

# Nuclear-modified TMD PDFs and FFs

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Alrashed, Anderle, Kang, Terry, Xing [PRL 129, 242001](#)

Alrashed, Kang, Terry, Xing, Zhang [2312.09226](#)

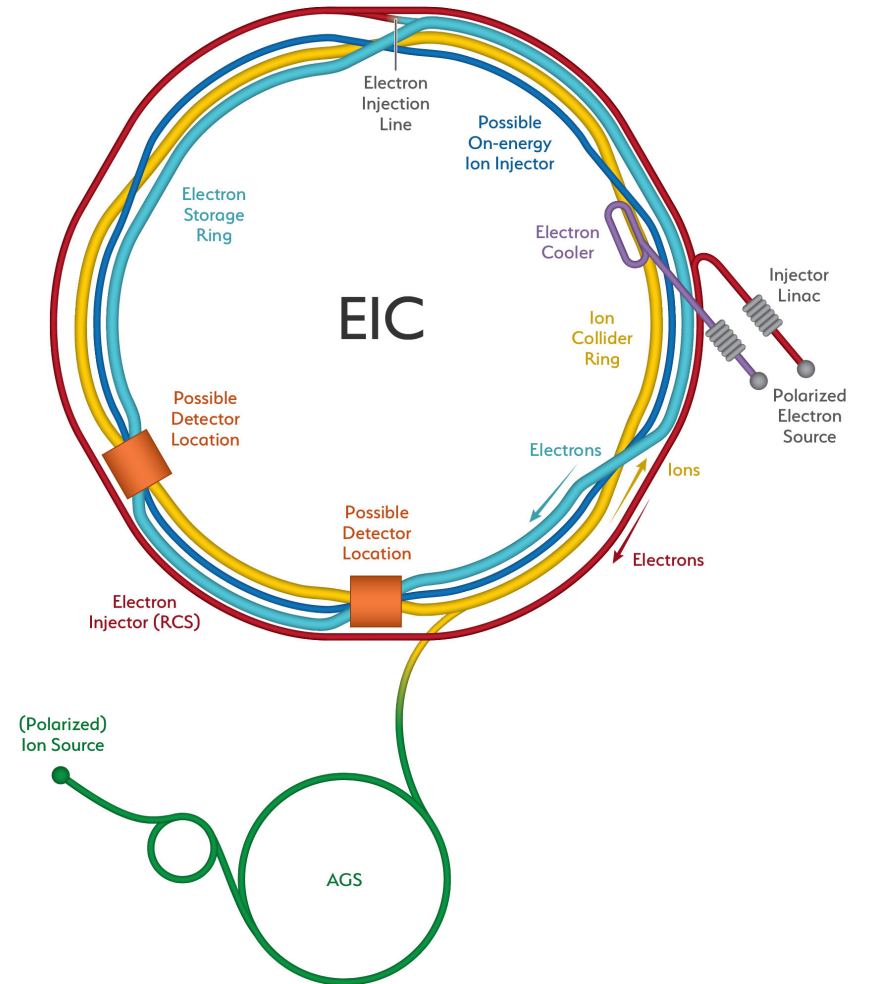
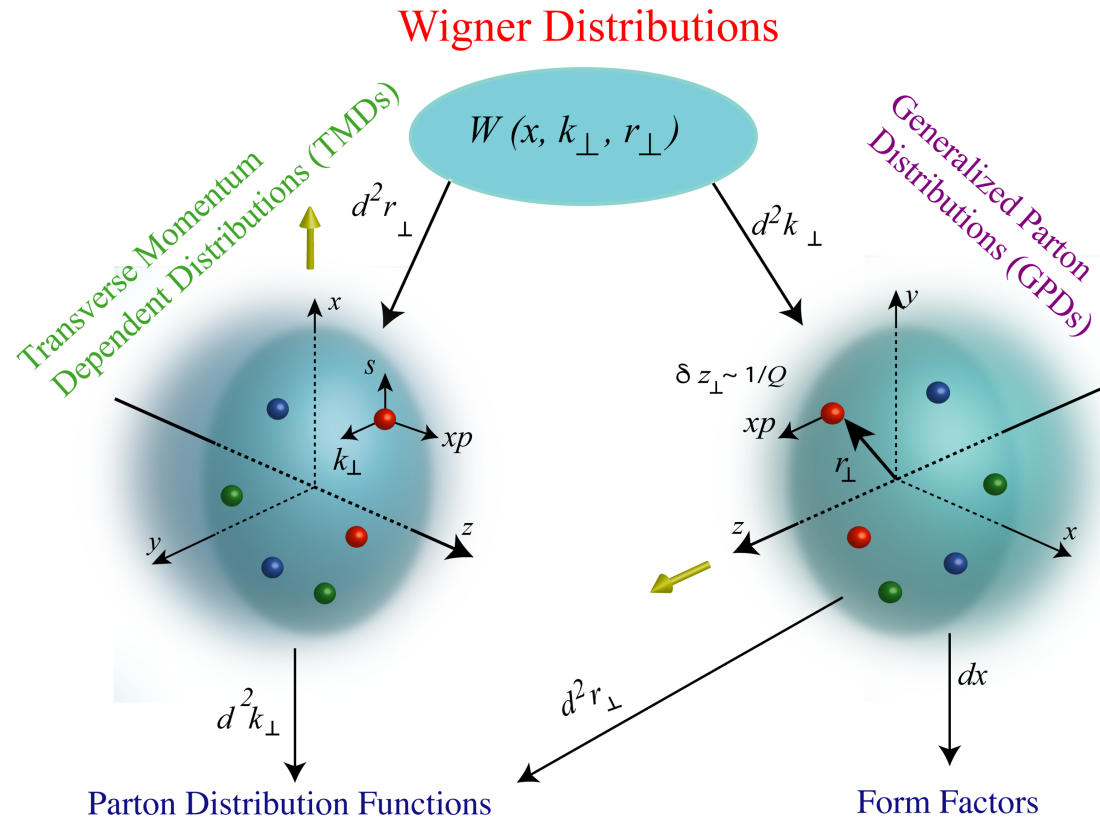
Ke, Shen, Shao, Terry [JHEP 05 \(2024\) 066](#)

Gao, Kang, Shao, Terry, Zhang [JHEP 10 \(2023\) 013](#)



# What can we do with the EIC?

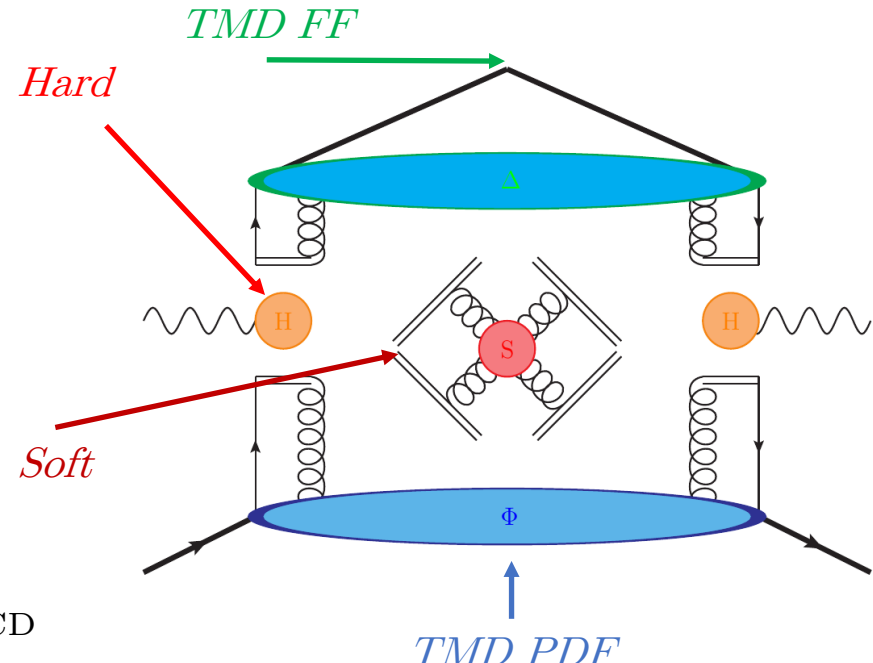
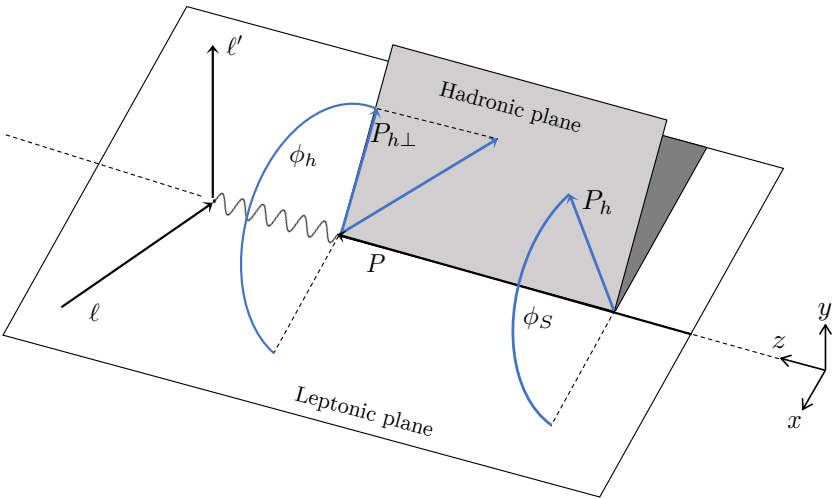
Provide information for distributions of partons in hadrons    A new machine for a new frontier in nuclear physics



# Factorization of physics at different scales

Factorization of the cross section in an OPE

$$Q \gg q_T \gtrsim \Lambda_{\text{QCD}}$$



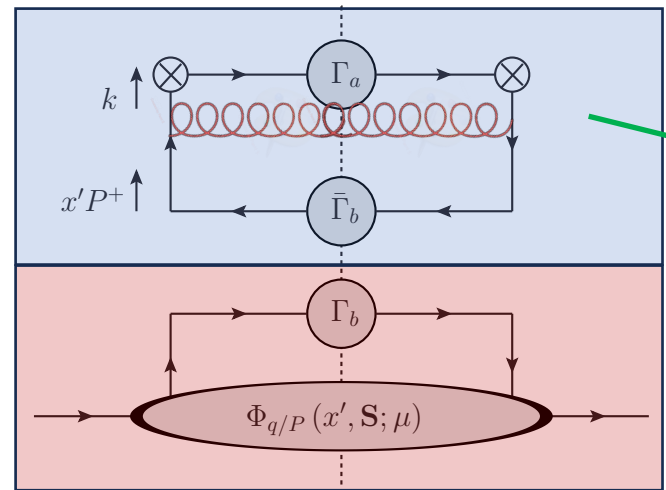
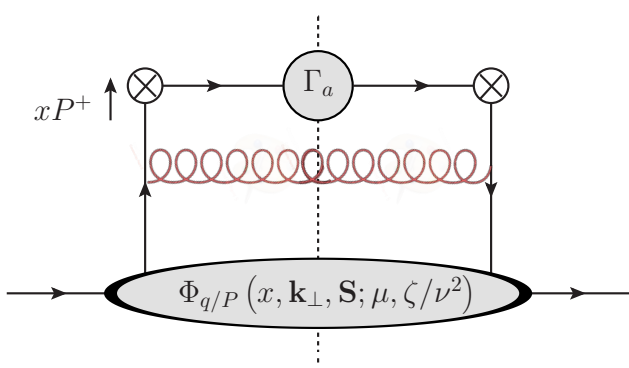
$$d\sigma \sim \sum_i \boxed{C(Q; \mu)} \otimes f \otimes D \otimes S(q_T, \mu)$$

↑  
Contains fixed order and large logs

$$\ln\left(\frac{Q}{\mu}\right)$$

Factorization of IR modes

$$q_T \gtrsim \Lambda_{\text{QCD}}$$



Matching coefficient contains fixed order and large logs

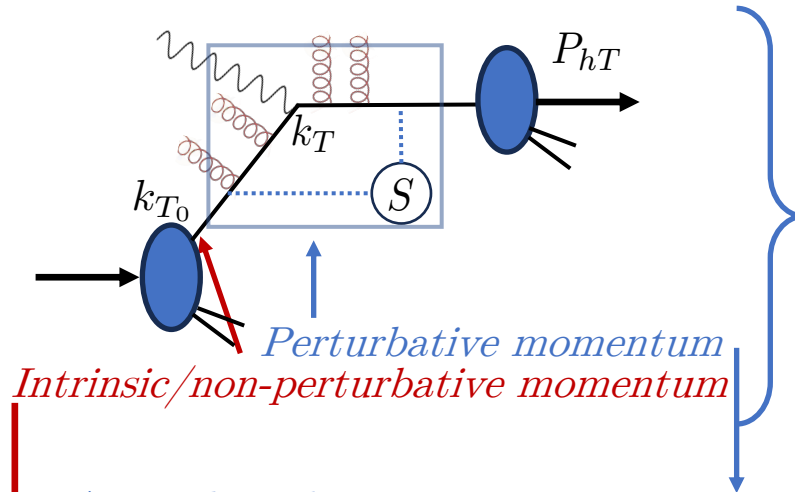
$$\ln\left(\frac{q_T}{\mu}\right)$$

$$f(x, q_T, \mu) \sim [C \otimes f](x, q_T, \mu)$$

Collinear PDF

# Perturbative background

*Perturbative Sudakov*: accounts for transverse momenta generated from soft and collinear emissions



Experiments involve mixture of Perturbative and non-perturbative momentum

*Intrinsic/non-perturbative momentum*

Anomalous dimensions

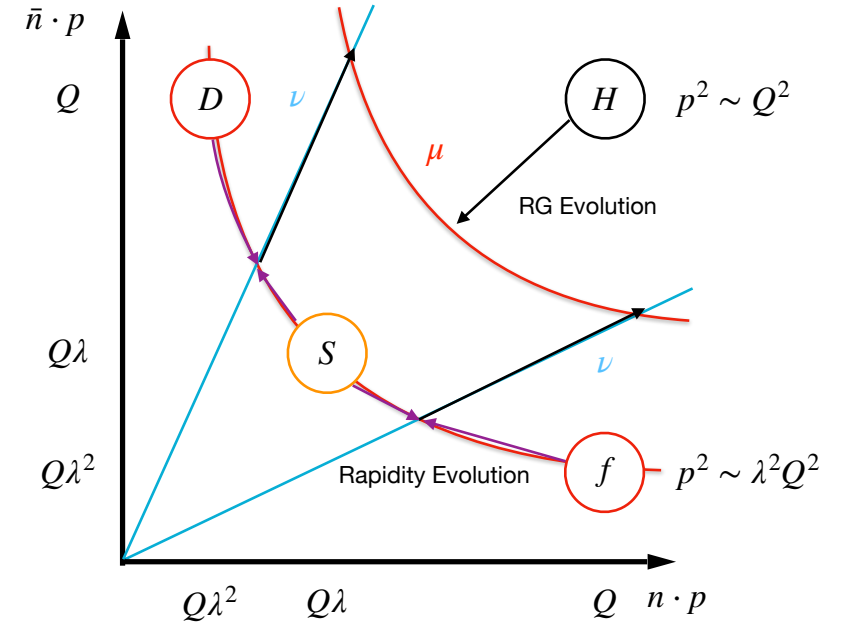
$$\mu \frac{d}{d\mu} \ln F(Q, \mu, \nu) = \gamma_{F\mu}^q(Q, \mu, \nu)$$

$$F \in \{H, f, D, S\}$$

$$\nu \frac{d}{d\nu} \ln G(Q, \mu, \nu) = \gamma_{G\nu}^q(Q, \mu, \nu)$$

$$G \in \{f, D, S\}$$

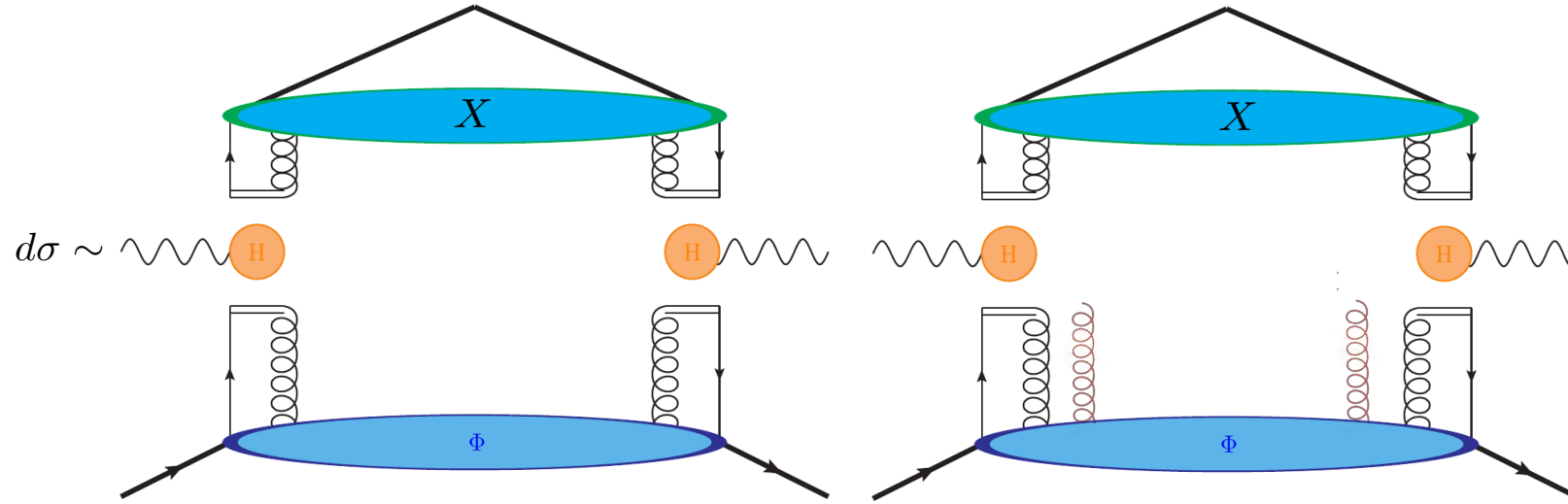
*Obtain intrinsic momentum through a fit to data*





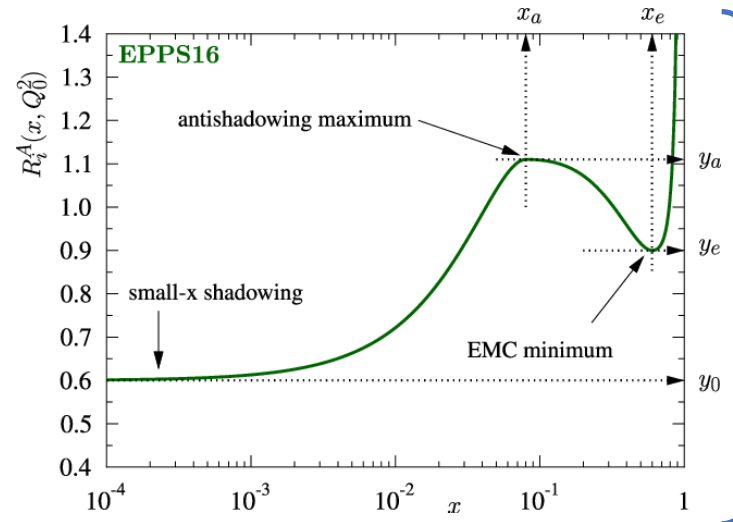
# Nuclear modifications to collinear PDFs

Nuclear medium modification via higher twist



Power suppressed by  $\frac{m^2}{Q^2}$   
Nuclear enhancement  $A^\#$

LP TMD factorization cannot address how multiple partons are correlated with one another

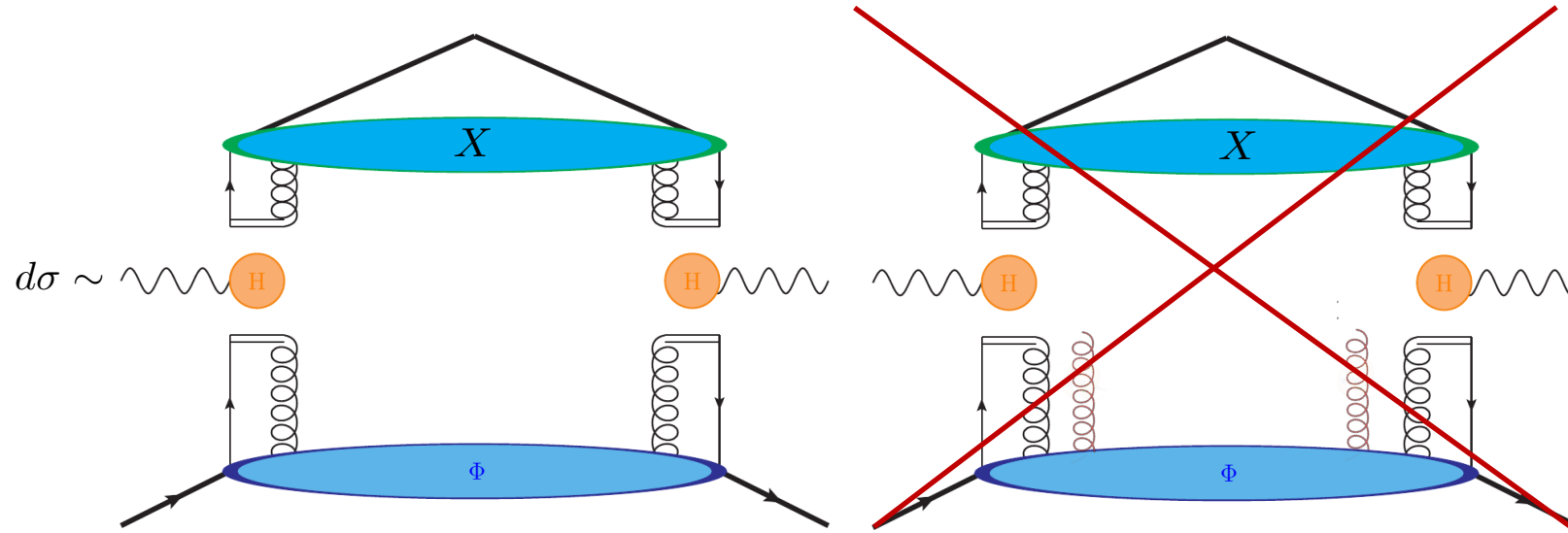


Eskola, Kolhinen, Ruuskanen (1998)

Eskola, Paakkinen, Paukkunen, Salgado (2017)

# Method of treating nuclear modifications

Nuclear medium modification via higher twist



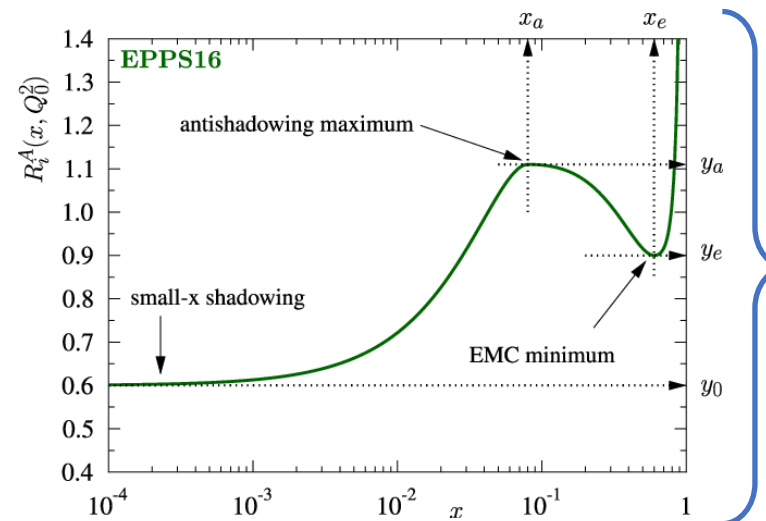
LP TMD factorization cannot address how multiple partons are correlated with one another

Eskola, Kolhinen, Ruuskanen (1998)

Eskola, Paakkinen, Paukkunen, Salgado (2017)

$$R_i^A(x, Q) = \frac{f_{i/p}^A(x; Q)}{f_{i/p}(x; Q)}$$

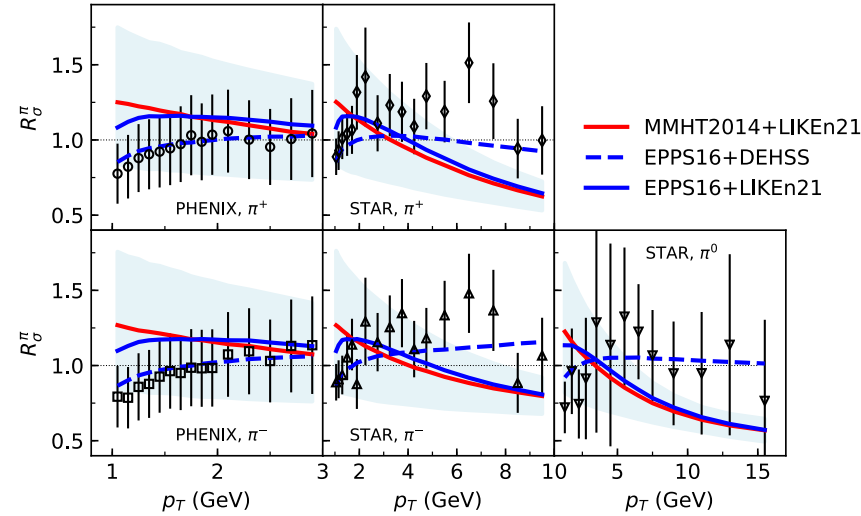
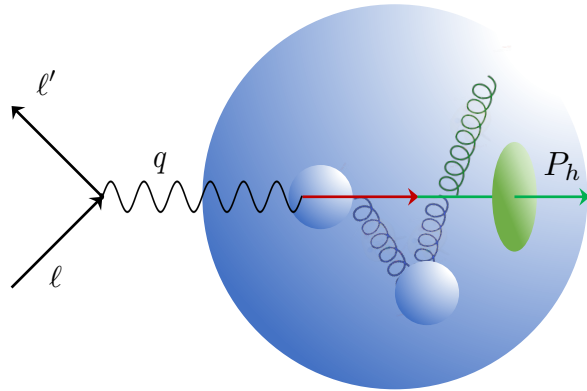
$$R_i^A(x, Q_0^2) = \begin{cases} a_0 + a_1(x - x_a)^2 & x \leq x_a \\ b_0 + b_1 x^\alpha + b_2 x^{2\alpha} + b_3 x^{3\alpha} & x_a \leq x \leq x_e \\ c_0 + (c_1 - c_2 x)(1 - x)^{-\beta} & x_e \leq x \leq 1, \end{cases}$$



Nuclear modifications are absorbed into the non-perturbative parameterization.

