Gluon Double-Spin Asymmetry at Small-x and k_T-Factorization

Ming Li

The Ohio State University

Y. Kovchegov and M. Li, JHEP **05** (2024) 177.

Outline

- Introduction and Motivation
- Gluon double-spin asymmetry at Small-x in Gluon+Proton Collisions
- Generalization of gluon double-spin asymmetry to Proton+Proton Collisions
- K_T-Factorization and Small-x Helicity Evolution

Origin of Nucleon Spin

Jaffe-Manohar spin sum rule for proton

The RHIC Spin Collaboration (2015)

$$S_q + L_q + S_G + L_G = \frac{1}{2}$$

Quark Spin

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2)$$

$S_q(Q^2 = 10\text{GeV}^2) \approx [0.15,\ 0.20]$

$x \in [0.001, 0.7]$

Gluon Spin

$$S_G(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

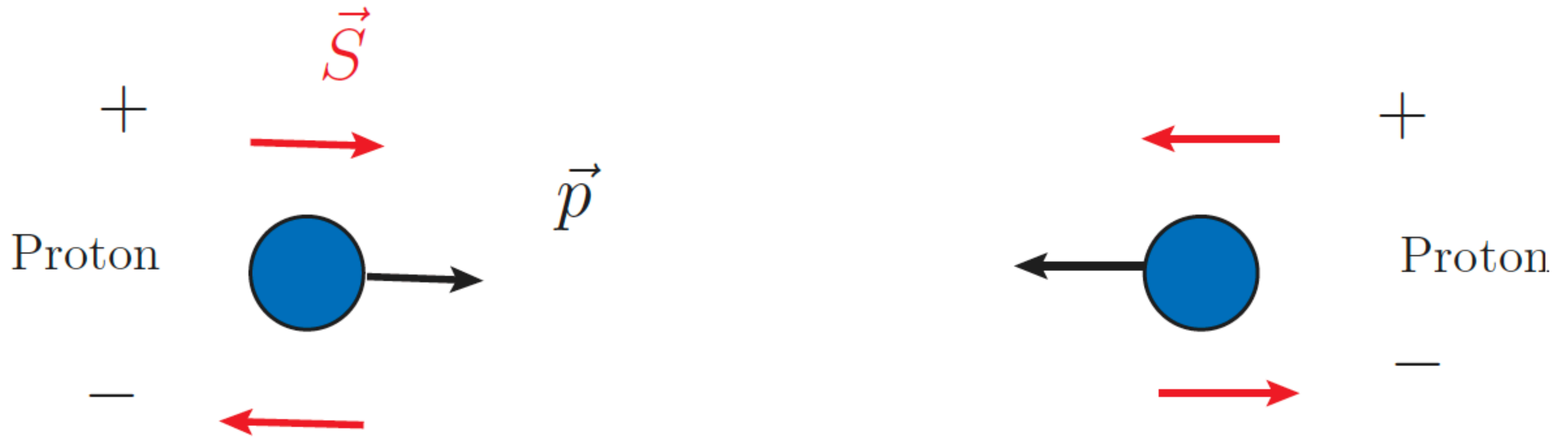
$S_G(Q^2 = 10\text{GeV}^2) \approx [0.13,\ 0.26]$

$x \in [0.05, 0.7]$

Missing spin of the proton maybe in quark and gluon orbital angular momentum L_q and L_G and/or smaller values of x

Longitudinal Double-Spin Asymmetry

How to measure quark and gluon intrinsic spin inside a proton?



$$A_{\rm LL} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

RHIC has measured $A_{\rm LL}$ for the productions of jets, dijets, π^0, $\pi^{\pm}$, direct photons…
at mid-rapidity, intermediate rapidity, forward rapidity…
at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$ and $\sqrt{s_{NN}} = 510\,\mathrm{GeV}$

RHIC Spin Collaboration, arXiv: 2302.00605

Longitudinal Double-Spin Asymmetry

Longitudinal double-spin asymmetry is related to parton helicity distribution.

$$A_{\mathrm{LL}} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

Collinear Factorization (also parity invariance)

Babcock, Monsay and Sivers (1979),
(DSSV) De Florian, Sassot, Stratmann and Vogelsang (2008)(2014)

$$d\Delta\sigma = \sum_{ab} \int dx_a \int dx_b \, \Delta f_a(x_a, Q^2) \, \Delta f_b(x_b, Q^2) \, d\Delta\hat{\sigma}_{ab}(x_a, x_b, p_T, \alpha_s(Q^2), p_T/Q)$$

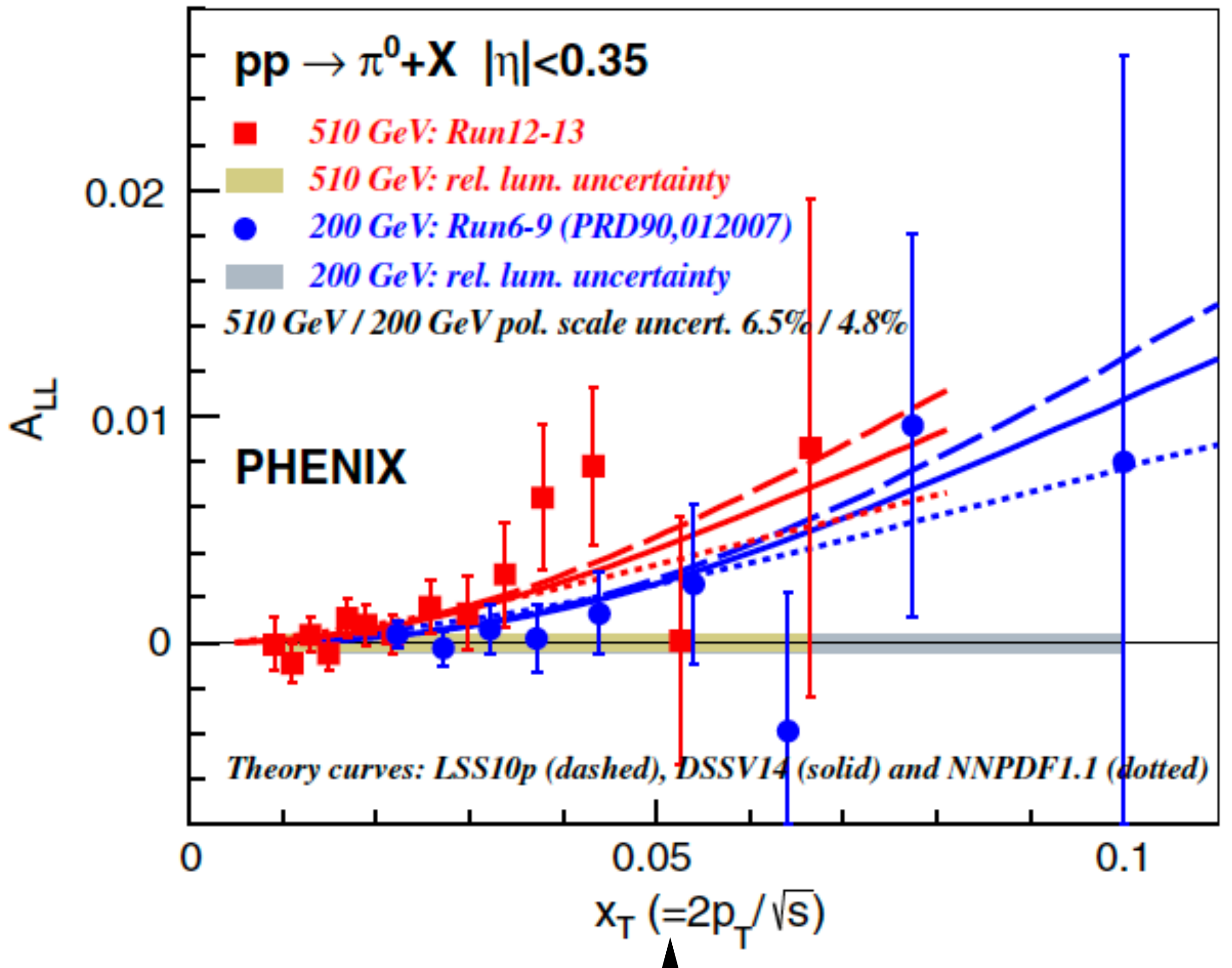
(Anti) quark and gluon helicity distribution $\Delta f_j(x, Q^2) \equiv f_j^+(x, Q^2) - f_j^-(x, Q^2)$

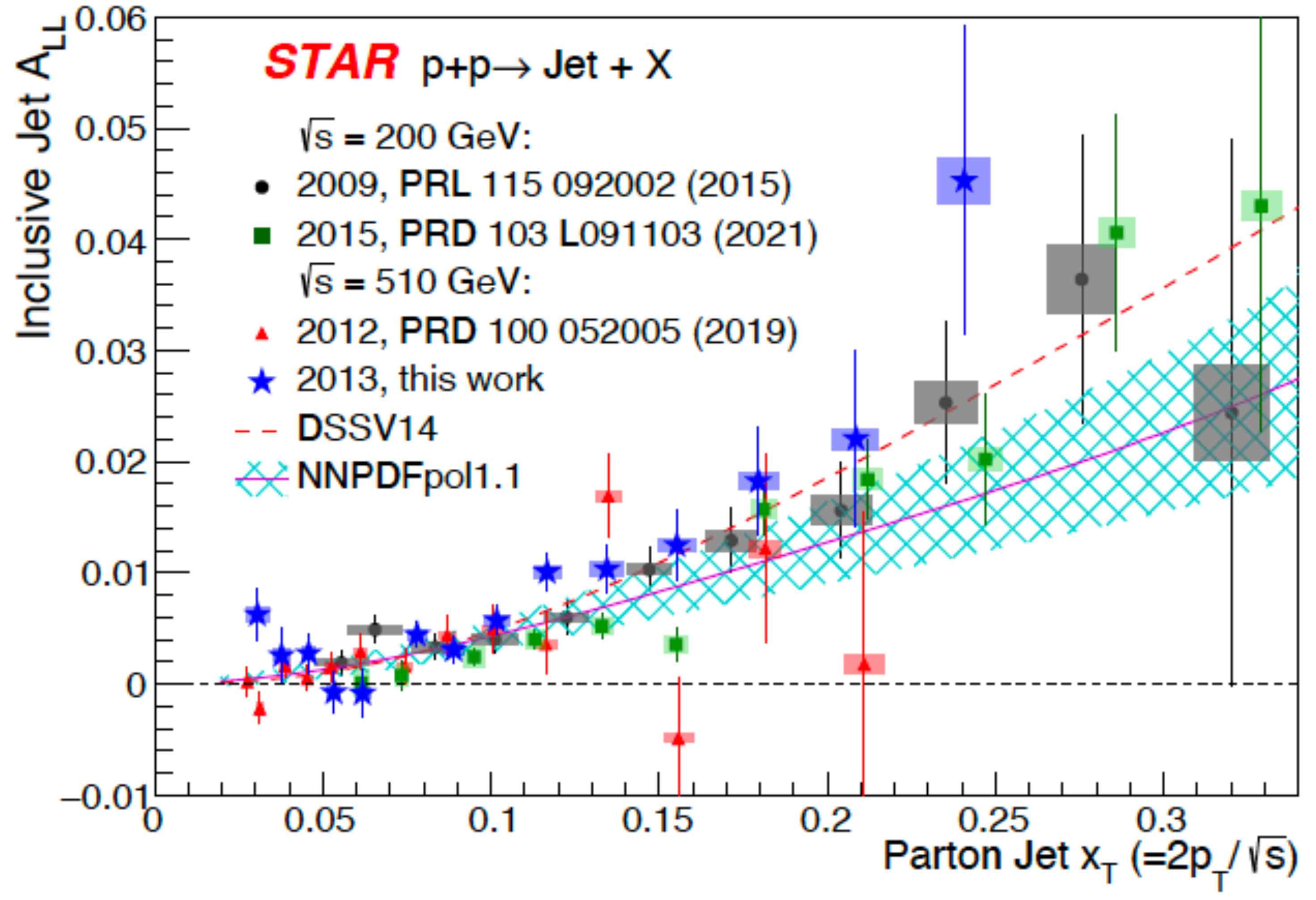
Partonic level double-spin asymmetry $d\Delta\hat{\sigma} = d\hat{\sigma}^{++} - d\hat{\sigma}^{+-}$

