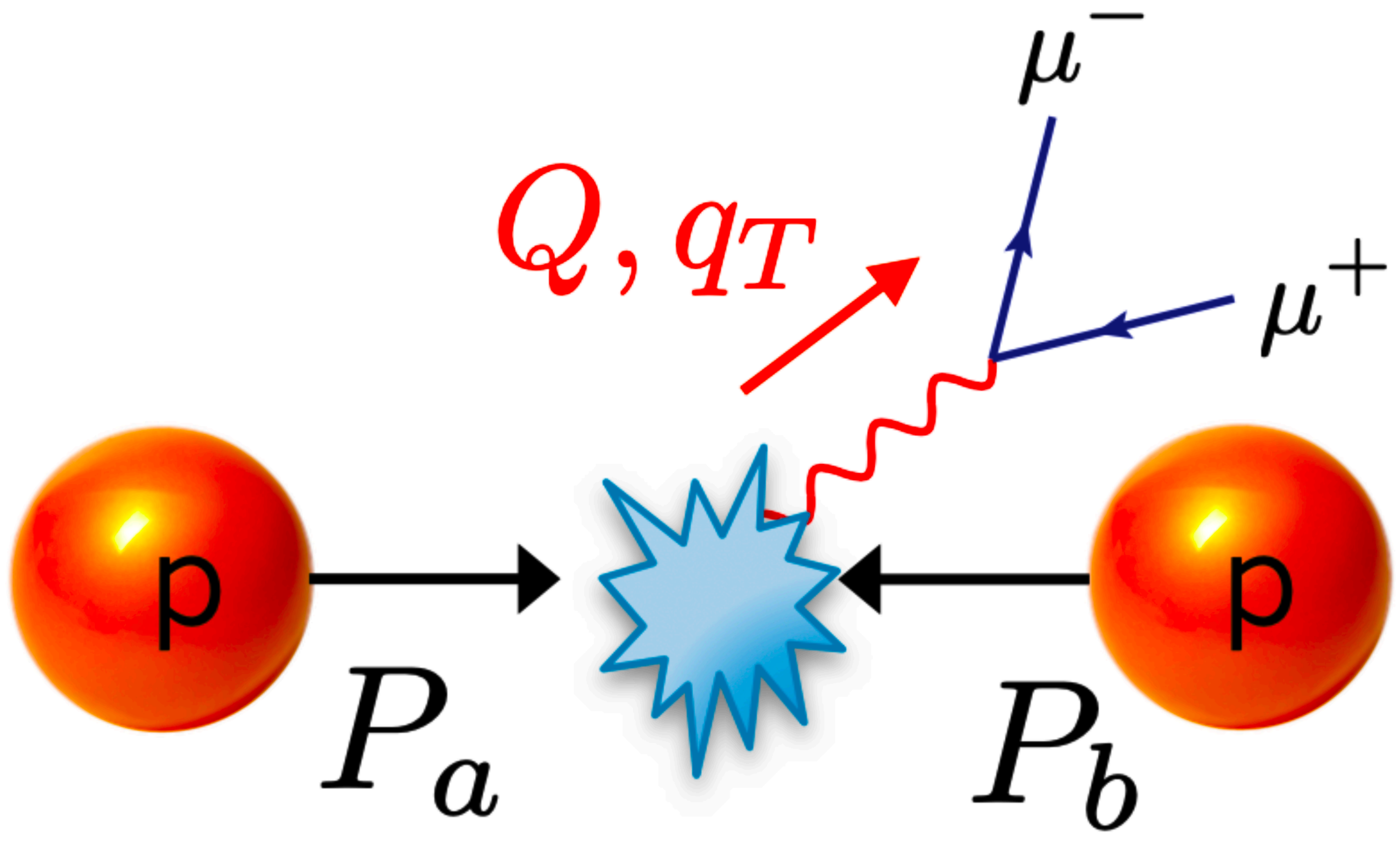
transverse momentum dependent (TMD) functions are ubiquitous

- TMD parton distribution function (TMDPDF)
  $\phi(x, k_\perp, \zeta, \mu)$

- TMD fragmentation function (TMDFF)
  $F(x, k_\perp, \zeta, \mu)$



transverse momentum dependent (TMD) functions are ubiquitous

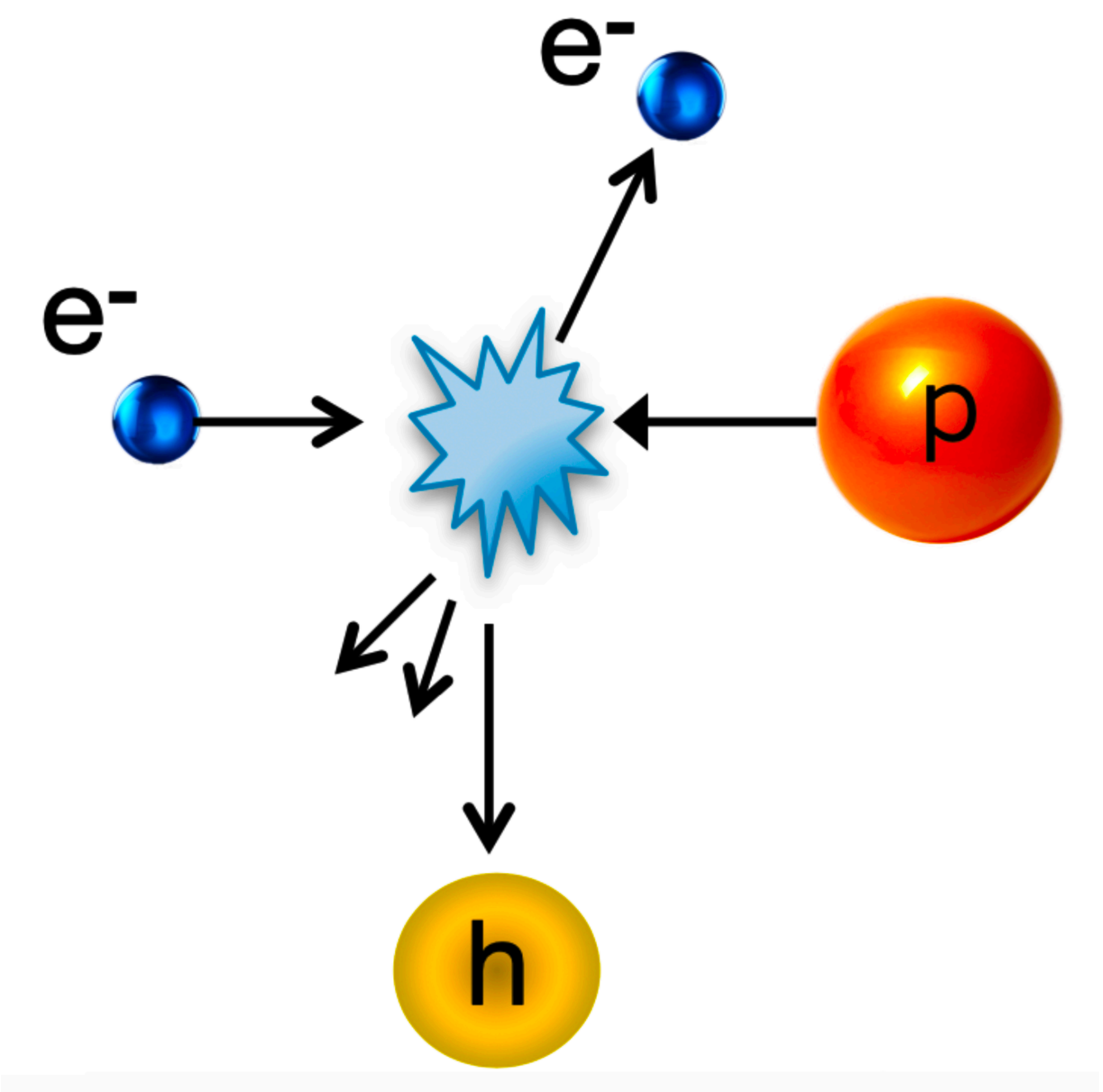


Drell-Yan processes

$$\sigma \sim \phi(x, k_\perp, \zeta, \mu) \otimes \phi(x, k_\perp, \zeta, \mu)$$

RHIC, LHC, …

transverse momentum dependent (TMD) functions are ubiquitous



semi-inclusive deep inelastic scatterings

$$\sigma \sim F(x, k_\perp, \zeta, \mu) \otimes \phi(x, k_\perp, \zeta, \mu)$$

JLAB, EIC, …

evolution of TMD functions across collision energies …

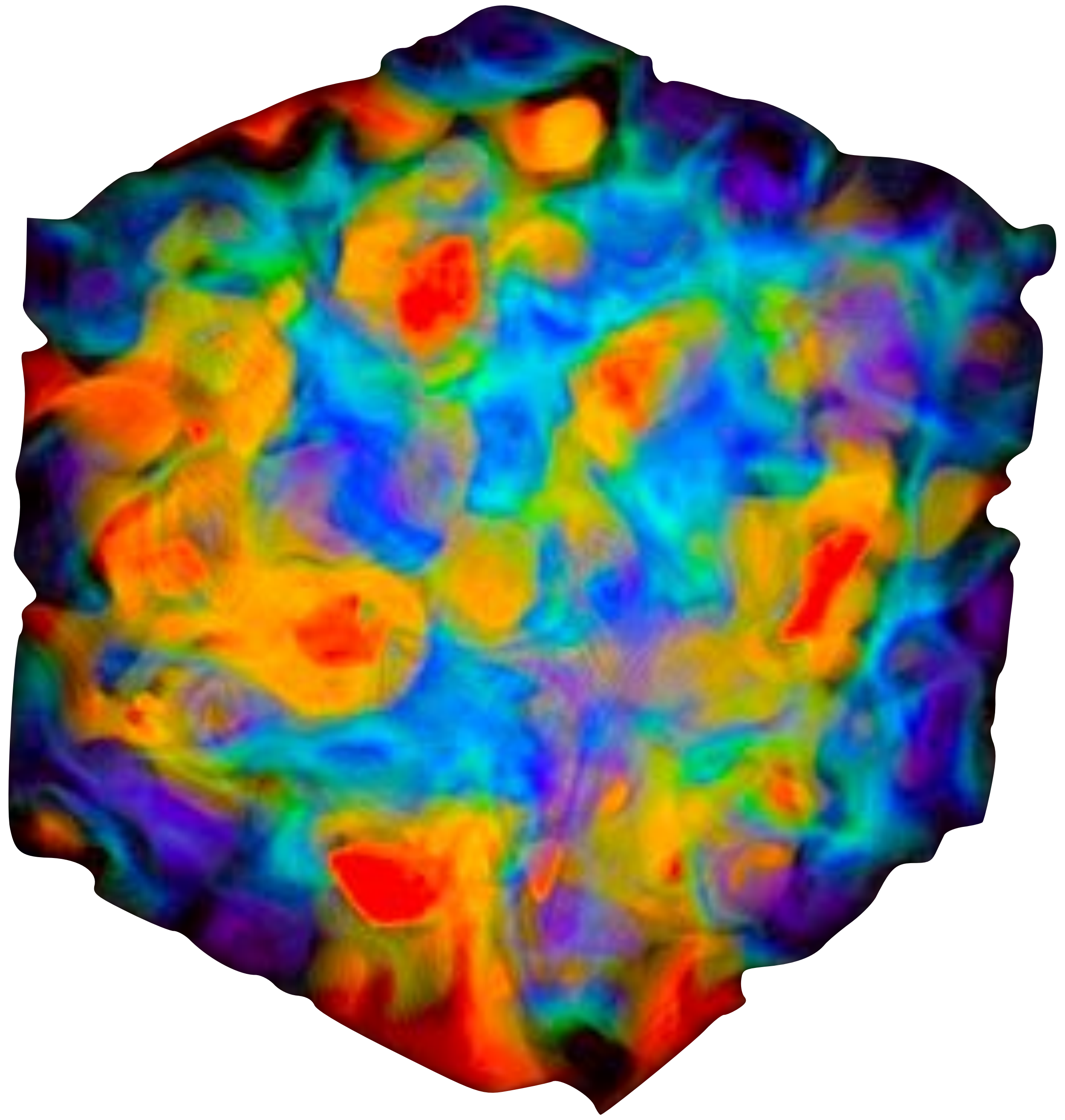
Collins Soper (CS) kernel

$$\boxed{\gamma^{\overline{\mathrm{MS}}}(b_\perp,\mu)} = \frac{\partial \phi(x, b_\perp, \zeta, \mu)}{\partial \ln \sqrt{\zeta}}$$