# Effective treatment of medium modifications

Ejected quark undergoes multiple scattering in the nuclear medium, modifies the fragmentation functions



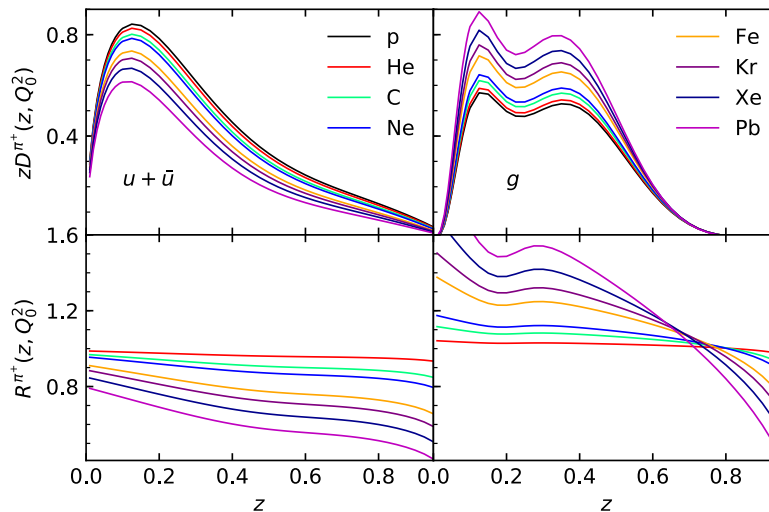
D. de Florian and R. Sassot (2004)  
Zurita (2021)

Simultaneous extraction from hadroproduction in p-A collisions from PHENIX and STAR, and Semi-Inclusive DIS (collinear) from HERMES

$$D_i^h(z, Q_0) = \tilde{N}_i z^{\alpha_i} (1-z)^{\beta_i} \left[ 1 + \gamma_i (1-z)^{\delta_i} \right]$$

$$\tilde{N}_i \rightarrow \tilde{N}_i \left[ 1 + N_{i,1} (1 - A^{N_{i,2}}) \right]$$

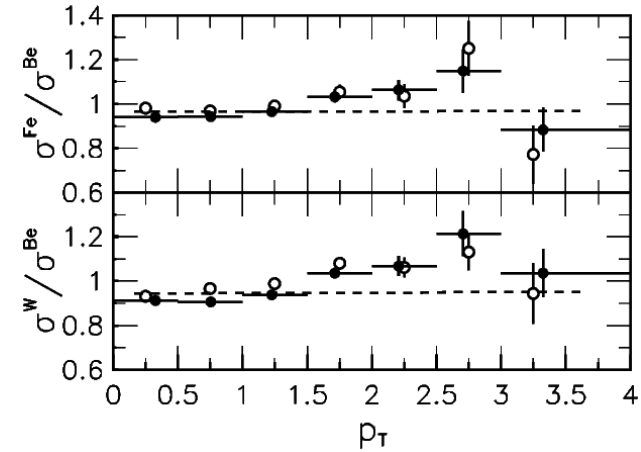
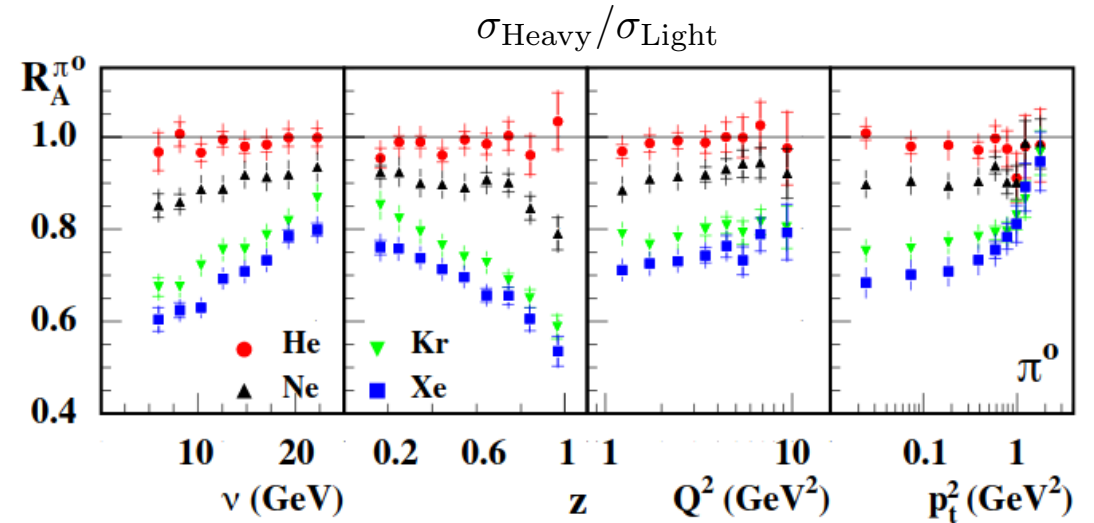
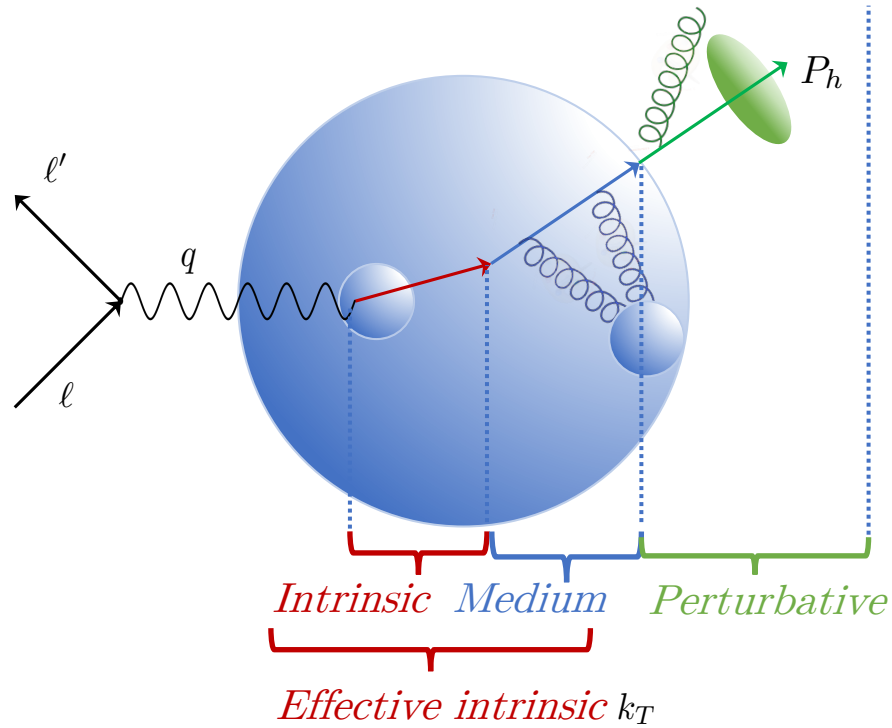
$$c_i \rightarrow c_i + c_{i,1} (1 - A^{c_{i,2}})$$



Abelev et al. (STAR) (2010)  
Adams et al. (STAR) (2006)  
Adare et al. (2013)  
Airapetian et al. (HERMES) (2007)

# Effective treatment of the transverse momentum broadening

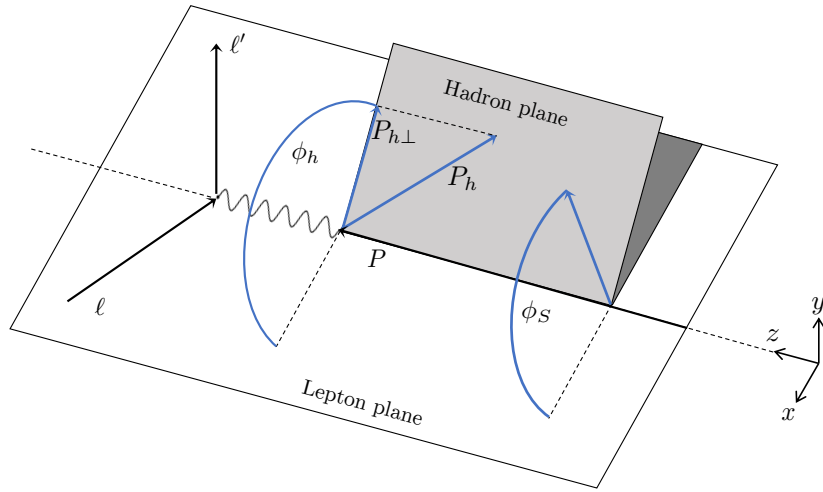
Interaction between partons and interact with the nuclear medium via Glauber exchange





# Available data

## Semi-Inclusive DIS for e-A collisions



### Multiplicity ratio

$$R_A^h = \frac{d\sigma_A^h/\mathcal{PS} d^2 P_{h\perp}}{d\sigma_A/\mathcal{PS}} \frac{d\sigma_D/\mathcal{PS}}{d\sigma_D^h/\mathcal{PS} d^2 P_{h\perp}}$$

SIDIS cross section  $\frac{d\sigma_A^h}{\mathcal{PS} d^2 P_{h\perp}}$

DIS cross section  $\frac{d\sigma_A}{\mathcal{PS}}$

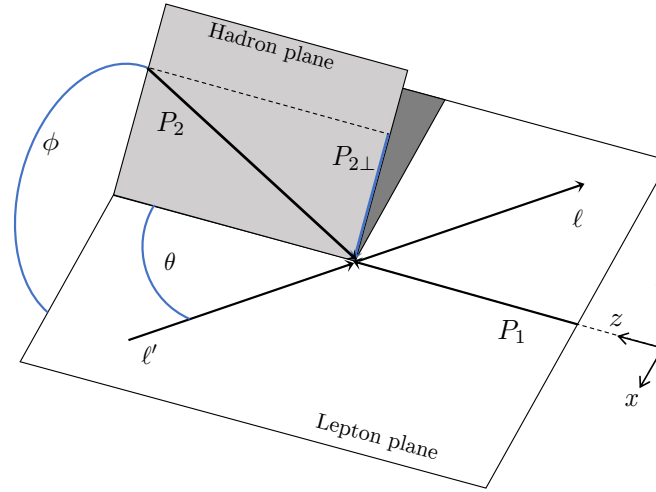
HERMES ratio for A = He, Ne, Kr, Xe

$$h = \pi^+, \pi^-, \pi^0, K^+, K^-, K^0$$

Jefferson Lab ratio for A = C, Fe, Pb

$$h = \pi^+, \pi^-$$

## Drell-Yan production in p-A collisions



### Cross section and cross section ratio for p-A collisions

$$R_{AB} = \frac{d\sigma_A}{d\mathcal{PS} d^2 q_\perp} \frac{d\mathcal{PS} d^2 q_\perp}{d\sigma_B}$$

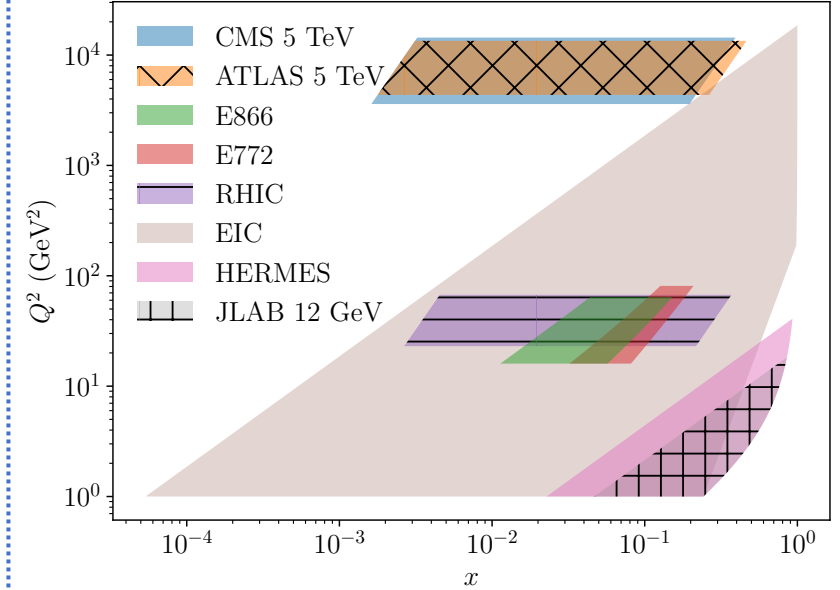
E772: A = C; B = D

E866: A = Fe, W; B = Be

RHIC: A = Au; B = p

ATLAS, CMS:  $q_\perp$  distribution p-Pb

## Kinematic coverage of the data



Airapetian et al. (HERMES), Nucl. Phys. B 780, 1 (2007)

Dudek et al., Eur. Phys. J. A 48, 187 (2012)

Burkert, in CLAS 12 RICH Detector Workshop (2008)

Alde et al., Phys. Rev. Lett. 64, 2479 (1990)

Vasilev et al. (NuSea), Phys. Rev. Lett. 83, 2304 (1999)

Leung (PHENIX), PoS HardProbes2018, 160 (2018)

Khachatryan et al. (CMS), Phys. Lett. B 759, 36 (2016)

Aad et al. (ATLAS), Phys. Rev. C 92, 044915 (2015) 8/30

# Available perturbative accuracy

## Anomalous dimensions

$$\mu \frac{d}{d\mu} \ln F(Q, \mu, \nu) = \gamma_F^q(Q, \mu, \nu)$$

$$F \in \{H, f, D, S\}$$

$$\mu \frac{d}{d\nu} \ln G(Q, \mu, \nu) = \gamma_G^q(Q, \mu, \nu)$$

$$G \in \{f, D, S\}$$

Anomalous dimensions are almost known up to N<sup>4</sup>LL at this point (no 5-loop cusp)

Accuracy	$H, \mathcal{J}$	$\Gamma_{\text{cusp}}(\alpha_s)$	$\gamma_H^q(\alpha_s)$	$\gamma_r^q(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level	1-loop			1-loop
NLL	Tree level	2-loop	1-loop	1-loop	2-loop
NLL'	1-loop	2-loop	1-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	2-loop	3-loop
NNLL'	2-loop	3-loop	2-loop	2-loop	3-loop
N <sup>3</sup> LL	2-loop	4-loop	3-loop	3-loop	4-loop
N <sup>3</sup> LL'	3-loop	4-loop	3-loop	3-loop	4-loop
N <sup>4</sup> LL	3-loop	5-loop	4-loop	4-loop	5-loop
N <sup>4</sup> LL'	4-loop	5-loop	4-loop	4-loop	5-loop

Lee, Smirnov, and Smirnov (2010)

Gehrmann, Glover, Huber, Ikizlerli, and Studerus (2010)

Ebert, Mistlberger, Vita (2020)

Ebert, Mistlberger, Vita (2020)

Agarwal, von Manteuffel, Panzer, and Schabinger (2021)

Duhr, Mistlberger, Vita (2022)

Moult, Zhu, Zhu (2022)

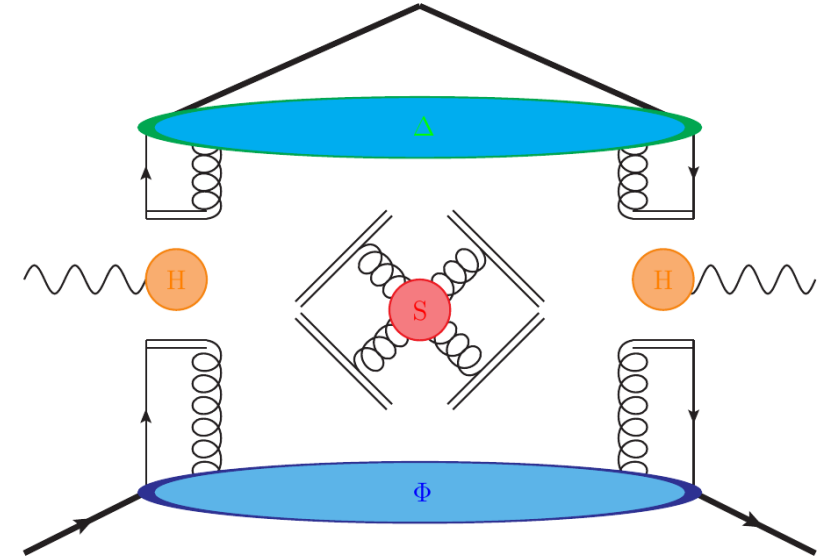
Herzog, Moch, Ruijl, Ueda, Vermaseren, and Vogt (2019)

Baikov, Chetyrkin, and Kuhn (2017)

# Factorization and resummation in the medium

Differential cross section for Semi-Inclusive DIS is given by

$$\frac{d\sigma}{d\mathcal{PS} d^2 P_{h\perp}} = \sigma_0 \underbrace{H(Q; \mu)}_{\text{Hard}} \sum_q e_q^2 \int \frac{bdb}{2\pi} J_0\left(\frac{bP_{h\perp}}{z}\right) \underbrace{f_{q/N}^A(x, b; \mu, \zeta_1)}_{n\text{TMD PDF}} \underbrace{D_{h/q}^A(z, b; \mu, \zeta_2)}_{n\text{TMD FF}}$$



TMDs can be matched onto the collinear distributions

$$f_{1q/A}(x, b, \mu, \zeta) = [C \otimes f]_{q/A}(x, b, \mu_i, \zeta_i) U(\mu_i, \mu; \zeta) Z(b, \zeta_i, \zeta; \mu_i) U_{\text{NP}}^{f^A}(x, b, \zeta, A)$$

$$D_{1h/q}^A(z, b, \mu, \zeta) = \frac{1}{z^2} [\hat{C} \otimes D^A]_{h/q}(z, b, \mu_i, \zeta_i) U(\mu_i, \mu; \zeta) Z(b, \zeta_i, \zeta; \mu_i) U_{\text{NP}}^{D^A}(z, b, \zeta, A)$$

Perturbative
Non-perturbative

Large logarithms are resummed to all orders in the perturbative Sudakov

$$U(\mu_i, \mu; \zeta) = \exp\left[\int_{\mu_i}^{\mu} \frac{d\mu'}{\mu'} \gamma_{\mu}(\mu', \zeta)\right], \quad Z(b, \mu_i, \mu; \zeta) = \left(\frac{\zeta}{\zeta_i}\right)^{\gamma_{\zeta}(b, \mu_i)}$$

# Non-perturbative treatment

Non-perturbative contributions given by

$$f_{1q/A}(x, b, \mu, \zeta) = [C \otimes f]_{q/A}(x, b, \mu_i, \zeta_i) U(\mu_i, \mu; \zeta) Z(b, \zeta_i, \zeta; \mu_i) U_{\text{NP}}^{f^A}(x, b, \zeta, A)$$

$$D_{1h/q}^A(z, b, \mu, \zeta) = \frac{1}{z^2} [\hat{C} \otimes D^A]_{h/q}(z, b, \mu_i, \zeta_i) U(\mu_i, \mu; \zeta) Z(b, \zeta_i, \zeta; \mu_i) U_{\text{NP}}^{D^A}(z, b, \zeta, A)$$

EPPS16      In house FF (new), previous analysis used LIKE<sub>n</sub>

Non-perturbative Sudakov given by

$$U_{\text{NP}}^{f^A}(x, b, \zeta) = U_{\text{NP}}^f(x, b, \zeta) \exp \left\{ -g_q^A \left( A^{1/3} - 1 \right) b^2 \left( \frac{\zeta_A}{\zeta} \right)^\Gamma \right\}$$

$$U_{\text{NP}}^{D^A}(x, b, \zeta) = U_{\text{NP}}^D(x, b, \zeta) \exp \left\{ -g_h^A \left( A^{1/3} - 1 \right) \frac{b^2}{z^2} \left( \frac{\zeta_A}{\zeta} \right)^\Gamma \right\}$$

Parameterization for the medium modified fragmentation

$$D_i^{\pi^+}(z, \mu_0) = \frac{N_i z^{\alpha_i} (1-z)^{\beta_i} [1 + \gamma_i (1-z)^{\delta_i}]}{B[2 + \alpha_i, \beta_i + 1] + \gamma_i B[2 + \alpha_i, \beta_i + \delta_i + 1]} .$$

$$\tilde{N}_i \rightarrow \tilde{N}_i \left[ 1 + N_{i,1} (1 - A^{N_{i,2}}) \right]$$