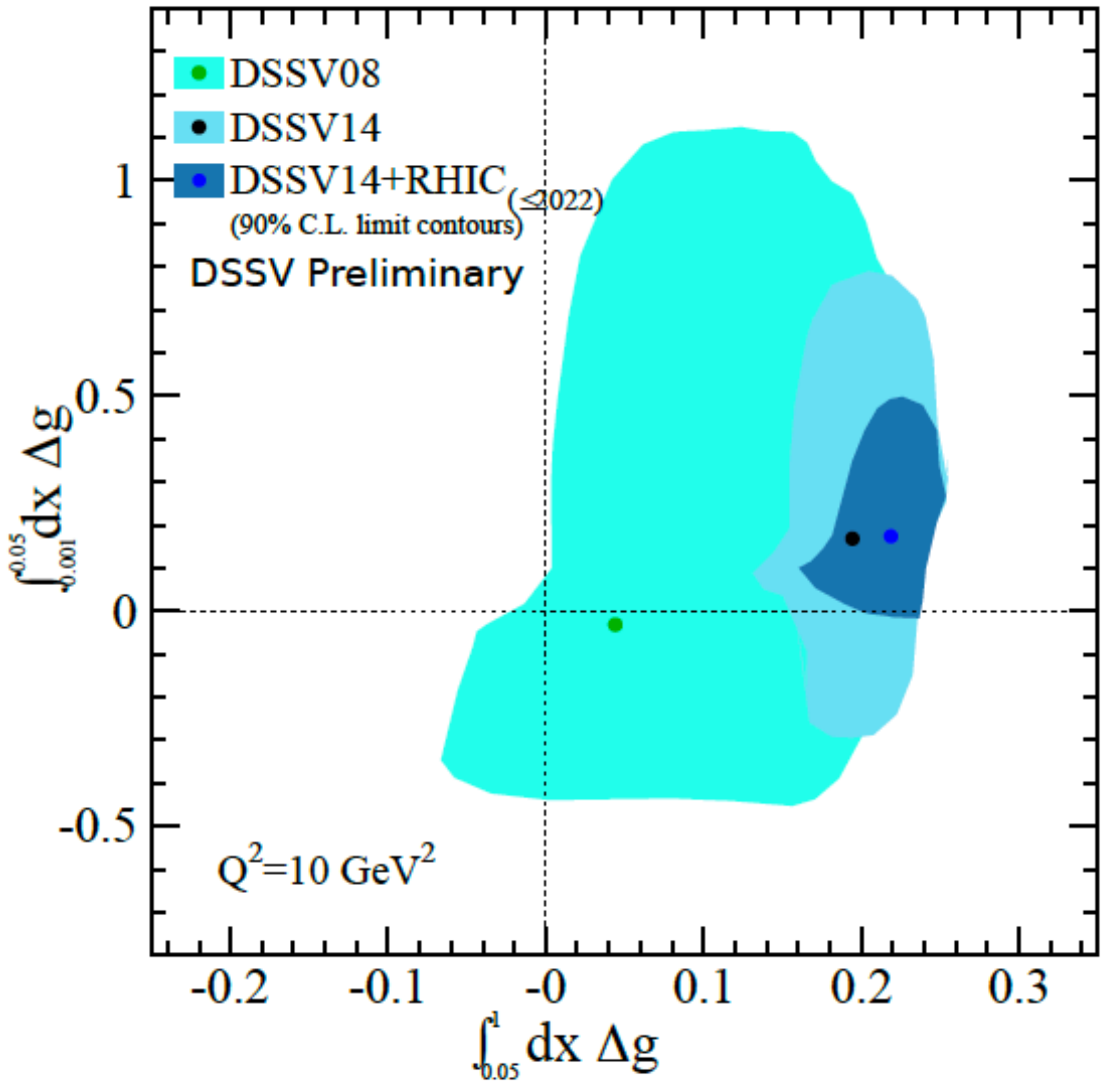
Longitudinal Double-Spin Asymmetry at small x

RHIC Spin Collaboration (2015, 2023)

A_{LL} **for inclusive neutral pion and inclusive jet productions at mid-rapidity**







5 GeV (12.5 GeV)

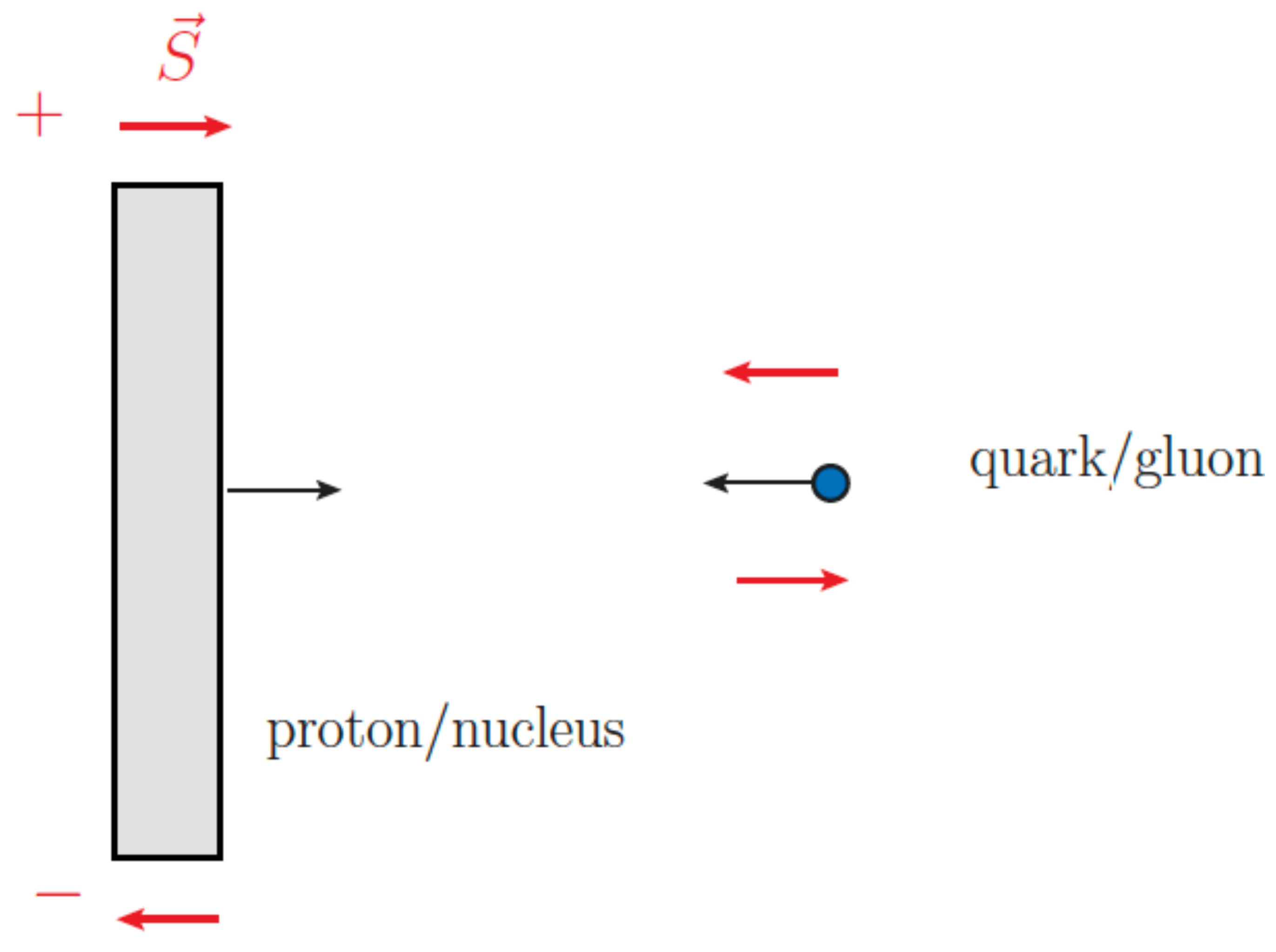
Low transverse momentum region, sensitive to small x gluons, collinear factorization probably breaks down.

very large uncertainty in constraining the small-x region of gluon helicity PDF using the collinear factorization formalism.

Transverse momentum dependent framework + Small-x helicity evolution equations, to describe A_{LL} in the low transverse momentum region and to constrain gluon helicity at smaller values of x.

