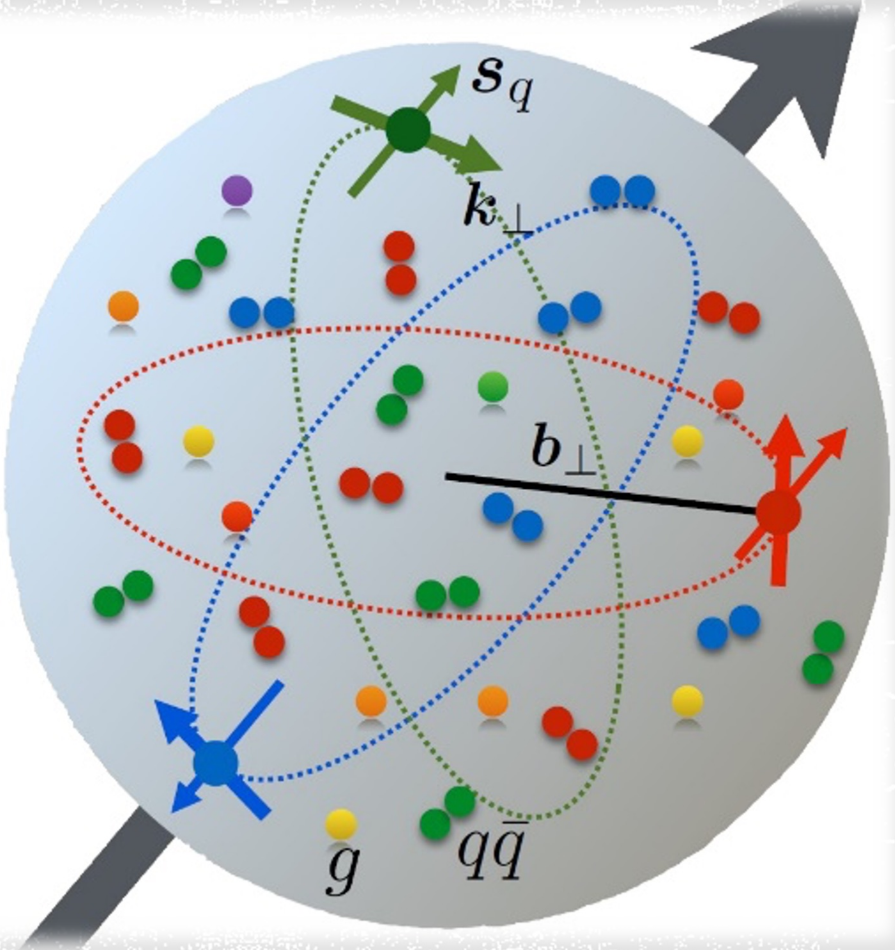


# A study of TMD evolution effect on $cos2\phi$ azimuthal asymmetry in a back-to-back $J/\psi$-jet pair production at the EIC

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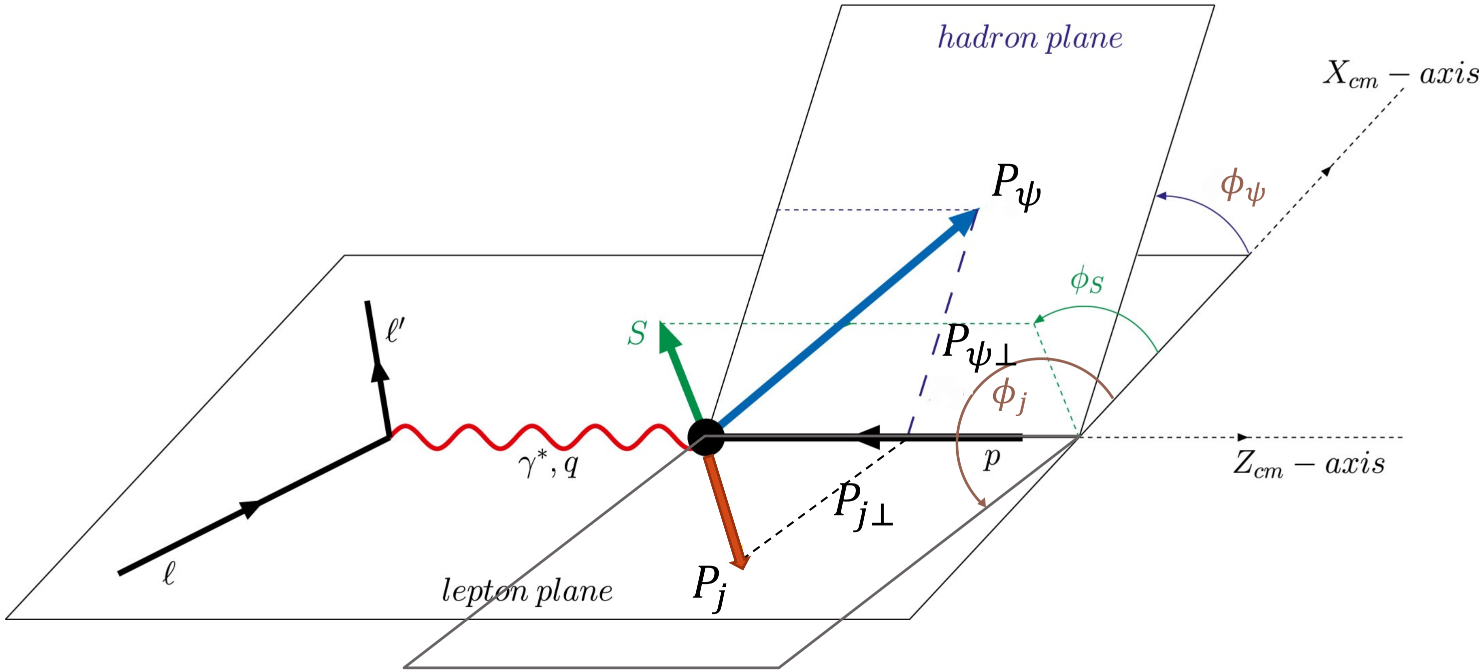
# Part1: $J/\psi - jet$ pair production in $ep$ scattering

Effect of TMD Evolution on $cos2\phi$ azimuthal asymmetry in $ep \to e' + J/\psi + X$ studied D.Boer, J. Bor (2022)

➢ Consider the electroproduction processes: $\boldsymbol{e(l) + p(P) \to e(l') + J/\psi(P_\psi) + jet(P_j) + X}$




$\gamma^* - p$ c.m. frame

SIDIS Kinematics

$$z\left(= \frac{P \cdot P_h}{P \cdot q}\right) < 1$$

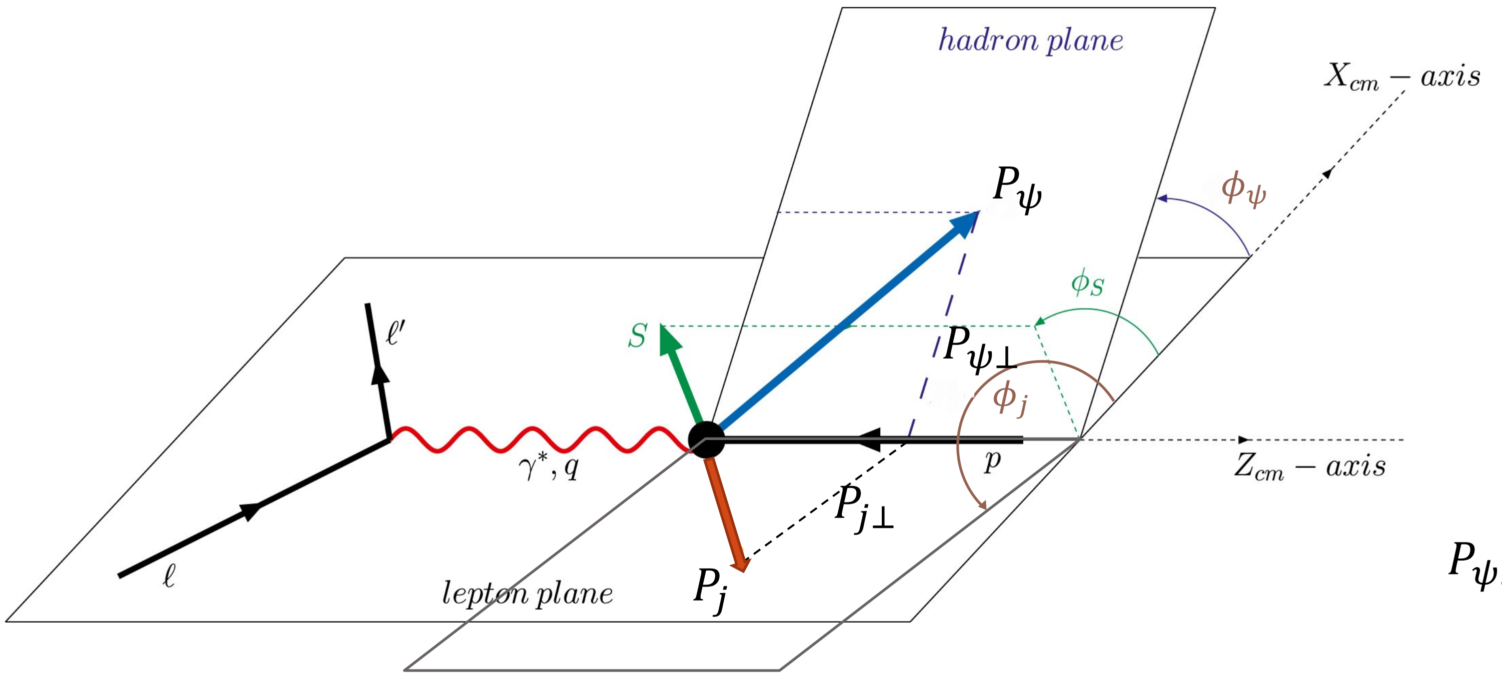
$z$ is fraction of virtual photon energy carried by $c\bar{c}$ in proton rest frame.