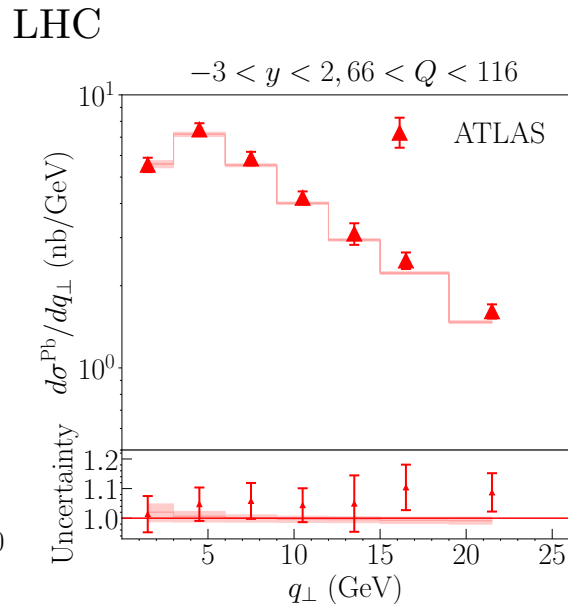
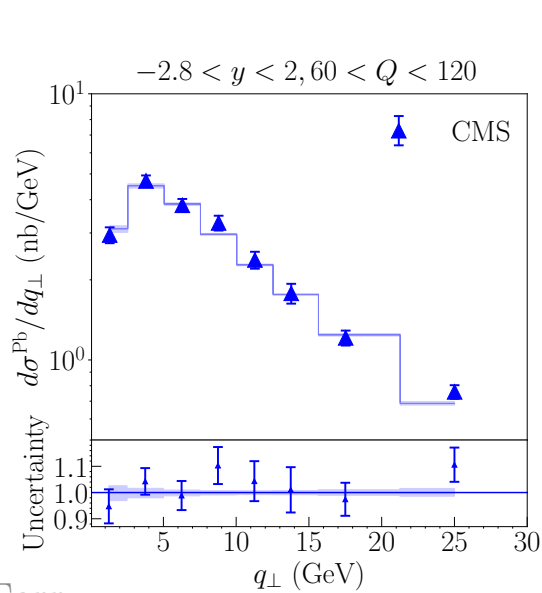
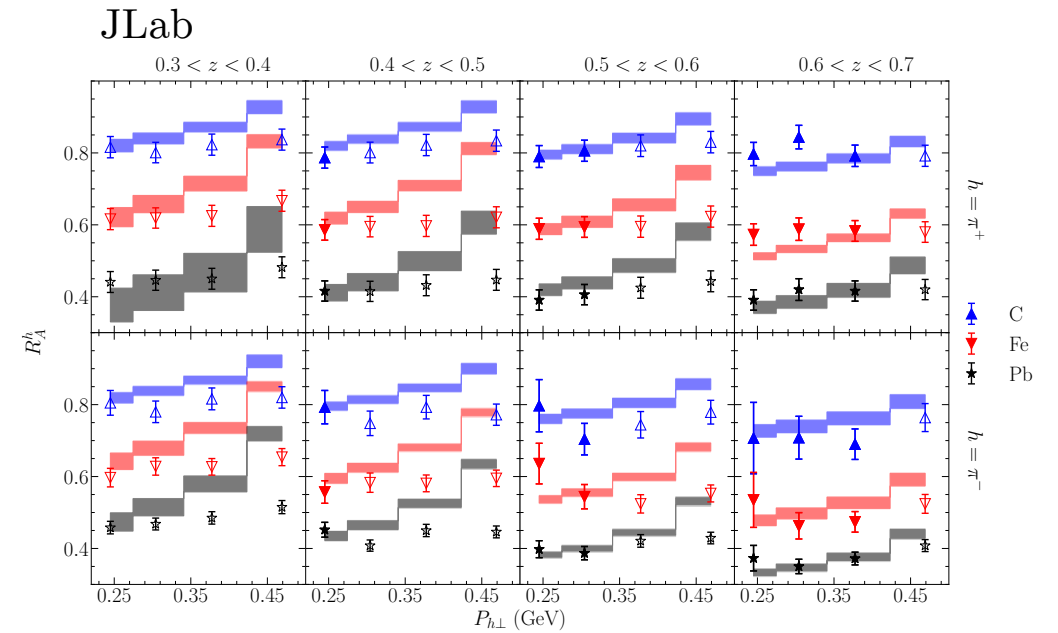
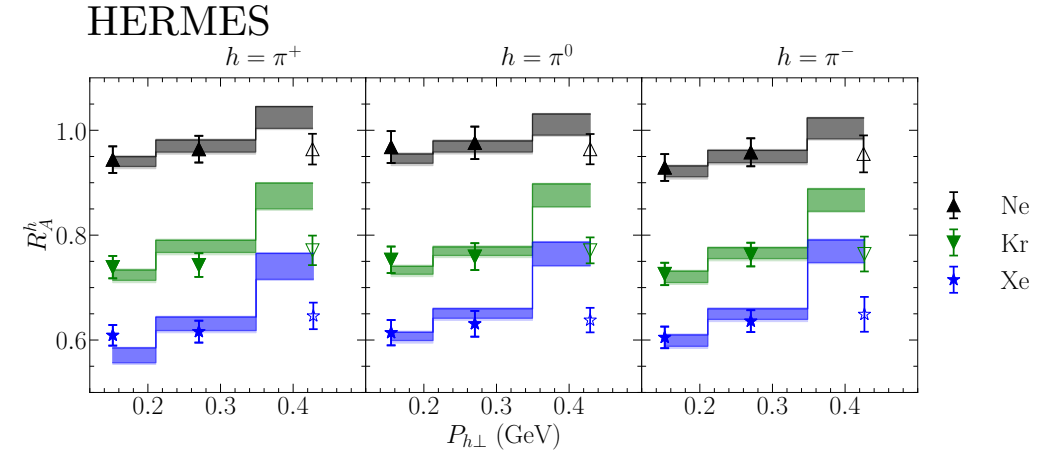
$$c_i \rightarrow c_i + c_{i,1} (1 - A^{c_{i,2}})$$

$$\mathbf{p} = \{ N_{q1}, N_{q2}, \gamma_{q1}, \gamma_{q2}, \delta_{q1}, \delta_{q2}, g_q^A, g_h^A, \Gamma \} ,$$



# Description of the experimental data

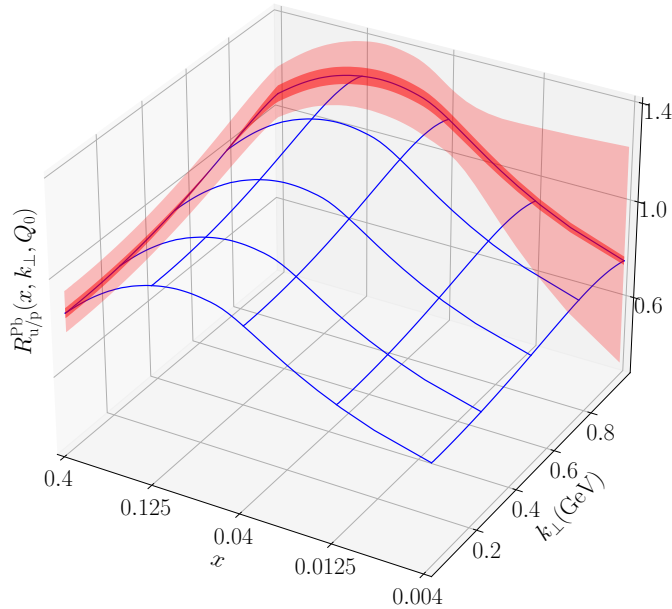
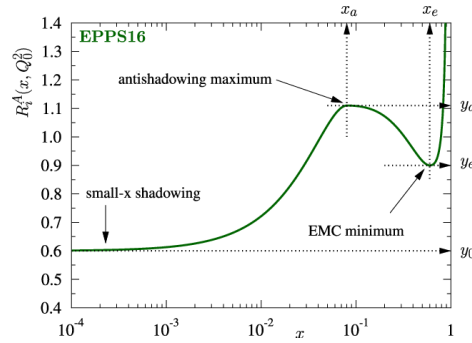
Collaboration	Process	Baseline	Nuclei	N <sub>data</sub>	$\chi^2$
JLAB [49]	SIDIS( $\pi$ )	D	C, Fe, Pb	36	41.7
HERMES [40]	SIDIS( $\pi$ )	D	Ne, Kr, Xe	18	10.2
RHIC [43]	DY	p	Au	4	1.3
E772 [41]	DY	D	C, Fe, W	16	40.2
E866 [42]	DY	Be	Fe, W	28	20.6
CMS [63]	$\gamma^*/Z$	N/A	Pb	8	10.4
ATLAS [83]	$\gamma^*/Z$	N/A	Pb	7	13.3
Total				117	137.8



# Three-dimensional images

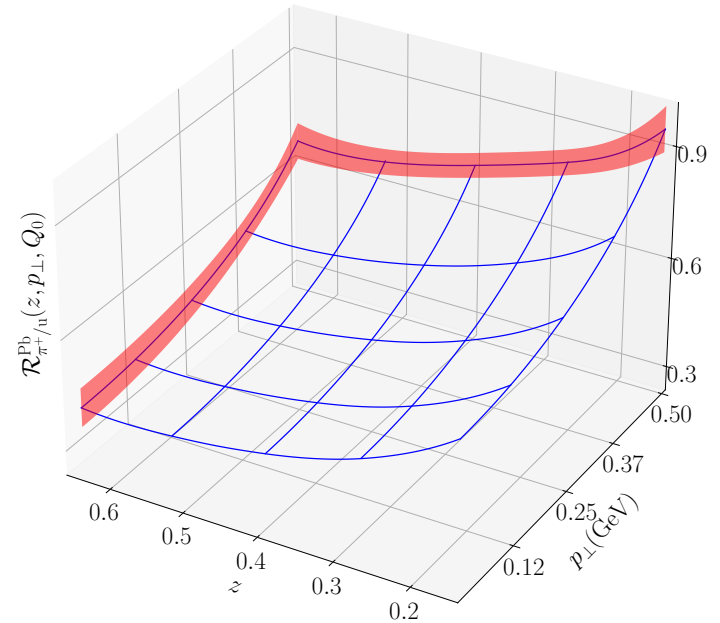
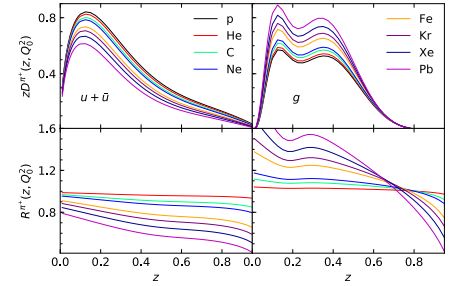
Ratios defined for nPDF and nFF

$$R_i^A(x, Q_0^2) = \frac{f_{i/p}^A(x, Q_0^2)}{f_{i/p}(x, Q_0^2)}$$



$$R_{u/p}^{Pb}(x, k_{\perp}, Q_0) = \frac{f_{u/p}^{Pb}(x, k_{\perp}, Q_0, Q_0^2)}{f_{u/p}(x, k_{\perp}, Q_0, Q_0^2)}$$

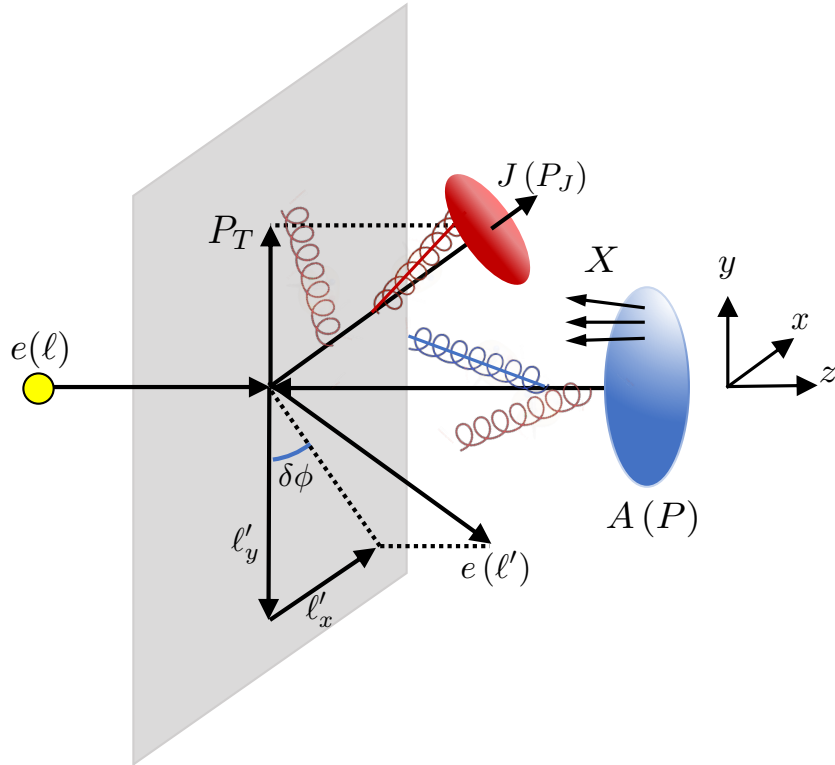
$$R_i^A(z, Q_0^2) = \frac{D_{h/i}^A(z, Q_0^2)}{D_{h/i}(z, Q_0^2)}$$



$$R_{\pi^+/u}^{Pb}(z, p_{\perp}, Q_0) = \frac{D_{\pi^+/u}^{Pb}(z, p_{\perp}, Q_0, Q_0^2)}{D_{\pi^+/u}(z, p_{\perp}, Q_0, Q_0^2)}$$

# Back-to-back lepton-jet production (a better observable than SIDIS)

Process proposed by: [Liu, Ringer, Vogelsang, Yuan \(2019\)](#)



Less sensitive to non-perturbative physics

Lepton-jet transverse momentum imbalance

$$\vec{q}_T = \vec{P}_{JT} + \vec{\ell}_T$$

TMD region

$$\frac{|\vec{q}_T|}{|\vec{P}_{JT}|} \ll 1$$

Better than SIDIS in that there is no sensitivity to FFs

Worse due to the perturbative accuracy

Novel factorization using recoil-free jets

Hard:  $P_{JT} (1, 1, 1)$

Collinear:  $P_{JT} (\lambda^2, 1, \lambda)$

Jet:  $P_{JT} (1, \lambda^2, \lambda)$

Global soft:  $P_{JT} (\lambda, \lambda, \lambda)$

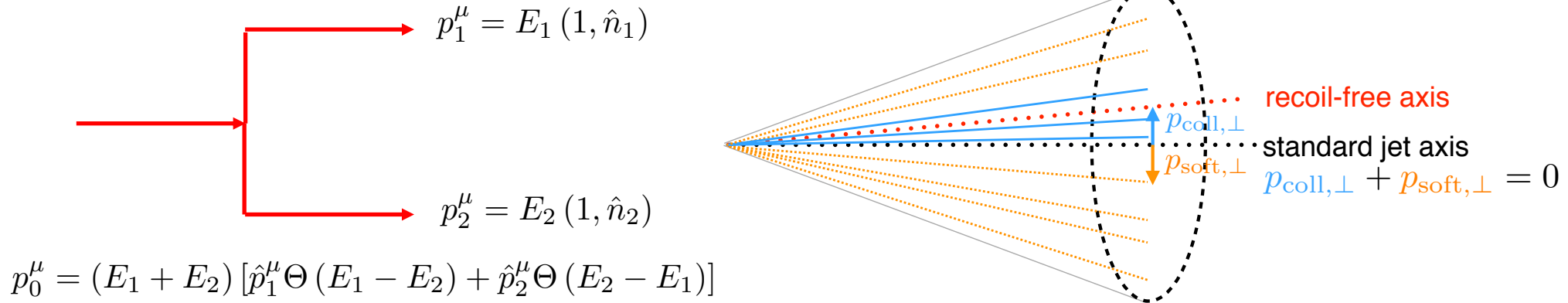
$$\frac{d\sigma_p}{d^2\ell'_T dy d\delta\phi} = \frac{\sigma_0 \ell'_T}{1-y} H(Q, \mu_H) \int \frac{db}{2\pi} \cos(bl'_T \delta\phi) \sum_q e_q^2 f_{q/p}(x_B, b, \mu_H, \zeta_B) \mathcal{J}_q(b, \mu_H, \zeta_J)$$

$$\frac{d\sigma_A}{d^2\ell'_T dy d\delta\phi} = \frac{\sigma_0 \ell'_T}{1-y} H(Q, \mu_H) \int \frac{db}{2\pi} \cos(bl'_T \delta\phi) \sum_q e_q^2 f_{q/A}(x_B, b, \mu_H, \zeta_B) \mathcal{J}_q^A(b, \mu_H, \zeta_J)$$

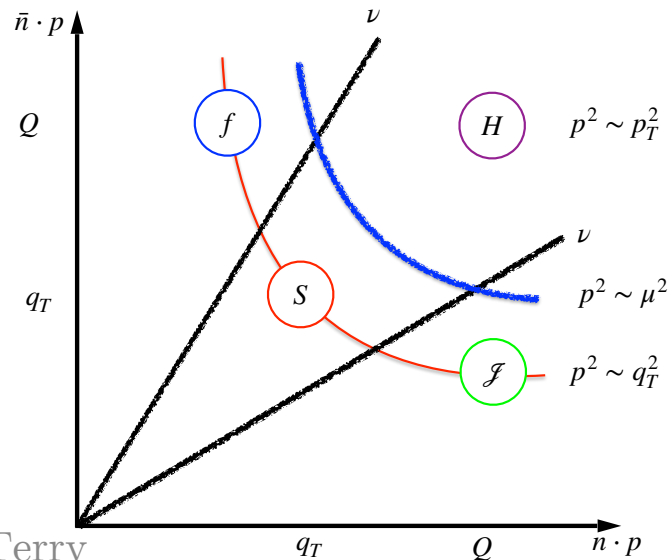
# Winner take all jet axis (a better jet axis than the standard)

Direction of recoil-free jet is insensitive all soft emissions, jet points in direction of most energetic hadron. Thus no NGLS

Larkoski, Neill, and Thaler (2014)



Direction of jet and total jet momentum have a transverse momentum relative to one another, contains rapidity divergence



$$\mu \frac{d}{d\mu} \ln \mathcal{J}(Q, \mu, \nu) = \gamma_{\mathcal{J}\mu}^q(Q, \mu, \nu) \quad \gamma_{\mathcal{J}\mu}^q(Q, \mu, \nu) = \gamma_{D\mu}^q(Q, \mu, \nu)$$

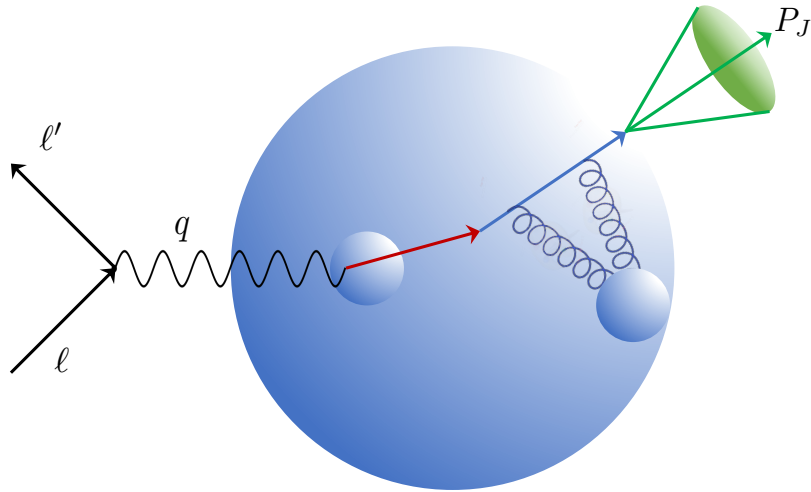
$$\nu \frac{d}{d\nu} \ln \mathcal{J}(Q, \mu, \nu) = \gamma_{\mathcal{J}\nu}^q(Q, \mu, \nu) \quad \gamma_{\mathcal{J}\nu}^q(Q, \mu, \nu) = \gamma_{D\nu}^q(Q, \mu, \nu)$$

$$\mathcal{J}_q(b, \mu, \zeta_{\mathcal{J}}/\nu^2) = 1 + \frac{\alpha_s C_F}{4\pi} \left[ 3L + 2L \ln \left( \frac{\nu^2}{\zeta_{\mathcal{J}}} \right) + 7 - \frac{2\pi^2}{3} - 6 \ln 2 \right] + \mathcal{O}(\alpha_s^2)$$



# Treatment of medium modifications to the jet

We want to take the jets to be energetic so that  $L/L_h \sim \frac{A^{1/3} \Lambda_{\text{QCD}}}{\nu} \ll 1$



Single gluon exchange

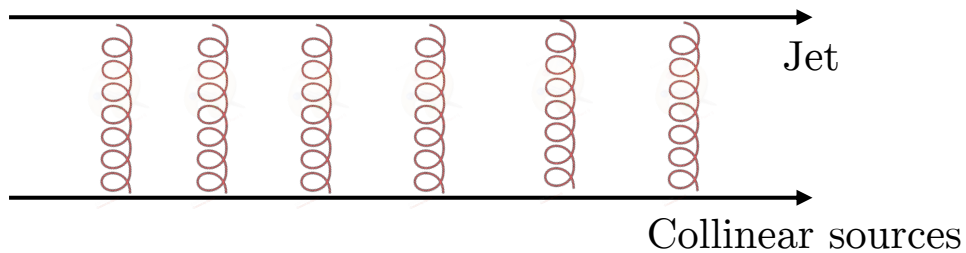
$$\frac{d\sigma}{d^2q_T} = \frac{\alpha_s C_F}{\pi} \frac{1}{(q_T^2 + m^2)^2}$$

Infinite number of gluon exchanges

$$\frac{d\sigma_{n \rightarrow \infty}}{db} = \exp\left(\frac{\rho_G L}{m^2} \alpha_s C_F (mb K_1(mb) - 1)\right)$$

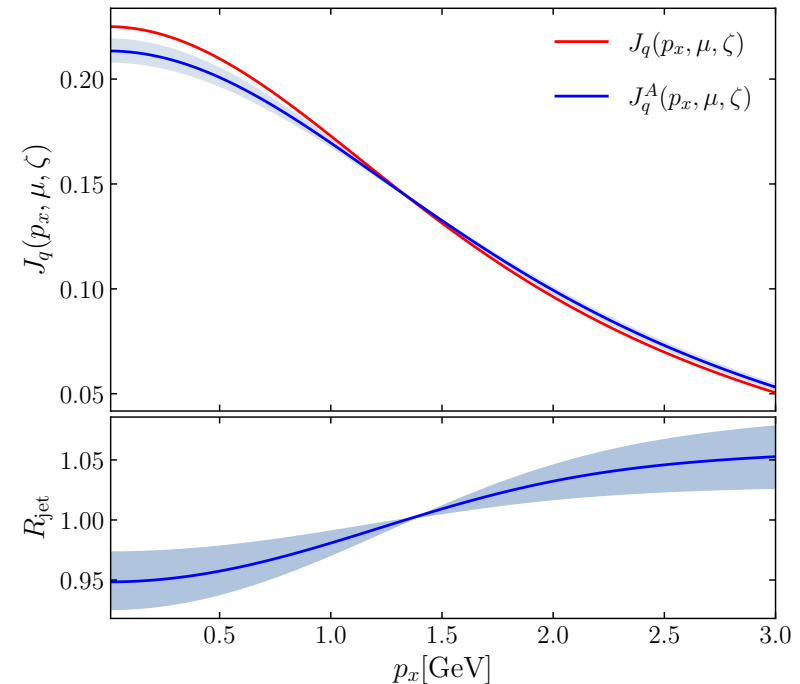
Modified jet function under this approximation