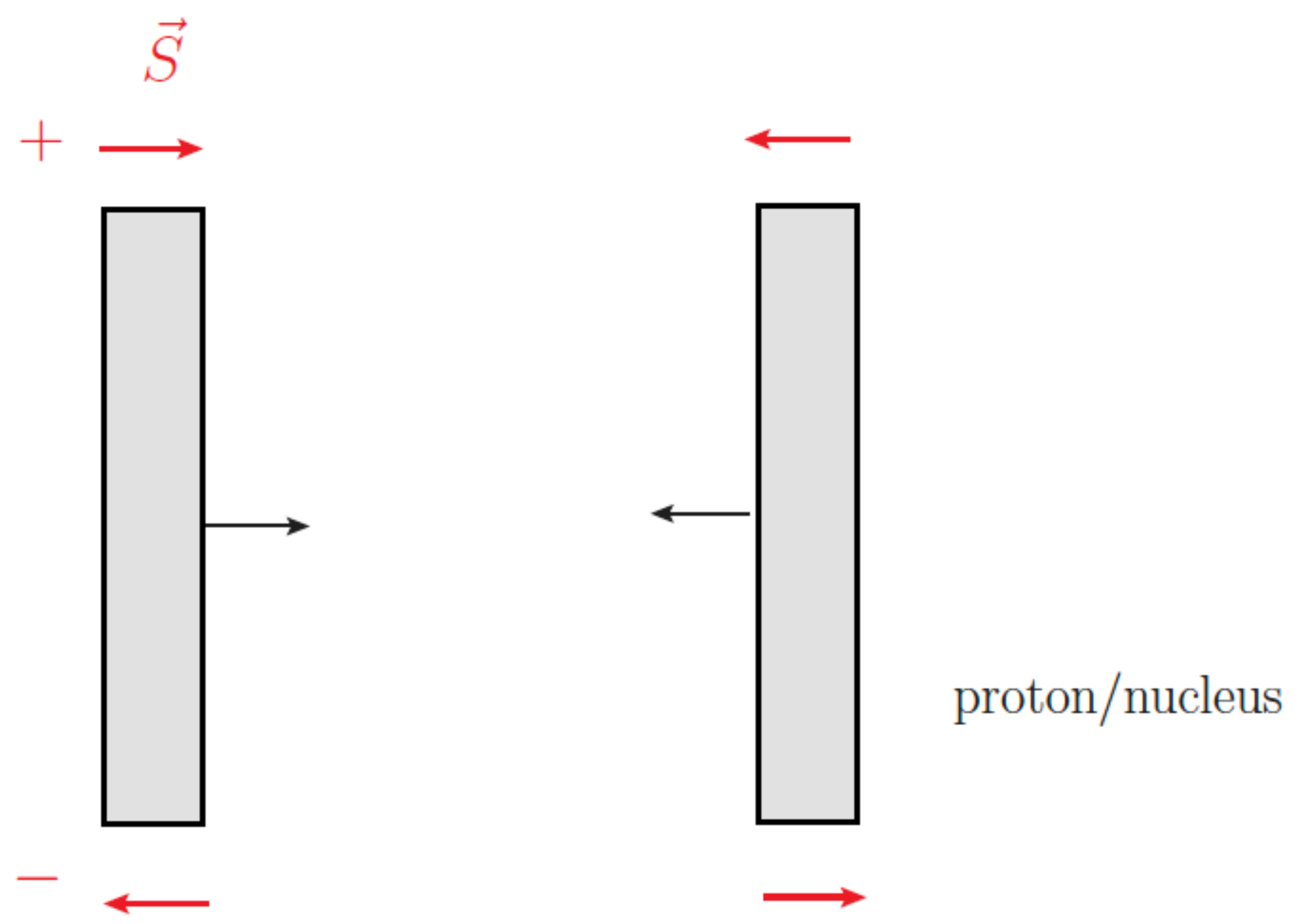
Gluon Double-Spin Asymmetry at Mid-Rapidity

Goal: A_{LL} at small-x for Gluon production at mid-rapidity



Gluon + Proton ⟶ Gluon + X

Generalization



Proton + Proton ⟶ Gluon + X

$$A_{\mathrm{LL}} \equiv \frac{d\Delta\sigma}{d\sigma} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

What we calculate

The leading order unpolarized gluon production at small-x has already been calculated.

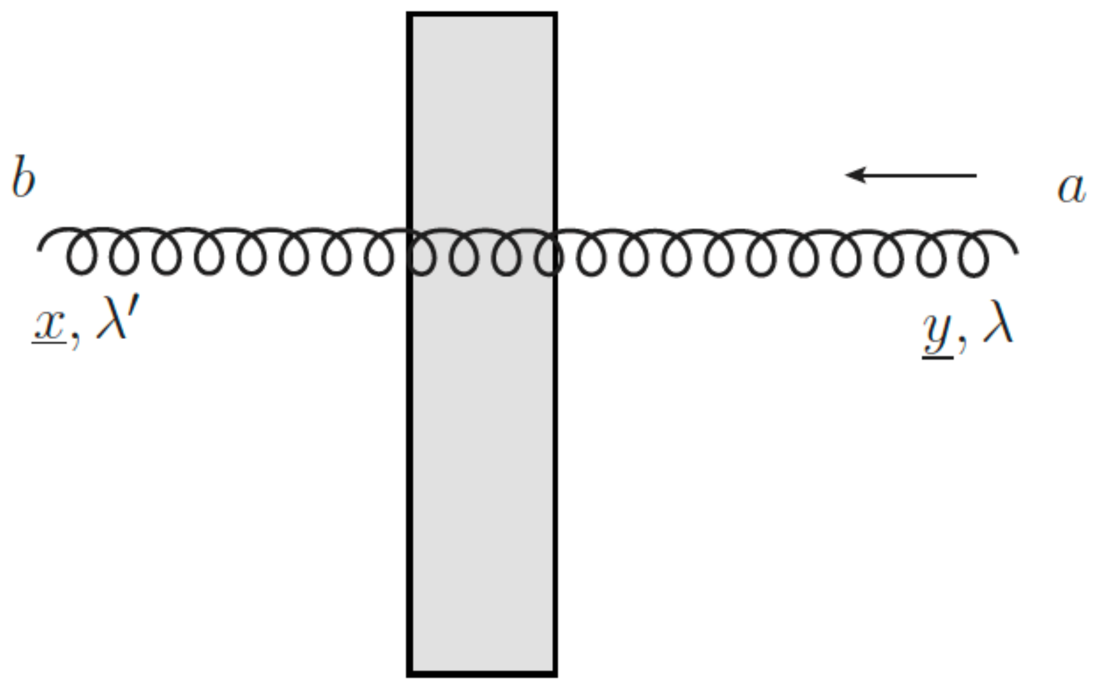
Kovchegov and Mueller (1998), Kopeliovich, Tarasov and Schafer(1999), Dumitru and McLerran (2002)

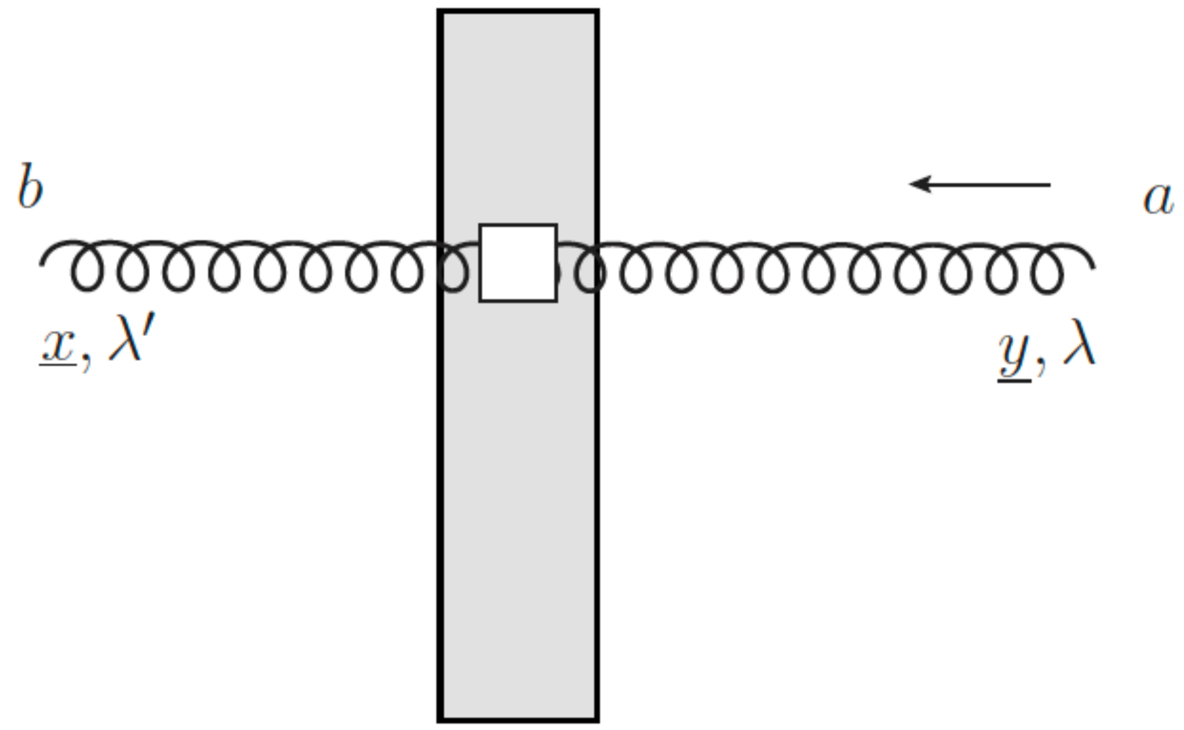
Wilson Lines at Sub-eikonal Order

We use the shockwave formalism for small-x physics and extend the analysis to sub-eikonal order for spin physics.

$$(U_{\underline{x},\underline{y};\lambda',\lambda})^{ba} \equiv (U_{\underline{x}})^{ba}\delta^{(2)}(\underline{x}-\underline{y})\delta_{\lambda,\lambda'} + \lambda\delta_{\lambda,\lambda'}(U_{\underline{x}}^{G[1]})^{ba}\delta^{(2)}(\underline{x}-\underline{y}) + \delta_{\lambda,\lambda'}(U_{\underline{x},\underline{y}}^{G[2]})^{ba} + \mathcal{O}\left(\frac{1}{s^2}\right)$$

Kovchegov, Chirilli, Altinoluk, Beuf….





$$U_{\underline{x}}[x_f^-, x_i^-] = \mathcal{P}\exp\left[ig\int_{x_i^-}^{x_f^-} dx^- \mathcal{A}^+(0^+, x^-, \underline{x})\right]$$

Chromomagnetic Field

$$U_{\underline{x}}^{G[1]} = \frac{2igp_1^+}{s}\int_{-\infty}^{\infty} dx^- U_{\underline{x}}[\infty, x^-]\mathcal{F}^{12}(x^-, \underline{x})U_{\underline{x}}[x^-, -\infty]$$

$$U_{\underline{x},\underline{y}}^{G[2]} = -\frac{ip_1^+}{s}\int_{-\infty}^{\infty} dz^- d^2z\, U_{\underline{x}}[\infty, z^-]\delta^2(\underline{x}-\underline{z})\overleftarrow{\underline{\mathcal{D}}}(z^-, \underline{z})\overrightarrow{\underline{\mathcal{D}}}(z^-, \underline{z})\delta^2(\underline{y}-\underline{z})U_{\underline{y}}[z^-, -\infty]$$

Background fields

Eikonal Order: $\mathcal{A}^+(x^-, \underline{x})$

Subeikonal Order: $\mathcal{A}^+(x^-, \underline{x}),\ \mathcal{A}^i(x^-, \underline{x})$