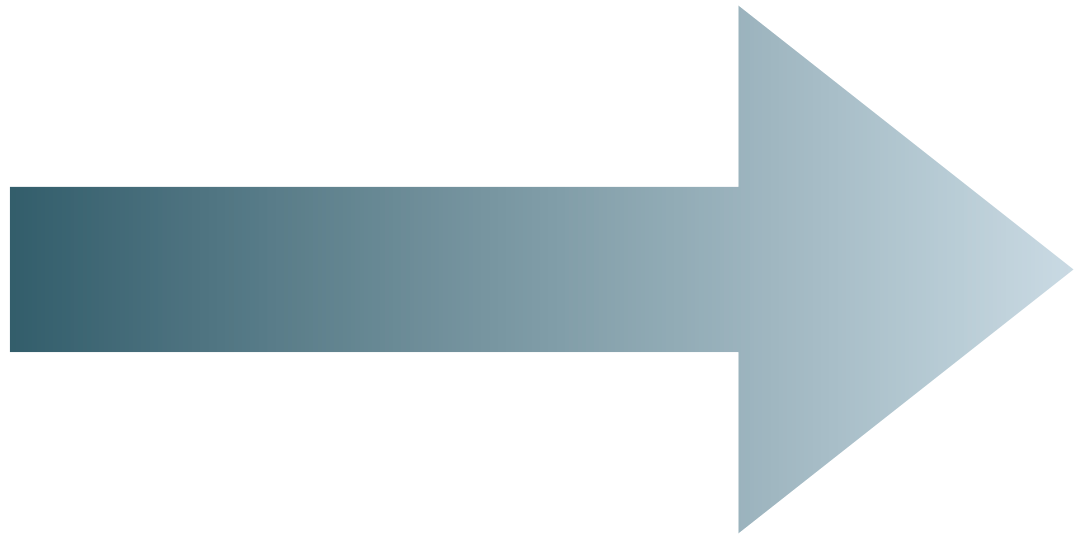
UV property of QCD— independent of hadronic state

nonperturbative CS kernel

our dream …



lattice QCD

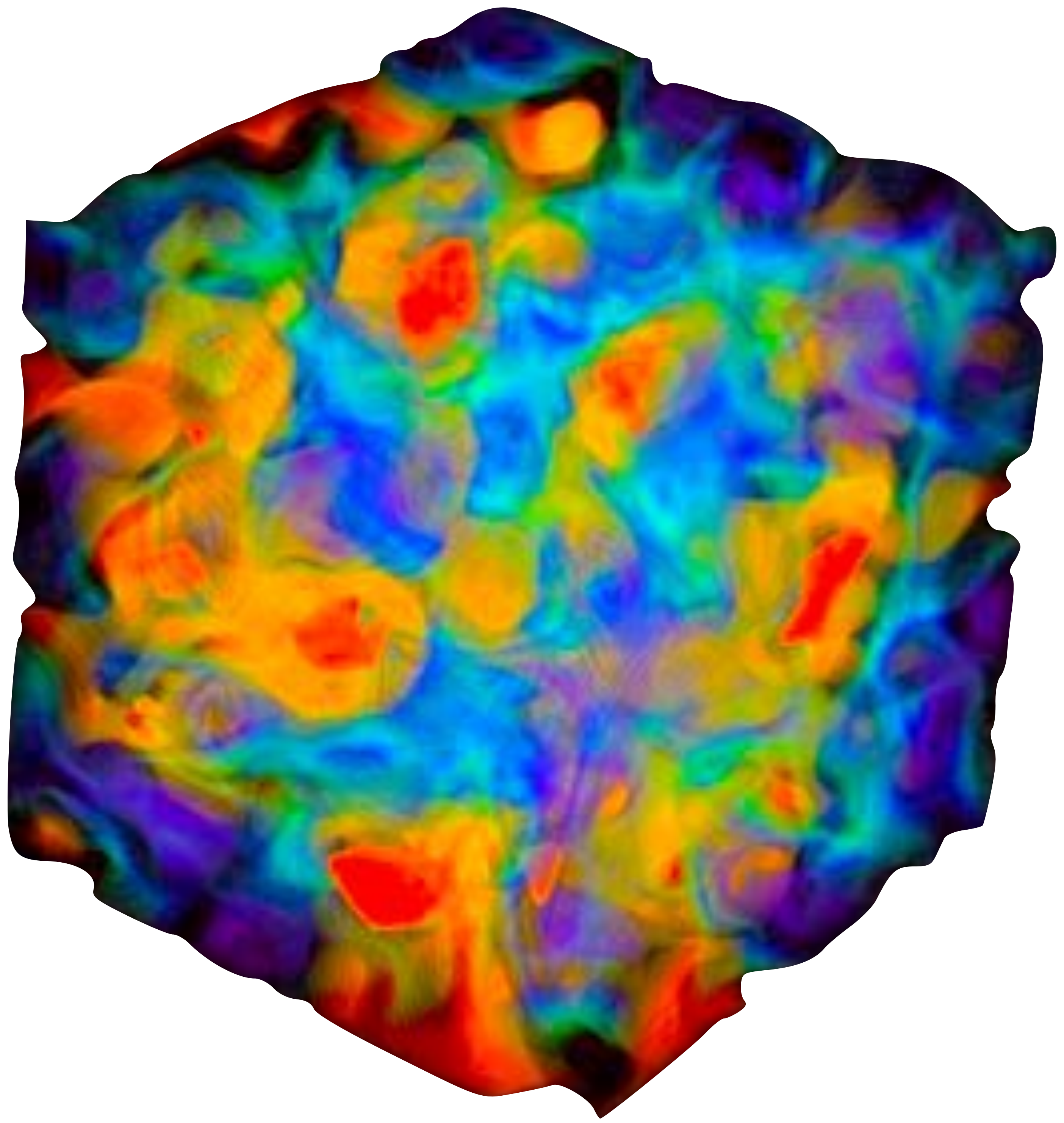


$$\gamma^{\overline{\mathrm{MS}}}(b_\perp, \mu)$$

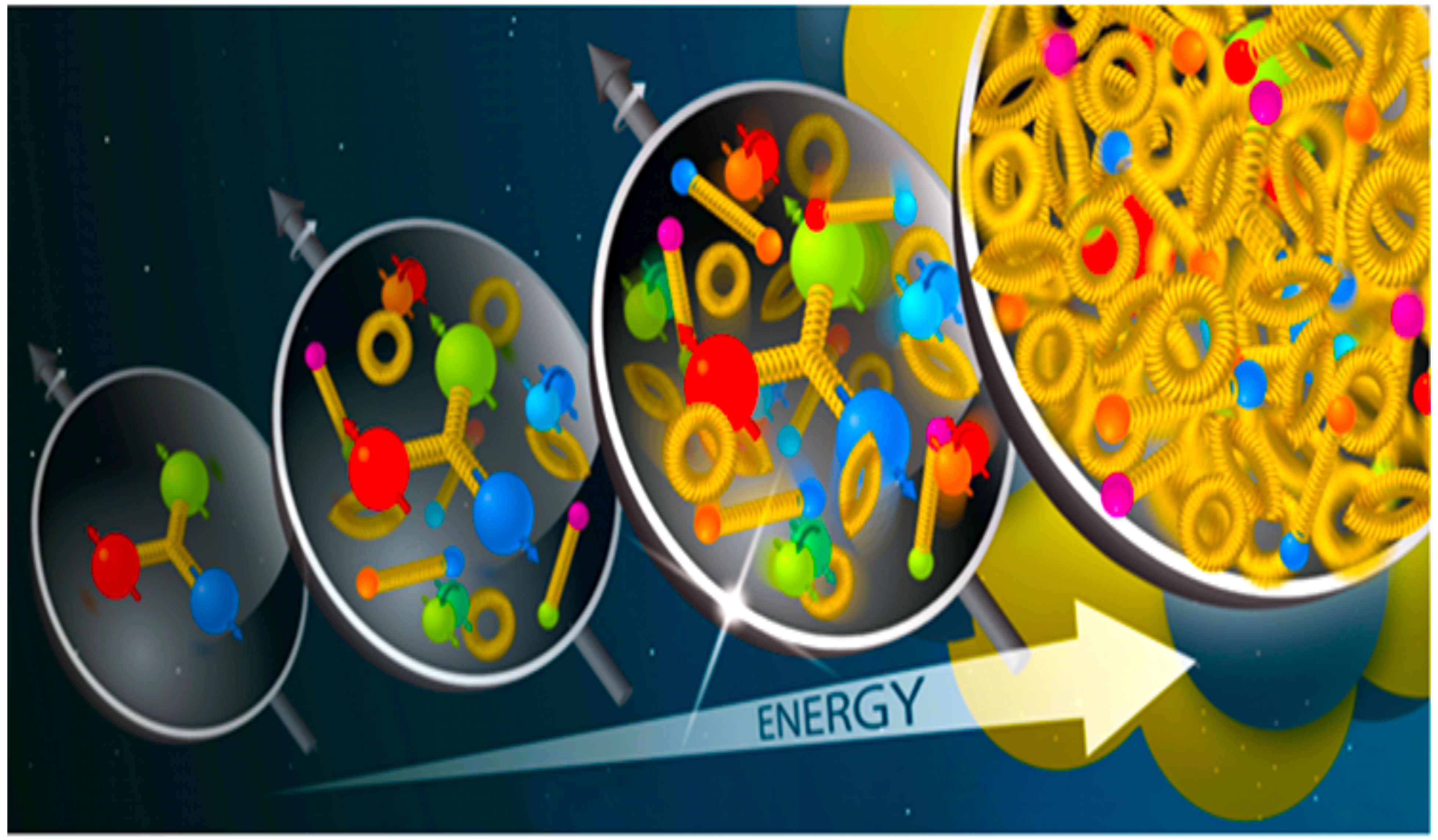
nonperturbative CS kernel

our challenge …

how to ‘see’ a parton on the lattice ?

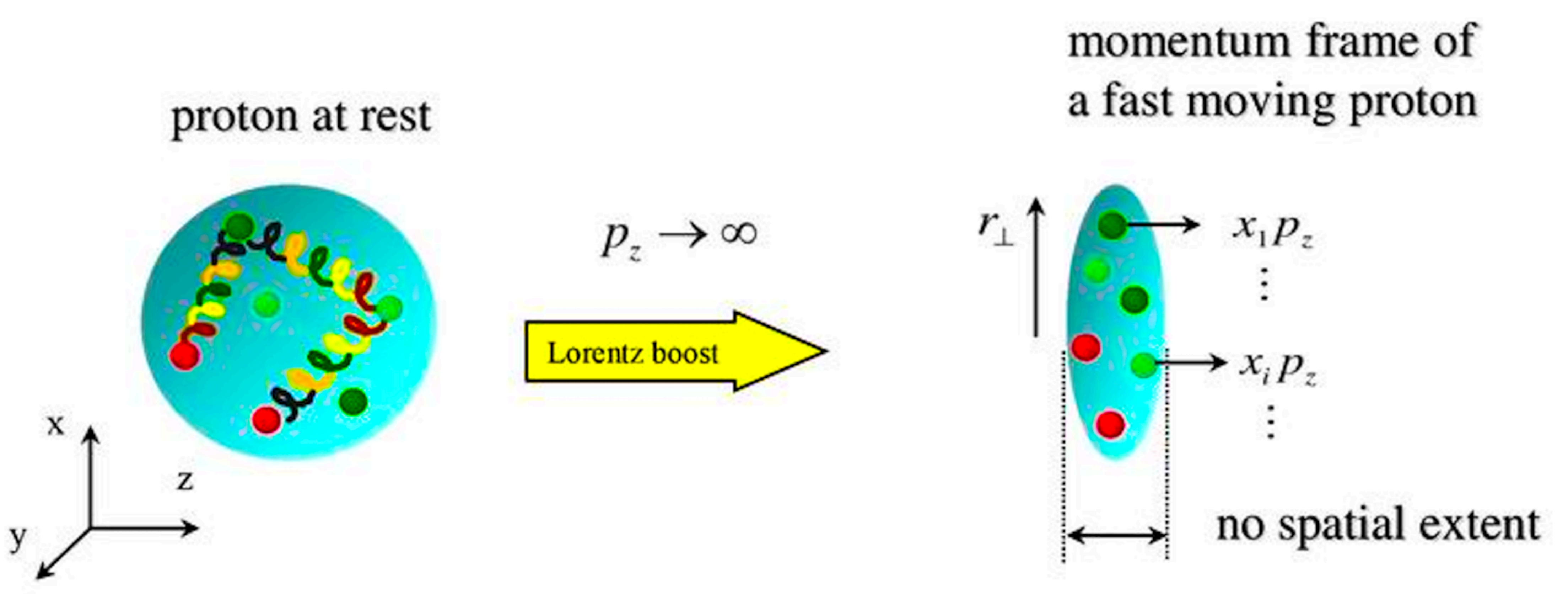


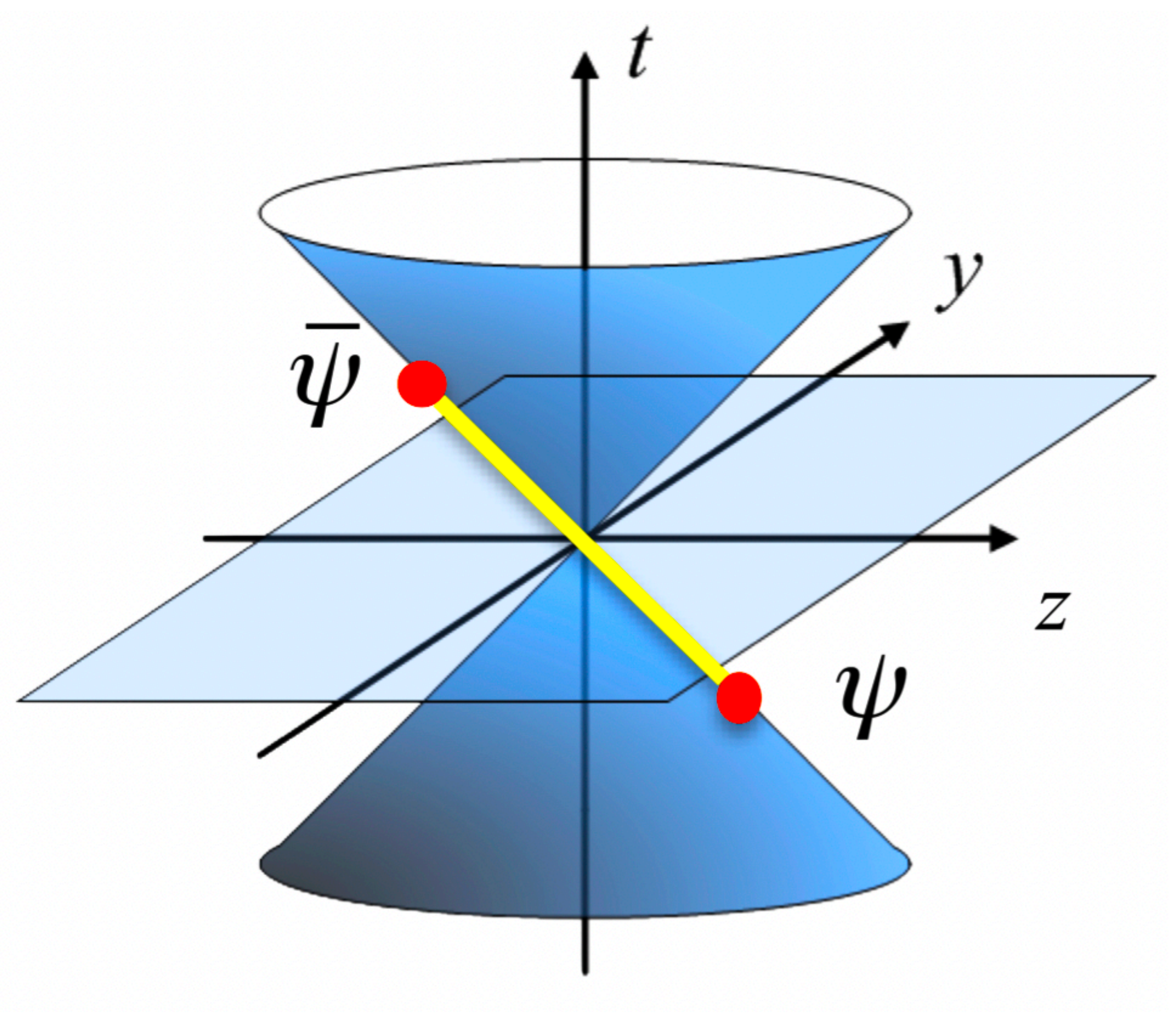
????