$q^2 = -Q^2$ (virtuality of photon)

$$x_B = \frac{Q^2}{2P \cdot q}, y = \frac{P \cdot q}{P \cdot l}$$

# $J/\psi - jet$ pair production in $ep$ scattering

- Consider the electroproduction processes: $\boldsymbol{e(l) + p(P) \to e(l') + J/\psi(P_\psi) + jet(P_j) + X}$




$\gamma^* - p$ c.m. frame

$$d\hat{\sigma} \equiv d\hat{\sigma}^{\gamma^* + g/q \to c\bar{c}[n] + g/q}$$

$[Quarkoniam: J/\psi]$

$P_{\psi\perp} \lesssim M_\psi$: Standard NRQCD: $\langle \boldsymbol{O_\psi[n]} \rangle \boldsymbol{\delta(\hat{z} - z)}$

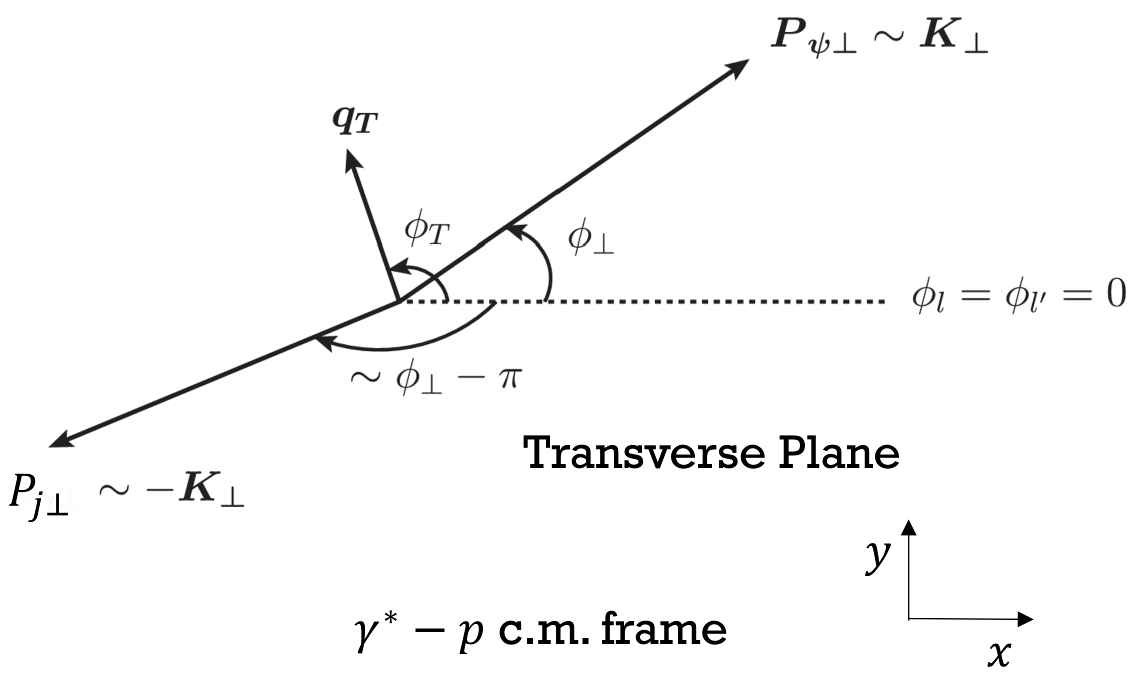
LDME

$\Lambda_{\text{QCD}} \ll P_{\psi\perp} \ll M_\psi$: Shape function: $\Delta_{[n]}(\hat{z}, k_\perp)$

$$d\sigma \propto [TMD - PDF] \otimes d\hat{\sigma} \otimes [Hadronization]$$

# $J/\psi - jet$: Back-to-back In The Transverse Plane

- $\boldsymbol{P}_{\psi\perp}$ and $\boldsymbol{P}_{j\perp}$ are transverse momentum of $J/\psi$ and $jet$ respectively in the plane orthogonal to the proton momentum.




- We define sum and difference of transverse momenta

$$\boldsymbol{q_T} = \boldsymbol{P}_{\psi\perp} + \boldsymbol{P}_{j\perp} \quad , \qquad \boldsymbol{K}_\perp = \frac{\boldsymbol{P}_{\psi\perp} - \boldsymbol{P}_{j\perp}}{2}$$

$\phi_T$ denotes azimuthal angle of $\boldsymbol{q_T}$

- In the case where $\boxed{|\boldsymbol{q_T}| \ll |\boldsymbol{K}_\perp|}$, the $J/\psi$ and jet are almost back-to-back in the transverse plane.

TMD Factorization

# CROSS SECTION: $ep \to e' + J\psi + jet + X$

$$d\sigma = \frac{1}{2s}\frac{d^3l'}{(2\pi)^3 2E_{l'}}\frac{d^3P_\psi}{(2\pi)^3 2E_\psi}\frac{d^3P_j}{(2\pi)^3 2E_j}\int dx d^2p_T (2\pi)^4\delta^4(q+p_g-P_\psi-P_j)\times$$
$$\frac{1}{Q^4}L^{\mu\mu'}(l,q)\Phi_g^{\nu\nu'}(x,p_T^2)M_{\mu\nu}^{g\gamma^*\to J/\psi\, g}M_{\mu'\nu'}^{*g\gamma^*\to J/\psi\, g}$$

Lepton tensor: $L^{\mu\mu'}(l,q) = e^2(-g^{\mu\mu'}Q^2 + 2(l^\mu l'^{\mu'} + l^{\mu'}l^\mu))$

Parameterization of gluon correlator for unpolarized proton target at 'leading twist'