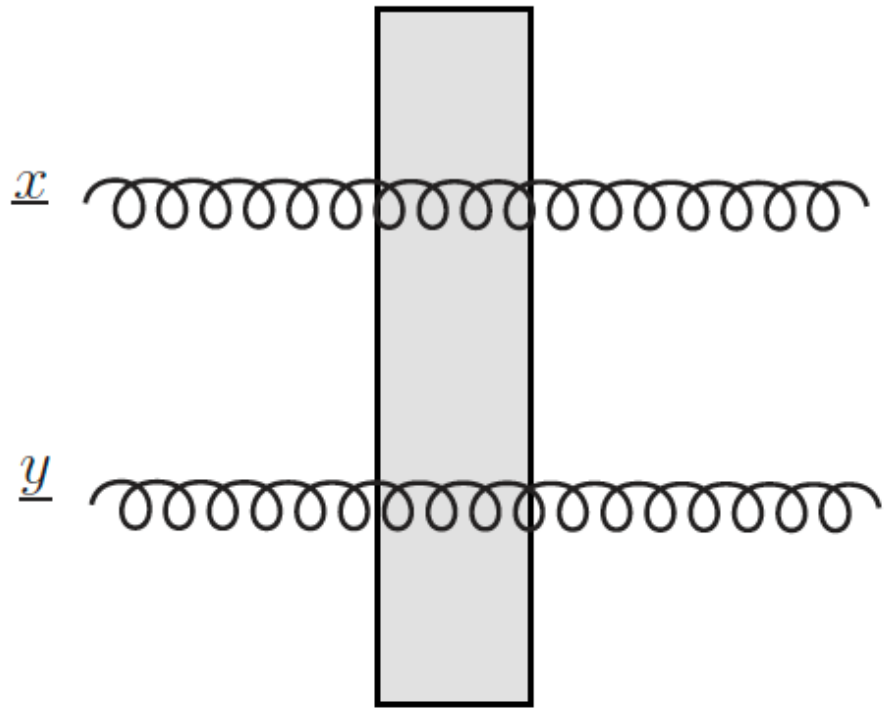
$$\Rightarrow\quad U_{\underline{x}}^{i\,\mathrm{G}[2]} \equiv \frac{igp_1^+}{s}\int_{-\infty}^{\infty} dx^- x^-\, U_{\underline{x}}[\infty, x^-]\,\mathcal{F}^{+i}(x^-, \underline{x})\,U_{\underline{x}}[x^-, -\infty]$$

Chromoelectric Field

Cougoulic and Kovchegov (2020), M.Li arXiv:2402.17568

Polarized Wilson Line Correlators

Unpolarized gluon dipole correlator



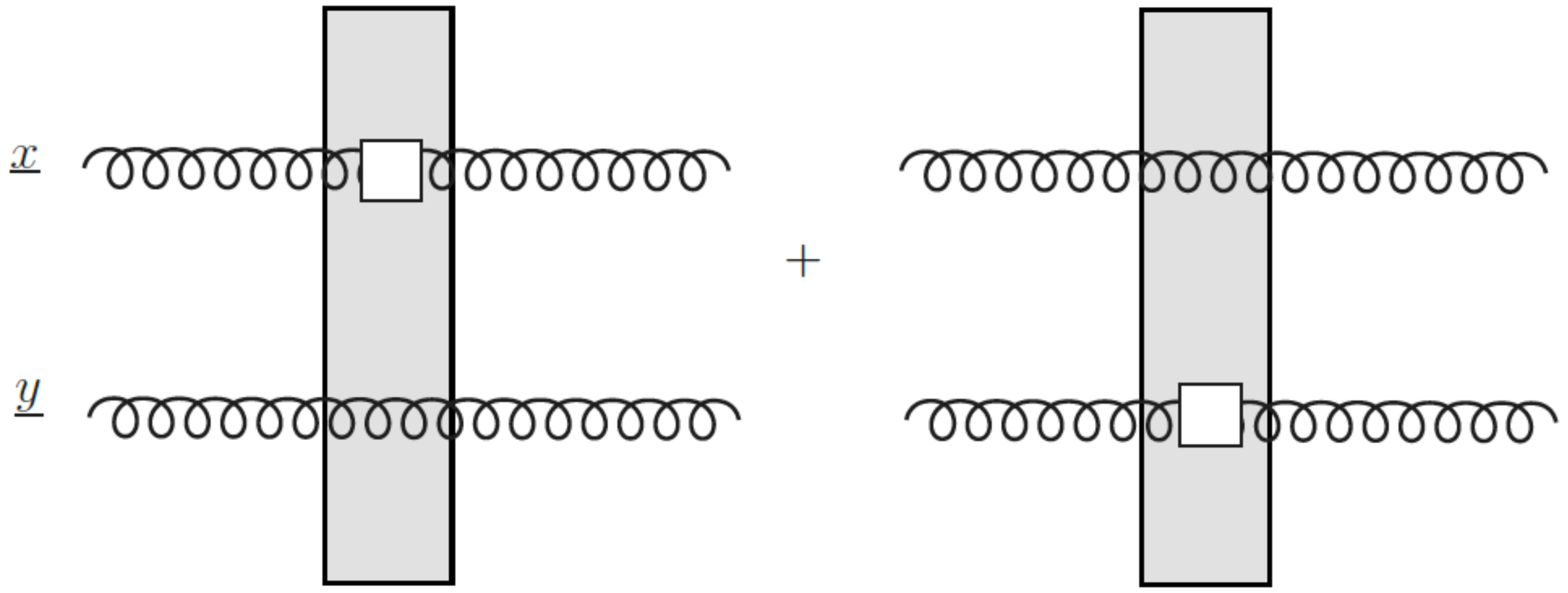
$$D_{\underline{x},\underline{y}} \equiv \frac{1}{(N_c^2-1)} \left\langle \mathrm{Tr}\left[U_{\underline{x}} U_{\underline{y}}^\dagger\right]\right\rangle$$

Averaging under Two-Gluon-Exchange Approximation

$$D_{\underline{x},\underline{y}} \simeq 1 - \frac{1}{2}\frac{\pi \alpha_s^2 N_c}{C_F} |\underline{x}-\underline{y}|^2 \ln \frac{1}{\Lambda^2 |\underline{x}-\underline{y}|^2} + \ldots$$

Quadratically approaches 1 as $\underline{y} \to \underline{x}$

Chromo-electromagnetically polarized gluon dipole correlators



$$G^{\mathrm{adj}}_{\underline{x},\underline{y}}(s) \equiv \frac{1}{2(N_c^2-1)} \left\langle\!\left\langle \mathrm{Tr}\left[U_{\underline{x}}^{\mathrm{G}[1]} U_{\underline{y}}^\dagger\right] + \mathrm{Tr}\left[U_{\underline{x}} U_{\underline{y}}^{\mathrm{G}[1]\,\dagger}\right]\right\rangle\!\right\rangle$$

$$G^{i,\mathrm{adj}}_{\underline{x},\underline{y}}(s) \equiv \frac{1}{2(N_c^2-1)} \left\langle\!\left\langle \mathrm{Tr}\left[U_{\underline{x}}^{i,\mathrm{G}[2]} U_{\underline{y}}^\dagger\right] - \mathrm{Tr}\left[U_{\underline{x}} U_{\underline{y}}^{i,\mathrm{G}[2]\,\dagger}\right]\right\rangle\!\right\rangle$$

$$G^{\mathrm{adj}}_{\underline{x},\underline{y}} \simeq \lambda' \, 2\frac{\pi \alpha_s^2 N_c}{C_F} \ln s |\underline{x}-\underline{y}|^2 + \ldots$$

$$G^{i,\mathrm{adj}}_{\underline{x},\underline{y}} \simeq \lambda' \frac{\pi \alpha_s^2 N_c}{C_F} \epsilon^{ij} (\underline{x}-\underline{y})^j \ln \frac{1}{\Lambda^2 |\underline{x}-\underline{y}|^2} + \ldots$$

Logarithmically/linearly approaches 0 as $\underline{y} \to \underline{x}$

Relating to Gluon TMDs at Small-x

Polarized Wilson line correlators are related to the small-x limit of various gluon helicity TMDs.

$$\Gamma^{\mu\nu;\rho\sigma}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle P,S|\mathrm{Tr}\left[F^{\mu\nu}(0)\,\mathcal{U}^{[+]}(0,\xi)\,F^{\rho\sigma}(\xi)\,\mathcal{U}^{[-]}(\xi,0)\right]|P,S\rangle$$

Mulders and Rodrigues (2001)

$\mu\nu;\rho\sigma = +i;+j$

$$\int dk^- \, \Gamma^{+i;+j}(k,P,S_L) = \frac{i}{4} x P^+ S_L \epsilon^{ij} g_{1L}^G(x,k_T^2)$$

$x \to 0$

$$g_{1L}^G(x,k_T^2) = -\frac{N_c}{\alpha_s 4\pi^4} i\epsilon^{ij}\underline{k}^i \int d^2\xi d^2\zeta e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} G^j_{\underline{\xi},\underline{\zeta}}(s)$$

Dipole Gluon helicity TMD

Cougoulic, Kovchegov, Tarasov and Tawabutr (2022)

$\mu\nu;\rho\sigma = ij;l+$

$$\int dk^- \, \Gamma^{ij;l+}(k;P,S_L) = -\frac{i}{4} S_L \,\epsilon^{ij}\, k^l \,\Delta H_{3L}^{\perp}(x,k_T^2)$$

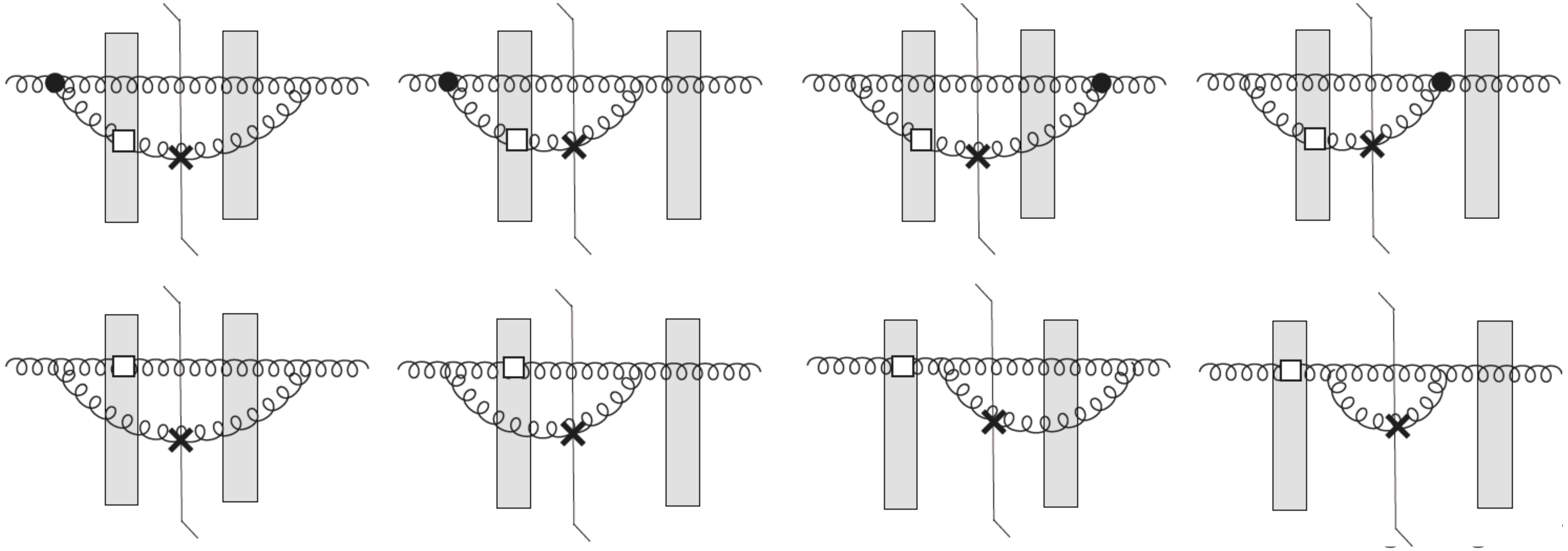
$x \to 0$

$$\Delta H_{3L}^{\perp}(x,k_T^2) = \frac{N_c}{\alpha_s 4\pi^4} \int d^2\xi d^2\zeta \, e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} G_{\underline{\xi},\underline{\zeta}}(s)$$

Twist-3 gluon helicity-flip TMD

Gluon + Proton $\longrightarrow$ Gluon + X

The calculation is performed in transverse coordinate space.



