Parton pseudo-distributions and their evolution

Joe Karpie (JLab) part of the HadStruc and JAM Collaboration

Jefferson Lab

Parton Structure

For various flavors and spin combinations

Wigner Distribution/
Generalized Transverse Momentum Distribution (GTMD)

$F(x, b_T, k_T)$

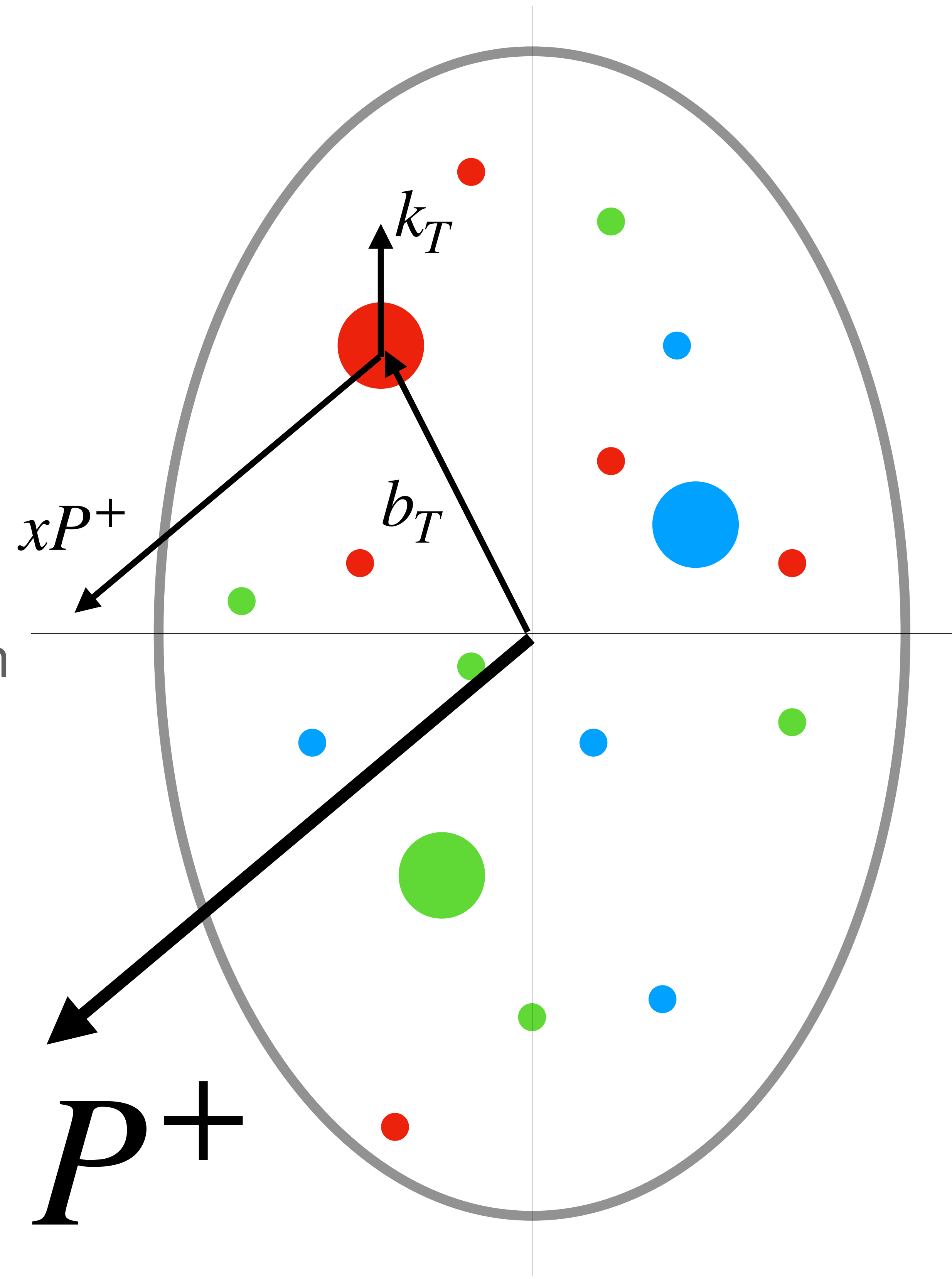
$\int d^2 b_t$ $\quad$ $\int d^2 k_t$

Transverse Momentum Distribution (TMD)
$f(x, k_T)$

Generalized Parton Distribution (GPD)
$f(x, b_T)$

$\int d^2 k_t$ $\quad$ $\int d^2 b_t$

Parton Distribution Function (PDF)
$f(x)$



Parton and Ioffe Time distributions

- Unpolarized Ioffe time distributions Ioffe time: $\nu = p \cdot z$

"Ioffe time distributions instead of parton momentum distributions in description of DIS"
V. Braun, P. Gornicki, L. Mankiewicz
Phys Rev D 51 (1995) 6036-6051

- $$I_q(\nu, \mu^2) = \frac{1}{2p^+} \langle p \,|\, \bar{\psi}_q(z^-) \gamma^+ W(z^-; 0) \psi_q(0) \,|\, p \rangle_{\mu^2}$$

$$z^2 = 0$$

$$I_g(\nu, \mu^2) = \frac{1}{(2p^+)^2} \langle p \,|\, F_{+i}(z^-) W(z^-; 0) F^i_+(0) \,|\, p \rangle_{\mu^2}$$

$$i = x, y$$

- Parton Distribution Functions

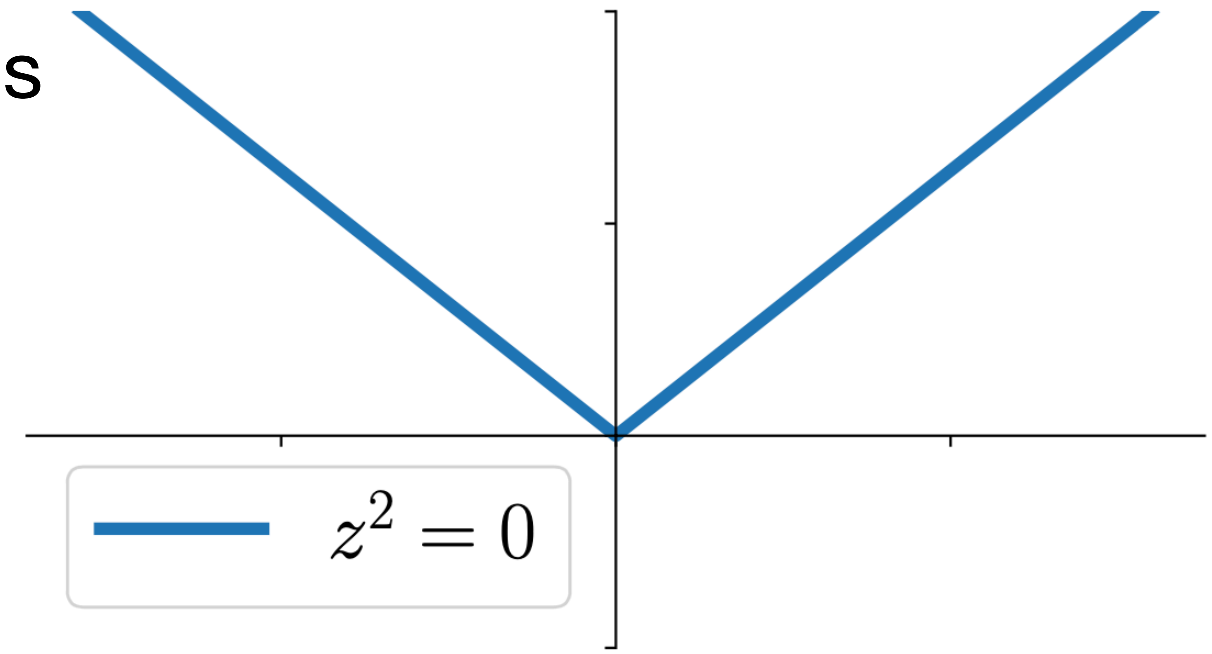
- $$I_q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{ix\nu} f_q(x, \mu^2)$$

- $$I_g(\nu, \mu^2) = \int_0^1 dx \, \cos(x\nu)\, x f_g(x, \mu^2)$$

Parton Distributions and the Lattice

- Parton Distributions are defined by operators with light-like separations

Minkowski



- Use space-like separations

  X. Ji *Phys Rev Lett* 110 (2013) 262002

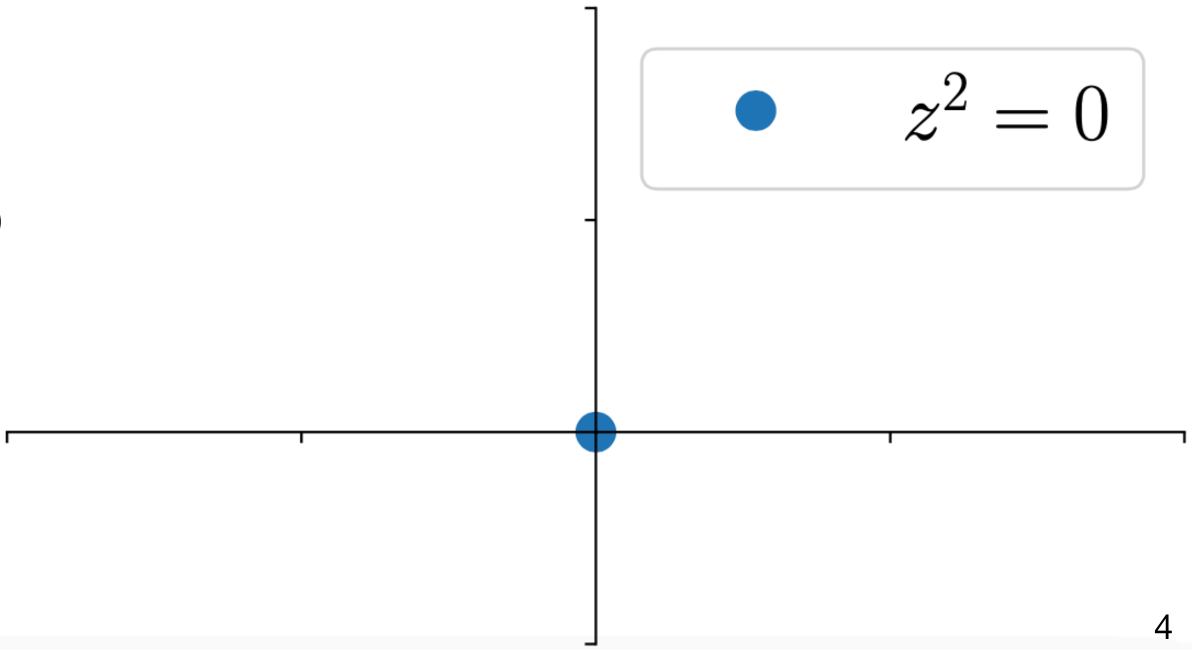
  - Wilson line operators

$$O_\Gamma^{\mathrm{WL}}(z) = \bar{\psi}(z)\Gamma W(z;0)\psi(0)$$

$$z^2 \neq 0$$

  - Factorizations exist analogous to cross sections

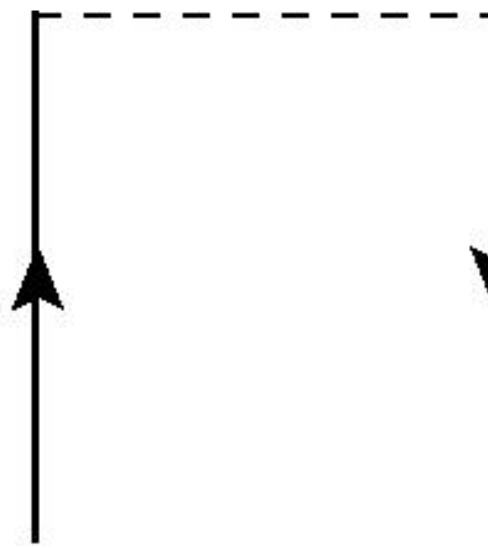
Euclidean



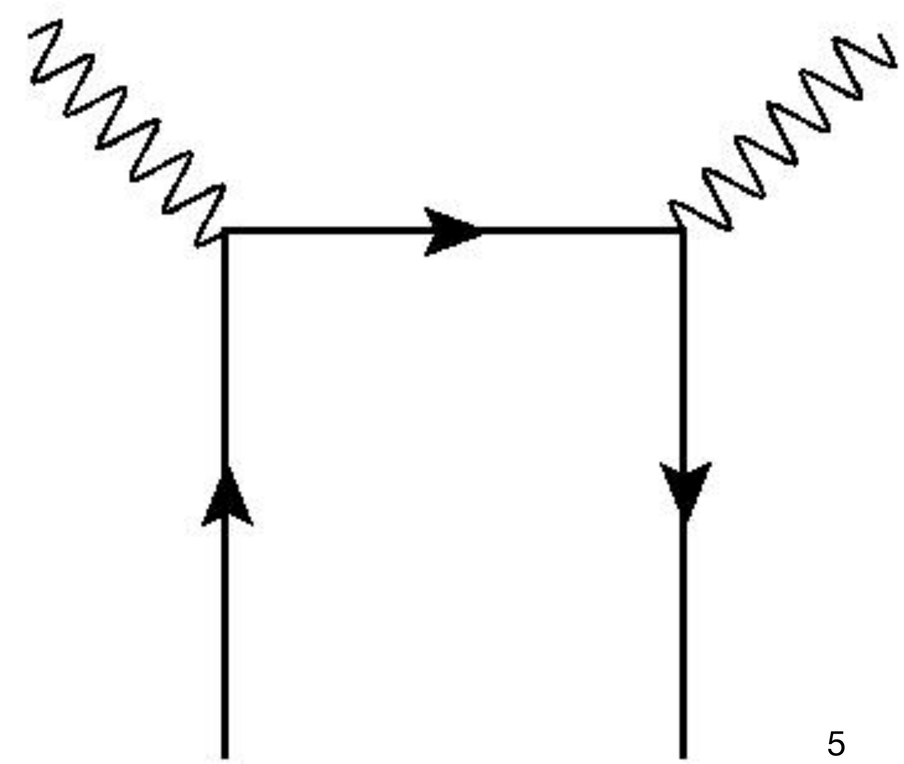
Many approaches

- **Wilson line operators**
    - LaMET X. Ji *Phys. Rev. Lett.* 110 (2013) 262002
    - Pseudo-PDF A. Radyushkin *Phys. Rev. D* 96 (2017) 3, 034025
- Two current correlators
    - Hadronic Tensor
      K.-F. Liu et al *Phys. Rev. Lett.* 72 1790 (1994)
      Phys. Rev. D 62 (2000) 074501
    - HOPE
      W. Detmold and C.-J. D. Lin, *Phys. Rev. D* 73 (2006) 014501
    - Short distance OPE
      V. Braun and D. Muller *Eur. Phys. J. C* 55 (2008) 349
    - OPE-without-OPE
      A. Chambers et al, *Phys. Rev. Lett.* 118 (2017) 242001
    - Good Lattice Cross Sections
      Y.-Q. Ma and J.-W. Qiu *Phys. Rev. Lett.* 120 (2018) 2, 022003

$$O_{WL}(x;z) = \bar{\psi}(x+z)\Gamma W(x+z;x)\psi(x)$$



$$O_{CC}(x,y) = J_{\Gamma}(x)J_{\Gamma'}(y)$$



Wilson Line Matrix Elements

- Matrix element $M^\alpha(p,z) = \langle p \,|\, \bar{\psi}(z)\gamma^\alpha W(z;0)\psi(0)\,|\, p\rangle$ $\quad z^2 < 0$

$$= 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

- Quasi-PDF: $\tilde{q}(y, p_z^2) = \dfrac{1}{2p_\alpha}\displaystyle\int dz e^{iyp_z z} M^\alpha(p_z, z)$ $\quad \alpha = t$ and $z^t = 0$