We consider an infinite chain of Glauber gluons



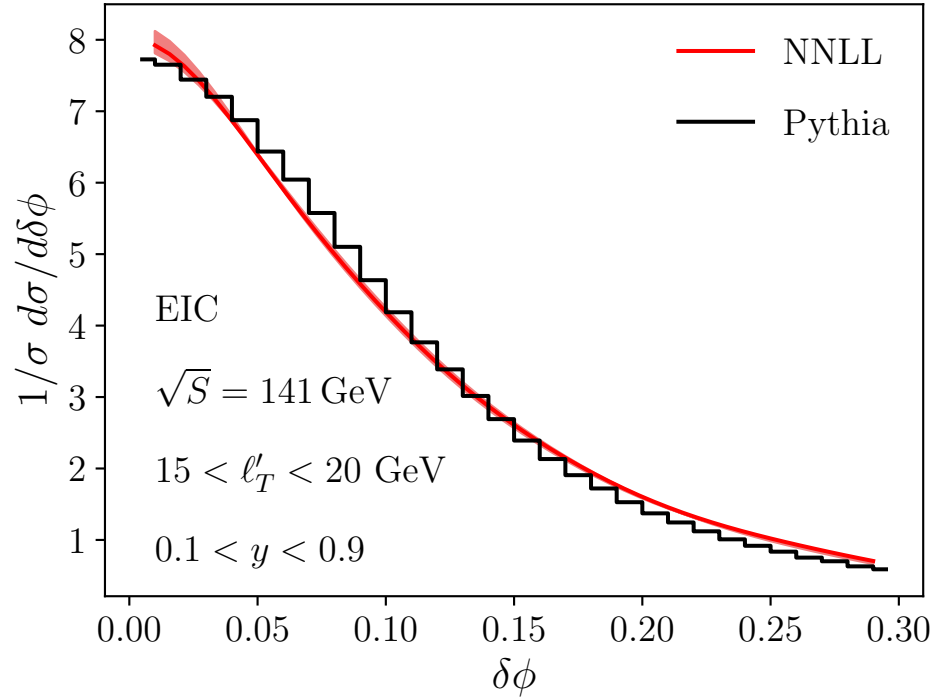
Modification to the jet

$$\mathcal{J}_q^A(b, \mu, \nu) = \frac{d\sigma_{n \rightarrow \infty}}{db} \mathcal{J}_q(b, \mu, \nu)$$

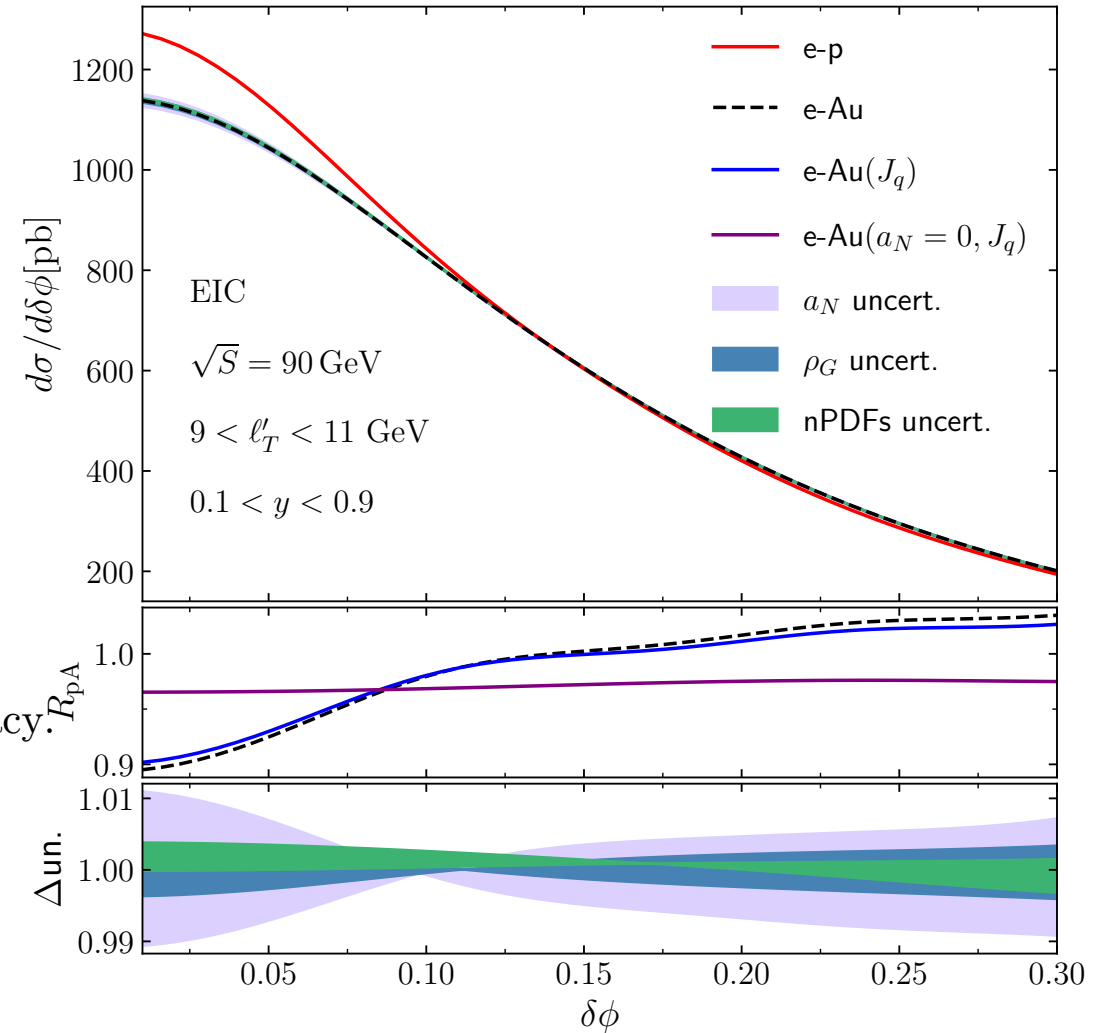


# Predictions at the EIC

Comparison of our results with Pythia at NNLL



Perturbative ingredients are known to have N3LL accuracy. Only missing the 3-loop jet function and the 5-loop cusp anomalous dimension to reach N4LL



# Di-jet decorrelations in pp

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

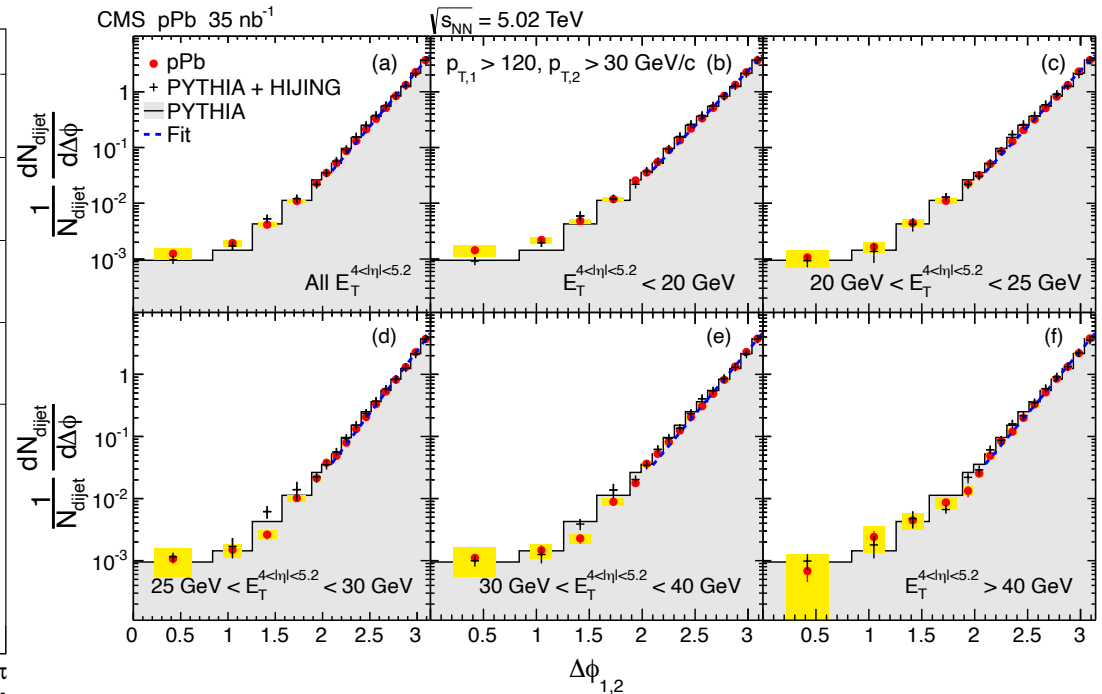
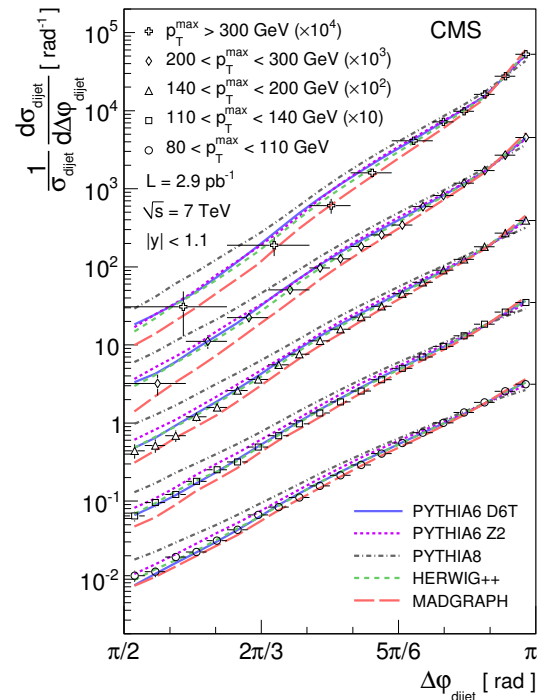
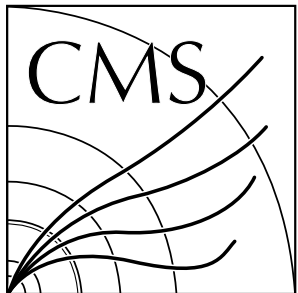
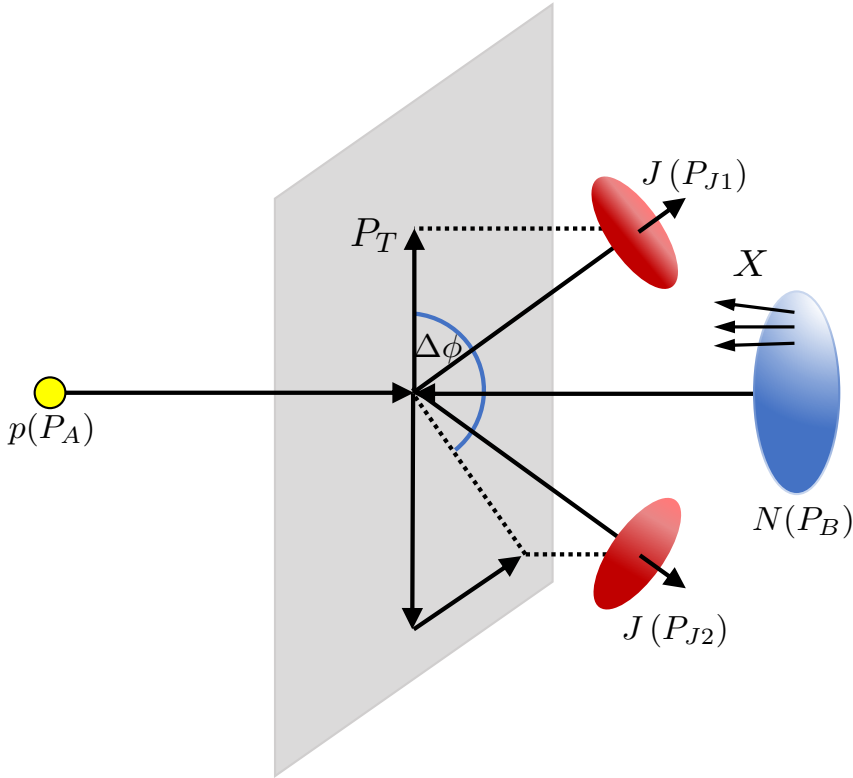
Back-to-back region is sensitive to the 1+1 dimensional TMDs

CMS Measurements in pp and pA collisions

[Phys.Rev.Lett.106:122003,2011](#)

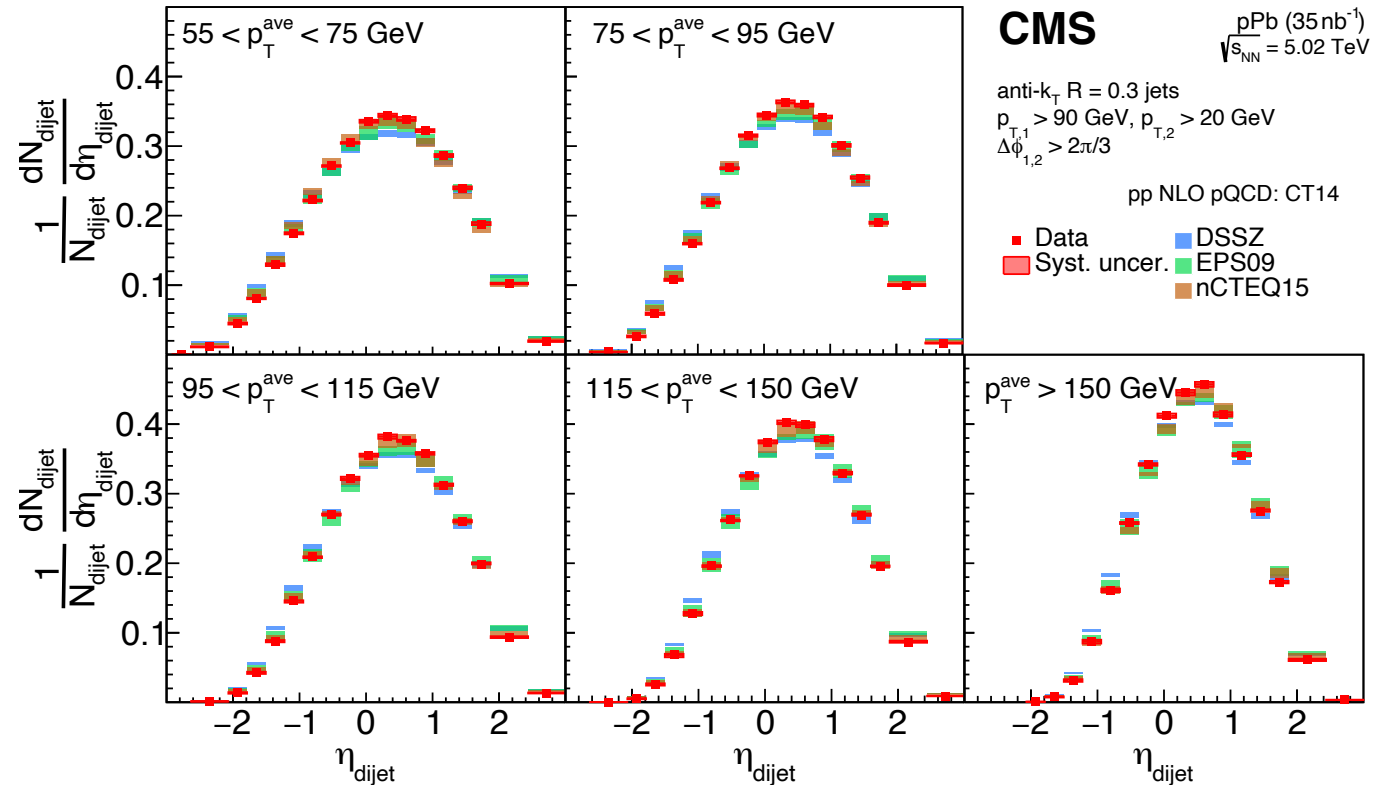
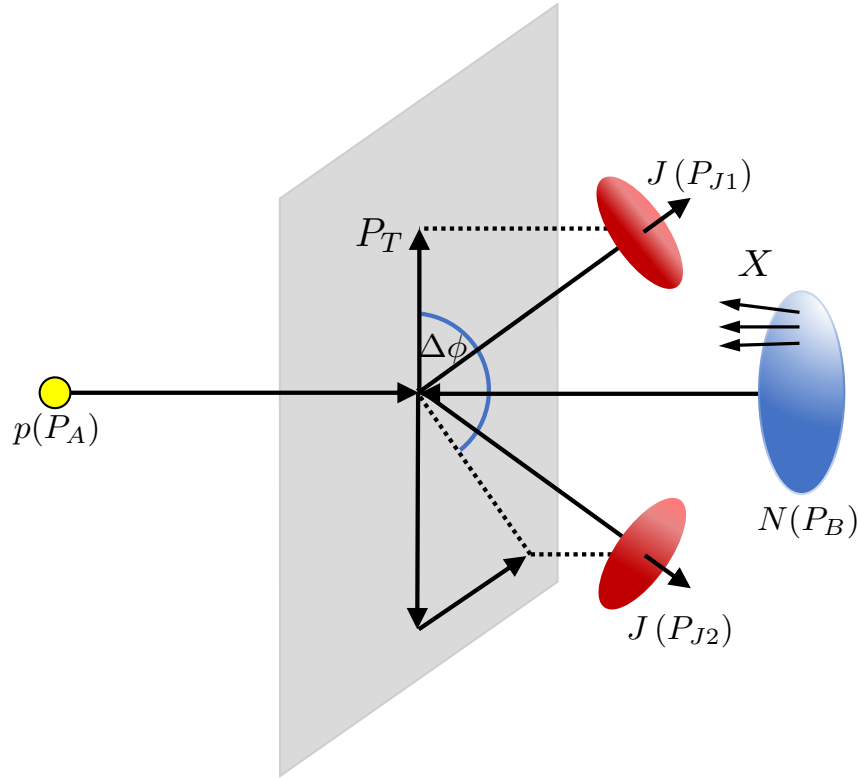
[Eur. Phys. J. C 74 \(2014\) 2951](#)

[Phys. Rev. Lett. 121, 062002 \(2018\)](#)



# Di-jet decorrelations in pp continued

Additional measurements of the integrated azimuthal angle decorrelation



Integration in region  $\Delta\phi > 2\pi/3$  performed using a collinear approximation. However, there are issues with this approach as  $\Delta\phi \rightarrow \pi$  due to large logarithms.



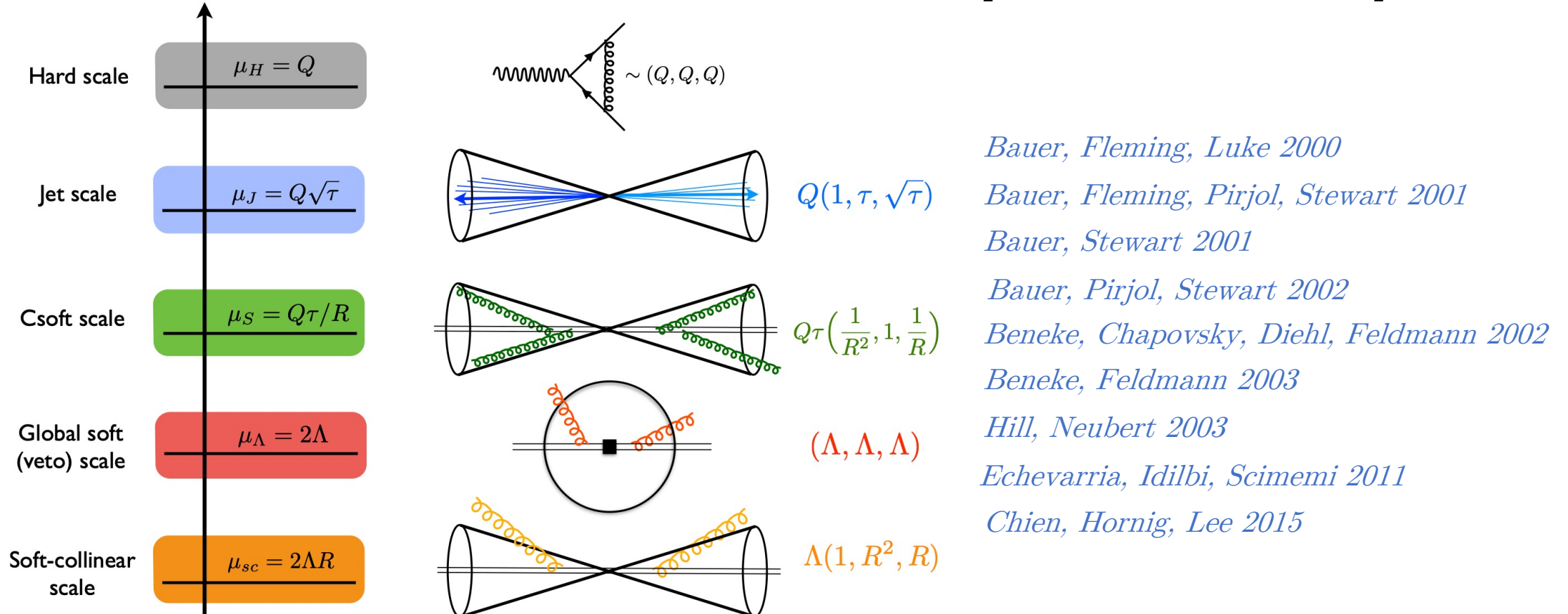
# QCD modes in SCET

SCET is an EFT which captures soft and collinear emissions along the directions

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{\psi} i \not{D} \psi - \frac{1}{4} G_{\mu\nu}^A G^{A\mu\nu} + \mathcal{L}_{\text{gauge-fix}} + \mathcal{L}_{\text{ghost}}$$

$$\psi \rightarrow \psi_s + \psi_c \quad A^\mu \rightarrow A_s^\mu + A_c^\mu \quad \longrightarrow \quad \mathcal{L}_{\text{SCET}} = \bar{\psi}_s i \not{D}_s \psi_s - \frac{1}{4} G_{\mu\nu s}^A G_s^{A\mu\nu}$$

$$+ \xi \frac{\not{n}}{2} \left[ i n \cdot D + i \not{D}_{c\perp} \frac{1}{i \bar{n} \cdot D_c} i \not{D}_{c\perp} \right] \xi - \frac{1}{4} G_{\mu\nu c}^A G_c^{A\mu\nu}$$