strongly coupled
quark, gluon fields

partonic picture

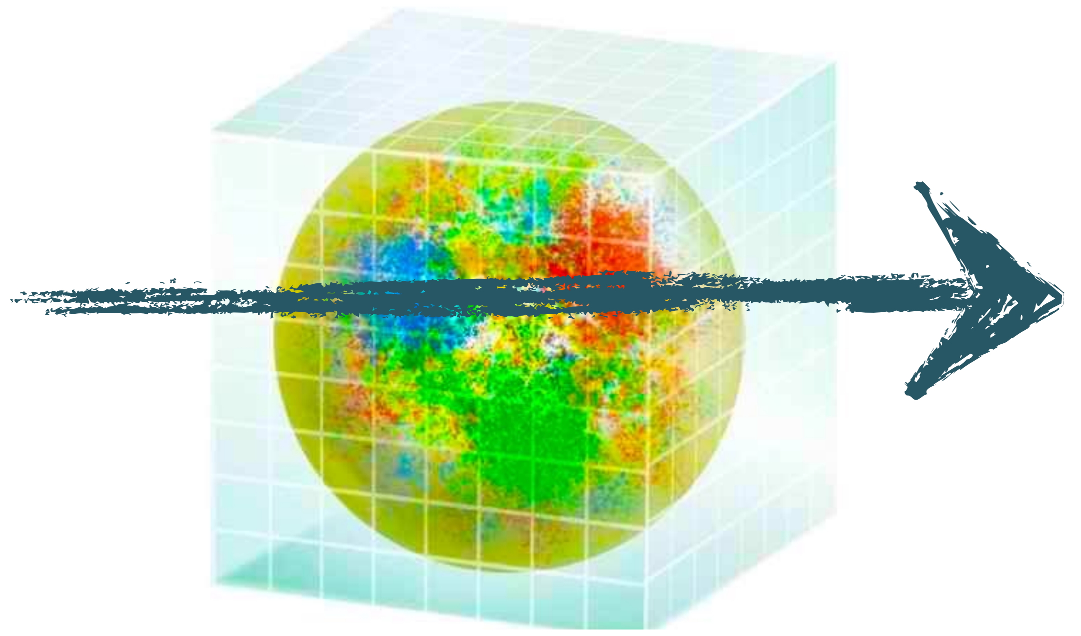




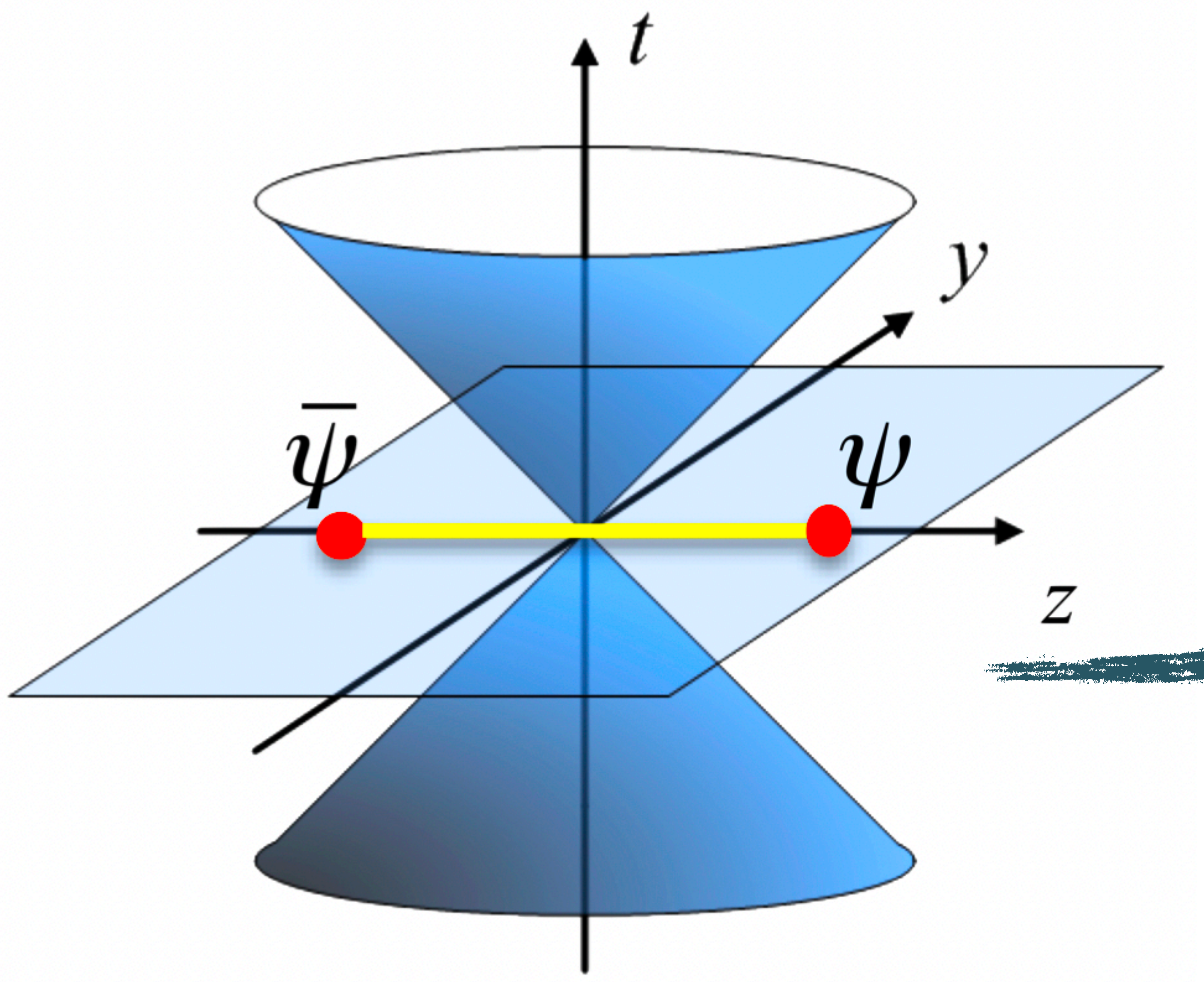
- QCD in infinite-momentum fame / probed at vanishingly short distances
- QCD simplified / effective description of on the lightcone
- $P_z \to \infty$ / $z^2 \to 0$ first, regularize QFT later