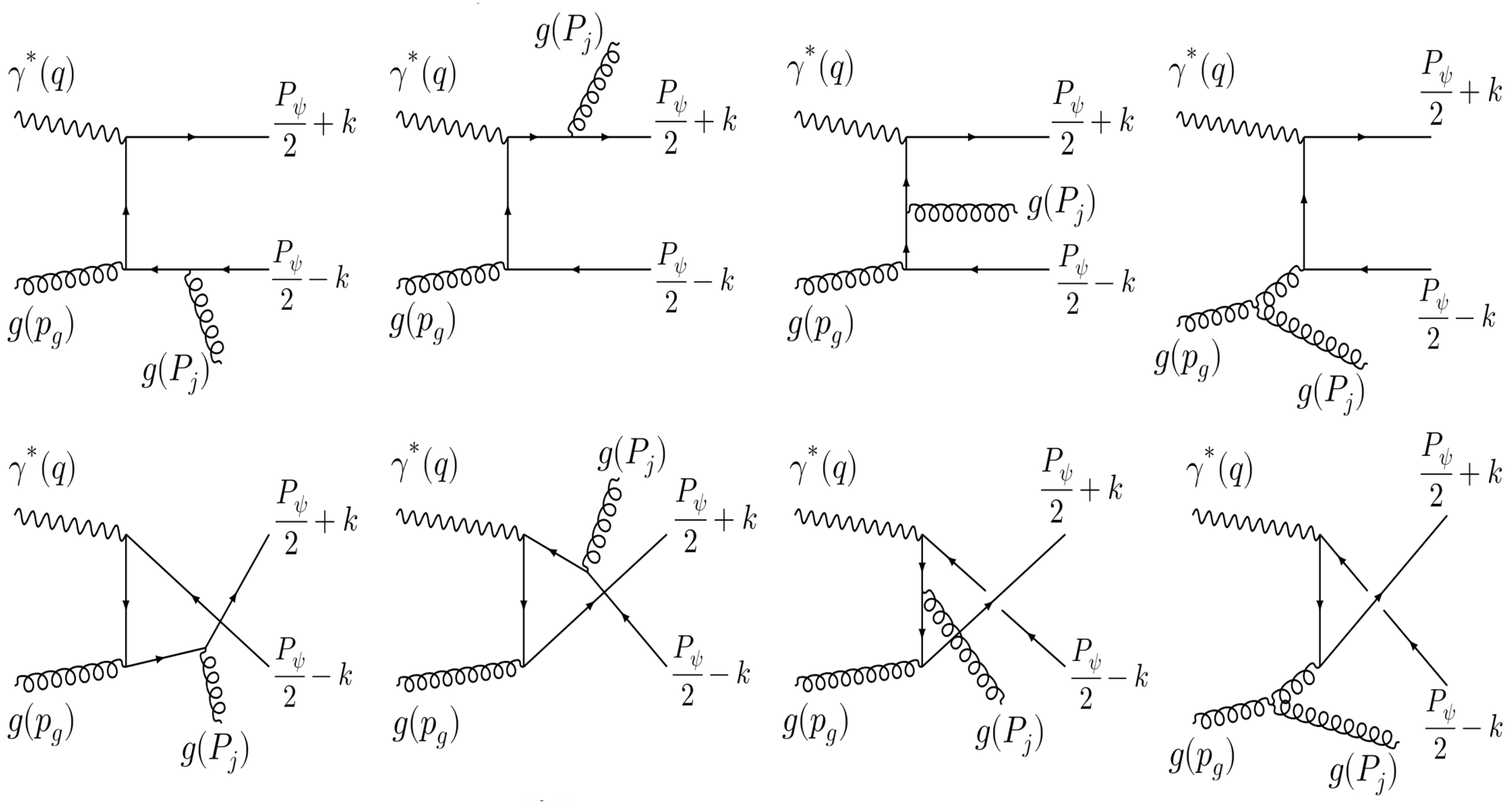
$$\Phi_g^{\nu\nu'}(x,\boldsymbol{p}_T^2) = \frac{1}{2x}\left[-g_\perp^{\nu\nu'} f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\nu p_T^{\nu'}}{M_p^2} + g_\perp^{\nu\nu'}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp g}(x,\boldsymbol{p}_T^2)\right]$$

Unpolarized gluon distribution

Linearly polarized gluon distribution

# Feynman diagrams

Gluon initiated hard process: $\gamma^* + g \to Q\bar{Q} + \ g$, contributes significantly over the quark(anti-quark) initiated hard process: $\gamma^* + q(\bar{q}) \to Q\bar{Q} + \ q(\bar{q})$, in the small$-x$ domain.




Tree level Feynman diagrams for the hard process: $\gamma^* + g \to c + \bar{c} + g$

# Amplitude Calculations Using NRQCD

The amplitude can be written as

$$M(\gamma^* g \to Q\bar{Q}[{}^{2S+1}L_J^{(1,8)}](P_\psi) + g)$$
$$= \sum_{L_z S_z} \int \frac{d^3k}{(2\pi)^3} \Psi_{LL_z}(k) \langle LL_z ; SS_z | JJ_z \rangle \mathrm{Tr}[\mathcal{O}(q,p,P_\psi,k) \mathcal{P}_{SS_z}(P_\psi,k)]$$

D. Boer and C. Pisano (2012)

$\mathcal{O}(q,p,P_\psi,k)$: amplitude for production of $Q\bar{Q}$ pair.

$$\mathcal{O}(q,p,P_\psi,k) = \sum_{m=1}^{8} C_m \mathcal{O}_m(q,p,P_\psi,k)$$

The spin projection operator, $\mathcal{P}_{SS_z}(P_\psi,k)$, projects the spin triplet and spin singlet states of $Q\bar{Q}$ pair

$$\mathcal{P}_{SS_z}(P_\psi,k) = \sum_{s_1 s_2} \left\langle \frac{1}{2} s_1 ; \frac{1}{2} s_2 \middle| SS_z \right\rangle v\left(\frac{P_\psi}{2} - k, s_1\right) \bar{u}\left(\frac{P_\psi}{2} + k, s_2\right)$$
$$= \frac{1}{4M_\psi^{3/2}} (-\not{P}_\psi + 2\not{k} + M_\psi) \Pi_{SS_z} (\not{P}_\psi + 2\not{k} + M_\psi) + O(k^2)$$

$\Pi_{SS_z} = \gamma^5$ for spin singlet ($S = 0$)

$\Pi_{SS_z} = \epsilon^\mu_{S_z}(P_\psi)\gamma_\mu$ for spin triplet ($S = 1$)

# Amplitude Calculations Using NRQCD

RK, Mukherjee, Pawar, Siddiqah, PRD 106 (2022)

Since, $k \ll P_h$ , amplitude expanded in Taylor series about $k = 0$

First term in the expansion gives the S-states $(L = 0, J = 0,1)$. The linear term in $k$ gives the P-states $(L = 1, J = 0,1,2)$.

The S-states amplitude :

$$M[{}^{2S+1}S_J^{(1,8)}](P_\psi, k) = \frac{1}{\sqrt{4\pi}} R_0(0) \mathrm{Tr}[\mathcal{O}(q,p,P_\psi,k)\mathcal{P}_{SS_z}(P_\psi,k)\Big|_{k=0}$$

The P-states amplitude :

$$M[{}^{2S+1}P_J^{(8)}](P_\psi, k) = -i\sqrt{\frac{3}{4\pi}} R_1'(0) \sum_L \epsilon_{L_z}^{\alpha}(P_\psi)\langle LL_z; SS_z|JJ_z\rangle \mathrm{Tr}[\mathcal{O}_\alpha(0)\mathcal{P}_{SS_z}(0) + \mathcal{O}(0)\mathcal{P}_{SS_z\alpha}(0)]$$

$$\mathcal{O}_\alpha(0) = \frac{\partial}{\partial k^\alpha}\mathcal{O}(q,p,P_\psi,k)\Big|_{k=0} \qquad \mathcal{P}_{SS_z\alpha}(0) = \frac{\partial}{\partial k^\alpha}\mathcal{P}_{SS_z}(q,p,P_\psi,k)\Big|_{k=0}$$

$R_0$ and $R_1'$ are related with the LDMEs

# Asymmetry Calculations

Final expression of the unpolarized differential cross section:

$$\frac{d\sigma}{dz\, dy\, d^2\boldsymbol{q}_T\, d^2\boldsymbol{K}_T} = \frac{1}{(2\pi)^4}\frac{1}{16sz(1-z)Q^4}\{(\mathbb{A}_0 + \mathbb{A}_1\cos\phi_\perp + \mathbb{A}_2\cos 2\phi_\perp)\boxed{f_1^g(x,\boldsymbol{q}_T^2)}$$
$$+ \frac{\boldsymbol{q}_T^2}{M_P^2}\boxed{h_1^{\perp g}(x,\boldsymbol{q}_T^2)}\big(\mathbb{B}_0\cos 2\phi_T + \mathbb{B}_1\cos(2\phi_T-\phi_\perp) + \mathbb{B}_2\cos 2(\phi_T-\phi_\perp) + \mathbb{B}_3\cos(2\phi_T-3\phi_\perp)$$
$$+ \mathbb{B}_4\cos(2\phi_T-4\phi_\perp)\big)\}$$

RK, Mukherjee, Pawar, Siddiqah, PRD 106 (2022)

Azimuthal modulation: $A^{W(\phi_S,\phi_T,\phi_\perp)} = 2\frac{\int d\phi_S\, d\phi_T\, d\phi_\perp W(\phi_S,\phi_T,\phi_\perp) d\sigma}{\int d\phi_S\, d\phi_T\, d\phi_\perp d\sigma}$