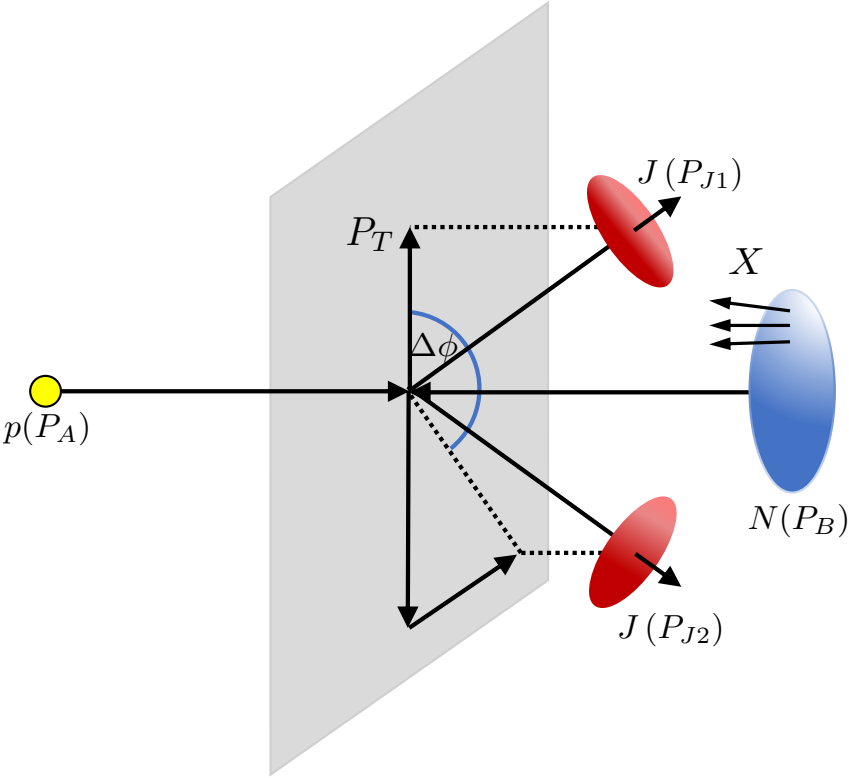


# Factorization in SCET

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

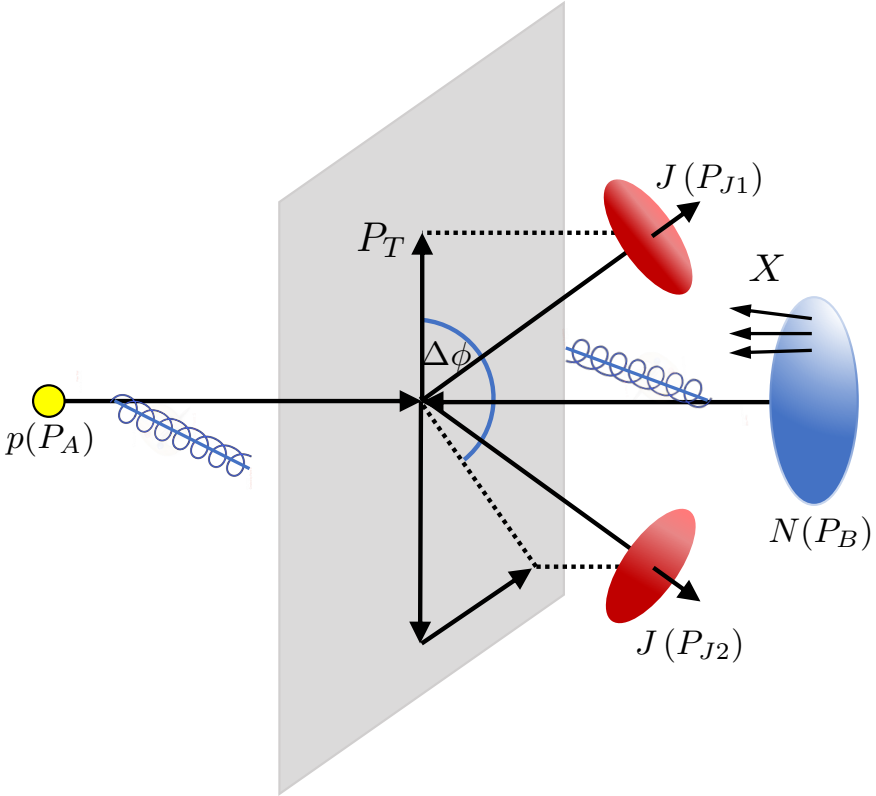
Factorization and resummation derived in a SCET framework

**hard** :  $p_h^\mu \sim p_T(1, 1, 1)$



# Factorization in SCET

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

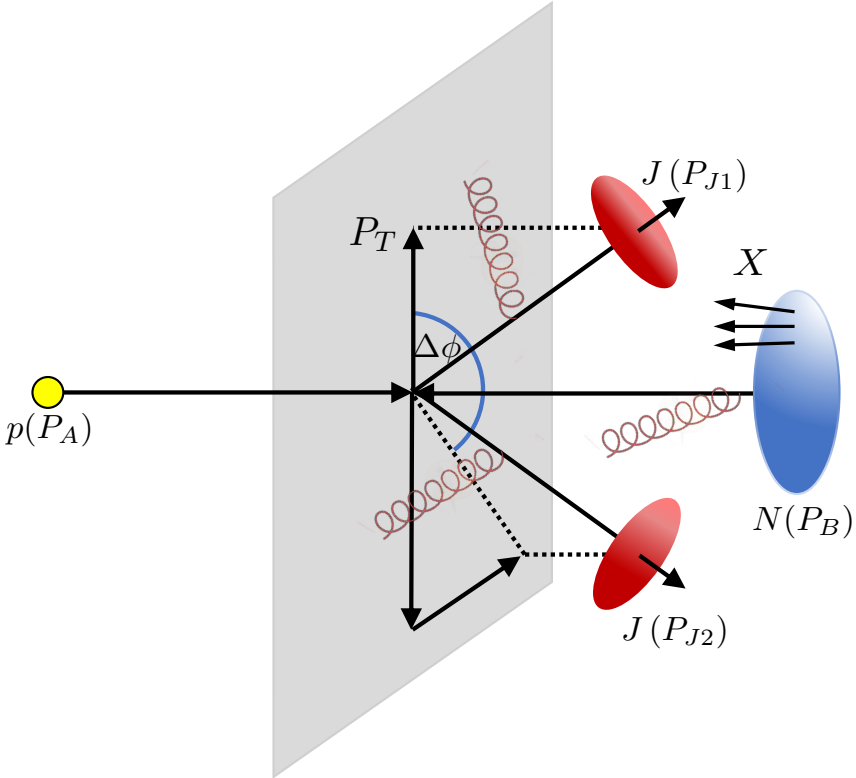


Factorization and resummation derived in a SCET framework

**hard** :  $p_h^\mu \sim p_T(1, 1, 1)$   
 **$n_{a,b}$ -collinear** :  $p_{c_i}^\mu \sim p_T(\delta\phi^2, 1, \delta\phi)n_i\bar{n}_i$ ,

# Factorization in SCET

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

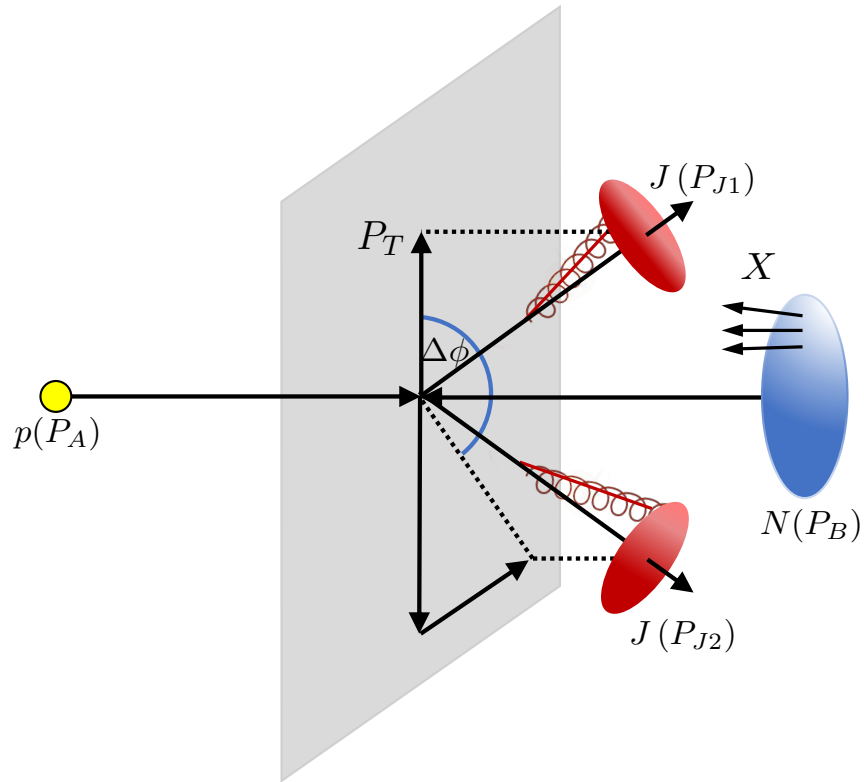


Factorization and resummation derived in a SCET framework

$$\begin{aligned}
 \text{hard} &: p_h^\mu \sim p_T(1, 1, 1) \\
 n_{a,b}\text{-collinear} &: p_{c_i}^\mu \sim p_T(\delta\phi^2, 1, \delta\phi)n_i\bar{n}_i, \\
 \text{soft} &: p_s^\mu \sim p_T(\delta\phi, \delta\phi, \delta\phi),
 \end{aligned}$$

# Factorization in SCET

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

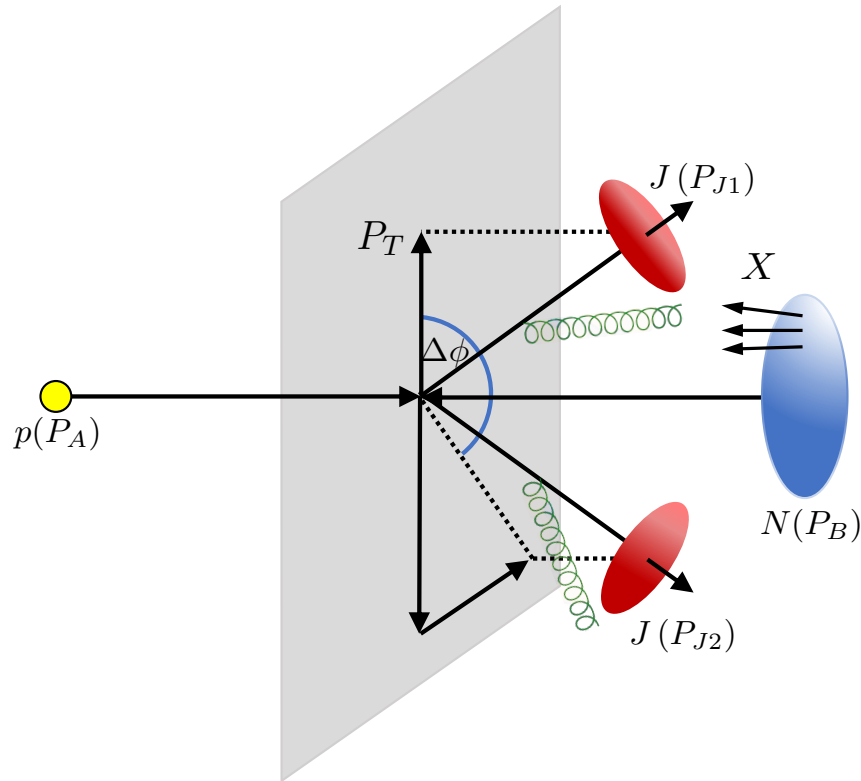


Factorization and resummation derived in a SCET framework

$$\begin{aligned} \text{hard} &: p_h^\mu \sim p_T(1, 1, 1) \\ n_{a,b}\text{-collinear} &: p_{c_i}^\mu \sim p_T(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i}, \\ \text{soft} &: p_s^\mu \sim p_T(\delta\phi, \delta\phi, \delta\phi), \\ n_{c,d}\text{-jet} &: p_{c_i}^\mu \sim p_T(R^2, 1, R)_{n_i\bar{n}_i}, \end{aligned}$$

# Factorization in SCET

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

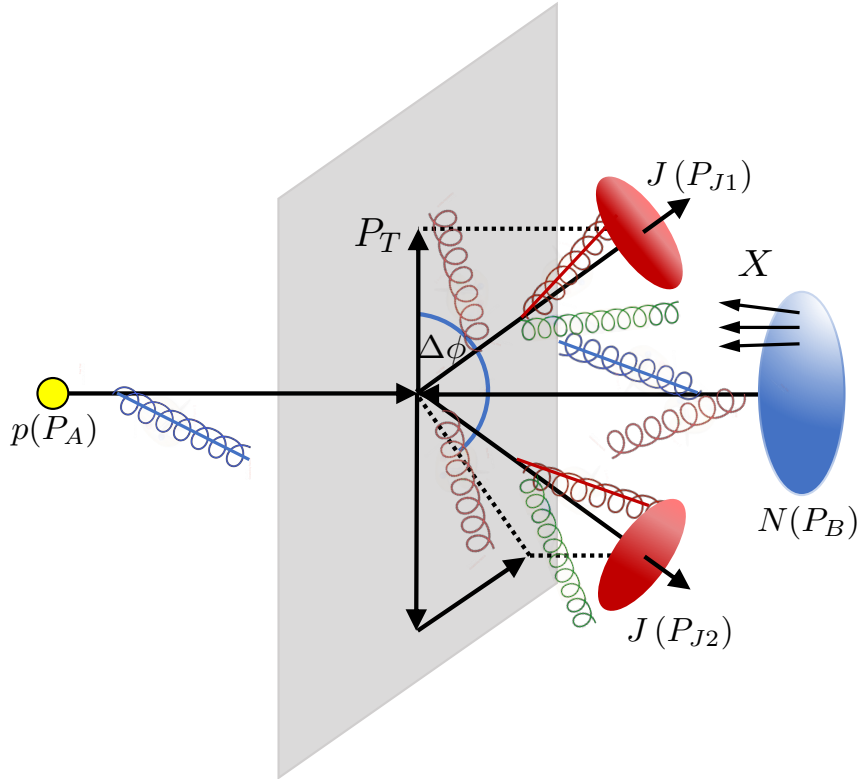


Factorization and resummation derived in a SCET framework

$$\begin{aligned}
 \text{hard} &: p_h^\mu \sim p_T(1, 1, 1) \\
 n_{a,b}\text{-collinear} &: p_{c_i}^\mu \sim p_T(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i}, \\
 \text{soft} &: p_s^\mu \sim p_T(\delta\phi, \delta\phi, \delta\phi), \\
 n_{c,d}\text{-jet} &: p_{c_i}^\mu \sim p_T(R^2, 1, R)_{n_i\bar{n}_i}, \\
 n_{c,d}\text{-collinear-soft} &: p_{cs_i}^\mu \sim \frac{p_T \delta\phi}{R}(R^2, 1, R)_{n_i\bar{n}_i},
 \end{aligned}$$

# Factorization in SCET

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs



Factorization and resummation at NLL:

Factorization and resummation derived in a SCET framework

$$\begin{aligned}
 \text{hard} &: p_h^\mu \sim p_T(1, 1, 1) \\
 n_{a,b}\text{-collinear} &: p_{c_i}^\mu \sim p_T(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i}, \\
 \text{soft} &: p_s^\mu \sim p_T(\delta\phi, \delta\phi, \delta\phi), \\
 n_{c,d}\text{-jet} &: p_{c_i}^\mu \sim p_T(R^2, 1, R)_{n_i\bar{n}_i}, \\
 n_{c,d}\text{-collinear-soft} &: p_{cs_i}^\mu \sim \frac{p_T \delta\phi}{R}(R^2, 1, R)_{n_i\bar{n}_i},
 \end{aligned}$$

$$\begin{aligned}
 \frac{d^4\sigma_{pp}}{dy_c dy_d dp_T^2 d\delta\phi} &= \sum_{abcd} \frac{p_T}{16\pi\hat{s}^2} \frac{1}{1+\delta_{cd}} \int_0^\infty \frac{2db}{\pi} \cos(bp_T\delta\phi) x_a \tilde{f}_{a/p}(x_a, \mu_{b_*}) x_b \tilde{f}_{b/p}(x_b, \mu_{b_*}) \\
 &\times \exp \left\{ - \int_{\mu_{b_*}}^{\mu_h} \frac{d\mu}{\mu} \left[ \gamma_{\text{cusp}}(\alpha_s) C_H \ln \frac{\hat{s}}{\mu^2} + 2\gamma_H(\alpha_s) \right] \right\} \\
 &\times \sum_{KK'} \exp \left[ - \int_{\mu_{b_*}}^{\mu_h} \frac{d\mu}{\mu} \gamma_{\text{cusp}}(\alpha_s) (\lambda_K + \lambda_{K'}^*) \right] H_{KK'}(\hat{s}, \hat{t}, \mu_h) W_{K'K}(b_*, \mu_{b_*}) \\
 &\times \exp \left[ - \int_{\mu_{b_*}}^{\mu_j} \frac{d\mu}{\mu} \Gamma^{J_c}(\alpha_s) - \int_{\mu_{b_*}}^{\mu_j} \frac{d\mu}{\mu} \Gamma^{J_d}(\alpha_s) \right] U_{\text{NG}}^c(\mu_{b_*}, \mu_j) U_{\text{NG}}^d(\mu_{b_*}, \mu_j) \\
 &\times \exp \left[ -S_{\text{NP}}^a(b, Q_0, \sqrt{\hat{s}}) - S_{\text{NP}}^b(b, Q_0, \sqrt{\hat{s}}) \right].
 \end{aligned}$$



# Do we observe factorization breaking effects?

Glauber mode not treated in our paper

**hard** :  $p_h^\mu \sim p_T(1, 1, 1)$

$n_{a,b}$ -**collinear** :  $p_{c_i}^\mu \sim p_T(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i}$ ,

**soft** :  $p_s^\mu \sim p_T(\delta\phi, \delta\phi, \delta\phi)$ ,

$n_{c,d}$ -**jet** :  $p_{c_i}^\mu \sim p_T(R^2, 1, R)_{n_i\bar{n}_i}$ ,

$n_{c,d}$ -**collinear-soft** :  $p_{cs_i}^\mu \sim \frac{p_T \delta\phi}{R}(R^2, 1, R)_{n_i\bar{n}_i}$ ,

**$n_G$ -Glauber** :  $p_G^\mu \sim p_T(\delta\phi^2, \delta\phi^2, \delta\phi)_{n_i\bar{n}_i}$

Factorization breaking effects in dijet production studied in

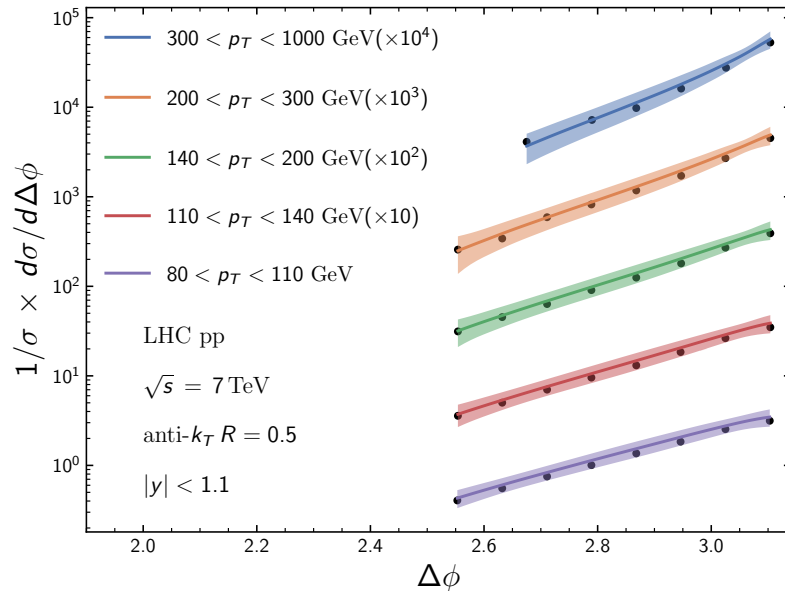
Collins, Qiu Phys.Rev.D75:114014,2007

Collins (2007)

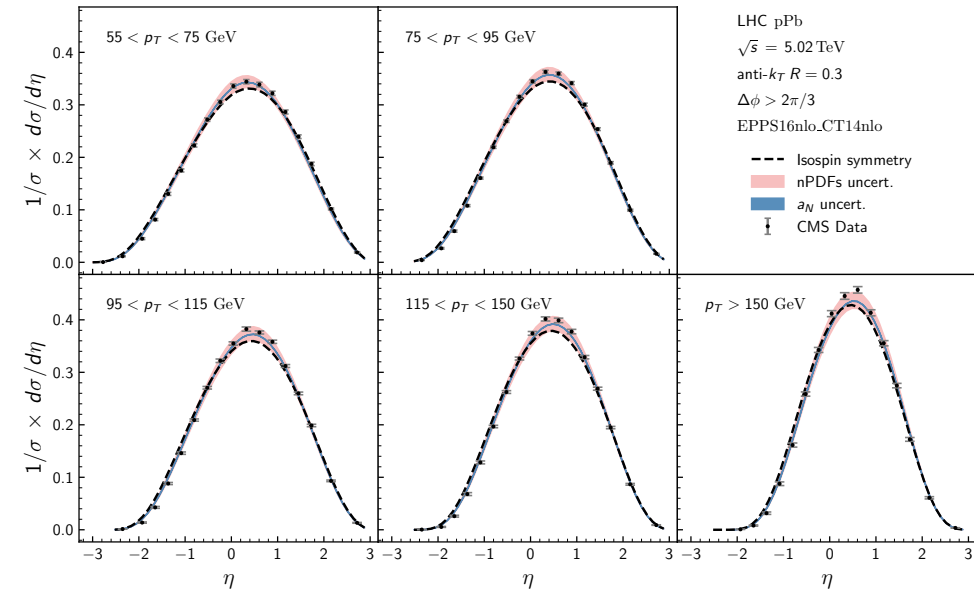
The experimental data is well-described by the experimental data in the back-to-back region, within the error bars

$$\frac{d^4\sigma_{pp}}{dy_c dy_d dp_T^2 d\delta\phi}$$

Phys.Rev.Lett.106:122003,2011



Phys. Rev. Lett. 121, 062002 (2018)



# Nuclear modifications to this process

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

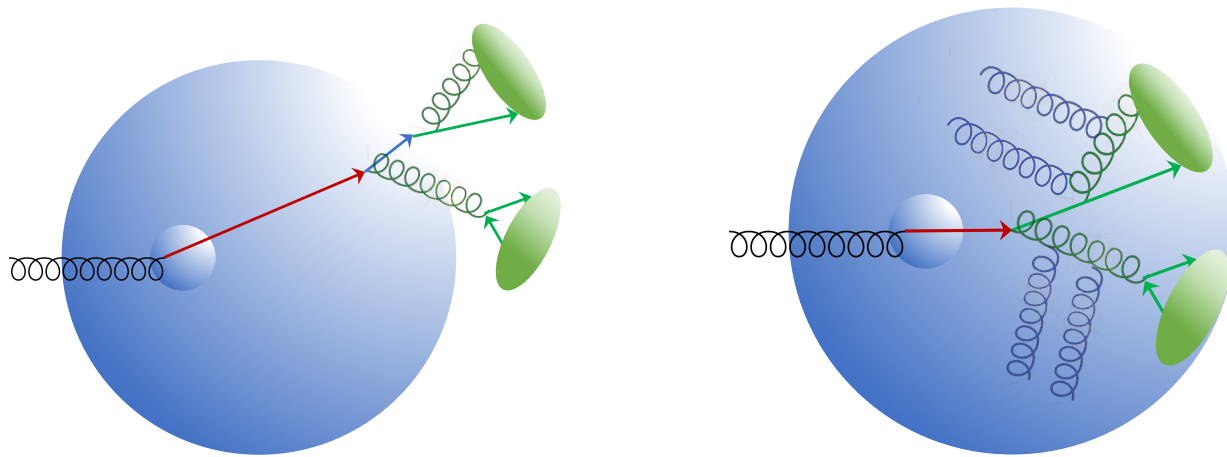
nTMDs can be matched onto the collinear distributions

$$f_{q/N}^A(b, x; \mu, \zeta_1) = [C \otimes f](x; \mu_i) \exp \left[ -S_{\text{pert}}(b; \mu_i, \mu, \zeta_i, \zeta_1) - S_{\text{NP}}^{fA}(b; Q_0, \mu, \zeta_i, \zeta) \right]$$

