Perturbative T-odd asymmetries in the Drell-Yan process revisited

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QCD Evolution in Pavia
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TODD Talk

Valery Lyubovitskij, Werner Vogelsang, FW, Alexey Zhevlakov,
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Outline

1. Intro to T(ime reversal)-odd observables
2. T-odd structure functions of the hadronic tensor
3. Analytical results for the partonic structure functions
4. Small-Q_T expansion of collinear results
5. Comparison to ATLAS data
6. Conclusion and Outlook

1. Intro to T(ime reversal)-odd observables

What is a T(ime reversal)-odd observable?

Foundations [Rujula 1971]

- Unitarity of S-matrix: $\mathbb{1} = \hat{S}\,\hat{S}^\dagger$
- Transition matrix $\hat{T}$: $\hat{S} = \mathbb{1} + i\,\hat{T}$ with $\langle f|\,\hat{T}\,|i\rangle = (2\pi)^4\delta^4(P_f - P_i)\mathcal{M}_{fi}$.
- (gen.) Optical Theorem: $\mathcal{M}_{fi} - \mathcal{M}^*_{if} = i\overbrace{\sum_X \mathcal{M}^*_{Xf}\mathcal{M}_{Xi}}^{\mathcal{A}_{fi}}$
- Time-reversal-symmetry under reversal of momenta & spin + $i \leftrightarrow f$: $|\mathcal{M}_{fi}|^2 = |\mathcal{M}_{\bar{i}\bar{f}}|^2$

- square OptTh: $|\mathcal{M}_{fi}|^2 = |\mathcal{M}_{if}|^2 - 2i\,\mathrm{Im}(\mathcal{M}_{if}\mathcal{A}_{fi}) - |\mathcal{A}_{fi}|^2$
- subtract $|\mathcal{M}_{\bar{f}\bar{i}}|^2$ on both sides:

$$\underbrace{|\mathcal{M}_{fi}|^2 - |\mathcal{M}_{\bar{f}\bar{i}}|^2}_{T\text{-odd effect}} = \underbrace{|\mathcal{M}_{if}|^2 - |\mathcal{M}_{\bar{f}\bar{i}}|^2}_{\text{vanishes by } T\text{-invariance}} - \underbrace{2i\,\mathrm{Im}(\mathcal{M}_{if}\mathcal{A}_{fi})}_{\sim\mathcal{M}^3} - \underbrace{|\mathcal{A}_{fi}|^2}_{\sim\mathcal{M}^4}$$

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Introduction

What are T(ime reversal)-odd observables?

- change sign under *naive* time reversal, i.e. reversal of mom. and spin w.o. interchange of initial and final state
- can occur in theories invariant under true time reversal, e.g. a wide range of QCD scattering phenomena

T-odd effect in Drell-Yan:

- T-odd effects appear as angular asymmetries in the angle between lepton and hadron plane ($\sim \sin\phi$ and $\sin 2\phi$)
- have been studied in: [Hagiwara 1984], [Mirkes 1992], [Yokoya 2007], [Benic 2024],...

What did we do?

- Order α_s^2 calc. of T-odd str. fcts. for charged and neutral current DY in col. factorization + small-Q_T expansion to NNLP in Q_T^2/Q^2

Introduction

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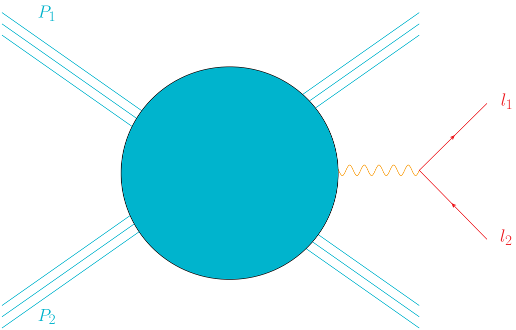
2. T-odd structure functions of the hadronic tensor

Drell-Yan process



Differential DY cross section

$$\frac{d\sigma_{pp\to\ell\bar{\ell}X}}{d^4q\, d\Omega} = \frac{\alpha^2}{2(2\pi)^4 s^2 Q^4} L_{\mu\nu} W^{\mu\nu}$$

