New insights on flavor dependence in TMD extractions from global fits

arXiv:2405.138833

Matteo Cerutti - MAP Collaboration

HAMPTON UNIVERSITY
1868

May 30, 2024

QCD Evolution 2024, Pavia (IT)

Transverse-Momentum Distributions (TMDs)

3-***dimensional map*** of the internal structure of the nucleon

Non-collinear framework

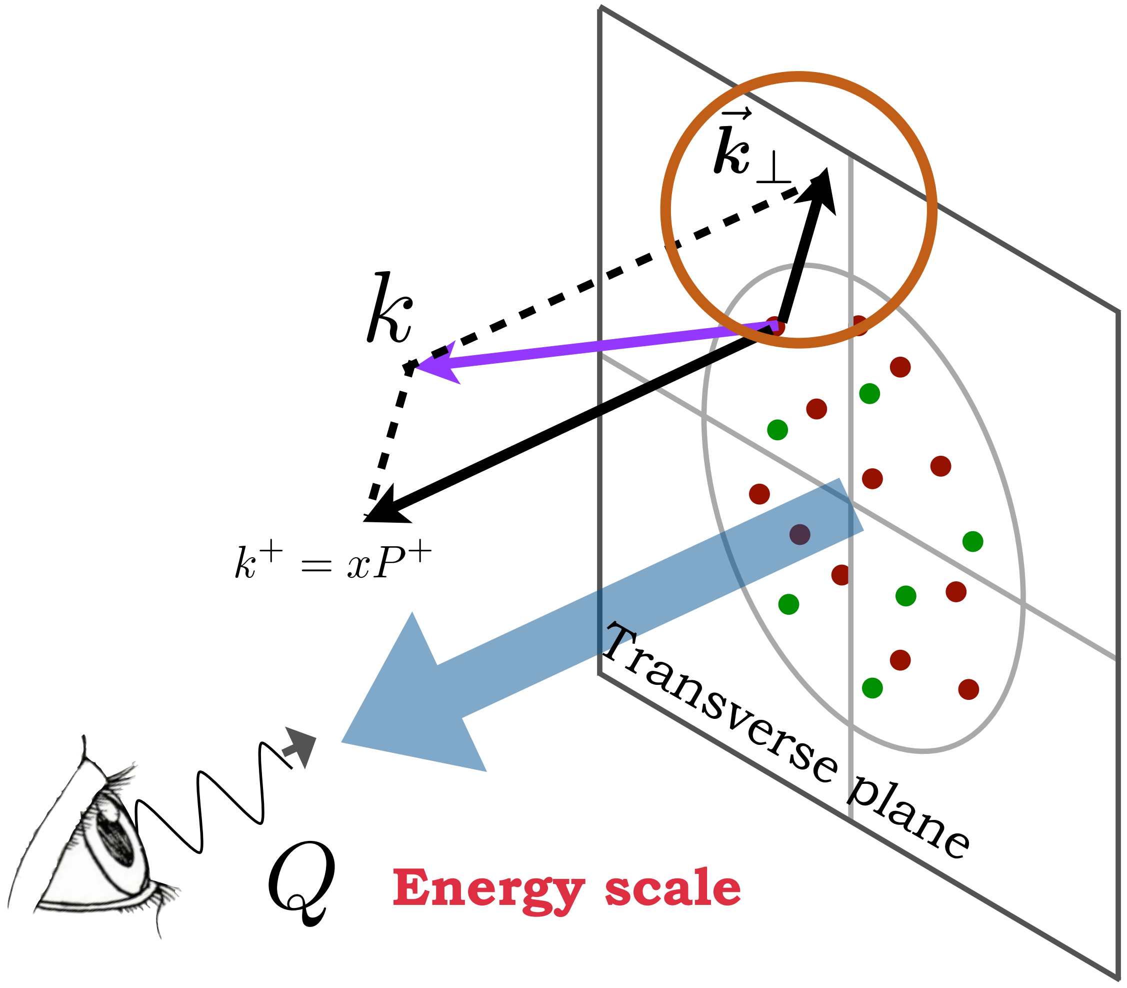
Quark Polarization

Nucleon Pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1 \, h_{1T}^\perp$

Time-reversal odd

Time-reversal even



TMD PDFs

$$F(x, \boldsymbol{k}_\perp^2, \mu, \zeta)$$

Transverse-Momentum Distributions (TMDs)

3-***dimensional*** ***map*** of the internal structure of the nucleon

Non-collinear framework

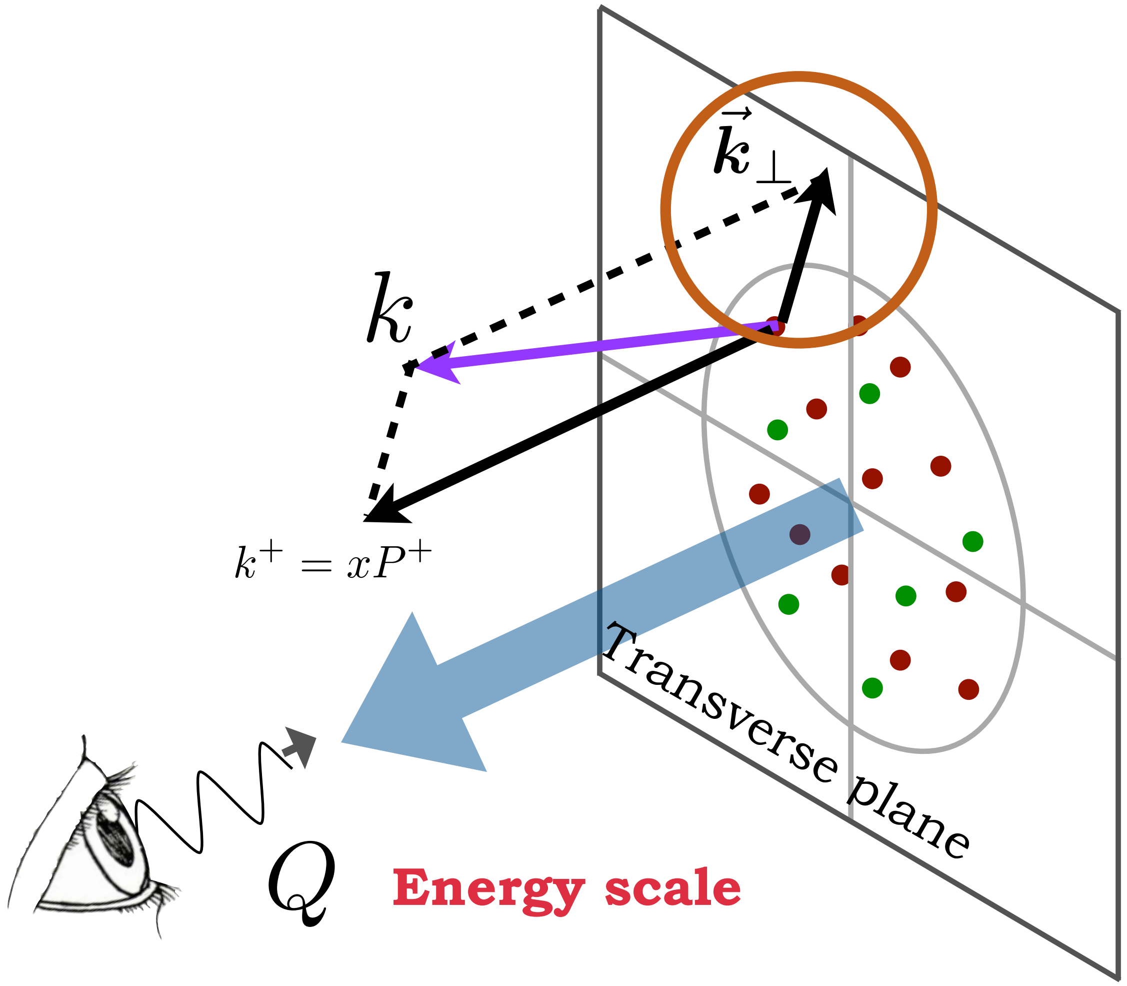
Quark Polarization

Nucleon Pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1 \, h_{1T}^\perp$

Time-reversal odd

Time-reversal even

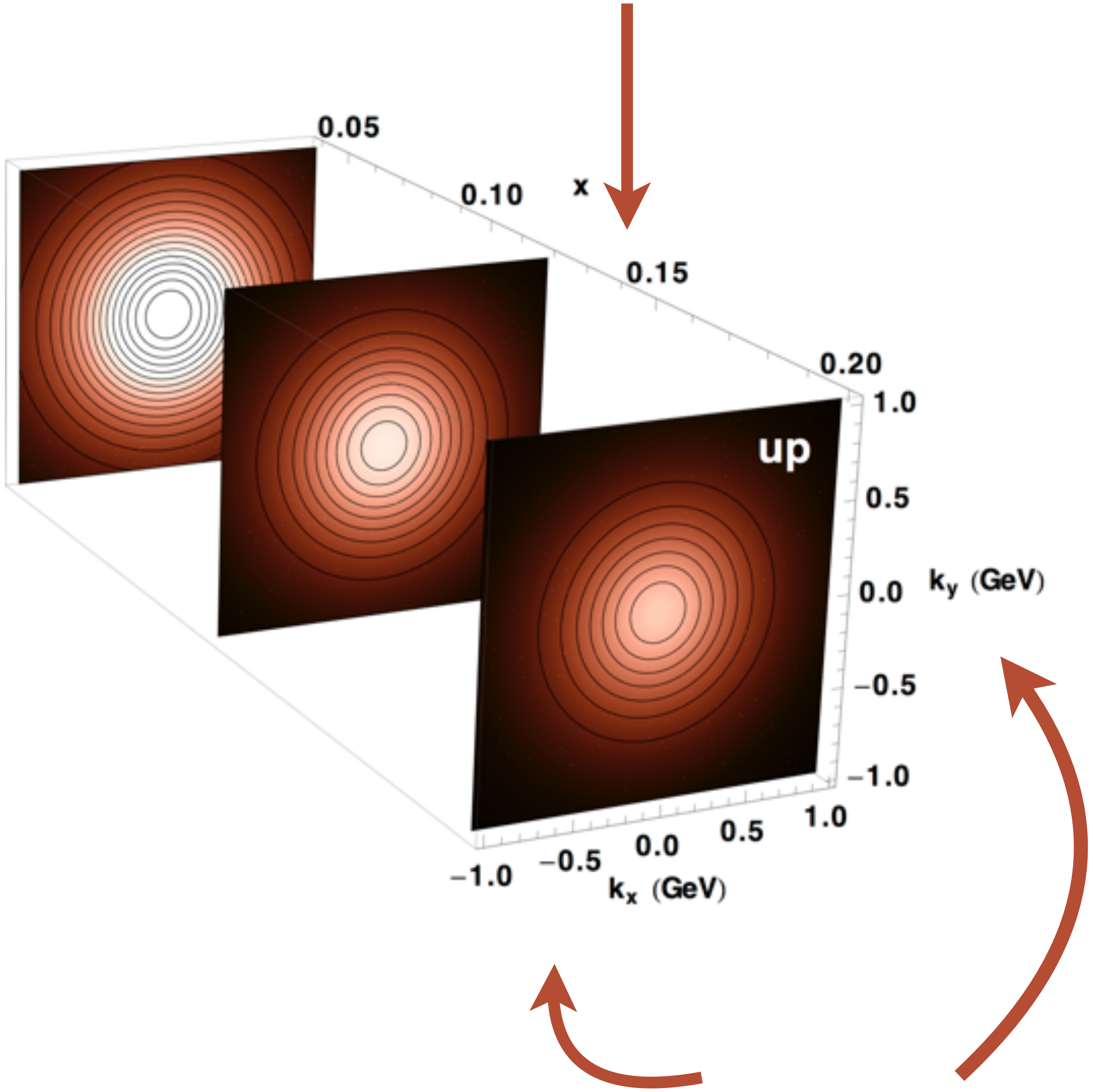


TMD PDFs

$$F(x, \boldsymbol{k}_\perp^2, \mu, \zeta)$$

Transverse-Momentum Distributions (TMDs)

Fraction of
longitudinal momentum



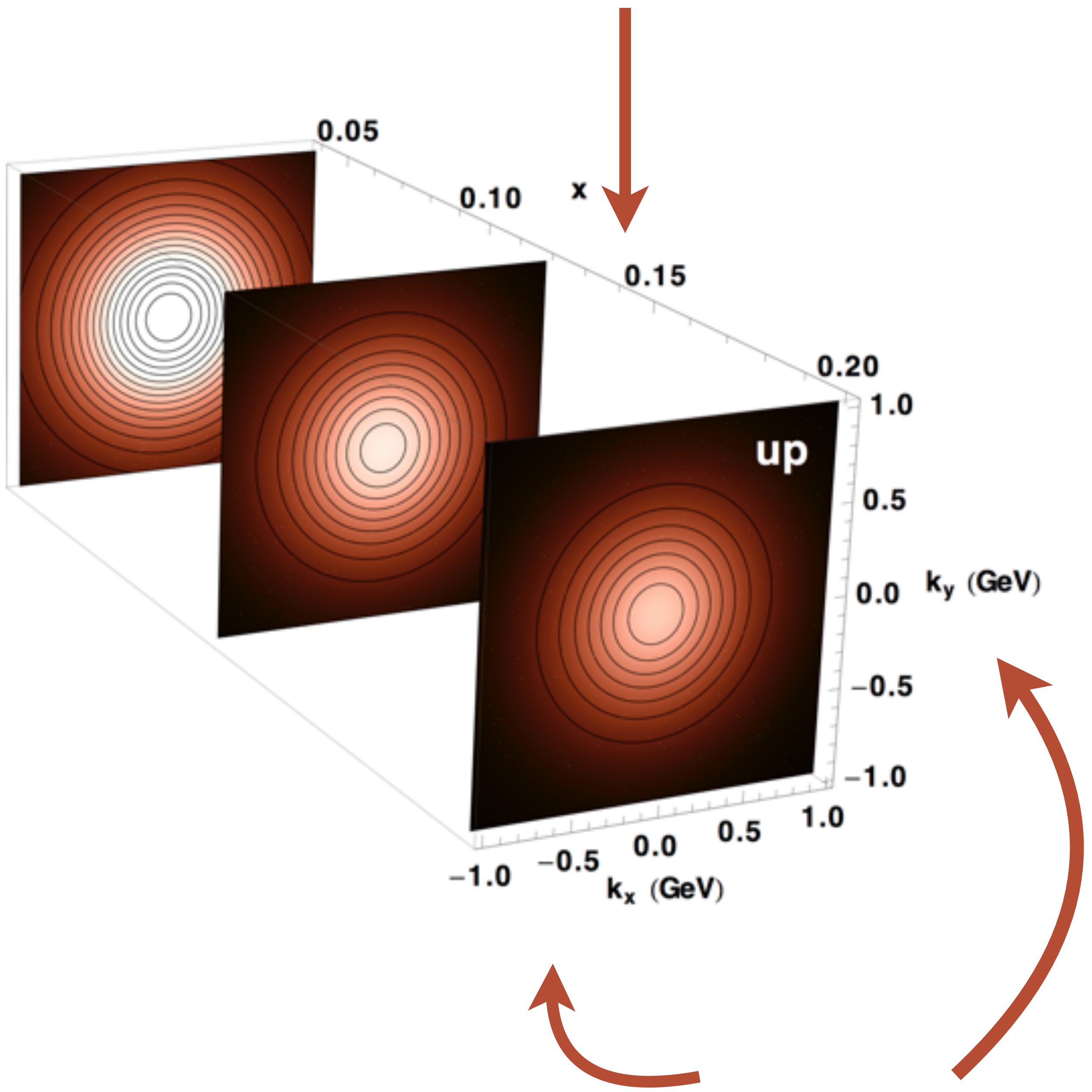
Transverse momentum

TMDs map the distribution of partons inside the nucleon in 3D in momentum space.

They can be extracted through ***global fits***
There are attempts to calculate them in lattice QCD

Transverse-Momentum Distributions (TMDs)

Fraction of longitudinal momentum



Transverse momentum

TMDs map the distribution of partons inside the nucleon in 3D in momentum space.

They can be extracted through ***global fits***
There are attempts to calculate them in lattice QCD

Are TMDs universal?

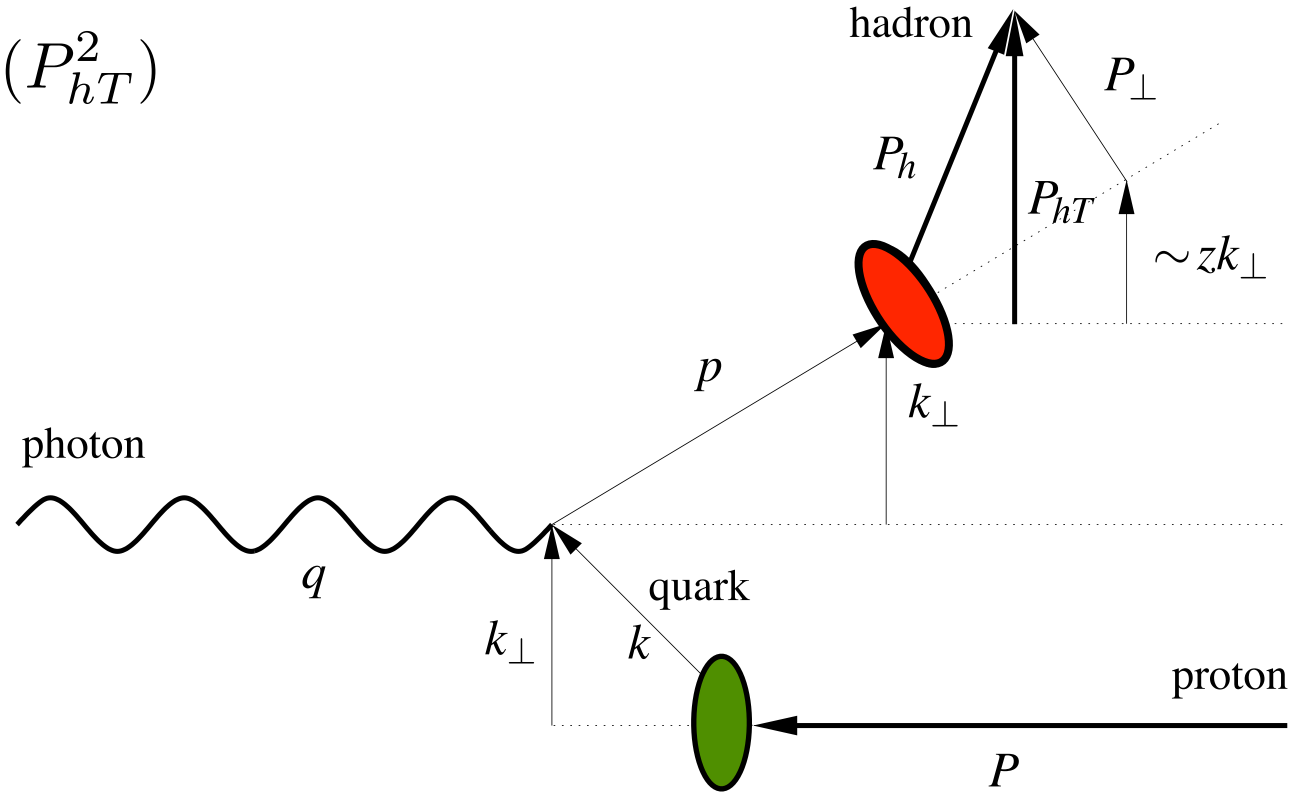
Do they depend on x?

Do they depend on the quark flavor?

TMD factorization: SIDIS

Semi-Inclusive Deep-Inelastic Scattering

If $Q^2 \gg M^2$ and $Q^2 \gg q_T^2 (P_{hT}^2)$



$$F_{UU,T}\big(x, z, |\boldsymbol{q}_T|, Q\big) = \frac{x}{2\pi}\,\mathcal{H}^{SIDIS}(Q,\mu) \sum_{a=q,\bar{q}} e_a^2 \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big)\, \hat{f}_1^a(x, b_T^2; \mu, \zeta_A)\, \hat{D}_1^{a\to h}(z, b_T^2; \mu, \zeta_B)$$

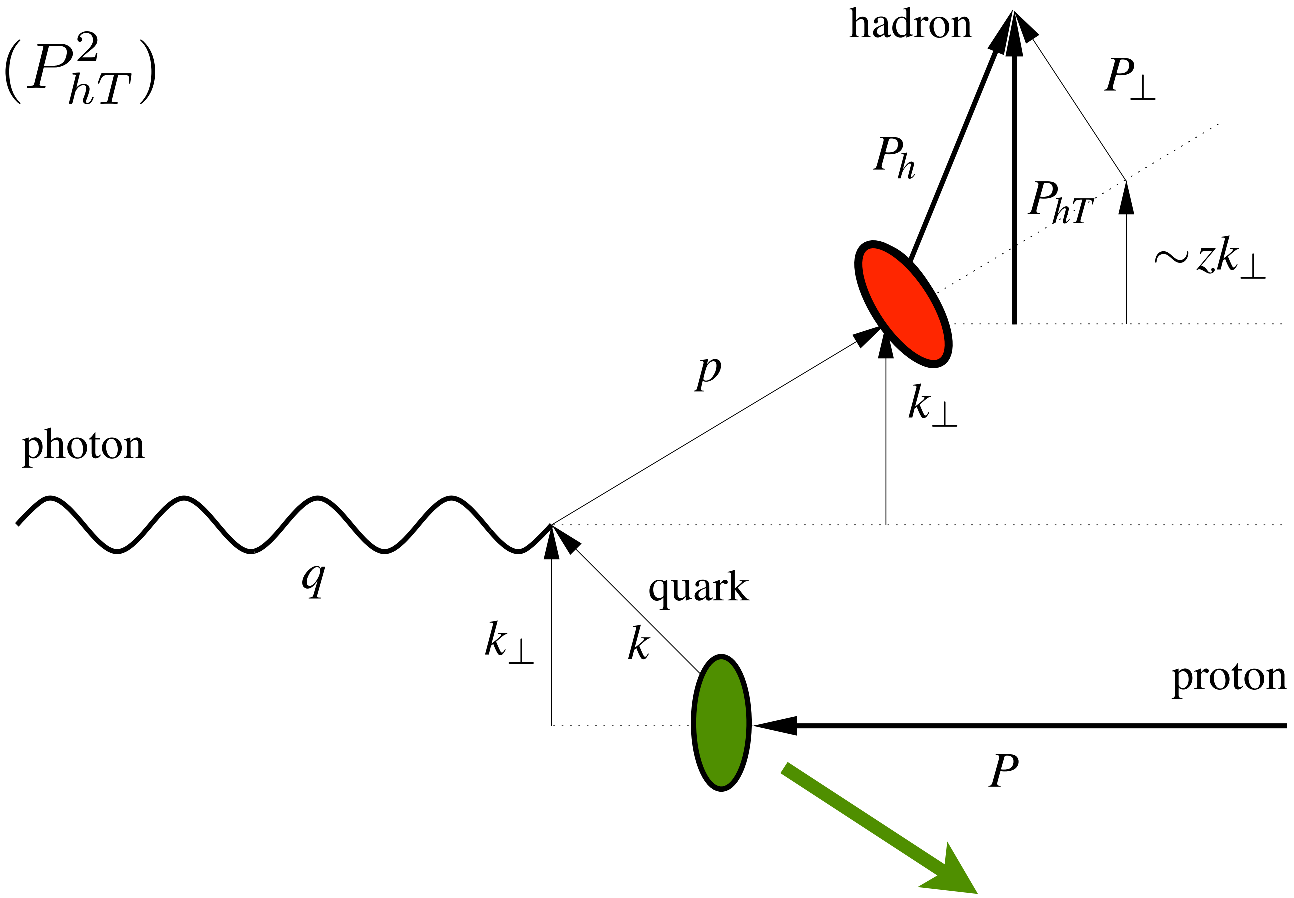
$$+Y_{UU,T}(Q^2, \mathbf{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

TMD factorization: SIDIS

Semi-Inclusive Deep-Inelastic Scattering

If $Q^2 \gg M^2$ and $Q^2 \gg q_T^2(P_{hT}^2)$



TMD PDF

$$F_{UU,T}\big(x, z, |\boldsymbol{q}_T|, Q\big) = \frac{x}{2\pi}\,\mathcal{H}^{SIDIS}(Q,\mu) \sum_{a=q,\bar{q}} e_a^2 \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big)\,\hat{f}_1^a(x, b_T^2; \mu, \zeta_A)\,\hat{D}_1^{a\to h}(z, b_T^2; \mu, \zeta_B)$$

$$+Y_{UU,T}(Q^2, \mathbf{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

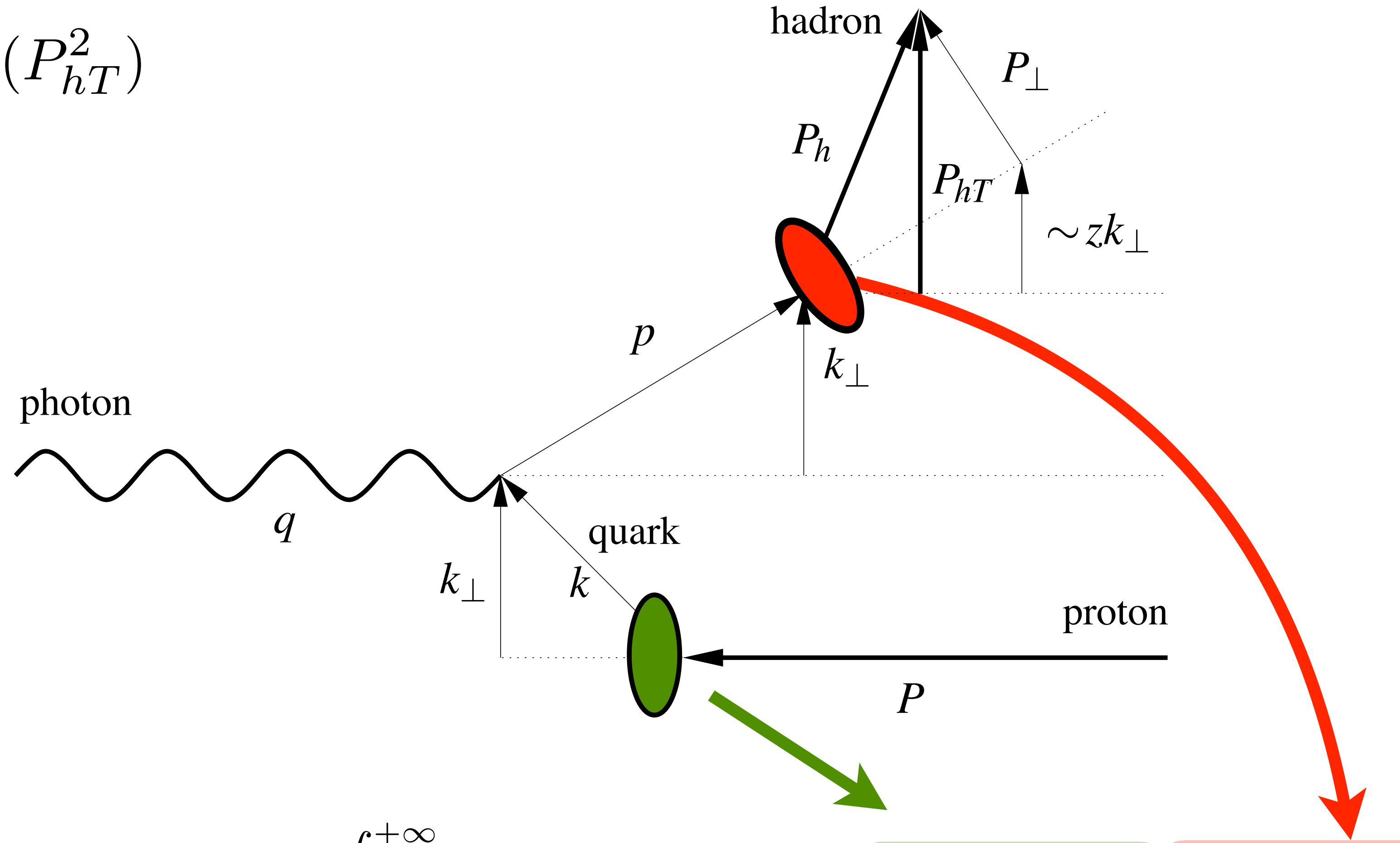
TMD factorization: SIDIS

Semi-Inclusive Deep-Inelastic Scattering

If $Q^2 \gg M^2$ and $Q^2 \gg q_T^2(P_{hT}^2)$

TMD FF

TMD PDF



$$F_{UU,T}\left(x,z,|\boldsymbol{q}_T|,Q\right)=\frac{x}{2\pi}\,\mathcal{H}^{SIDIS}(Q,\mu)\sum_{a=q,\bar{q}}e_a^2\int_0^{+\infty}d|\boldsymbol{b}_T||\boldsymbol{b}_T|J_0\left(|\boldsymbol{b}_T||\boldsymbol{q}_T|\right)\hat{f}_1^a(x,b_T^2;\mu,\zeta_A)\hat{D}_1^{a\to h}(z,b_T^2;\mu,\zeta_B)$$

$$+Y_{UU,T}(Q^2,\mathbf{P}_{hT}^2)+\mathcal{O}(M^2/Q^2)$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

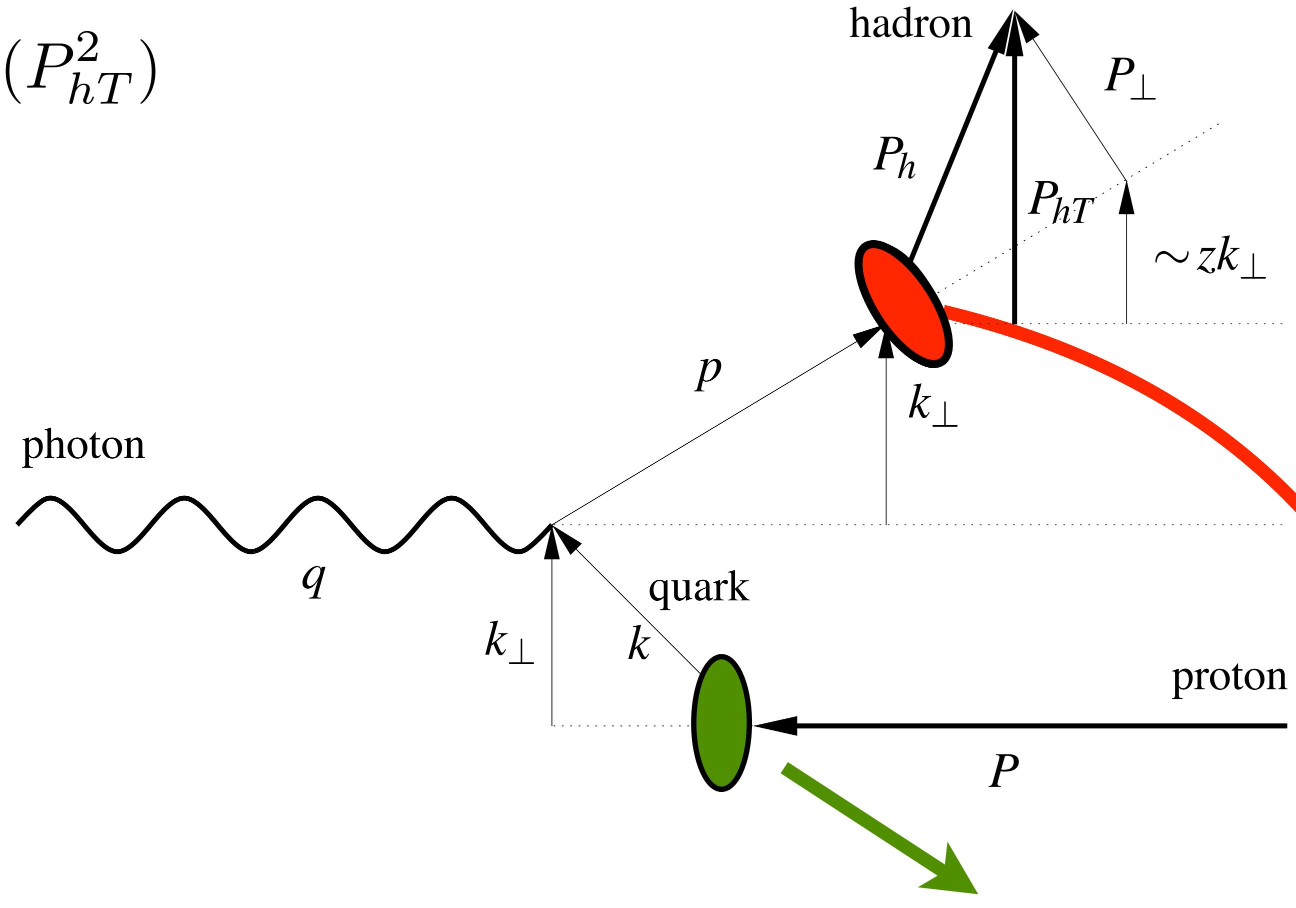
TMD factorization: SIDIS

Semi-Inclusive Deep-Inelastic Scattering

If $Q^2 \gg M^2$ and $Q^2 \gg q_T^2(P_{hT}^2)$

TMD FF

TMD PDF



$$F_{UU,T}\left(x,z,|\boldsymbol{q}_T|,Q\right) = \frac{x}{2\pi}\,\mathcal{H}^{SIDIS}(Q,\mu)\sum_{a=q,\bar{q}} e_a^2 \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T|J_0\left(|\boldsymbol{b}_T||\boldsymbol{q}_T|\right)\hat{f}_1^a(x,b_T^2;\mu,\zeta_A)\hat{D}_1^{a\to h}(z,b_T^2;\mu,\zeta_B)$$