Boer, Mulders, Pisano, Zhou, JHEP 1608 (2016)

$\cos 2\phi_T$ azimuthal asymmetry as function of $z, x_B, y$ and $K_t$:

$$\langle\cos 2\phi_T\rangle \equiv A^{\cos 2\phi_T} = \frac{\int dq_T\, q_T\, \frac{q_T^2}{M_P^2}\mathbb{B}_0 \boxed{h_1^{\perp g}(x,q_T^2)}}{\int dq_T\, q_T\, \mathbb{A}_0 \boxed{f_1^g(x,q_T^2)}}$$

# Results: $cos2\phi_T$ asymmetry

RK, Mukherjee, Pawar, Siddiqah, PRD 106 (2022)

$\cos2\phi_t$ azimuthal asymmetry in

(A) Gaussian Parameterization

D. Boer and C. Pisano (2012)

(B) Spectator Model

Bacchetta, Celiberto, Radici, Taels, EPJC 80 (2020)

Kinematics: $\sqrt{s} = 140$ GeV

$Z = 0.7, \quad 0 < q_t < 1$ GeV

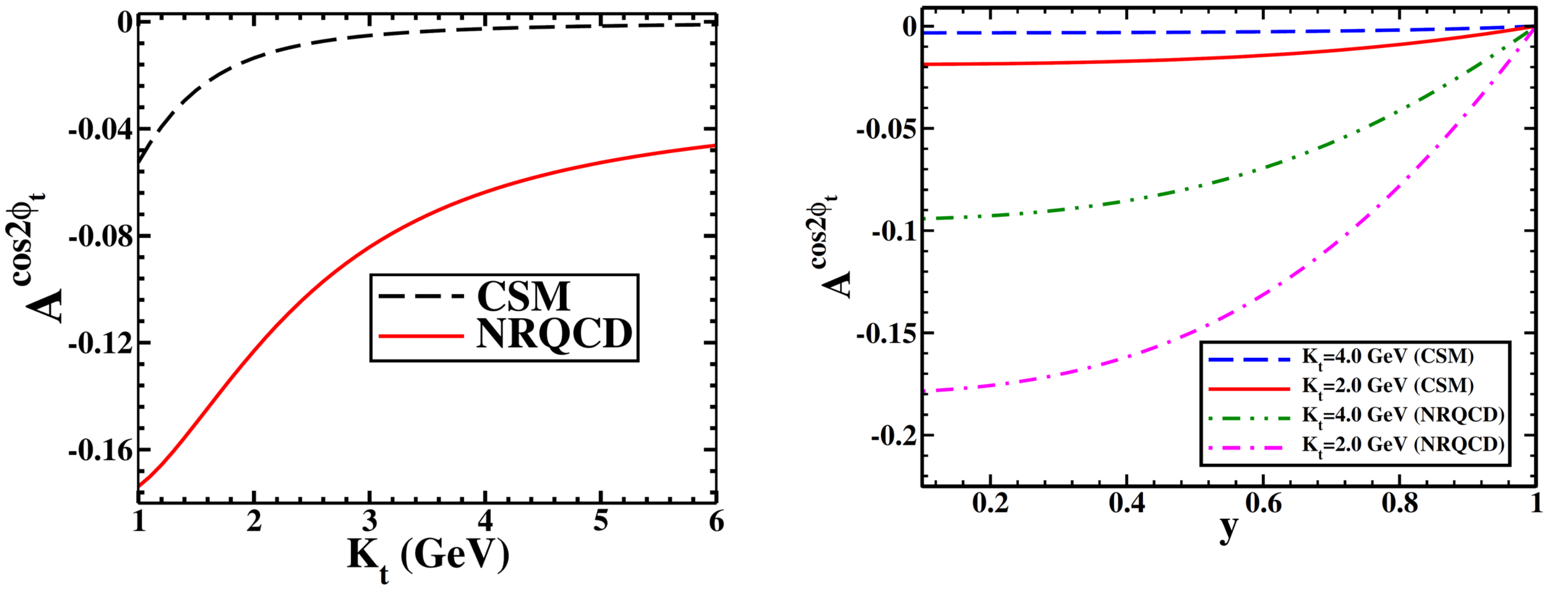
$$Q = \sqrt{M_\psi^2 + K_t^2}$$

$q_T \equiv q_t$ , $K_\perp \equiv K_t$ and $\phi_T \equiv \phi_t$

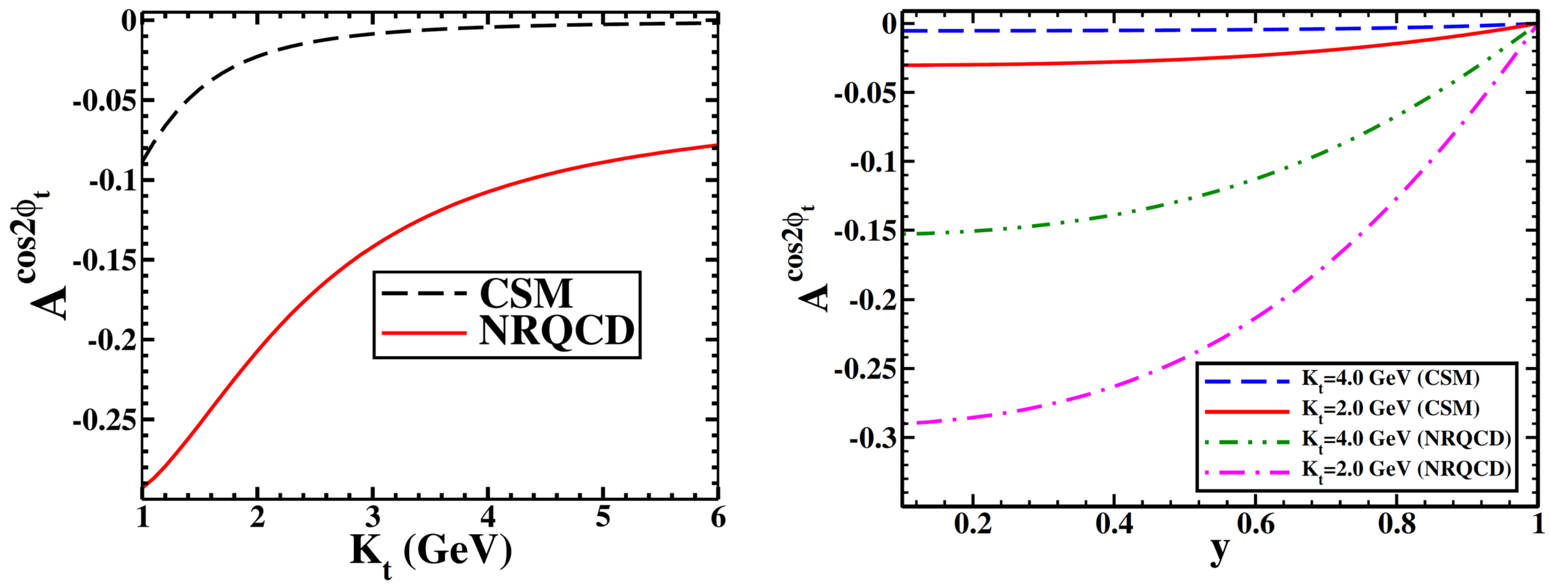
We used CSMWZ set of LDME

Chao (2012)

**(A)**




**(B)**



# Part2: Evolution Of TMDs

Aybat and Rogers (2011)
Echevarria, Scimemi, Vladimirov JHEP 09(2016)004

Scale evolution of TMDs can be obtained by solving the Collins-Soper evolution equation and renormalization group equation.