Parametrization of hadronic tensor

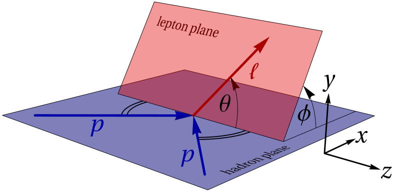
$$
\begin{aligned}
W^{\mu\nu} &= (X^\mu X^\nu + Y^\mu Y^\nu) W_T + i(X^\mu Y^\nu - Y^\mu X^\nu) W_{T_P} + Z^\mu Z^\nu W_L \\
&+ (Y^\mu Y^\nu - X^\mu X^\nu) W_{\Delta\Delta} - (X^\mu Y^\nu + Y^\mu X^\nu) W_{\Delta\Delta_P} \\
&- (X^\mu Z^\nu + Z^\mu X^\nu) W_\Delta - (Y^\mu Z^\nu + Z^\mu Y^\nu) W_{\Delta_P} \\
&+ i(Z^\mu X^\nu - X^\mu Z^\nu) W_\nabla + i(Y^\mu Z^\nu - Z^\mu Y^\nu) W_{\nabla_P}
\end{aligned}
$$

T-odd (and P-odd) helicity structure functions

- Tranverse-transverse interference (double-spin flip) $W_{\Delta\Delta_P}$
- Transverse-longitudinal int. (single-spin flip) W_{Δ_P}, W_∇

Angular Distributions

$$\frac{dN}{d\Omega} \equiv \frac{\frac{d\sigma}{d^4q\,d\Omega}}{\frac{d\sigma}{d^4q}} = \frac{3}{8\pi(2W_T + W_L)} \times \Big[g_T\, W_T + g_L\, W_L + g_\Delta\, W_\Delta + g_{\Delta\Delta}\, W_{\Delta\Delta} + g_{T_P}\, W_{T_P} + g_{\nabla_P}\, W_{\nabla_P} + g_\nabla\, W_\nabla + g_{\Delta\Delta_P}\, W_{\Delta\Delta_P} + g_{\Delta_P}\, W_{\Delta_P} \Big],$$



Collins-Soper frame

where $g_i = g_i(\theta, \phi)$ denote the angular coefficients

$$g_T = 1 + \cos^2\theta\,, \qquad g_L = 1 - \cos^2\theta\,, \qquad g_{T_P} = \cos\theta\,,$$
$$g_{\Delta\Delta} = \sin^2\theta\, \cos 2\phi\,, \qquad g_\Delta = \sin 2\theta\, \cos\phi\,, \qquad g_{\nabla_P} = \sin\theta\, \cos\phi\,,$$
$$g_{\Delta\Delta_P} = \sin^2\theta\, \sin 2\phi\,, \qquad g_{\Delta_P} = \sin 2\theta\, \sin\phi\,, \qquad g_\nabla = \sin\theta\, \sin\phi\,,$$

3. Analytical results for the partonic structure functions

Collinear factorization

We employ a collinear factorization approach with $p_i = \xi_i P_i$, valid at $Q \sim Q_\mathrm{T} \gg \Lambda_\mathrm{QCD}$.

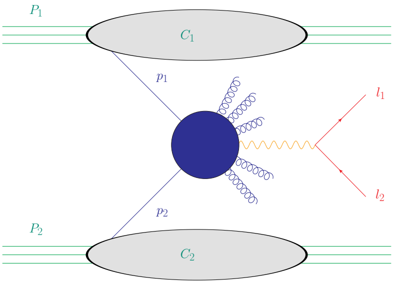
$$\underbrace{W(x_1, x_2, \rho^2)}_{\text{hadronic structure fct.}} = \frac{1}{x_1 x_2} \sum_{a,b} \int_{x_1}^{1} dz_1 \int_{x_2}^{1} dz_2 \, \underbrace{w^{ab}(z_1, z_2, \rho^2)}_{\text{partonic structure fct.}} \underbrace{f_{a/H_1}\left(\frac{x_1}{z_1}\right) f_{b/H_2}\left(\frac{x_2}{z_2}\right)}_{\text{parton distributions}}$$

with

$$\rho^2 = \frac{Q_\mathrm{T}^2}{Q^2},$$

$$x_{1,2} = e^{\pm y} \sqrt{\frac{Q^2 + Q_\mathrm{T}^2}{s}},$$

$$z_{1,2} = \frac{x_{1,2}}{\xi_{1,2}}.$$



Partonic structure functions – Orthogonal basis

Use convenient orthogonal basis:

$$
\begin{aligned}
P^\mu &= (p_1 + p_2)^\mu\,, \\
R^\mu &= (p_1 - p_2)^\mu\,, \\
K^\mu &= k_1^\mu - P^\mu \frac{P\cdot k_1}{P^2} - R^\mu \frac{R\cdot k_1}{R^2} = -q^\mu + P^\mu \frac{P\cdot q}{P^2} + R^\mu \frac{R\cdot q}{R^2}\,,
\end{aligned}
$$

satisfying

$$
P^2 = -R^2 = \hat{s}\,,\; K^2 = -\frac{\hat{u}\hat{t}}{\hat{s}}\,,\; P\cdot R = P\cdot K = R\cdot K = 0\,.
$$