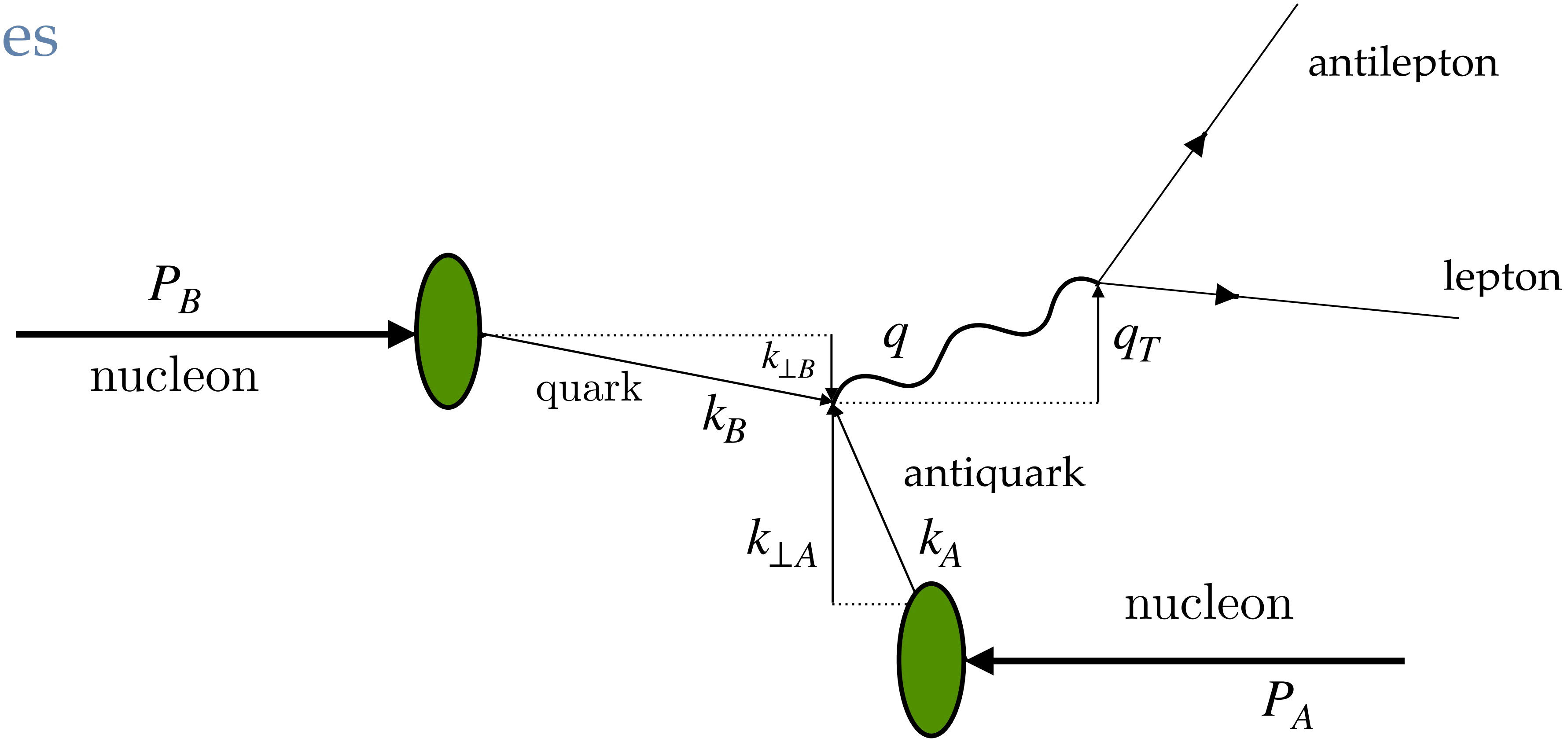
$$+Y_{UU,T}(Q^2,\mathbf{P}_{hT}^2)+\mathcal{O}(M^2/Q^2)$$

- The **W term** dominates in the region where $q_T \ll Q$
- The Y term has been excluded in the analysis

Bacchetta, Diehl, et al., JHEP 02 (2007)

TMD factorization: Drell-Yan

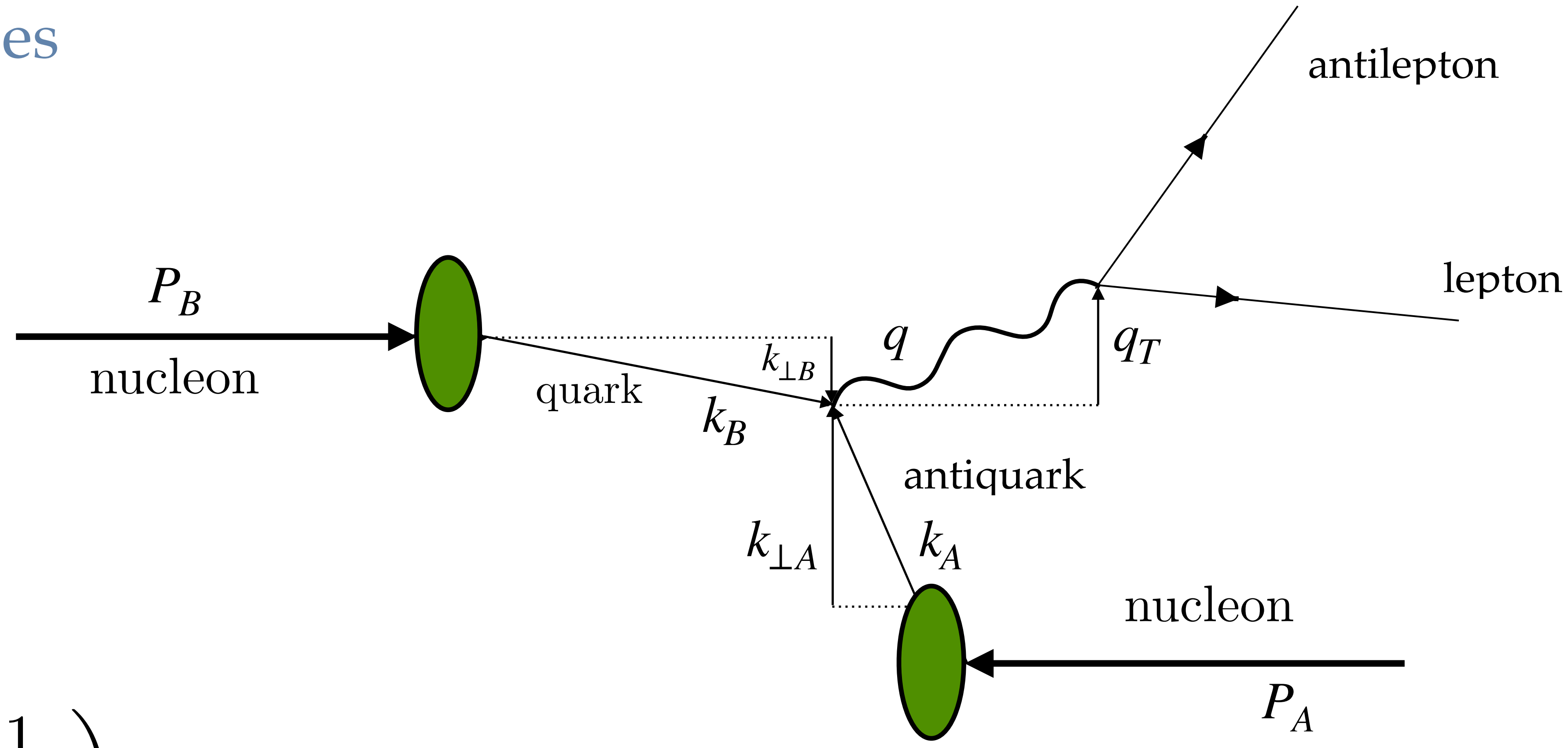
Drell-Yan processes



Collins, Metz, PRL 93 (2004)

TMD factorization: Drell-Yan

Drell-Yan processes

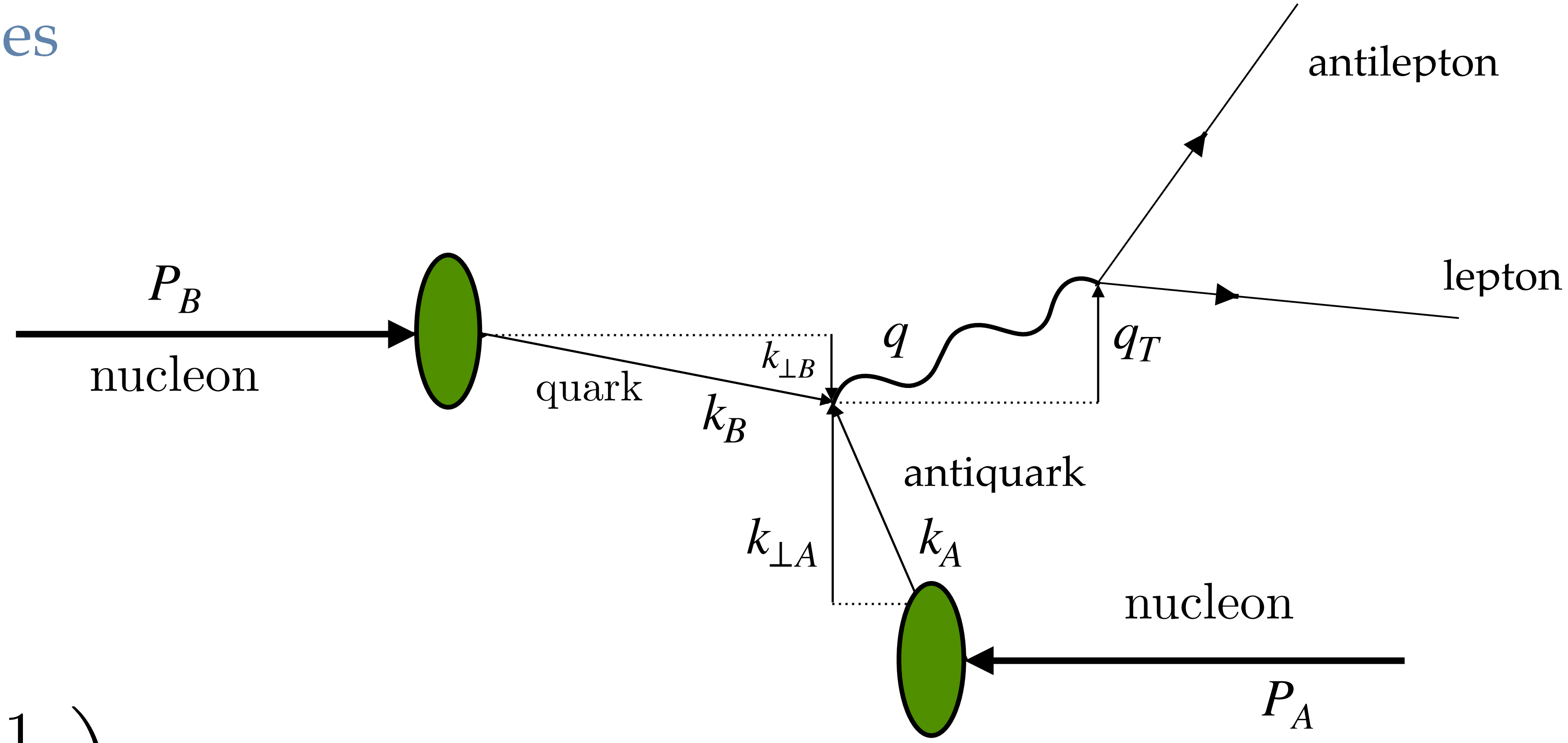


$$d\sigma \sim F^1_{UU} + O\left(\frac{1}{Q^2}\right)$$

Collins, Metz, PRL 93 (2004)

TMD factorization: Drell-Yan

Drell-Yan processes



$$d\sigma \sim F_{UU}^1 + O\left(\frac{1}{Q^2}\right)$$

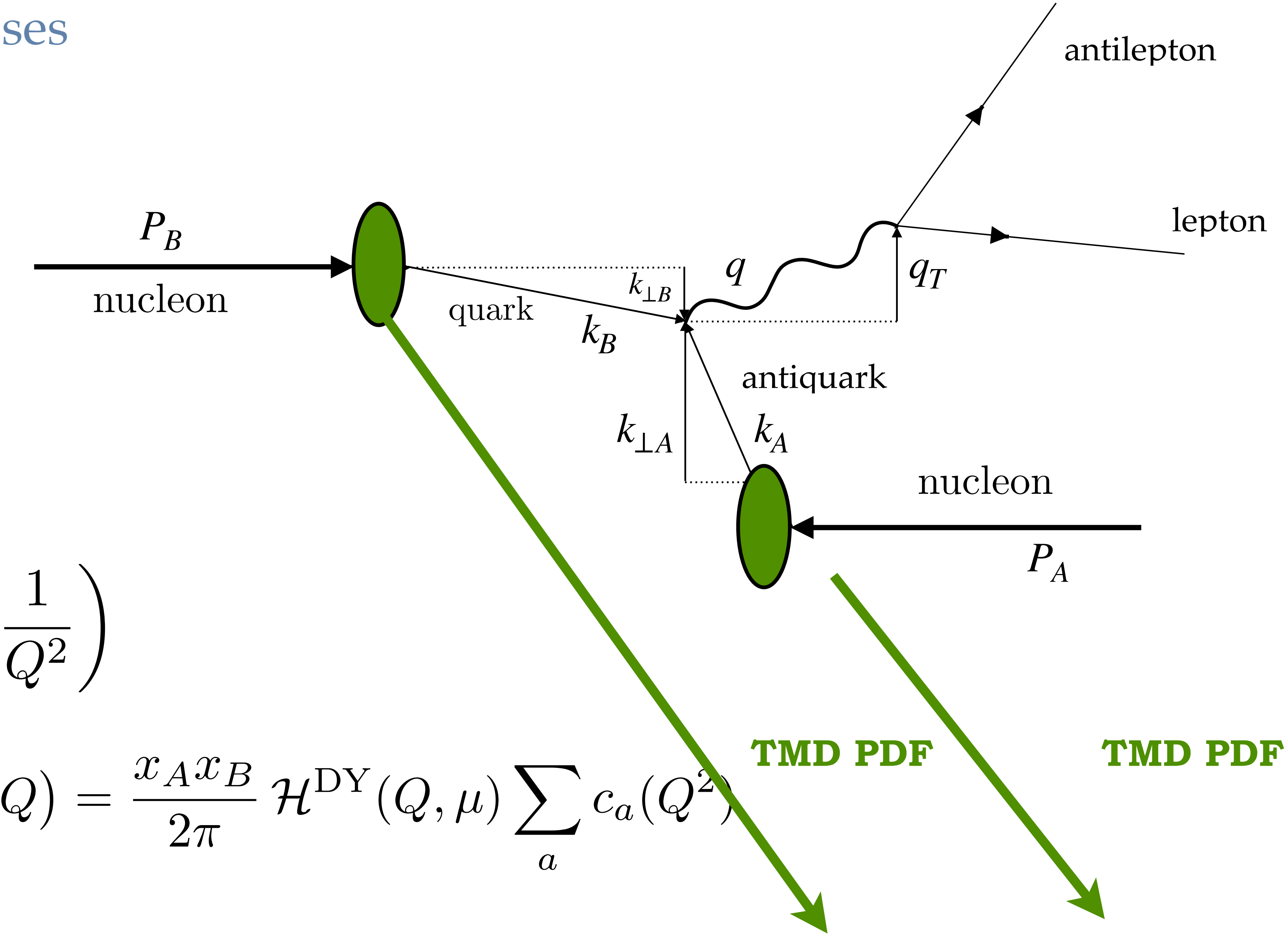
$$F_{UU}^1\left(x_A, x_B, |\boldsymbol{q}_T|, Q\right) = \frac{x_A x_B}{2\pi} \mathcal{H}^{\mathrm{DY}}(Q,\mu) \sum_a c_a(Q^2)$$

$$\int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0\left(|\boldsymbol{b}_T||\boldsymbol{q}_T|\right) \hat{f}_1^a(x_A, b_T^2; \mu, \zeta_A) \hat{f}_1^{\bar{a}}(x_B, b_T^2; \mu, \zeta_B)$$

Collins, Metz, PRL 93 (2004)

TMD factorization: Drell-Yan

Drell-Yan processes



$$d\sigma \sim F_{UU}^1 + O\left(\frac{1}{Q^2}\right)$$

$$F_{UU}^1\big(x_A, x_B, |\boldsymbol{q}_T|, Q\big) = \frac{x_A x_B}{2\pi}\,\mathcal{H}^{\mathrm{DY}}(Q,\mu)\sum_a c_a(Q^2)$$

$$\int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T|J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big)\,\hat{f}_1^a(x_A, b_T^2; \mu, \zeta_A)\,\hat{f}_1^{\bar{a}}(x_B, b_T^2; \mu, \zeta_B)$$

Collins, Metz, PRL 93 (2004)

Structure of a TMD

See R. Kishore's talk

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

J. Collins, "Foundation of Perturbative QCD"

Structure of a TMD

See R. Kishore's talk

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2) \otimes f_1^j(x, \mu_{b_*})$$

J. Collins, "Foundation of Perturbative QCD"

Structure of a TMD

See R. Kishore's talk

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2 \boldsymbol{k}_\perp}{(2\pi)^2} e^{i \boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

b$_*$-prescription

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2) \otimes f_1^j(x, \mu_{b_*})$$

: A

Perturbative TMD at the initial scale

J. Collins, "Foundation of Perturbative QCD"

Structure of a TMD

See R. Kishore's talk

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

b$_*$-prescription

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2) \otimes f_1^j(x, \mu_{b_*}) \qquad : \mathrm{A}$$

Perturbative TMD at the initial scale

$$\times \exp\left\{ K(b_*; \mu_{b_*}) \ln \frac{\sqrt{\zeta}}{\mu_{b_*}} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} \qquad : \mathrm{B}$$

Evolution to final scale (of the process)

J. Collins, "Foundation of Perturbative QCD"

Structure of a TMD

See R. Kishore's talk

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

b$_*$-prescription

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2) \otimes f_1^j(x, \mu_{b_*}) \qquad : \text{A}$$

Perturbative TMD at the initial scale

$$\times \exp\left\{ K(b_*; \mu_{b_*}) \ln \frac{\sqrt{\zeta}}{\mu_{b_*}} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} \qquad : \text{B}$$

Evolution to final scale (of the process)

$$\times f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \text{C}$$

Non-perturbative part of the TMD

J. Collins, "Foundation of Perturbative QCD"

Structure of a TMD

See R. Kishore's talk

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

b_*-prescription

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j \boxed{C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2)} \otimes f_1^j(x, \mu_{b_*}) \qquad : \mathrm{A}$$

Perturbative TMD at the initial scale

Perturbative

$$\times \exp\left\{\boxed{K(b_*;\mu_{b_*})}\ln\frac{\sqrt{\zeta}}{\mu_{b_*}} + \int_{\mu_{b_*}}^{\mu}\frac{d\mu'}{\mu'}\left[\boxed{\gamma_F} - \boxed{\gamma_K}\ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\} \qquad : \mathrm{B}$$

Evolution to final scale (of the process)

$$\times f_{NP}(x, b_T^2)\exp\left\{g_K(b_T^2)\ln\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\right\} \qquad : \mathrm{C}$$

Non-perturbative part of the TMD

J. Collins, "Foundation of Perturbative QCD"

Structure of a TMD

See R. Kishore's talk

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

Collinear extractions

b$_*$-prescription

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j \boxed{C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2)} \otimes \boxed{f_1^j(x, \mu_{b_*})} \qquad : \mathrm{A}$$

Perturbative TMD at the initial scale

Perturbative

$$\times \exp\left\{ \boxed{K(b_*; \mu_{b_*})} \ln \frac{\sqrt{\zeta}}{\mu_{b_*}} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left[\boxed{\gamma_F} - \boxed{\gamma_K} \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} \qquad : \mathrm{B}$$

Evolution to final scale (of the process)

$$\times f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

Non-perturbative part of the TMD

J. Collins, "Foundation of Perturbative QCD"

Structure of a TMD

See R. Kishore's talk

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

Collinear extractions

b$_*$-prescription

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j \boxed{C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2)} \otimes \boxed{f_1^j(x, \mu_{b_*})} \qquad : \mathrm{A}$$

Perturbative TMD at the initial scale

Perturbative

$$\times \exp\left\{ \boxed{K(b_*; \mu_{b_*})} \ln \frac{\sqrt{\zeta}}{\mu_{b_*}} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left[\boxed{\gamma_F} - \boxed{\gamma_K} \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} \qquad : \mathrm{B}$$

Evolution to final scale (of the process)

$$\times \boxed{f_{NP}(x, b_T^2)} \exp\left\{ \boxed{g_K(b_T^2)} \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

Parameterization

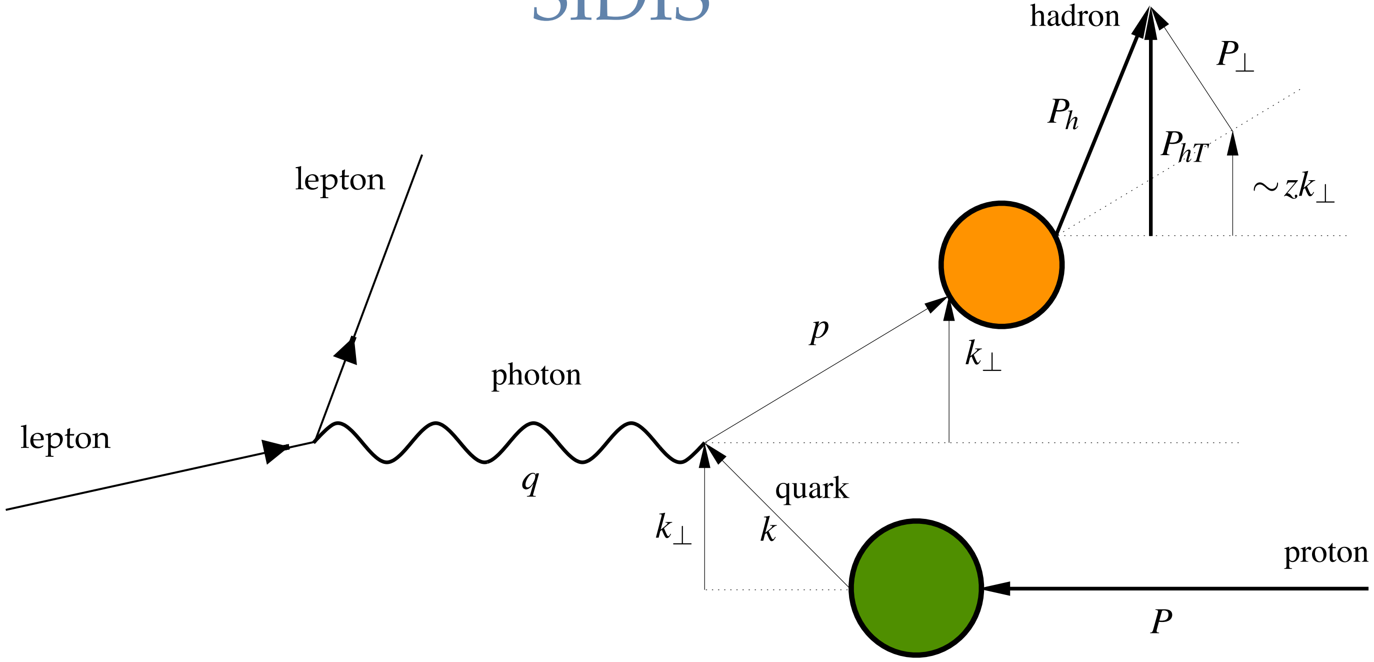
Non-perturbative part of the TMD

J. Collins, "Foundation of Perturbative QCD"

TMD factorization: Universality

TMD factorization: Universality

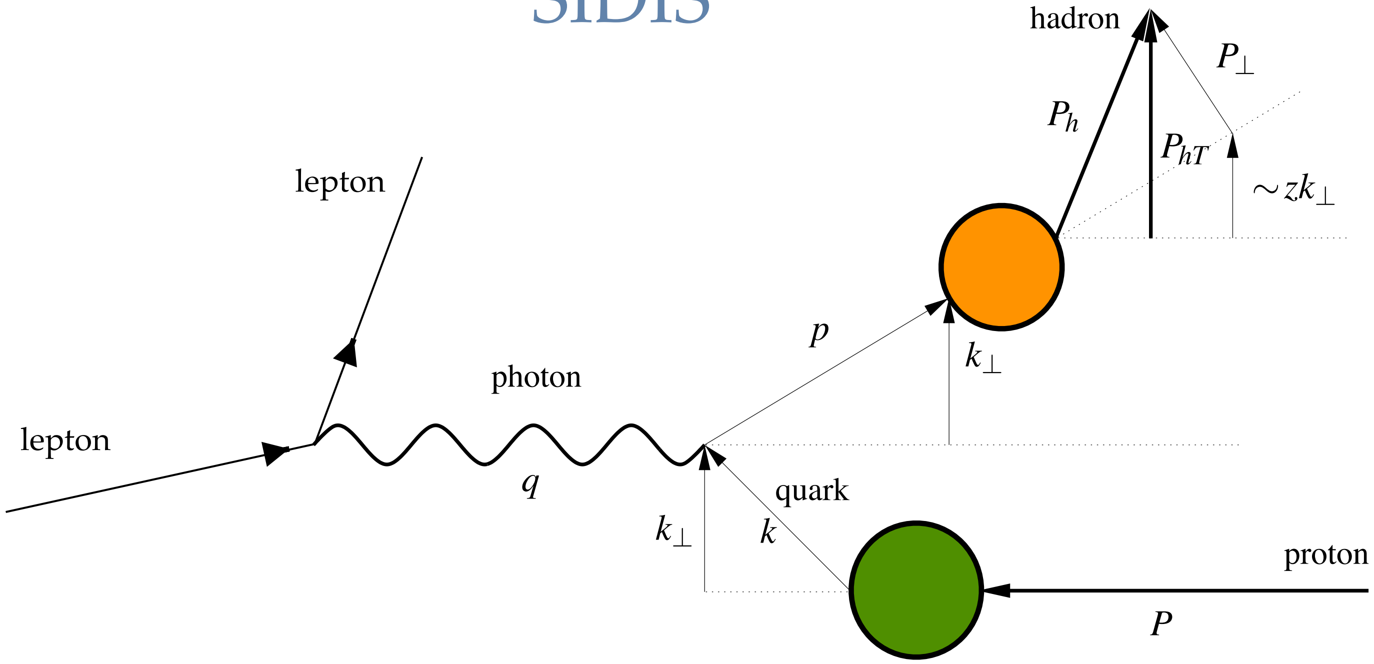
SIDIS



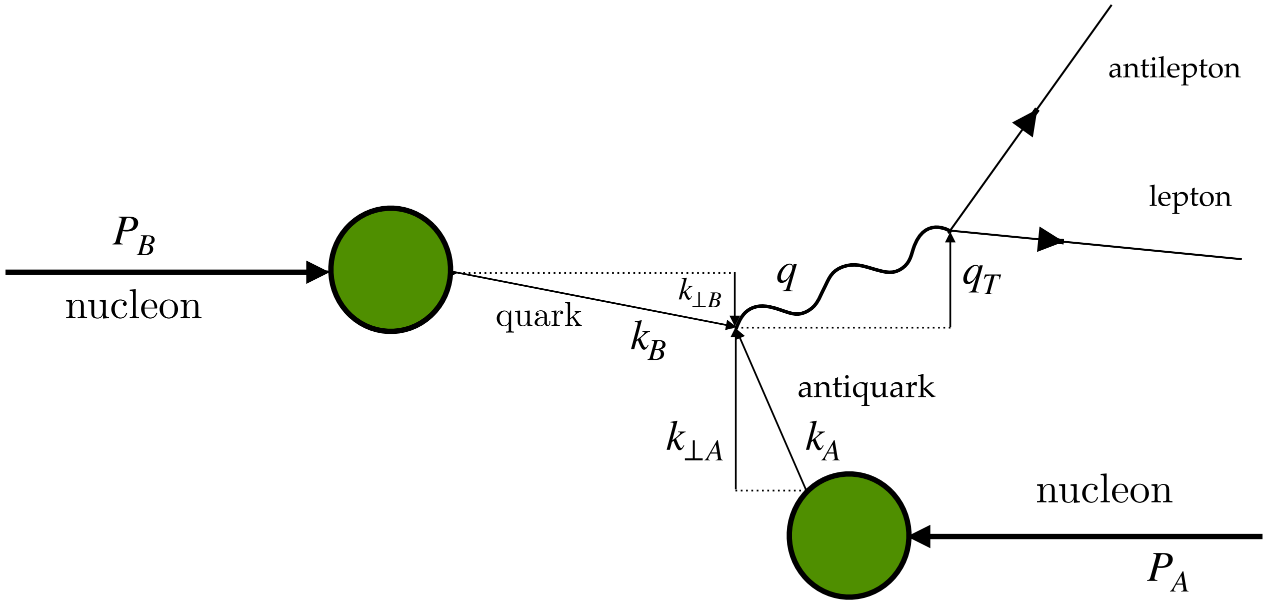
$$F_{UU,T}\big(x,z,|\boldsymbol{q}_T|,Q\big) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T|J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big)\,\hat{f}_1^a(x,b_T^2;\mu,\zeta_A)\,\hat{D}_1^{a\to h}(z,b_T^2;\mu,\zeta_B)$$

TMD factorization: Universality

SIDIS



Drell-Yan

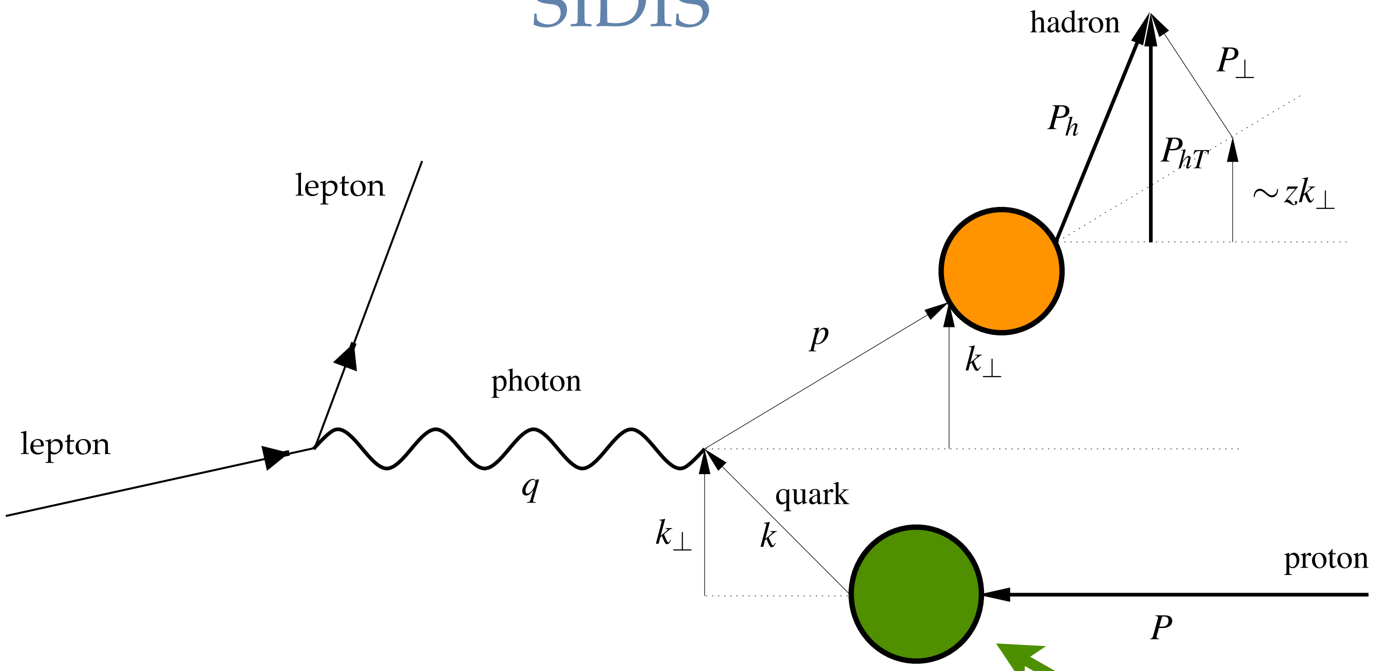


$$F_{UU,T}\big(x, z, |\boldsymbol{q}_T|, Q\big) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big)\, \hat{f}_1^a(x, b_T^2; \mu, \zeta_A)\, \hat{D}_1^{a\to h}(z, b_T^2; \mu, \zeta_B)$$

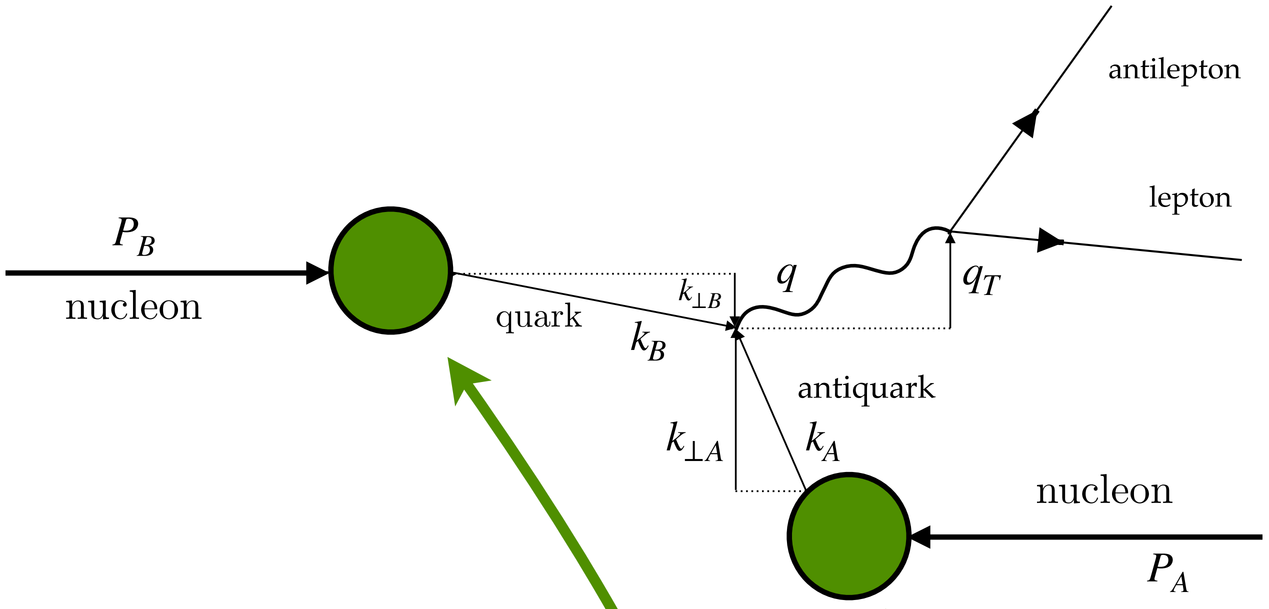
$$F_{UU}^1\big(x_A, x_B, |\boldsymbol{q}_T|, Q\big) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big)\, \hat{f}_1^a(x_A, b_T^2; \mu, \zeta_A)\, \hat{f}_1^{\bar{a}}(x_B, b_T^2; \mu, \zeta_B)$$