- Large Momentum Effective Theory: X. Ji *Phys. Rev. Lett.* 110 (2013) 262002

- $$\tilde{q}(y, p_z^2) = \int \frac{dx}{|x|} K\left(\frac{y}{x}, \frac{\mu^2}{(xp_z)^2}\right) q(x, \mu^2) + O\left(\frac{\Lambda^2_{\text{QCD}}}{(xp_z)^2}, \frac{\Lambda^2_{\text{QCD}}}{((1-x)p_z)^2}\right)$$

- Pseudo-PDF: A. Radyushkin *Phys. Rev. D* 96 (2017) 3, 034025

$$\mathcal{M}(\nu, z^2) = \int dx\, C(x\nu, \mu^2 z^2) q(x, \mu^2) + O(\Lambda^2_{\text{QCD}} z^2)$$

$$= \int du C'(u, \mu^2 z^2) I_q(u\nu, \mu^2) + O(\Lambda^2_{\text{QCD}} z^2)$$

The Role of Separation and Momentum

- In Structure Functions, quasi-PDF, and pseudo-PDF, variables have common roles

Scale:

$Q^2 \,/\, p_z^2 \,/\, z^2$

- Scale for factorization to PDF
- Scale in power expansion
- Keep away from Λ_{QCD}^2
- Technically only requires single value, use many to study systematics

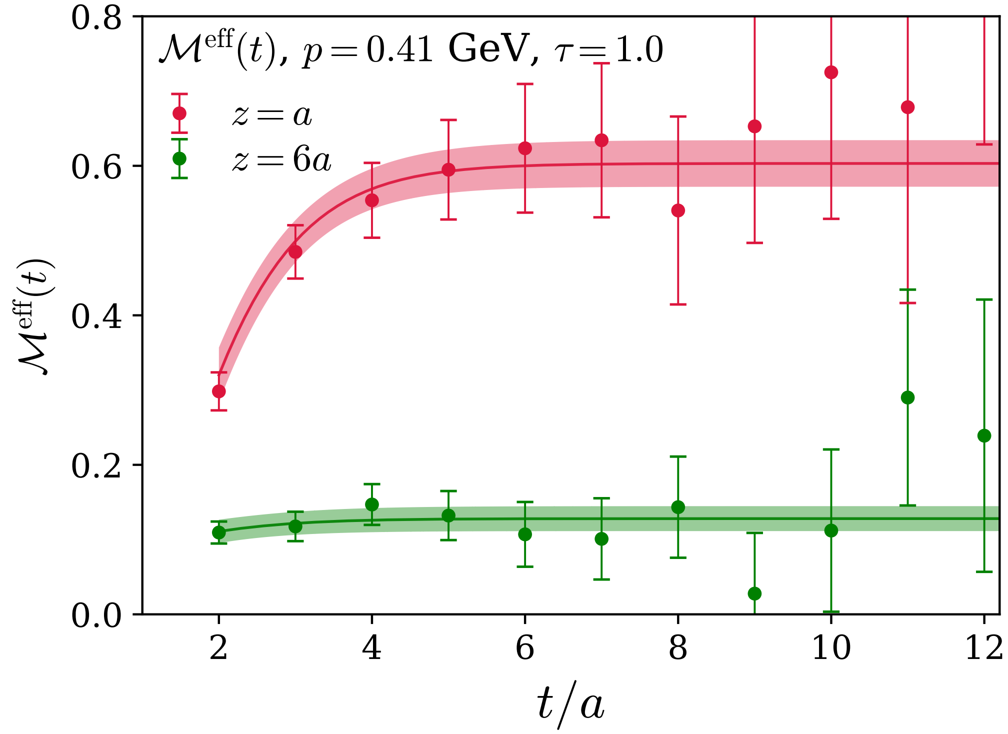
Dynamical variable:

$x_B \,/\, z \,/\, p_z$, or $\nu = p \cdot z$

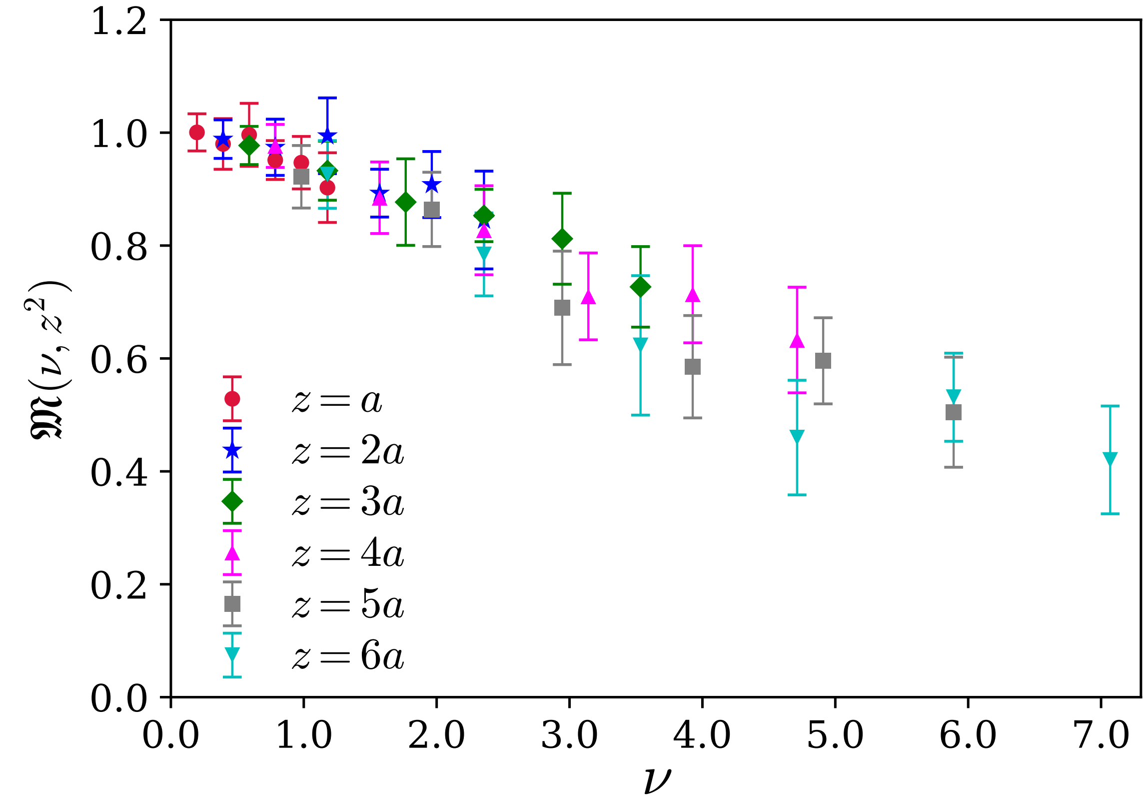
- Variable describes non-perturbative dynamics
- Can take large or small value
- Want as many as are available
- Wider range improves the inverse problem

From Lattice QCD to PDFs

Lattice Correlation Functions



Hadron Matrix Elements



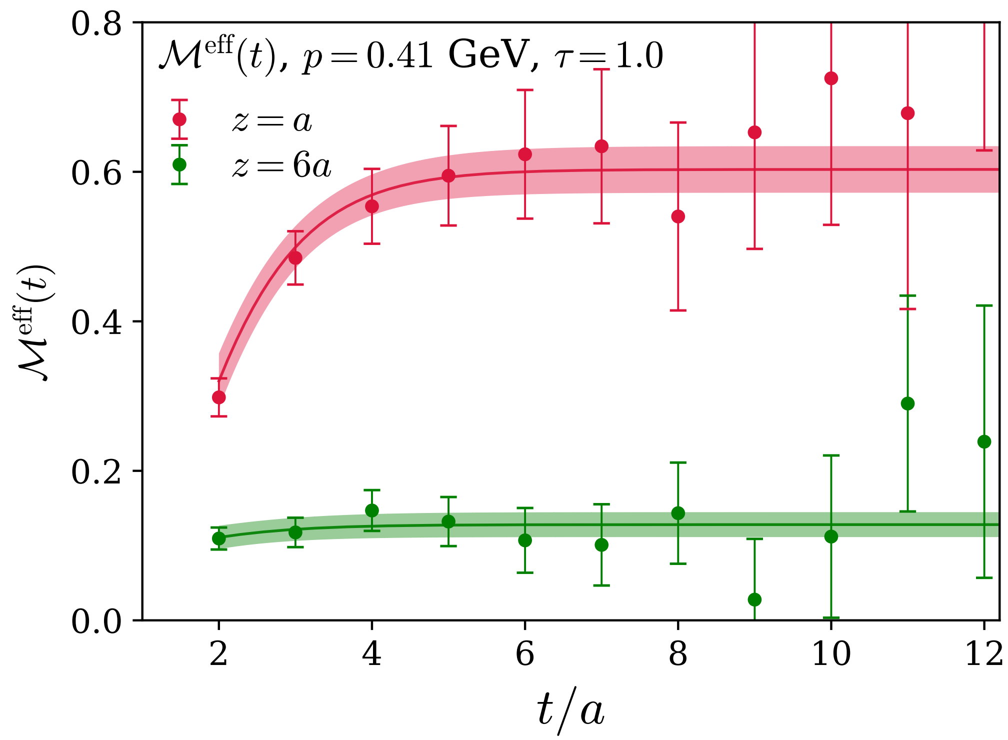
- Correlators (vacuum expectation values of time separated operators) are described as sums over an exponential for each energy eigenstate.
- Coefficients are matrix elements and exponential rates are energy levels
- Model and/or remove subdominant states by using large time but noise grows exponentially

Unpolarized Gluon PDF

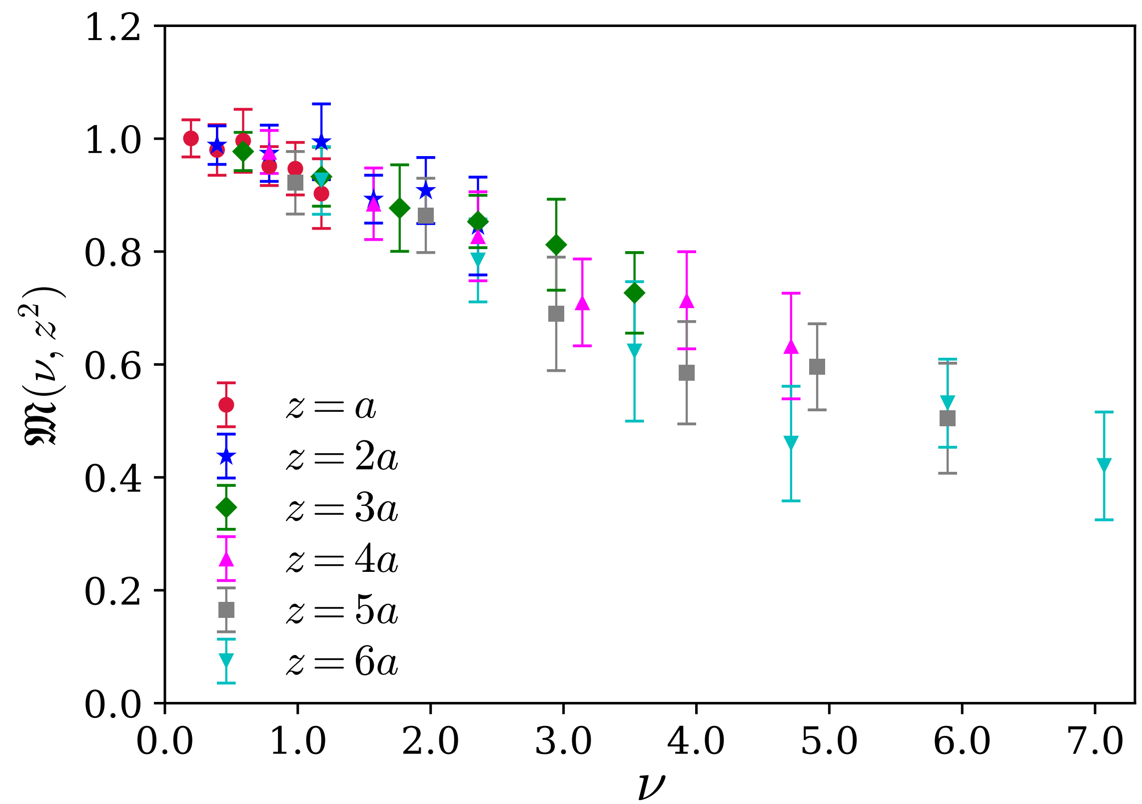
T. Khan, R. Sufian, JK, C. Monahan, C. Egerer, B. Joo, W. Morris, K. Orginos, A. Radyushkin, D. Richards, E. Romero, S. Zafeiropoulos PRD 104 (2021) 9, 094516