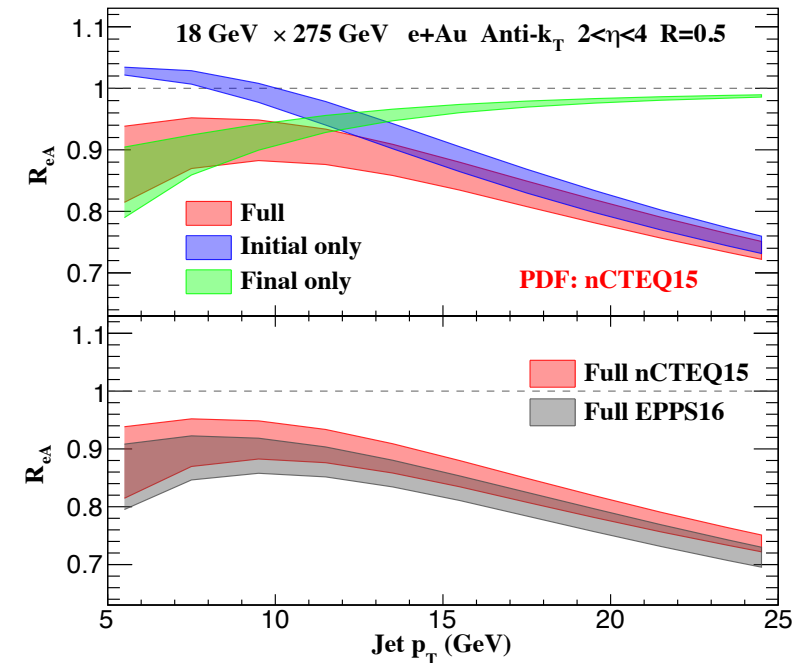
*Perturbative*

*Non-perturbative: Contains all medium contributions*

We ignore all final-state interactions between the jets and the medium.  
High energy jets are not expected to be affected by the medium

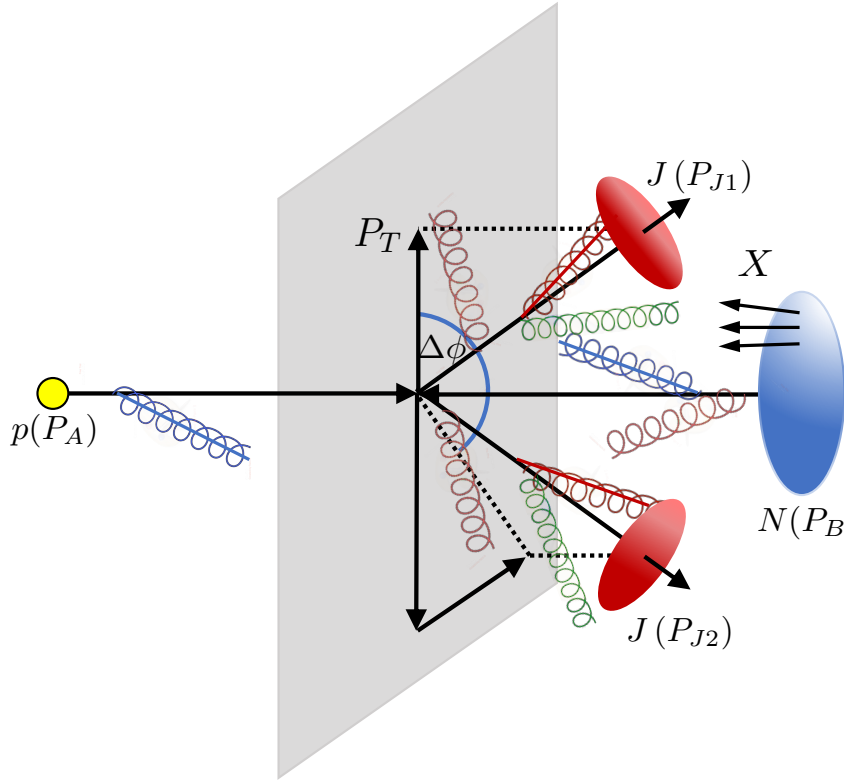


Li, Vitev (2021)



# Factorization in pA

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs



Factorization and resummation:

Factorization and resummation derived in a SCET framework

$$\text{hard} : p_h^\mu \sim p_T(1, 1, 1)$$

$$n_{a,b}\text{-collinear} : p_{c_i}^\mu \sim p_T(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i},$$

$$\text{soft} : p_s^\mu \sim p_T(\delta\phi, \delta\phi, \delta\phi),$$

$$n_{c,d}\text{-jet} : p_{c_i}^\mu \sim p_T(R^2, 1, R)_{n_i\bar{n}_i},$$

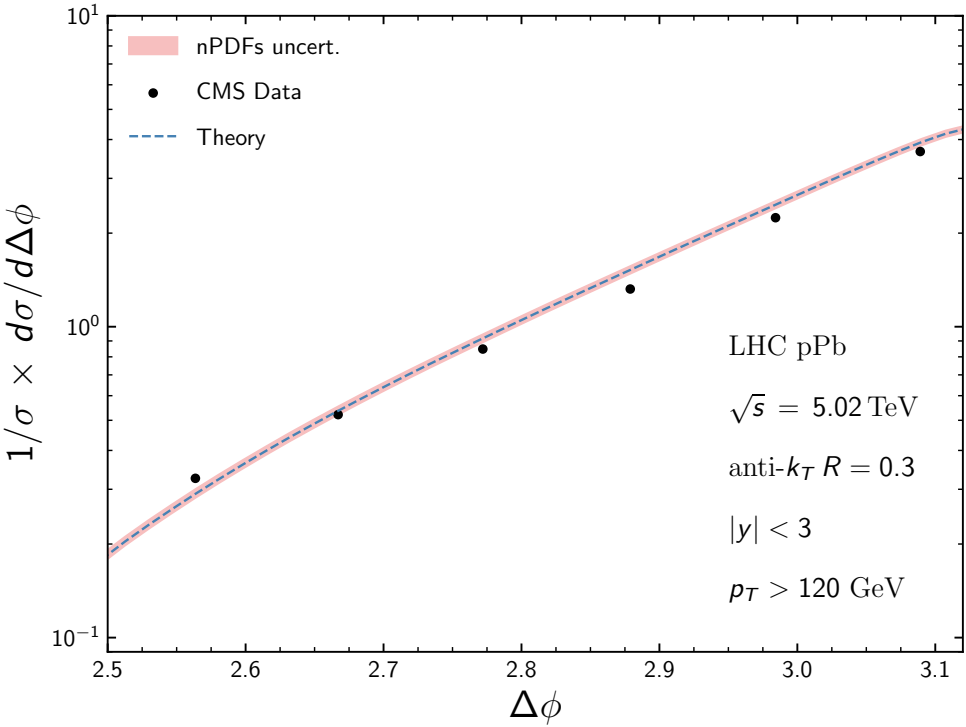
$$n_{c,d}\text{-collinear-soft} : p_{cs_i}^\mu \sim \frac{p_T \delta\phi}{R}(R^2, 1, R)_{n_i\bar{n}_i},$$

$$\begin{aligned} \frac{d^4\sigma_{pA}}{dy_c dy_d dp_T^2 d\delta\phi} &= \sum_{abcd} \frac{p_T}{16\pi\hat{s}^2} \frac{1}{1+\delta_{cd}} \int_0^\infty \frac{2db}{\pi} \cos(bp_T\delta\phi) x_a \tilde{f}_{a/p}(x_a, \mu_{b_*}) x_b \tilde{f}_{b/A}(x_b, \mu_{b_*}) \\ &\times \exp \left\{ - \int_{\mu_{b_*}}^{\mu_h} \frac{d\mu}{\mu} \left[ \gamma_{\text{cusp}}(\alpha_s) C_H \ln \frac{\hat{s}}{\mu^2} + 2\gamma_H(\alpha_s) \right] \right\} \\ &\times \sum_{KK'} \exp \left[ - \int_{\mu_{b_*}}^{\mu_h} \frac{d\mu}{\mu} \gamma_{\text{cusp}}(\alpha_s) (\lambda_K + \lambda_{K'}^*) \right] H_{KK'}(\hat{s}, \hat{t}, \mu_h) W_{K'K}(b_*, \mu_{b_*}) \\ &\times \exp \left[ - \int_{\mu_{b_*}}^{\mu_j} \frac{d\mu}{\mu} \Gamma^{J_c}(\alpha_s) - \int_{\mu_{b_*}}^{\mu_j} \frac{d\mu}{\mu} \Gamma^{J_d}(\alpha_s) \right] U_{\text{NG}}^c(\mu_{b_*}, \mu_j) U_{\text{NG}}^d(\mu_{b_*}, \mu_j) \\ &\times \exp \left[ -S_{\text{NP}}^a(b, Q_0, \sqrt{\hat{s}}) - S_{\text{NP}}^{b,A}(b, Q_0, \sqrt{\hat{s}}) \right] \end{aligned}$$

# Description of pA data

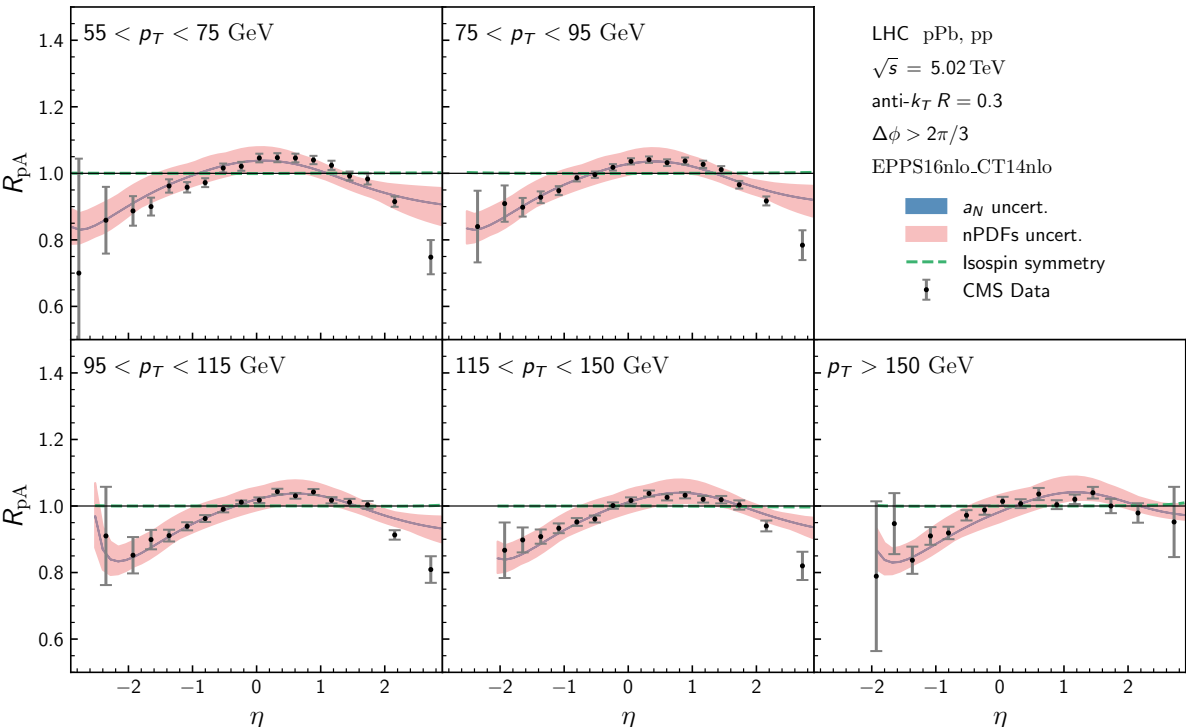
Strong consistency with the CMS measurements of the azimuthal angle decorrelation in pA and the ratio of the integrated azimuthal angle decorrelation.

$$\frac{d^4\sigma_{pA}}{dy_c dy_d dp_T^2 d\delta\phi}$$



[Eur. Phys. J. C 74 \(2014\) 2951](#)

$$R_{pA} = \frac{1}{A} \frac{d^4\sigma_{pA}}{dy_c dy_d dp_T^2 d\Delta\phi} \bigg/ \frac{d^4\sigma_{pp}}{dy_c dy_d dp_T^2 d\Delta\phi}$$

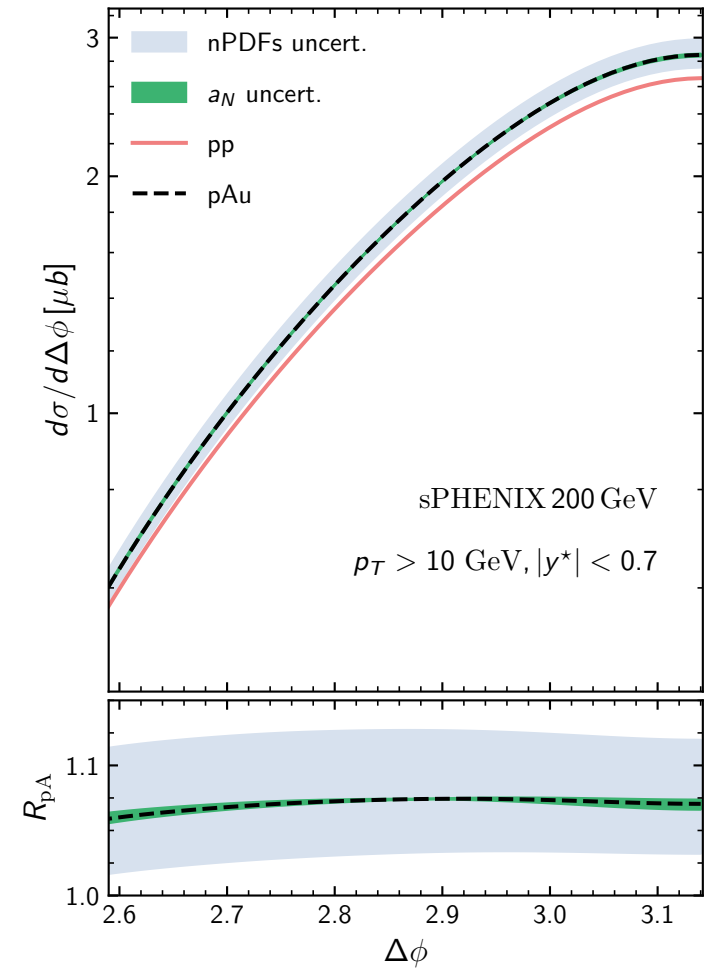
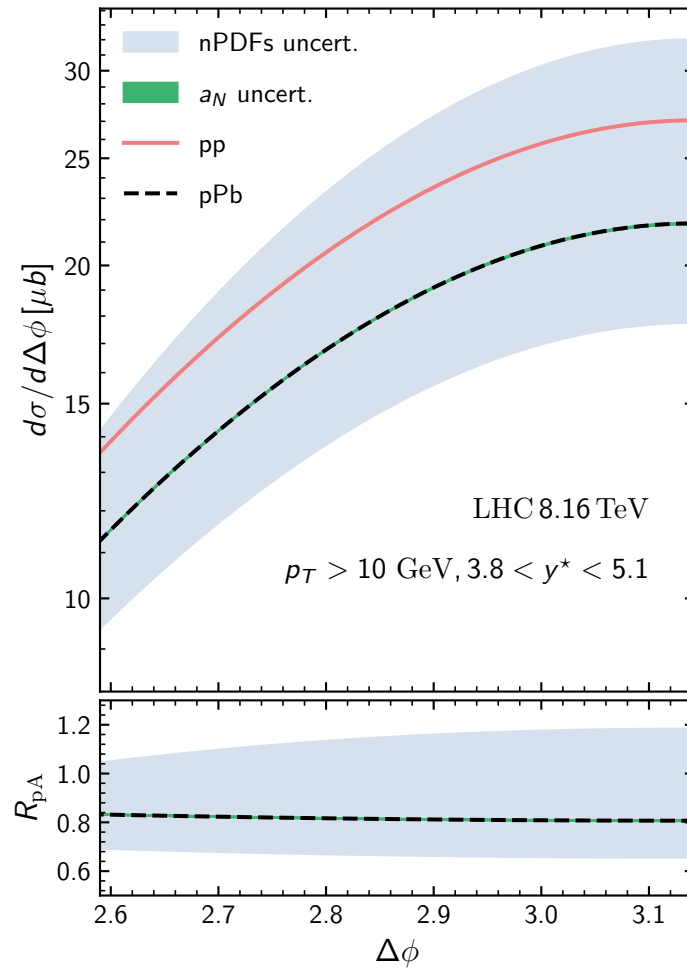
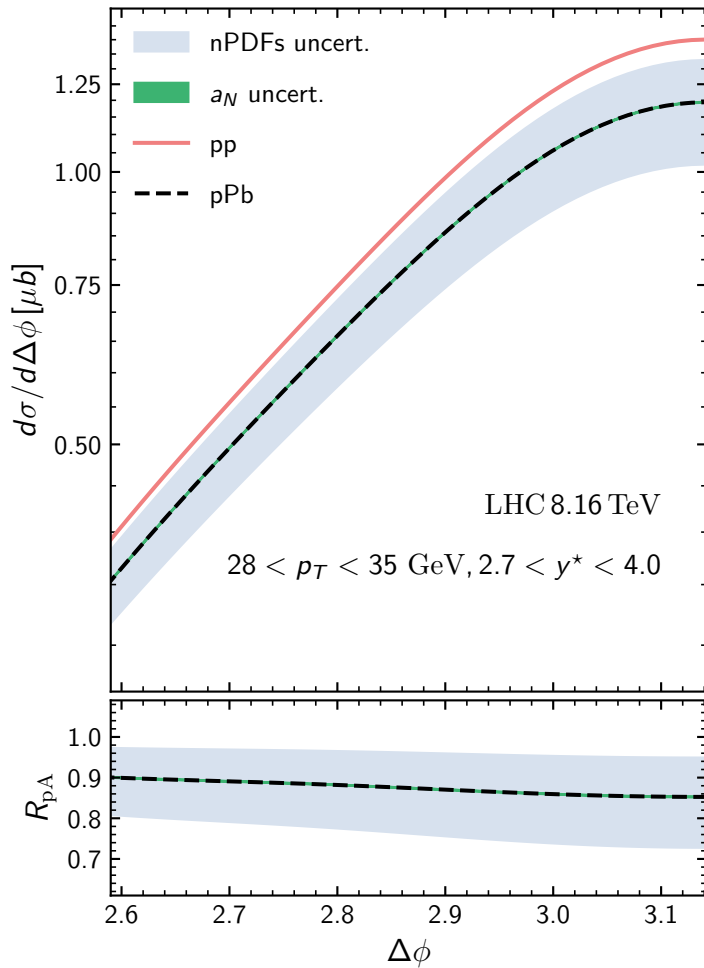


[Phys. Rev. Lett. 121, 062002 \(2018\)](#)

Red band is the uncertainty from the EPPS sets, small blue band from the uncertainty of the nTMDs and is very small for high  $p_T$  jet production.

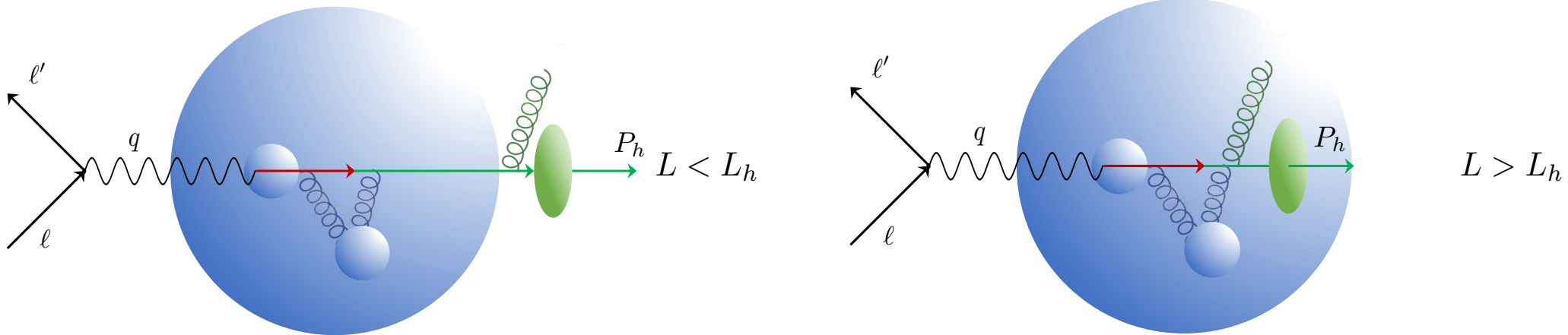
# Predictions at ATLAS, ALICE, and sPHENIX

At the LHC, the collinear uncertainties are more dominant due to the large perturbative transverse momenta that are generated. Uncertainty band of the broadening becomes larger at lower center of mass energies



# Fragmentation in the medium from first principles

Medium introduces three length scales to the problem  $L \sim A^{1/3}/\Lambda_{\text{QCD}}$   $L_h \sim \nu/m_h^2$   $\lambda$



Thin medium: NP input only from medium properties

Large medium: requires additional NP input from hadronization  
also require input from hadronic collisions

Number of collisions goes like  $\chi = L/\lambda$

Dilute limit: Opacity expansion or higher twist  $\chi \lesssim 1$

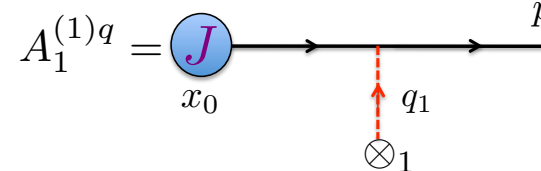
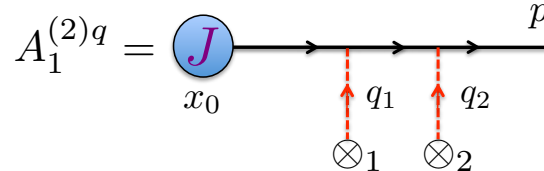
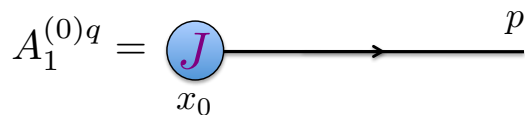
Dense limit: Opacity expansion or higher twist  $\chi \gg 1$

Gyulassy-Levai-Vitev (2000) Guo, Wang (2000)

Baier, Dokshitzer, Mueller, Peigné, Schiff (1996) Zakharov (1997)

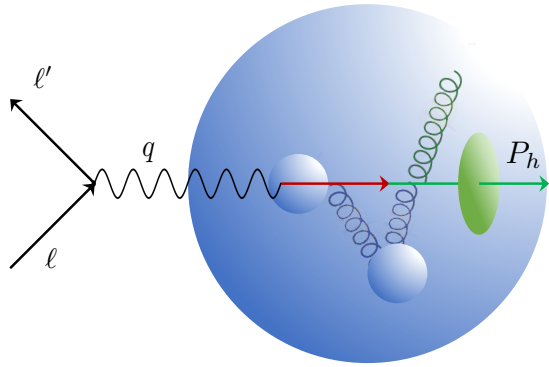
Can be treated in Glauber SCET [Ovanesyan and Vitev \(2011\)](#)

$$p_G^\mu \sim Q(\lambda^2, \lambda^2, \lambda)$$



# Medium modified DGLAP evolution

Previous work has been done in QCD and SCET to derive medium modified evolution equations



$$\frac{\partial \tilde{D}_{h/j}(z; \mu)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu)}{2\pi} \sum_i \left[ \tilde{P}_{ij} \otimes \tilde{D}_{h/i} \right] (z; \mu)$$

$$\tilde{P}_{ij}(z; \mu) = \tilde{P}_{ij}(z) + \Delta \tilde{P}_{ij}(z; \mu)$$

$$\frac{dN}{dx} \sim \left| \text{[Feynman diagrams]} \right|^2 + 2\text{Re} \left[ \text{[Feynman diagrams]} \right] \times \text{[Feynman diagram]}$$

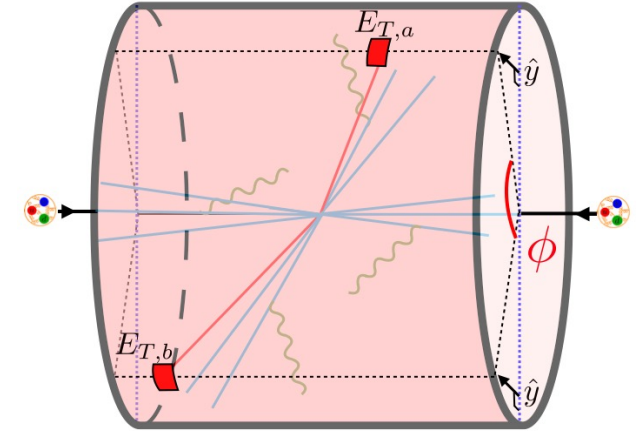
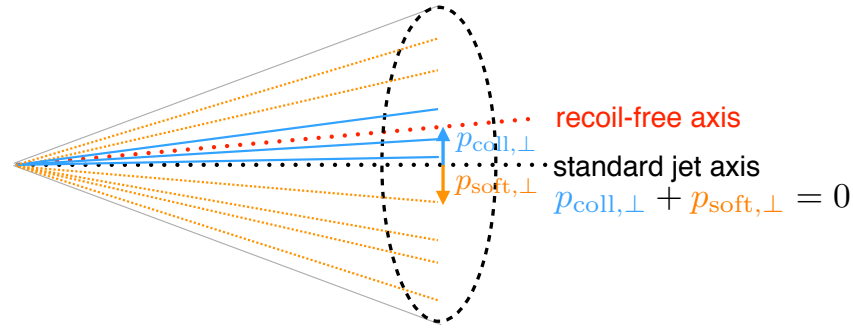
Ovanesyan, Vitev (2012)

Medium modification can be implemented into the fit, but introduces additional scales. Future work in this community will involve including the medium modified DGLAP into the fit, as well as calculating the medium modifications to the RG and Collins-evolution of the TMDs.