TMD factorization: Universality

SIDIS



Drell-Yan

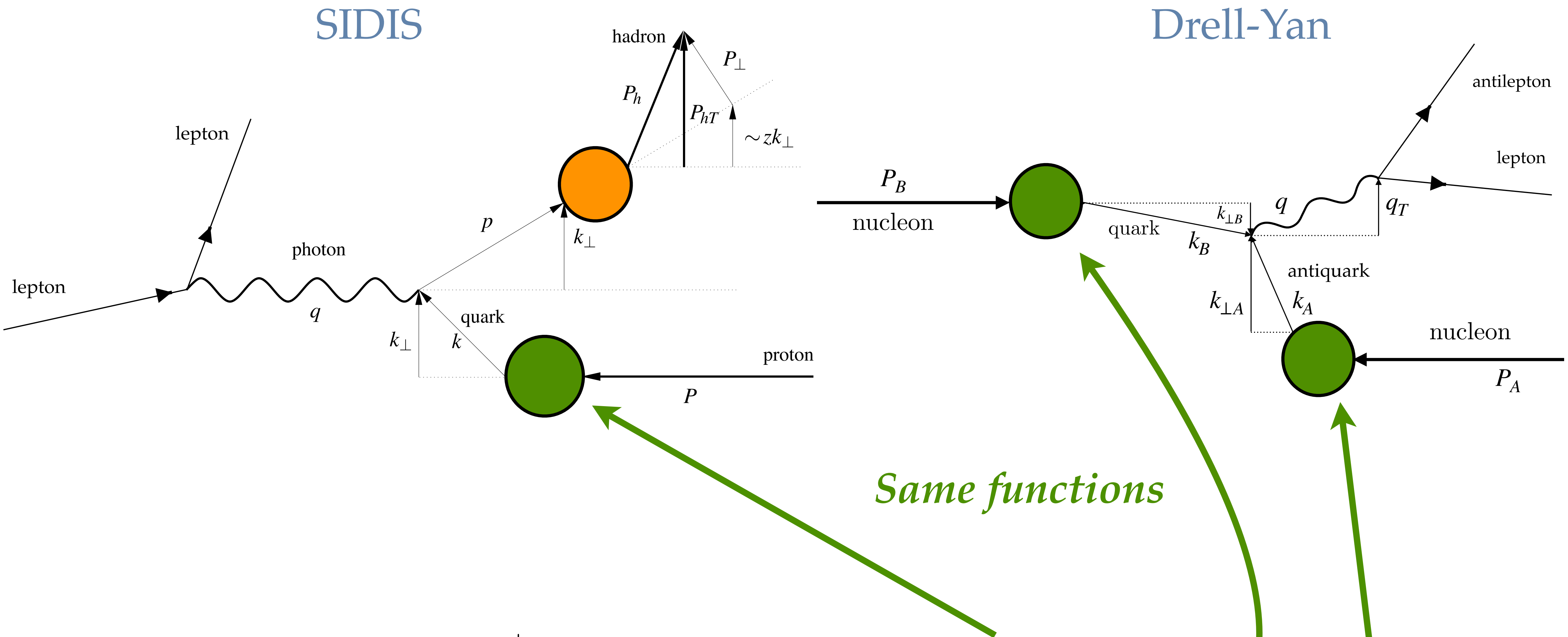


Same functions

$$F_{UU,T}\big(x,z,|\boldsymbol{q}_T|,Q\big) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big) \boxed{\hat{f}_1^a(x,b_T^2;\mu,\zeta_A)} \hat{D}_1^{a\to h}(z,b_T^2;\mu,\zeta_B)$$

$$F_{UU}^1\big(x_A,x_B,|\boldsymbol{q}_T|,Q\big) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big) \boxed{\hat{f}_1^a(x_A,b_T^2;\mu,\zeta_A)} \boxed{\hat{f}_1^{\bar{a}}(x_B,b_T^2;\mu,\zeta_B)}$$

TMD factorization: Universality



Same functions

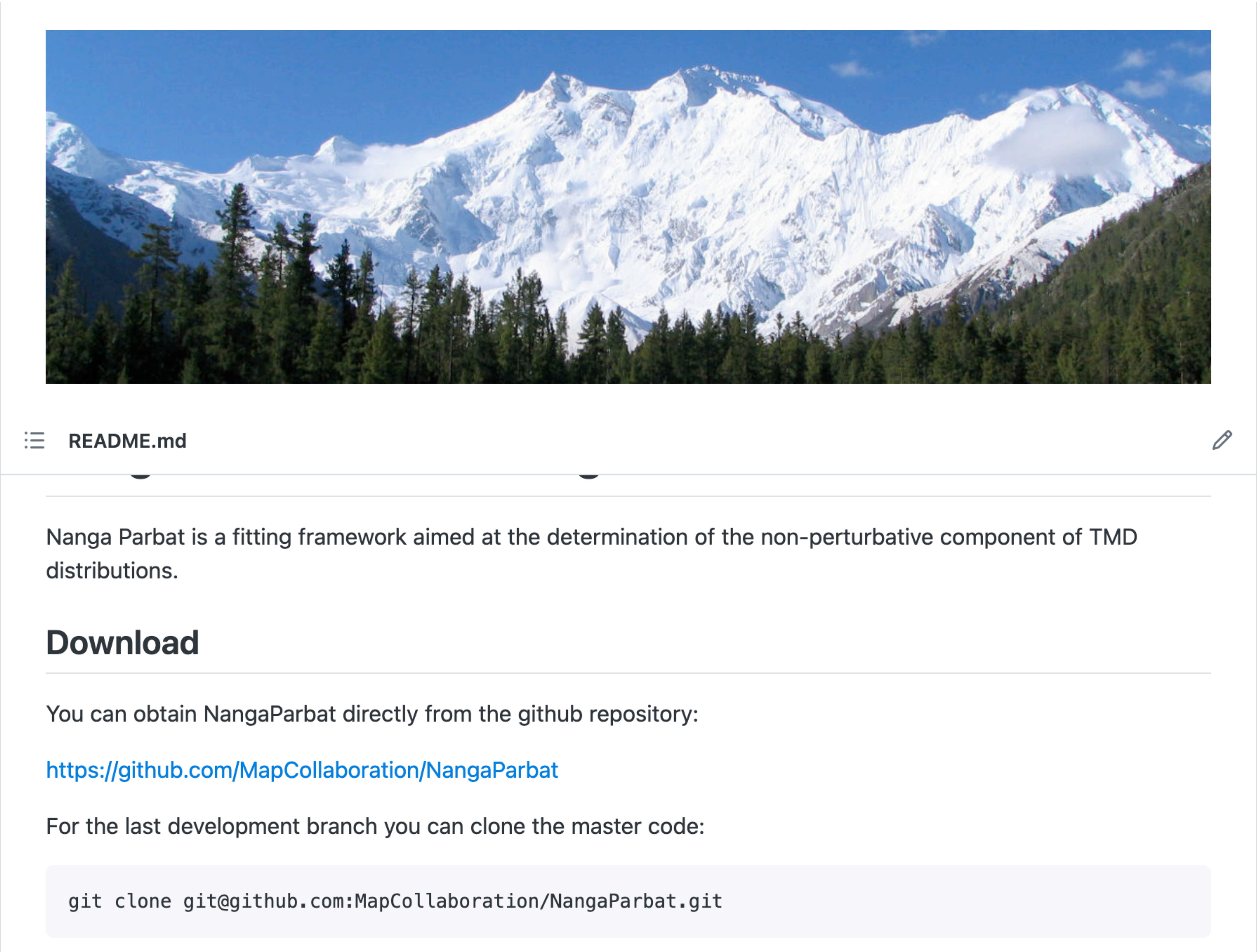
$$F_{UU,T}(x, z, |\boldsymbol{q}_T|, Q) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0(|\boldsymbol{b}_T||\boldsymbol{q}_T|) \boxed{\hat{f}_1^a(x, b_T^2; \mu, \zeta_A)} \hat{D}_1^{a \to h}(z, b_T^2; \mu, \zeta_B)$$

$$F_{UU}^1(x_A, x_B, |\boldsymbol{q}_T|, Q) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0(|\boldsymbol{b}_T||\boldsymbol{q}_T|) \boxed{\hat{f}_1^a(x_A, b_T^2; \mu, \zeta_A)} \boxed{\hat{f}_1^{\bar{a}}(x_B, b_T^2; \mu, \zeta_B)}$$

GLOBAL FITs

MAP TMD fitting framework

https://github.com/MapCollaboration/NangaParbat



README.md

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

https://github.com/MapCollaboration/NangaParbat

For the last development branch you can clone the master code:

```
git clone git@github.com:MapCollaboration/NangaParbat.git
```

Available Global Fits

	Accuracy	SIDIS	DY	N of points	χ^2/N_{data}
Pavia 2017 Bacchetta, Delcarro, et al., JHEP 06 (2017)	NLL	✓	✓	8059	1.55
SV 2019 Scimemi, Vladimirov, JHEP 06 (2020)	N^3LL^-	✓	✓	1039	1.06
MAPTMD22 Bacchetta, Bertone, et al., JHEP 10 (2022)	***N^3LL^-***	✓	✓	***2031***	***1.06***

MAPTMD22 global fit: features

MAP Collaboration, JHEP 10 (2022)

MAPTMD22 global fit: features

- Global analysis of Drell-Yan and SIDIS data sets: ***2031*** data points

MAP Collaboration, JHEP 10 (2022)

MAPTMD22 global fit: features

- Global analysis of Drell-Yan and SIDIS data sets: ***2031*** data points

- Perturbative accuracy: N^3LL^-

MAP Collaboration, JHEP 10 (2022)

MAPTMD22 global fit: features

- Global analysis of Drell-Yan and SIDIS data sets: ***2031*** data points
- Perturbative accuracy: N^3LL^-
- ***Normalization prefactor*** for SIDIS observables

MAP Collaboration, JHEP 10 (2022)

MAPTMD22 global fit: features

- Global analysis of Drell-Yan and SIDIS data sets: ***2031*** data points
- Perturbative accuracy: N^3LL^-
- ***Normalization prefactor*** for SIDIS observables
- Number of fitted parameters: ***21***

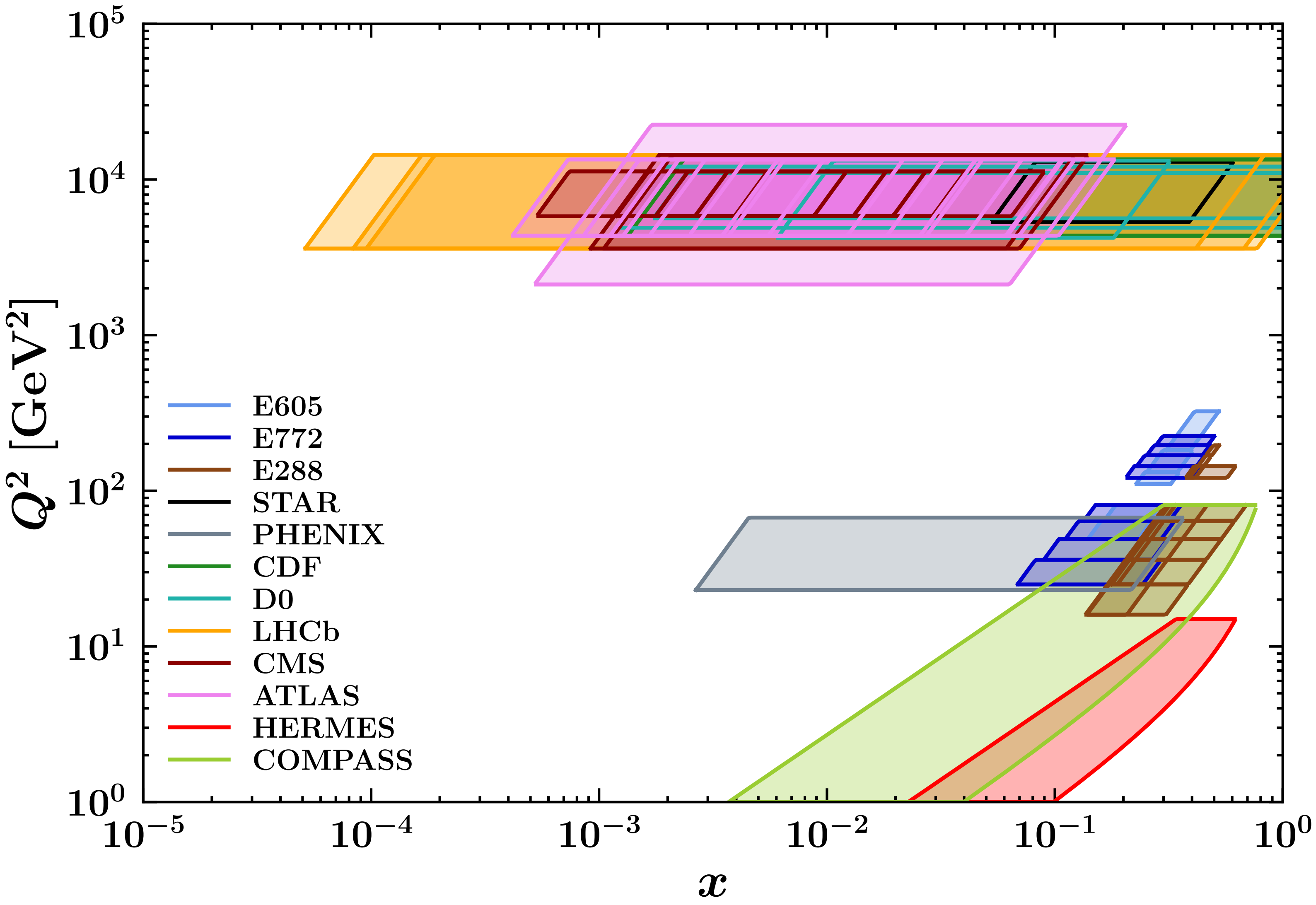
MAP Collaboration, JHEP 10 (2022)

MAPTMD22 global fit: features

- Global analysis of Drell-Yan and SIDIS data sets: ***2031*** data points
- Perturbative accuracy: N^3LL^-
- ***Normalization prefactor*** for SIDIS observables
- Number of fitted parameters: ***21***
- Agreement with data: $\chi^2/N_{data} = 1.06$

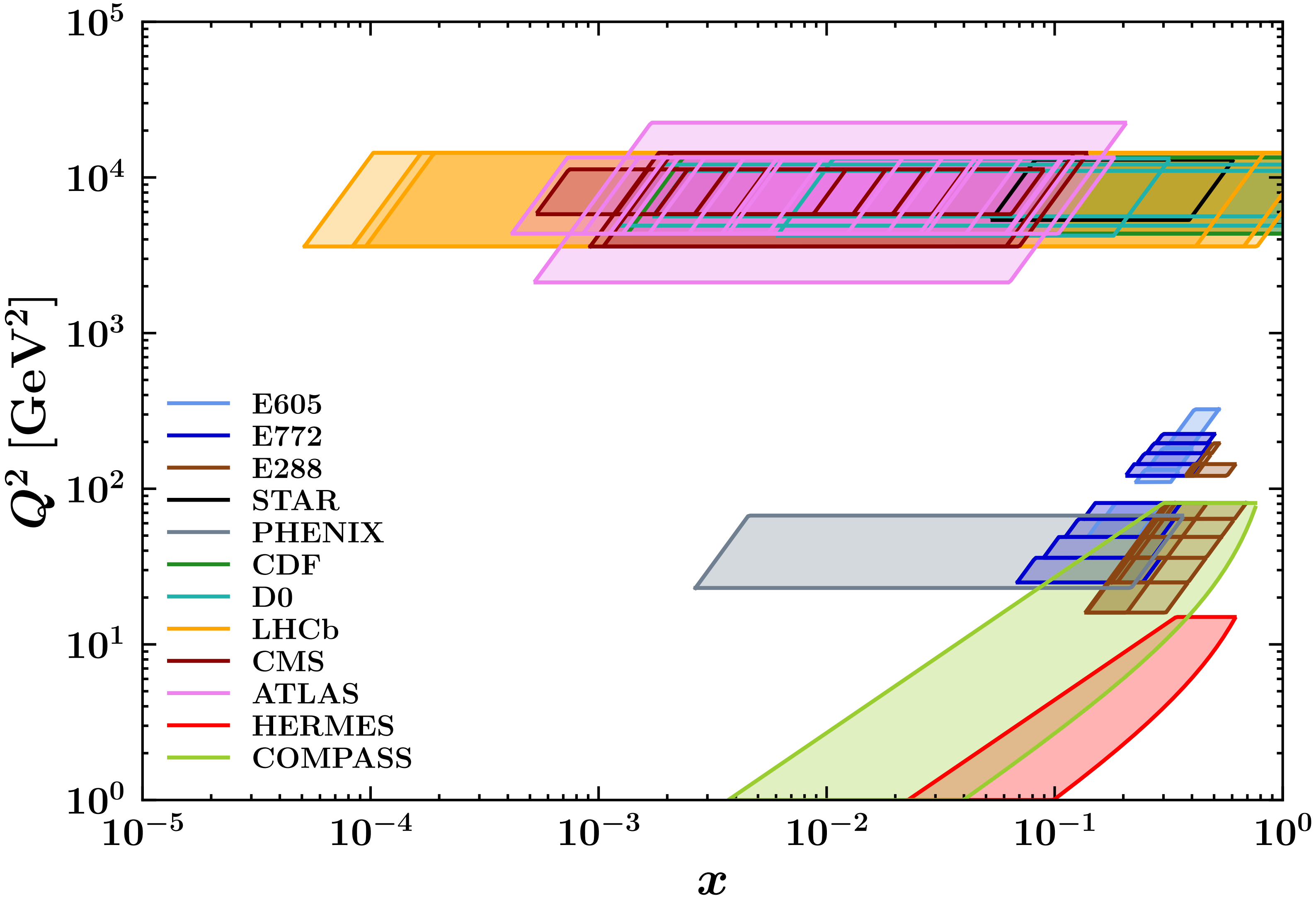
MAP Collaboration, JHEP 10 (2022)

MAP22: included data sets



MAP22: included data sets

Drell-Yan data 484

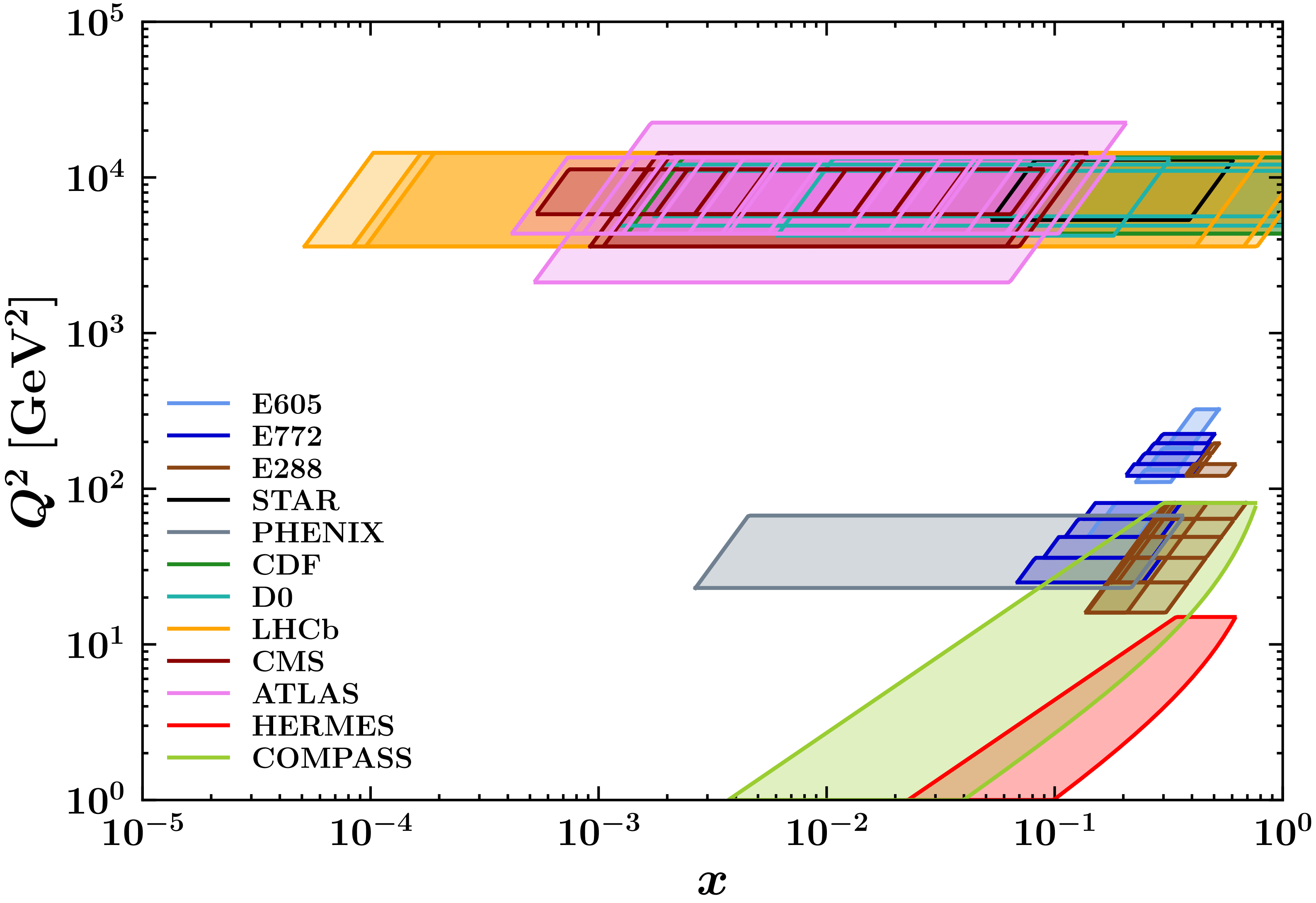


MAP22: included data sets

Drell-Yan data 484

Fixed-target:
E288, E605, E772

Collider mode:
RHIC, Tevatron, LHC



MAP22: included data sets

Drell-Yan data 484

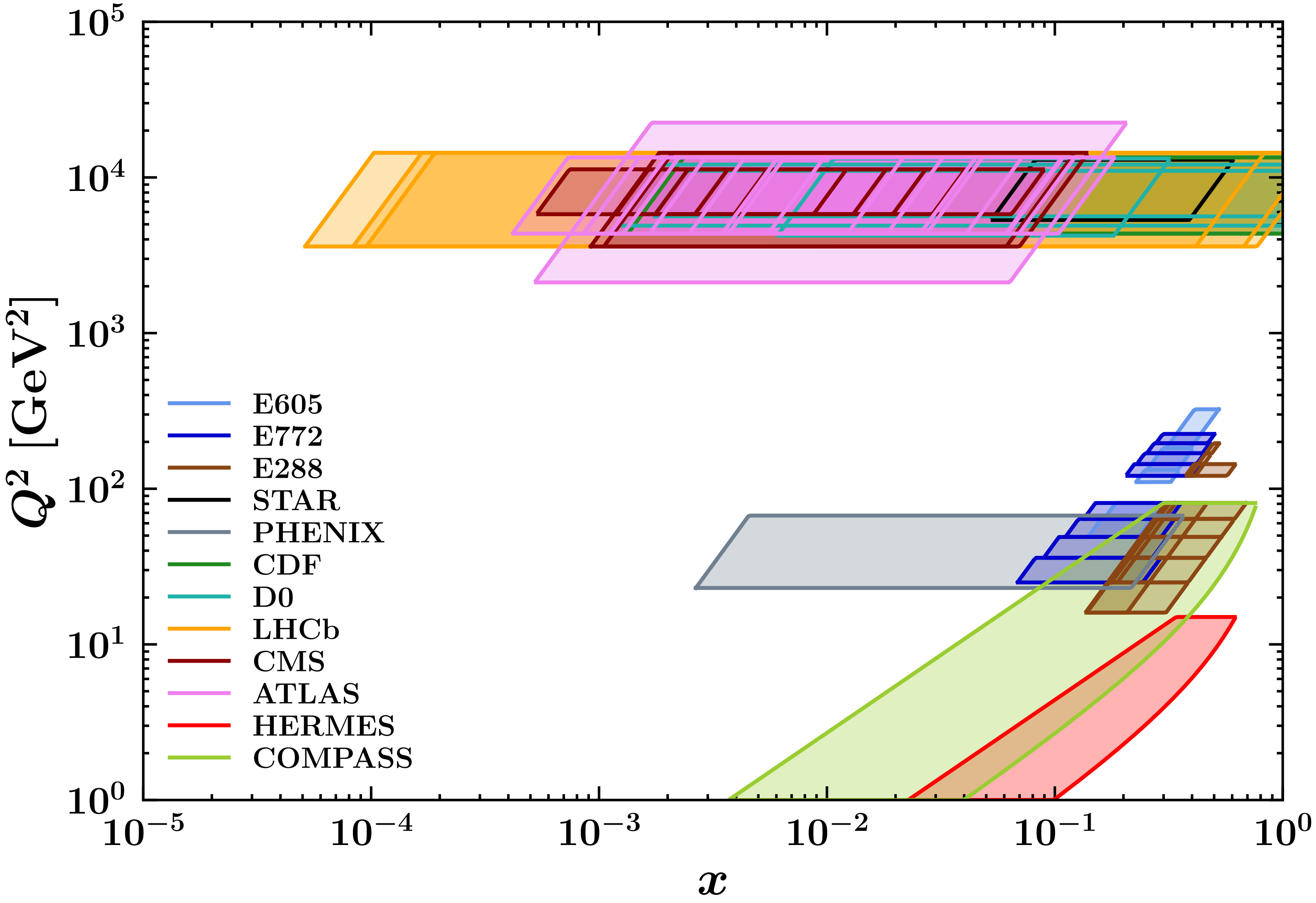
Fixed-target:

E288, E605, E772

Collider mode:

RHIC, Tevatron, LHC

SIDIS data 1547



MAP22: included data sets

Drell-Yan data 484

Fixed-target:

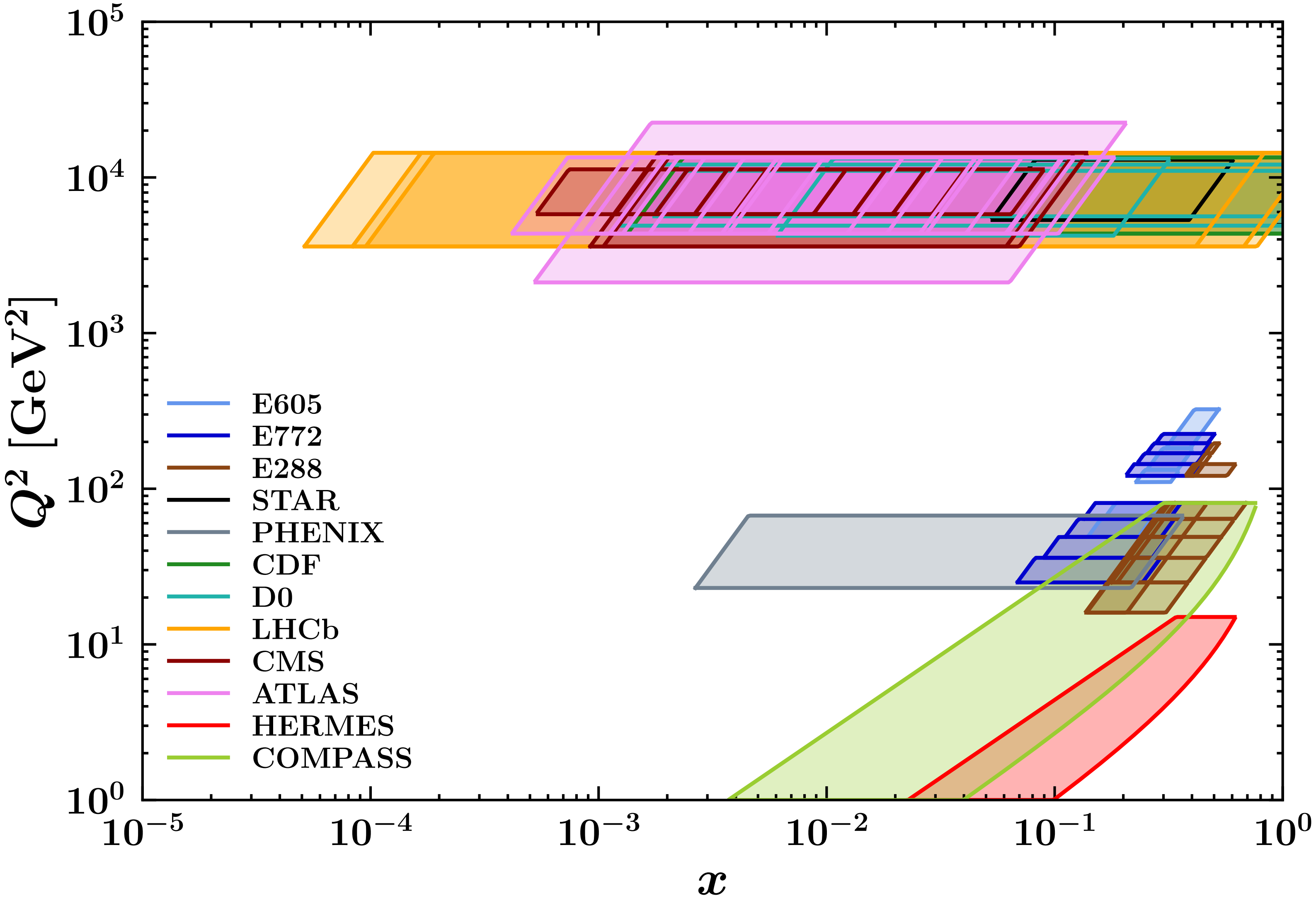
E288, E605, E772

Collider mode:

RHIC, Tevatron, LHC

SIDIS data 1547

HERMES, COMPASS



MAP22: included data sets

Drell-Yan data **484**

Fixed-target:

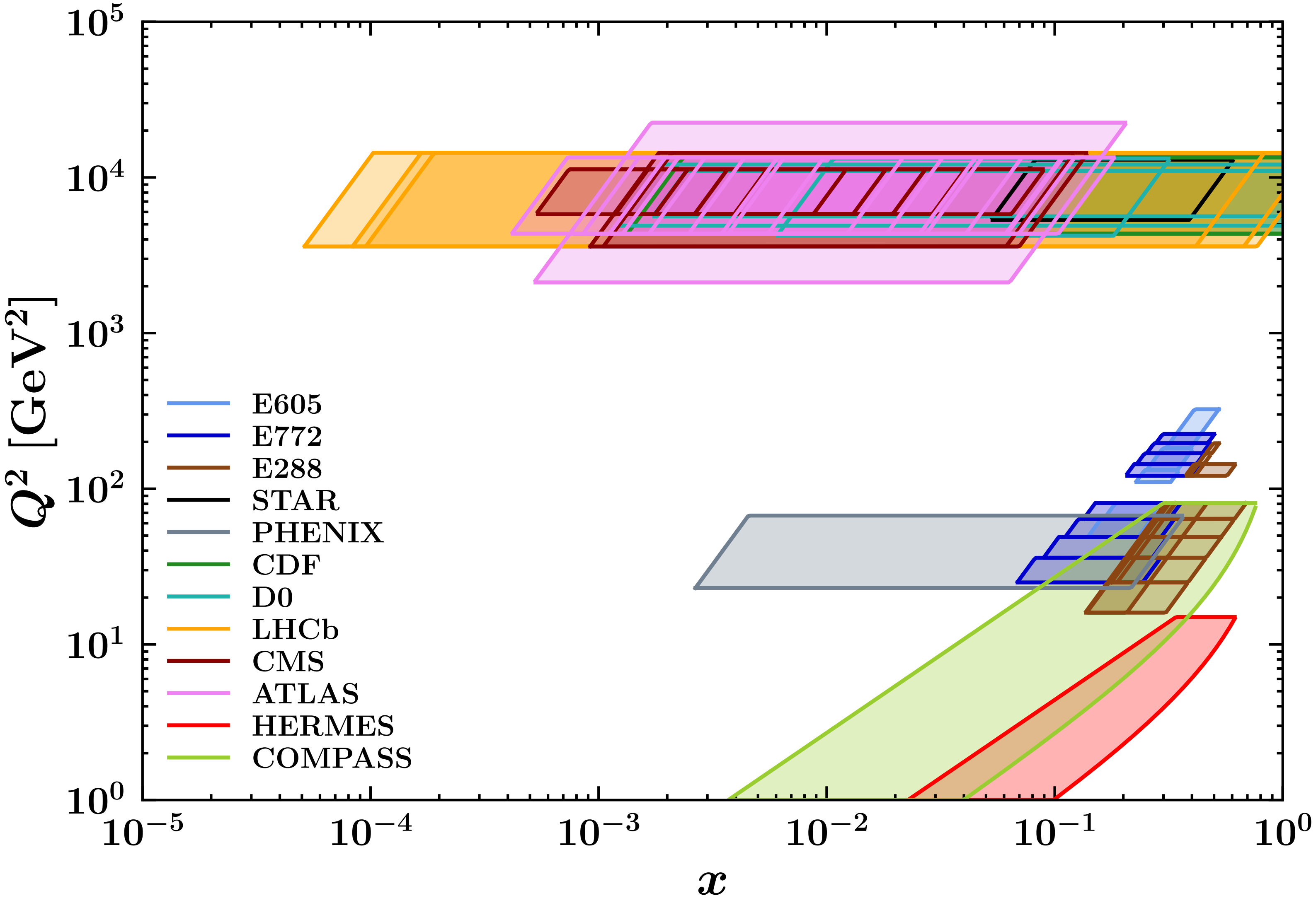
E288, E605, E772

Collider mode:

RHIC, Tevatron, LHC

SIDIS data **1547**

HERMES, COMPASS



Total number of data: 2031

MAP22: Perturbative accuracy

Resummation of large logs

-

MAP22: Perturbative accuracy

Resummation of large logs

$$S_{\text{pert}}(\mu_b,\mu) = 1 + \sum_{k=0}^{\infty} \boxed{R_{\text{N}^k\text{LL}}}$$

$$S_{\text{pert}}(\mu_b,\mu) = 1 + \sum_{k=0}^{\infty} \sum_{n=1+[k/2]}^{\infty} \left(\frac{\alpha_S(\mu)}{4\pi}\right)^n \sum_{k=1}^{2n} L^{2n-k} R^{(n,2n-k)}$$

-

MAP22: Perturbative accuracy

Resummation of large logs

$$S_{\text{pert}}(\mu_b, \mu) = 1 + \sum_{k=0}^{\infty} \boxed{R_{\text{N}^k\text{LL}}}$$

$$S_{\text{pert}}(\mu_b, \mu) = 1 + \sum_{k=0}^{\infty} \sum_{n=1+[k/2]}^{\infty} \left(\frac{\alpha_S(\mu)}{4\pi}\right)^n \sum_{k=1}^{2n} \boxed{L^{2n-k}} R^{(n,2n-k)} \qquad L = \ln\left(\frac{\mu^2}{\mu_b^2}\right)$$

-

MAP22: Perturbative accuracy

Resummation of large logs

$$S_{\mathrm{pert}}(\mu_b, \mu) = 1 + \sum_{k=0}^{\infty} \boxed{R_{\mathrm{N}^k\mathrm{LL}}}$$

$$S_{\mathrm{pert}}(\mu_b, \mu) = 1 + \sum_{k=0}^{\infty} \sum_{n=1+[k/2]}^{\infty} \left(\frac{\alpha_S(\mu)}{4\pi}\right)^n \sum_{k=1}^{2n} \boxed{L^{2n-k}} R^{(n,2n-k)} \qquad L = \ln\left(\frac{\mu^2}{\mu_b^2}\right)$$

Accuracy	*H* and *C*	*K* and γ_F	γ_K	PDF/FF and α_S evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL’	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL’	2	2	3	NNLO
N^{3}LL$^-$	**2**	**3**	**4**	**NNLO/NLO**
N^{3}LL	2	3	4	NNLO
N^{3}LL’	3	3	4	N^{3}LO

Bacchetta, Bertone, Bissolotti, et al., JHEP 07 (2020)
TMD handbook, Boussarie, et al., 2023

MAP22: NP parametrization

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\}$$

Bacchetta, Gamberg, Goldstein, et al., PLB 659 (2008)
Bacchetta, Conti, Radici, PRD 78 (2008)
Pasquini, Cazzaniga, Boffi, PRD 78 (2008)
Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)
Burkardt, Pasquini, EPJA (2016)
Grewal, Kang, Qiu, Signori, PRD 101 (2020)

MAP22: NP parametrization

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\}$$

Bacchetta, Gamberg, Goldstein, et al., PLB 659 (2008)
Bacchetta, Conti, Radici, PRD 78 (2008)
Pasquini, Cazzaniga, Boffi, PRD 78 (2008)
Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)
Burkardt, Pasquini, EPJA (2016)
Grewal, Kang, Qiu, Signori, PRD 101 (2020)

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \mathrm{F.T.\ of} \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

MAP22: NP parametrization

Bacchetta, Gamberg, Goldstein, et al., PLB 659 (2008)
Bacchetta, Conti, Radici, PRD 78 (2008)
Pasquini, Cazzaniga, Boffi, PRD 78 (2008)
Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)
Burkardt, Pasquini, EPJA (2016)
Grewal, Kang, Qiu, Signori, PRD 101 (2020)

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\}$$

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \mathrm{F.T.\ of} \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

$$g_1(x) = N_1 \frac{(1-x)^\alpha \; x^\sigma}{(1-\hat{x})^\alpha \; \hat{x}^\sigma}$$

MAP22: NP parametrization

Bacchetta, Gamberg, Goldstein, et al., PLB 659 (2008)

Bacchetta, Conti, Radici, PRD 78 (2008)

Pasquini, Cazzaniga, Boffi, PRD 78 (2008)

Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)

Burkardt, Pasquini, EPJA (2016)

Grewal, Kang, Qiu, Signori, PRD 101 (2020)

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\}$$

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

$$g_1(x) = N_1 \, \frac{(1-x)^\alpha \; x^\sigma}{(1-\hat{x})^\alpha \; \hat{x}^\sigma}$$

$$D_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

$$g_3(z) = N_3 \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

MAP22: NP parametrization

Bacchetta, Gamberg, Goldstein, et al., PLB 659 (2008)

Bacchetta, Conti, Radici, PRD 78 (2008)

Pasquini, Cazzaniga, Boffi, PRD 78 (2008)

Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)

Burkardt, Pasquini, EPJA (2016)

Grewal, Kang, Qiu, Signori, PRD 101 (2020)

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\}$$

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

$$g_1(x) = N_1 \frac{(1-x)^\alpha \; x^\sigma}{(1-\hat{x})^\alpha \; \hat{x}^\sigma}$$

$$D_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

$$g_3(z) = N_3 \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

MAP22: NP parametrization

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\}$$

Bacchetta, Gamberg, Goldstein, et al., PLB 659 (2008)
Bacchetta, Conti, Radici, PRD 78 (2008)
Pasquini, Cazzaniga, Boffi, PRD 78 (2008)
Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)
Burkardt, Pasquini, EPJA (2016)
Grewal, Kang, Qiu, Signori, PRD 101 (2020)

$$\boxed{f_{1\mathrm{NP}}(x, b_T^2)} \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

$$g_1(x) = N_1 \frac{(1-x)^\alpha \; x^\sigma}{(1-\hat{x})^\alpha \; \hat{x}^\sigma}$$

$$\boxed{D_{1\mathrm{NP}}(x, b_T^2)} \propto \text{F.T. of } \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

$$g_3(z) = N_3 \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

$$\boxed{g_K(b_T^2)} = -g_2^2 \frac{b_T^2}{4}$$

11 parameters for TMD PDF

+ 1 for NP evolution + 9 for TMD FF

= 21 free parameters

MAPTMD22 global fit $\Rightarrow$ MAPTMD24

- Global analysis of Drell-Yan and SIDIS data sets: *2031* data points
- Perturbative accuracy: N^3LL^-
- *Normalization prefactor* for SIDIS observables
- Number of fitted parameters: *21*
- Agreement with data: $\chi^2/N_{data} = 1.06$

MAP Collaboration, JHEP 10 (2022)

MAPTMD22 global fit ⇒ MAPTMD24

- Global analysis of Drell-Yan and SIDIS data sets: *2031* data points

  Same global data set

- Perturbative accuracy: N^3LL^-

- *Normalization prefactor* for SIDIS observables

- Number of fitted parameters: *21*

- Agreement with data: $\chi^2/N_{data} = 1.06$

MAP Collaboration, JHEP 10 (2022)

MAPTMD22 global fit ⇒ MAPTMD24

- Global analysis of Drell-Yan and SIDIS data sets: *2031* data points

  Same global data set

- Perturbative accuracy: N^3LL^-

  N^3LL

- *Normalization prefactor* for SIDIS observables

- Number of fitted parameters: *21*

- Agreement with data: $\chi^2/N_{data} = 1.06$

MAP Collaboration, JHEP 10 (2022)

MAPTMD22 global fit ⇒ MAPTMD24

- Global analysis of Drell-Yan and SIDIS data sets: *2031* data points

  Same global data set

- Perturbative accuracy: N^3LL^-

  N^3LL

MMHT
DSS

⇒

NNPDF
MAPFF

- *Normalization prefactor* for SIDIS observables

- Number of fitted parameters: *21*

- Agreement with data: $\chi^2/N_{data} = 1.06$

MAP Collaboration, JHEP 10 (2022)

MAPTMD22 global fit ⇒ MAPTMD24

- Global analysis of Drell-Yan and SIDIS data sets: *2031* data points

  Same global data set

- Perturbative accuracy: N^3LL^- MMHT ⇒ **NNPDF**

  N^3LL DSS **MAPFF**

- *Normalization prefactor* for SIDIS observables

  Same approach

- Number of fitted parameters: *21*

- Agreement with data: $\chi^2/N_{data} = 1.06$

MAP Collaboration, JHEP 10 (2022)

MAPTMD22 global fit ⇒ MAPTMD24

- Global analysis of Drell-Yan and SIDIS data sets: *2031* data points

  Same global data set

- Perturbative accuracy: N^3LL^- MMHT DSS ⇒ **NNPDF** **MAPFF**

  N^3LL

- *Normalization prefactor* for SIDIS observables

  Same approach

- Number of fitted parameters: *21*

  Same parameterization

- Agreement with data: $\chi^2/N_{data} = 1.06$

MAP Collaboration, JHEP 10 (2022)

MAPTMD22 global fit ⇒ MAPTMD24

- Global analysis of Drell-Yan and SIDIS data sets: *2031* data points

  Same global data set

- Perturbative accuracy: N^3LL^- MMHT ⇒ **NNPDF**

  N^3LL DSS **MAPFF**

- *Normalization prefactor* for SIDIS observables

  Same approach

- Number of fitted parameters: *21*

  Same parameterization

- Agreement with data: $\chi^2/N_{data} = 1.06$

  Worse agreement $\chi^2/N_{data} = 1.40$

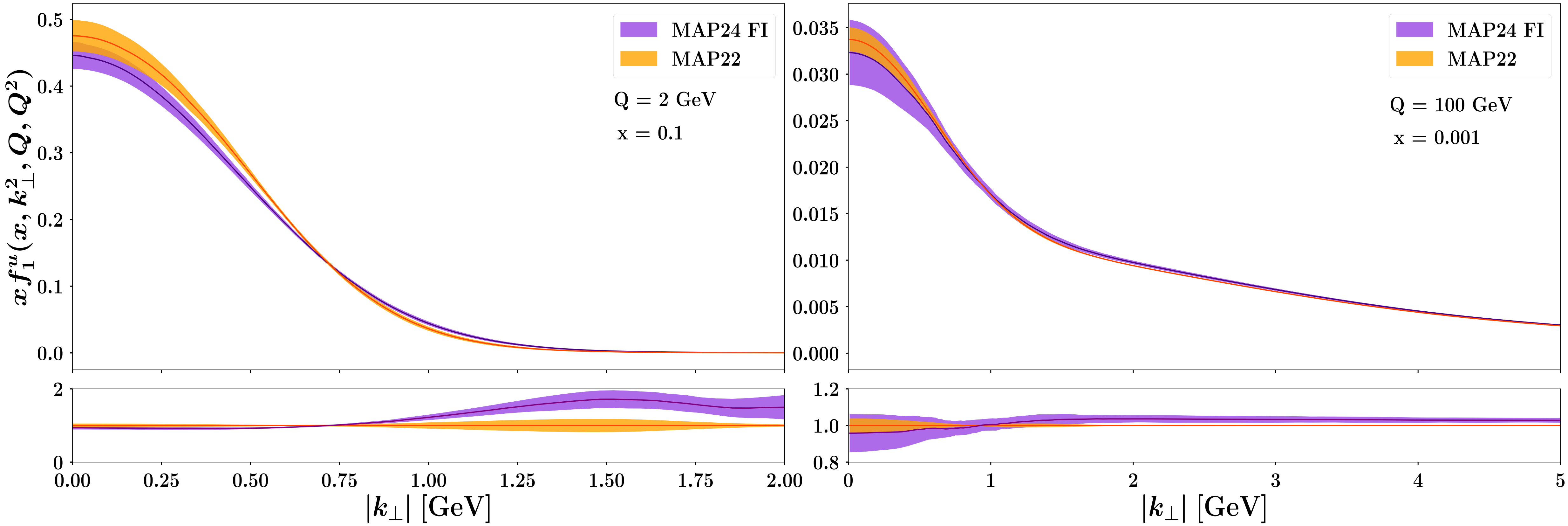
MAP Collaboration, JHEP 10 (2022)

MAPTMD24

	χ^2/N_{data}		
Configuration	DY	SIDIS	Total
MMHT+DSS (MAP22)	1.66	0.87	**1.06**
NNPDF+MAPFF (MAP24 FI)	1.58	1.34	**1.40**

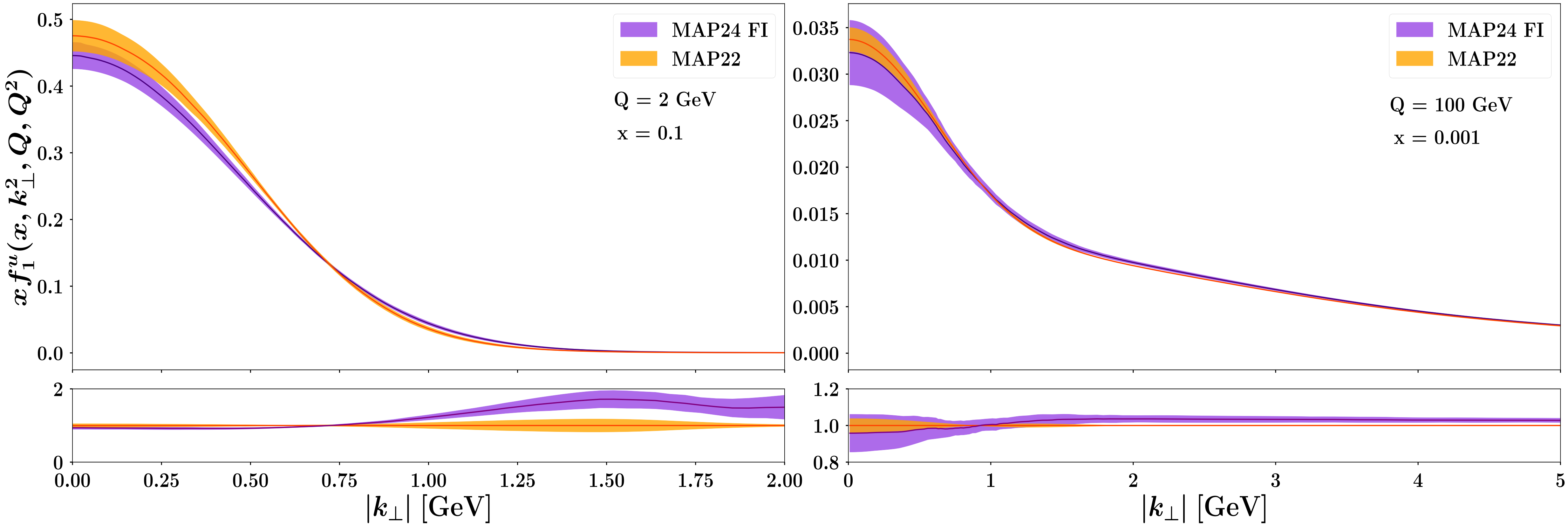
MAPTMD24

	χ^2/N_{data}		
Configuration	DY	SIDIS	Total
MMHT+DSS (MAP22)	1.66	0.87	**1.06**
NNPDF+MAPFF (MAP24 FI)	1.58	1.34	**1.40**



MAPTMD24

	χ^2/N_{data}		
Configuration	DY	SIDIS	Total
MMHT+DSS (MAP22)	1.66	0.87	**1.06**
NNPDF+MAPFF (MAP24 FI)	1.58	1.34	**1.40**



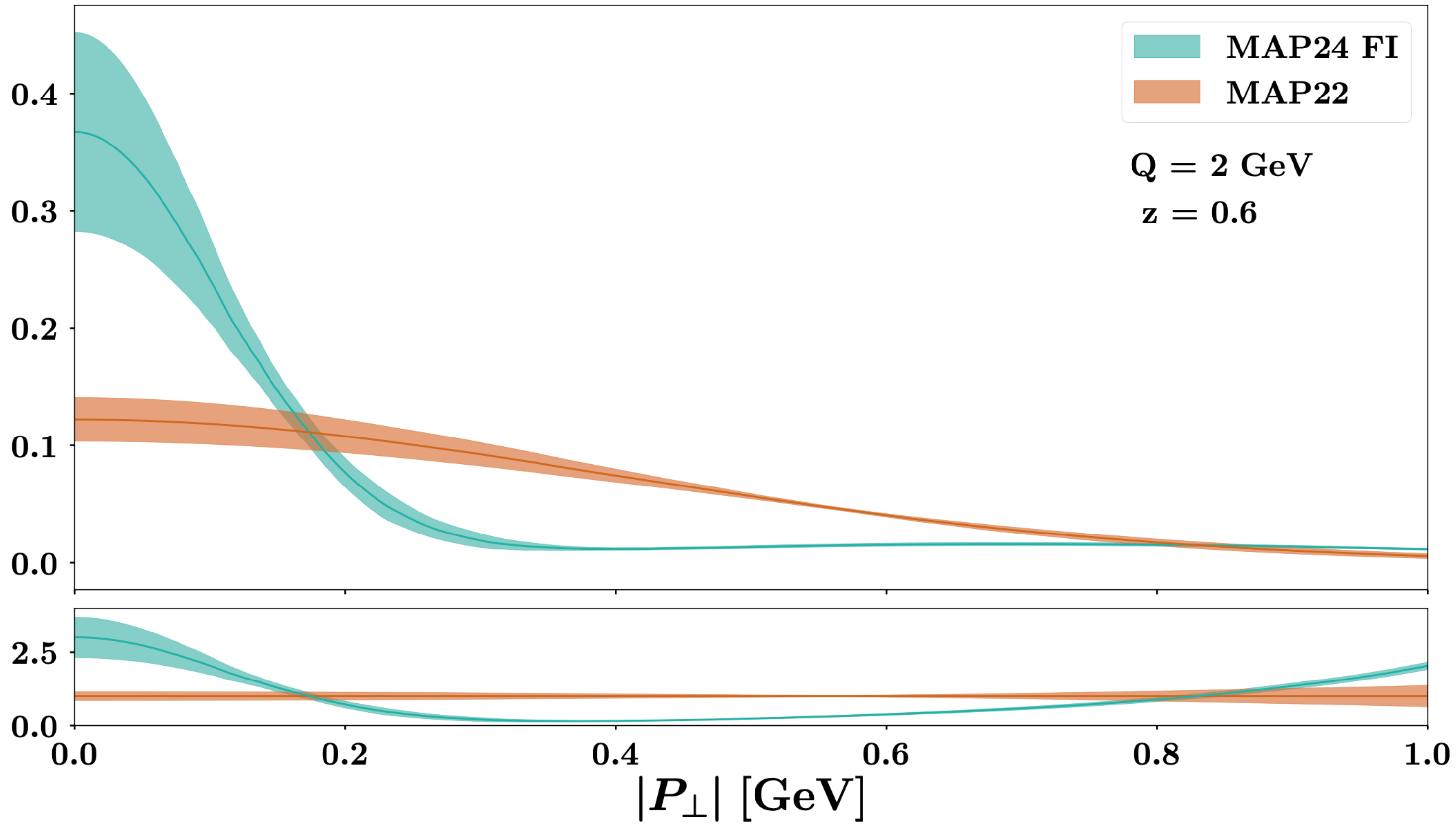
TMD PDFs are compatible with MAP22

MAPTMD24

	χ^2/N_{data}		
Configuration	DY	SIDIS	Total
MMHT+**DSS** (MAP22)	1.66	0.87	**1.06**
NNPDF+**MAPFF** (MAP24 FI)	1.58	1.34	**1.40**

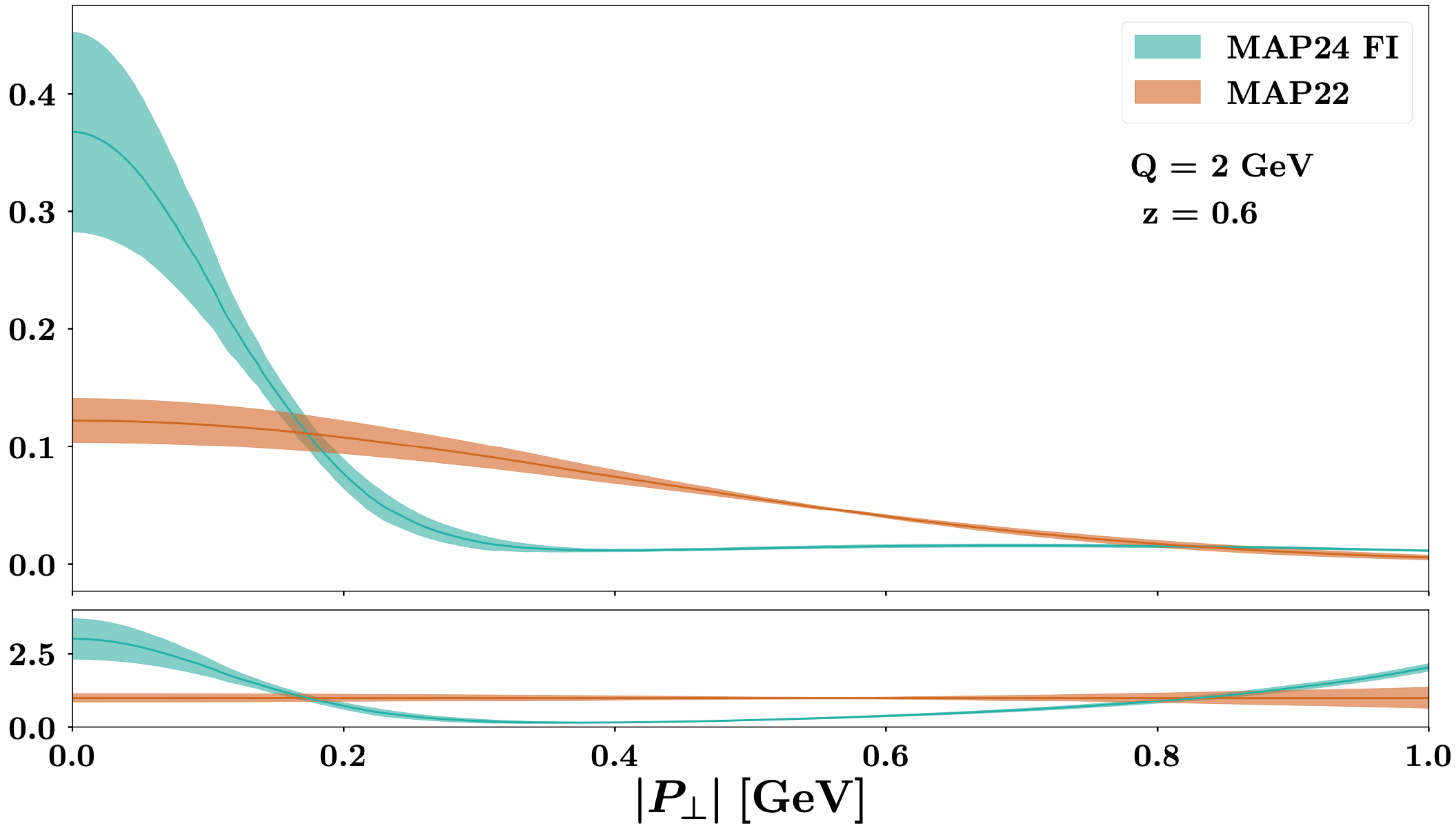
MAPTMD24

	χ^2/N_{data}		
Configuration	DY	SIDIS	Total
MMHT+**DSS** (MAP22)	1.66	0.87	**1.06**
NNPDF+**MAPFF** (MAP24 FI)	1.58	1.34	**1.40**



MAPTMD24

	χ^2/N_{data}		
Configuration	DY	SIDIS	Total
MMHT+**DSS** (MAP22)	1.66	0.87	**1.06**
NNPDF+**MAPFF** (MAP24 FI)	1.58	1.34	**1.40**

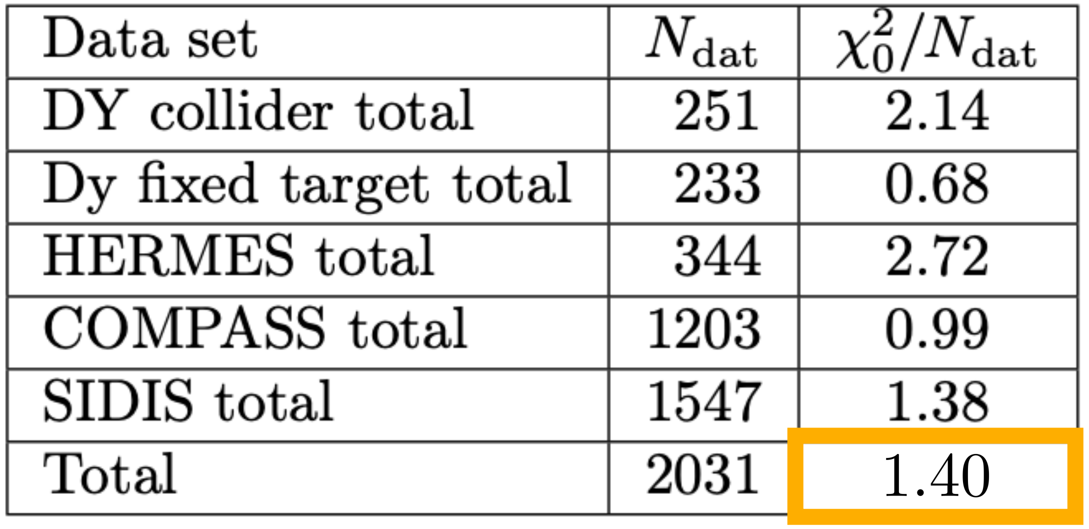


MAPFF1.0nnlo

- approx NNLO
- NN approach
- New behaviors
- Smaller uncertainties

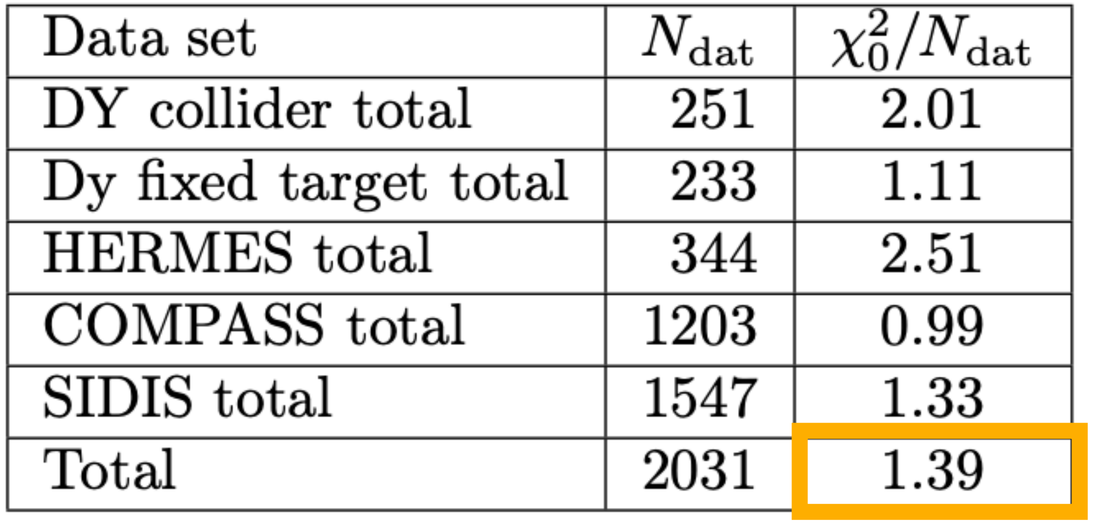
MAPTMD24

NNPDF + MAPFF (MAP24 FI)

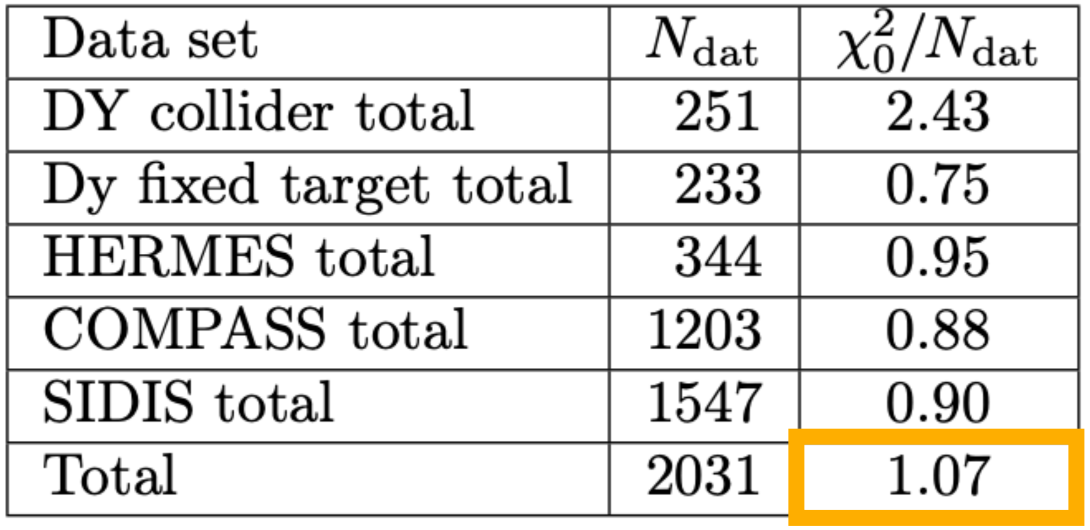


Data set	N_{dat}	χ_0^2/N_{dat}
DY collider total	251	2.14
Dy fixed target total	233	0.68
HERMES total	344	2.72
COMPASS total	1203	0.99
SIDIS total	1547	1.38
Total	2031	1.40

MMHT + MAPFF

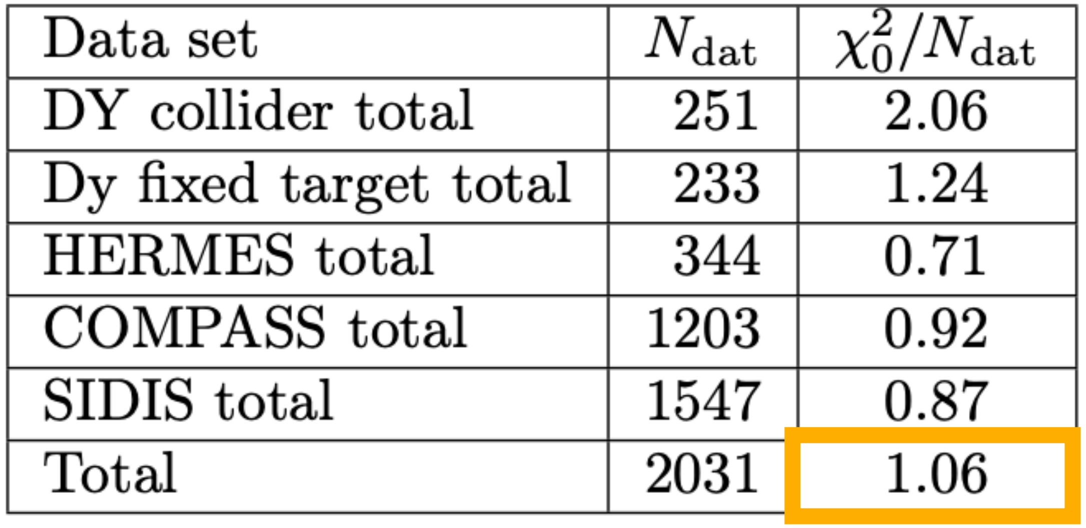


Data set	N_{dat}	χ_0^2/N_{dat}
DY collider total	251	2.01
Dy fixed target total	233	1.11
HERMES total	344	2.51
COMPASS total	1203	0.99
SIDIS total	1547	1.33
Total	2031	1.39



Data set	N_{dat}	χ_0^2/N_{dat}
DY collider total	251	2.43
Dy fixed target total	233	0.75
HERMES total	344	0.95
COMPASS total	1203	0.88
SIDIS total	1547	0.90
Total	2031	1.07

NNPDF + DSS

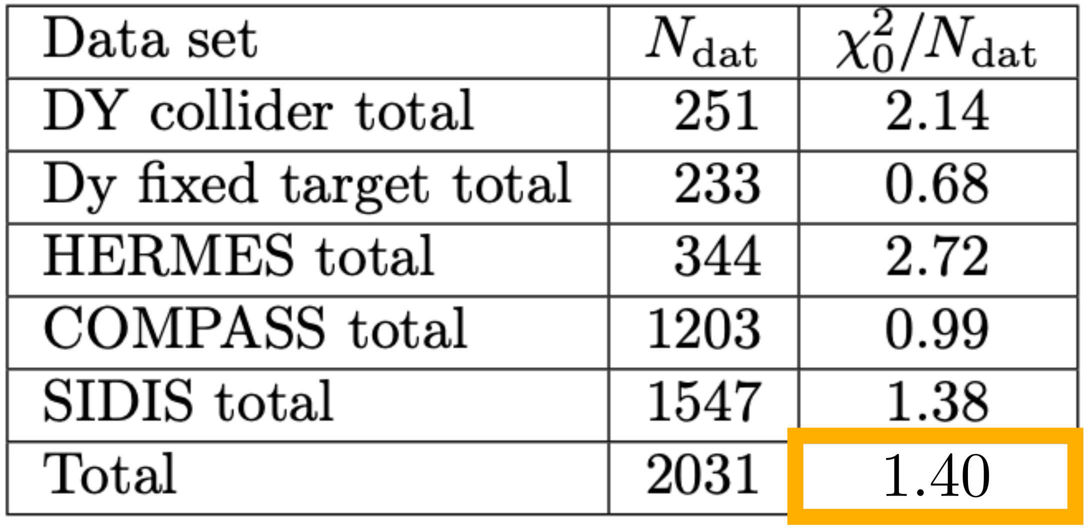


Data set	N_{dat}	χ_0^2/N_{dat}
DY collider total	251	2.06
Dy fixed target total	233	1.24
HERMES total	344	0.71
COMPASS total	1203	0.92
SIDIS total	1547	0.87
Total	2031	1.06