From Lattice QCD to PDFs

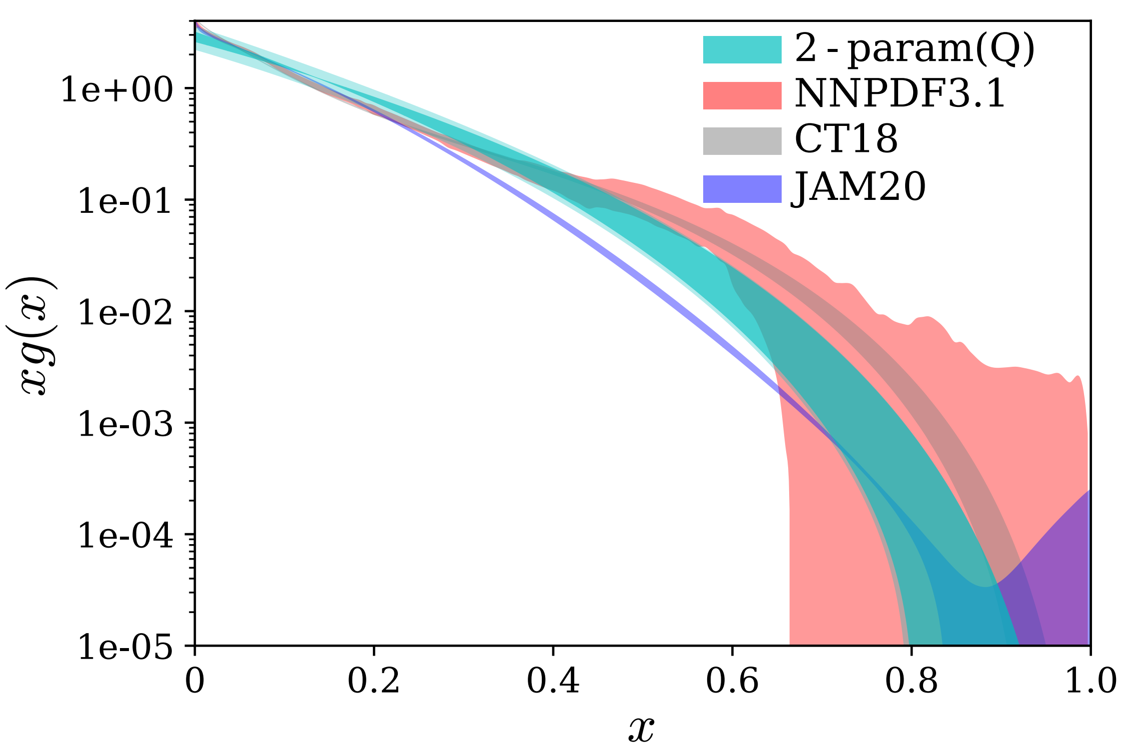
Lattice Correlation Functions



Hadron Matrix Elements



Parton Distributions



- Incomplete information gives integral inverse problem

$$M(\nu) = \int dx C(x\nu) xg(x)$$

$$xg(x) = x^a(1-x)^b / B(a+1, b+1)$$

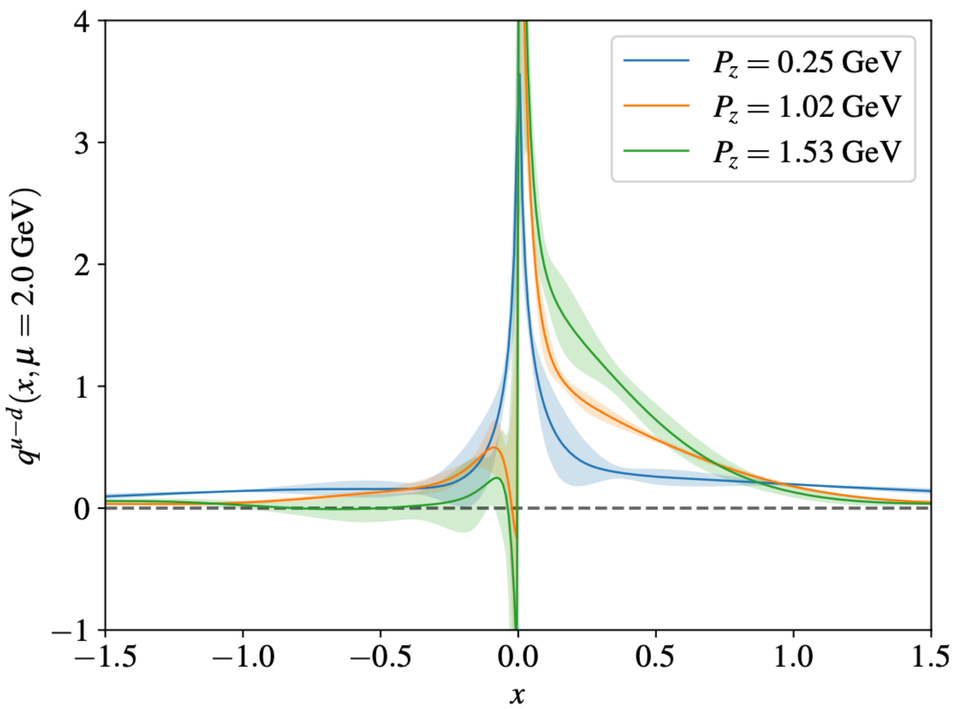
- To more accurately infer PDF, we need larger ν

Unpolarized Gluon PDF

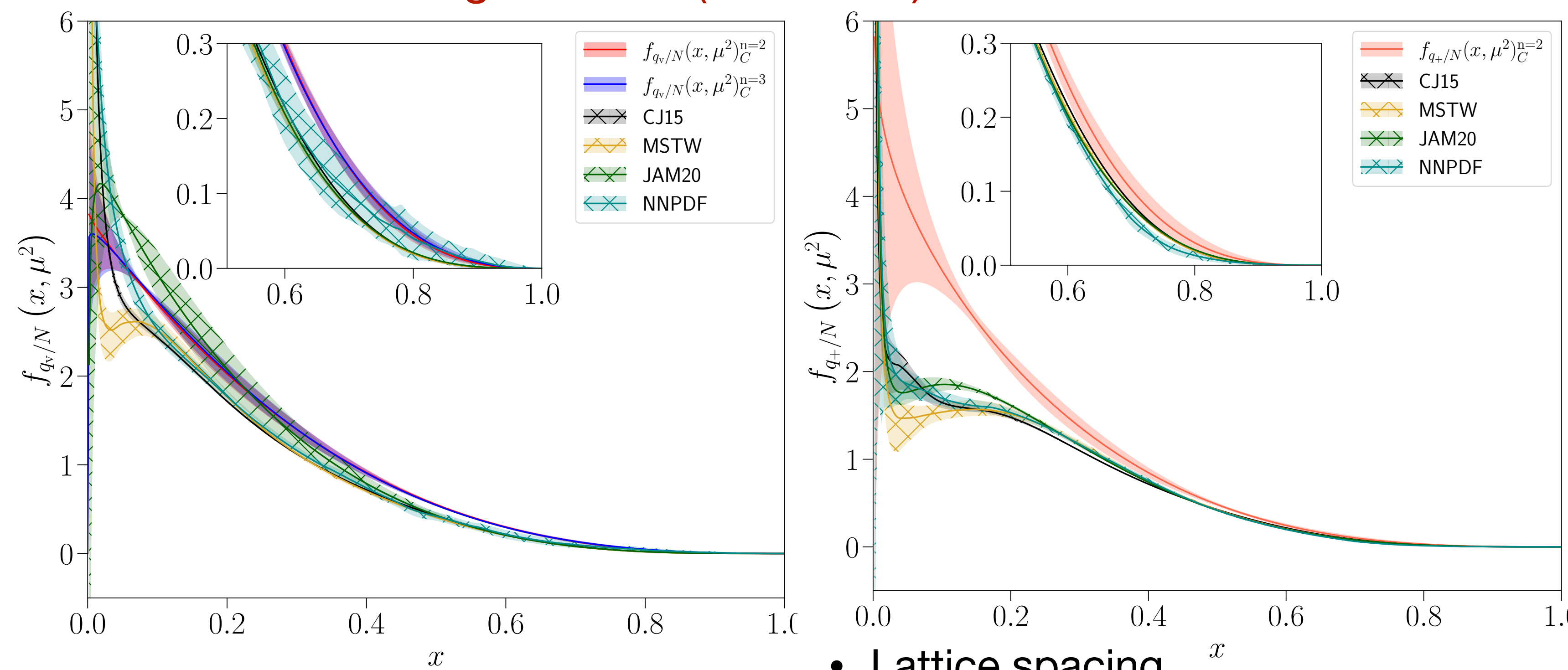
T. Khan, R. Sufian, JK, C. Monahan, C. Egerer, B. Joo, W. Morris, K. Orginos, A. Radyushkin, D. Richards, E. Romero, S. Zafeiropoulos
PRD 104 (2021) 9, 094516

Nucleon Unpolarized Quark PDF

X. Gao et al (ANL/BNL) 2212.12569



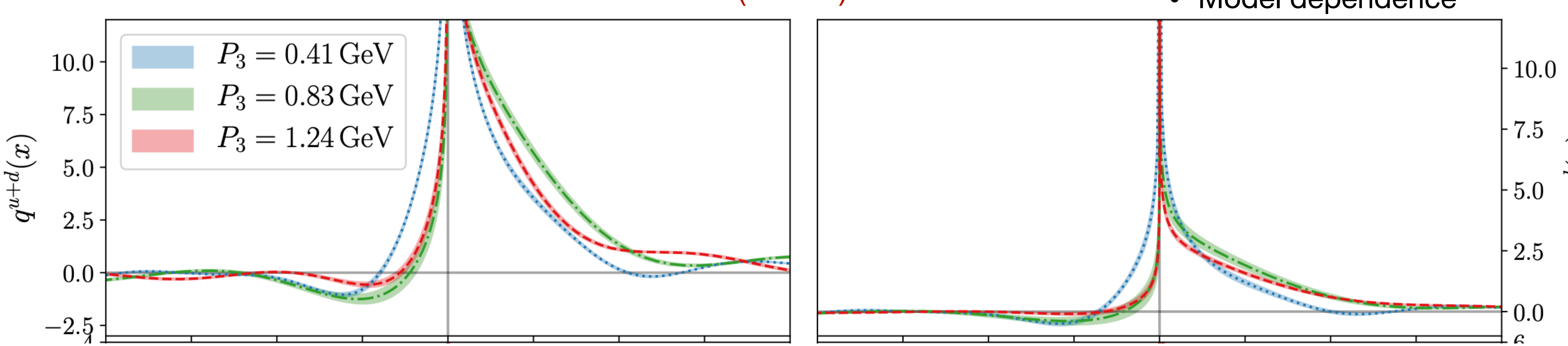
C. Egerer et al (HadStruc) 2107.05199



- Lattice spacing
- Pion mass
- Excited States
- Finite Volume
- Higher order matching
- Power Corrections
- Model dependence

- Decade since first works
- Systematics have been continually improved

C. Alexandrou et al (ETMC) 2106.16065



What can we do beyond looking at nice PDF fits?

If PDFs are universal….

*If the **same** PDFs are factorizable from lattice and experiment, and if power corrections can be controlled for both*

Why not analyze both simultaneously?

- Factorization of hadronic cross sections

$$d\sigma_h = d\sigma_q \otimes f_{h/q} + P.C.$$

- Factorization of Lattice observables

$$M_h = M_q \otimes f_{h/q} + P.C.$$

Consider Lattice data as a theoretical prior to the experimental Global Fit

Complementarity in Lattice and Experiment

LATTICE

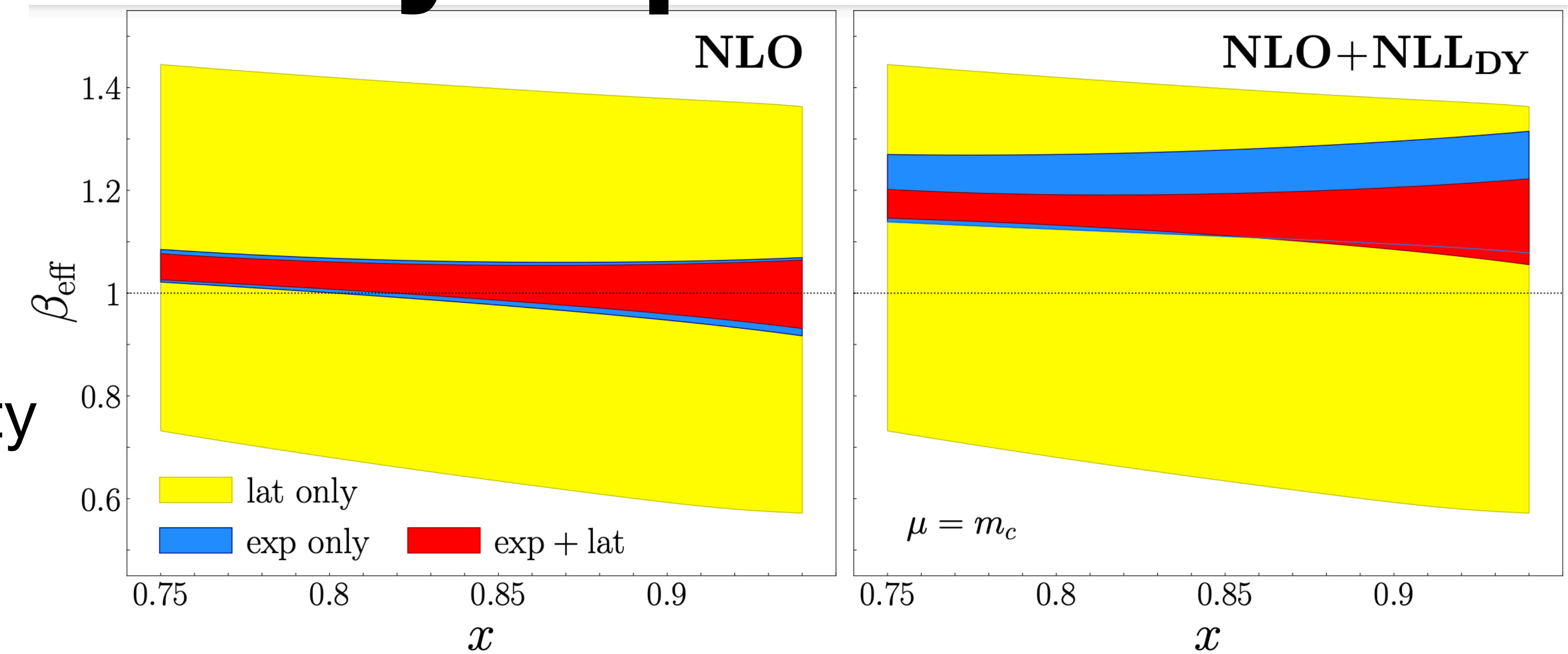
- Lattice limited to low ν, inverse Fourier gives to $x \gtrsim 0.2$, but higher sensitivity to large x
- Lattice matching relation is integral over all x
- Low p_z data can reach high signal-to-noise compared to experiment
- Lattice can evaluate independently each spin, flavor, and hadron