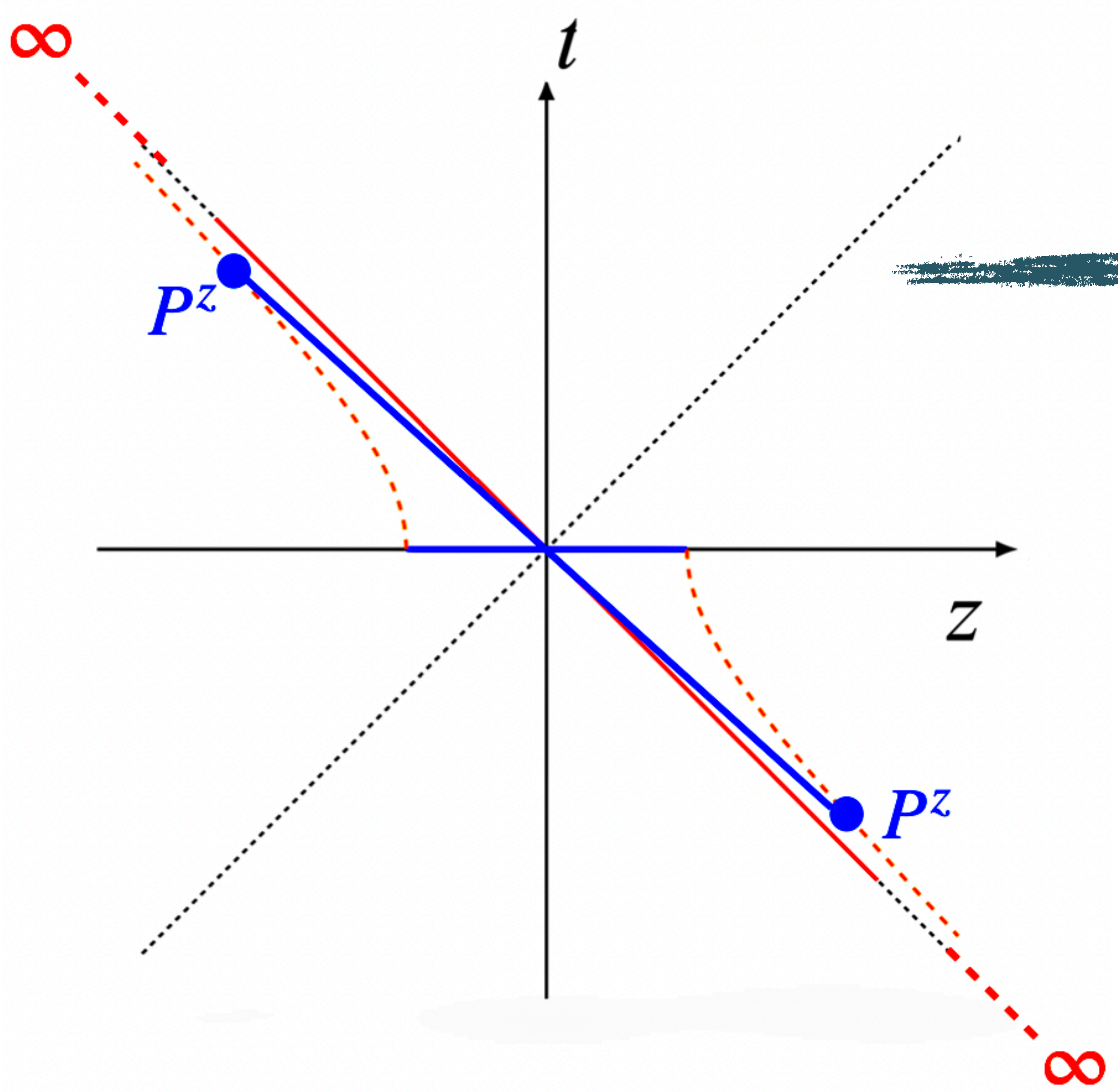
partonic structure from lattice QCD

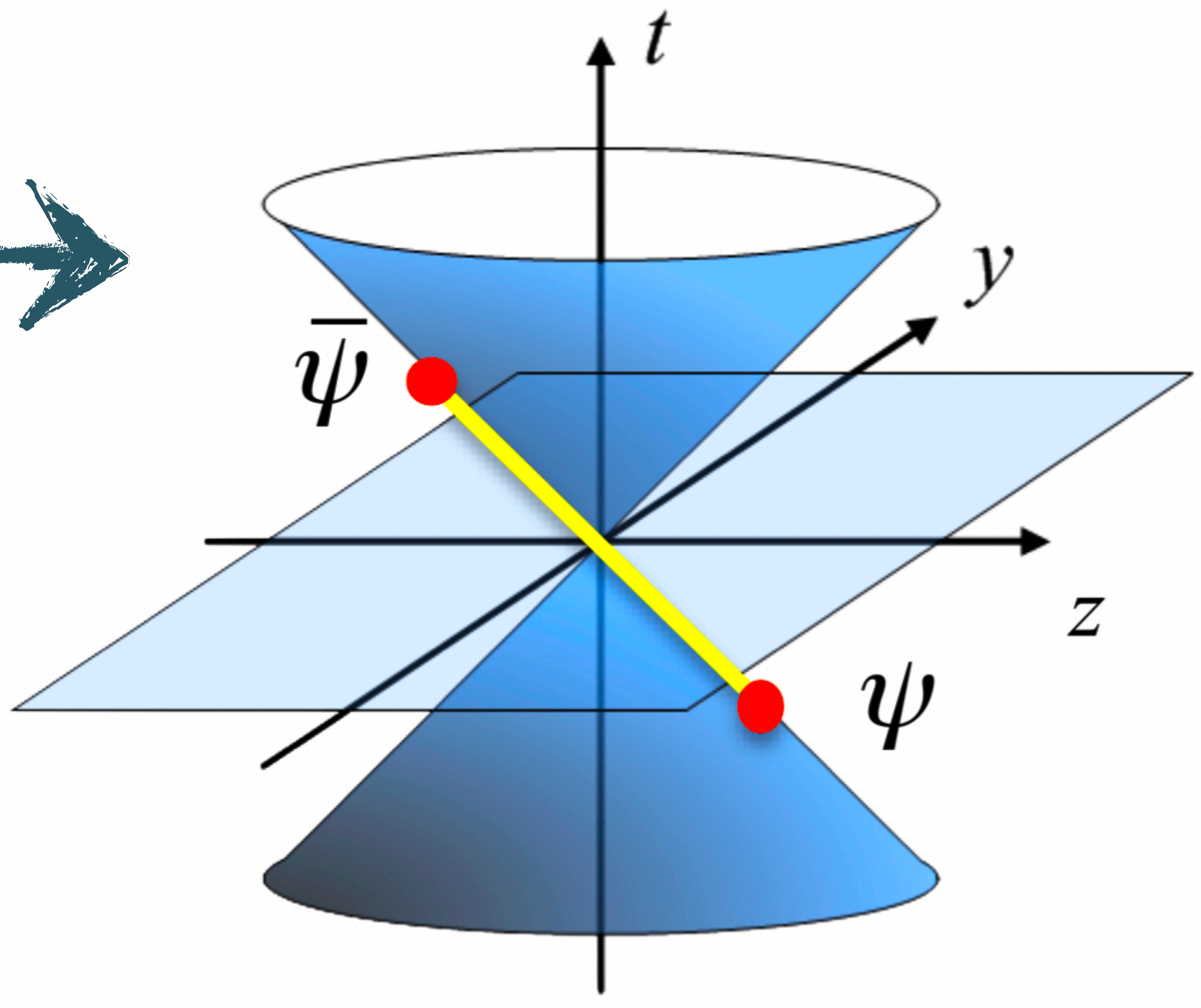
fast-moving hadron



$P_z \approx E$





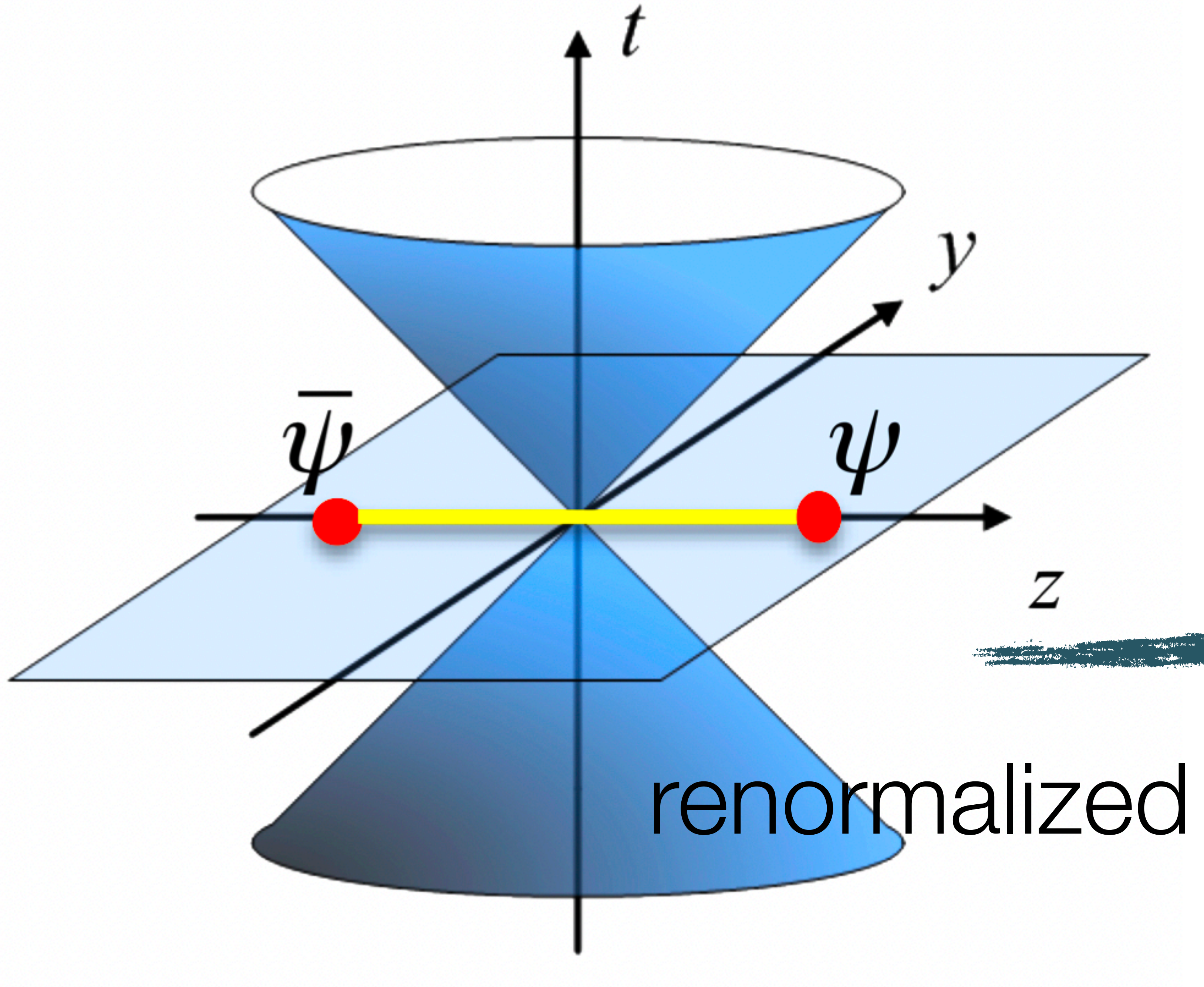


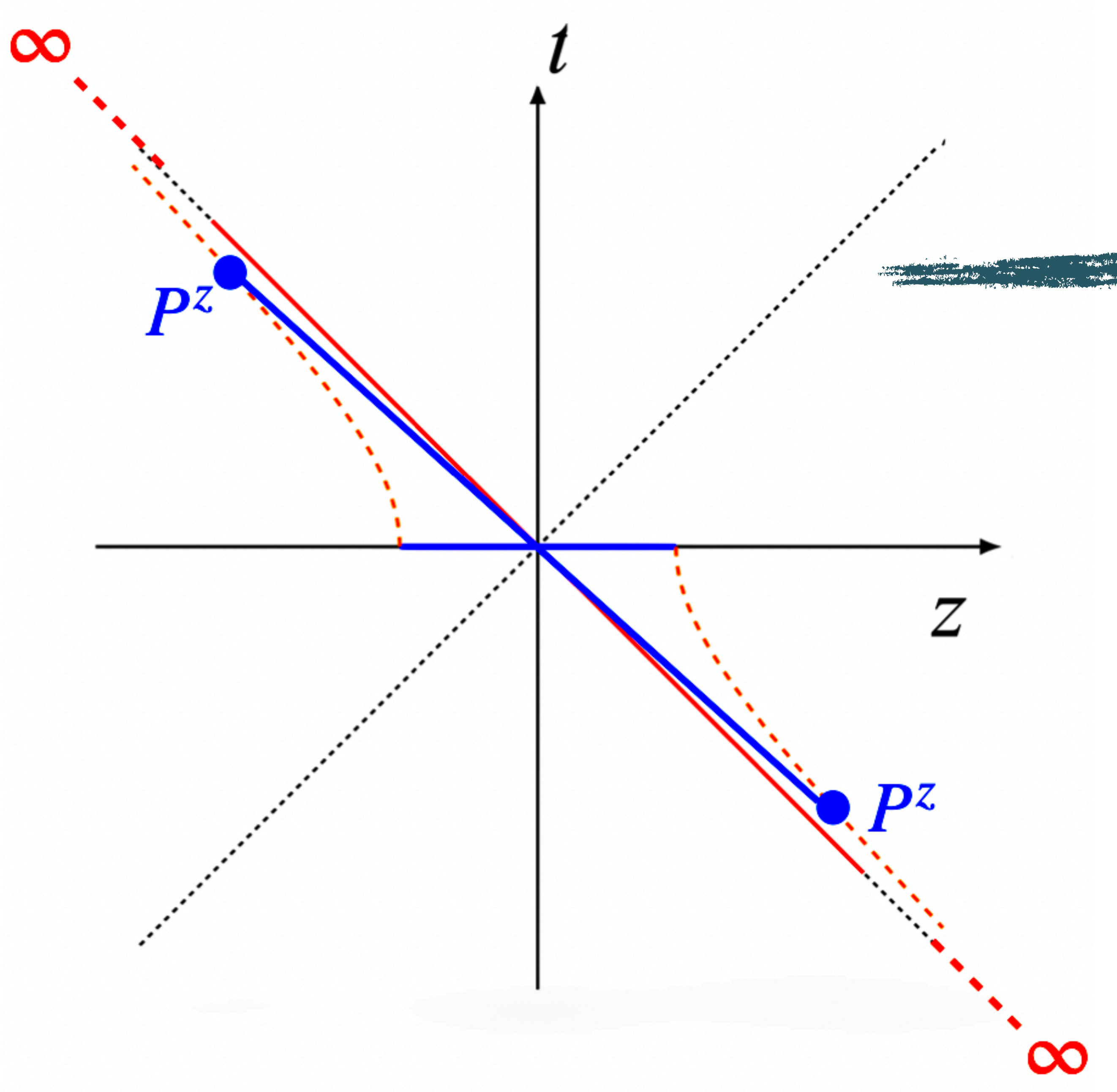
renormalize

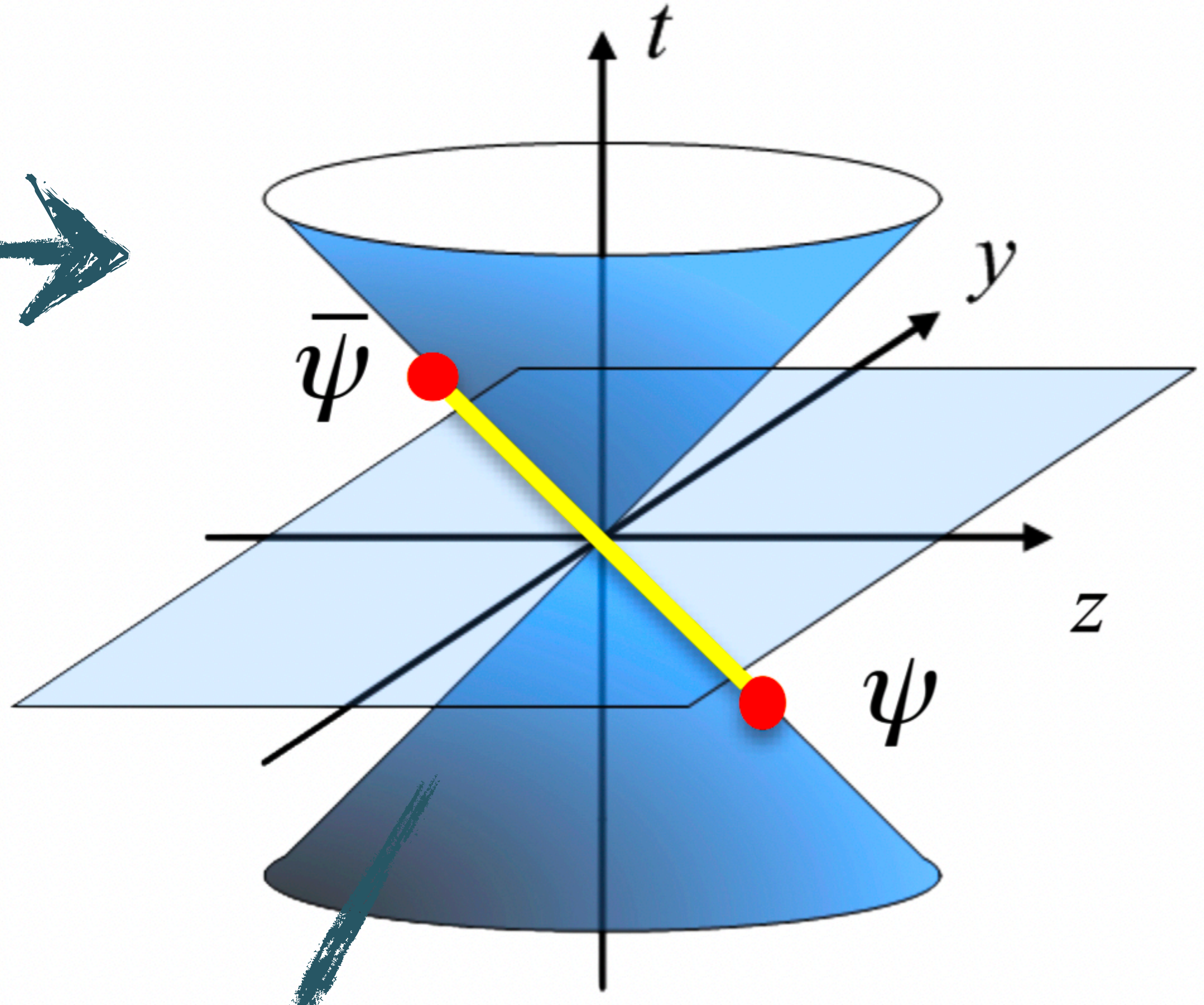
- first regularize QCD on a lattice, then $P_z \to \infty$ / $z^2 \to 0$