In impact parameter space($b_T$ −space ), TMDs can be expressed as

$\hat{F}(x, b_T, \zeta, \mu) = e^{-\frac{1}{2}S_A(b_T;\zeta,\zeta_0,\mu,\mu_0)}\hat{F}(x, b_T, \zeta_0, \mu_0)$ : Perturbative part

$\hat{F}(x, b_T, \zeta, \mu)$ represents TMDs in $b_T$ −space

$S_A$ is a perturbative sudakove factor; valid in the perturbative domain: $|b_T| \ll 1/\Lambda_{QCD}$

$e^{-S_A}$ resumms the leading logarithms; to avoid large logarithms: $\mu \sim \sqrt{\zeta} \sim Q$ and $\mu_0 \sim \sqrt{\zeta_0} \sim \mu_b \sim 1/b_T$

$S_A$ at LO is given by $S_A(b_T, \mu) = 2\frac{C_A}{\pi}\int_{\mu_b^2}^{\mu^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\alpha_S(\bar{\mu}^2)\left(\ln\frac{\mu^2}{\bar{\mu}^2} - \frac{11-2n_f/C_A}{6}\right)$

# Evolution Of TMDs

For $b_T \ll \Lambda_{QCD}^{-1}$, perturbative tails of the TMDs can be given by OPE:

Aybat and Rogers (2011)

$$\hat{F}_{g/A}(x, b_T, \mu_0, \zeta_0) = \sum_{j=q,\bar{q},g} C_{g/j}(x, b_T; \mu_0, \zeta_0) \otimes f_{j/A}(\hat{x}, \mu_0)$$

$C_{g/a}(x; \mu_b)$: Wilson coefficient function which are different for each TMDs, expands in powers of $\alpha_S$ as

$$C_{g/a}(x; \mu_b^2) = \delta_{ga}\delta(1-x) + \sum_{k=1}^{\infty} C_{g/a}^k(x)\left(\frac{\alpha_S(\mu_b)}{\pi}\right)^k$$

For large $b_T$, we need to introduce a non-perturbative Sudakov factor that freezes the perturbative contribution slowely as $b_T$ gets larger.

$$\hat{F}(x, b_T, \zeta, \mu) = e^{-\frac{1}{2}S_A(b_T^*; \zeta, \zeta_0, \mu, \mu_0)} \hat{F}(x, b_T^*, \zeta_0, \mu_0) e^{-S_{NP}(x, b_T)}$$

$b_T \xrightarrow{\text{Perturbative part}} b_T^*$

$\to b_0/Q \approx 0$ for $b_T \to 0$

$\to b_{Tmax}$ for $b_T \to \infty$

$b_T^* - prescription$

*to ensure that* $b_T \le b_{Tmax}$

# Evolution of TMDs with leading terms: B

Perturbative tails of $f_1^g$ and $h_1^{\perp g}$ given by integrated PDF; only leading order terms:

$$\hat{f}_1^g(x, b_T; \mu_0, \zeta_0) = f_{g/A}(x; \mu_0) + \mathcal{O}(\alpha_S) + \mathcal{O}(b_T \Lambda_{QCD})$$

$h_1^{\perp g}$ requires a helicity flip and therefore an additional gluon exchange; perturbative tails starts at $\mathcal{O}(\alpha_S)$

$$\hat{h}_1^{\perp g}(x, b_T; \mu_0, \zeta_0) = \frac{C_A \alpha_S(\mu_0)}{\pi} \int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1\right) f_{g/A}(\hat{x}, \mu_0^2) + \frac{C_F \alpha_S(\mu_0)}{\pi} \sum_{j=q,\bar{q}} \int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1\right) f_{j/A}(\hat{x}, \mu_0^2) + \mathcal{O}(\alpha_S) + \mathcal{O}(b_T \Lambda_{QCD})$$

P. Sun, B.-W. Xiao, F. Yuan (2011)

Perturbative Sudakov factor $S_A$, with inclusion of one-loop running of $\alpha_S$; we set $\mu \sim \sqrt{\zeta} \sim Q$ and $\mu_0 \sim \sqrt{\zeta_0} \sim \mu_b$

$$S_A(b_T; Q, \mu_b) = \frac{36}{33 - 2n_f} \left[\ln \frac{Q^2}{\mu_b^2} + \ln \frac{Q^2}{\Lambda_{QCD}^2} \ln\left(1 - \frac{\ln(Q^2/\mu_b^2)}{\ln(Q^2/\Lambda_{QCD}^2)}\right) + \left(\frac{11 - 2n_f/C_A}{6}\right) \ln\left(\frac{\ln(Q^2/\Lambda_{QCD}^2)}{\ln(\mu_b^2/\Lambda_{QCD}^2)}\right)\right] + \mathcal{O}(\alpha_S^2)$$

D.Boer, J. Bor (2022)

$$b_T^*(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{Tmax})^2}}$$

$0$ for $b_T \to 0$

$b_{Tmax}$ for $b_T \to \infty$

$$\mu_b \sim 1/b_T \Rightarrow \quad \mu_b = \frac{Q b_0}{Q b_T + b_0} \Rightarrow \mu_b < Q$$

# Evolution of TMDs with leading terms: B

Evolved TMDs at final scale, $\mu = Q$

$$\hat{f}_1^{g}(x, b_T; Q) = f_{g/A}(x; \mu_b^*)e^{-\frac{1}{2}S_A(b_T; Q, \mu_b^*)}e^{-S_{NP}(x,b_T)} + \mathcal{O}(\alpha_S)$$

$$\hat{h}_1^{\perp g}(x, b_T; Q) = \left[\frac{C_A \alpha_S(\mu_b^*)}{\pi}\int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}}\left(\frac{\hat{x}}{x} - 1\right) f_{g/A}(\hat{x}, \mu_b^*) + \frac{C_F \alpha_S(\mu_b^*)}{\pi}\sum_{j=q,\bar{q}}\int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}}\left(\frac{\hat{x}}{x} - 1\right) f_{j/A}(\hat{x}, \mu_b^*)\right] e^{-\frac{1}{2}S_A(b_T; Q, \mu_b^*)}e^{-S_{NP}(x,b_T)} + \mathcal{O}(\alpha_S^2)$$

Fourier Transformation to the momentum space

$$\hat{f}_1^{g}(x, q_T; Q) = \frac{1}{2\pi}\int_0^\infty db_T\, b_T J_0(b_T q_T) f_{g/A}(x; \mu_b^*)e^{-\frac{1}{2}S_A(b_T^*; Q, \mu_b^*)}e^{-S_{NP}(x,b_T)}$$

$$\frac{q_T^2}{2M_p^2}\hat{h}_1^{\perp g}(x, q_T; Q) = \frac{1}{2\pi}\int_0^\infty db_T\, b_T J_2(b_T q_T)\left[\frac{C_A \alpha_S(\mu_b^*)}{\pi}\int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}}\left(\frac{\hat{x}}{x} - 1\right) f_{g/A}(\hat{x}, \mu_b^*) + \frac{C_F \alpha_S(\mu_b^*)}{\pi}\sum_{j=q,\bar{q}}\int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}}\left(\frac{\hat{x}}{x} - 1\right) f_{j/A}(\hat{x}, \mu_b^*)\right] e^{-\frac{1}{2}S_A(b_T^*; Q, \mu_b^*)}e^{-S_{NP}(x,b_T)} + \mathcal{O}(\alpha_S^2)$$

# TMD Evolution; expanded to $\mathcal{O}(\alpha_S)$: A

For $b_T \ll \Lambda_{QCD}^{-1}$, perturbative tails of TMDs can be given by OPE:

Aybat and Rogers (2011)

$$\hat{F}_{g/A}(x, b_T, \mu_0, \zeta_0\,) = \sum_{j=q,\bar{q},g} C_{g/j}(x, b_T; \mu_0, \zeta_0\,) \otimes f_{j/A}\,(\hat{x}, \mu_0\,)$$

$C_{g/a}(x; \mu_b)$: Wilson coefficient function which are different for each TMDs, expands in powers of $\alpha_S$ as

$$C_{g/a}(x; \mu_b^2) = \delta_{ga}\delta(1-x) + \sum_{k=1}^{\infty} C_{g/a}^{k}(x) \left(\frac{\alpha_S(\mu_b)}{\pi}\right)^k$$

For large $b_T$, we need to introduce a non-perturbative Sudakov factor that freezes the perturbative contribution slowly as $b_T$ gets larger.

$$\hat{F}(x, b_T, \zeta, \mu\,) = \boxed{e^{-\frac{1}{2}S_A(b_T^*;\zeta,\zeta_0,\mu,\mu_0)}\,\hat{F}(x, b_T^*, \zeta_0, \mu_0)}\,e^{-S_{NP}(x,b_T)}$$

$$b_T \xrightarrow{\text{Perturbative part}} b_T^*$$

$b_0/Q \approx 0$ for $b_T \to 0$

$b_{Tmax}$ for $b_T \to \infty$

$b_T^* - prescription$

*to ensure that* $b_T \leq b_{Tmax}$

# TMD Evolution; expanded to $\mathcal{O}(\alpha_S)$: A

Expansion in resummation, $e^{-\frac{1}{2}S_A} \to 1 - S_A/2$ and coefficient fucntion $C_{g/j}$ to $\mathcal{O}(\alpha_S)$

$$\hat{f}_1^g(x, b_T; \mu)$$
$$= f_{g/A}(x; \mu_b^*)$$
$$- \frac{\alpha_S}{2\pi}\left[\left(\frac{C_A}{2}\ln^2\frac{\mu^2}{\mu_b^{*2}} - \frac{11C_A - 2n_f}{6}\ln\frac{\mu^2}{\mu_b^{*2}}\right) f_{g/A}(x; \mu_b^*) + \sum_{i=q,\bar{q},g}\int_x^1 \frac{d\hat{x}}{\hat{x}} C_{g/i}^1(x; \mu_b^*) f_{i/A}\left(\frac{\hat{x}}{x}; \mu^*{}_b\right)\right] e^{-S_{NP}(x,b_T)} + \mathcal{O}(\alpha_S^2)$$