# Future work

Graphs for medium modified evolution can be applied to final-state functions for TMD measurements (exclusive jet functions, EECs)

$$\frac{dN}{dx} \sim \left| \text{[Feynman diagrams]} \right|^2 + 2\text{Re} \left[ \text{[Feynman diagrams]} \right] \times \text{[Feynman diagram]}$$

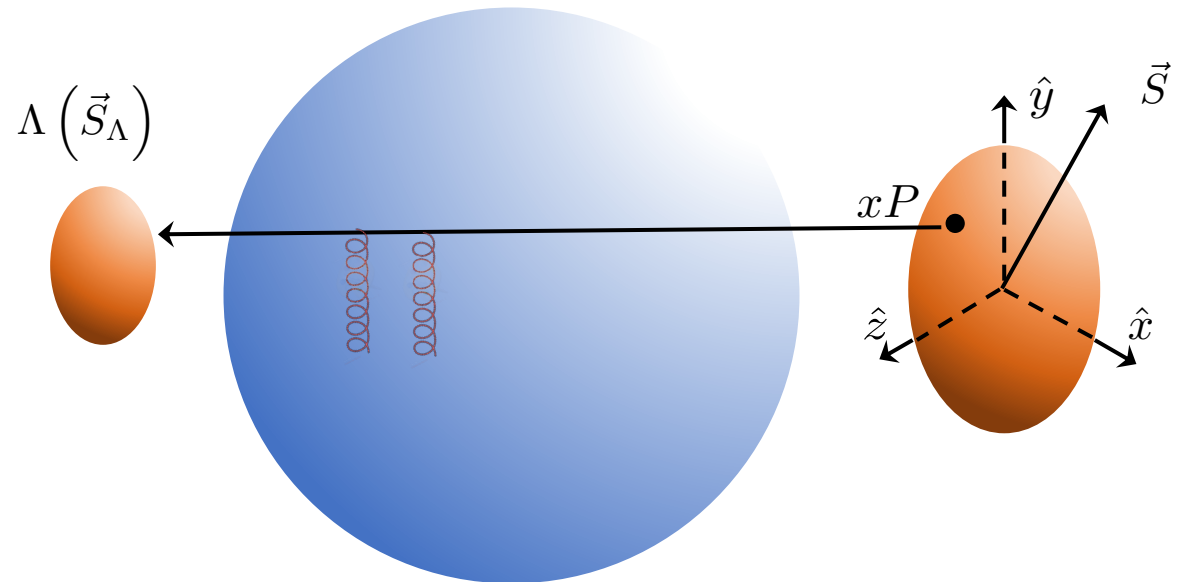


Medium modified DGLAP can be applied in a global analysis

Medium modified spin physics as a new probe of medium properties

$$\frac{\partial \tilde{D}_{h/j}(z; \mu)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu)}{2\pi} \sum_i \left[ \tilde{P}_{ij} \otimes \tilde{D}_{h/i} \right] (z; \mu)$$

$$\tilde{P}_{ij}(z; \mu) = \tilde{P}_{ij}(z) + \Delta \tilde{P}_{ij}(z; \mu)$$

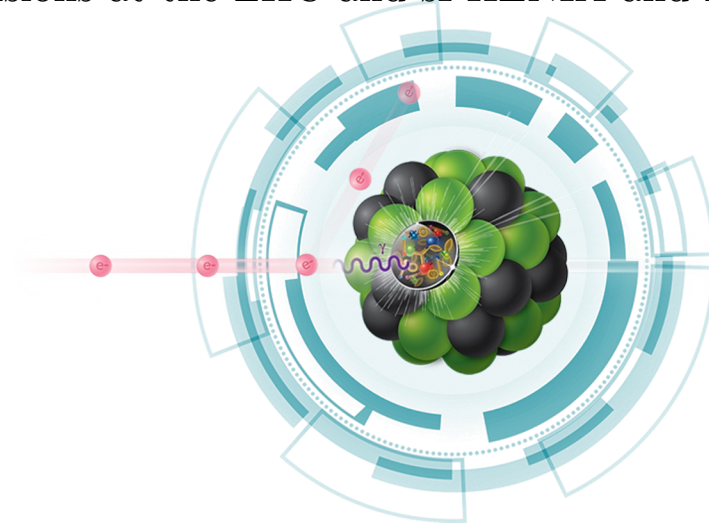




# Conclusion

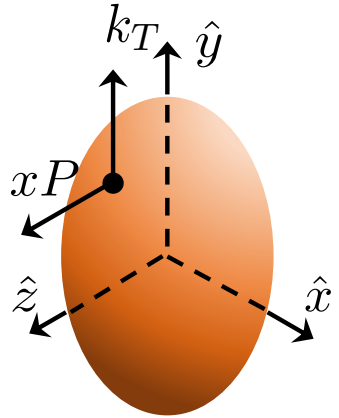
---

- We develop a formalism for approximating broadening effects in Drell-Yan and Semi-Inclusive DIS.
- We find that we can absorb medium modifications into the intrinsic widths of the TMDs to define nTMDs.
- We perform the first extraction of both the nTMD PDF and nTMD FF from the world data of Semi-Inclusive DIS and Drell-Yan.
- Our new analysis took into consideration the JLab data which allowed us to extract the collinear nFFs.
- We have improved the perturbative accuracy of lepton-jet correlations and used this process to make predictions at the EIC.
- We studied jet production in pA collisions at the LHC and sPHENIX and find good agreement with the data.



# Power counting and factorization

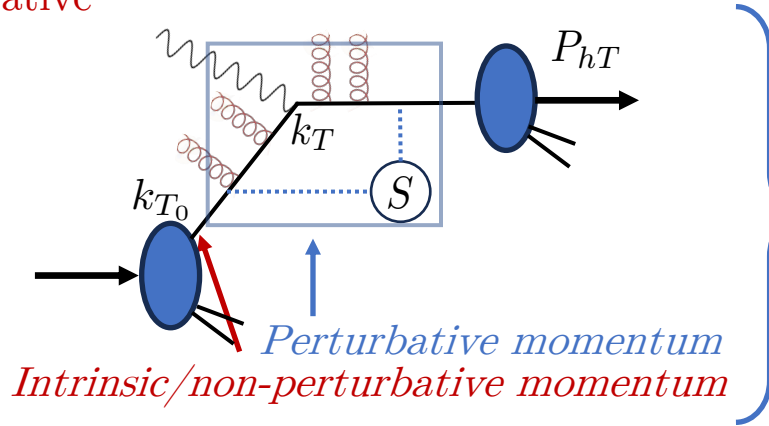
Intrinsic transverse momentum is non-perturbative



Inject large current

$$xP^+ \sim Q \gg \Lambda_{\text{QCD}}$$

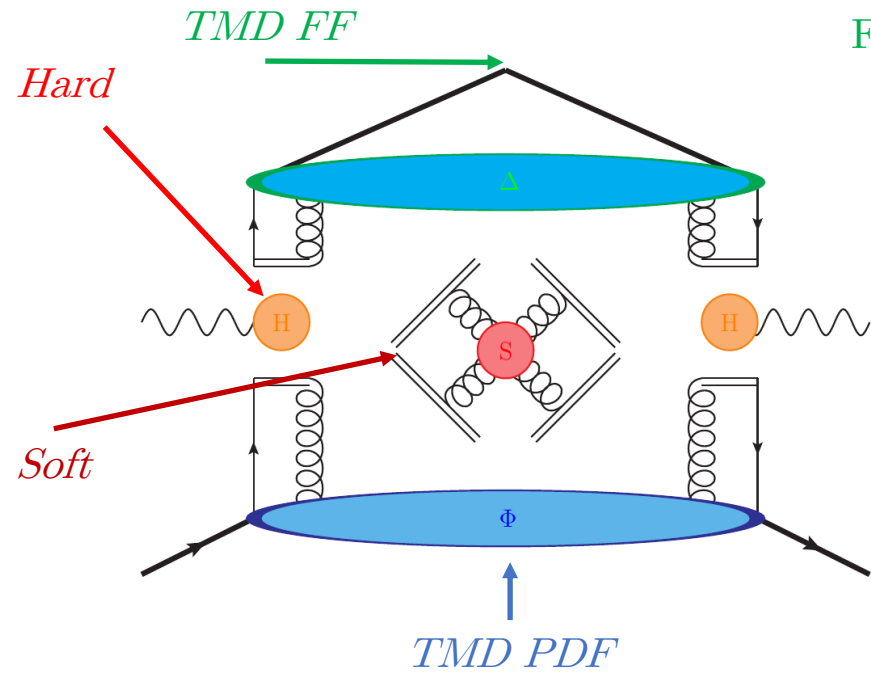
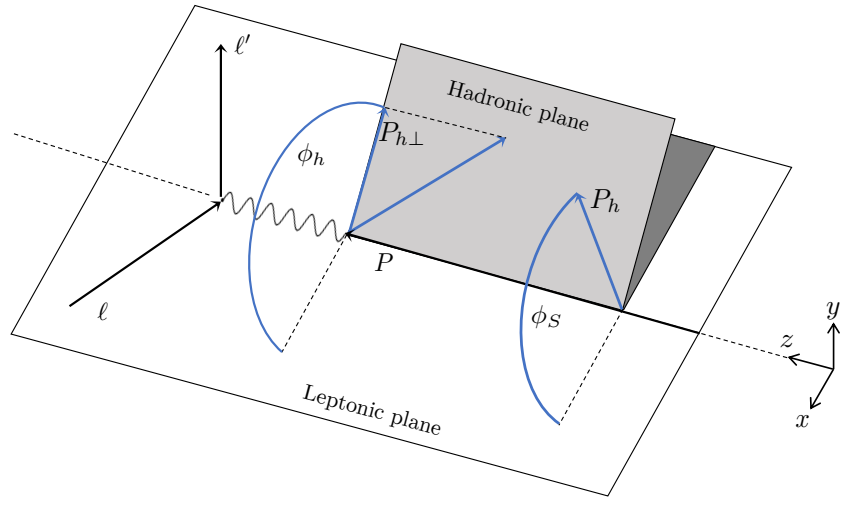
$$k_T \sim \Lambda_{\text{QCD}}$$



Experiments involve mixture of Perturbative and non-perturbative momentum

$$xP^+ \gg k_T \gtrsim \Lambda_{\text{QCD}}$$

Perturbative and non-perturbative contributions decouple using *factorization theorems*



Fit

$$S_{\text{NP}}^D(z, b, Q_0, Q)$$

$$S_{\text{pert}}(\mu_i, \mu_f, \zeta_i, \zeta_f) = \int_{\mu_i}^{\mu_f} \frac{d\mu'}{\mu'} \left[ \gamma^V + \Gamma_{\text{cusp}} \ln \left( \frac{\zeta_f}{\mu'^2} \right) \right]$$

$$+ D(b; \mu_i) \ln \left( \frac{\zeta_f}{\zeta_i} \right)$$

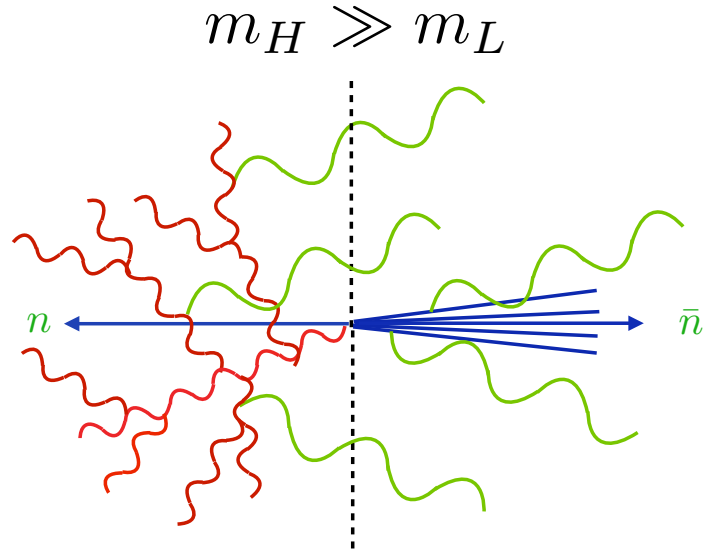
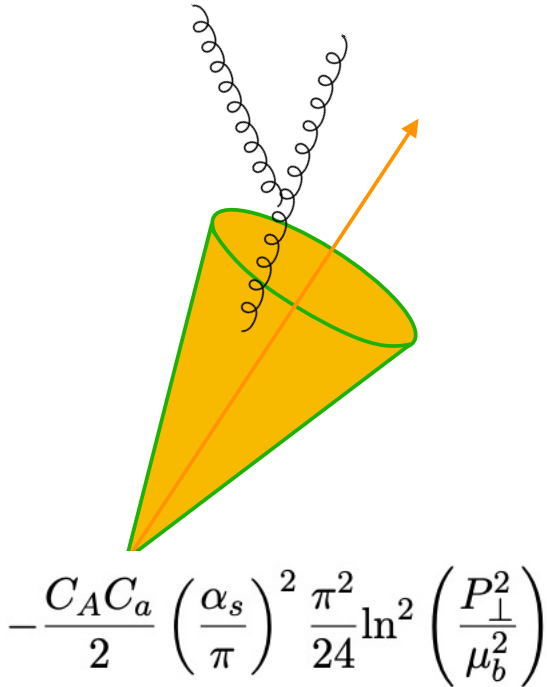
$$S_{\text{NP}}^f(x, b, Q_0, Q)$$

Calculate

# Non-global logs

Standard jets complicate resummation

Dasgupta, Salam (2001) Larkoski, Moult, Neill (2016)



NGLs for jet at NLL is the same as jet mass in e+ e-

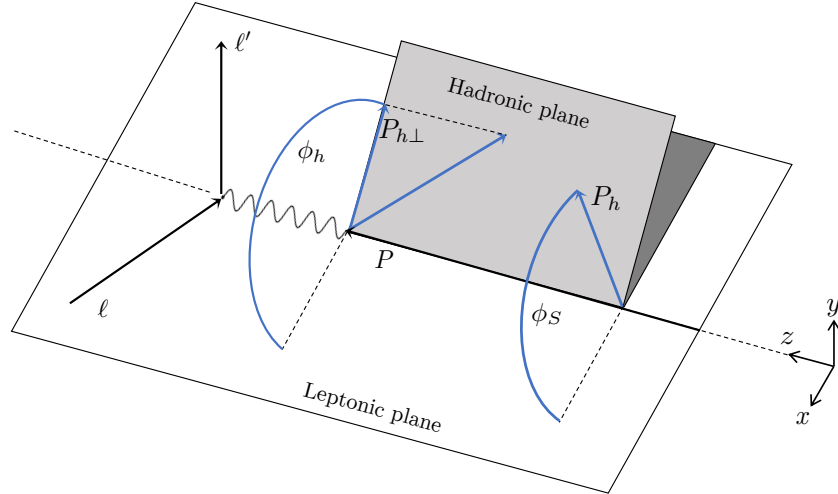
Becher, Neubert, Rothen and Shao (2016)

$$U_{\text{NG}}^k(\mu_{cs}, \mu_j) = \exp\left[-C_A C_k \frac{\pi^2}{3} u^2 \frac{1 + (au)^2}{1 + (bu)^c}\right],$$

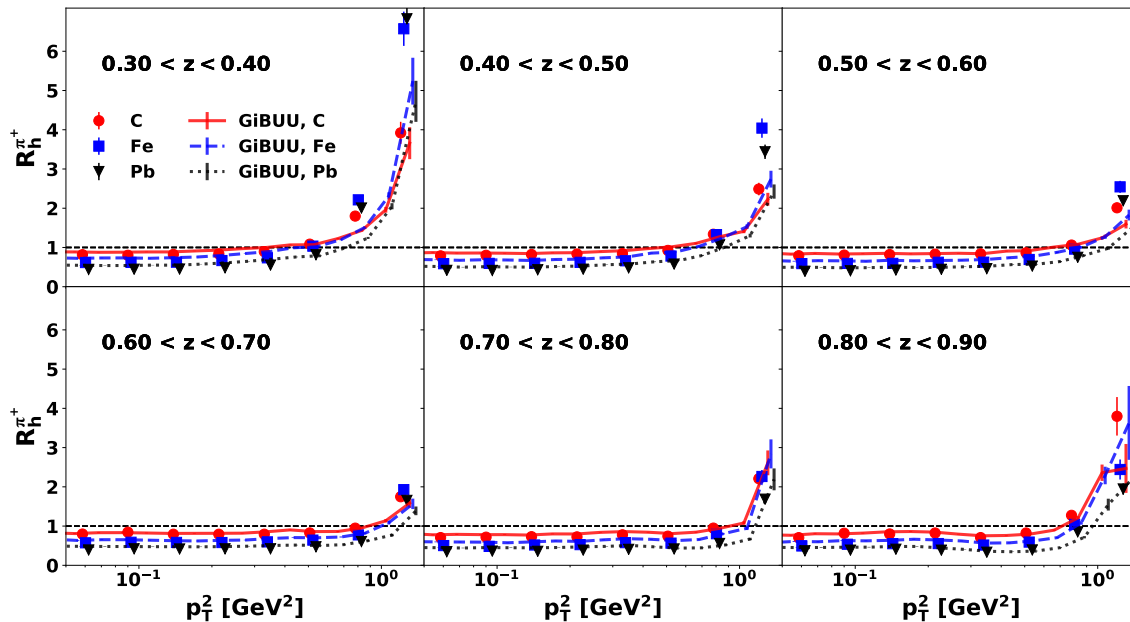
# CLAS measurements

## Semi-Inclusive DIS

Morán *et al.* (CLAS Collaboration) Phys. Rev. C **105**, 015201

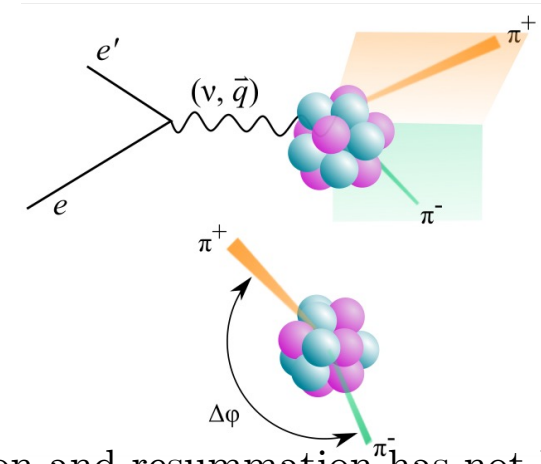


Hadron-multiplicity data can be incorporated into the fit

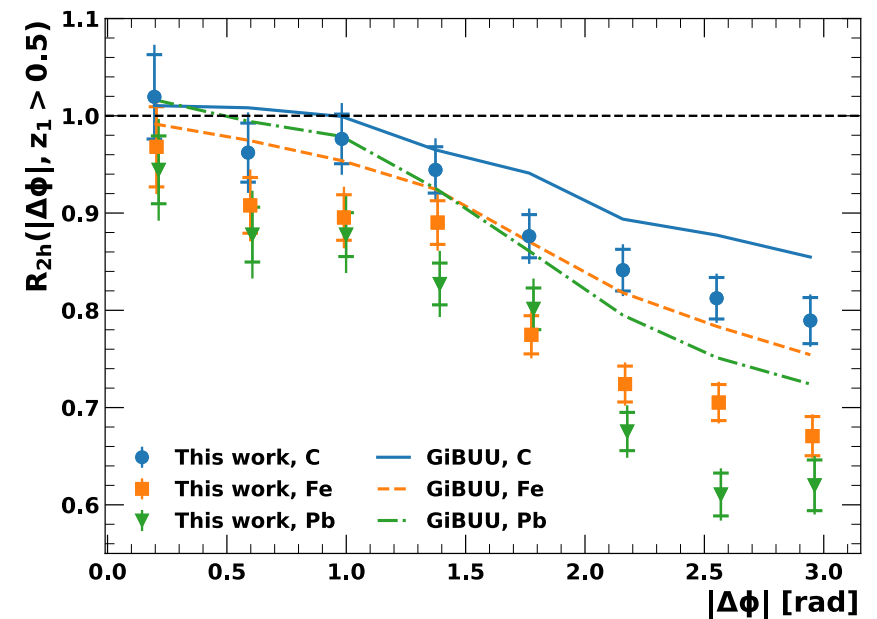


Measurements have been performed for angular decorrelation

Paul *et al.* (CLAS Collaboration) Phys. Rev. Lett. **129**, 182501

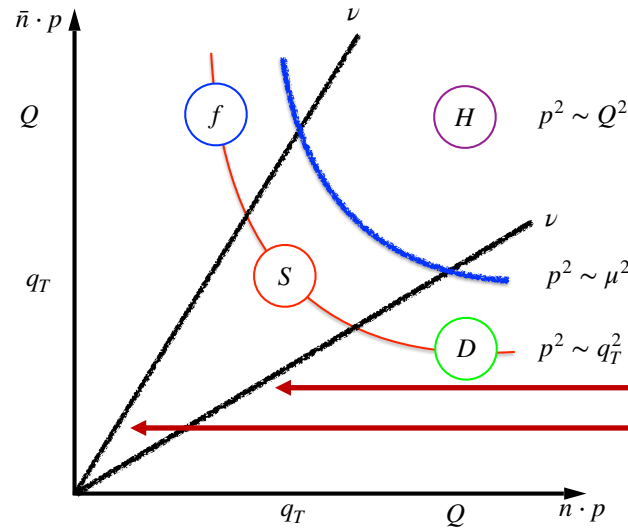
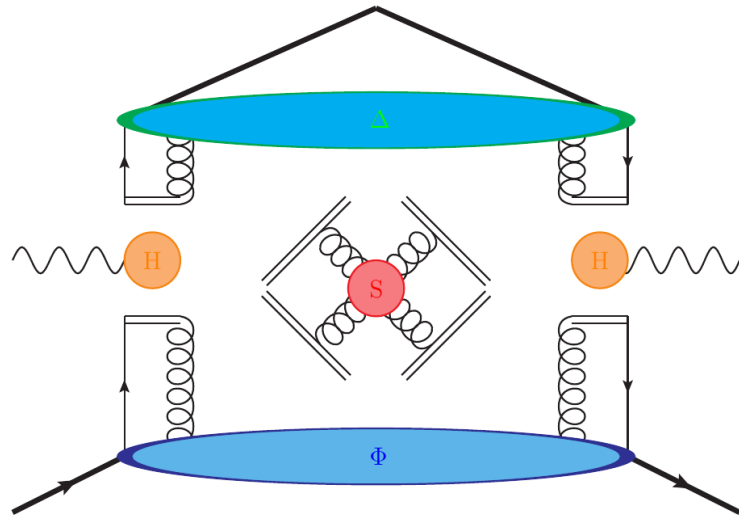