Projectors for T-odd structure functions

Construct projectors on T-odd structure functions:

$$\begin{aligned} w_{\Delta\Delta_P} &= -\frac{1}{2}(X^\mu Y^\nu + X^\nu Y^\mu)\, w_{\mu\nu} \\ &= \frac{z_1 z_2}{4Q^4\,(1+\rho^2)^{3/2}} \left[\epsilon^{\mu PRK}\left(P^\nu z_{12}^+ + R^\nu z_{12}^-\right) + \epsilon^{\nu PRK}\left(P^\mu z_{12}^+ + R^\mu z_{12}^-\right)\right] w_{\mu\nu}\,, \end{aligned}$$

$$\begin{aligned} w_{\Delta_P} &= -\frac{1}{2}(Y^\mu Z^\nu + Y^\nu Z^\mu)\, w_{\mu\nu} \\ &= \frac{z_1 z_2}{4Q^4\,\rho\,(1+\rho^2)^{3/2}} \left[\epsilon^{\mu PRK}\left(P^\nu z_{12}^- + R^\nu z_{12}^+\right) + \epsilon^{\nu PRK}\left(P^\mu z_{12}^- + R^\mu z_{12}^+\right)\right] w_{\mu\nu}\,, \end{aligned}$$

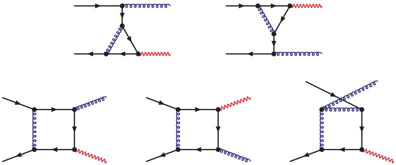
$$w_\nabla = \frac{i}{2}(X^\mu Z^\nu - X^\nu Z^\mu)\, w_{\mu\nu} = \frac{i\rho\, z_1 z_2}{2Q^2\,(1+\rho^2)} \left(P^\nu R^\mu - P^\mu R^\nu\right) w_{\mu\nu}\,.$$

Contributing channels (at 1-loop)

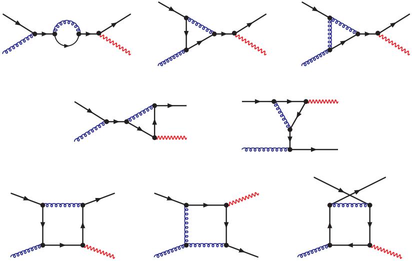
Partonic T-odd structure functions

Only loop diagrams with non-vanishing imaginary parts give contributions.

Quark-quark contribution



Quark-gluon contribution



Imaginary parts of 1-loop integrals

Scalar bubble B_0, triangle C_0, and box D_0:

$$\mathrm{Im}B_0(Q^2) = \mathrm{Im}B_0(\hat{s}) = \pi\,, \quad \mathrm{Im}B_0(\hat{u}) = \mathrm{Im}B_0(\hat{t}) = 0\,,$$

$$\mathrm{Im}C_0(\hat{s},0) = \frac{\pi}{\hat{s}}\left(\frac{1}{\bar{\epsilon}} - \log\frac{\hat{s}}{\mu^2}\right), \quad \mathrm{Im}C_0(\hat{u},0) = \mathrm{Im}C_0(\hat{t},0) = 0\,,$$

$$\mathrm{Im}C_0(Q^2,0) = \frac{\pi}{Q^2}\left(\frac{1}{\bar{\epsilon}} - \log\frac{Q^2}{\mu^2}\right), \quad \mathrm{Im}C_0(Q^2,\hat{s}) = -\frac{\pi}{Q^2-\hat{s}}\log\frac{Q^2}{\hat{s}}\,,$$

$$\mathrm{Im}C_0(Q^2,\hat{u}) = \frac{\pi}{Q^2-\hat{u}}\left(\frac{1}{\bar{\epsilon}} - \log\frac{Q^2}{\mu^2}\right), \quad \mathrm{Im}C_0(Q^2,\hat{t}) = \frac{\pi}{Q^2-\hat{t}}\left(\frac{1}{\bar{\epsilon}} - \log\frac{Q^2}{\mu^2}\right),$$