EXPERIMENT

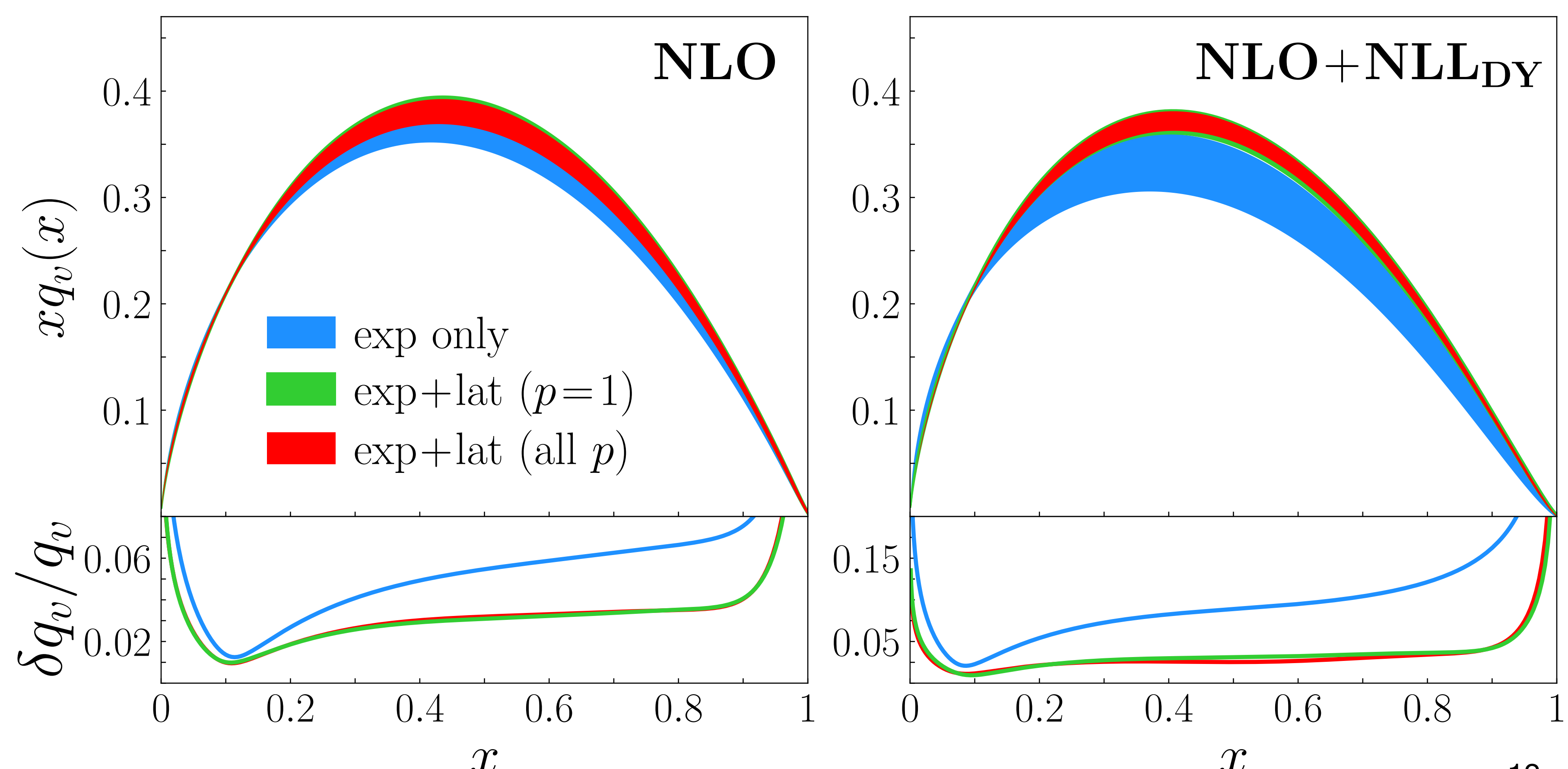
- Cross Sections limited to specific max but can reach low x_B
- Cross Section matching is integral from x_B to 1
  - Creates sensitively to hard kernel in large x region
- Wealth of decades of experimental data outweigh modern lattice in both number and systematic error control

Complementarity in pion PDF

- Lattice can readily access different hadrons
- Lattice lacks sensitivity to threshold logs and can be used to study theoretical kernels
- Improves precision in large x where experimental data does not exist
- Low momentum pion data are extremely precise



P. Barry et al, *Phys. Rev. D* 105 (2022) 11, 114051

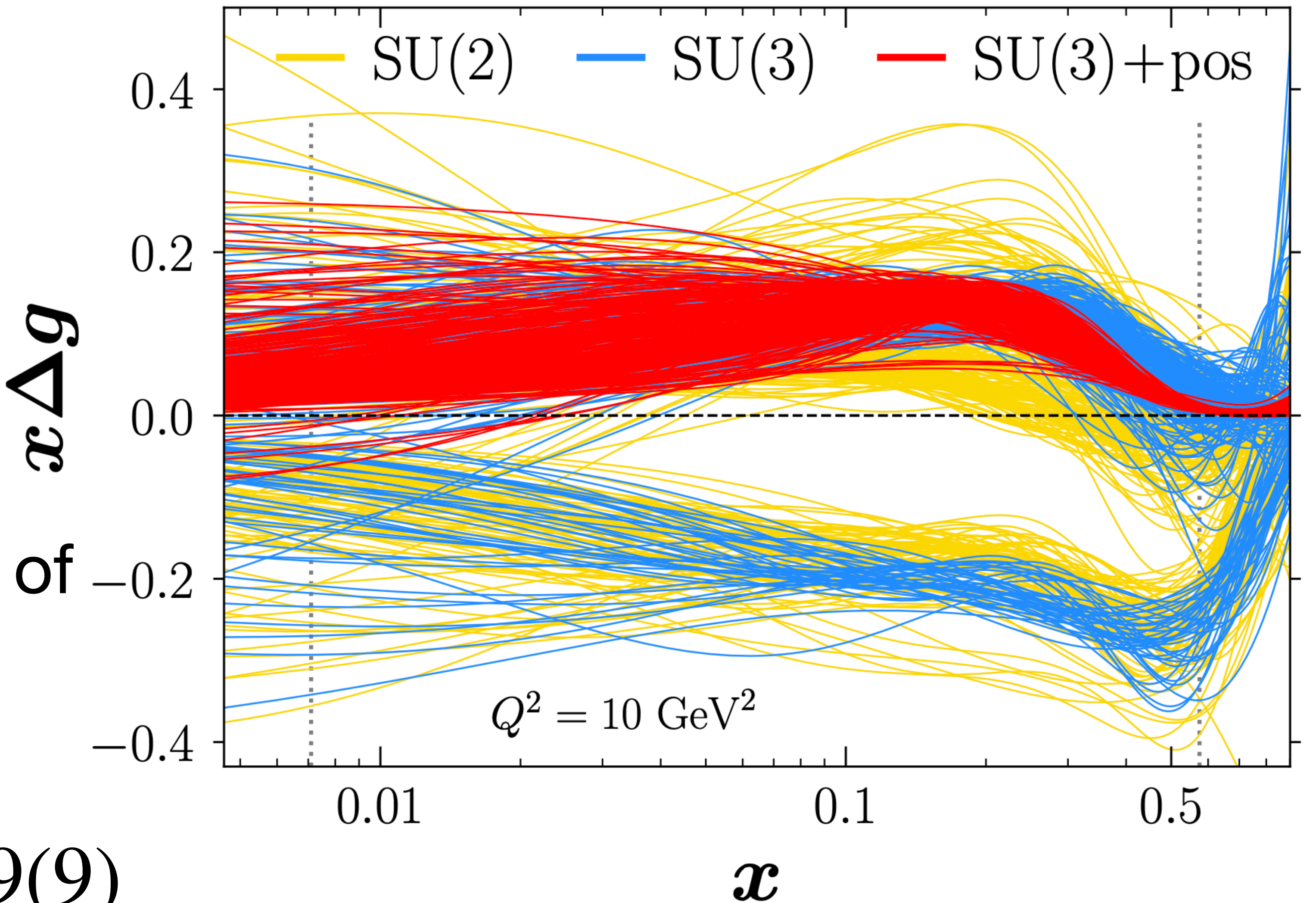


Spinning gluons

Y. Zhou et al (JAM) Phys. Rev. D 105, 074022 (2022)

- Positivity assumed in many analyses

$$|\Delta g| \leq g(x) \qquad g_\uparrow = \frac{1}{2}(g + \Delta g) \qquad g_\downarrow = \frac{1}{2}(g - \Delta g)$$



  - Removing reveals new band of solutions

- With constraint: $\Delta G = 0.39(9)$

R. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990)

- Without constraint: $\Delta G = 0.3(5)$

$$J = \frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + L_G + \Delta G$$

- Lattice: $\Delta G = 0.251(47)(16)$

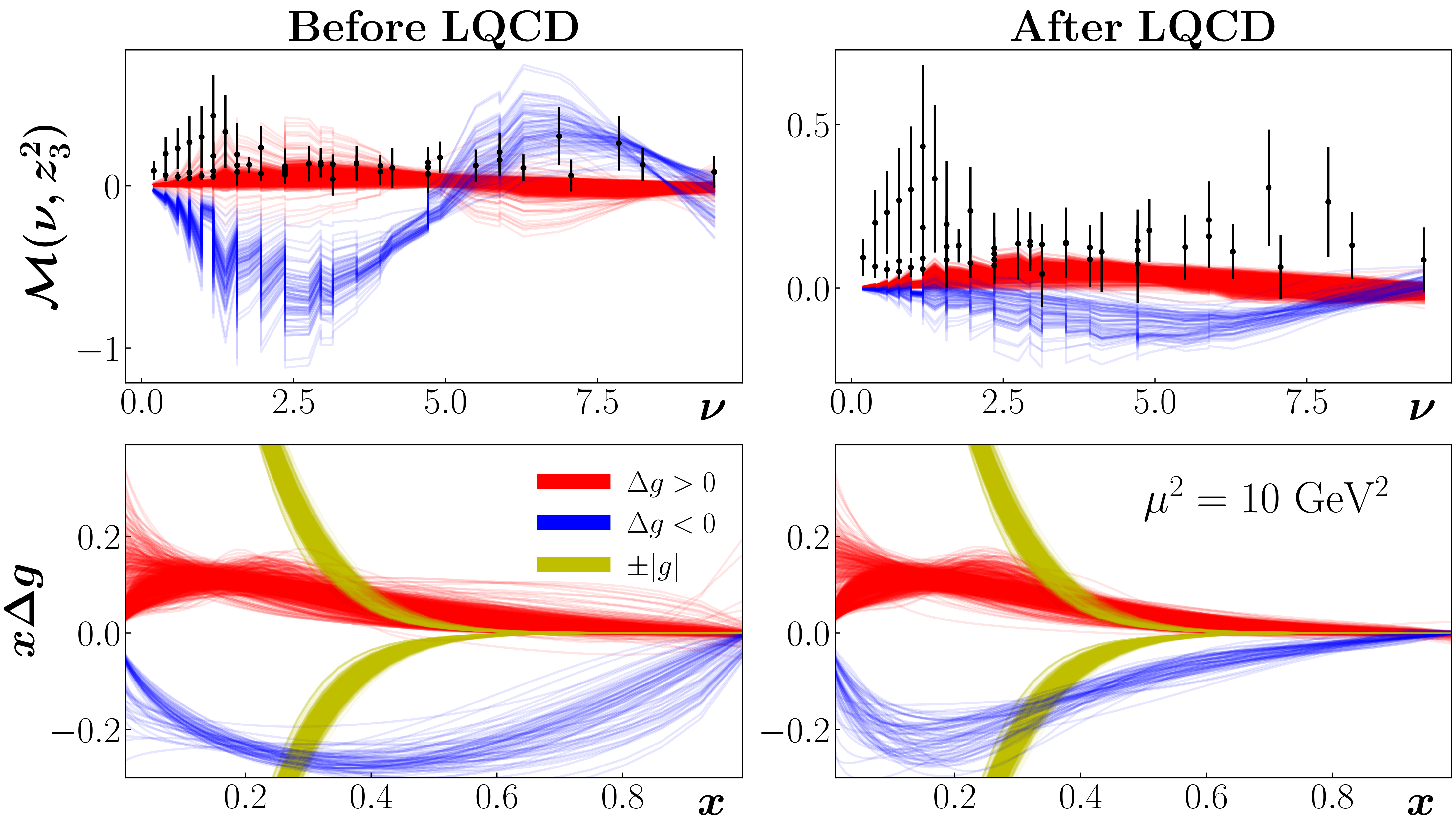
$$\Delta G = \int dx\, \Delta g(x)$$

Y-B. Yang et al (χ-QCD) Phys. Rev. Lett. 118, 102001 (2017)
K-F. Liu arXiv: 2112.08416

Spinning gluons

Y. Zhou et al Phys. Rev. D 105, 074022 (2022)
C. Egerer et al (HadStruc) arXiv:2207.08733
JK et al arXiv:2310.18179

Can lattice data affect phenomenological polarized gluon analysis?