Black dot: Subeikonal order gluon splitting wavefunction.

Gluon momentum $p_2 = (0^+, p_2^-, \underline{0})$

Proton momentum $p_1 = (p_1^+, 0^-, \underline{0})$

$$\beta = \frac{k^-}{p_2^-}, \quad \alpha = \frac{k^+}{p_1^+}$$

impact parameter

$$\int d^2b\, G^{\text{adj}}_{\underline{b},\underline{b}-\underline{x}}(\beta s) = G^{\text{adj}}(x_\perp^2, \beta s),$$

$$\int d^2b\, G^{i,\text{adj}}_{\underline{b},\underline{b}-\underline{x}}(\beta s) = x^i\, G_1^{\text{adj}}(x_\perp^2, \beta s) + \epsilon^{ij}\, x^j\, G_2^{\text{adj}}(x_\perp^2, \beta s).$$

$$\frac{d\sigma(\lambda)}{d^2k_T dy} = \lambda \frac{\alpha_s N_c}{\pi^4} \frac{1}{s} \int d^2x\, d^2y\, d^2b e^{-i\underline{k}\cdot(\underline{x}-\underline{y})} \Bigg\{ \frac{\underline{x}-\underline{b}}{|\underline{x}-\underline{b}|^2} \cdot \frac{\underline{y}-\underline{b}}{|\underline{y}-\underline{b}|^2} \Bigg[\Big(G^{\text{adj}}_{\underline{x},\underline{y}}(\beta s) - G^{\text{adj}}_{\underline{x},\underline{b}}(\beta s) \Big)$$

$$- \frac{1}{4} \Big(G^{\text{adj}}_{\underline{b},\underline{y}}(\beta s) + G^{\text{adj}}_{\underline{b},\underline{x}}(\beta s) \Big) \Bigg] - 2i\, k^i \frac{\underline{x}-\underline{b}}{|\underline{x}-\underline{b}|^2} \times \frac{\underline{y}-\underline{b}}{|\underline{y}-\underline{b}|^2}\, G^{i\,\text{adj}}_{\underline{x},\underline{b}}(\beta s) \Bigg\}$$

$$\frac{d\sigma(\lambda)}{d^2k_T\, dy} = \lambda \frac{2\alpha_s N_c}{\pi^3} \frac{1}{s} \int d^2x e^{-i\underline{k}\cdot\underline{x}} \left[\ln\left(\frac{1}{x_\perp \Lambda}\right) G^{\text{adj}}(x_\perp^2, \beta s) - i \frac{\underline{x}}{|\underline{x}|^2} \cdot \frac{\underline{k}}{|\underline{k}|^2} \left(\frac{3}{2}\, G^{\text{adj}}(x_\perp^2, \beta s) + 2\, G_2^{\text{adj}}(x_\perp^2, \beta s) \right) \right]$$

Sanity Check: Leading Perturbative Result

We calculated A_{LL} for gluon production in *Gluon+Proton* collisions:

$$\frac{d\sigma(\lambda)}{d^2k_T\,dy} = \lambda \frac{2\alpha_s N_c}{\pi^3}\frac{1}{s}\int d^2x e^{-i\underline{k}\cdot\underline{x}}\left[\ln\left(\frac{1}{x_\perp \Lambda}\right) G^{\text{adj}}(x_\perp^2,\beta s) - i\frac{\underline{x}}{|\underline{x}|^2}\cdot\frac{\underline{k}}{|\underline{k}|^2}\left(\frac{3}{2}G^{\text{adj}}(x_\perp^2,\beta s) + 2\,G_2^{\text{adj}}(x_\perp^2,\beta s)\right)\right]$$

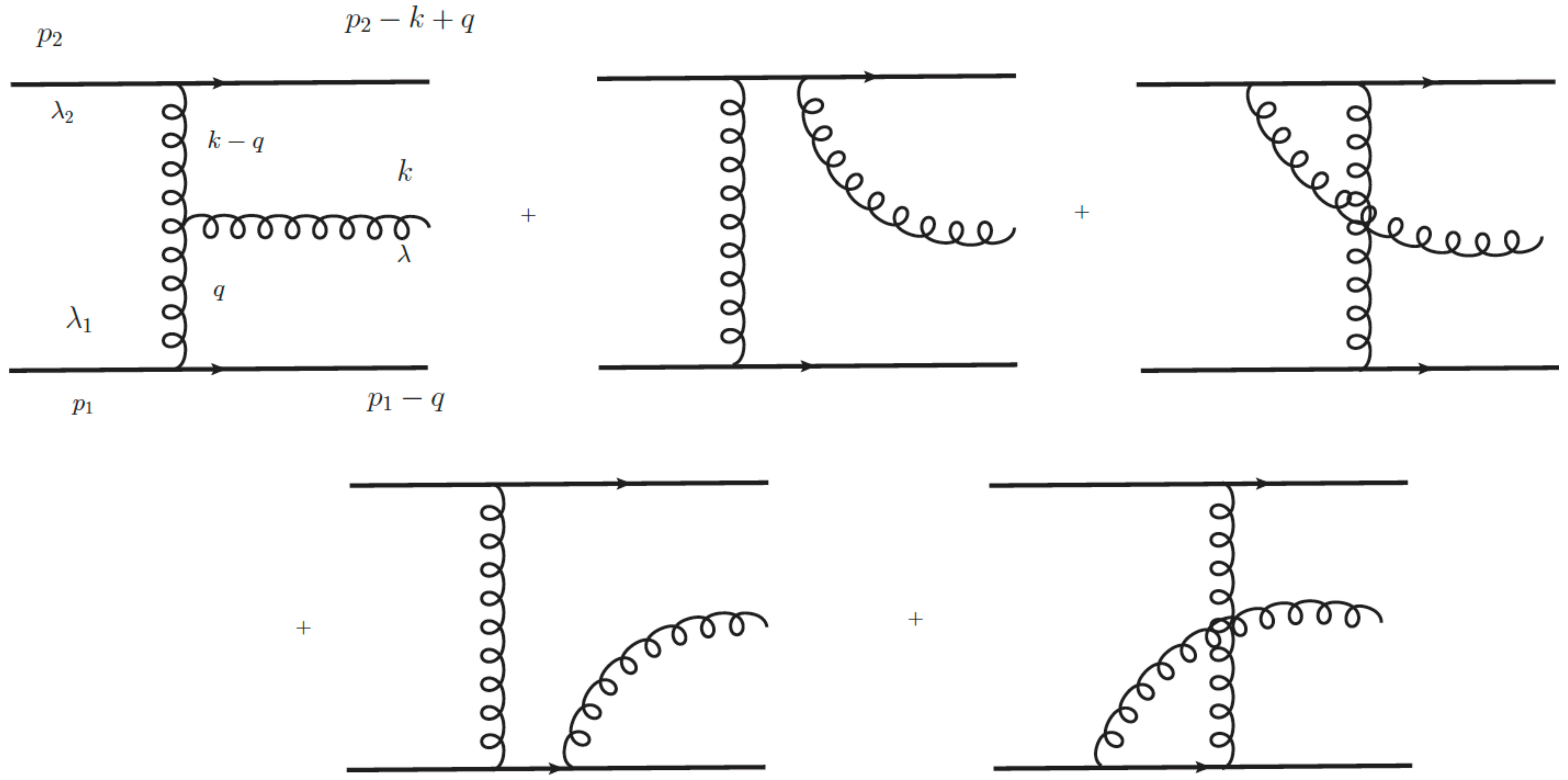
How to obtain A_{LL} for gluon production in *Gluon+Gluon* collisions?

Cougoulic and Kovchegov (2020)

Use the Born level expressions:

$$G^{\text{adj}\,(0)}(x_\perp^2,\beta s) = 2\,\alpha_s^2\,\pi\,\frac{N_c}{C_F}\,\ln(\beta s\, x_\perp^2),$$

$$G_2^{\text{adj}\,(0)}(x_\perp^2,\beta s) = \alpha_s^2\,\pi\,\frac{N_c}{C_F}\,\ln\left(\frac{1}{x_\perp^2\,\Lambda^2}\right).$$



$$\frac{d\sigma_{LO}^{GG\to GGG}}{d^2k_T\,dy} = \frac{8\,\alpha_s^3}{\pi}\frac{N_c}{s\,k_T^2}\left\{3\,\ln\frac{k_T^2}{\Lambda^2} + \ln\left(\frac{\min\{\alpha,\beta\}\,s}{\Lambda^2}\right)\right\}$$

$$\frac{d\sigma_{LO,\,\text{unpolarized}}^{GG\to GGG}}{d^2k_T\,dy} = \frac{4\,\alpha_s^3\,N_c^2}{\pi\,C_F}\frac{1}{k_T^4}\ln\frac{k_T^2}{\Lambda^2}$$

→ $A_{LL} \sim \frac{k_T^2}{s}$

Quadratically dependent on k_T at large external transverse momentum

Proton + Proton $\longrightarrow$ Gluon + X

We calculated A_{LL} for gluon production in *Gluon+Proton* collisions:

$$\frac{d\sigma(\lambda)}{d^2k_T\,dy} = \lambda\frac{2\alpha_s N_c}{\pi^3}\frac{1}{s}\int d^2x e^{-i\underline{k}\cdot\underline{x}}\left[\ln\left(\frac{1}{x_\perp\Lambda}\right)G_T^{\text{adj}}(x_\perp^2,\beta s) - i\frac{\underline{x}}{|\underline{x}|^2}\cdot\frac{\underline{k}}{|\underline{k}|^2}\left(\frac{3}{2}G_T^{\text{adj}}(x_\perp^2,\beta s)+2\,G_{2T}^{\text{adj}}(x_\perp^2,\beta s)\right)\right]$$

It is projectile-target asymmetric!!!