$$\mathrm{Im}C_0(\hat{s},\hat{u}) = \frac{\pi}{\hat{s}-\hat{u}}\left(\frac{1}{\bar{\epsilon}} - \log\frac{\hat{s}}{\mu^2}\right), \quad \mathrm{Im}C_0(\hat{s},\hat{t}) = \frac{\pi}{\hat{s}-\hat{t}}\left(\frac{1}{\bar{\epsilon}} - \log\frac{\hat{s}}{\mu^2}\right),$$

$$\mathrm{Im}D_0(Q^2,\hat{s},\hat{u}) = -\frac{2\pi}{\hat{s}\hat{u}}\log\frac{Q^2-\hat{u}}{Q^2}\,, \quad \mathrm{Im}D_0(Q^2,\hat{s},\hat{t}) = -\frac{2\pi}{\hat{s}\hat{t}}\log\frac{Q^2-\hat{t}}{Q^2}\,,$$

$$\mathrm{Im}D_0(Q^2,\hat{t},\hat{u}) = \frac{2\pi}{\hat{u}\hat{t}}\left(-\frac{1}{\bar{\epsilon}} + \log\frac{Q^2}{\mu^2} - \log\frac{(Q^2-\hat{u})(Q^2-\hat{t})}{\hat{u}\hat{t}}\right).$$

Analytical results in terms of $z_{1,2}$ and ρ^2

Variables:

$$\hat{s} = \frac{Q^2 + Q_T^2}{z_1 z_2}\,, \quad Q^2 - \hat{t} = \frac{Q^2 + Q_T^2}{z_1}\,, \quad Q^2 - \hat{u} = \frac{Q^2 + Q_T^2}{z_2}\,,$$

$$\frac{\hat{u}\hat{t}}{Q^2\hat{s}} = \rho^2\,, \quad \frac{(Q^2 - \hat{u})(Q^2 - \hat{t})}{Q^2\hat{s}} = 1 + \rho^2\,.$$

For $q\bar{q}$-channel $\left(w^{ab}(z_1, z_2, \rho^2) = \tilde{w}^{ab}(z_1, z_2, \rho^2)\,\delta\big((\hat{s} + \hat{t} + \hat{u} - Q^2)/\hat{s}\big)\right)$:

$$\tilde{w}^{q\bar{q}}_{\Delta\Delta_P} = -\frac{g_{q\bar{q};1}}{4z_1 z_2}\,\frac{1}{\sqrt{1+\rho^2}}\left[C_A\frac{z_1^2 + z_2^2}{2} + C_1\Big(z_1^2 F_1(z_2) + z_2^2 F_1(z_1)\Big)\right],$$

$$\tilde{w}^{q\bar{q}}_{\Delta_P} = -\frac{g_{q\bar{q};1}}{2z_1 z_2}\,\frac{1}{\rho\sqrt{1+\rho^2}}\left[C_A\frac{z_1^2 - z_2^2}{2} + C_1\Big(z_1^2 F_2(z_2) - z_2^2 F_2(z_1)\Big)\right],$$

$$\tilde{w}^{q\bar{q}}_{\nabla} = -\frac{g_{q\bar{q};2}}{z_1 z_2}\,\frac{1}{\rho}\left[\left(C_A - \frac{\rho^2}{1+\rho^2}C_F\right)\frac{z_1^2 - z_2^2}{2} + C_1\Big(z_1 F_2(z_2) - z_2 F_2(z_1)\Big)\right].$$

where $C_1 = C_F - N_c/2 = -1/(2N_c)$.

Analytical results

For qg-channel $\left(w^{ab}(z_1,z_2,\rho^2) = \tilde{w}^{ab}(z_1,z_2,\rho^2)\,\delta\big((\hat{s}+\hat{t}+\hat{u}-Q^2)/\hat{s}\big)\right)$:

$$\begin{aligned}
\tilde{w}^{qg}_{\Delta\Delta_P} &= -\frac{g_{qg;1}}{2}\,\frac{1-z_2}{z_1 z_2}\,\frac{1}{\sqrt{1+\rho^2}}\left[\frac{C_F}{2}z_1\left(1+\frac{2z_1 z_2}{1+\rho^2}\right)\right.\\
&+ \left. C_1 z_1 z_2\left(\left(F_1(z_1)-\frac{1-\rho^2}{1+\rho^2}\right)\frac{z_2}{2}+z_1\log\frac{\rho^2}{1+\rho^2}\right)\right],\\
\tilde{w}^{qg}_{\Delta_P} &= -\frac{g_{qg;1}}{2}\,\frac{1-z_2}{z_1 z_2}\,\frac{1}{\rho\sqrt{1+\rho^2}}\left[\frac{C_F}{2}z_1(1+z_1-2z_2)\right.\\
&+ \left. C_1 z_2\left(1-z_2-\frac{z_1^2 z_2}{1+\rho^2}+z_1^2(1-z_2)\log\frac{\rho^2}{1+\rho^2}\right)\right],\\
\tilde{w}^{qg}_{\nabla} &= -g_{qg;2}\frac{1-z_2}{z_1 z_2}\,\frac{1}{\rho}\left[C_F z_1\left(\frac{z_1(1-z_2)}{1+\rho^2}+\frac{1-z_1}{2z_2}\right)\right.\\
&+ \left. C_1 z_2\left(\frac{z_1^2-2z_2}{1+\rho^2}+z_1(1-z_2)\log\frac{\rho^2}{1+\rho^2}-(1-z_2)F_2(z_1)\right)\right].
\end{aligned}$$