* We neglected the running of $\alpha_S$ here

Boer, D'Alesio,Murgai,Pisano,Taels (2020)
Boer,Bor,Maxia,Pisano,Yuan (2023)

At inpute scale, $\mu_b$: $\quad C_{g/g}^1 = -\frac{\pi^2}{12}\delta(1-x) \qquad C_{g/q}^1 = C_{g/\bar{q}}^1 = C_F x$

$$\hat{h}_1^{\perp g}(x, b_T; \mu)$$
$$= \left[\frac{C_A\alpha_S(\mu_b^*)}{\pi}\int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}}\left(\frac{\hat{x}}{x} - 1\right) f_{g/A}(\hat{x}, \mu_b^{*2}) + \frac{C_F\alpha_S(\mu_b)}{\pi}\sum_{j=q,\bar{q}}\int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}}\left(\frac{\hat{x}}{x} - 1\right) f_{j/A}(\hat{x}, \mu_b^{*2})\right] e^{-S_{NP}(x,b_T)} + \mathcal{O}(\alpha_S^2)$$

# Non-perturbative Sudakov factor $S_{NP}$

$S_{NP}$ is introduced to suppress perturbative contribution in the large $b_T$ region (non-perturbative)

General Charectistics: $e^{-S_{NP}(x,b_T,Q)}$ → 1 for $b_T \to 0$

→ 0 for large $b_T$ → Confinement distance

Functional form of $S_{NP}$ is largely unknown, however often choosen to be gaussian in $b_T$

Case 1: $S_{NP}(b_T;Q) = A\ln\left(\frac{Q}{Q_{NP}}\right) b_c^2(b_T)\,,\qquad Q_{NP} = 1\ \mathrm{GeV}\qquad b_c(b_T) = \sqrt{b_T^2 + \left(\frac{b_0}{Q}\right)^2}\quad and\quad b_T^*(b_T) = \frac{b_c}{\sqrt{1+(b_c/b_{Tmax})^2}}$

Scarpa,Boer,Echevarria,Lensberg,pisano,Schlegel (2020)

$A$ is fixed by defining a $b_{Tlim}$ such that $e^{-S_{NP}}$ becomes negligible ($\sim 10^{-3}$) for given $Q$

To estimate uncertainty, we consider $b_{Tlim} = 2, 4$ and $8\ \mathrm{GeV}^{-1}$, which roughly spans the region from $b_{Tmax} = 1.5\ \mathrm{GeV}^{-1}$ to the charge radius of the proton.

Case 2: $S_{NP}(x,b_T;Q) = \left[A\ln\left(\frac{Q}{Q_{NP}}\right) + B(x)\right] b_T^2,\qquad Q_{NP} = 1.6\ \mathrm{GeV}$

J. Bor and D. Boer, PRD 106, 014030 (2022), 2204.01527

Inspired by parameterization obtained from fitting SIDIS, DY and Z-boson production data.

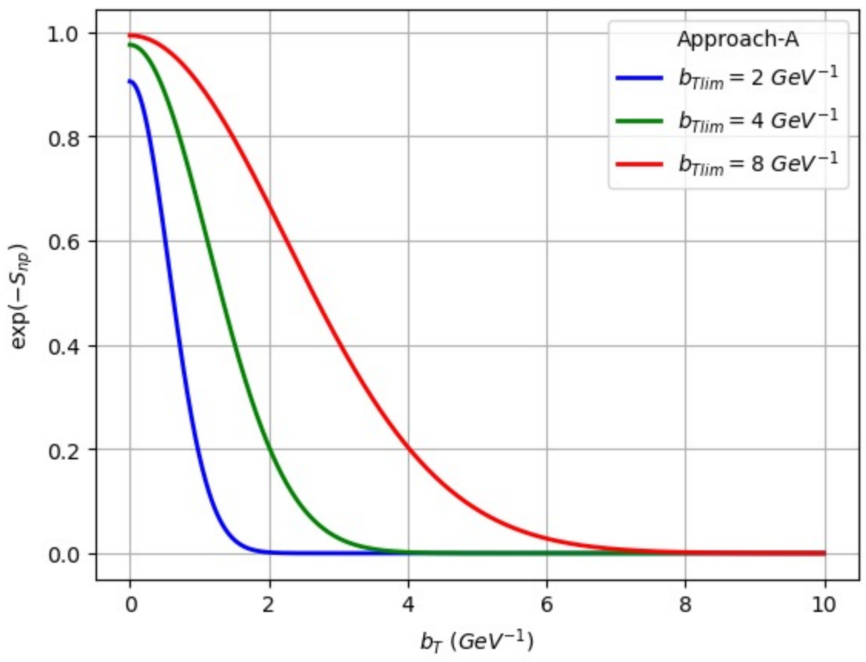
S. Aybat and T. Rogers, PRD 83,114042 (2011)

# Non-perturbative Sudakov factor $S_{NP}$

**Case 1:**

$$b_c(b_T) = \sqrt{b_T^2 + \left(\frac{b_0}{Q}\right)^2}$$

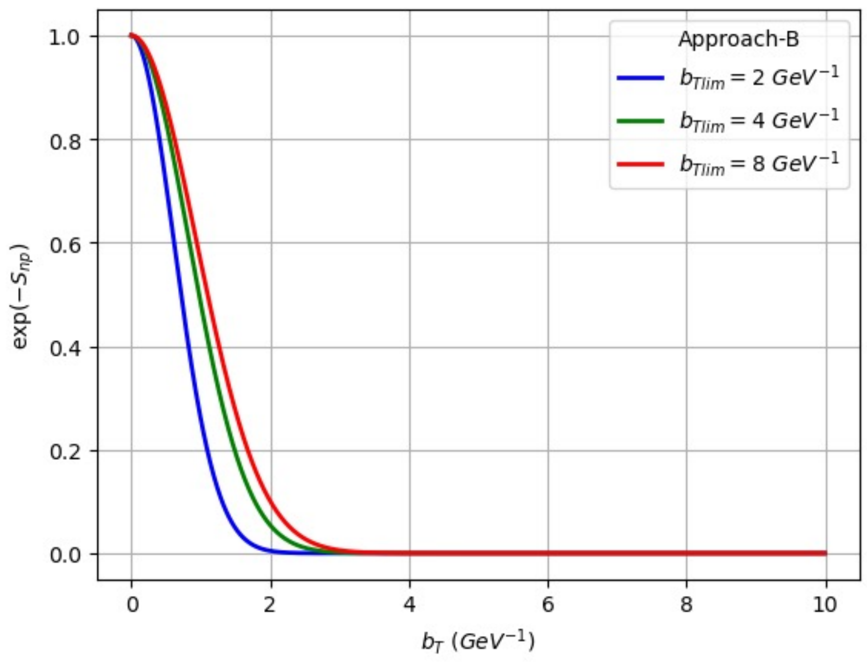
$$S_{NP}(b_T;Q) = A \ln\left(\frac{Q}{Q_{NP}}\right) b_c^2(b_T), \qquad Q_{NP} = 1\ \text{GeV}$$




Scarpa,Boer,Echevarria,Lensberg,pisano,Schlegel (2020)