For unpolarized gluon production, Kovchegov and Tuchin(2002)

How to obtain A_{LL} for gluon production in *Proton+Proton* collisions?

$$\ln\left(\frac{1}{x_\perp\Lambda}\right)\sim G_{2P}^{\text{adj}\,(0)}(x_\perp^2,\alpha s)$$

$$\frac{\underline{x}^i}{|\underline{x}|^2}\sim c_1\,\partial^i G_P^{\text{adj}(0)}(x_\perp^2,\alpha s)+c_2\,\partial^i G_{2P}^{\text{adj}(0)}(x_\perp^2,\alpha s)$$

Born level expressions for projectile proton:

$$G_P^{\text{adj}\,(0)}(x_\perp^2,\alpha s) = 2\,\alpha_s^2\,\pi\,\frac{N_c}{C_F}\,\ln(\alpha s\,x_\perp^2),$$

$$G_{2P}^{\text{adj}\,(0)}(x_\perp^2,\alpha s) = \alpha_s^2\,\pi\,\frac{N_c}{C_F}\,\ln\left(\frac{1}{x_\perp^2\Lambda^2}\right).$$

Step 0: Integration by parts.

$$\int d^2x e^{-i\underline{k}\cdot\underline{x}}\left[\ln\left(\frac{1}{x_\perp\Lambda}\right)-2i\,\frac{\underline{x}}{|\underline{x}|^2}\cdot\frac{\underline{k}}{|\underline{k}|^2}\right]G^{\text{adj}}(x_\perp^2,\beta s)$$

$$= -\int d^2x\,e^{-i\underline{k}\cdot\underline{x}}\ln\left(\frac{1}{x_\perp\Lambda}\right)\frac{1}{k_T^2}\nabla_\perp^2 G^{\text{adj}}(x_\perp^2,\beta s).$$

$$G_P^{\text{adj}\,(0)}\longrightarrow G_P^{\text{adj}},\quad G_{2P}^{\text{adj}\,(0)}\longrightarrow G_{2P}^{\text{adj}}$$

The final expression should be projectile and target symmetric ($T\leftrightarrow P$).

k_T-Factorization

The A_{LL} for gluon production in *Proton+Proton* collisions:

$$\frac{d\sigma}{d^2 k_T\, dy} = \frac{C_F}{\alpha_s\, \pi^4} \frac{1}{s\, k_T^2} \int d^2x\, e^{-i\underline{k}\cdot\underline{x}} \left(G_P^{\text{adj}}(x_\perp^2, \alpha s) \quad G_{2P}^{\text{adj}}(x_\perp^2, \alpha s) \right) \begin{pmatrix} \frac{1}{4} \overleftarrow{\nabla}_\perp \cdot \overrightarrow{\nabla}_\perp & \overleftarrow{\nabla}_\perp^2 + \overleftarrow{\nabla}_\perp \cdot \overrightarrow{\nabla}_\perp \\ \overrightarrow{\nabla}_\perp^2 + \overleftarrow{\nabla}_\perp \cdot \overrightarrow{\nabla}_\perp & 0 \end{pmatrix} \begin{pmatrix} G_T^{\text{adj}}(x_\perp^2, \beta s) \\ G_{2T}^{\text{adj}}(x_\perp^2, \beta s) \end{pmatrix}$$

In momentum space:

$$\frac{d\sigma}{d^2 k_T\, dy} = -\frac{C_F}{\alpha_s\, \pi^4} \frac{1}{s\, k_T^2} \int \frac{d^2q}{(2\pi)^2} \left(G_P^{\text{adj}}(q_T^2, \alpha s) \quad G_{2P}^{\text{adj}}(q_T^2, \alpha s) \right) \begin{pmatrix} \frac{1}{4} \underline{q} \cdot (\underline{k} - \underline{q}) & \underline{q} \cdot \underline{k} \\ \underline{k} \cdot (\underline{k} - \underline{q}) & 0 \end{pmatrix} \begin{pmatrix} G_T^{\text{adj}}((\underline{k} - \underline{q})^2, \beta s) \\ G_{2T}^{\text{adj}}((\underline{k} - \underline{q})^2, \beta s) \end{pmatrix}$$

In terms of dipole gluon helicity TMD and twist-3 helicity-flip TMD:

$$\frac{d\sigma}{d^2 k_T\, dy} = -\frac{32\pi^4\, \alpha_s}{N_c} \frac{1}{s\, k_T^2} \int \frac{d^2q}{(2\pi)^2} \left(\Delta H_{3L}^{\perp,P}(q_T^2, \tfrac{k_T^2}{\alpha s}) \quad g_{1L}^{G,P}(q_T^2, \tfrac{k_T^2}{\alpha s}) \right) \begin{pmatrix} \underline{q} \cdot (\underline{k} - \underline{q}) & \underline{q} \cdot \underline{k} \\ \underline{k} \cdot (\underline{k} - \underline{q}) & 0 \end{pmatrix} \begin{pmatrix} \Delta H_{3L}^{\perp,T}((\underline{k} - \underline{q})^2, \tfrac{k_T^2}{\beta s}) \\ g_{1L}^{G,T}((\underline{k} - \underline{q})^2, \tfrac{k_T^2}{\beta s}) \end{pmatrix}$$

This equation is only applicable in the small-x regime.

$$\alpha s = 2p_2^- k^+ = \sqrt{2} p_2^- k_T\, e^{-y},$$
$$\beta s = 2p_1^+ k^- = \sqrt{2} p_1^+ k_T\, e^{y}$$

k_T-Factorization

The A_{LL} for gluon production in *Proton+Proton* collisions:

$$\frac{d\sigma}{d^2k_T\,dy} = -\frac{32\pi^4\,\alpha_s}{N_c}\frac{1}{s\,k_T^2}\int\frac{d^2q}{(2\pi)^2}\left(\Delta H_{3L}^{\perp,P}(q_T^2,\tfrac{k_T^2}{\alpha s})\quad g_{1L}^{G,P}(q_T^2,\tfrac{k_T^2}{\alpha s})\right)\begin{pmatrix}\underline{q}\cdot(\underline{k}-\underline{q}) & \underline{q}\cdot\underline{k}\\ \underline{k}\cdot(\underline{k}-\underline{q}) & 0\end{pmatrix}\begin{pmatrix}\Delta H_{3L}^{\perp,T}((\underline{k}-\underline{q})^2,\tfrac{k_T^2}{\beta s})\\ g_{1L}^{G,T}((\underline{k}-\underline{q})^2,\tfrac{k_T^2}{\beta s})\end{pmatrix}$$

1. $\Delta H_{3,L}^{\perp}(k_T^2,s)$ is a pure TMD effect that doesn't contribute to gluon helicity PDF.

$$\int_0^{1/Q^2} d^2k\Delta H_{3L}^{\perp}(k_T^2,s)\approx 0.$$

2. How to understand the 2×2 matrix? Similar to the usual k_T-factorization approach? *Collins and Ellis (1991)*

$$\sigma(s)=\int_0^1 dx_1\int_0^1 dx_2\int d^2k_1d^2k_2\,\mathcal{F}(x_1,\underline{k}_1,\mu)\,\mathcal{F}(x_2,\underline{k}_2,\mu)\,I_2(x_1x_2s,\underline{k}_1,\underline{k}_2).$$

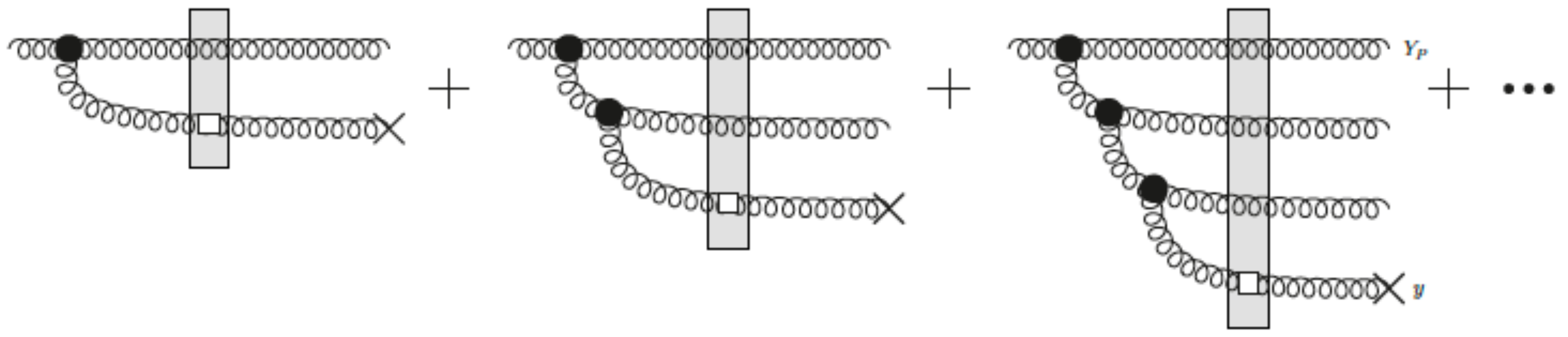
I_2 : Impact factor (Off-shell, gauge-invariant partonic cross-section)