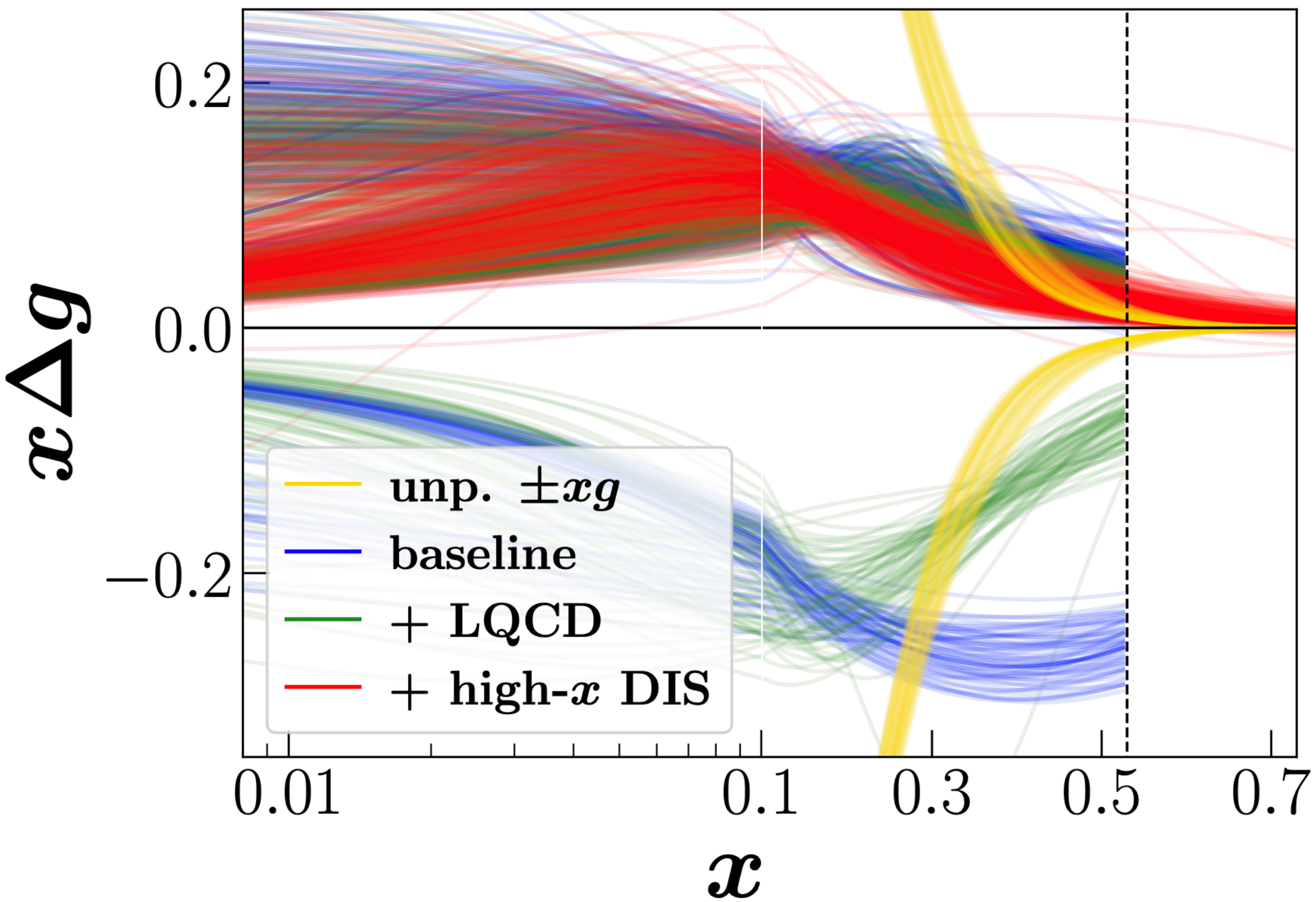


- The positive and negative solutions without positivity constraints
- Only positive band "consistent" with lattice data, but is too noisy to constrain.

Resolution of the helicity sign

- Rejection of negative helicity gluon PDF requires 3 datasets
  - **RHIC Spin Asymmetries**
    - Linear and quadratic in Δg
  - **Lattice QCD matrix element**
    - Linear in Δg
  - **JLab high-x DIS from relaxing cuts on Final state mass**
    - Linear in Δg
    - $W^2 > 10\,\text{GeV}^2 \rightarrow W^2 > 4\,\text{GeV}^2$

N.T. Hunt-Smith et al arXiv:2403.08117



Evolution of parton distributions

- Standard DGLAP evolution
    - Parton model: Splitting of partons into smaller x

$$\mu^2 \frac{d}{d\mu^2} q(x,\mu^2) = \int_x^1 dy\, P_{qq}(y) q(\frac{x}{y},\mu^2)$$

- MSbar Step Scaling function
    - Integrated or discretized version of evolution

$$q(x,\mu^2) = \int_x^1 dy\, \mathcal{E}(y,\mu^2,\mu_0^2) q(\frac{x}{y},\mu_0^2)$$

PDF at high scale $\mu \sim Q$

PDF at low input scale $\mu_0 \sim m_c$

H. Dutrieux, JK, C. Monahan, K. Orginos, S. Zafeiropoulos arXiv:2310.19926

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- MSbar Step Scaling function
  - Integrated or discretized version of evolution

$$q(x,\mu^2) = \int_x^1 dy\, \mathscr{E}(y,\mu^2,\mu_0^2) q(\frac{x}{y},\mu_0^2)$$

- pseudo-PDF evolution
  - $\mathfrak{M}(\nu,z^2) = \int_0^1 du\, C(u,\mu^2 z^2)\, I(u\nu,\mu^2) + O(z^2)$
- Data does not know about MSbar scale

$$\mu^2 \frac{d}{d\mu^2} \mathfrak{M}(\nu,z^2) = 0$$

$$\Downarrow$$

$$z^2 \frac{d}{dz^2} \mathfrak{M}(\nu,z^2) = \int_0^1 d\alpha \mathscr{P}(\alpha,z^2) \mathfrak{M}(\alpha\nu,z^2) + O(z^2)$$

$$\mathfrak{M}(\nu,z^2) = \int_0^1 d\alpha\, \Sigma(\alpha,z^2,z_0^2) \mathfrak{M}(\alpha\nu,z_0^2) + O(z^2,z_0^2)$$

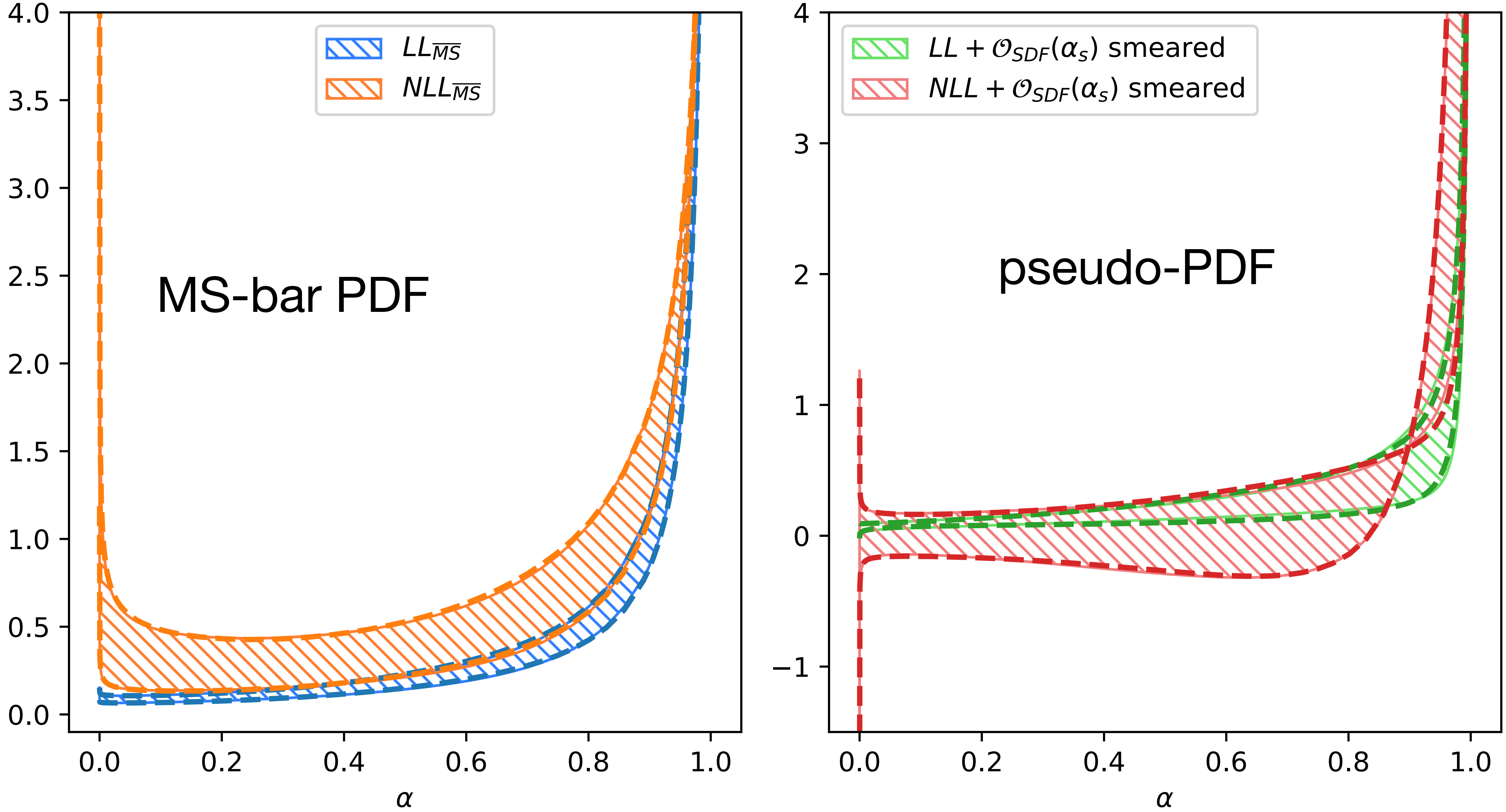
$$\mathscr{E}(\mu^2,\mu_0^2) = C^{-1}(\mu^2 z^2) \otimes \Sigma(z^2,z_0^2) \otimes C(\mu_0^2 z_0^2)$$

H. Dutrieux, JK, C. Monahan, K. Orginos, S. Zafeiropoulos arXiv:2310.19926

Evolution of parton distributions

H. Dutrieux, JK, C. Monahan, K. Orginos, S. Zafeiropoulos arXiv:2310.19926

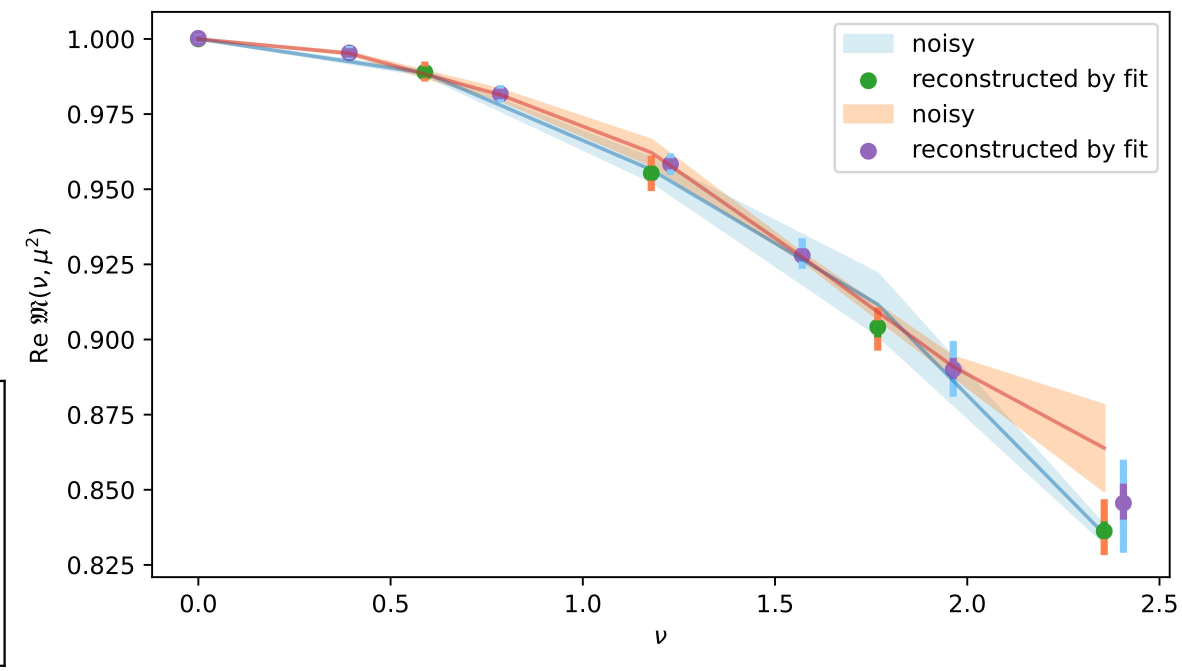
- Perturbative evolution from ~700 MeV (0.282 fm) to ~1GeV (0.188 fm)
- Bands from varying scale by factor of 2 to estimate higher order effects