**Case 2:**

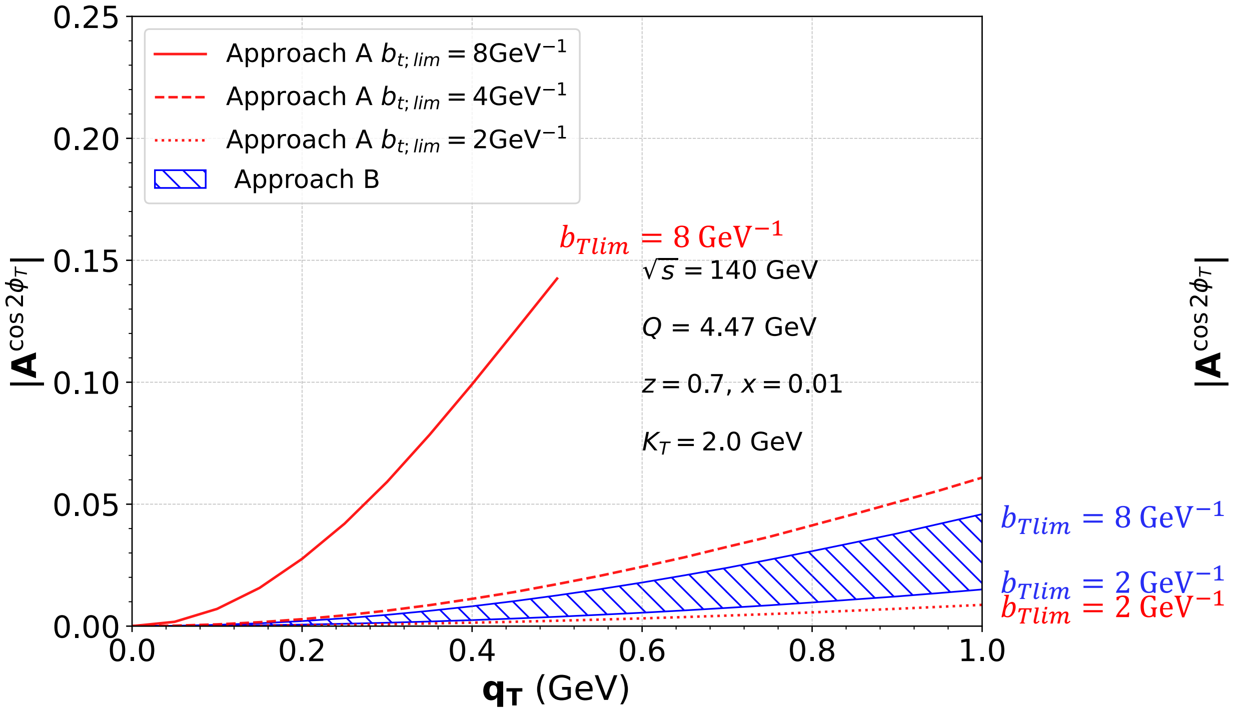
$$S_{NP}(x, b_T;Q) = \left[A \ln\left(\frac{Q}{Q_{NP}}\right) + B(x)\right] b_T^2, \qquad Q_{NP} = 1.6\ \text{GeV}$$



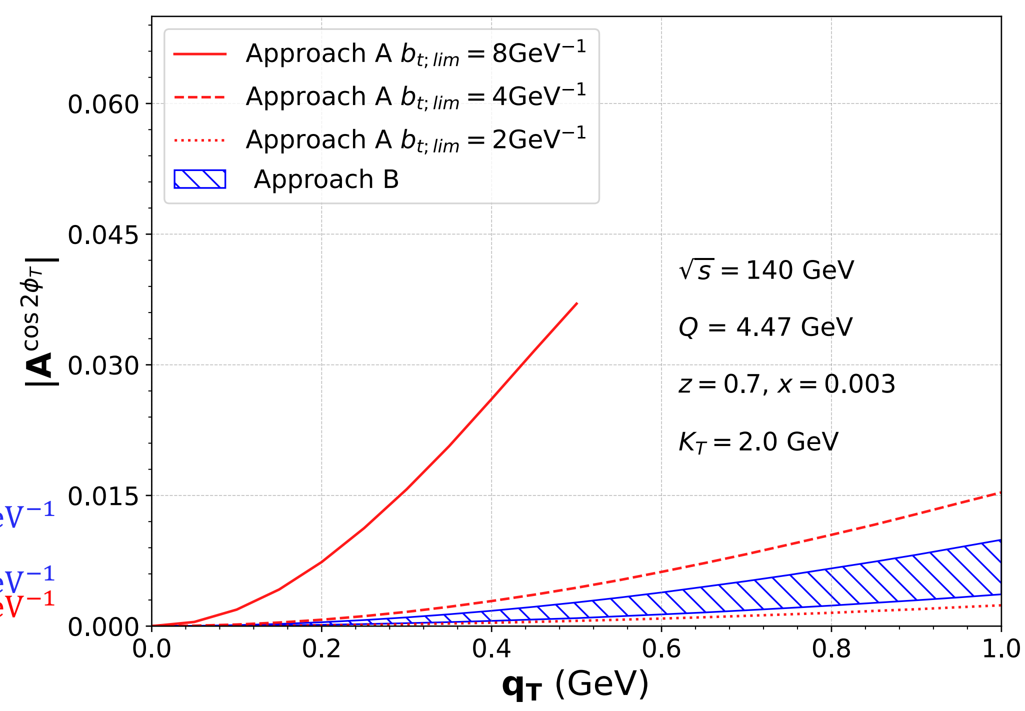

D.Boer, J. Bor (2022)

$$x = 0.01, \qquad Q^2 = 20\ \text{GeV}^2$$

# Results




$x = 0.01$




$x = 0.003$

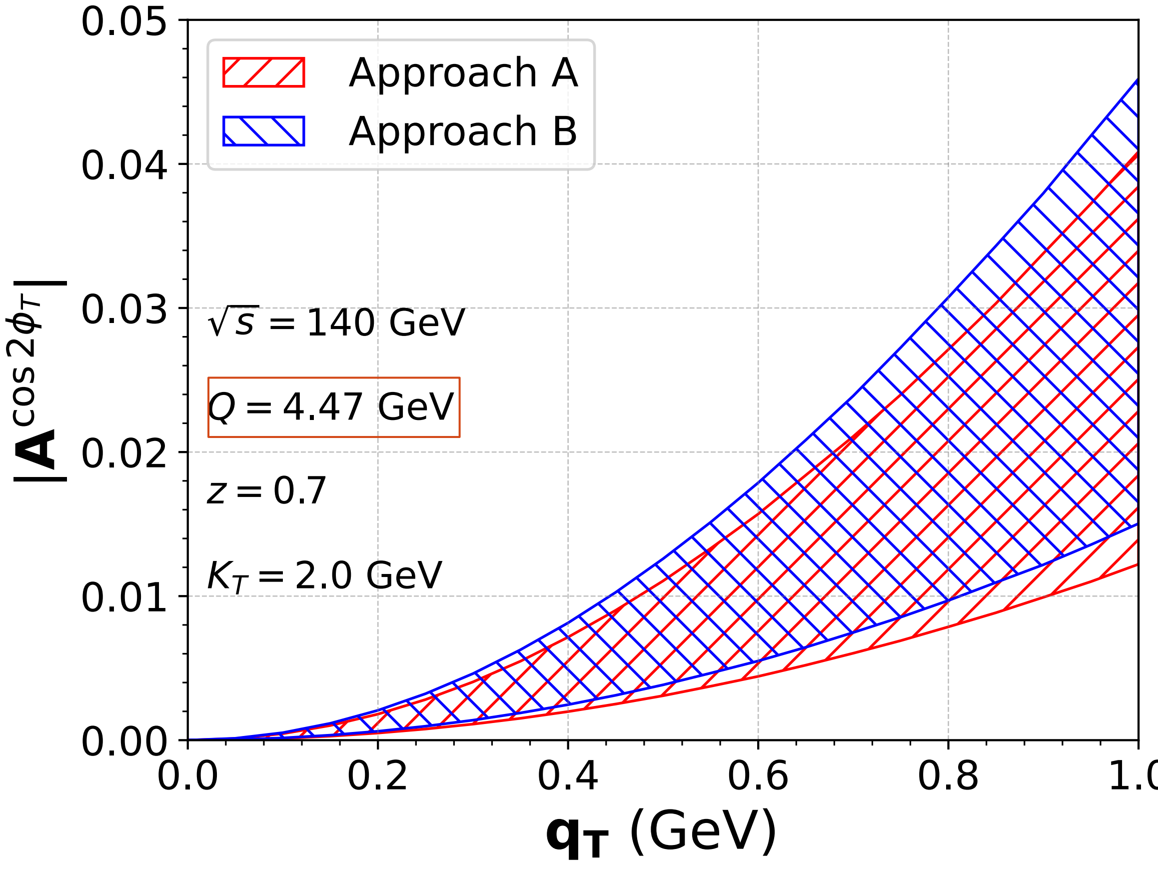
Approach-A : $S_{NP}(b_T; Q) = A \ln\left(\frac{Q}{Q_{NP}}\right) b_c^2(b_T)$ (case-1)