4. Small-Q_T expansion of collinear results

Leading power expansion

Plug partonic results in collinear factorization formula:

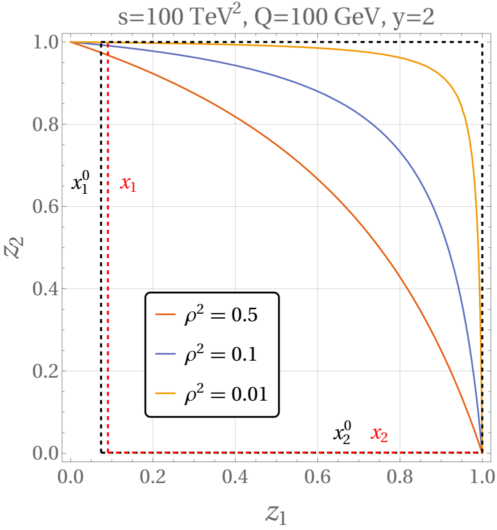
$$W(x_1, x_2, \rho^2) = \frac{1}{x_1 x_2} \sum_{a,b} \int_{x_1}^{1} dz_1 \int_{x_2}^{1} dz_2\, f_{a/H_1}\left(\frac{x_1}{z_1}\right) f_{b/H_2}\left(\frac{x_2}{z_2}\right)$$

$$\times \underbrace{\delta\left((1-z_1)(1-z_2) - \frac{\rho^2}{1+\rho^2} z_1 z_2\right)}_{\text{from 2 particle phase space}} \tilde{w}^{ab}(z_1, z_2, \rho^2)\,.$$

Delta function expansion

$$\delta\left((1-z_1)(1-z_2) - \frac{z_1 z_2 \rho^2}{1+\rho^2}\right) = \frac{\delta(1-z_1)}{(1-z_1)_+} + \frac{\delta(1-z_2)}{(1-z_2)_+}$$
$$-\delta(1-z_1)\delta(1-z_2)\log\rho^2 + \mathcal{O}(\rho^2)$$

Q_T dependence of phase space



Delta function expansion

$$\delta\left((1-z_1)(1-z_2) - \frac{z_1 z_2 \rho^2}{1+\rho^2}\right)$$
$$= \frac{\delta(1-z_1)}{(1-z_1)_+} + \frac{\delta(1-z_2)}{(1-z_2)_+}$$
$$- \delta(1-z_1)\delta(1-z_2)\log\rho^2 + \mathcal{O}(\rho^2)$$

Q_T dependent lower boundary

$$x_{1,2} = e^{\pm y}\sqrt{\frac{Q^2(1+\rho^2)}{s}}$$

$$x_{1,2}^0 = e^{\pm y}\sqrt{\frac{Q^2}{s}}$$

$$x_{1,2} = x_{1,2}^0 + \mathcal{O}(\rho^2)$$

Expansion beyond leading power

$$\delta\left((1-z_1)(1-z_2)-\frac{z_1 z_2 \rho^2}{1+\rho^2}\right)=\frac{\delta(1-z_2)}{(1-z_1)_+}+\frac{\delta(1-z_1)}{(1-z_2)_+}-\log\rho^2\,\delta(1-z_1)\delta(1-z_2)$$
$$+\rho^2\Bigg[\frac{\delta^{(1)}(1-z_2)}{(1-z_1)^2_{+,1}}-\frac{\delta^{(1)}(1-z_2)}{(1-z_1)_+}+\frac{\delta^{(1)}(1-z_1)}{(1-z_2)^2_{+,1}}-\frac{\delta^{(1)}(1-z_1)}{(1-z_2)_+}$$
$$+\frac{\delta(1-z_2)}{(1-z_1)_+}-\frac{\delta(1-z_2)}{(1-z_1)^2_{+,1}}+\frac{\delta(1-z_1)}{(1-z_2)_+}-\frac{\delta(1-z_1)}{(1-z_2)^2_{+,1}}+\delta(1-z_1)\delta(1-z_2)$$
$$+\log\rho^2\Big(-\delta^{(1)}(1-z_1)\delta^{(1)}(1-z_2)+\delta^{(1)}(1-z_1)\delta(1-z_2)$$
$$+\delta(1-z_1)\delta^{(1)}(1-z_2)-\delta(1-z_1)\delta(1-z_2)\Big)\Bigg]+\mathcal{O}(\rho^4\log\rho^2)\,.$$