H. Dutrieux, JK, C. Monahan, K. Orginos, S. Zafeiropoulos arXiv:2310.19926

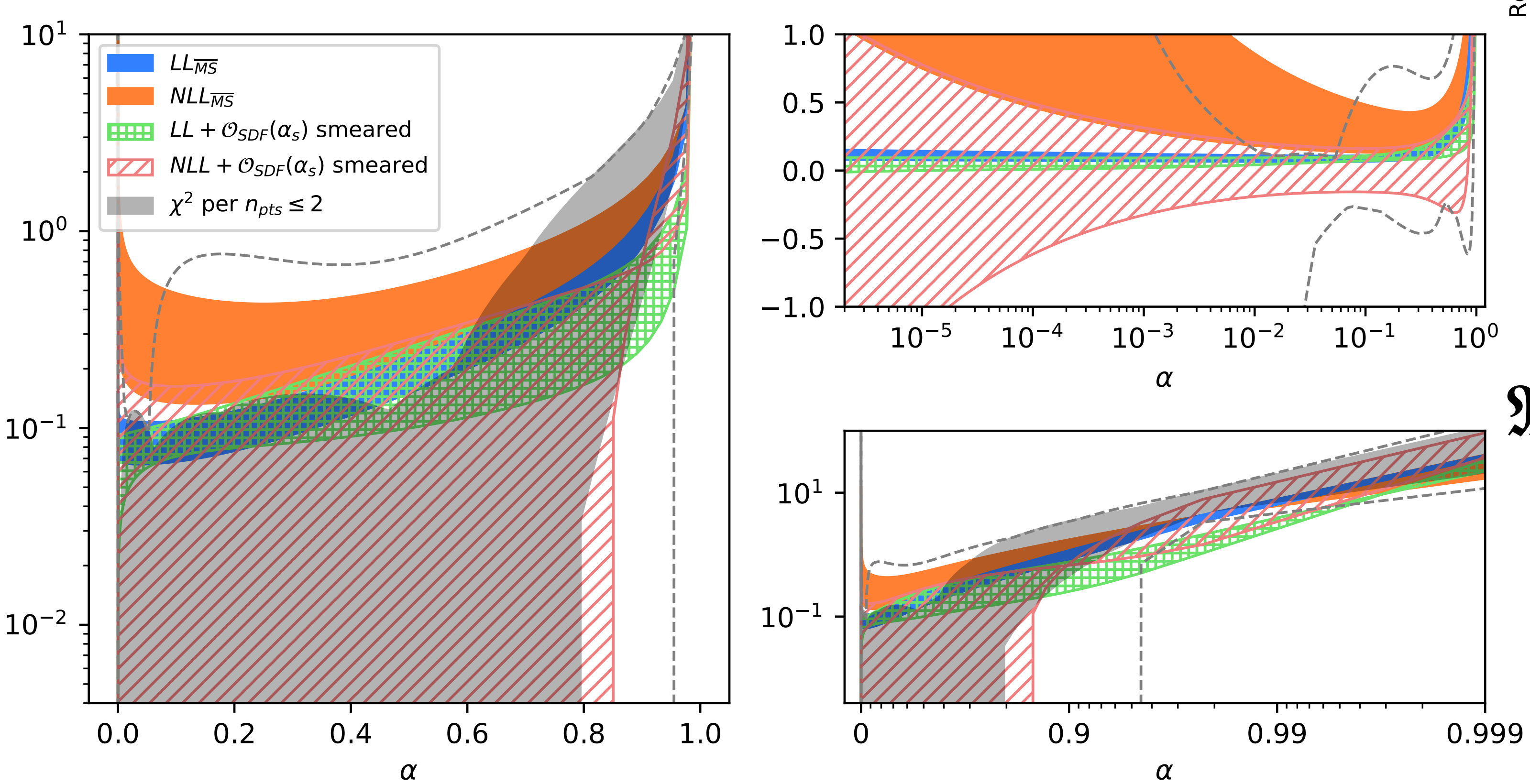
Step Scaling from the lattice

- Requires data in same range of ν and different z



- Model Function

$$\Sigma(\alpha) = A\alpha^{-\delta}(1 + r\alpha) + B(-\ln(\alpha))^{-\eta}\ln^2(1-\alpha) + \sigma\alpha(1-\alpha)$$



$$\mathfrak{M}(\nu, z^2) = \int_0^1 d\alpha\, \Sigma(\alpha, z^2, z_0^2)\mathfrak{M}(\alpha\nu, z_0^2)$$

- Catch: Requires assumption of leading twist dominance and ranges of ν are limited
  - Need very fine lattices to study systematics
  - Test universality by studying pion, kaon, nucleon, quark (in fixed gauge)

Non-Parametric Bayesian inferences

- Take advantage of single dimension and limited range
- Approximate unknown by value on grid and interpolate for integrals
- Maximize the posterior distribution

$$P\left[q\,|\,\mathfrak{M}, I\right] \propto P\left[\mathfrak{M}\,|\,q, I\right] P\left[q\,|\,I\right]$$

- Add prior information to regulate the inverse problem

$$P\left[q\,|\,I\right] \propto \exp[-S(q)]$$

Y. Burnier and A. Rothkopf (2013) 1307.6106

Shannon-Jaynes entropy

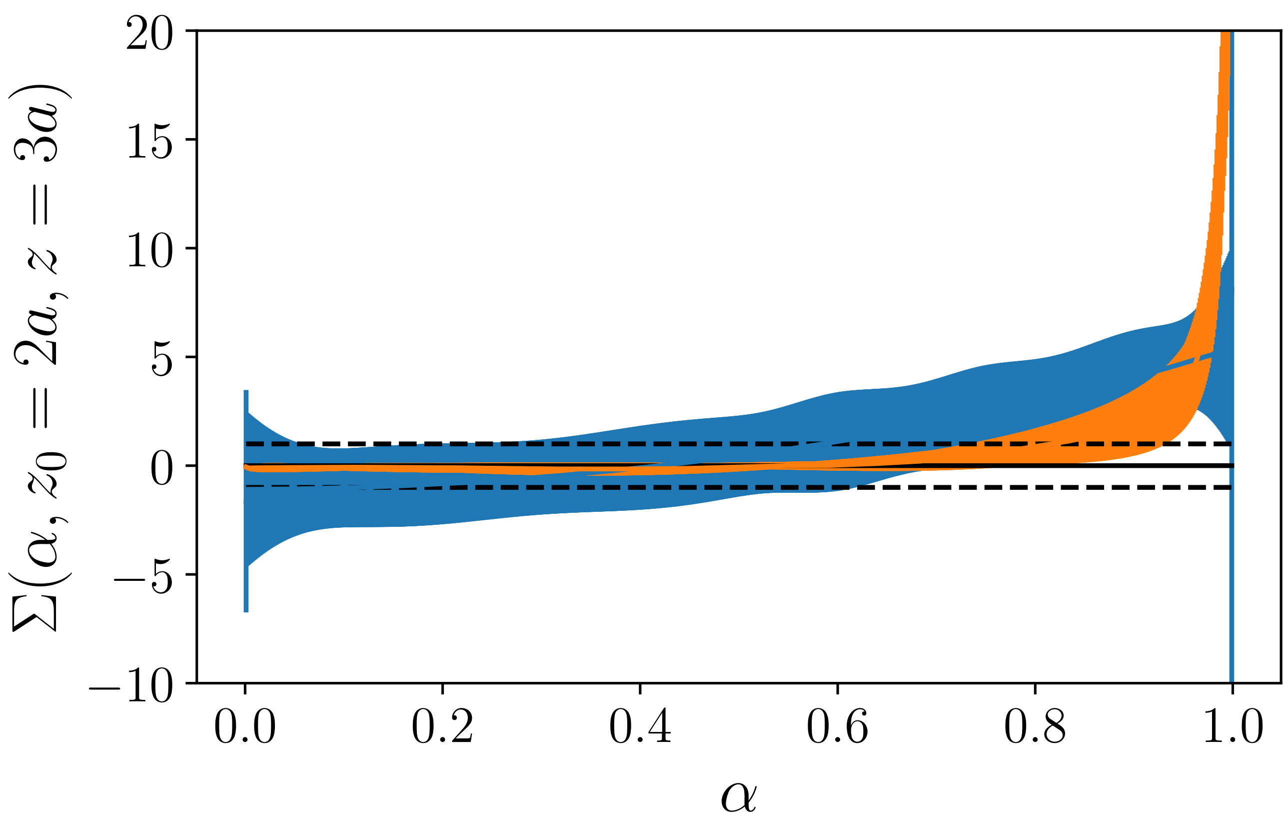
$$S(q) = \alpha \int_0^1 dx \left(q(x) - m(x) - q(x)\log(\frac{q(x)}{m(x)}) \right)$$

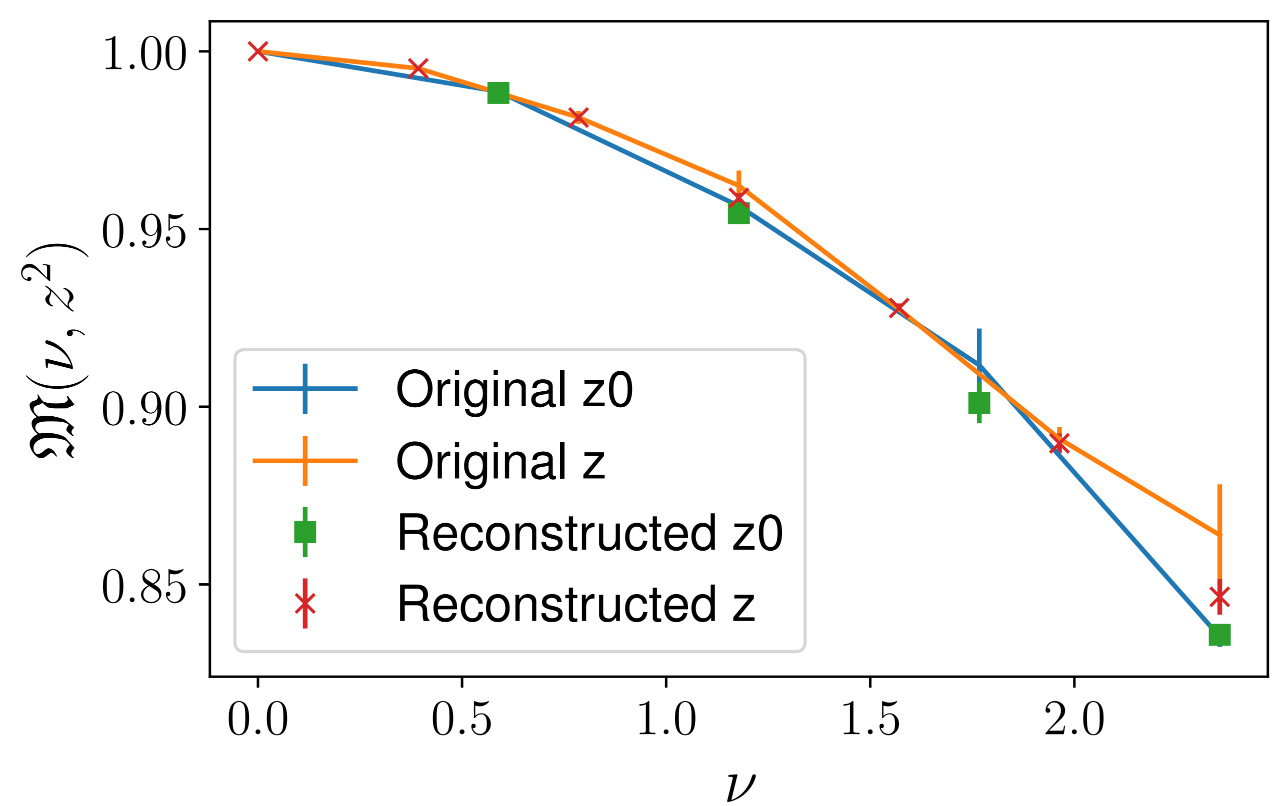
Burnier-Rothkopf

$$S(q) = \alpha \int_0^1 dx \left(1 - \frac{q(x)}{m(x)} + \log(\frac{q(x)}{m(x)}) \right)$$

Non-Parametric Bayesian inferences

- Use different priors to study model dependencies
- First prior with easily understood biases
  - Quadratic Difference Ratio (QDR) $S(\Sigma) = u \int_0^1 d\alpha \, \frac{\left(\Sigma(\alpha) - h(\alpha)\right)^2}{\sigma(\alpha)^2}$





$$u = 1 \quad h(\alpha) = 0 \quad \sigma(\alpha) = 1$$

  - Large errors from prior with no correlations at different α
    Need for better choices

H. Dutrieux, JK, C. Monahan, K. Orginos, S. Zafeiropoulos arXiv:2310.19926

Conclusions

- Lattice matrix elements can be related to PDFs and their calculation have matured over the decade
- With control of systematic errors, lattice PDFs are approaching accuracy of global fits
- Non-perturbative PDF evolution can be determined from lattice data
- Adding Lattice data into global fits give better results than either could do alone
- All lessons can be extended to TMDs and GPDs

Back up slides

Helicity Gluon matrix element

I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
C. Egerer et al (HadStruc) arXiv:2207.08733

- Helicity Gluon Matrix Element:

$$\widetilde{M}_{\mu\alpha;\nu\beta}(z,p,s) = \frac{1}{2}\epsilon_{\nu\beta\rho\sigma}M_{\mu\alpha;\rho\sigma} = \langle p,s\,|\,\mathrm{Tr}\left[F^{\mu\alpha}(z)W(z;0)\widetilde{F}^{\nu\beta}(0)\right]|p,s\rangle$$

- Useful Combination $\widetilde{\mathscr{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
    - Gives **two** amplitudes, one has no leading twist contribution

Helicity Gluon matrix element

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- Useful Combination $\widetilde{\mathscr{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
    - Gives **two** amplitudes, one has no leading twist contribution
- Use ratio with finite continuum limit

$$\widetilde{\mathfrak{M}}(\nu,z^2) = i\frac{\left[\widetilde{\mathscr{M}}(z,p)/p_z p_0\right]/Z_L(z/a)}{\mathscr{M}(0,z^2)/m^2}$$

Helicity Gluon matrix element

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- Relation to gluon and quark singlet ITD

$$\langle x\rangle_g\,\widetilde{\mathfrak{M}}(\nu,z^2) = \int_0^1 \widetilde{C}^{gg}(u,\mu^2z^2)\widetilde{I}_g(u\nu,\mu^2) + \widetilde{C}^{qg}(u,\mu^2z^2)\widetilde{I}_s(u\nu,\mu^2)$$

Helicity Gluon matrix element

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$$\langle x\rangle_g\,\widetilde{\mathfrak{M}}(\nu,z^2) = \int_0^1 \widetilde{C}^{gg}(u,\mu^2z^2)\widetilde{I}_g(u\nu,\mu^2) + \widetilde{C}^{qg}(u,\mu^2z^2)\widetilde{I}_s(u\nu,\mu^2)$$