- opposite order of limits; two limits don't commute

- difference is UV physics, can be taken care of through pQCD







parton physics

pQCD

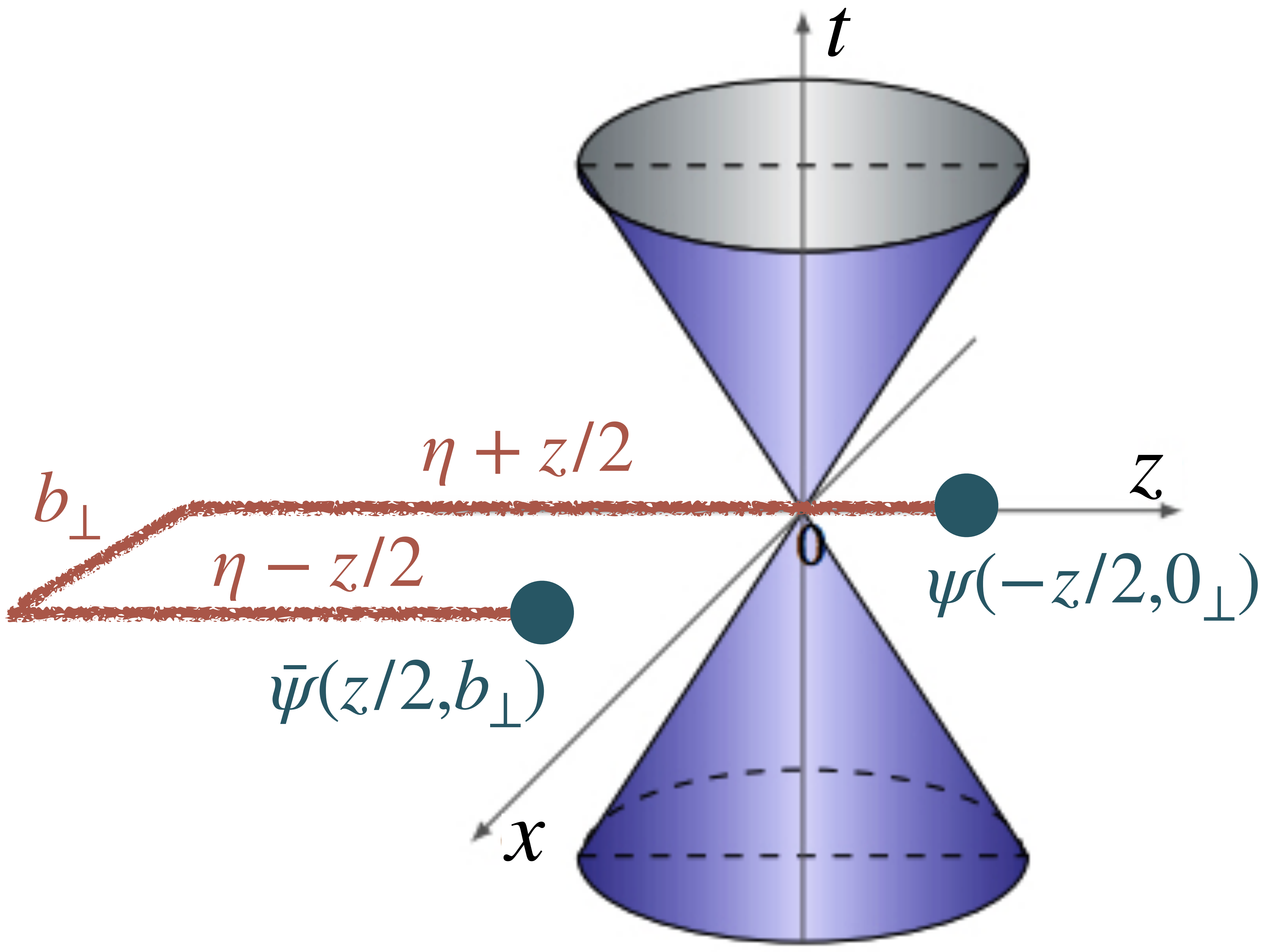
$$+\mathcal{O}\left[\frac{\Lambda_{\rm QCD}^2}{x^2P_z^2},\frac{\Lambda_{\rm QCD}}{(1-x)P_z},\frac{M_H^2}{P_z^2},\ldots\right]$$

$C(x, P_z, \mu) \otimes$ momentum space

$$+\mathcal{O}\left[z^2\Lambda_{\rm QCD}^2, z^2M_H^2, \ldots\right]$$

$C(\alpha, z^2, \mu) \otimes$ position space

TMD distributions from lattice QCD

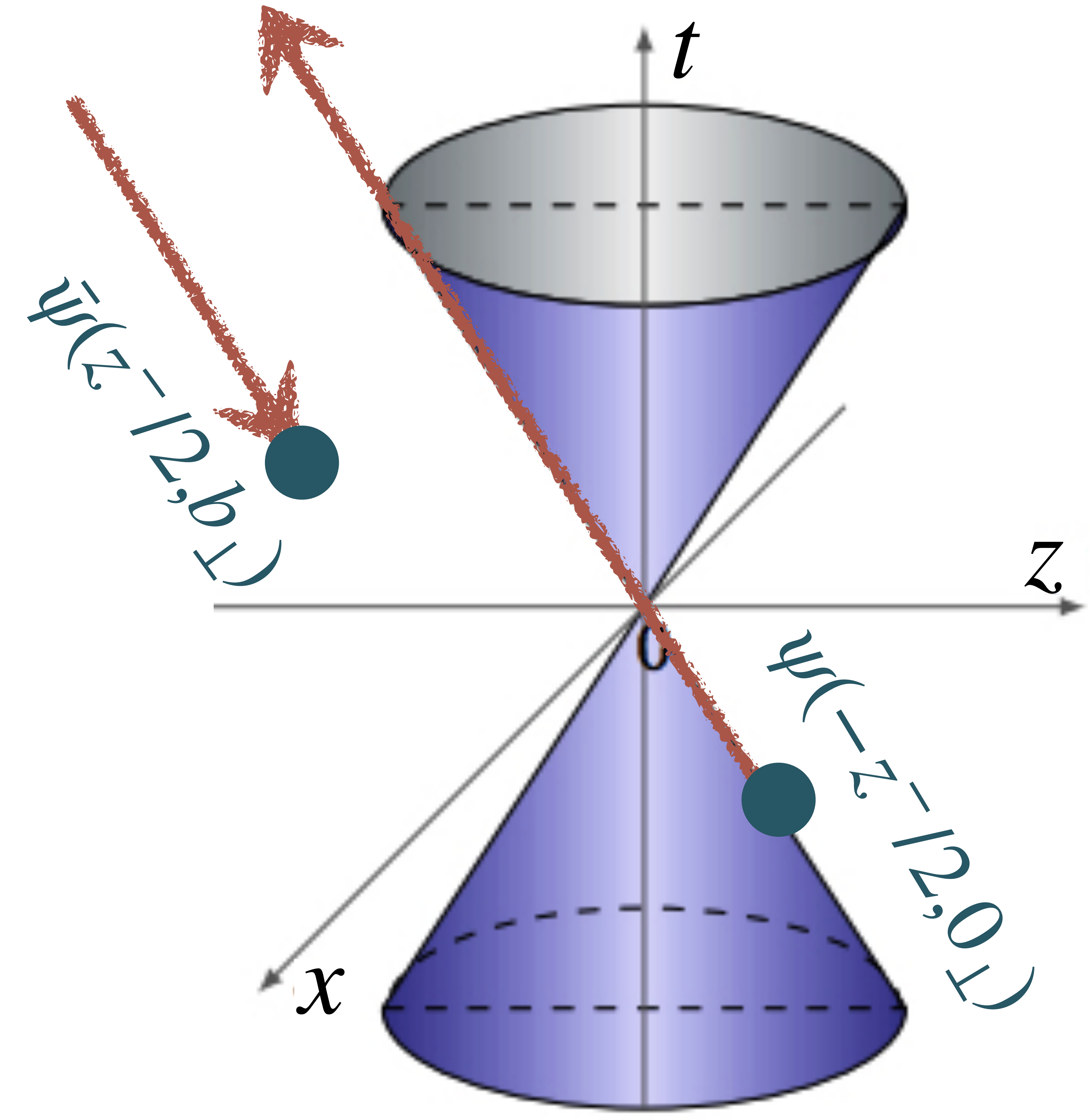


$\tilde\phi(z, b_\perp, \eta, P_z)$

quasi-TMD beam function

$P_z \to \infty$

$\eta \to \infty$



lighcone-TMD beam function

- large η; check for η-independence: $\tilde{\phi}(b_z, b_\perp, \eta, P_z) \longrightarrow \tilde{\phi}(b_z, b_\perp, P_z)$

- renormalize: $\tilde{\phi}(b_z, b_\perp, P_z) \longrightarrow \tilde{\phi}(b_z, b_\perp, P_z, \mu)$

- Fourier transform to momentum (x) space: $\tilde{\phi}(b_z, b_\perp, P_z, \mu) \longrightarrow \tilde{\phi}(x, b_\perp, P_z, \mu)$

- Perturbative matching:

Collins Soper kernel

$$\frac{\tilde{\phi}_\Gamma(x, b_\perp, P_z, \mu)}{\sqrt{S_r(b_\perp, \mu)}} = H(x, \bar{x}, P_z, \mu)\phi(x, b_\perp, \zeta, \mu)\exp\left[\frac{1}{4}\left(\ln\frac{(2xP_z)^2}{\zeta} + \ln\frac{(2\bar{x}P_z)^2}{\zeta}\right)\gamma^{\overline{\mathrm{MS}}}(b_\perp, \mu)\right]$$

$$+\,\mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(xP_z)^2}, \frac{1}{(b_\perp(xP_z))^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(\bar{x}P_z)^2}, \frac{1}{(b_\perp(\bar{x}P_z))^2}\right)$$

soft function

perturbative kernel

$\bar{x} = 1 - x$

lightcone TMD distribution

power corrections

CS kernel from lattice QCD

- ratios of quasi-TMD beam functions for 2 different boost momenta, P_1 & P_2

Collins Soper kernel

perturbative kernel

$$\gamma^{\overline{\mathrm{MS}}}(b_\perp, \mu) = \frac{1}{\ln(P_2/P_1)} \ln\left[\frac{\tilde{\phi}(x, b_\perp, P_2, \mu)}{\tilde{\phi}(x, b_\perp, P_1, \mu)}\right] + \delta\gamma^{\overline{\mathrm{MS}}}(x, \mu, P_1, P_2)$$

independent of x, P_1, P_2

+ power corrections

soft function cancels

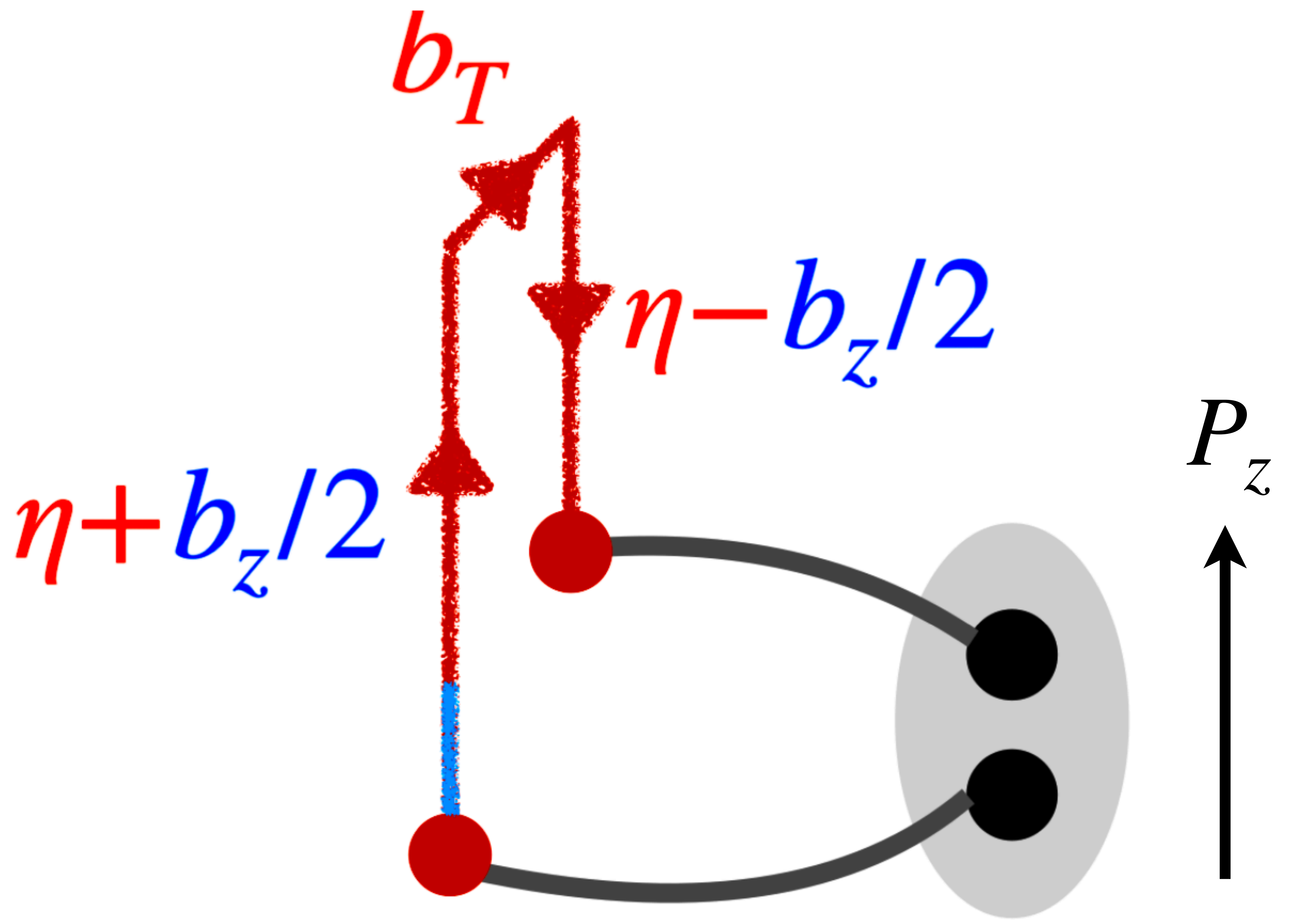
P_1 & P_2 both must be large to suppress power corrections, such that CS kernel is indep. of those

lattice QCD calculations of CS kernel

- simplest choice for the quasi-TMD beam function $\tilde{\phi}(b_z, b_\perp, \eta, P_z)$