Generalized plus distributions

$$\int_0^1 \mathrm{d}z\left[\frac{\log^l(1-z)}{(1-z)^m}\right]_{+;k} f(z)=\int_0^1 \mathrm{d}z\,\frac{\log^l(1-z)}{(1-z)^m}\left[f(z)-\mathcal{T}_1^m f(z)\right],$$

where $\mathcal{T}_1^m f(z)$ means the mth order Taylor polynomial of f about $z=1$,

$$\mathcal{T}_1^m f(z)=\sum_{j=0}^{m}\frac{(-1)^j}{j!}(1-z)^j\,\frac{\partial^j f}{\partial z^j}(1)$$

Expansion of lower boundary

- after expansion of delta functions we receive convolutions of the form $I(x) = \int_x^1 \frac{dz}{z}\, w(z)\, q\left(\frac{x}{z}\right)$
- we can expand the convolution about $x_0 \equiv x(\rho^2 = 0)$ using

$$I(x) = \sum_{n=0}^{\infty} \frac{(x - x_0)^n}{n!} \int_{x_0}^{1^+} \frac{dz}{z} \left(\frac{d^n}{dz^n} \left[w(z)\Theta(1-z) \right] \right) \left(\frac{z}{x_0} \right)^n q\left(\frac{x_0}{z} \right).$$

- here derivatives of plus distributions are calculated as

$$\frac{d}{dz} \left[\frac{\log^l(1-z)}{(1-z)^{n+1}} \right]_{+,n} = (n+1) \left[\frac{\log^l(1-z)}{(1-z)^{n+2}} \right]_{+,n+1} - l \left[\frac{\log^{l-1}(1-z)}{(1-z)^{n+2}} \right]_{+,n+1} - \delta_{l0} \sum_{k=0}^{n+1} \frac{1}{k!}\, \delta^{(k)}(1-z).$$

Hadronic results

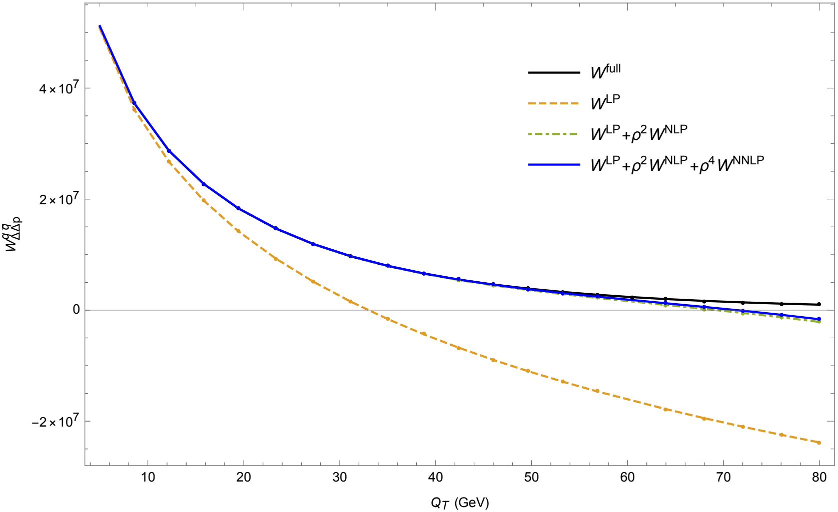
$q\bar{q}$ contribution, leading power:

$$
\begin{aligned}
W^{\mathrm{LP};q\bar{q}}_{\Delta\Delta_P}(x_1^0, x_2^0, \log\rho^2) &= \frac{g_{q\bar{q};1}}{4x_1^0 x_2^0}\, C_A \left(\log\rho^2 + \frac{3}{2}\right) q_1(x_1^0)\, \bar{q}_2(x_2^0) \\
&- \frac{g_{q\bar{q};1}}{4x_1^0 x_2^0}\, \frac{C_A}{2C_F} \left[q_1(x_1^0) \left(P_{qq} \otimes \bar{q}_2\right)(x_2^0) + \left(P_{qq} \otimes q_1\right)(x_1^0)\, \bar{q}_2(x_2^0) \right] \\
&- \frac{g_{q\bar{q};1}}{4x_1^0 x_2^0}\, C_1 \left[q_1(x_1^0) \left(f_1 \otimes \bar{q}_2\right)(x_2^0) + \left(f_1 \otimes q_1\right)(x_1^0)\, \bar{q}_2(x_2^0) \right], \\
W^{\mathrm{LP};q\bar{q}}_{\nabla}(x_1^0, x_2^0, \log\rho^2) &= 2\,\beta\, W^{\mathrm{LP};q\bar{q}}_{\Delta_P}(x_1^0, x_2^0) \\
&= -\frac{g_{q\bar{q};1}}{\rho\, x_1^0 x_2^0}\, \frac{C_A}{2C_F} \left[q_1(x_1^0) \left(\tilde{P}_{qq} \otimes \bar{q}_2\right)(x_2^0) - \left(\tilde{P}_{qq} \otimes q_1\right)(x_1^0)\, \bar{q}_2(x_2^0) \right] \\
&- \frac{g_{q\bar{q};1}}{\rho\, x_1^0 x_2^0}\, C_1 \left[q_1(x_1^0) \left(f_2 \otimes \bar{q}_2\right)(x_2^0) - \left(f_2 \otimes q_1\right)(x_1^0)\, \bar{q}_2(x_2^0) \right].
\end{aligned}
$$

Relation at leading power

$$W^{\mathrm{LP}}_{\nabla} = 2\,\beta\, W^{\mathrm{LP}}_{\Delta_P} \qquad \text{with a constant } \beta = \frac{g_{\mathrm{EW};\,1}}{g_{\mathrm{EW};\,2}}$$

Small-Q_T expansion of $W_{\Delta\Delta p}$

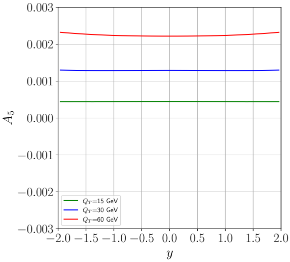


5. Comparison to ATLAS data

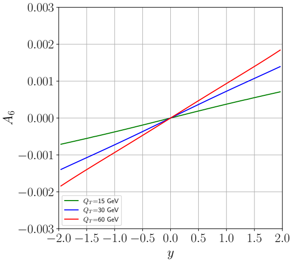
T-odd angular coefficients $A_{5,6,7}$ at various Q_T

as functions of lepton pair rapidity y at $Q \sim M_Z$, $\sqrt{s} = 8\,\mathrm{TeV}$:

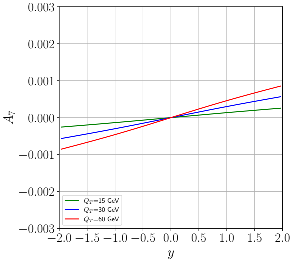
$$A_5(y) \sim \frac{W_{\Delta\Delta_P}}{2W_T + W_L}$$



$$A_6(y) \sim \frac{W_{\Delta_P}}{2W_T + W_L}$$



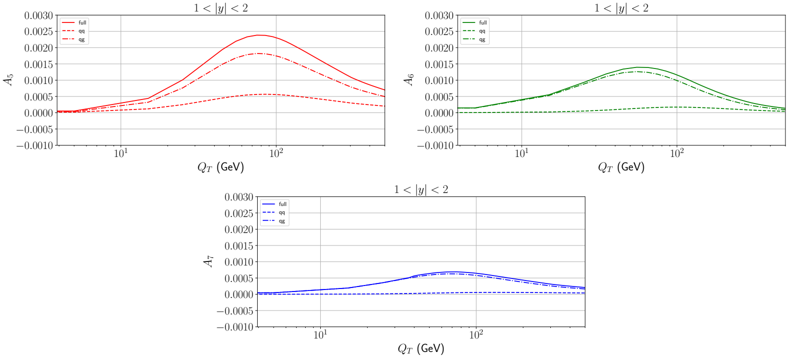
$$A_7(y) \sim \frac{W_{\nabla}}{2W_T + W_L}$$