3. Collinear limit? From $2\to 3$ process to $2\to 2$ process?

4. Solving $\Delta H_{3L}^{\perp}(k_T^2,x)$ and $g_{1L}^{G}(k_T^2,x)$ from the small-x helicity evolution equations.

Including Small-x Helicity Evolutions

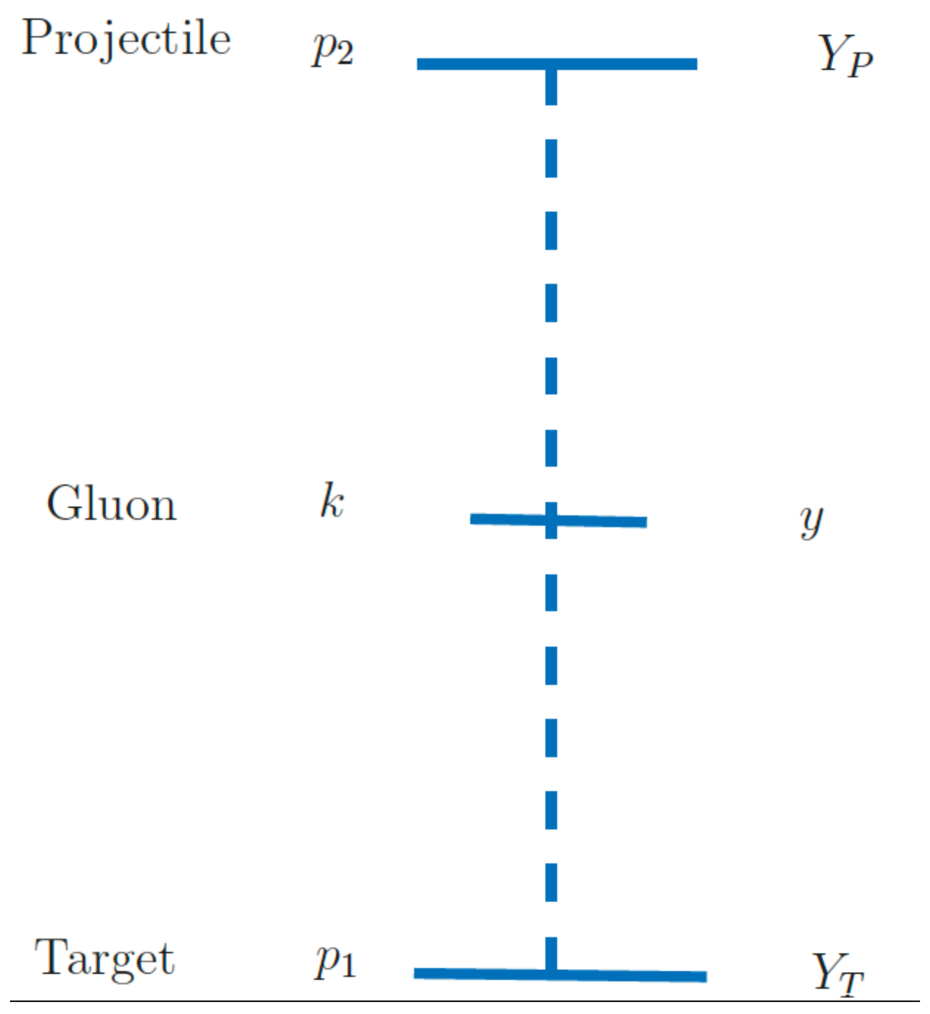
$$\frac{d\sigma}{d^2k_T\,dy} = \frac{C_F}{\alpha_s\,\pi^4}\frac{1}{s\,k_T^2}\int d^2x\, e^{-i\underline{k}\cdot\underline{x}} \left(G_P^{\text{adj}}(x_\perp^2, Y_P - y) \quad G_{2P}^{\text{adj}}(x_\perp^2, Y_P - y)\right) \frac{1}{4}\begin{pmatrix} \overleftarrow{\nabla}_\perp\cdot\overrightarrow{\nabla}_\perp & \overleftarrow{\nabla}_\perp^2 + \overleftarrow{\nabla}_\perp\cdot\overrightarrow{\nabla}_\perp \\ \overrightarrow{\nabla}_\perp^2 + \overleftarrow{\nabla}_\perp\cdot\overrightarrow{\nabla}_\perp & 0 \end{pmatrix} \begin{pmatrix} G_T^{\text{adj}}(x_\perp^2, y - Y_T) \\ G_{2T}^{\text{adj}}(x_\perp^2, y - Y_T) \end{pmatrix}$$



In the double-logarithmic approximation:

$$\alpha_s \ll 1, \qquad \ln\frac{1}{x} \gg 1. \quad \longrightarrow \quad \alpha_s \ln\frac{1}{x} \ll 1, \qquad \boxed{\alpha_s \ln^2\frac{1}{x} \sim 1.}$$

single-logarithmic terms can be discarded.



When $\alpha_s(Y_P - y)^2 \sim 1$, including small-x helicity evolution on the projectile side.

When $\alpha_s(y - Y_T)^2 \sim 1$, including small-x helicity evolution on the target side.

The small-x helicity evolution equations under double-logarithmic approximation, which close at large-N_c, have been derived.

Kovchegov, Pitonyak and Sievert (2015-2019)
Cougoulic, Kovchegov, Tarasov and Tawabutr (2022)

Including Small-x Helicity Evolutions

$$\frac{d\sigma}{d^2 k_T\, dy} = \frac{C_F}{\alpha_s\, \pi^4} \frac{1}{s\, k_T^2} \int d^2 x\, e^{-i\underline{k}\cdot\underline{x}} \begin{pmatrix} G_P^{\mathrm{adj}}(x_\perp^2, Y_P - y) & G_{2P}^{\mathrm{adj}}(x_\perp^2, Y_P - y) \end{pmatrix} \begin{pmatrix} \frac{1}{4} \overleftarrow{\nabla}_\perp \cdot \overrightarrow{\nabla}_\perp & \overleftarrow{\nabla}_\perp^2 + \overleftarrow{\nabla}_\perp \cdot \overrightarrow{\nabla}_\perp \\ \overrightarrow{\nabla}_\perp^2 + \overleftarrow{\nabla}_\perp \cdot \overrightarrow{\nabla}_\perp & 0 \end{pmatrix} \begin{pmatrix} G_T^{\mathrm{adj}}(x_\perp^2, y - Y_T) \\ G_{2T}^{\mathrm{adj}}(x_\perp^2, y - Y_T) \end{pmatrix}$$

In the double-logarithmic approximation, large-N_c and dilute limit:

$$G(x_{10}^2, zs) = G^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{sx_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z's) + 3G(x_{21}^2, z's) + 2G_2(x_{21}^2, z's) + 2\Gamma_2(x_{10}^2, x_{21}^2, z's) \right],$$

Kovchegov, Pitonyak and Sievert (2015-2019)
Cougoulic, Kovchegov, Tarasov and Tawabutr (2022)

$$\Gamma(x_{10}^2, x_{21}^2, z's) = G^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{sx_{10}^2}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z''s}}^{\min[x_{10}^2, x_{21}^2 \frac{z'}{z''}]} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z''s) + 3G(x_{32}^2, z''s) + 2G_2(x_{32}^2, z''s) + 2\Gamma_2(x_{10}^2, x_{32}^2, z''s) \right],$$

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max[x_{10}^2, \frac{1}{z's}]}^{\min[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}]} \frac{dx_{21}^2}{x_{21}^2} \left[G(x_{21}^2, z's) + 2G_2(x_{21}^2, z's) \right],$$

$$\Gamma_2(x_{10}^2, x_{21}^2, z's) = G_2^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int\limits_{\max[x_{10}^2, \frac{1}{z''s}]}^{\min[\frac{z'}{z''} x_{21}^2, \frac{1}{\Lambda^2}]} \frac{dx_{32}^2}{x_{32}^2} \left[G(x_{32}^2, z''s) + 2G_2(x_{32}^2, z''s) \right].$$

$$G^{\mathrm{adj}} = 4G, \quad G_2^{\mathrm{adj}} = 2G_2\,.$$

Γ and Γ_2 have the same operator definition as G and G_2, respectively. But they have different life time ordering constraints.

Conclusions

- We derived the first-ever transverse momentum dependent small-x expression for double-spin asymmetry of gluon production at mid-rapidity in longitudinally polarized proton-proton collisions.

- In the pure glue case, the expression contains dipole gluon helicity TMDs and twist-3 helicity-flip TMDs from both the projectile and the target in a projectile-target symmetric form.

- The expression exhibits k_T- factorization. Together with the small-x helicity evolution equations under double-logarithmic approximation, it can be used to constrain gluon helicity distribution at small-x using experimental data from RHIC on A_{LL} for inclusive jet and neutral pion productions. (ongoing work by Nicholas Baldonado and Matthew Sievert)

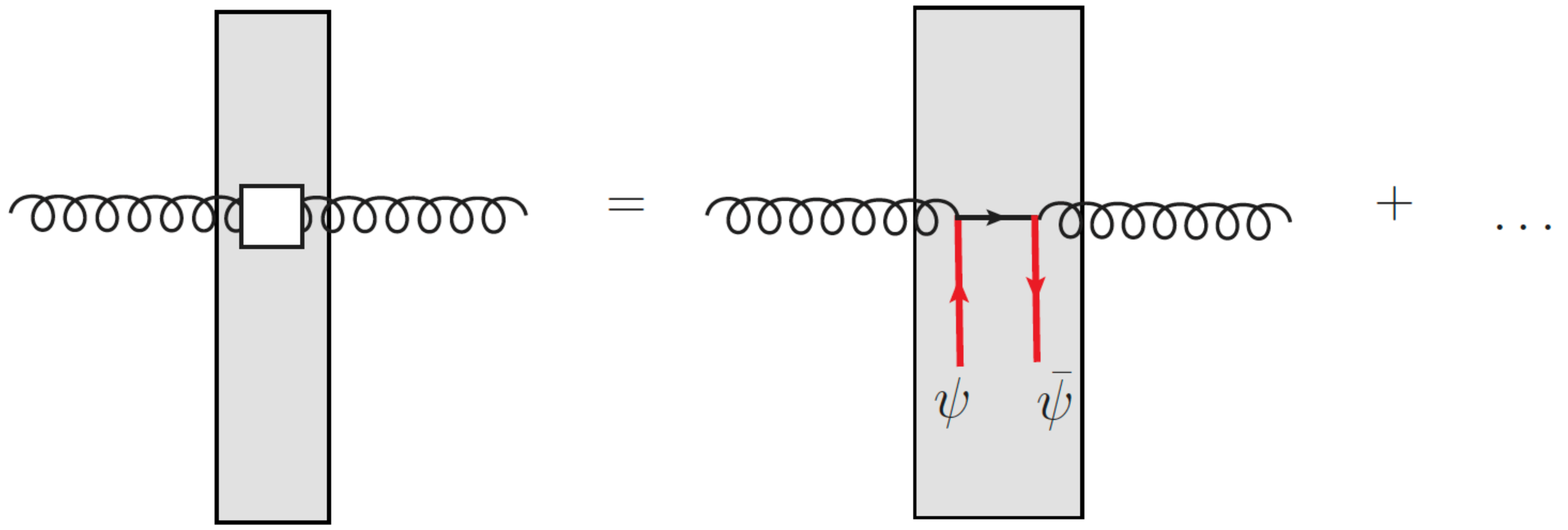
- Including quarks.

Backup

Including Quarks (work in progress)

Scattering amplitudes depend on background (anti) quark fields.

$$(U_{\underline{x},\underline{y};\lambda',\lambda})^{ba} \equiv (U_{\underline{x}})^{ba}\delta^{(2)}(\underline{x}-\underline{y})\delta_{\lambda,\lambda'} + \lambda\delta_{\lambda,\lambda'}\left(U_{\underline{x}}^{G[1]} + U_{\underline{x}}^{q[1]}\right)^{ba}\delta^{(2)}(\underline{x}-\underline{y}) + \delta_{\lambda,\lambda'}\left(U_{\underline{x},\underline{y}}^{G[2]} + U_{\underline{x}}^{q[2]}\delta^{(2)}(\underline{x}-\underline{y})\right)^{ba}$$