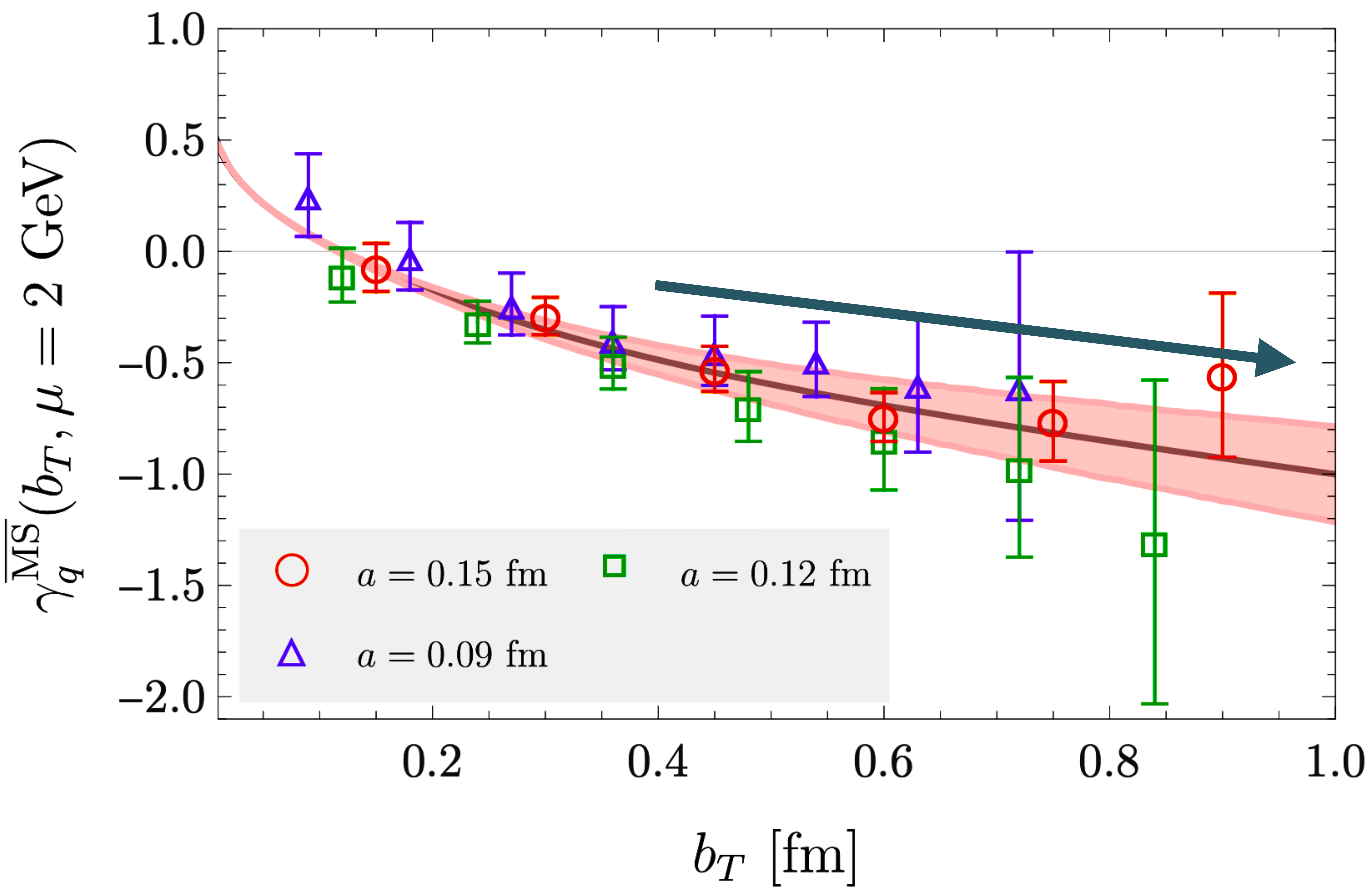
pion TMD wave function (TMDWF)

$$\langle\Omega|\bar{\psi}(\frac{b_z}{2}, b_\perp)\Gamma W_\sqsupset(\frac{\mathbf{b}}{2}, -\frac{\mathbf{b}}{2}, \eta)\psi(-\frac{b_z}{2}, 0)|\pi^+, P_z\rangle$$



difficulties in lattice QCD calculations

rapidly growing errors with increasing $b_\perp$

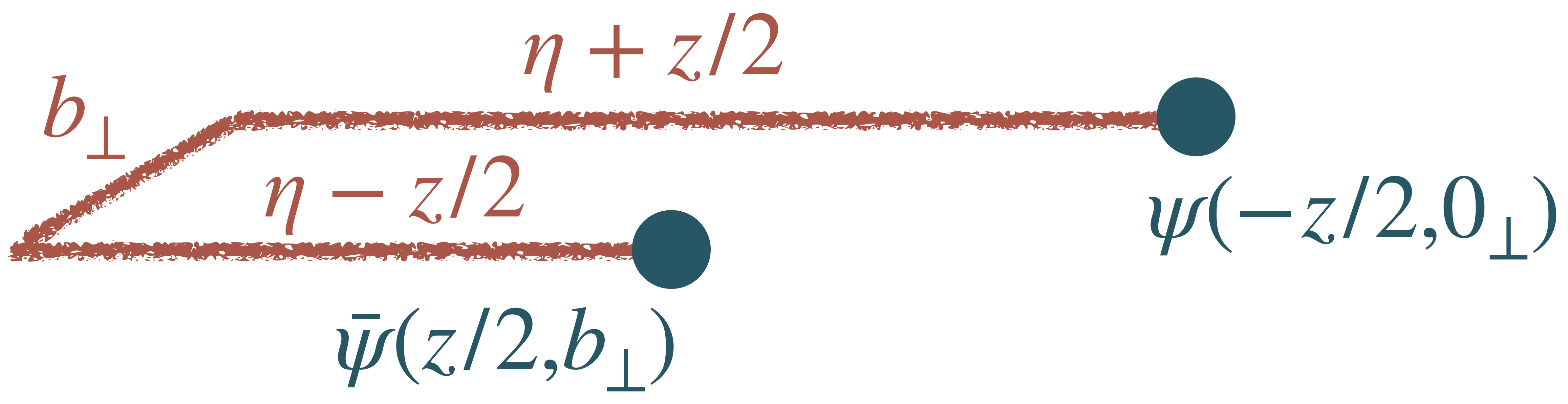


Avkhadiev et al.: 2402.06725

why difficult ?

multiplicative renormalization factor of the Wilson line:

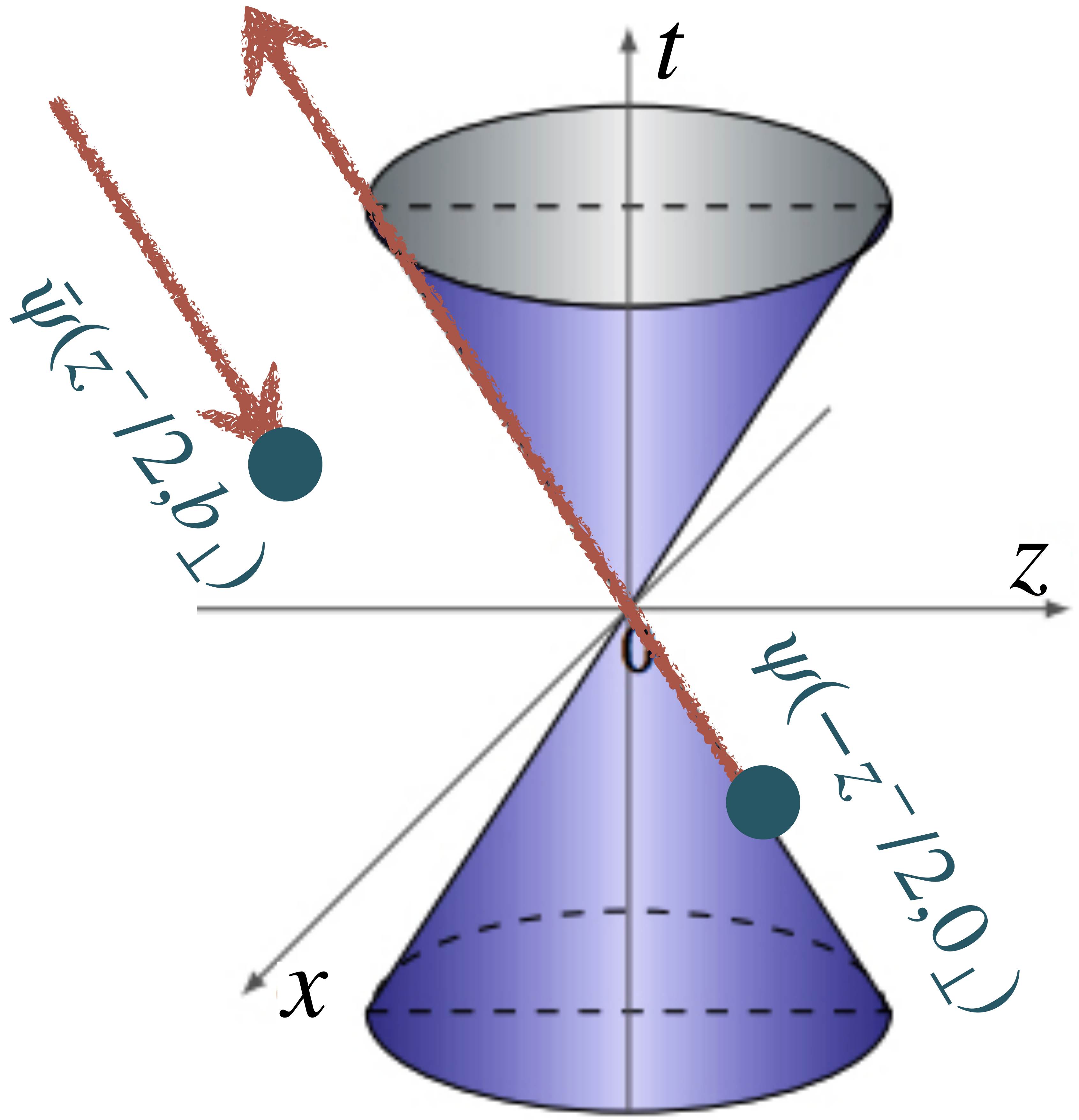
$$\sim e^{-\delta m(\eta + b_\perp)}$$



exponential decrease of signal for large η and increasing $b_\perp$

overcoming difficulties

physical lightcone gauge $A^+ = 0$



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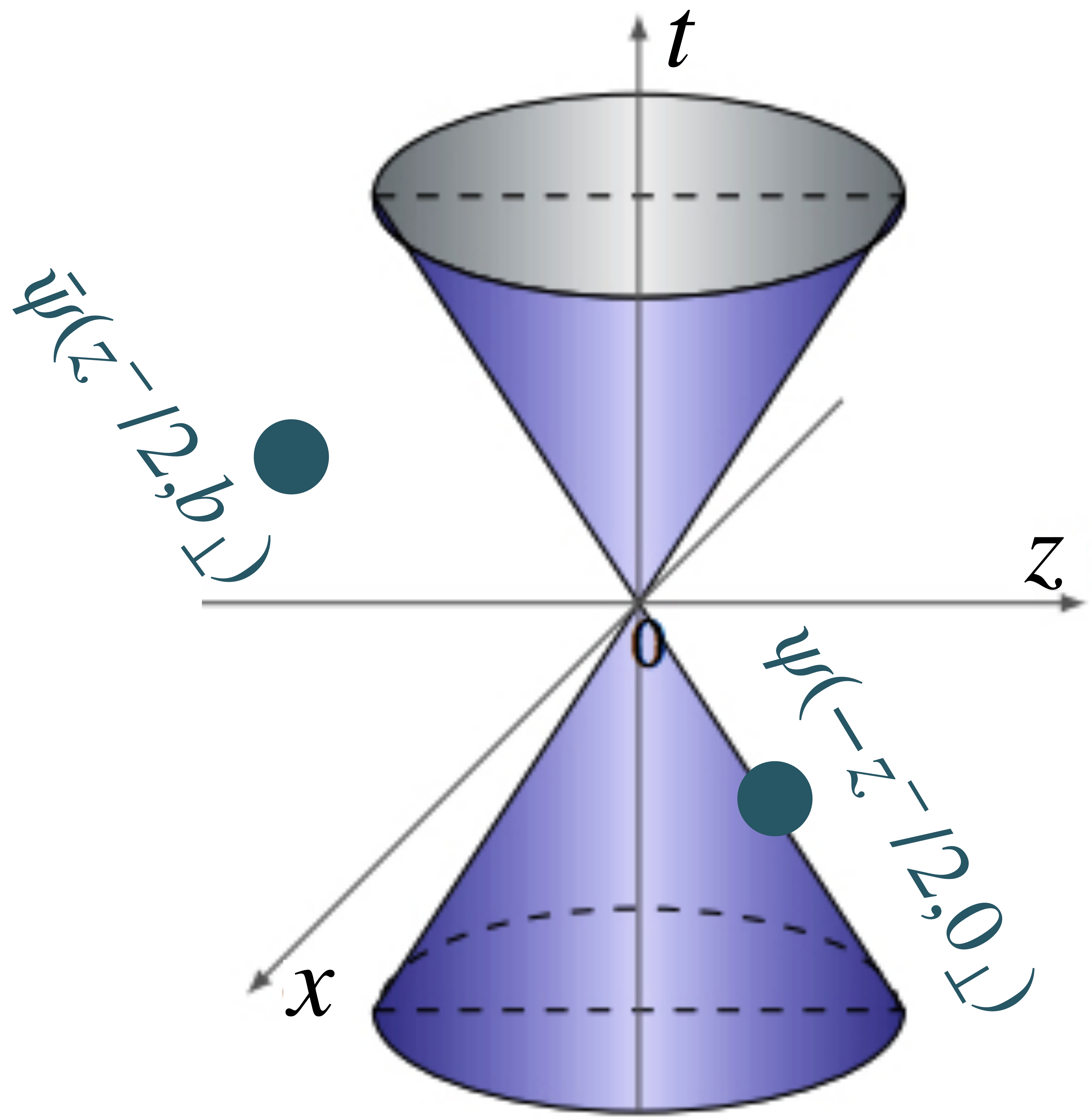


overcoming difficulties

but how can we access $A^+ = 0$ in lattice QCD calculations ?