Factorization and resummation has not been established



# Resummation of large logs

## TMD factorization in SCET



UV and IR modes are separated in dim reg

IR modes all have same invariant mass, requires additional scale

TMDs and soft functions have double scale evolution

$$\frac{d}{d \ln \mu} F(b, \mu, \nu) = \gamma_F^\mu(\mu, \dots) F(b, \mu, \nu),$$

$$\frac{d}{d \ln \nu} F(b, \mu, \nu) = \gamma_F^\nu(b, \mu) F(b, \mu, \nu),$$

Accuracy	$H, \mathcal{J}$	$\Gamma_{\text{cusp}}(\alpha_s)$	$\gamma_H^q(\alpha_s)$	$\gamma_r^q(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level	1-loop	–	–	1-loop
NLL	Tree level	2-loop	1-loop	1-loop	2-loop
NLL'	1-loop	2-loop	1-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	2-loop	3-loop
NNLL'	2-loop	3-loop	2-loop	2-loop	3-loop
N <sup>3</sup> LL	2-loop	4-loop	3-loop	3-loop	4-loop
N <sup>3</sup> LL'	3-loop	4-loop	3-loop	3-loop	4-loop
N <sup>4</sup> LL	3-loop	5-loop	4-loop	4-loop	5-loop
N <sup>4</sup> LL'	4-loop	5-loop	4-loop	4-loop	5-loop

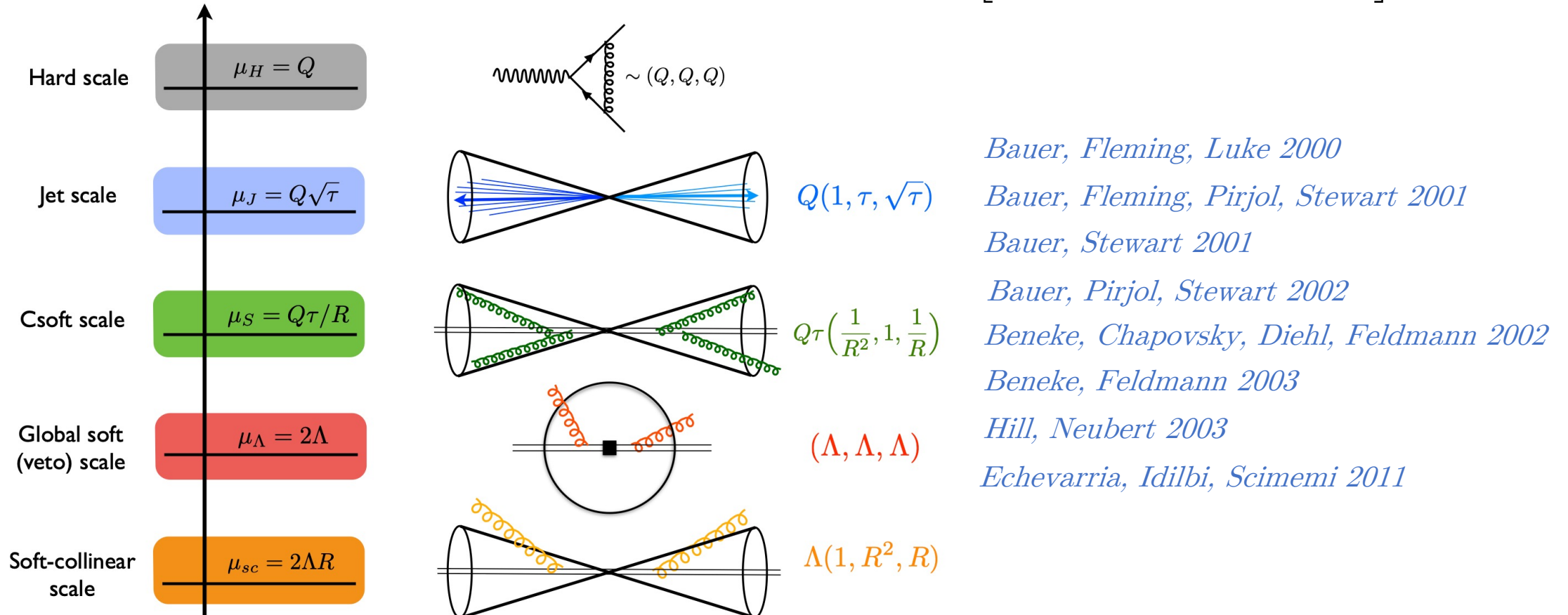
# Modes in SCET

SCET is an EFT which captures soft and collinear emissions along the directions

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{\psi} i \not{D} \psi - \frac{1}{4} G_{\mu\nu}^A G^{A\mu\nu} + \mathcal{L}_{\text{gauge-fix}} + \mathcal{L}_{\text{ghost}}$$

$$\psi \rightarrow \psi_s + \psi_c \quad A^\mu \rightarrow A_s^\mu + A_c^\mu \quad \longrightarrow \quad \mathcal{L}_{\text{SCET}} = \bar{\psi}_s i \not{D}_s \psi_s - \frac{1}{4} G_{\mu\nu s}^A G_s^{A\mu\nu}$$

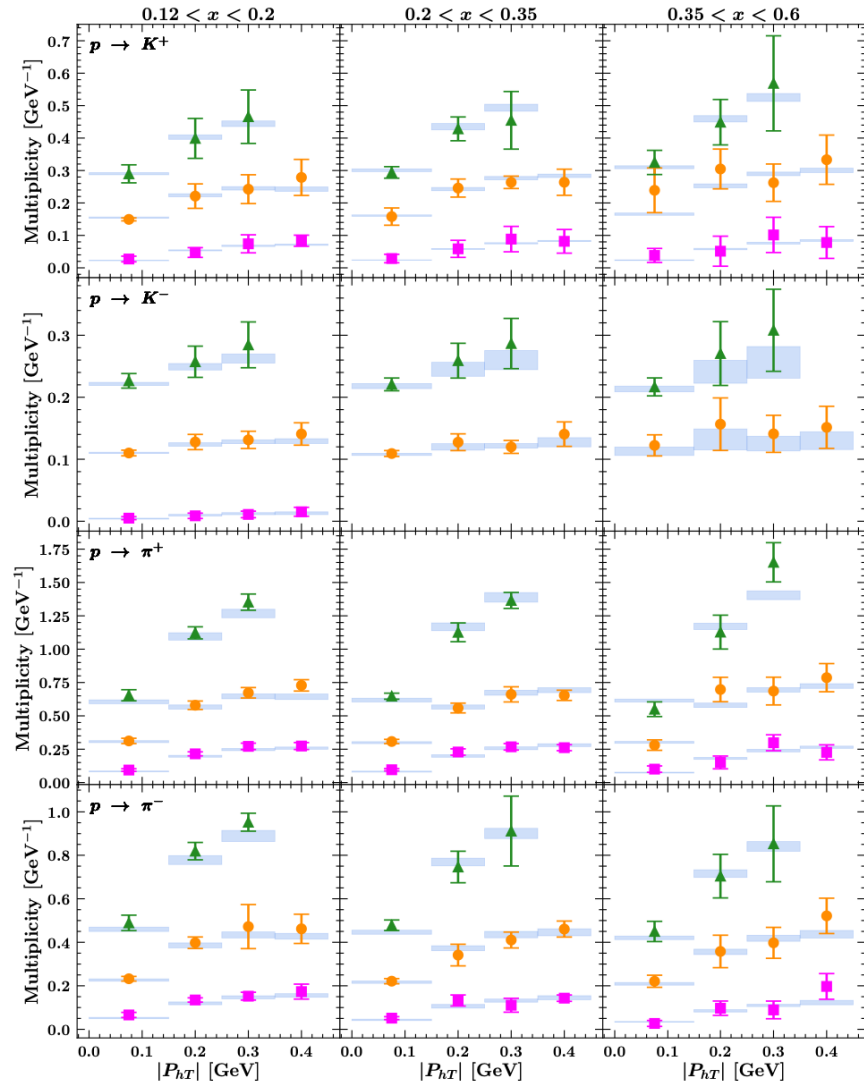
$$+ \xi \bar{\not{n}} \left[ in \cdot D + i \not{D}_{c\perp} \frac{1}{i \bar{n} \cdot D_c} i \not{D}_{c\perp} \right] \xi - \frac{1}{4} G_{\mu\nu c}^A G_c^{A\mu\nu}$$



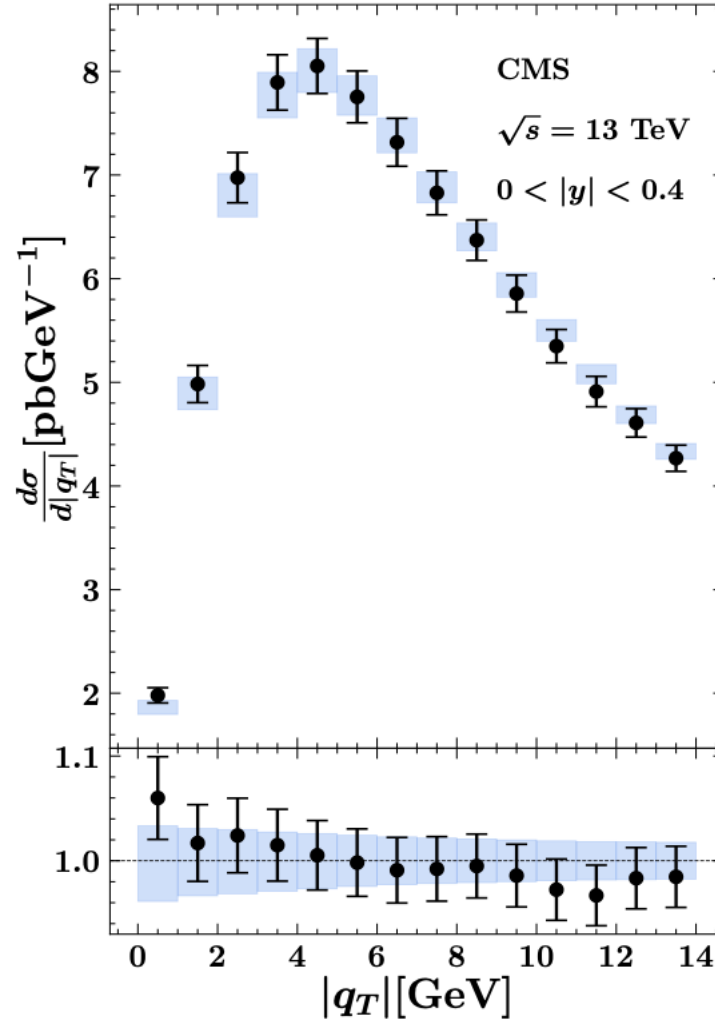
# The MAP collaboration (Bachetta et al 2022)

Extraction at NNLO+N<sup>3</sup>LL, SIDIS+DY

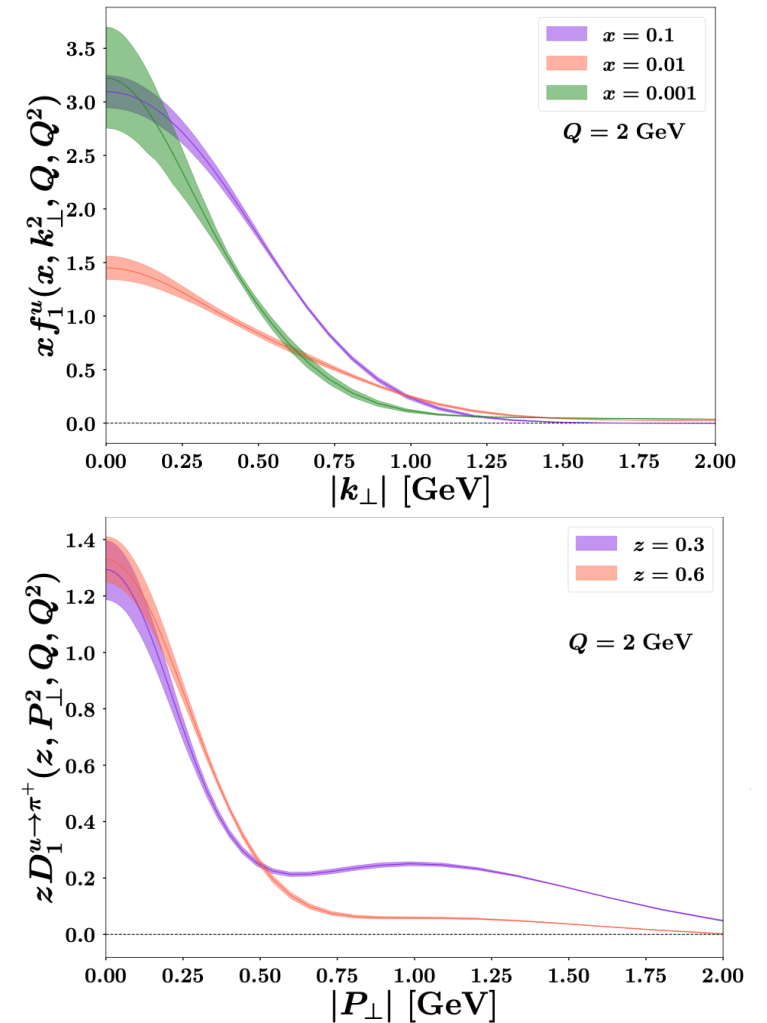
*Hermes multiplicity data*



*CMS  $q_{\perp}$  distribution*



*TMDs*

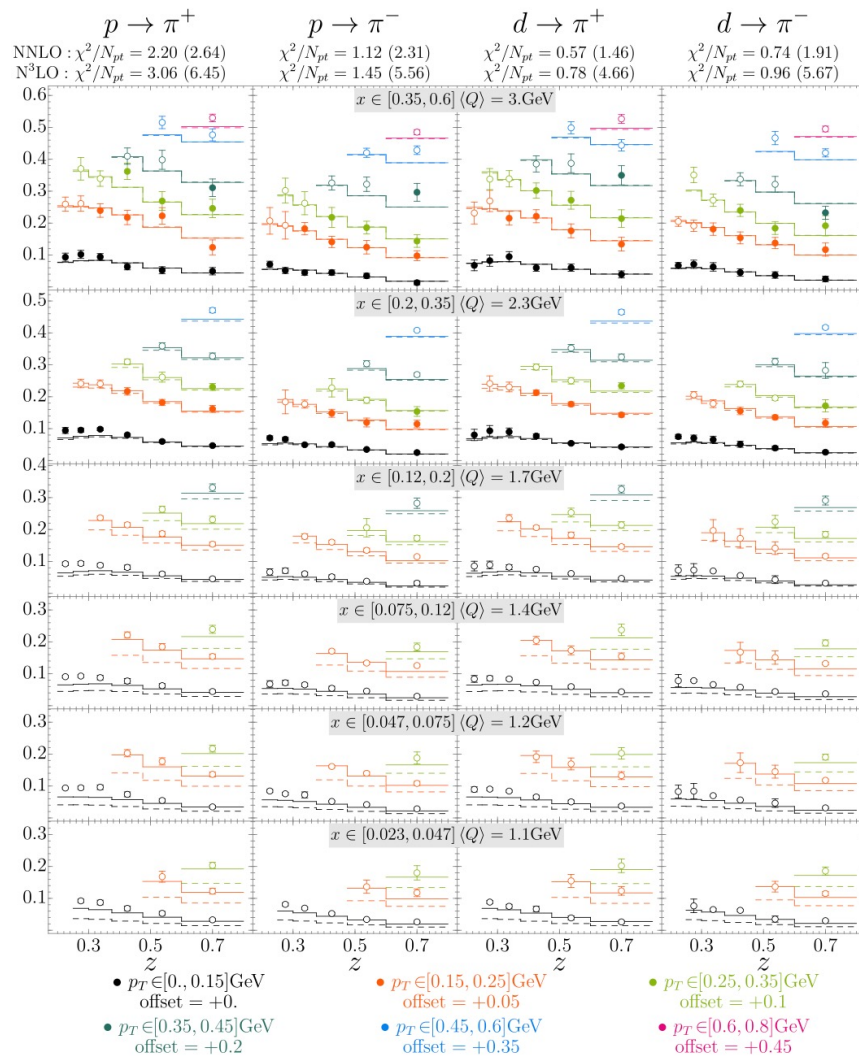




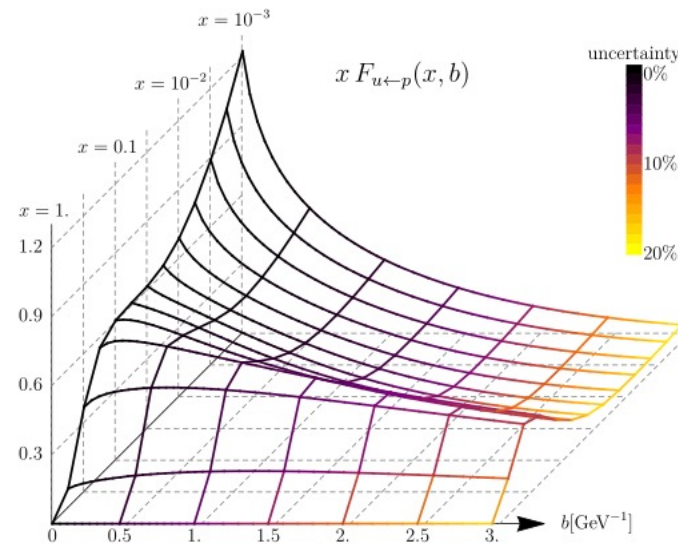
# Artemide (Scimemi, Vladimirov 2019)

Extraction obtained at NNLO+N<sup>3</sup>LL, SIDIS+DY

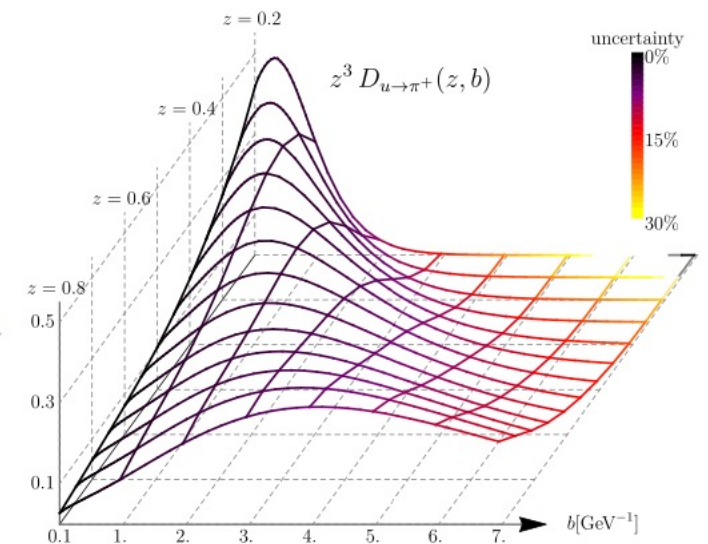
## Hermes Multiplicity data



## TMD PDF



## TMD FF

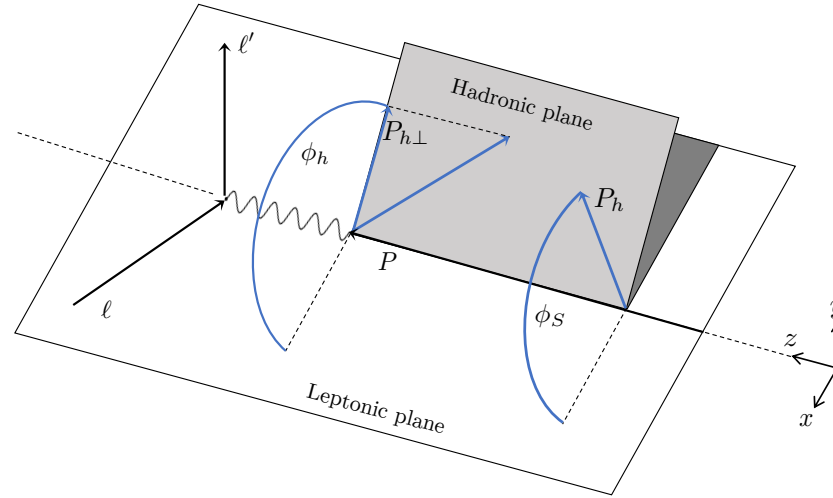


$$b \sim 1/q_T$$



# Standard processes and power counting

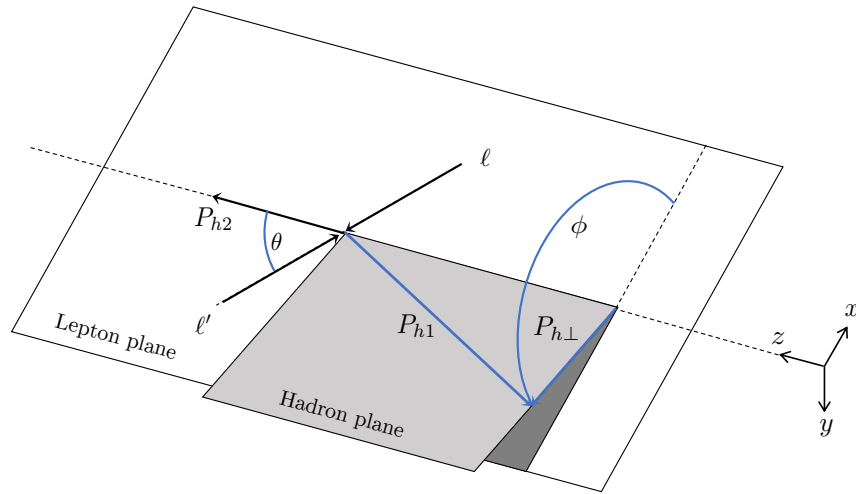
*TMD PDFs and TMD FFs*



$$Q \gg q_T \gtrsim \Lambda_{\text{QCD}}$$

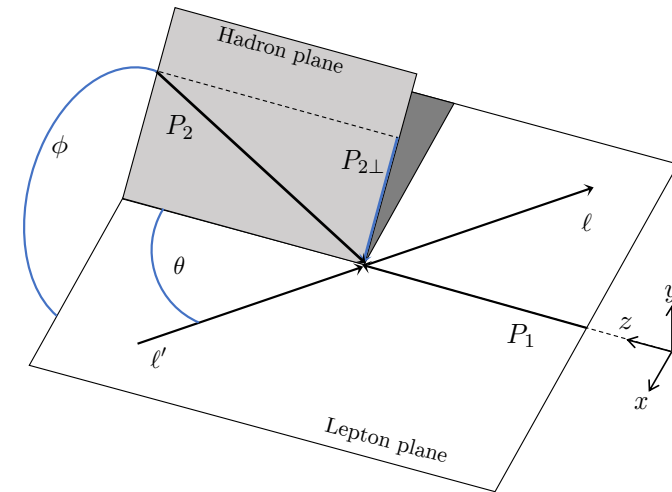
*TMD FFs*

*Sivers, Collins asymmetries  
COMPASS, HERMES, JLab data*



*Collins asymmetries  
BELLE, BaBar, BESIII data*

*TMD PDFs*



*Sivers asymmetries  
COMPASS, STAR data*