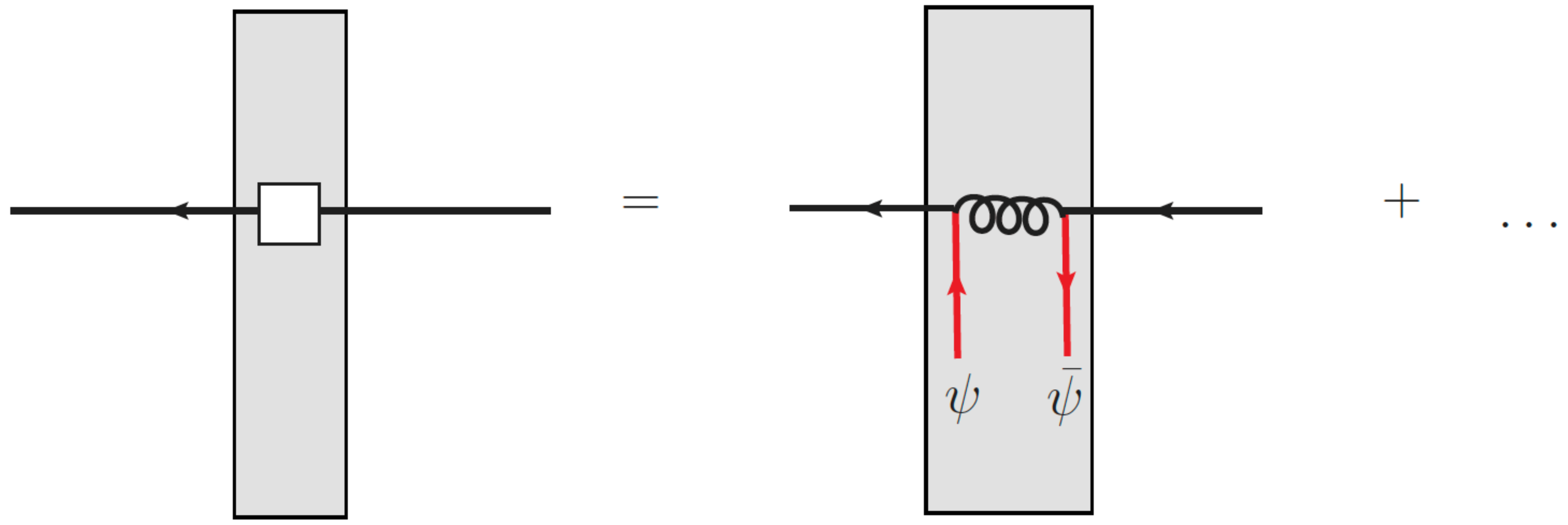


$$(U_{\underline{x}}^{\mathrm{q}[1]})^{ba} = \frac{g^2 p_1^+}{s}\int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \, (U_{\underline{x}}[\infty, x_2^-])^{bb'}\bar{\psi}(x_2^-,\underline{x})t^{b'} V_{\underline{x}}[x_2^-, x_1^-]\frac{\gamma^+\gamma^5}{2}t^{a'}\psi(x_1^-,\underline{x})(U_{\underline{x}}[x_1^-,-\infty])^{a'a} + c.c.$$

$$(U_{\underline{x}}^{\mathrm{q}[2]})^{ba} = -\frac{g^2 p_1^+}{s}\int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \, (U_{\underline{x}}[\infty, x_2^-])^{bb'}\bar{\psi}(x_2^-,\underline{x})t^{b'} V_{\underline{x}}[x_2^-, x_1^-]\frac{\gamma^+}{2}t^{a'}\psi(x_1^-,\underline{x})(U_{\underline{x}}[x_1^-,-\infty])^{a'a} - c.c.$$

We also need the subeikonal order quark Wilson lines:

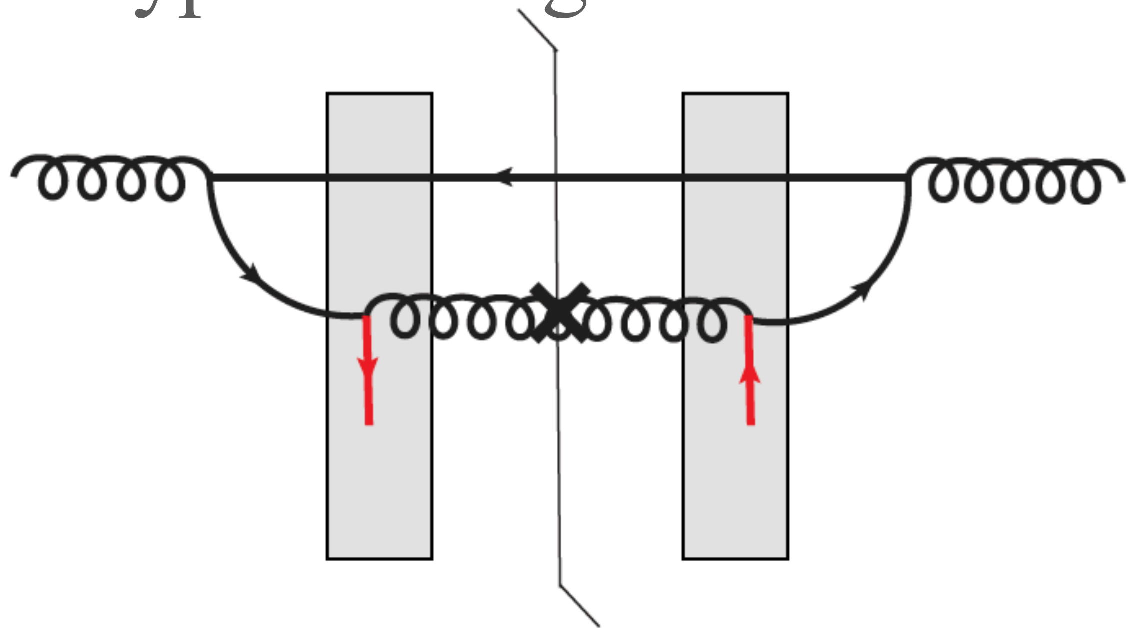
$$(V_{\underline{x},\underline{y};\sigma',\sigma})^{ij} \equiv (V_{\underline{x}})^{ij}\delta^{(2)}(\underline{x}-\underline{y})\delta_{\sigma,\sigma'} + \sigma\delta_{\sigma,\sigma'}\left(V_{\underline{x}}^{G[1]} + V_{\underline{x}}^{q[1]}\right)^{ij}\delta^{(2)}(\underline{x}-\underline{y}) + \delta_{\sigma,\sigma'}\left(V_{\underline{x},\underline{y}}^{G[2]} + V_{\underline{x}}^{q[2]}\delta^{(2)}(\underline{x}-\underline{y})\right)^{ij}$$

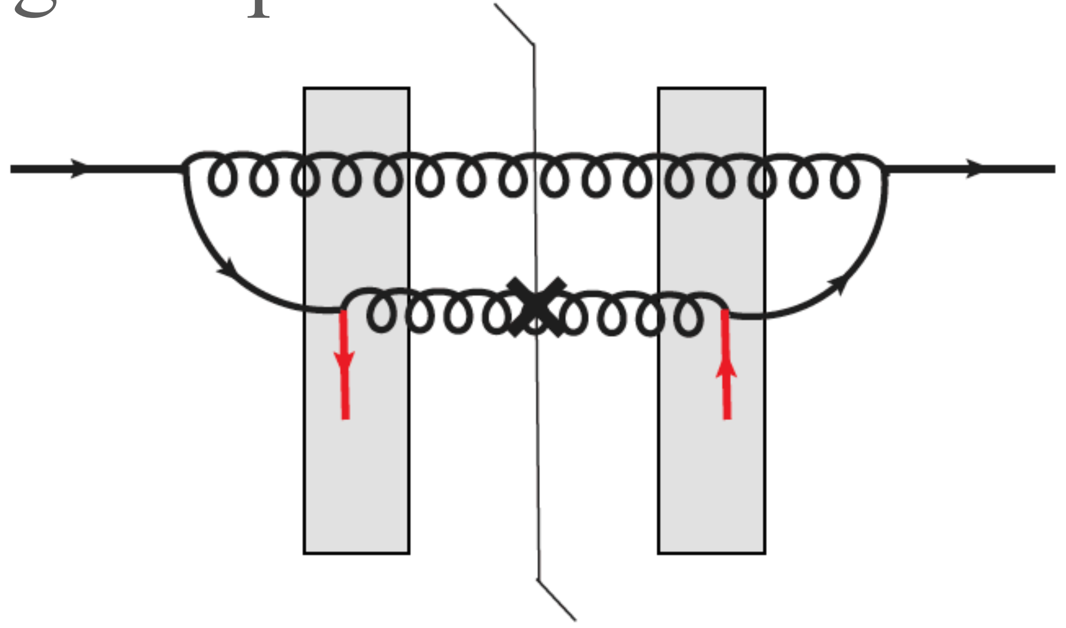


Quark initiated channels: Quark + Proton $\longrightarrow$ Gluon + X

Including Quarks (work in progress)

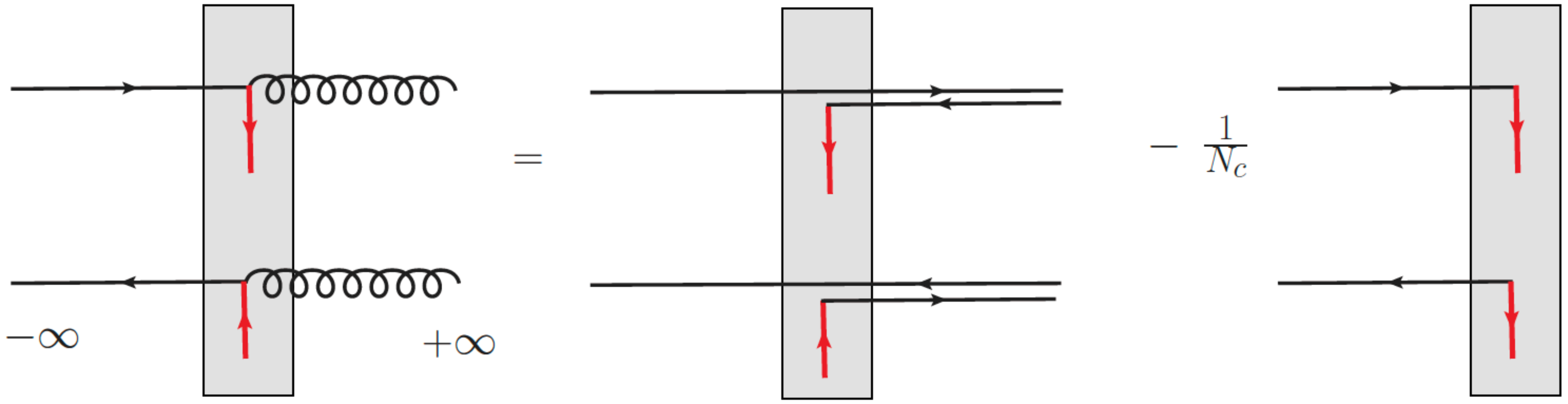
New types of diagrams contributing to gluon production.





Altinoluk, Armesto and Beuf (2023)

$$\hat{O}(\underline{x},\underline{y}) = \frac{g^2 P^+}{s} \int_{-\infty}^{+\infty} dx^- \int_{-\infty}^{+\infty} dy^- \, U_{\underline{y}}^{ce}[+\infty, y^-] \, \bar{\psi}(y^-, \underline{y}) \left(t^e V_{\underline{y}}[y^-, -\infty] V_{\underline{x}}^{\dagger}[x^-, -\infty] t^d \right) \left[\frac{\gamma^- \gamma^5}{2} \right] \psi(x^-, \underline{x}) \, U_{\underline{x}}^{cd}[+\infty, x^-] + c\,.c\,.$$



The new operator is related to the small-x limit of quark helicity TMD.

Chirilli (2021)

We have extended the small-x helicity evolution equations to include the quark-gluon (gluon-quark) transition operators in the large N_c & N_f limit.

Borden, Kovchegov and Li, work in progress.