Pol Gluon Lorentz decomposition

I. Balitsky, W. Morris, A. Radyushkin
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$$\widetilde{M}^{(2)}_{\mu\alpha;\lambda\beta}(z,p) = (sz)\big(g_{\mu\lambda}p_\alpha p_\beta - g_{\mu\beta}p_\alpha p_\lambda - g_{\alpha\lambda}p_\mu p_\beta + g_{\alpha\beta}p_\mu p_\lambda\big)\widetilde{\mathcal{M}}_{pp}$$
$$+(sz)\big(g_{\mu\lambda}z_\alpha z_\beta - g_{\mu\beta}z_\alpha z_\lambda - g_{\alpha\lambda}z_\mu z_\beta + g_{\alpha\beta}z_\mu z_\lambda\big)\widetilde{\mathcal{M}}_{zz}$$
$$+(sz)\big(g_{\mu\lambda}z_\alpha p_\beta - g_{\mu\beta}z_\alpha p_\lambda - g_{\alpha\lambda}z_\mu p_\beta + g_{\alpha\beta}z_\mu p_\lambda\big)\widetilde{\mathcal{M}}_{zp}$$
$$+(sz)\big(g_{\mu\lambda}p_\alpha z_\beta - g_{\mu\beta}p_\alpha z_\lambda - g_{\alpha\lambda}p_\mu z_\beta + g_{\alpha\beta}p_\mu z_\lambda\big)\widetilde{\mathcal{M}}_{pz}$$
$$+(sz)\big(p_\mu z_\alpha - p_\alpha z_\mu\big)\big(p_\lambda z_\beta - p_\beta z_\lambda\big)\widetilde{\mathcal{M}}_{ppzz}$$
$$+(sz)\big(g_{\mu\lambda}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\lambda}\big)\widetilde{\mathcal{M}}_{gg}$$

$$\widetilde{M}^{(1)}_{\mu\alpha;\lambda\beta}(z,p) = \Big(g_{\mu\lambda}s_\alpha p_\beta - g_{\mu\beta}s_\alpha p_\lambda - g_{\alpha\lambda}s_\mu p_\beta + g_{\alpha\beta}s_\mu p_\lambda\Big)\widetilde{\mathcal{M}}_{sp}$$
$$+\Big(g_{\mu\lambda}p_\alpha s_\beta - g_{\mu\beta}p_\alpha s_\lambda - g_{\alpha\lambda}p_\mu s_\beta + g_{\alpha\beta}p_\mu s_\lambda\Big)\widetilde{\mathcal{M}}_{ps}$$
$$+\Big(g_{\mu\lambda}s_\alpha z_\beta - g_{\mu\beta}s_\alpha z_\lambda - g_{\alpha\lambda}s_\mu z_\beta + g_{\alpha\beta}s_\mu z_\lambda\Big)\widetilde{\mathcal{M}}_{sz}$$
$$+\Big(g_{\mu\lambda}z_\alpha s_\beta - g_{\mu\beta}z_\alpha s_\lambda - g_{\alpha\lambda}z_\mu s_\beta + g_{\alpha\beta}z_\mu s_\lambda\Big)\widetilde{\mathcal{M}}_{zs}$$
$$+(p_\mu s_\alpha - p_\alpha s_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{pspz}$$
$$+(p_\mu z_\alpha - p_\alpha z_\mu)(p_\lambda s_\beta - p_\beta s_\lambda)\widetilde{\mathcal{M}}_{pzps}$$
$$+(s_\mu z_\alpha - s_\alpha z_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{szpz}$$
$$+(p_\mu z_\alpha - p_\alpha z_\mu)(s_\lambda z_\beta - s_\beta z_\lambda)\widetilde{\mathcal{M}}_{pzsz}$$

Want: $M_{\Delta g}(\nu, z^2) = \big[\widetilde{\mathcal{M}}^{(+)}_{sp} - \nu\,\widetilde{\mathcal{M}}_{pp}\big]$

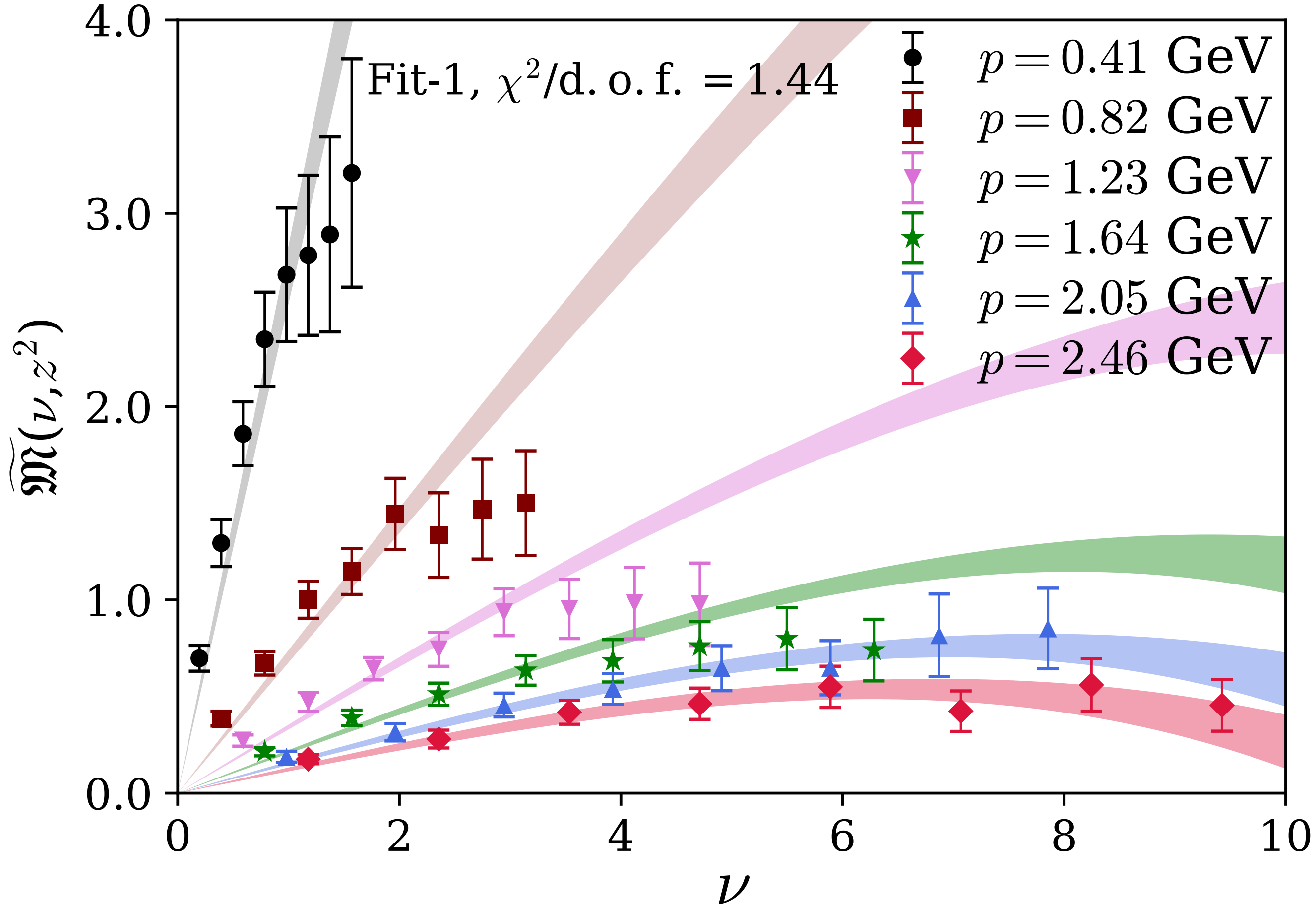
Can get: $\widetilde{\mathcal{M}}(z,p) = \big[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\big]$

$$= M_{\Delta g} - \frac{m^2 z^2}{\nu}\widetilde{\mathcal{M}}_{pp}$$

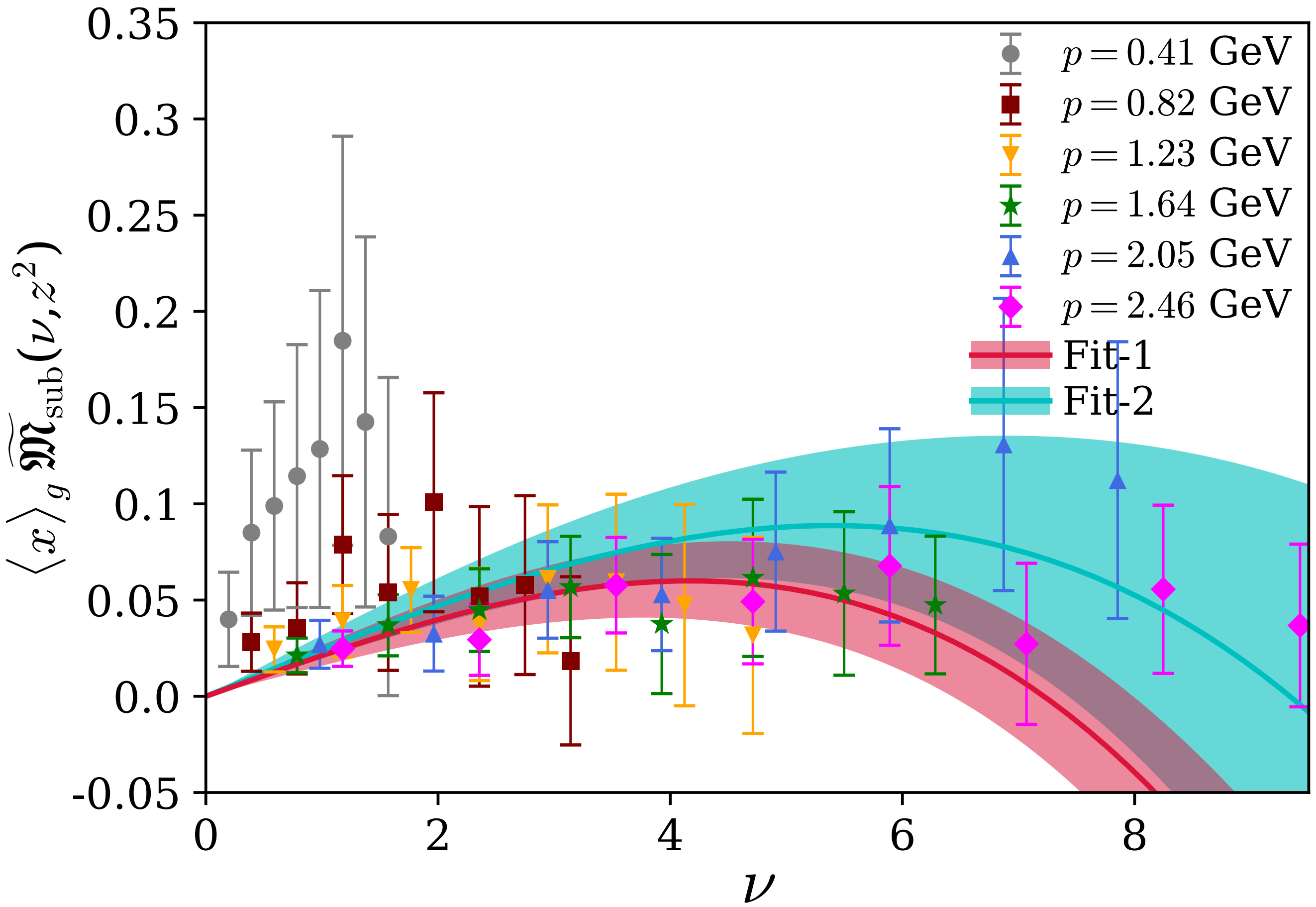
$$= M_{\Delta g} - \frac{m^2}{p_z^2}\nu\,\widetilde{\mathcal{M}}_{pp}$$