MMHT + DSS (MAP22)

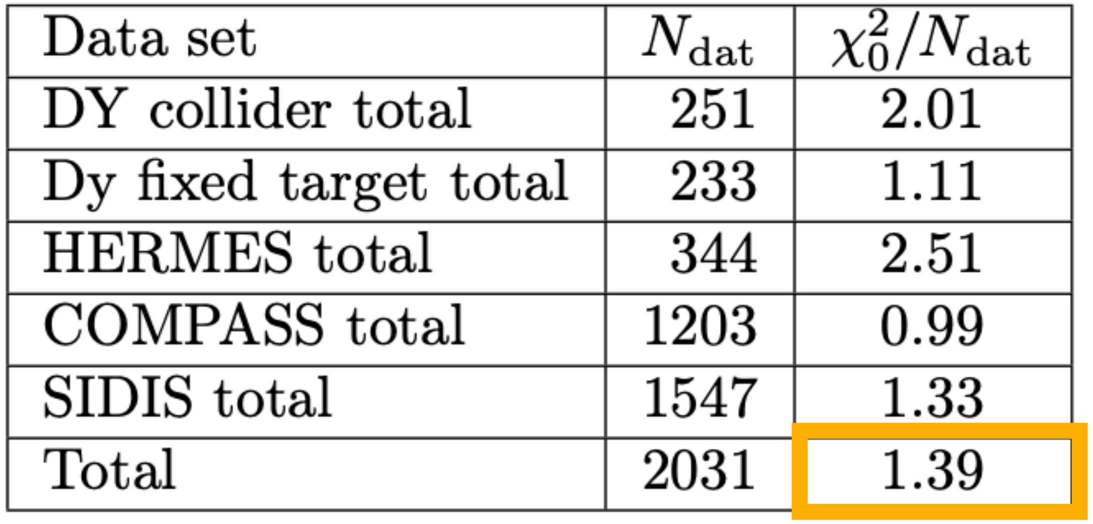
MAPTMD24

NNPDF + MAPFF (MAP24 FI)



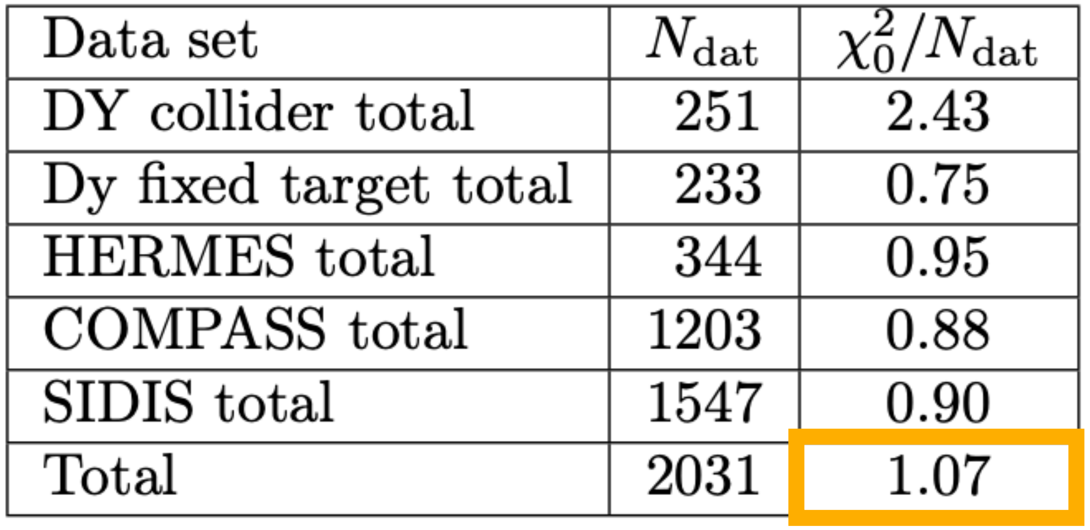
Data set	$N_{\rm dat}$	$\chi_0^2/N_{\rm dat}$
DY collider total	251	2.14
Dy fixed target total	233	0.68
HERMES total	344	2.72
COMPASS total	1203	0.99
SIDIS total	1547	1.38
Total	2031	1.40

MMHT + MAPFF

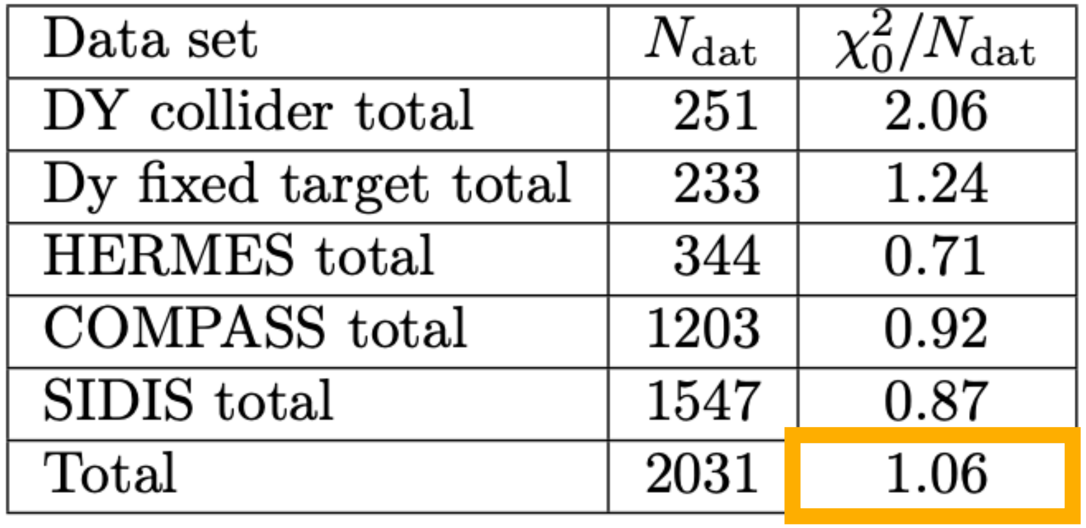


Data set	$N_{\rm dat}$	$\chi_0^2/N_{\rm dat}$
DY collider total	251	2.01
Dy fixed target total	233	1.11
HERMES total	344	2.51
COMPASS total	1203	0.99
SIDIS total	1547	1.33
Total	2031	1.39

COMPATIBILITY



Data set	$N_{\rm dat}$	$\chi_0^2/N_{\rm dat}$
DY collider total	251	2.43
Dy fixed target total	233	0.75
HERMES total	344	0.95
COMPASS total	1203	0.88
SIDIS total	1547	0.90
Total	2031	1.07



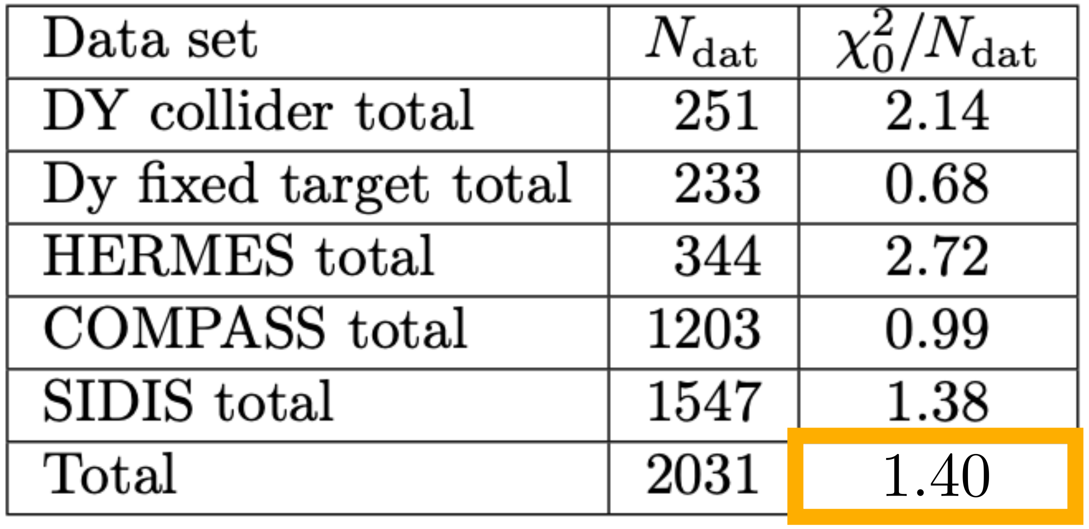
Data set	$N_{\rm dat}$	$\chi_0^2/N_{\rm dat}$
DY collider total	251	2.06
Dy fixed target total	233	1.24
HERMES total	344	0.71
COMPASS total	1203	0.92
SIDIS total	1547	0.87
Total	2031	1.06

NNPDF + DSS

MMHT + DSS (MAP22)

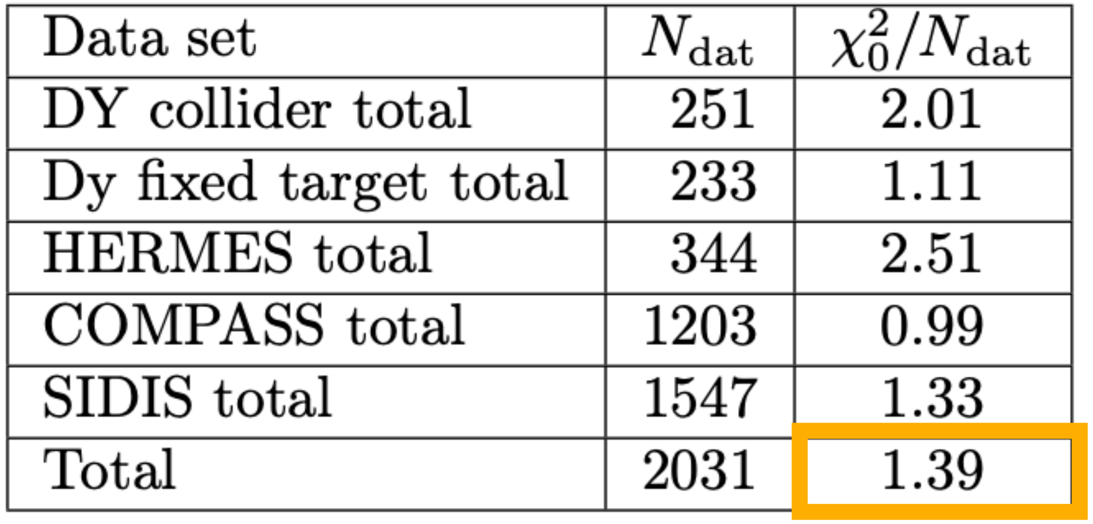
MAPTMD24

NNPDF + MAPFF (MAP24 FI)



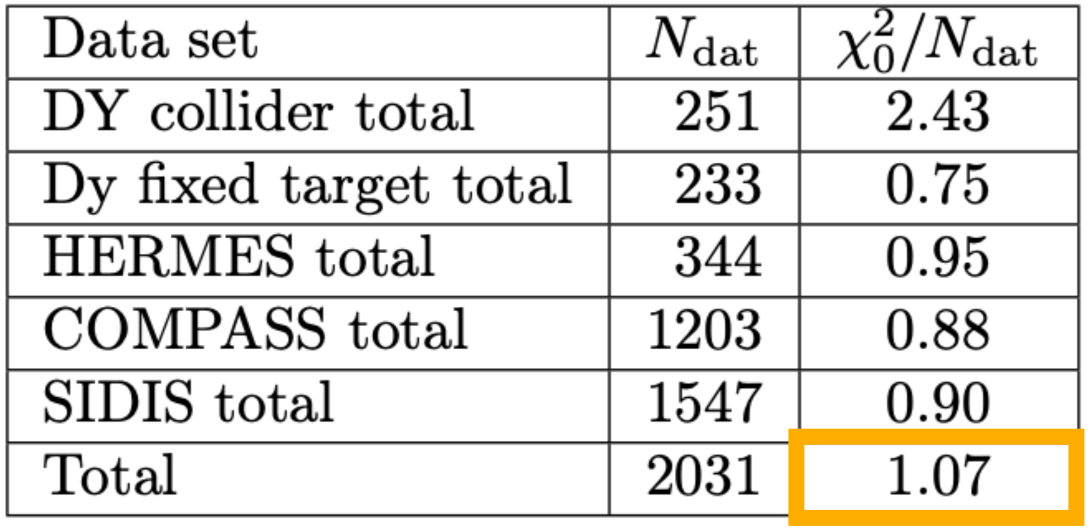
Data set	N_{dat}	$\chi_0^2/N_{\mathrm{dat}}$
DY collider total	251	2.14
Dy fixed target total	233	0.68
HERMES total	344	2.72
COMPASS total	1203	0.99
SIDIS total	1547	1.38
Total	2031	1.40

MMHT + MAPFF



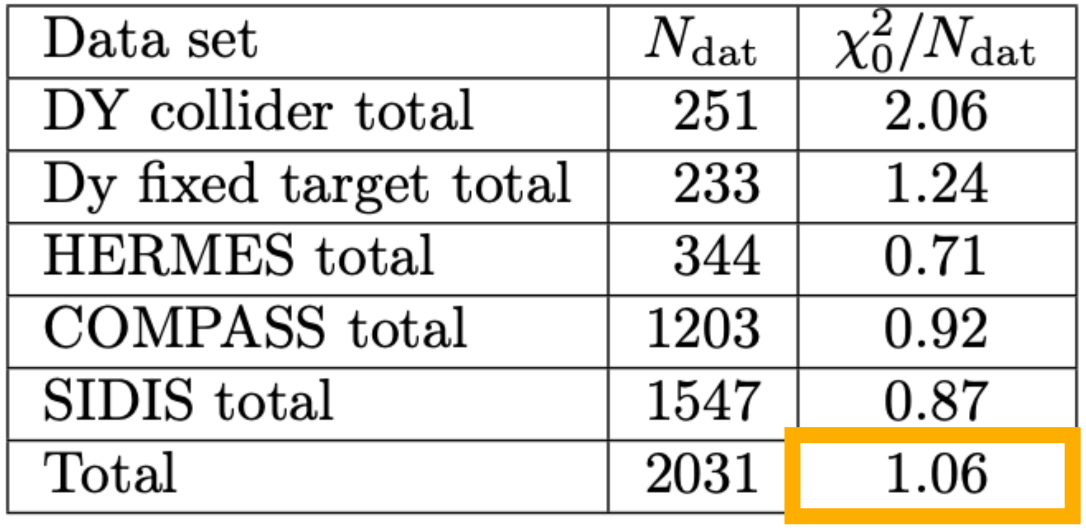
Data set	N_{dat}	$\chi_0^2/N_{\mathrm{dat}}$
DY collider total	251	2.01
Dy fixed target total	233	1.11
HERMES total	344	2.51
COMPASS total	1203	0.99
SIDIS total	1547	1.33
Total	2031	1.39

INCOMPATIBILITY



Data set	N_{dat}	$\chi_0^2/N_{\mathrm{dat}}$
DY collider total	251	2.43
Dy fixed target total	233	0.75
HERMES total	344	0.95
COMPASS total	1203	0.88
SIDIS total	1547	0.90
Total	2031	1.07

NNPDF + DSS

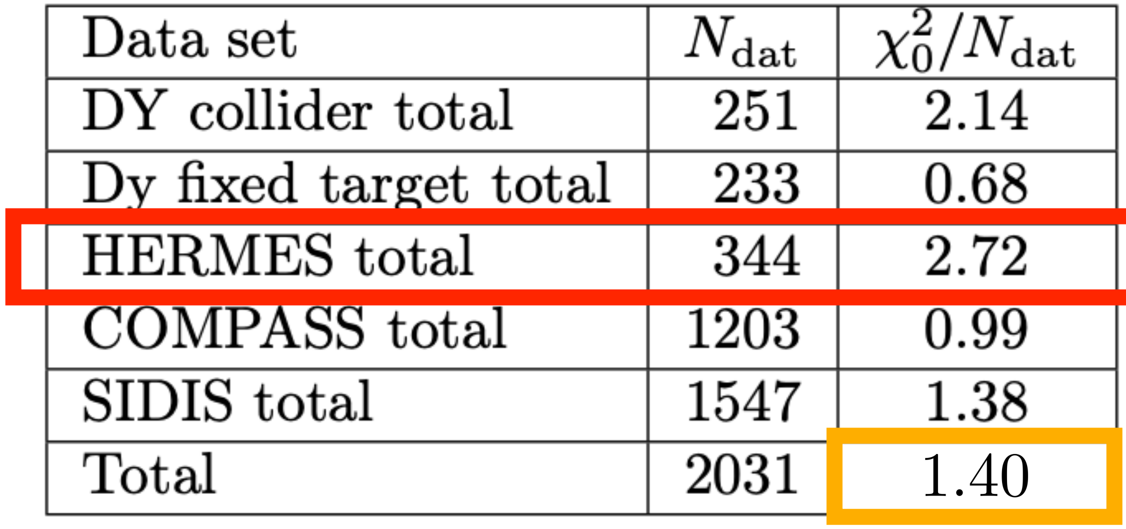


Data set	N_{dat}	$\chi_0^2/N_{\mathrm{dat}}$
DY collider total	251	2.06
Dy fixed target total	233	1.24
HERMES total	344	0.71
COMPASS total	1203	0.92
SIDIS total	1547	0.87
Total	2031	1.06

MMHT + DSS (MAP22)

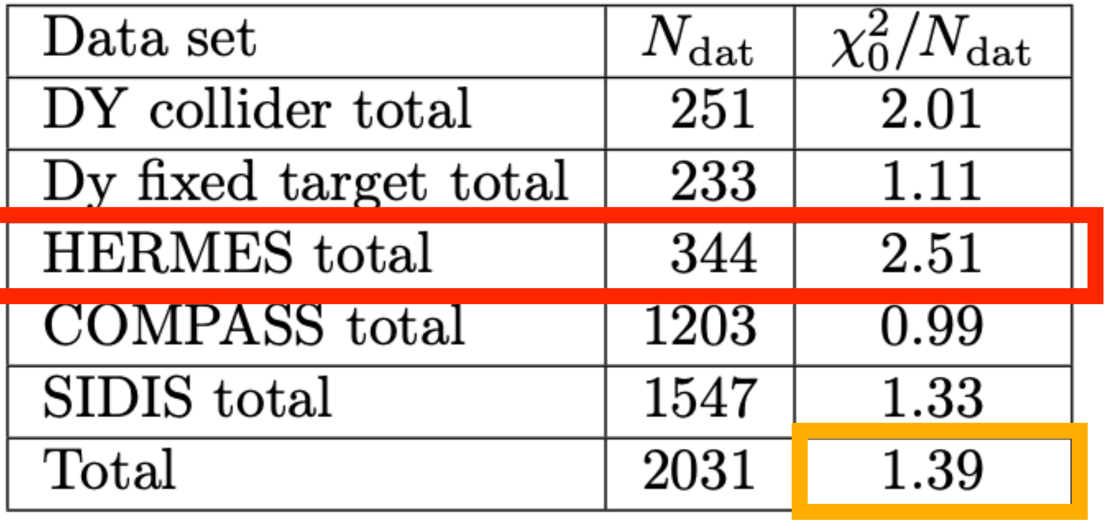
MAPTMD24

NNPDF + MAPFF (MAP24 FI)



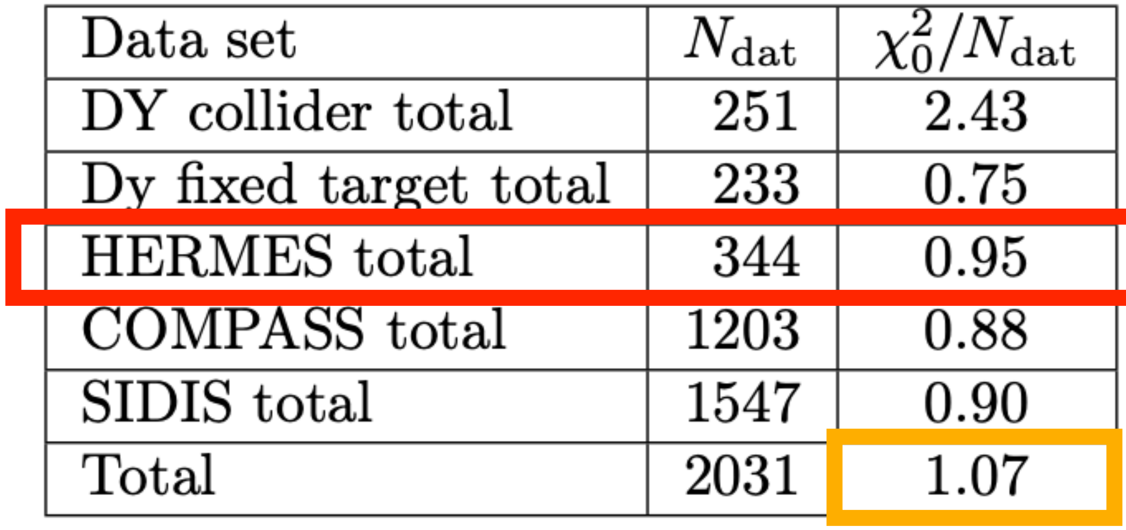
Data set	$N_{\rm dat}$	$\chi_0^2/N_{\rm dat}$
DY collider total	251	2.14
Dy fixed target total	233	0.68
HERMES total	344	2.72
COMPASS total	1203	0.99
SIDIS total	1547	1.38
Total	2031	1.40

MMHT + MAPFF



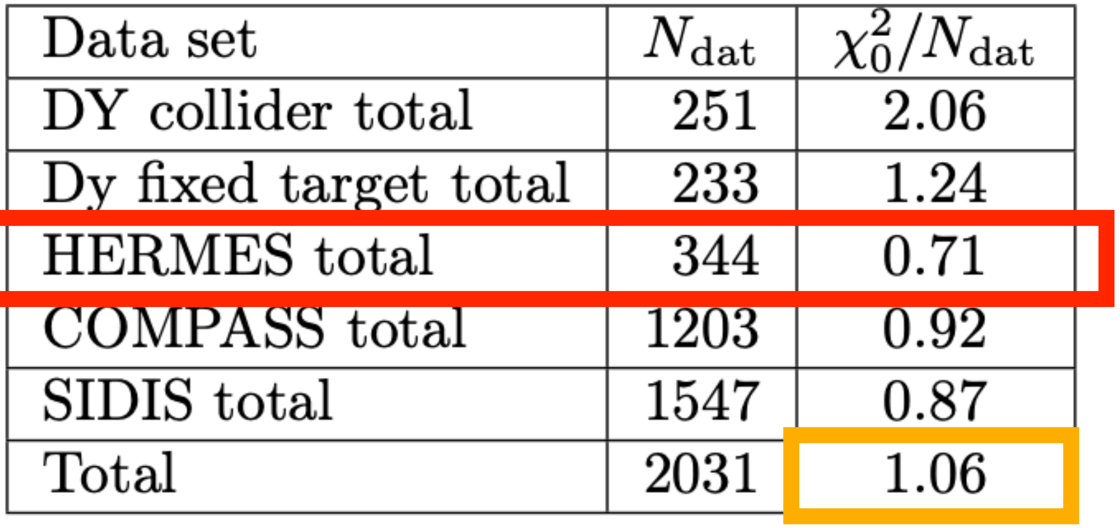
Data set	$N_{\rm dat}$	$\chi_0^2/N_{\rm dat}$
DY collider total	251	2.01
Dy fixed target total	233	1.11
HERMES total	344	2.51
COMPASS total	1203	0.99
SIDIS total	1547	1.33
Total	2031	1.39

INCOMPATIBILITY



Data set	$N_{\rm dat}$	$\chi_0^2/N_{\rm dat}$
DY collider total	251	2.43
Dy fixed target total	233	0.75
HERMES total	344	0.95
COMPASS total	1203	0.88
SIDIS total	1547	0.90
Total	2031	1.07

NNPDF + DSS



Data set	$N_{\rm dat}$	$\chi_0^2/N_{\rm dat}$
DY collider total	251	2.06
Dy fixed target total	233	1.24
HERMES total	344	0.71
COMPASS total	1203	0.92
SIDIS total	1547	0.87
Total	2031	1.06

MMHT + DSS (MAP22)

TU QUOQUE,

~~BRUTE~~

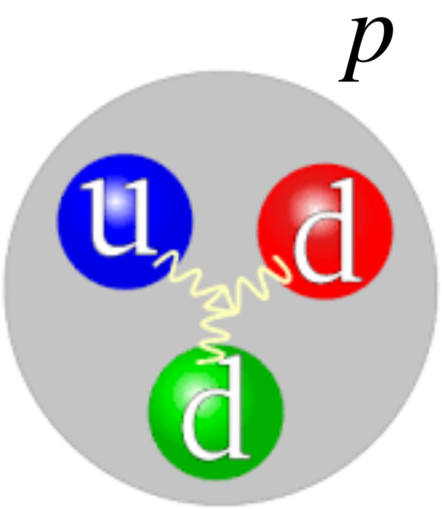
HERMES

MAPTMD24: new approach

Solution: we need **flavor dependence** to obtain a good agreement between theory and experiments

MAPTMD24: new approach

Solution: we need **flavor dependence** to obtain a good agreement between theory and experiments



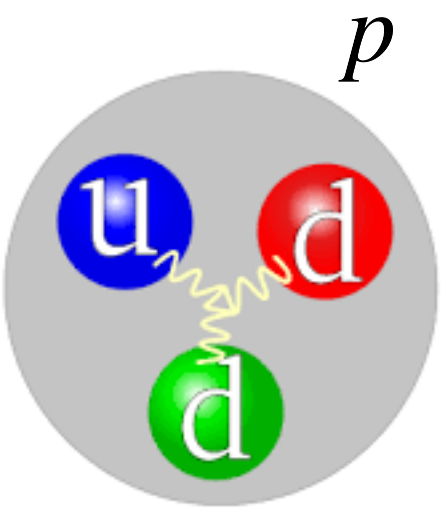
u, d

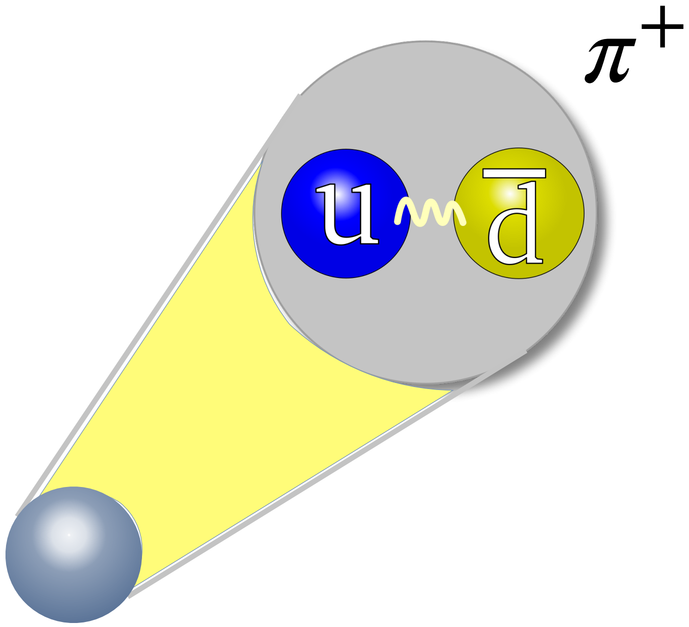
$\bar{u}, \bar{d}$

$s\ \ (sea)$

MAPTMD24: new approach

Solution: we need **flavor dependence** to obtain a good agreement between theory and experiments





u, d

$\bar{u}, \bar{d}$

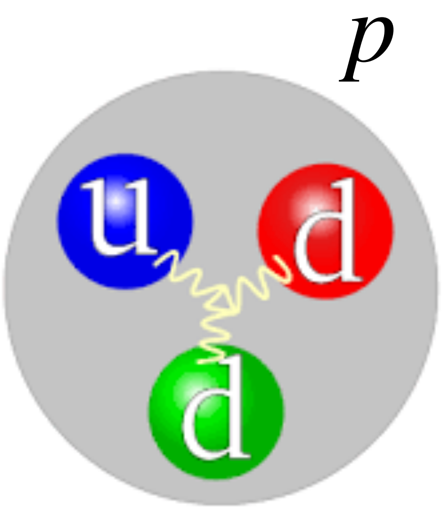
$s \ (sea)$

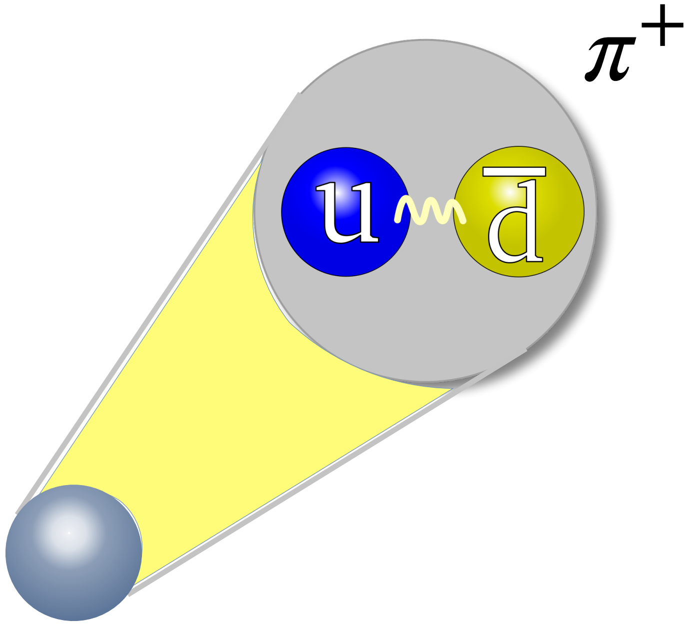
$u \to \pi^+, \ldots$

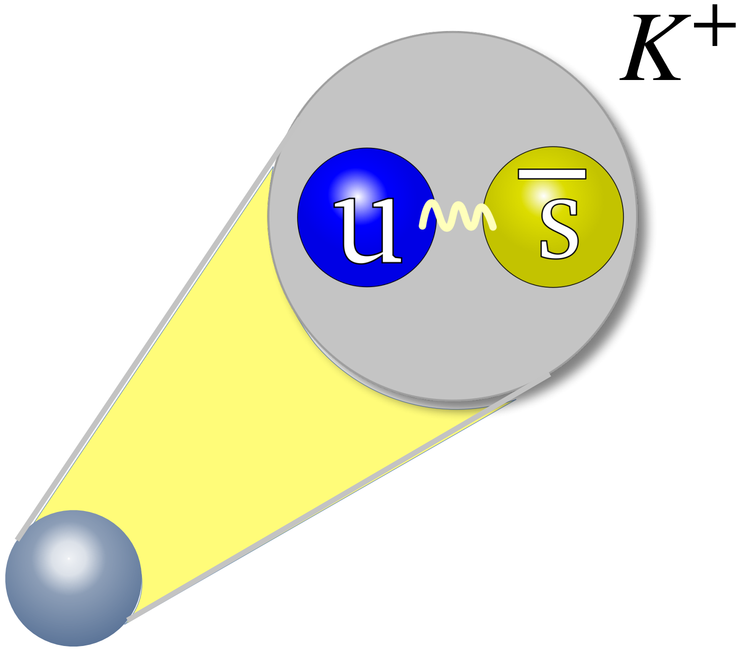
$d \to \pi^+, \ldots$

MAPTMD24: new approach

Solution: we need **flavor dependence** to obtain a good agreement between theory and experiments