find a gauge that becomes equivalent to $A^+ = 0$ in the limit $P_z \to \infty$

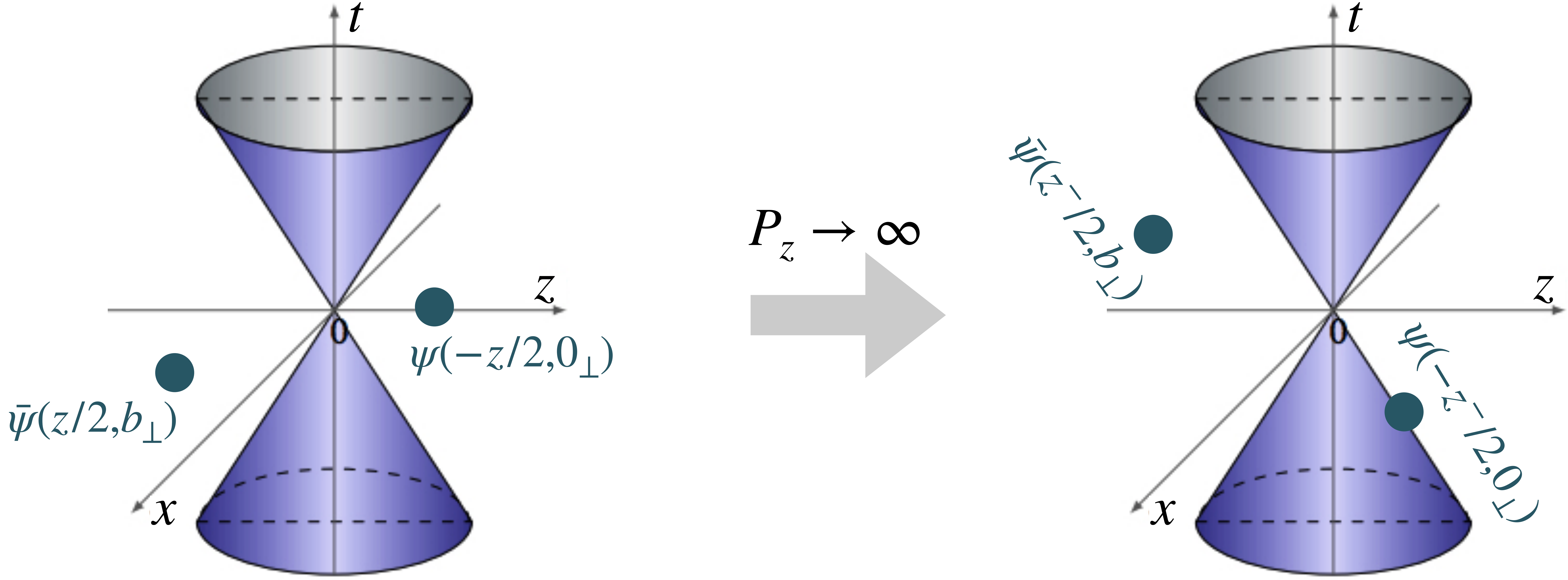
Coulomb gauge: $\vec{\nabla} \cdot \vec{A} = 0$ $\xrightarrow{P_z \to \infty}$ $A^+ = 0$

Ji et al., Phys. Rev. Lett. 111, 112002 (2013)

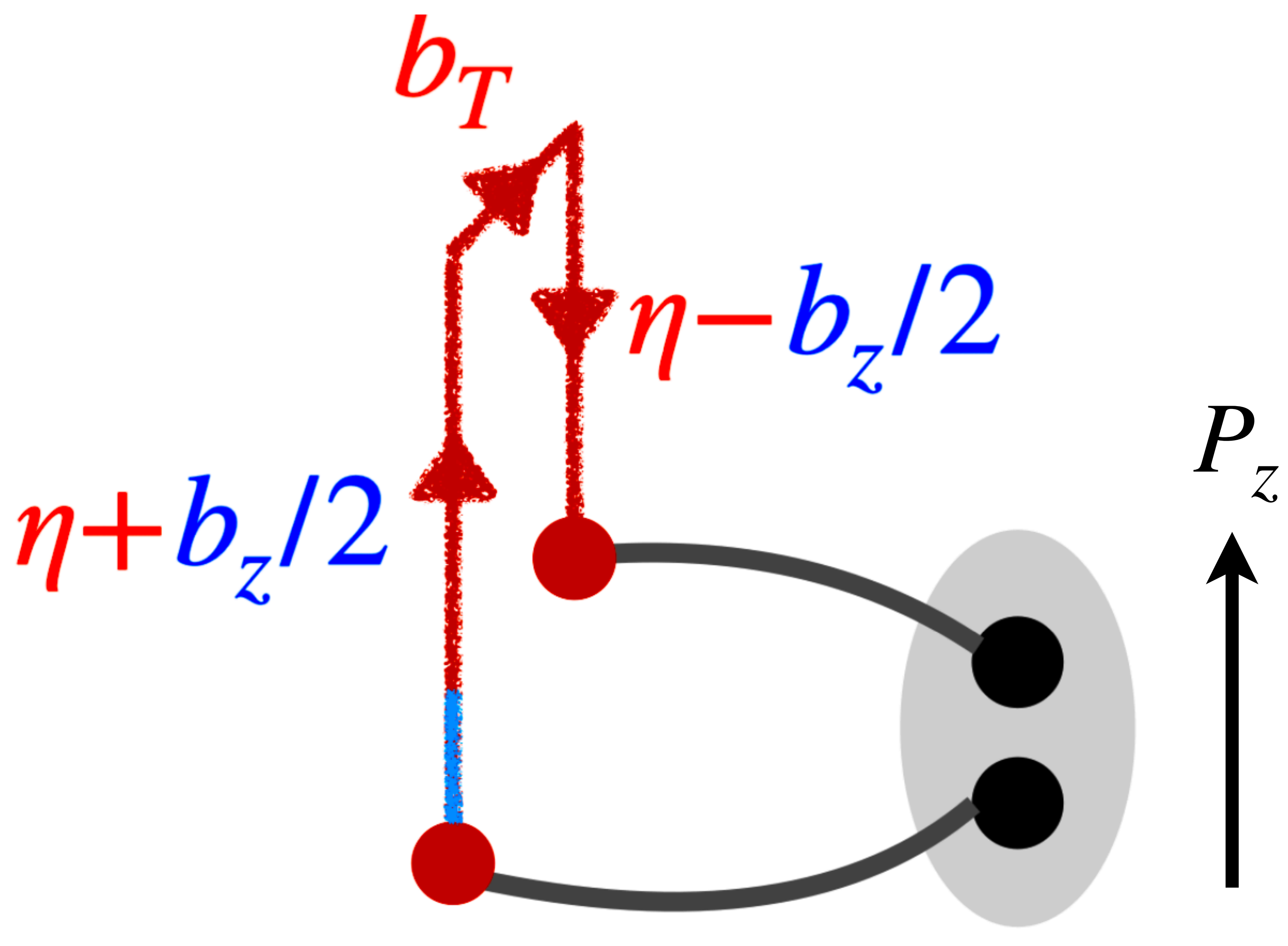
Hatta et al., Phys. Rev. D 89, no.8, 085030 (2014)

quasi-TMD beam function in Coulomb gauge (CG)

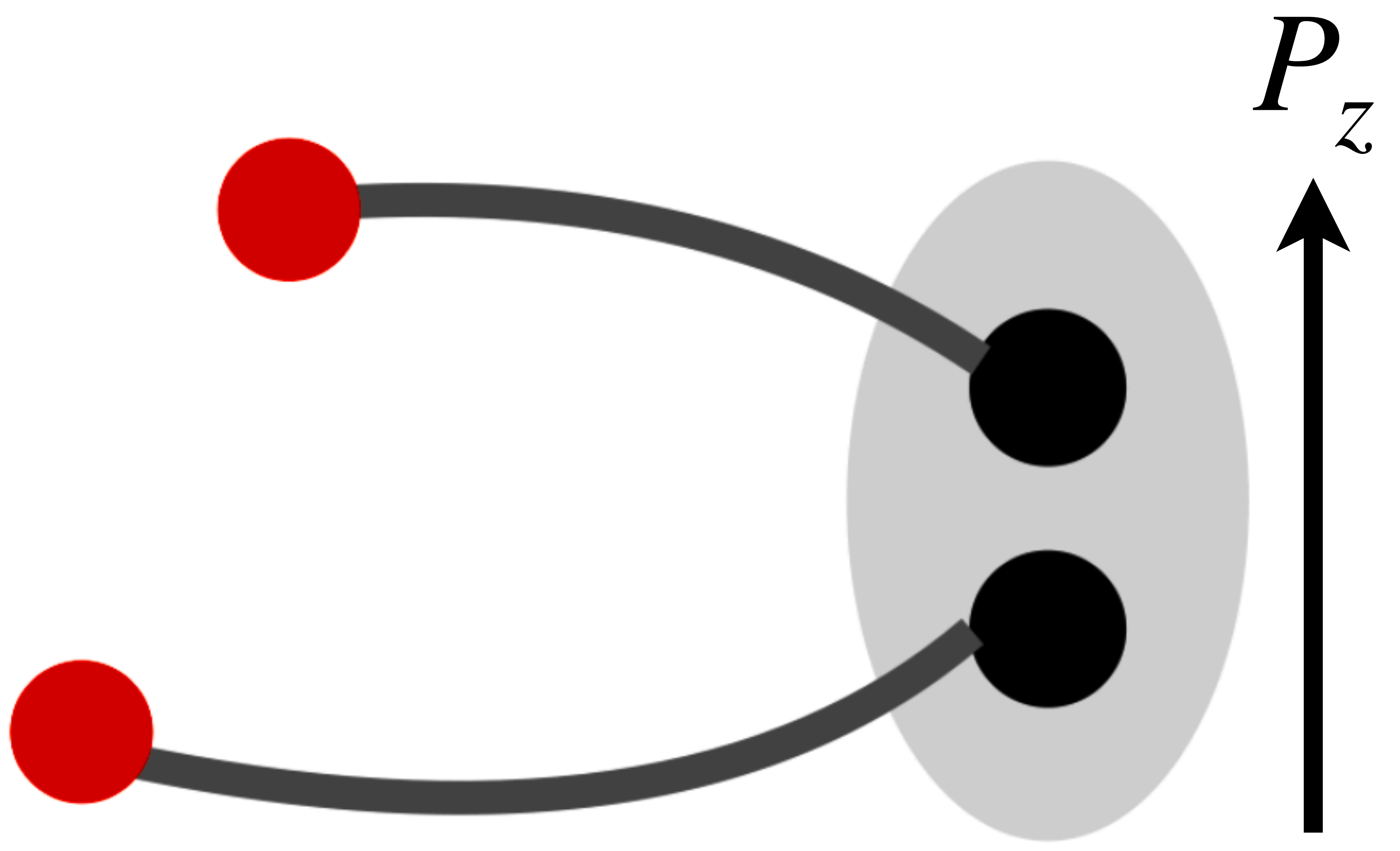
$$\overline{\psi}(\frac{\mathbf{b}}{2})\Gamma\psi(-\frac{\mathbf{b}}{2})|_{\nabla\cdot\mathbf{A}=0}$$



CG quasi-TMD beam function



$$\langle \Omega | \overline{\psi}(\frac{b_z}{2}, b_\perp) \Gamma W_\sqsupset(\frac{\mathbf{b}}{2}, -\frac{\mathbf{b}}{2}, \eta) \psi(-\frac{b_z}{2}, 0) | \pi^+, P_z \rangle$$



$$\langle \Omega | \overline{\psi}(\frac{b_z}{2}, b_\perp) \Gamma \psi(-\frac{b_z}{2}, 0) |_{\vec{\nabla} \cdot \vec{A} = 0} | \pi^+, P_z \rangle$$

+ re-computation of pQCD matching function $\delta\gamma^{\overline{\mathrm{MS}}}(x, \mu, P_1, P_2)$