Helicity Gluon PDF

- Model both terms



- Subtract rest frame



$a = 0.094$ fm

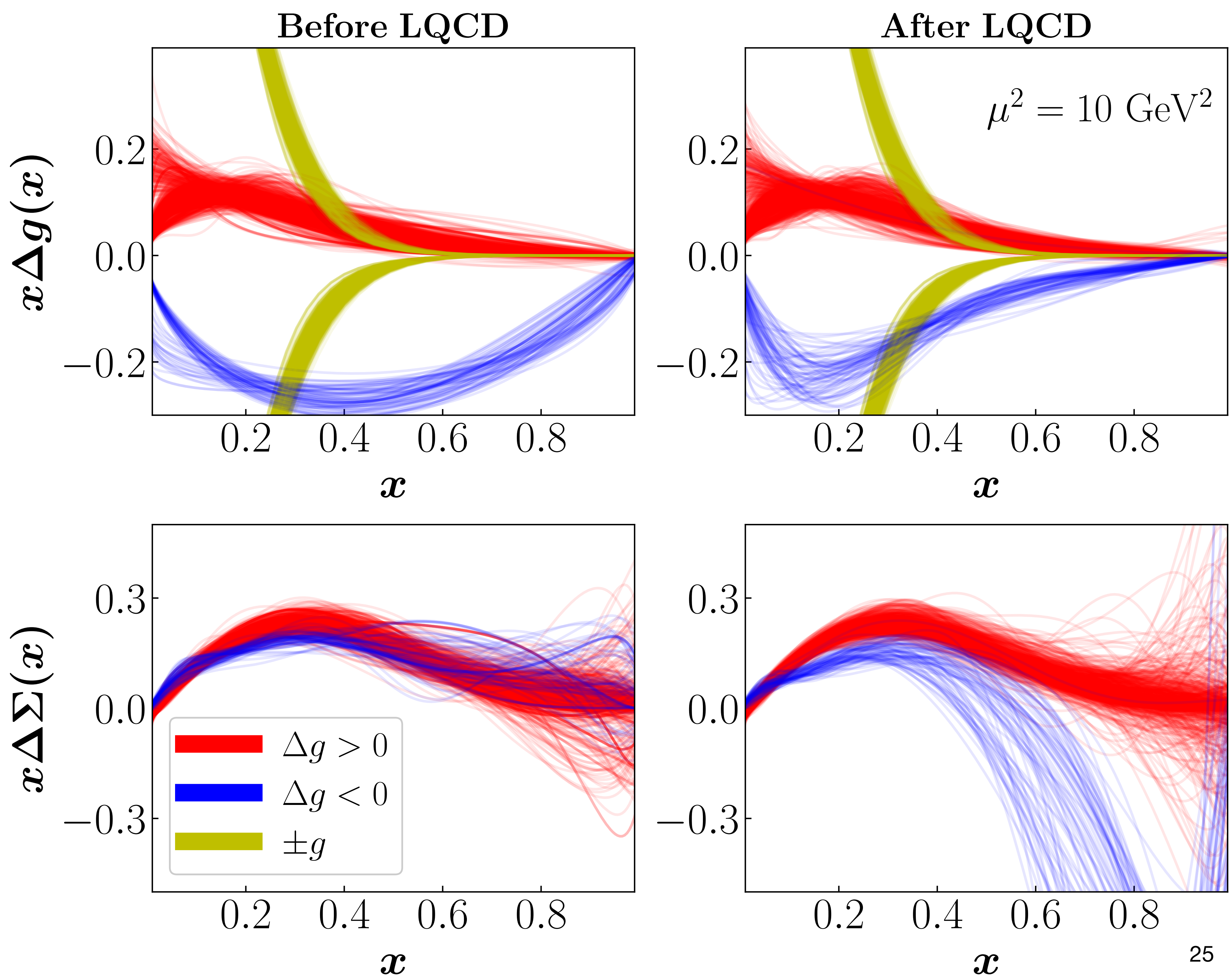
$m_\pi = 358$ MeV

C. Egerer et al (HadStruc) arXiv: 2207.08733

Lattice gluon data impacts quarks

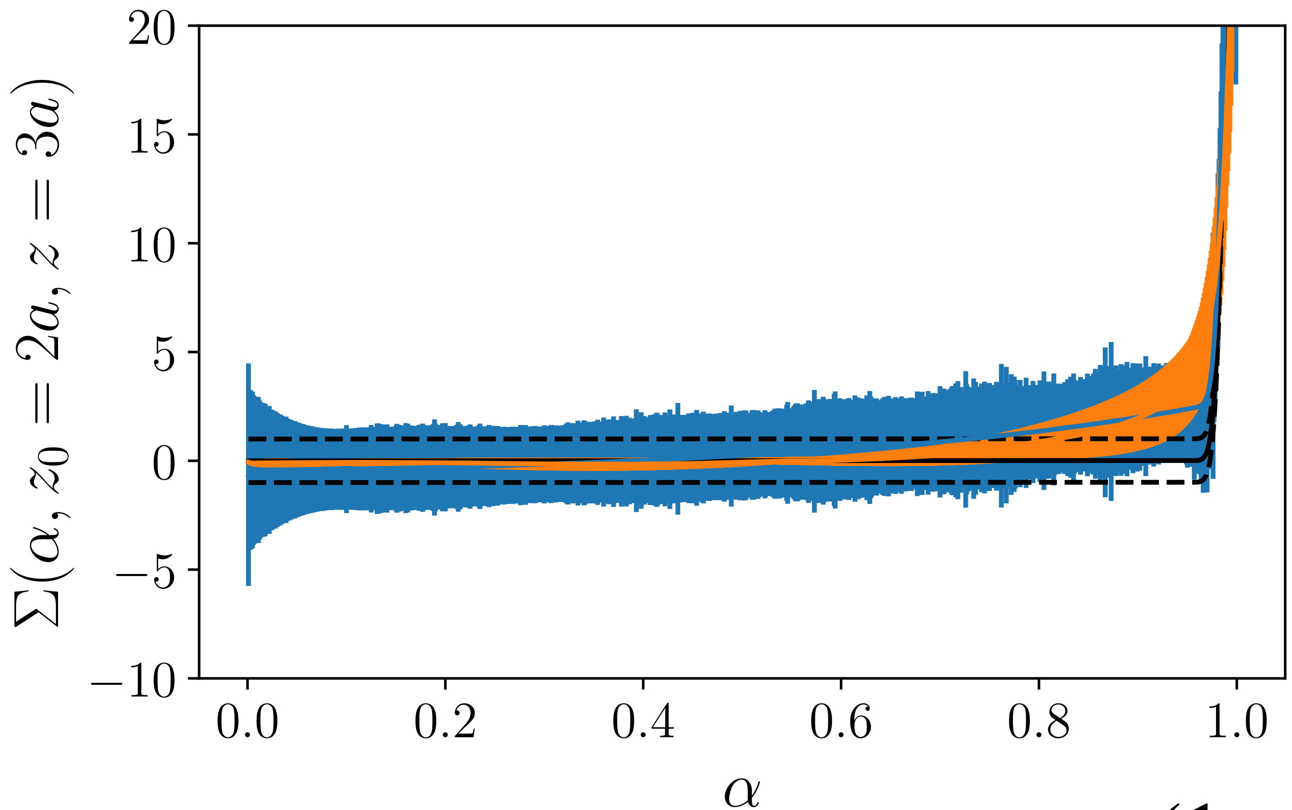
C. Egerer et al (HadStruc) arXiv:2207.08733

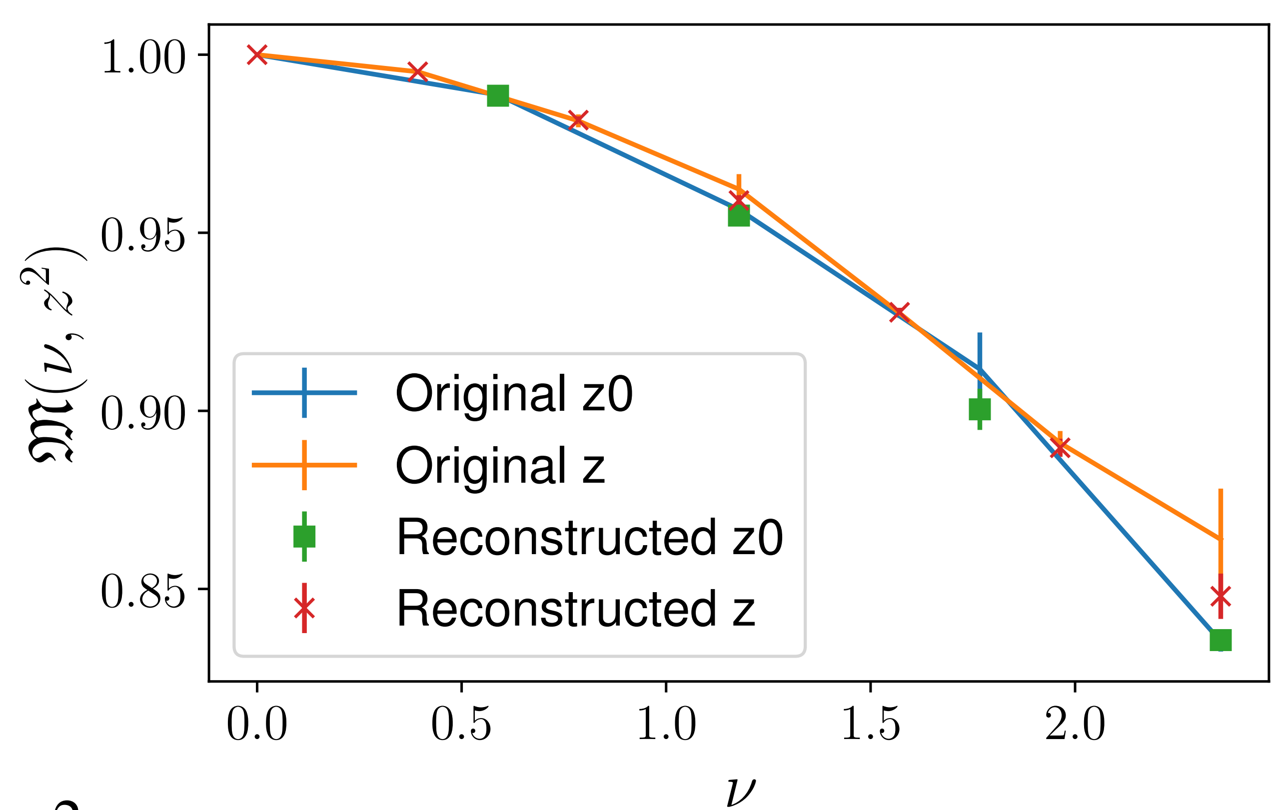
- Quark gluon mixing leads to impact on singlet
- Unexpected change in extrapolation region
- Compensates reduced magnitude of Δg in relation to cross sections



Non-Parametric Bayesian inferences

- Use different priors to study model dependencies
- First prior with easily understood biases
  - Quadratic Difference Ratio (QDR) $S(\Sigma) = u \int_0^1 d\alpha \, \frac{\left(\Sigma(\alpha) - h(\alpha)\right)^2}{\sigma(\alpha)^2}$





$$u = 1 \quad h(\alpha) = \exp\left(-\frac{(1-\alpha)^2}{w^2}\right)/(w\sqrt{2\pi}) \quad \sigma(\alpha) = 1$$

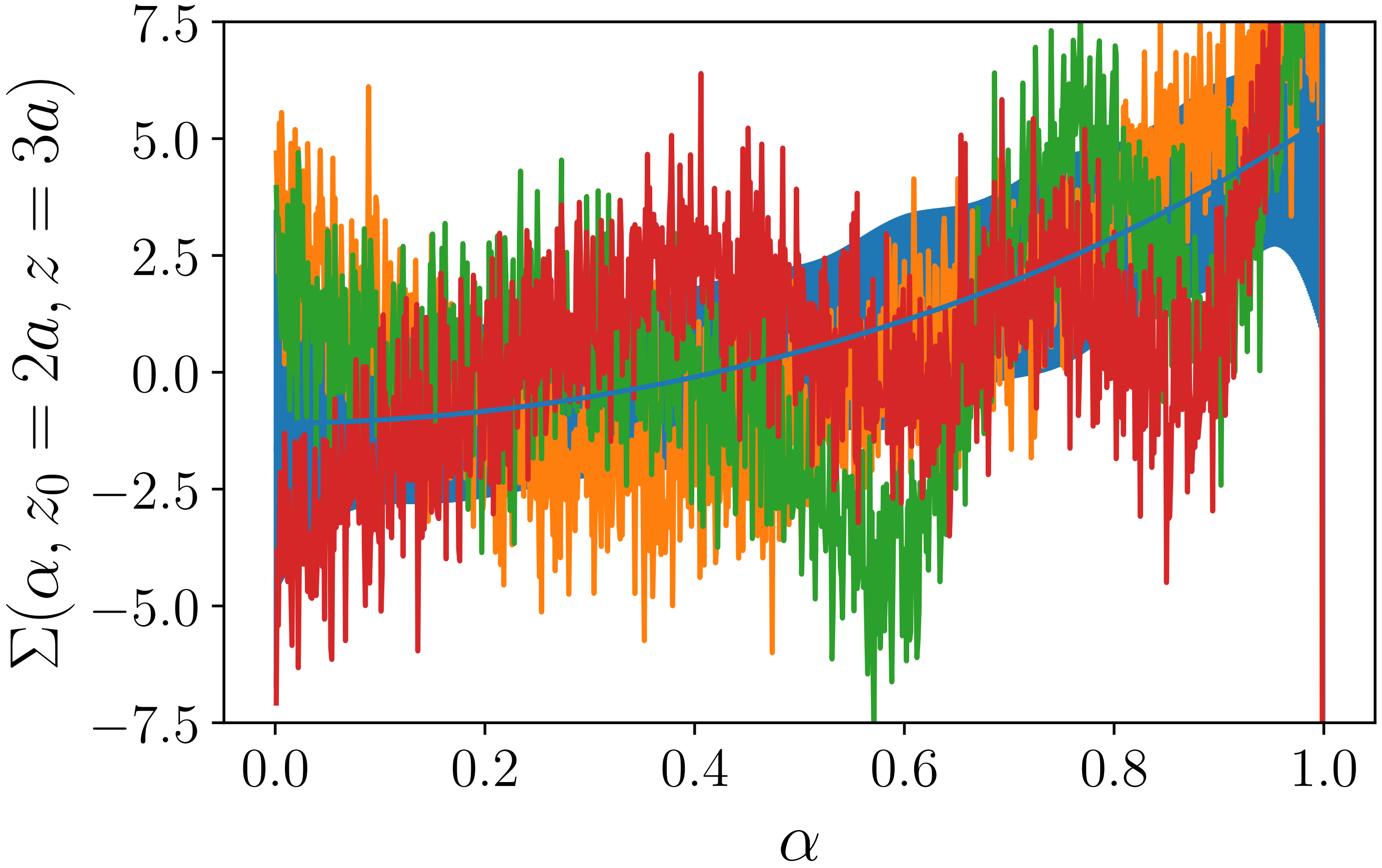
$$w = 0.01$$

H. Dutrieux, JK, C. Monahan, K. Orginos, S. Zafeiropoulos arXiv:2310.19926

“I’m sorry, Nature hates Wiggles”

-A. Radyushkin

- Characteristic curves from fit
- QDR has no correlations between neighbors
- Need better priors!



H. Dutrieux, JK, C. Monahan, K. Orginos, S. Zafeiropoulos arXiv:2310.19926

“I’m sorry, Nature hates Wiggles”

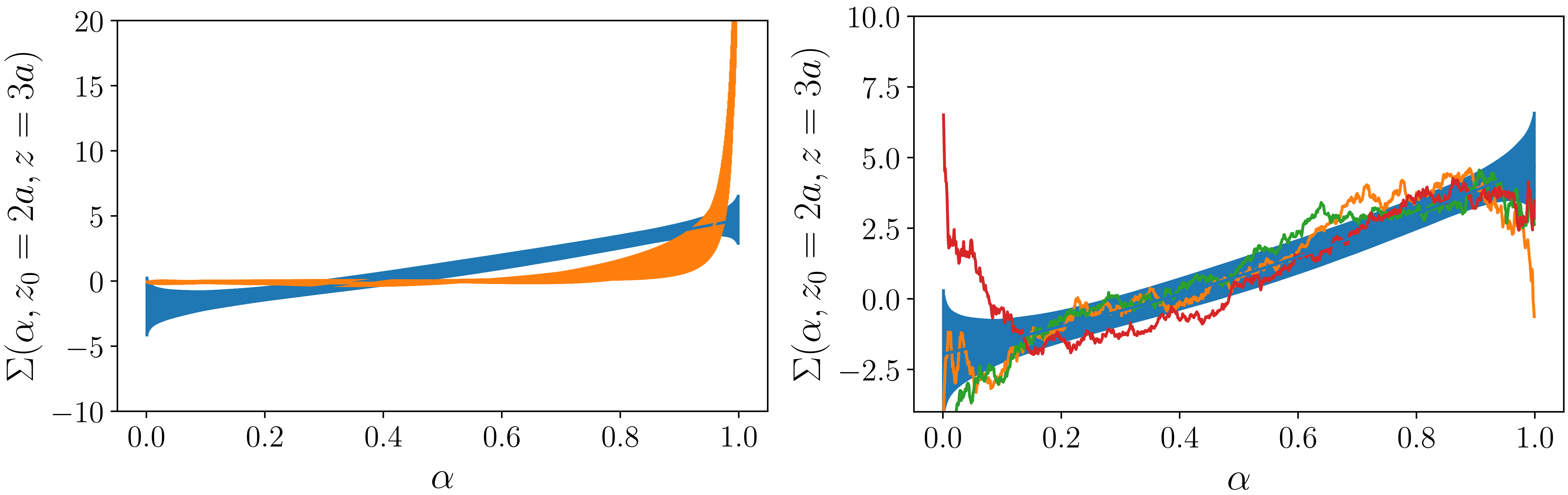
-A. Radyushkin

- Use different priors to study model dependencies
- Can we remove the wiggles?
    - A smoothing prior

$$S(\Sigma) = u \int_0^1 d\alpha\, \alpha(1-\alpha) \left(\frac{\partial \Sigma}{\partial \alpha}\right)^2$$

$$u = 1$$

- Set u too large and it forces a flat result.
- Alternative to correlate α’s is to use Gaussian Processes



H. Dutrieux, JK, C. Monahan, K. Orginos, S. Zafeiropoulos arXiv:2310.19926