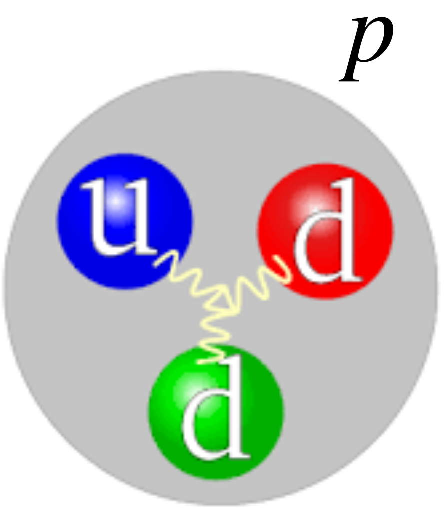


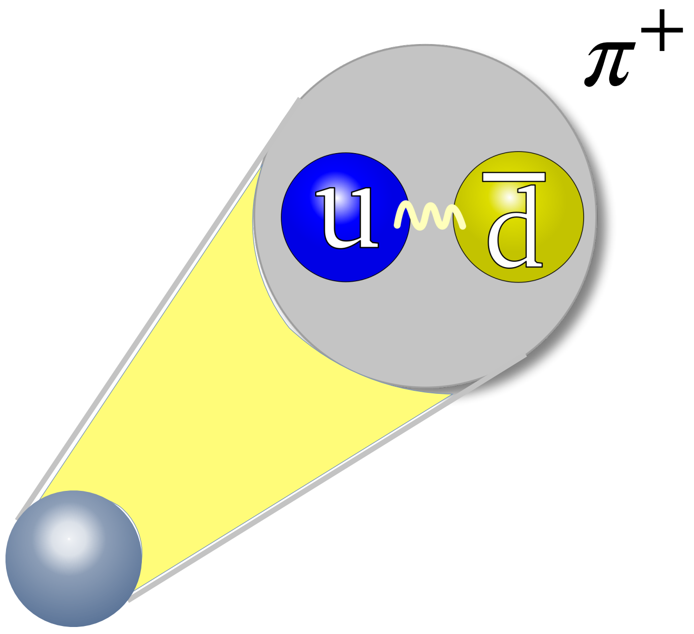


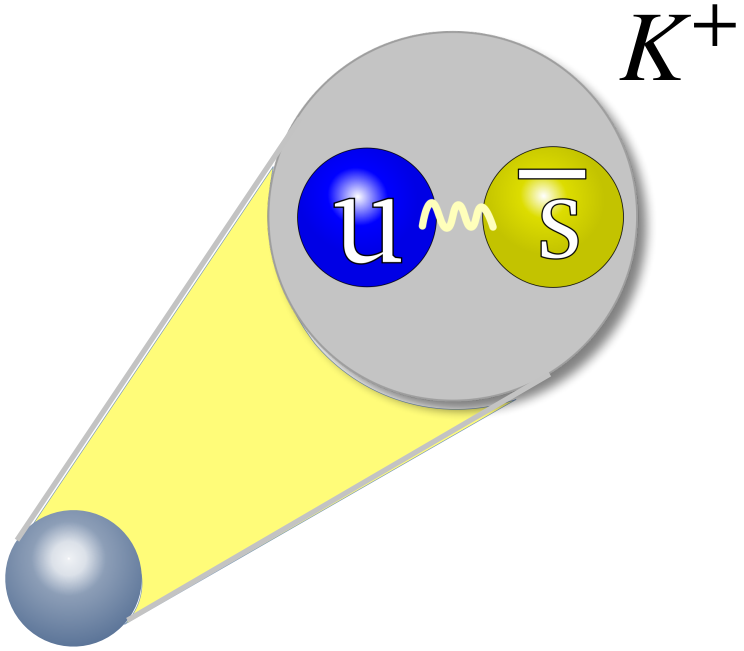
u, d	$u \to \pi^+, \ldots$	$u \to K^+, \ldots$
$\bar{u}, \bar{d}$	$d \to \pi^+, \ldots$	$\bar{s} \to K^+, \ldots$
$s \;\; (sea)$		$d \to K^+, \ldots$

MAPTMD24: new approach

Solution: we need **flavor dependence** to obtain a good agreement between theory and experiments







u, d

$\bar{u}, \bar{d}$

$s \ (sea)$

$u \to \pi^+, \dots$

$d \to \pi^+, \dots$

$u \to K^+, \dots$

$\bar{s} \to K^+, \dots$

$d \to K^+, \dots$

charge conjugation

MAPTMD24: new approach

HERMES

$$e + p \to e' + \pi^+ + X$$
$$e + p \to e' + \pi^- + X$$
$$e + p \to e' + K^+ + X$$
$$e + p \to e' + K^- + X$$

MAPTMD24: new approach

HERMES

$$e + p \to e' + \pi^+ + X$$
$$e + p \to e' + \pi^- + X$$
$$e + p \to e' + K^+ + X$$
$$e + p \to e' + K^- + X$$

high sensitivity to flavor dependence

MAPTMD24: new approach

HERMES

$$e + p \to e' + \pi^+ + X$$

$$e + p \to e' + \pi^- + X$$

$$e + p \to e' + K^+ + X$$

$$e + p \to e' + K^- + X$$

+ deuteron target

high sensitivity to flavor dependence

COMPASS deuteron target & unidentified final state hadron

Drell-Yan $q\bar{q}$ in the initial state

MAPTMD24: new approach

HERMES

$$e + p \to e' + \pi^+ + X$$
$$e + p \to e' + \pi^- + X$$
$$e + p \to e' + K^+ + X$$
$$e + p \to e' + K^- + X$$

+ deuteron target

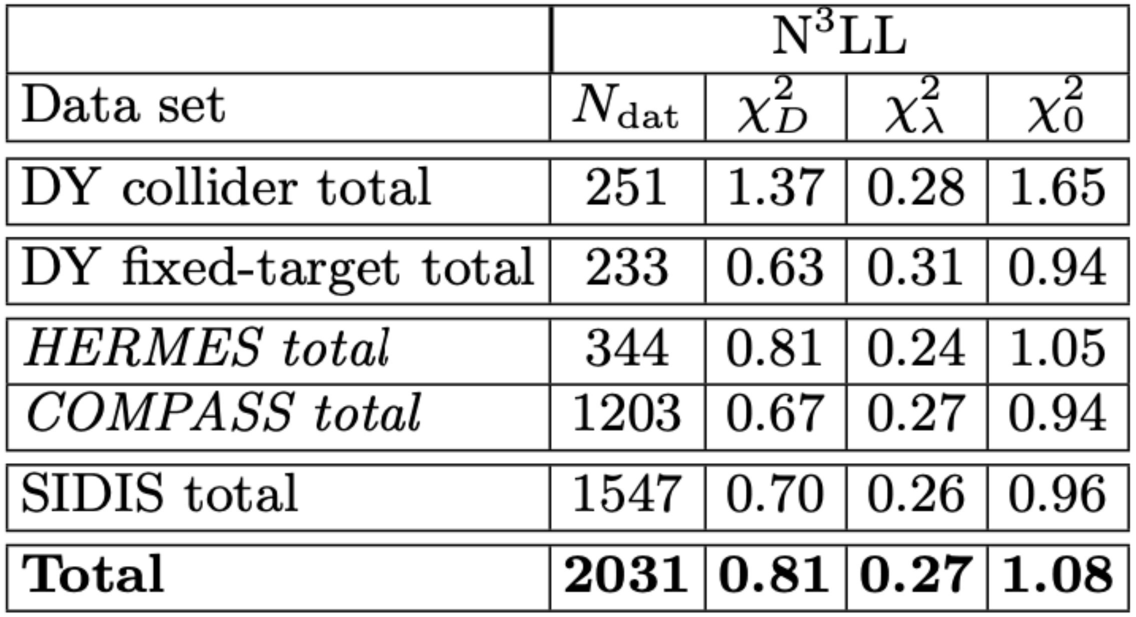
high sensitivity to flavor dependence

COMPASS deuteron target & unidentified final state hadron

Drell-Yan $q\bar{q}$ in the initial state

low sensitivity to flavor dependence

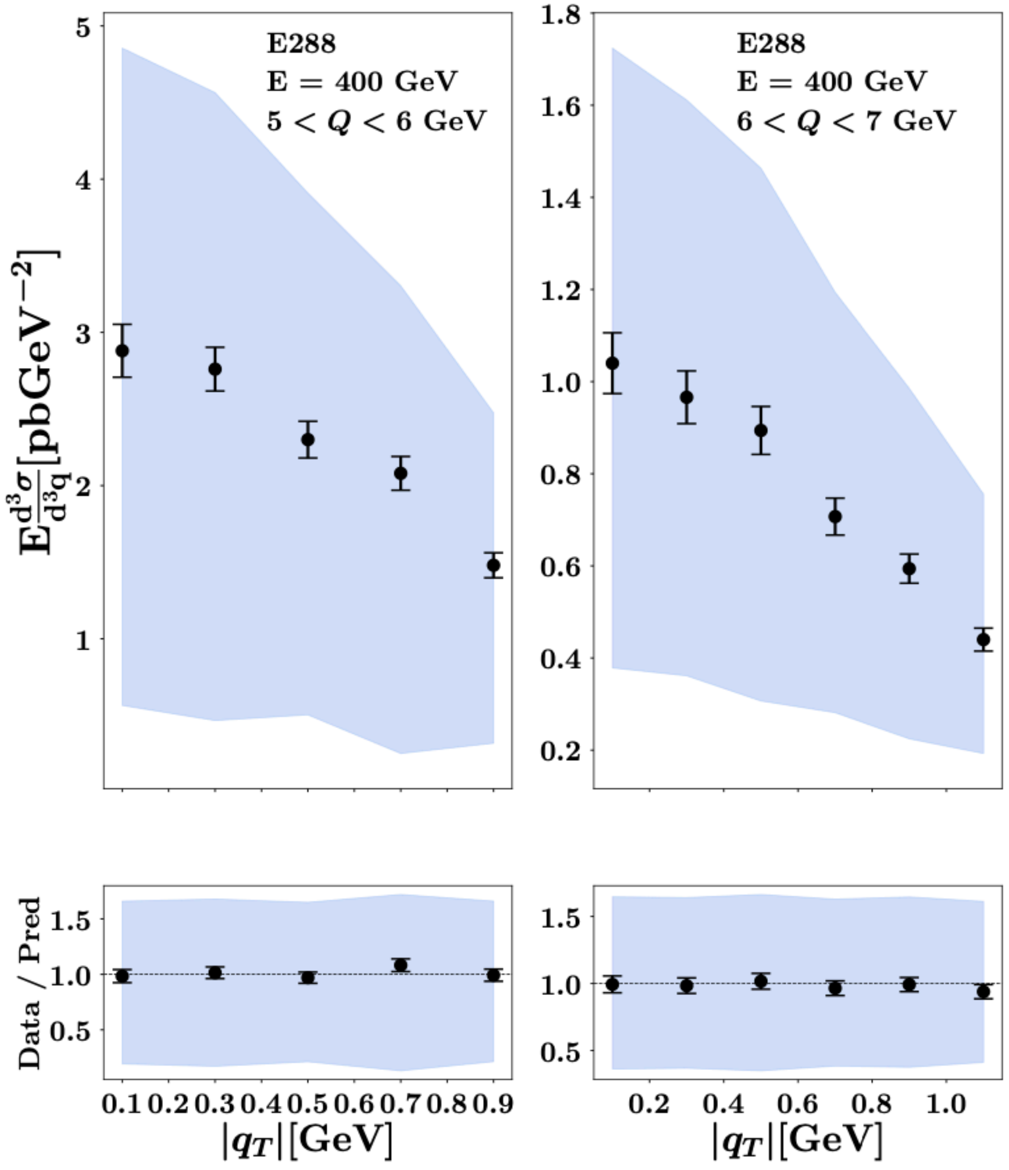
MAPTMD24: results



	N^{3}LL			
Data set	N_{dat}	χ^2_D	χ^2_λ	χ^2_0
DY collider total	251	1.37	0.28	1.65
DY fixed-target total	233	0.63	0.31	0.94
HERMES total	344	0.81	0.24	1.05
COMPASS total	1203	0.67	0.27	0.94
SIDIS total	1547	0.70	0.26	0.96
Total	**2031**	**0.81**	**0.27**	**1.08**

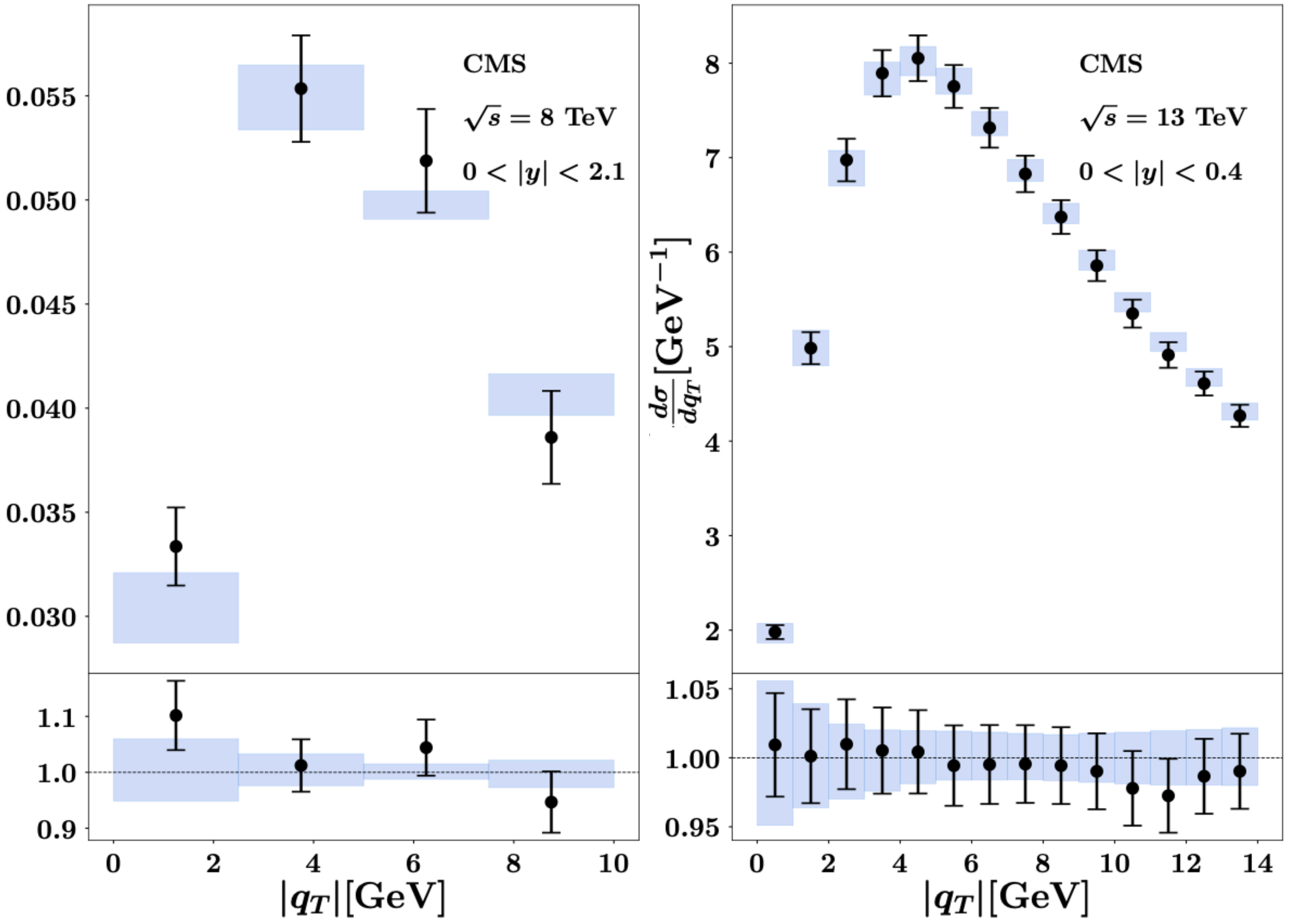
MAPTMD24: results

	N^{3}LL			
Data set	N_{dat}	χ^2_D	χ^2_λ	χ^2_0
DY collider total	251	1.37	0.28	1.65
DY fixed-target total	233	0.63	0.31	0.94
HERMES total	344	0.81	0.24	1.05
COMPASS total	1203	0.67	0.27	0.94
SIDIS total	1547	0.70	0.26	0.96
Total	**2031**	**0.81**	**0.27**	**1.08**



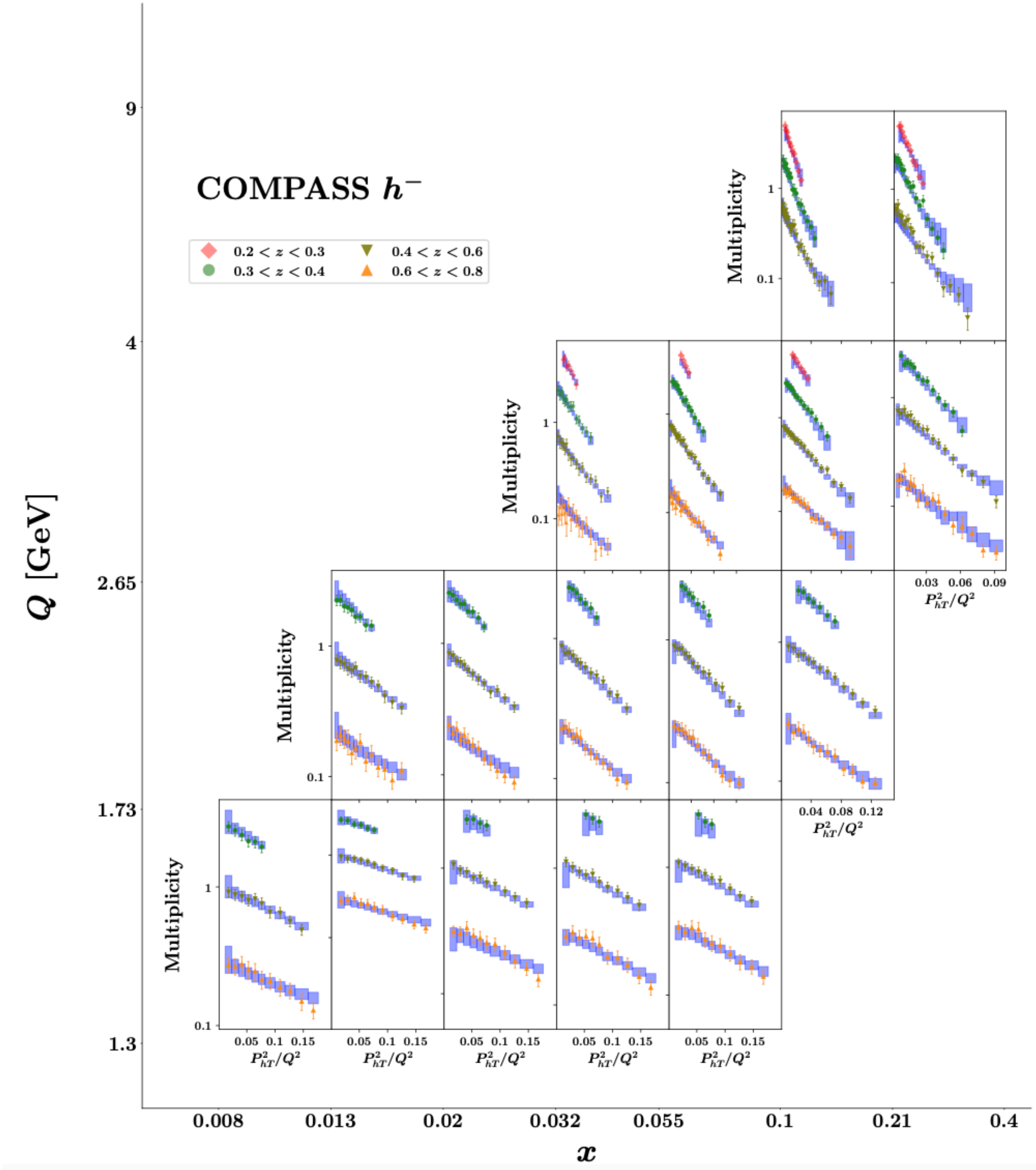
MAPTMD24: results

	N^{3}LL			
Data set	$N_{\rm dat}$	χ_D^2	χ_λ^2	χ_0^2
DY collider total	251	1.37	0.28	1.65
DY fixed-target total	233	0.63	0.31	0.94
HERMES total	344	0.81	0.24	1.05
COMPASS total	1203	0.67	0.27	0.94
SIDIS total	1547	0.70	0.26	0.96
Total	**2031**	**0.81**	**0.27**	**1.08**



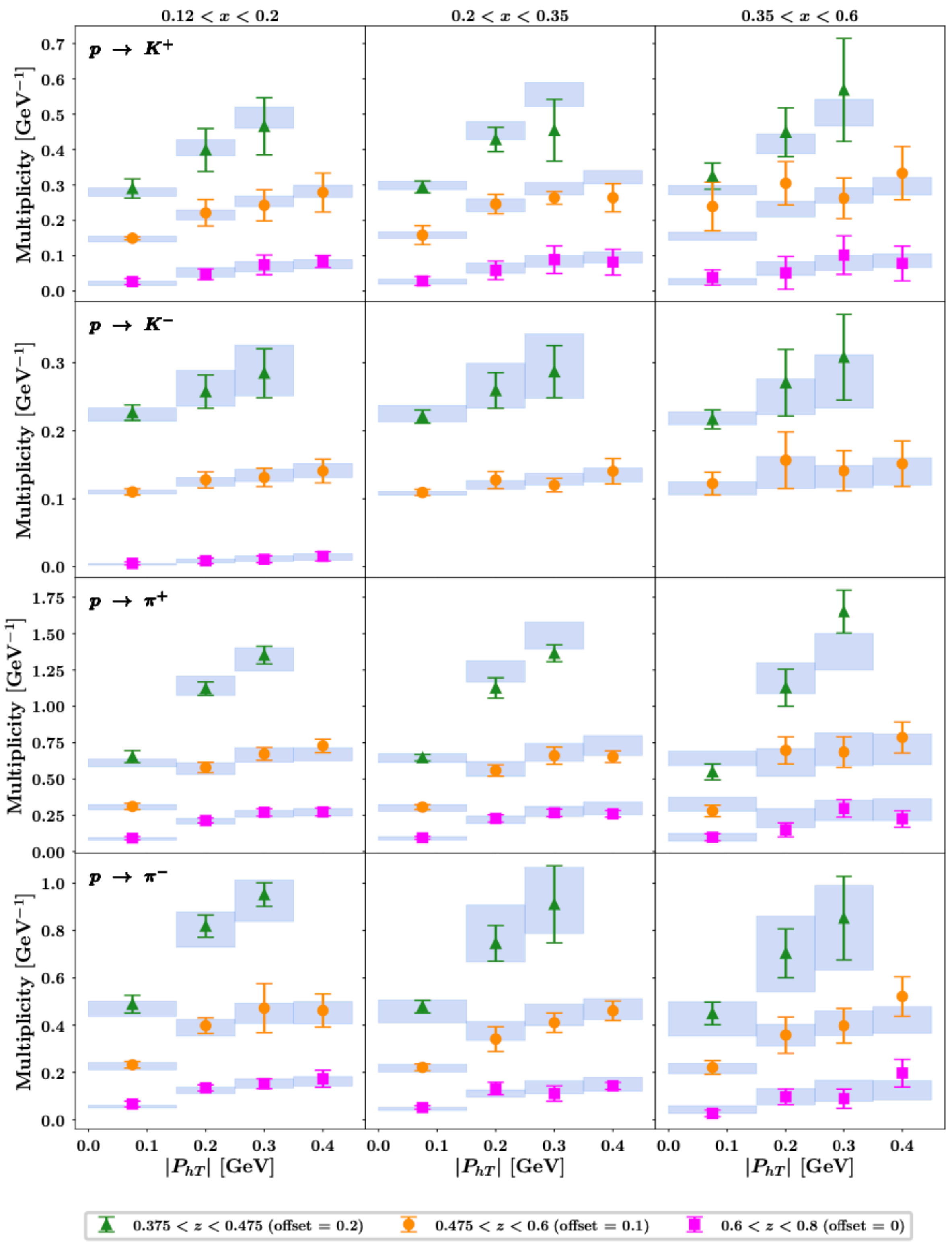
MAPTMD24: results

	N^3LL			
Data set	N_{dat}	χ^2_D	χ^2_λ	χ^2_0
DY collider total	251	1.37	0.28	1.65
DY fixed-target total	233	0.63	0.31	0.94
HERMES total	344	0.81	0.24	1.05
COMPASS total	1203	0.67	0.27	0.94
SIDIS total	1547	0.70	0.26	0.96
Total	**2031**	**0.81**	**0.27**	**1.08**

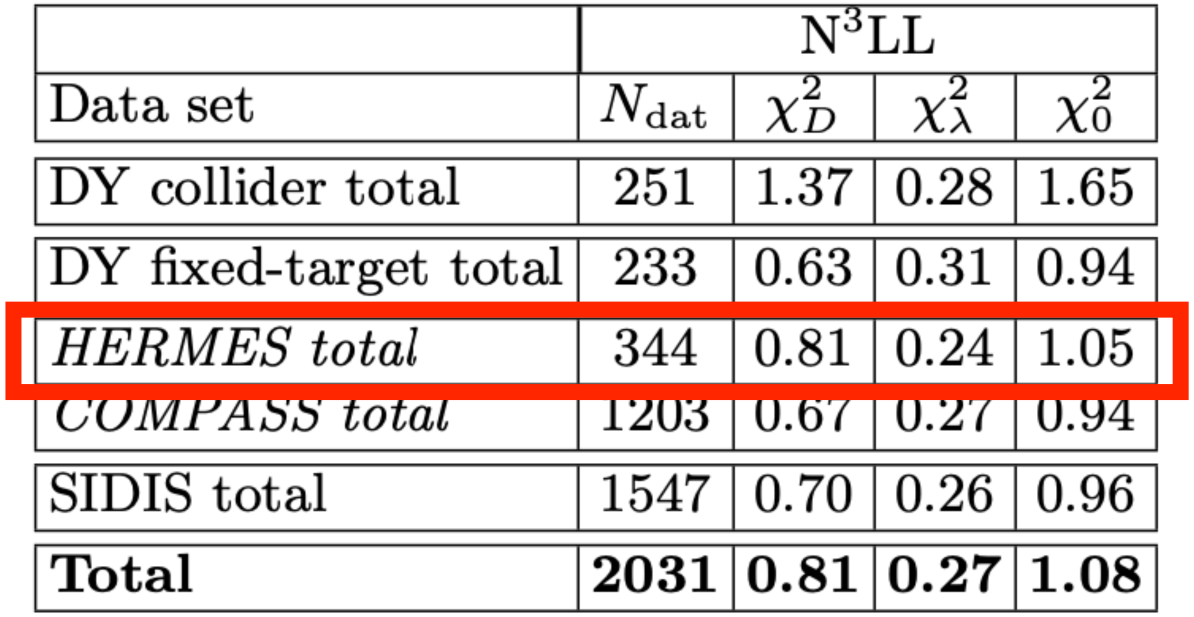


MAPTMD24: results

	N^{3}LL			
Data set	N_{dat}	χ_D^2	χ_λ^2	χ_0^2
DY collider total	251	1.37	0.28	1.65
DY fixed-target total	233	0.63	0.31	0.94
HERMES total	344	0.81	0.24	1.05
COMPASS total	1203	0.67	0.27	0.94
SIDIS total	1547	0.70	0.26	0.96
Total	**2031**	**0.81**	**0.27**	**1.08**

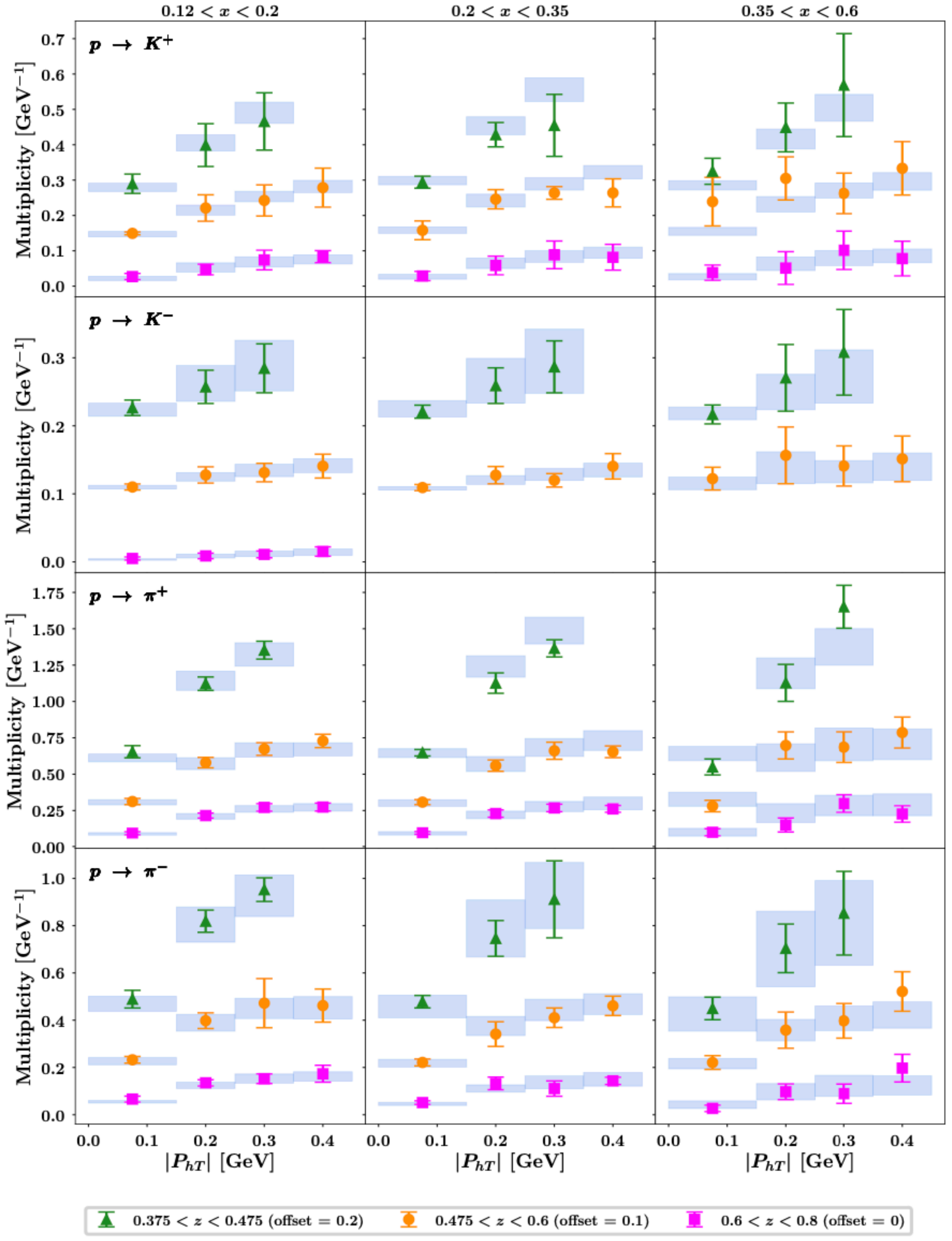


MAPTMD24: results



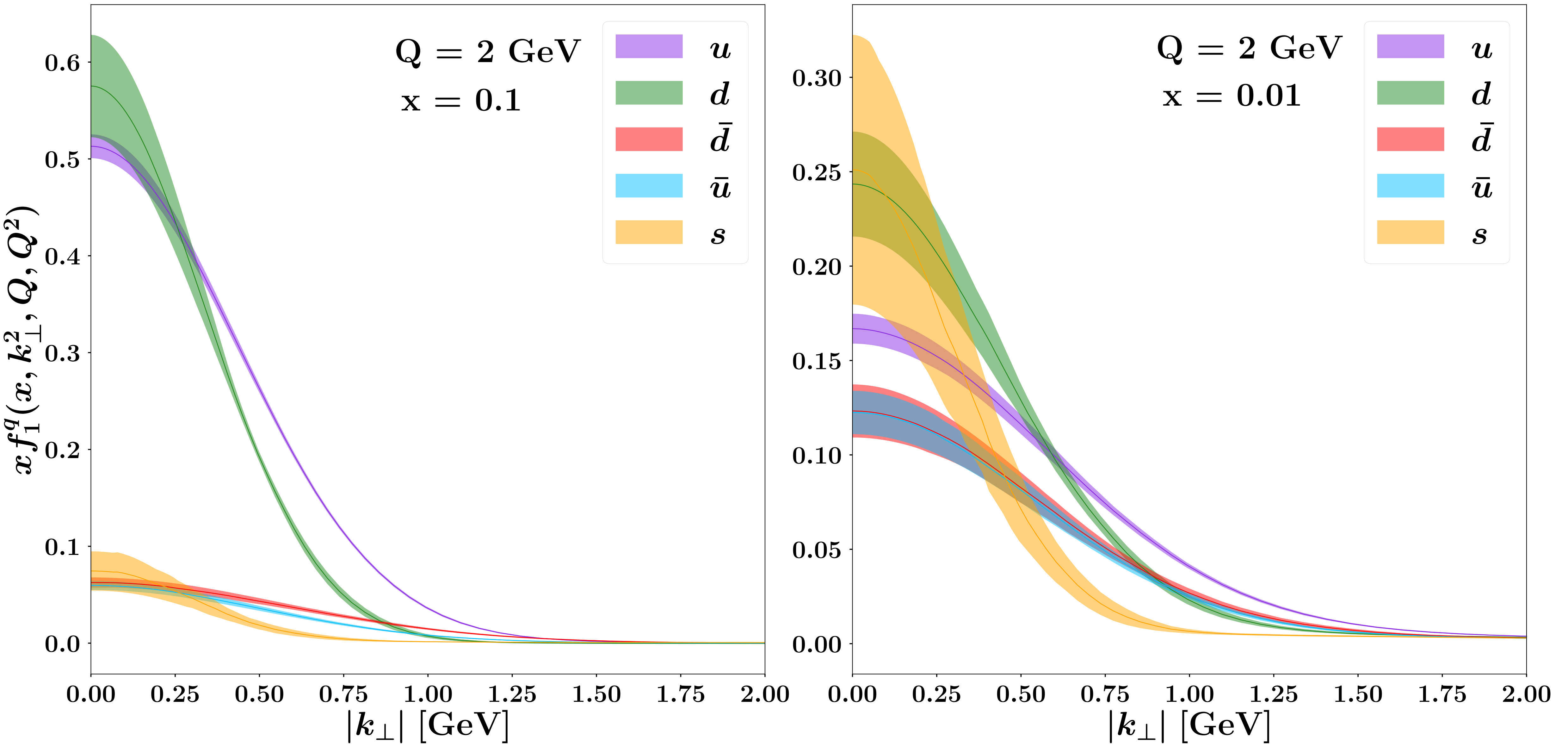
	N^{3}LL			
Data set	N_{dat}	χ^2_D	χ^2_λ	χ^2_0
DY collider total	251	1.37	0.28	1.65
DY fixed-target total	233	0.63	0.31	0.94
HERMES total	344	0.81	0.24	1.05
COMPASS total	1203	0.67	0.27	0.94
SIDIS total	1547	0.70	0.26	0.96
Total	**2031**	**0.81**	**0.27**	**1.08**

The agreement between theory and HERMES data has increased a lot!