- A_5 roughly constant in y
- A_6 and A_7 growing linearly with y

Transeverse momentum dependence of $A_{5,6,7}$

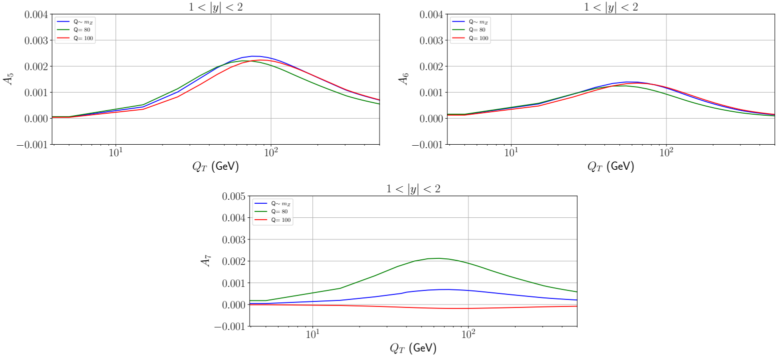
Relative importance of $q\bar{q}$ vs. qg at $Q \sim M_Z$:



- dominant contribution from qg channel

Transverse momentum dependence of $A_{5,6,7}$

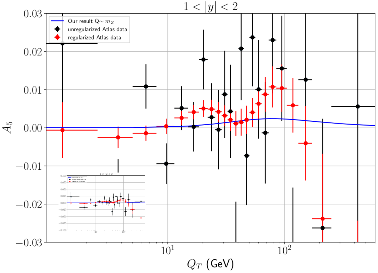
Q dependence of the Q_T spectrum around $Q \sim M_Z$:



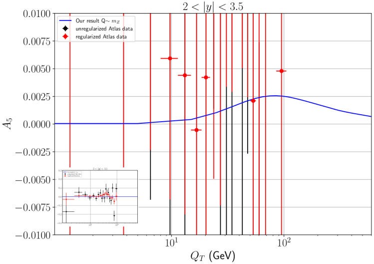
- moderate Q dependence of A_5 and A_6
- large Q dependence for A_7, sign change between $Q = 80\,\text{GeV}$ and $100\,\text{GeV}$

Comparison of A_5 with ATLAS data [ATLAS 2016]

Rapidity dependence of the Q_T spectrum at $Q \sim M_Z$:



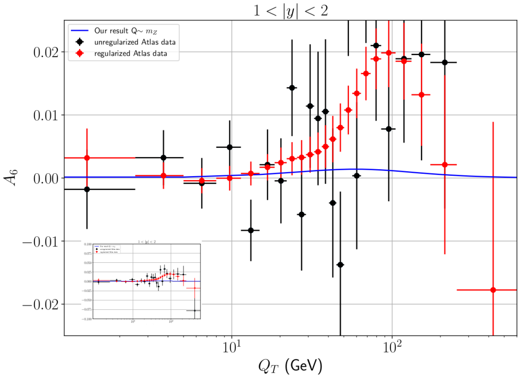
small y



large y

Comparison of A_6 with ATLAS data [ATLAS 2016]

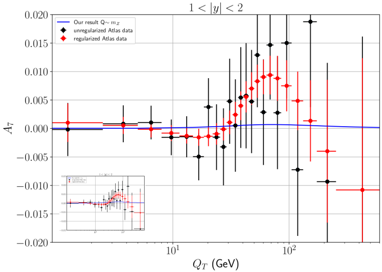
Rapidity dependence of the Q_T spectrum at $Q \sim M_Z$: (no data available for larger y yet)



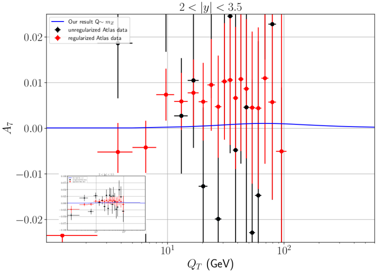
small y

Comparison of A_7 with ATLAS data [ATLAS 2016]

Rapidity dependence of the Q_T spectrum at $Q \sim M_Z$:



small y



large y

6. Conclusion and Outlook

Conclusion

- performed study of perturbative T-odd effects in charged & neutral current DY at order α_s^2
- T-odd effects induced by imaginary parts of loop integrals from hard matrix elements
- Q_{T} expansion for matching to small-Q_{T} TMD factorization
- NLP (i.e. $\sim Q_{\mathrm{T}}^2/Q^2$) contribution numerically significant for intermediate Q_{T}
- data available only for Z-boson near $Q = M_Z$, agreement regarding non-vanishing of T-odd effects, beyond that visually bad agreement between data and theory prediction