NLL accuracy

Zhao: 2311.01391

renormalized quasi-TMD beam functions

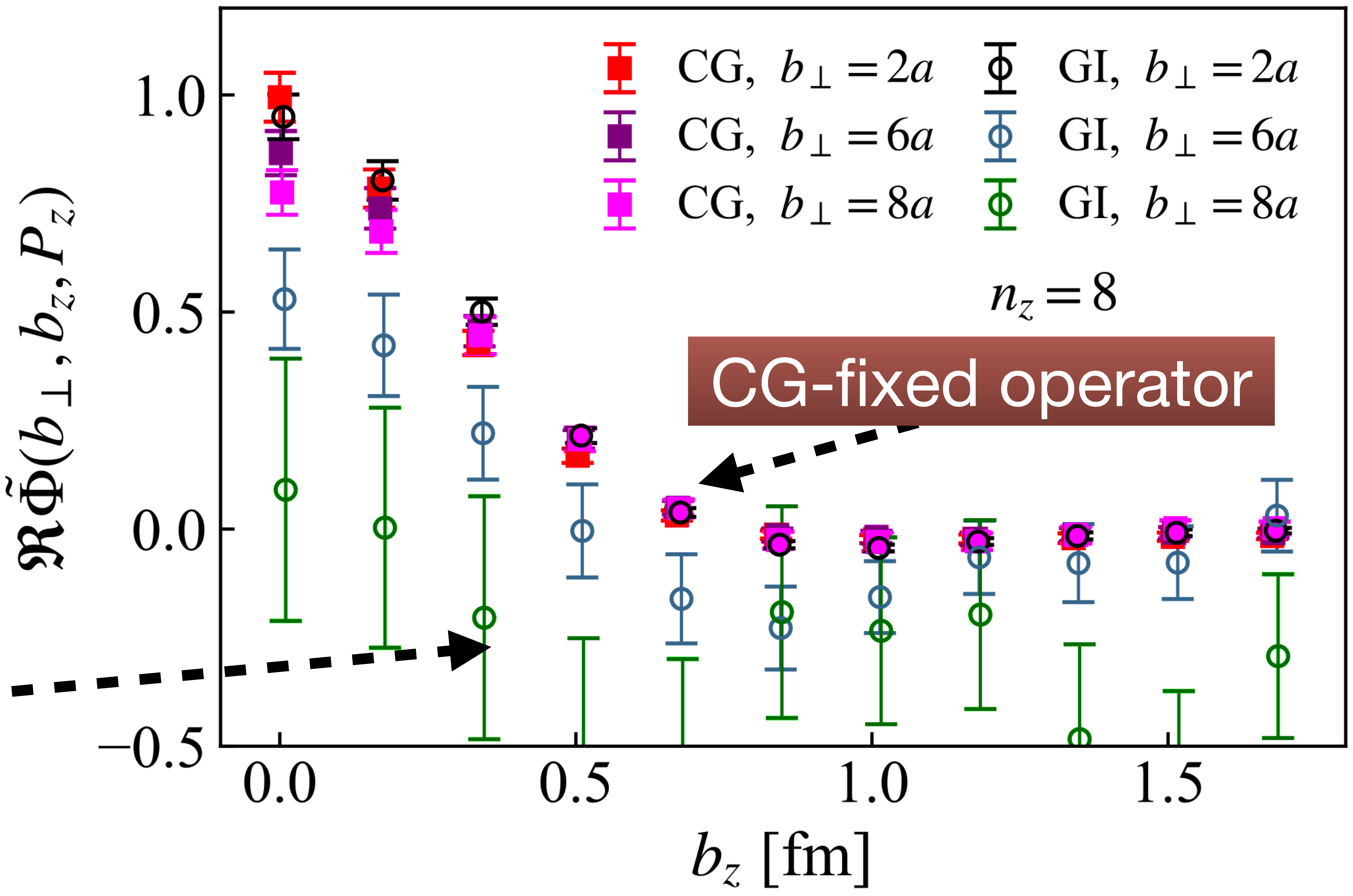
Bollweg et al.: Phys. Lett. B 852, 138617 (2024)

unitary chiral (Domain Wall) fermions, physical pion mass

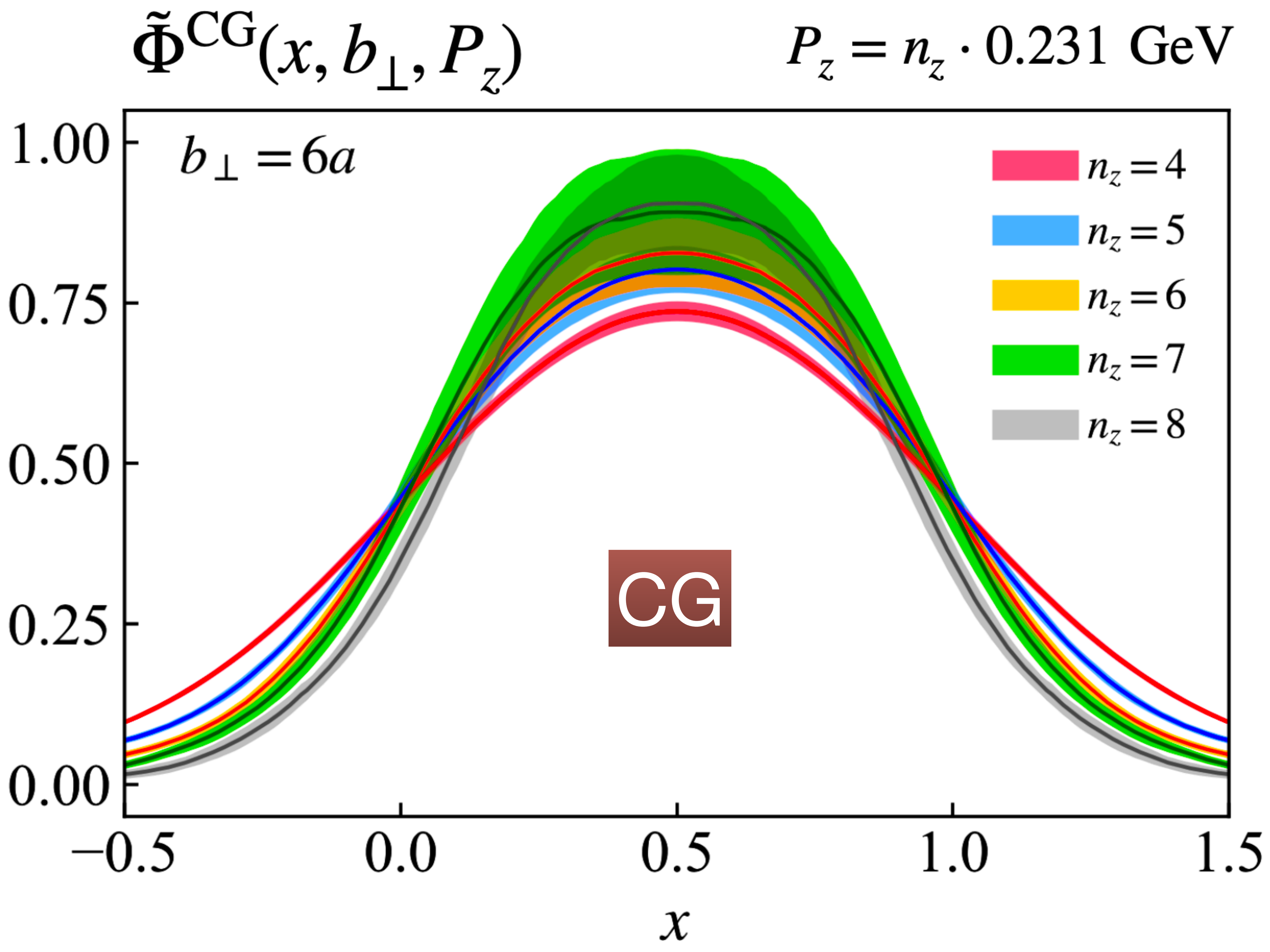
lattice spacing a=0.085 fm

gauge invariant (GI) operator

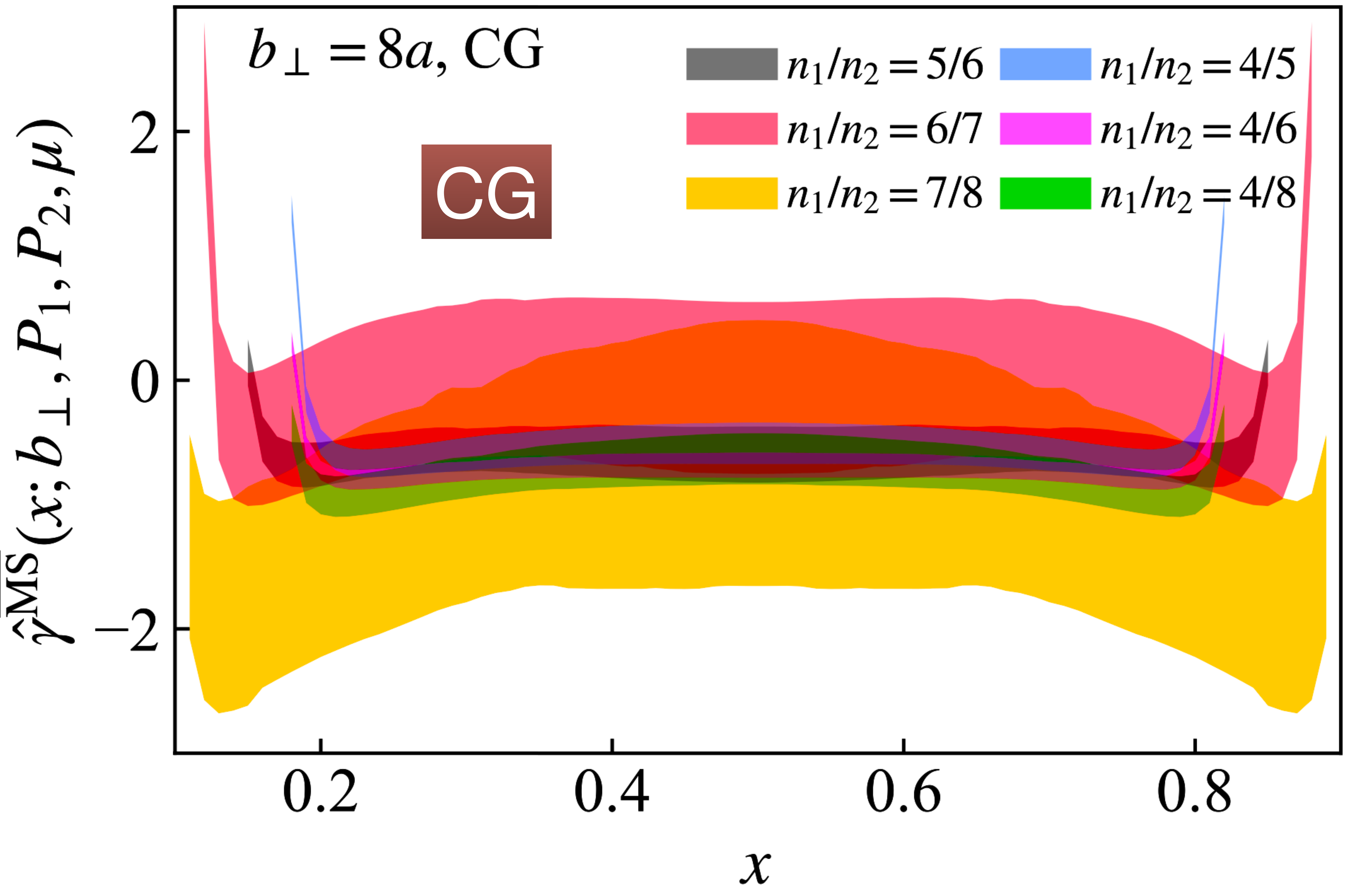
$P_z = 0.231 n_z$ GeV



quasi-TMD beam functions in momentum space



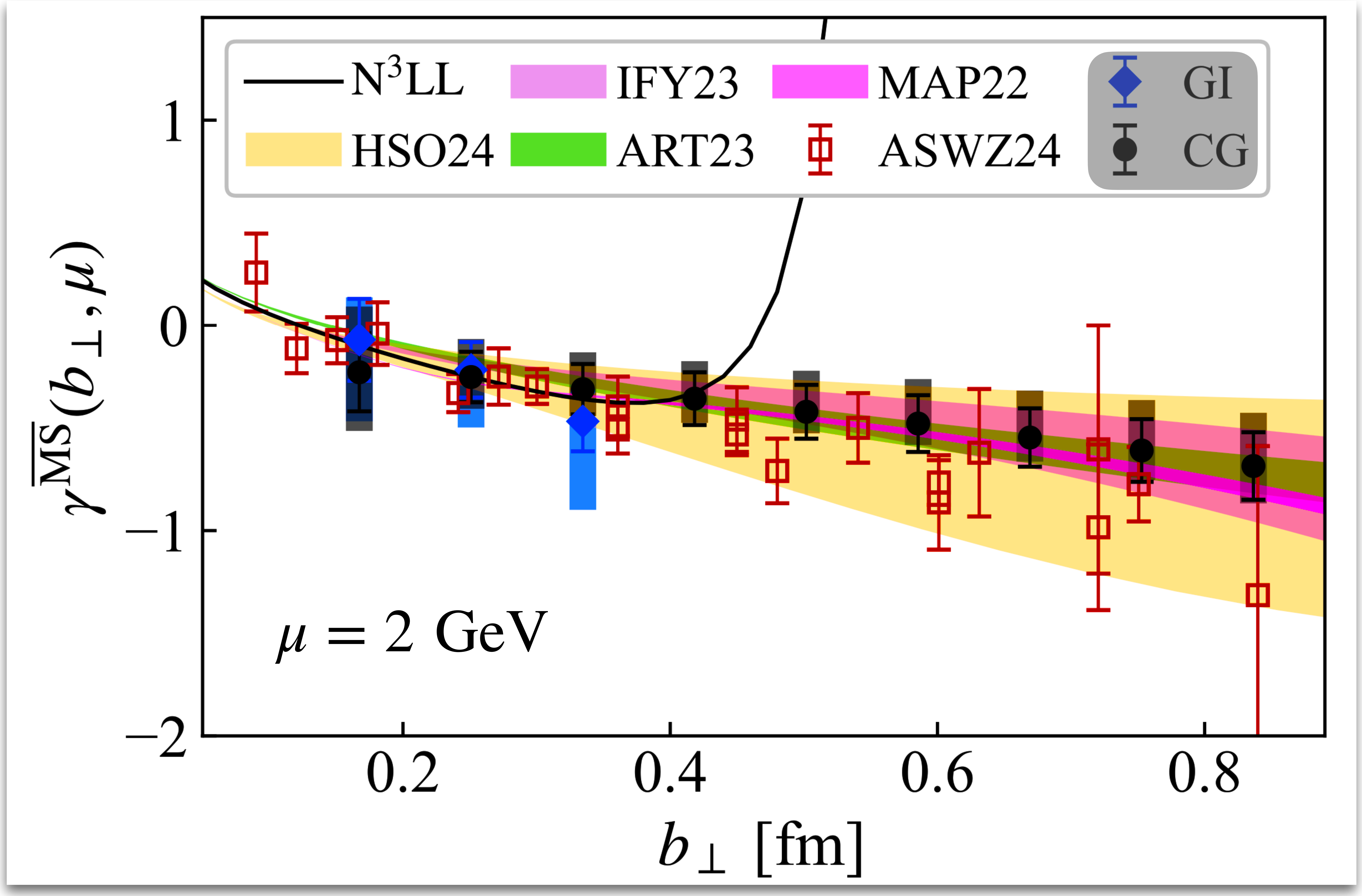
x and P independence of CS kernel



$P_i = 0.231 n_i$ GeV, μ = 2 GeV

Summary: nonperturbative CS kernel from lattice QCD

Bollweg et al.: Phys. Lett. B 852, 138617 (2024)



Summary: nonperturbative CS kernel from lattice QCD

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