MAPTMD24: results

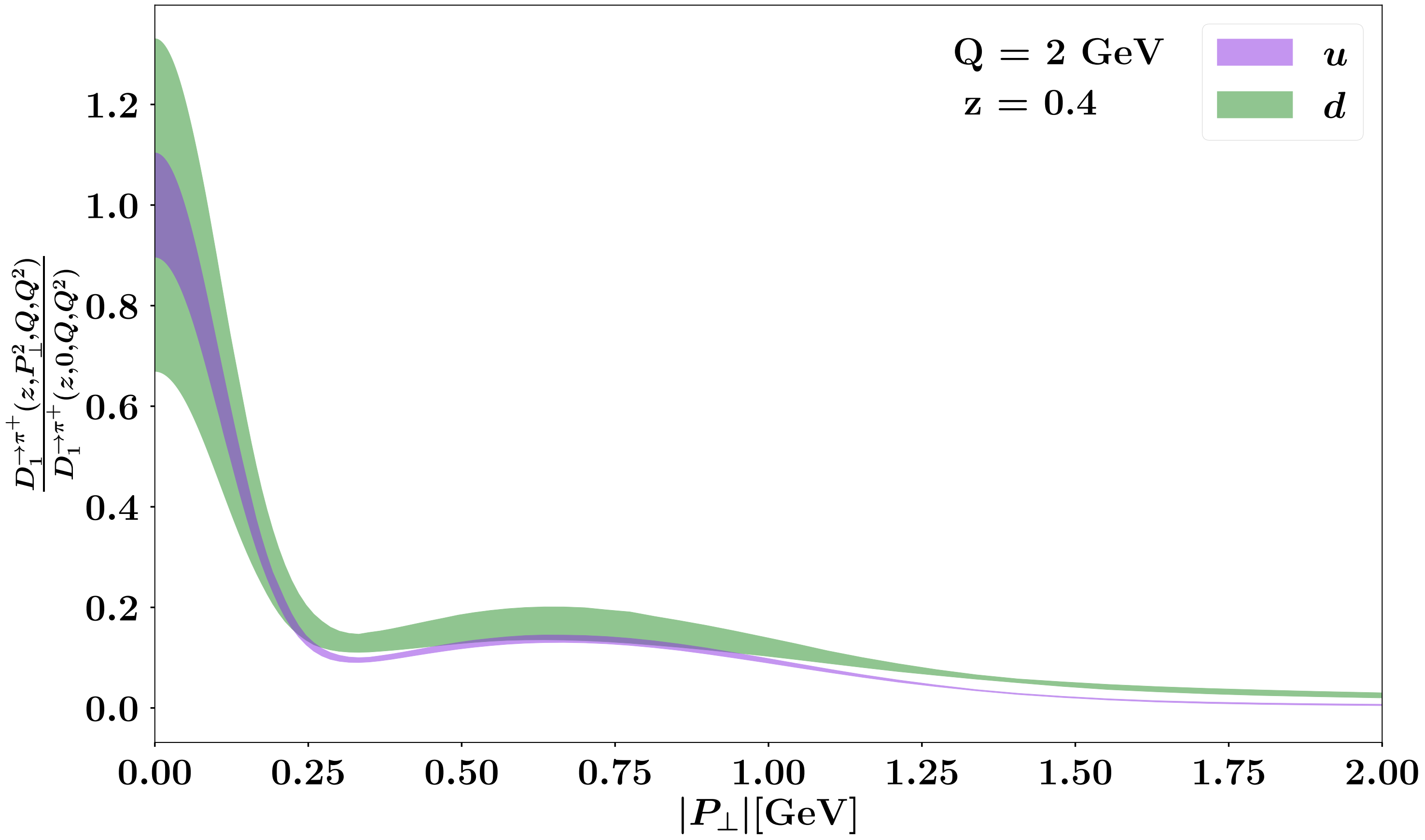
Flavor-dependent TMD PDFs



Evidence of different behaviors for different flavors

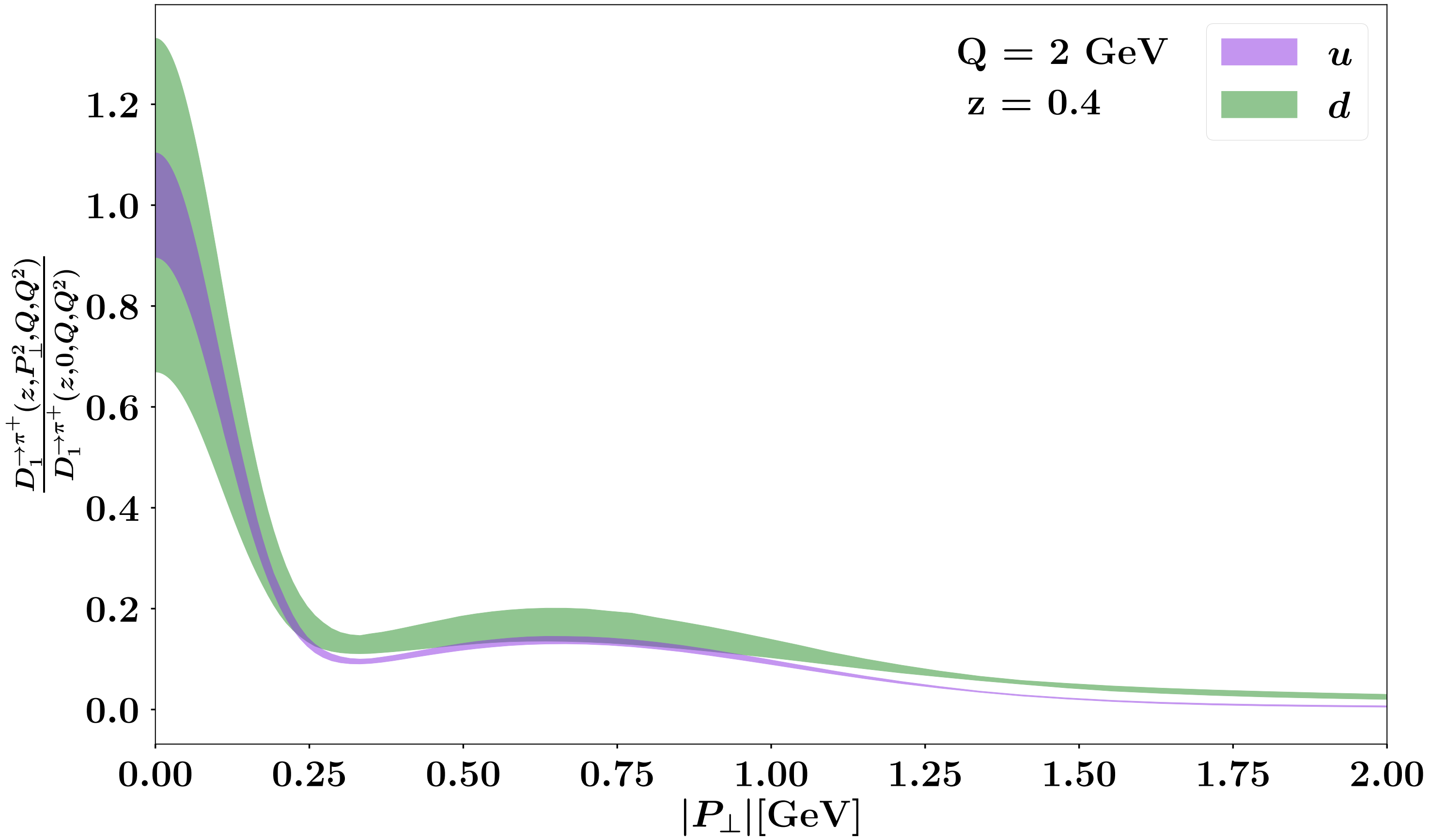
MAPTMD24: results

Flavor-dependent TMD FFs



MAPTMD24: results

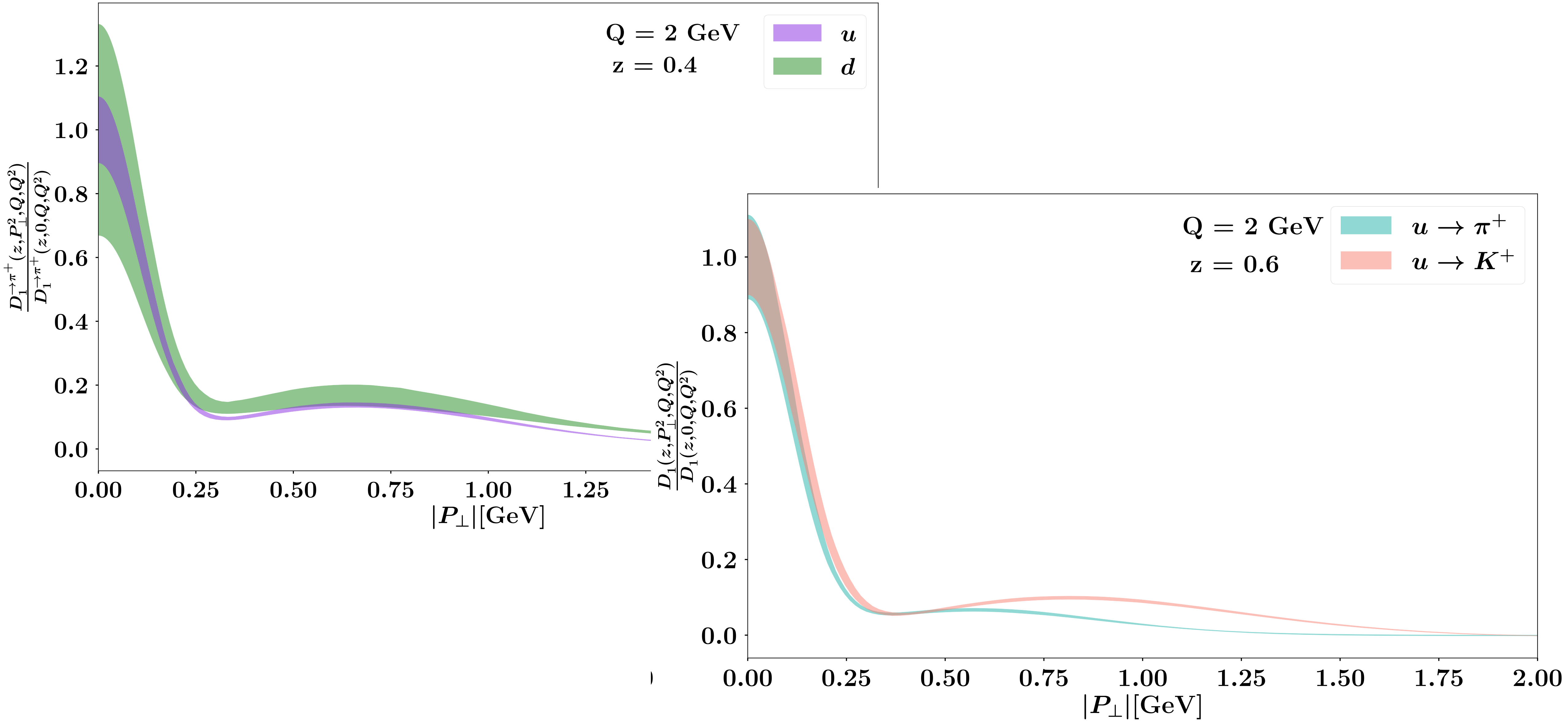
Flavor-dependent TMD FFs



Small evidence of different behaviors for different flavors

MAPTMD24: results

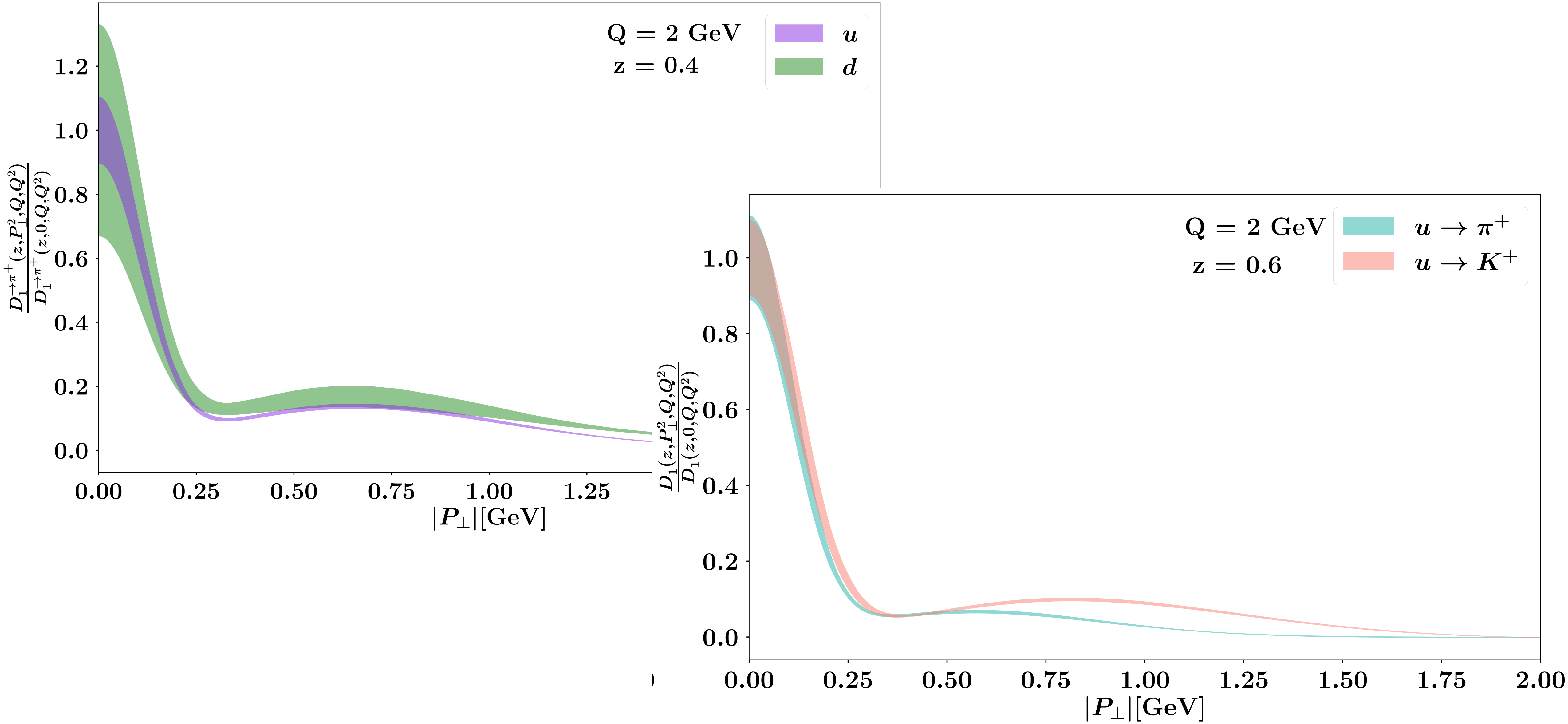
Flavor-dependent TMD FFs



Small evidence of different behaviors for different flavors

MAPTMD24: results

Flavor-dependent TMD FFs

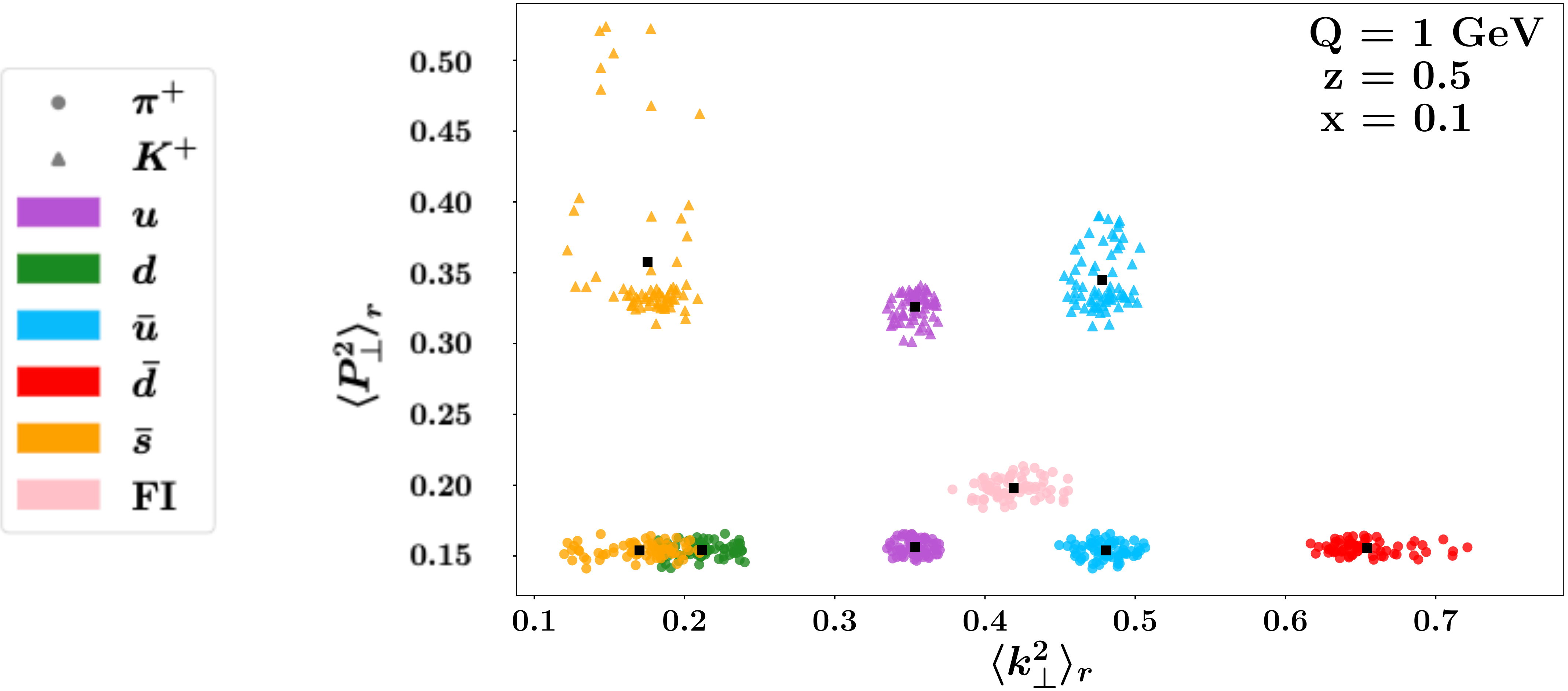


Small evidence of different behaviors for different flavors

Some evidence of different behaviors for different measured hadrons

MAPTMD24: results

TMD's "effective width"



Evidence of different behaviors for different flavors

Evidence of different behaviors for different measured hadrons

MAPTMD24: results

Collins-Soper kernel:

MAPTMD24: results

Collins-Soper kernel: kernel of the rapidity evolution equation

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(b_T, \mu)$$

MAPTMD24: results

Collins-Soper kernel: kernel of the rapidity evolution equation

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(b_T, \mu)$$

$$K(b_T, \mu_{b_*}) = K(b_*, \mu_{b_*}) + g_K(b_T)$$

to be fitted

perturbatively calculable

MAPTMD24: results

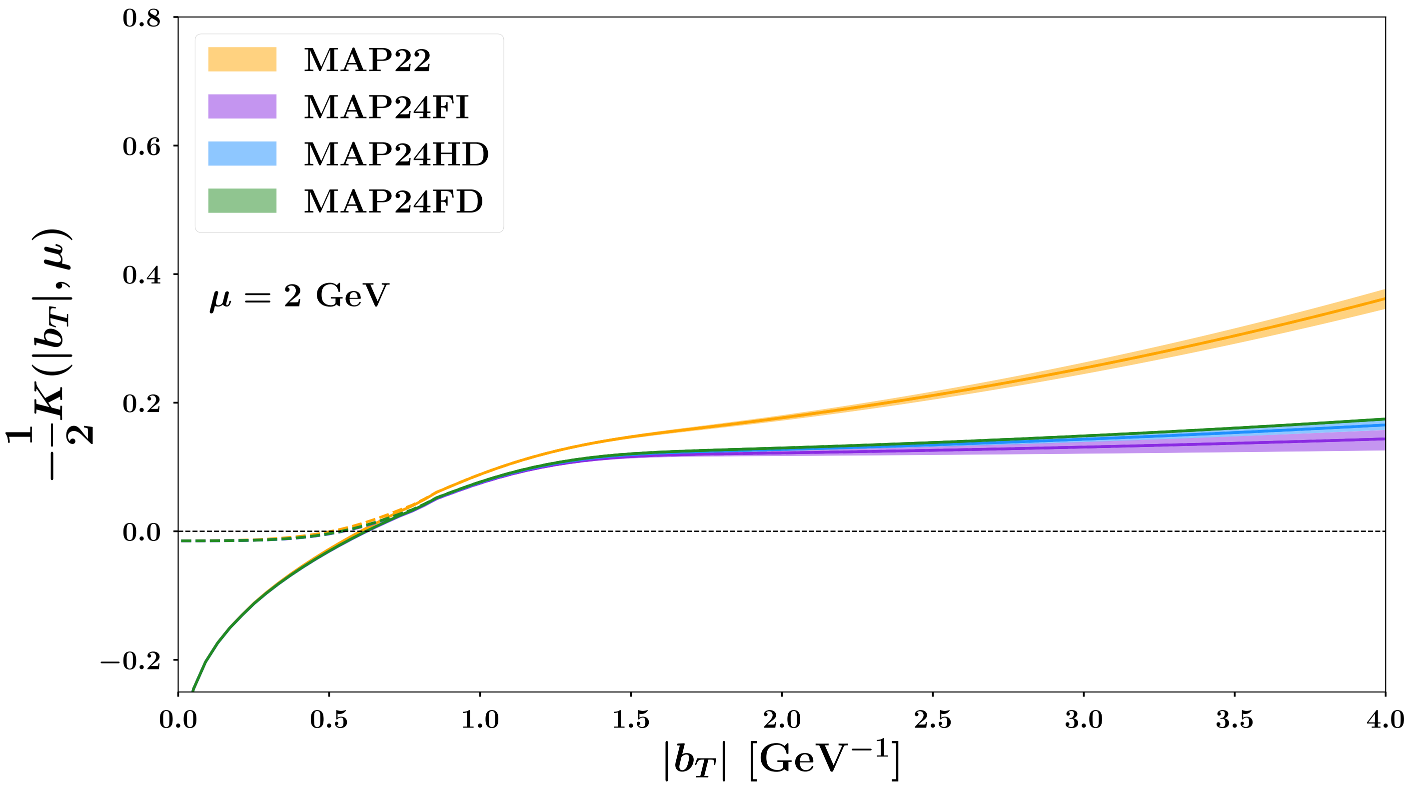
Collins-Soper kernel: kernel of the rapidity evolution equation

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(b_T, \mu)$$

$$K(b_T, \mu_{b_*}) = K(b_*, \mu_{b_*}) + g_K(b_T)$$

to be fitted

perturbatively calculable



See S. Mukerjee's talk

- New feature: almost-linear behaviour at large bT

Conclusions

Conclusions

- The extractions of **unpolarized quark TMDs** through global fits of experimental data have reached very high accuracy (NNNLL)

Conclusions

- The extractions of **unpolarized quark TMDs** through global fits of experimental data have reached very high accuracy (NNNLL)

- With the inclusion of recent NNLO collinear distributions, we need to introduce **flavor dependence** to obtain good theory / data agreement

Conclusions

- The extractions of **unpolarized quark TMDs** through global fits of experimental data have reached very high accuracy (NNNLL)

- With the inclusion of recent NNLO collinear distributions, we need to introduce **flavor dependence** to obtain good theory/data agreement

- **MAPTMD24** is the **first** simultaneous **extraction** of flavor-dependent unpolarized TMD PDFs and FF **through a global fit**

Conclusions

- The extractions of **unpolarized quark TMDs** through global fits of experimental data have reached very high accuracy (NNNLL)

- With the inclusion of recent NNLO collinear distributions, we need to introduce **flavor dependence** to obtain good theory / data agreement

- **MAPTMD24** is the **first** simultaneous **extraction** of flavor-dependent unpolarized TMD PDFs and FF **through a global fit**

- We observe **non-trivial differences** in the transverse momentum distribution of partons inside hadrons

Backup

Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp \left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}_T|}$$

Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}_T|} \quad \xrightarrow{b_T \gg 1} \quad 0$$

Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}_T|} \quad \xrightarrow{b_T \gg 1} \quad 0 \qquad \alpha_S(\mu_b) \to +\infty$$

Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

$$\mu_b > \mu \qquad \infty \quad \xleftarrow{b_T \ll 1} \quad \mu_b = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}_T|} \quad \xrightarrow{b_T \gg 1} \quad 0 \qquad \alpha_S(\mu_b) \to +\infty$$

Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \text{C}$$

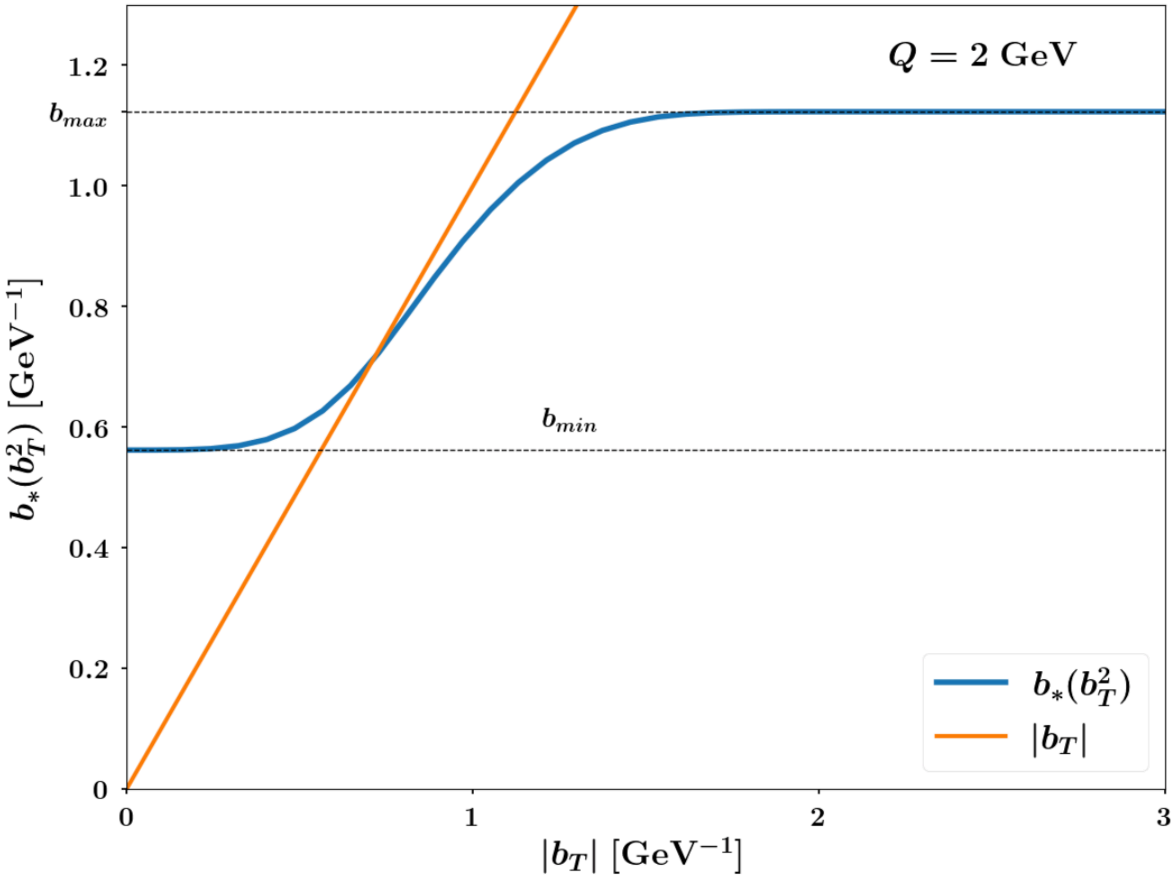
$$\boxed{\mu_b > \mu} \qquad \infty \quad \xleftarrow{b_T \ll 1} \quad \mu_b = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}_T|} \quad \xrightarrow{b_T \gg 1} \quad 0 \qquad \boxed{\alpha_S(\mu_b) \to +\infty}$$

Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

$$\boxed{\mu_b > \mu} \qquad \infty \xleftarrow{b_T \ll 1} \mu_b = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}_T|} \xrightarrow{b_T \gg 1} 0 \qquad \boxed{\alpha_S(\mu_b) \to +\infty}$$

b$_*$-prescription



Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

$$\boxed{\mu_b > \mu} \qquad \infty \xleftarrow{b_T \ll 1} \mu_b = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}_T|} \xrightarrow{b_T \gg 1} 0 \qquad \boxed{\alpha_S(\mu_b) \to +\infty}$$

b_*-prescription

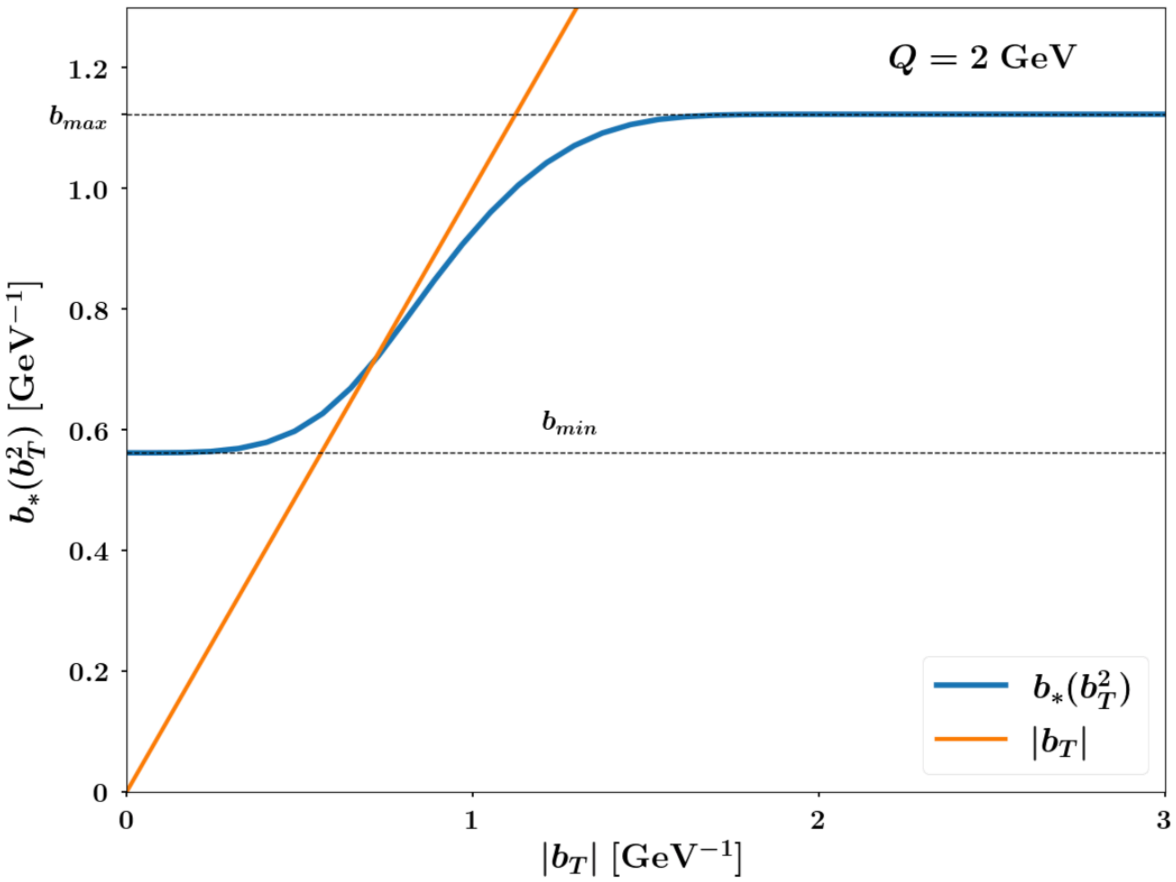
$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = \frac{2e^{-\gamma_E}}{\mu}$$

Collins, Soper, Sterman, Nucl. Phys. B250 (1985)

Collins, Gamberg, et al., PRD (2016)

Bacchetta, Echevarria, Mulders, et al., JHEP 11 (2015)



Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

$$\boxed{\mu_b > \mu} \qquad \infty \xleftarrow{b_T \ll 1} \mu_b = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}_T|} \xrightarrow{b_T \gg 1} 0 \qquad \boxed{\alpha_S(\mu_b) \to +\infty}$$

b$_*$-prescription

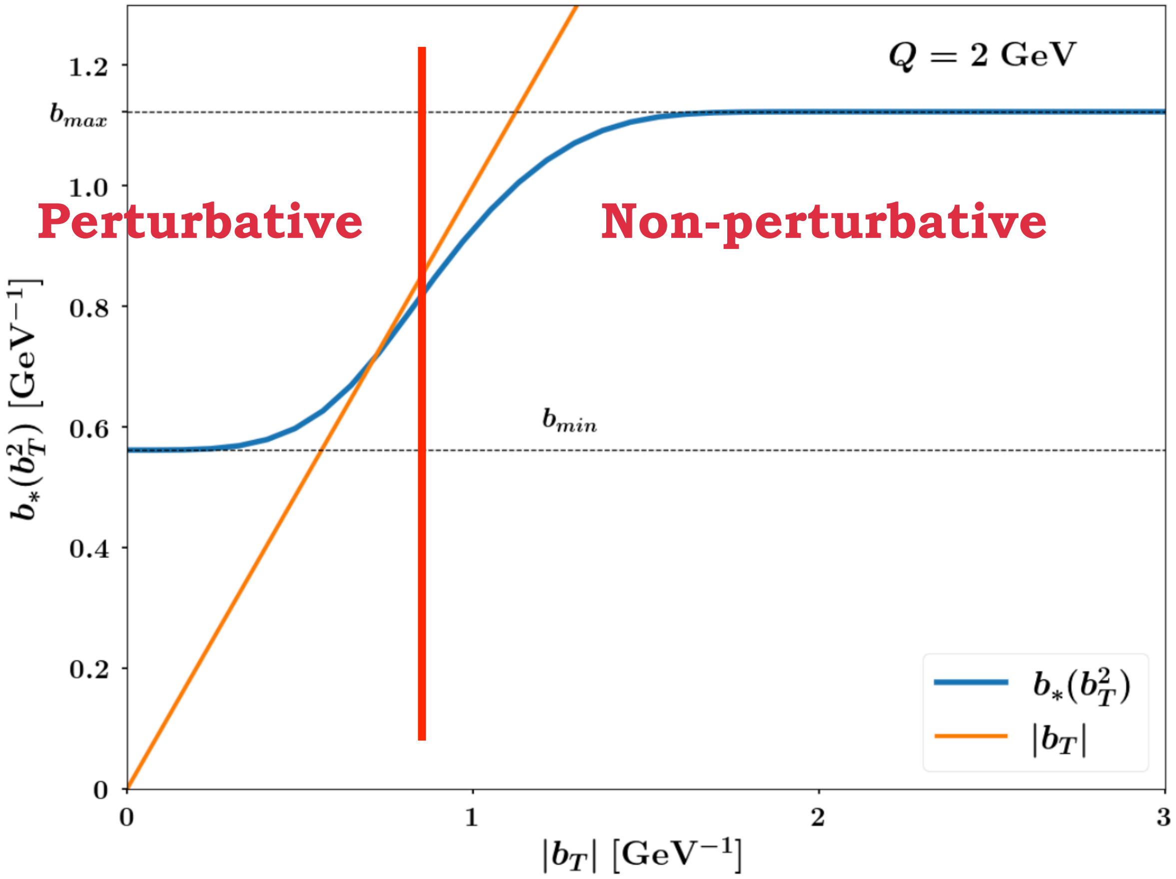
$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = \frac{2e^{-\gamma_E}}{\mu}$$

Collins, Soper, Sterman, Nucl. Phys. B250 (1985)

Collins, Gamberg, et al., PRD (2016)

Bacchetta, Echevarria, Mulders, et al., JHEP 11 (2015)



$$\hat{f}_1(x, b_T^2; \mu, \zeta) = \left[\frac{\hat{f}_1(x, b_T^2; \mu, \zeta)}{\hat{f}_1(x, b_*(b_T^2); \mu, \zeta)} \right] \hat{f}_1(x, b_*(b_T^2); \mu, \zeta) \equiv f_{\mathrm{NP}}(x, b_T^2; \zeta) \hat{f}_1(x, b_*(b_T^2); \mu, \zeta)$$

Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

$$\boxed{\mu_b > \mu} \qquad \infty \xleftarrow{\;b_T \ll 1\;} \mu_b = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}_T|} \xrightarrow{\;b_T \gg 1\;} 0 \qquad \boxed{\alpha_S(\mu_b) \to +\infty}$$

b_*-prescription

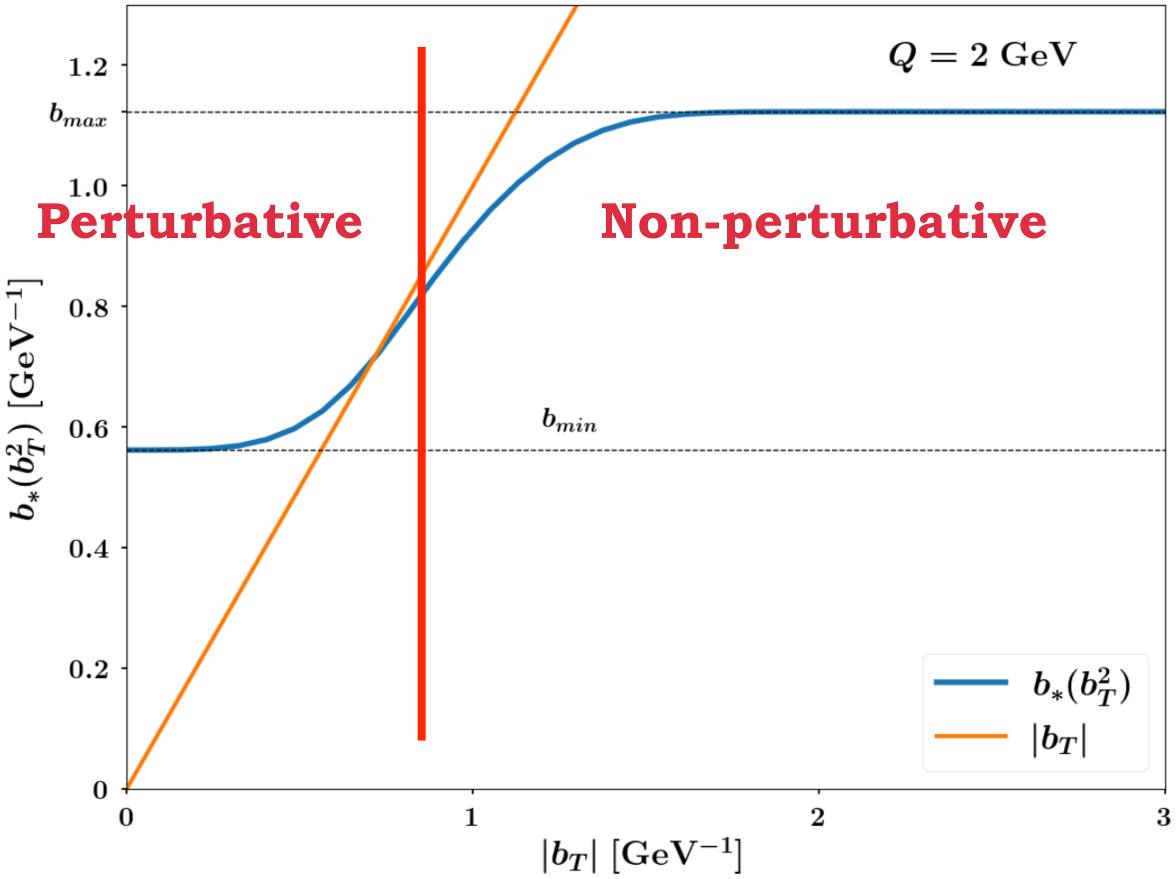
$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = \frac{2e^{-\gamma_E}}{\mu}$$

Collins, Soper, Sterman, Nucl. Phys. B250 (1985)

Collins, Gamberg, et al., PRD (2016)

Bacchetta, Echevarria, Mulders, et al., JHEP 11 (2015)

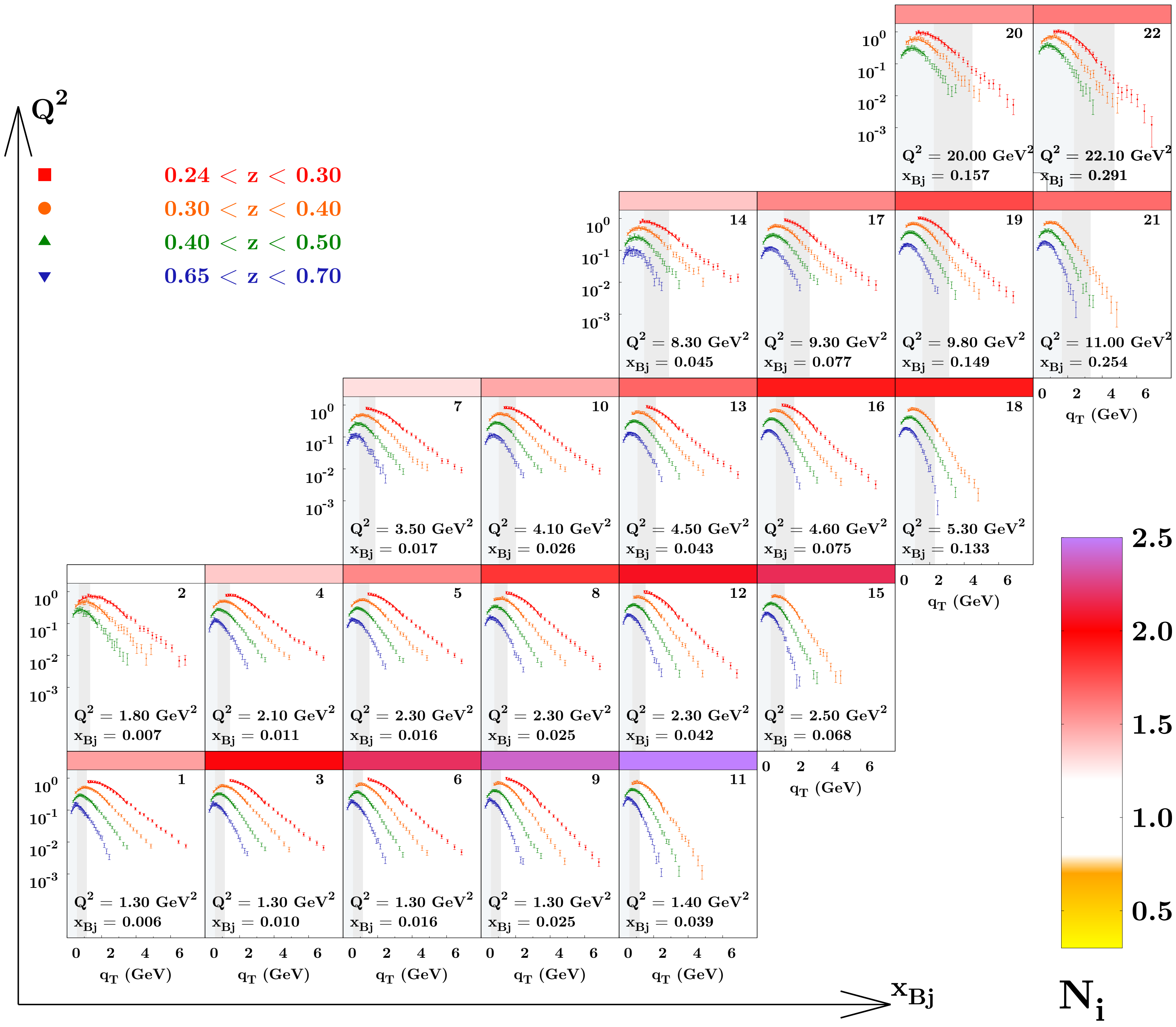


$$\hat{f}_1(x, b_T^2; \mu, \zeta) = \left[\frac{\hat{f}_1(x, b_T^2; \mu, \zeta)}{\hat{f}_1(x, b_*(b_T^2); \mu, \zeta)} \right] \hat{f}_1(x, b_*(b_T^2); \mu, \zeta) \equiv \boxed{f_{\mathrm{NP}}(x, b_T^2; \zeta)} \hat{f}_1(x, b_*(b_T^2); \mu, \zeta)$$

Normalization of SIDIS calculation

Normalization issue confirmed also in other analyses from different collaborations

Vladimirov, JHEP 12 (2023)



Gonzalez-Hernandez, PoS DIS2019 (2019)

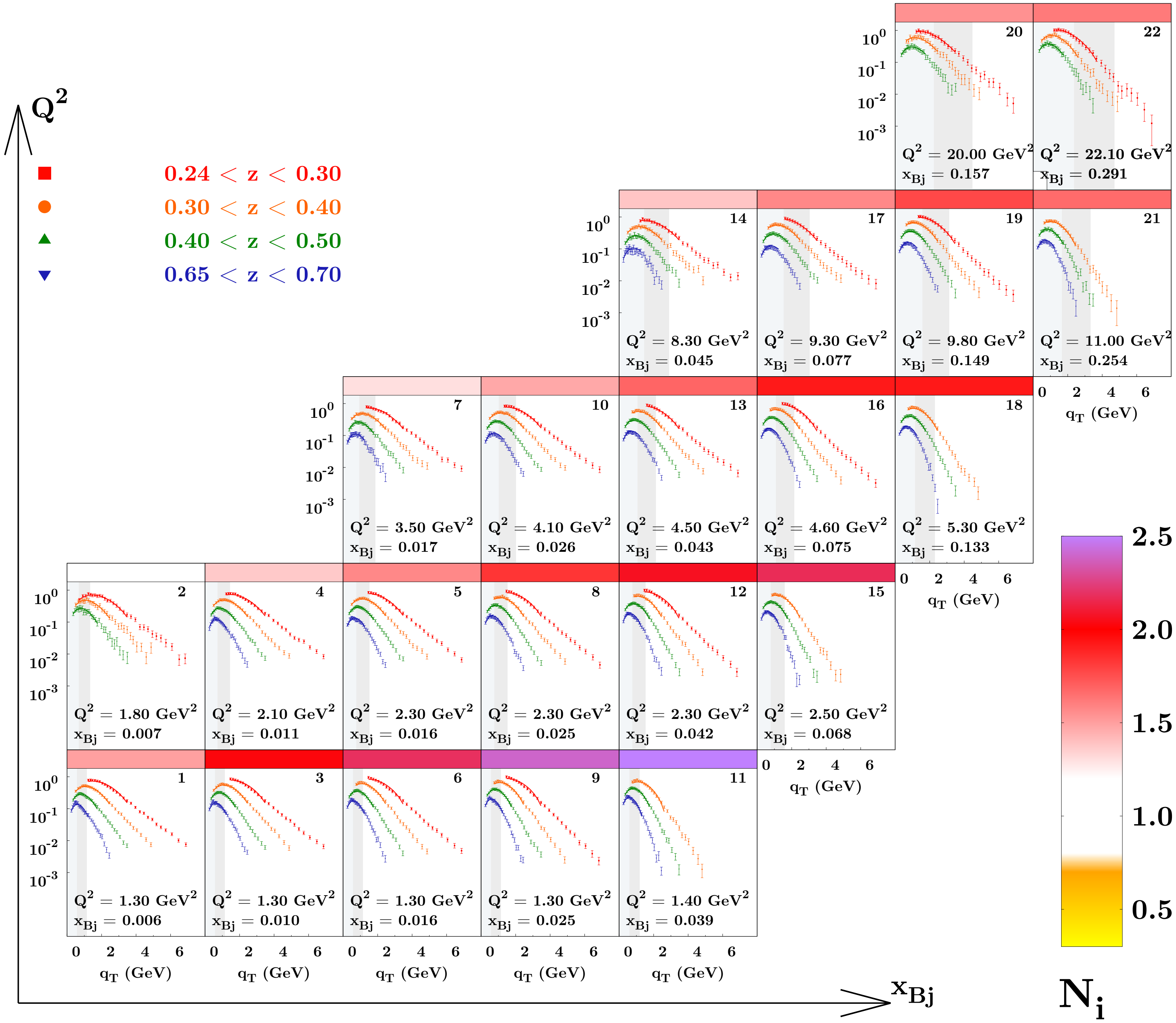
Normalization of SIDIS calculation

Normalization issue confirmed also in other analyses from different collaborations

Sun, Isaacson, Yuan, Yuan, IJNP A (2014)

Gonzalez-Hernandez, PoS DIS2019 (2019)

Vladimirov, JHEP 12 (2023)



Gonzalez-Hernandez, PoS DIS2019 (2019)

Normalization of SIDIS calculation

MAP22 work solution

Good agreement for almost all bins

MAP Collaboration, JHEP 10 (2022)

Normalization of SIDIS calculation

MAP22 work solution

SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx\,dQ\,{\color{red}dz\,dP_{hT}}} \Bigg/ \frac{d\sigma}{dx\,dQ}$$

Good agreement for almost all bins

MAP Collaboration, JHEP 10 (2022)

Normalization of SIDIS calculation

MAP22 work solution

SIDIS multiplicity
$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \Big/ \frac{d\sigma}{dx dQ}$$

Collinear SIDIS cross section
$$\frac{d\sigma}{dx dQ dz}$$

Good agreement for almost all bins

MAP Collaboration, JHEP 10 (2022)

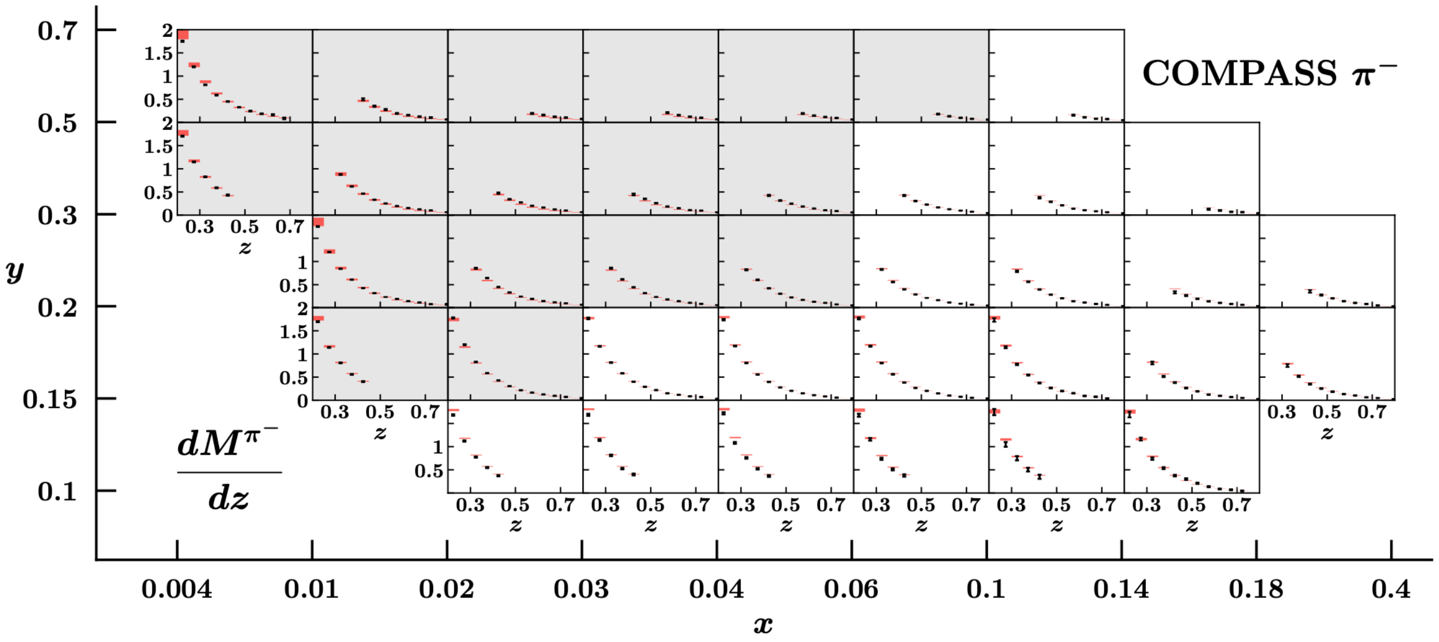
Normalization of SIDIS calculation

MAP22 work solution

SIDIS multiplicity $$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \Big/ \frac{d\sigma}{dx dQ}$$

Collinear SIDIS cross section $$\frac{d\sigma}{dx dQ dz}$$

Good agreement theory/data



Khalek, Bertone, Nocera, et al., PRD 104 (2021)

Good agreement for almost all bins

MAP Collaboration, JHEP 10 (2022)

Normalization of SIDIS calculation

MAP22 work solution

SIDIS multiplicity $$M(x,z,P_{hT},Q) = \frac{d\sigma}{dxdQ\color{red}{dzdP_{hT}}} \Big/ \frac{d\sigma}{dxdQ}$$

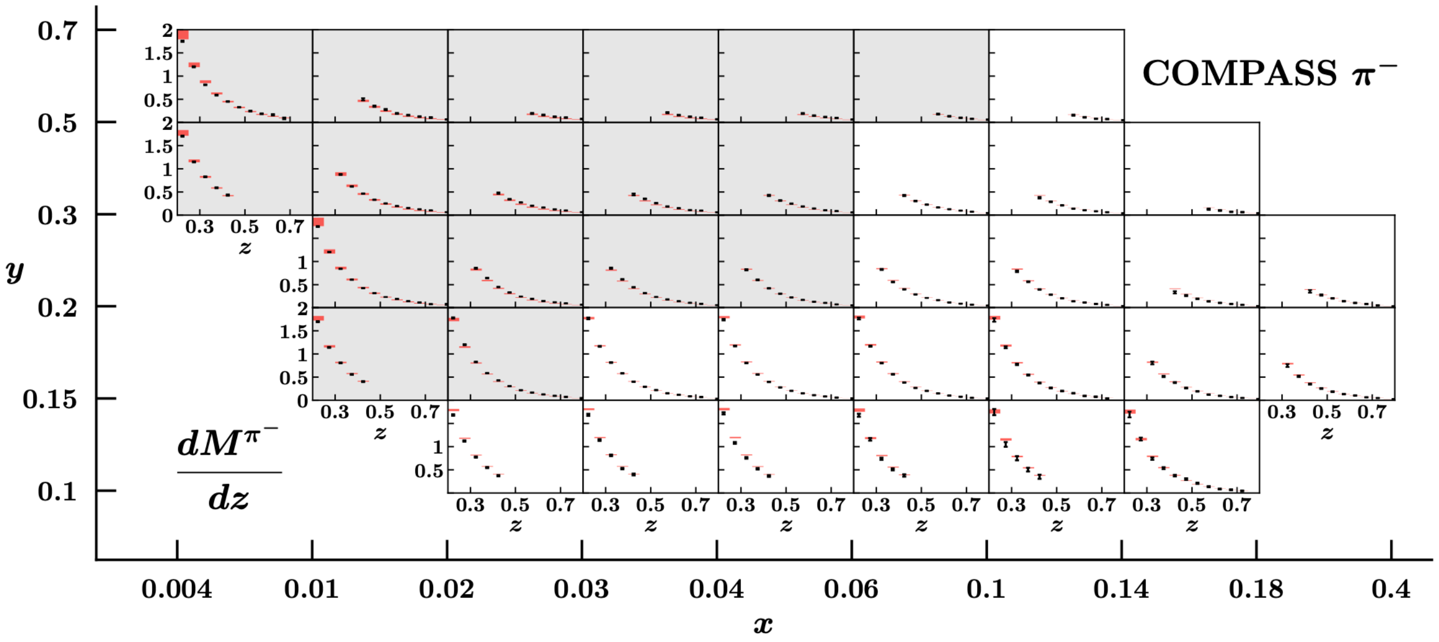
Collinear SIDIS cross section $$\frac{d\sigma}{dxdQ\color{red}{dz}}$$

Normalization of prediction such that

$$\int d\color{red}{P_{hT}}\color{black}{W(x,z,Q,}\color{red}{P_{hT}}\color{black}{)} = \frac{d\sigma}{dxdQdz}$$

Piacenza, PhD thesis (2020)

Good agreement theory/data



Khalek, Bertone, Nocera, et al., PRD 104 (2021)

Good agreement for almost all bins

MAP Collaboration, JHEP 10 (2022)

Normalization of SIDIS calculation

MAP22 work solution

SIDIS multiplicity
$$M(x,z,P_{hT},Q) = \frac{d\sigma}{dxdQdzdP_{hT}} \Big/ \frac{d\sigma}{dxdQ}$$

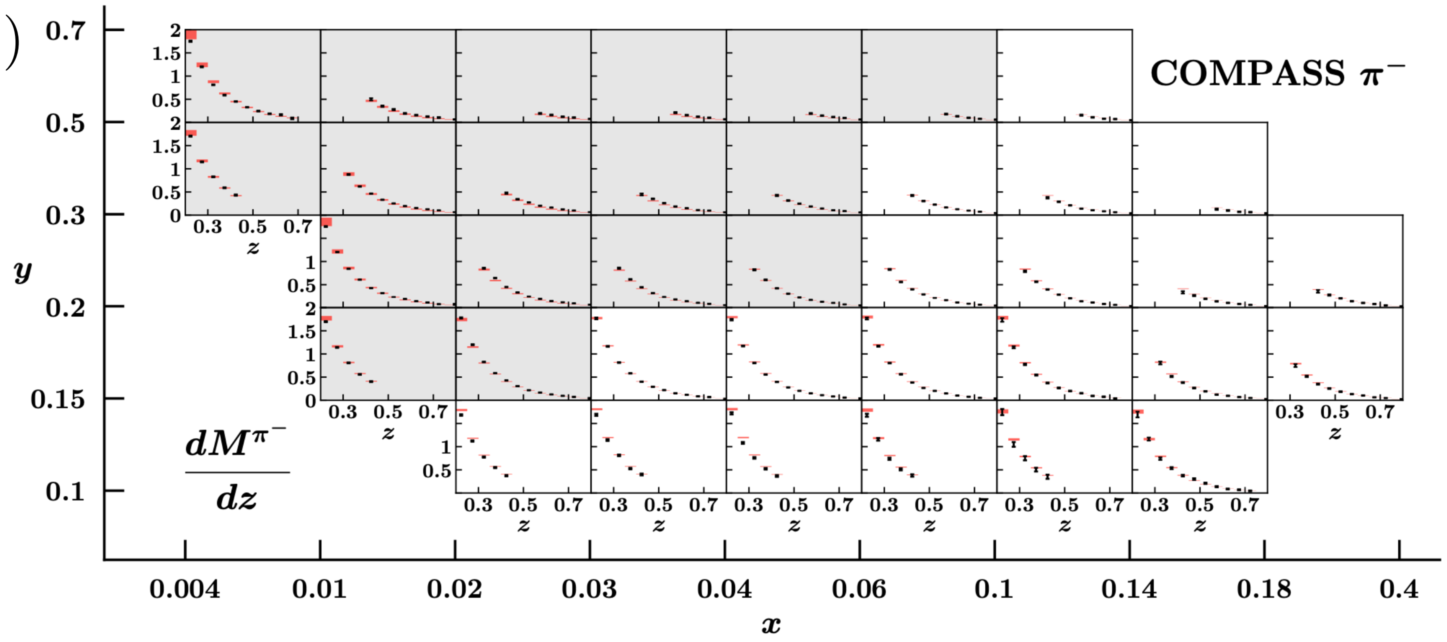
Collinear SIDIS cross section
$$\frac{d\sigma}{dxdQdz}$$

Normalization of prediction such that

$$n(x,z,Q) = \frac{d\sigma}{dxdQdz} \Big/ \int dP_{hT} W(x,z,Q,P_{hT})$$

Piacenza, PhD thesis (2020)

Good agreement theory/data



Khalek, Bertone, Nocera, et al., PRD 104 (2021)

Good agreement for almost all bins

MAP Collaboration, JHEP 10 (2022)

Normalization of SIDIS calculation

MAP22 work solution

SIDIS multiplicity
$$M(x,z,P_{hT},Q)=\frac{d\sigma}{dxdQdzdP_{hT}}\Big/\frac{d\sigma}{dxdQ}$$

Collinear SIDIS cross section
$$\frac{d\sigma}{dxdQdz}$$

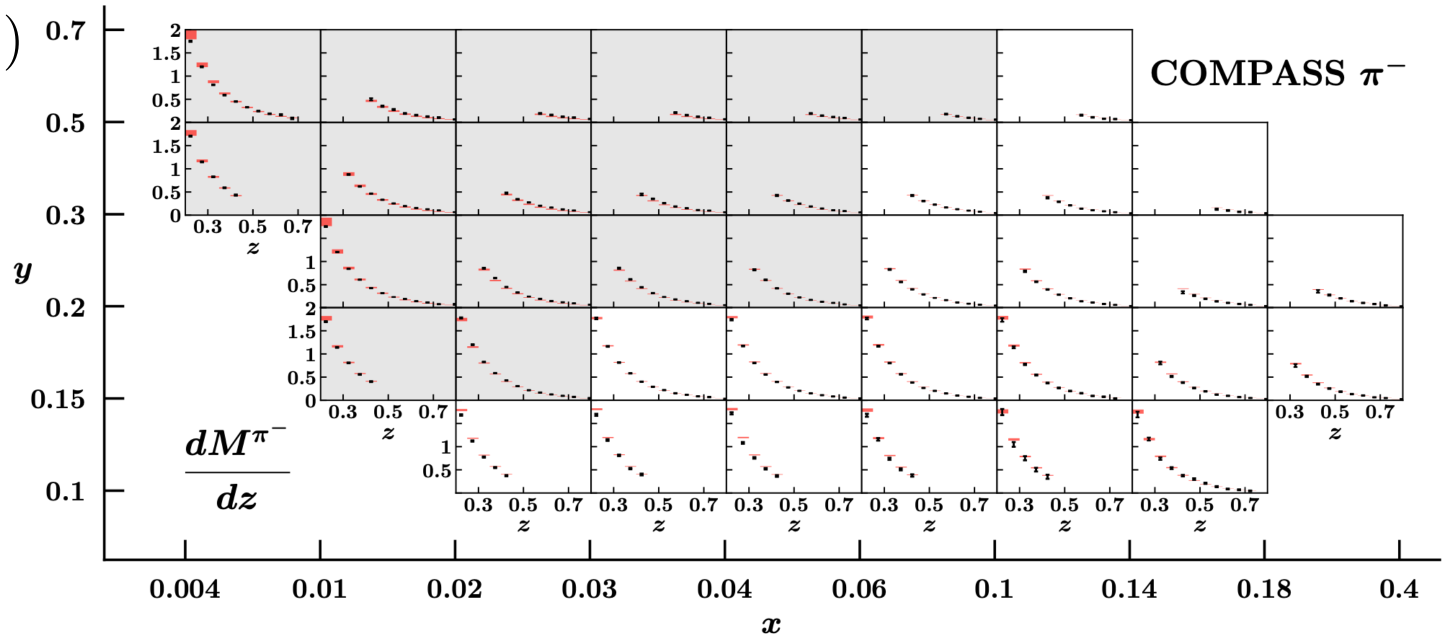
Normalization of prediction such that

$$n(x,z,Q)=\frac{d\sigma}{dxdQdz}\Big/\int dP_{hT}W(x,z,Q,P_{hT})$$

Piacenza, PhD thesis (2020)

$$M(x,z,P_{hT},Q)=n(x,z,Q)W(x,z,Q,P_{hT})\Big/\frac{d\sigma}{dxdQ}$$

Good agreement theory/data



Khalek, Bertone, Nocera, et al., PRD 104 (2021)

Good agreement for almost all bins

MAP Collaboration, JHEP 10 (2022)

Normalization of SIDIS calculation

MAP22 work solution

SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dxdQdzdP_{hT}} \Big/ \frac{d\sigma}{dxdQ}$$

Collinear SIDIS cross section

$$\frac{d\sigma}{dxdQdz}$$

Normalization of prediction such that

$$n(x, z, Q) = \frac{d\sigma}{dxdQdz} \Big/ \int dP_{hT} W(x, z, Q, P_{hT})$$

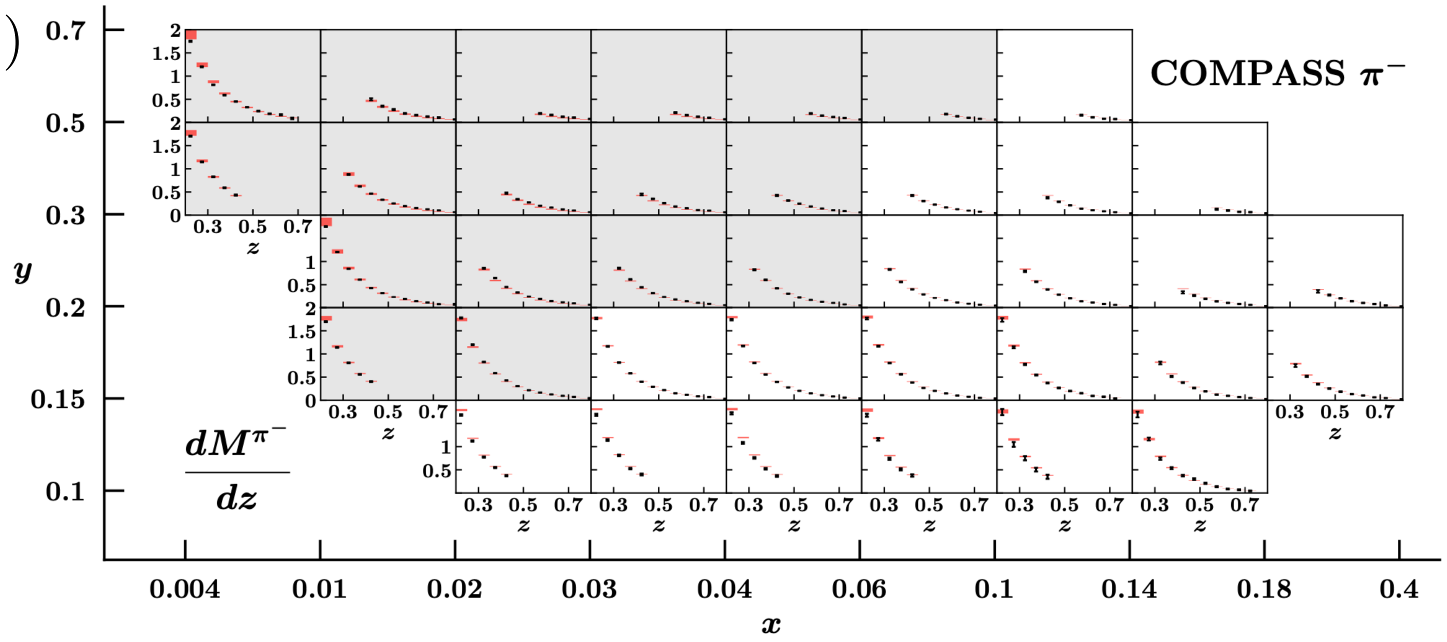
Piacenza, PhD thesis (2020)

$$M(x, z, P_{hT}, Q) = \boxed{n(x, z, Q)} W(x, z, Q, P_{hT}) \Big/ \frac{d\sigma}{dxdQ}$$

Calculable before the fit

Good agreement for almost all bins

Good agreement theory/data



Khalek, Bertone, Nocera, et al., PRD 104 (2021)

MAP Collaboration, JHEP 10 (2022)

MAPTMD22 — Error analysis

Error propagation

↓

100 Monte Carlo replicas of data

100 Monte Carlo replicas of PDFs

100 Monte Carlo replicas of FFs