Approach-B: $S_{NP}(x, b_T; Q) = \left[A \ln\left(\frac{Q}{Q_{NP}}\right) + B(x)\right] b_T^2$ (case-2)

- Reduced range of $q_T$ for which the positivity bound is satisfied

# Results

$$S_{NP}(x, b_T; Q) = \left[A \ln\left(\frac{Q}{Q_{NP}}\right) + B(x)\right] b_T^2$$



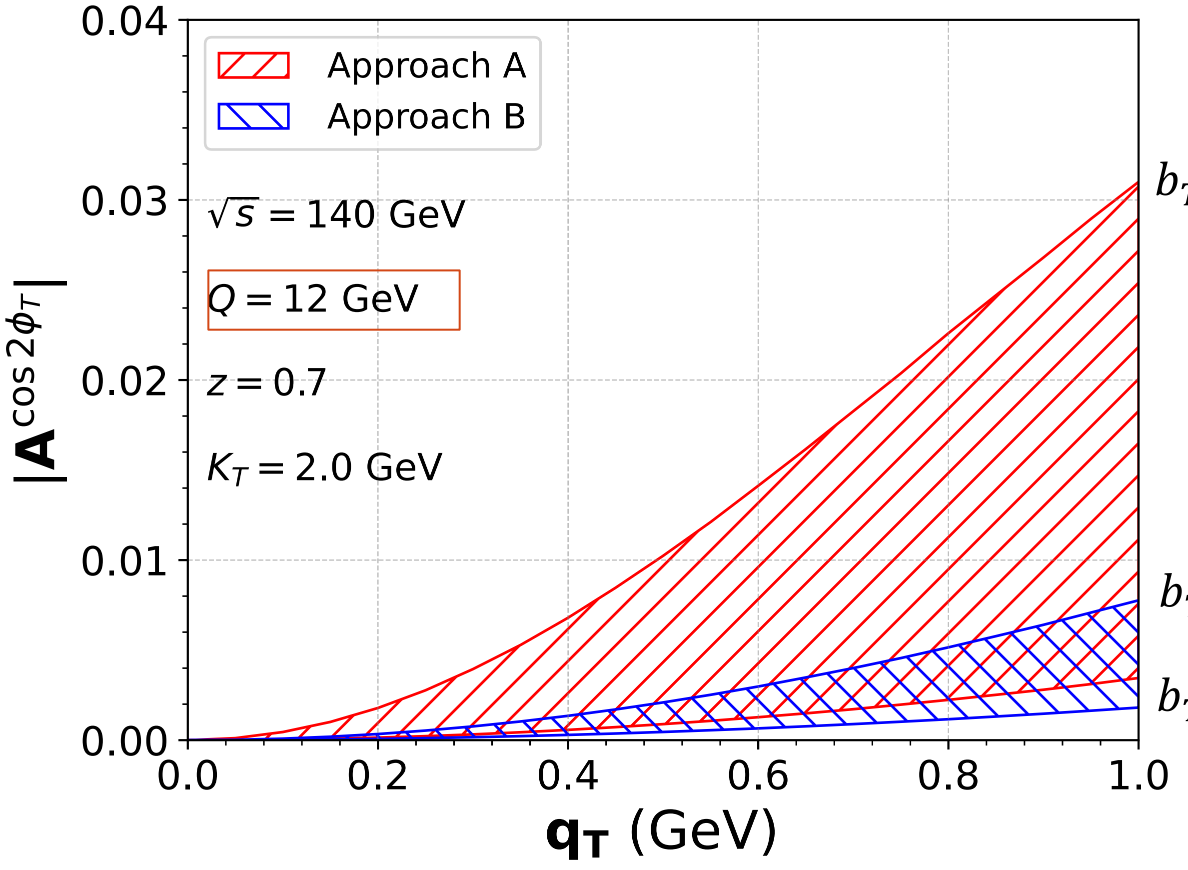

$b_{Tlim} = 8\ \mathrm{GeV}^{-1}$

$b_{Tlim} = 2\ \mathrm{GeV}^{-1}$

$x = 0.01$

# Results

$$S_{NP}(x, b_T; Q) = \left[A \ln\left(\frac{Q}{Q_{NP}}\right) + B(x)\right] b_T^2$$




$b_{Tlim} = 8\ \mathrm{GeV}^{-1}$

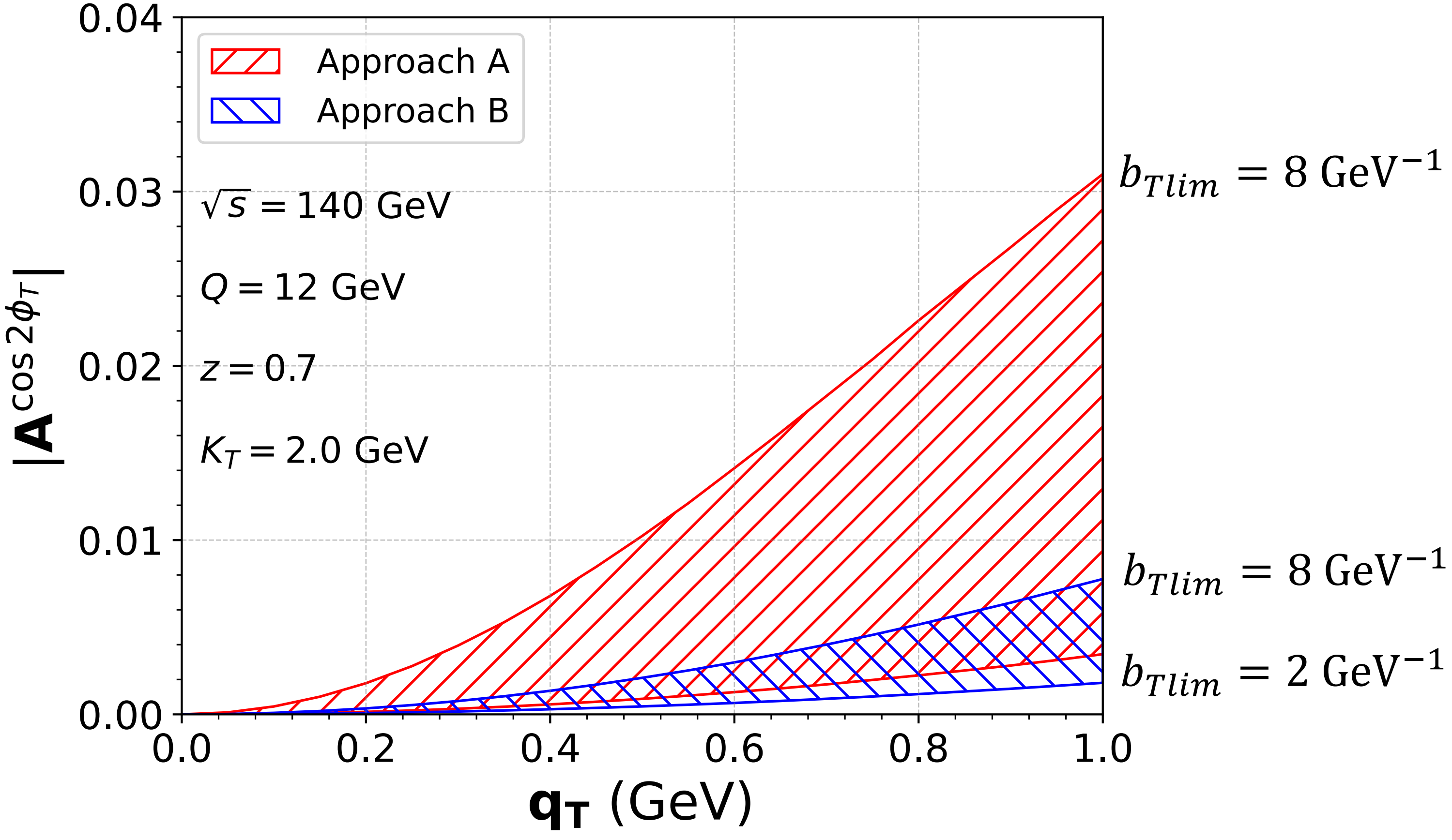
$b_{Tlim} = 8\ \mathrm{GeV}^{-1}$

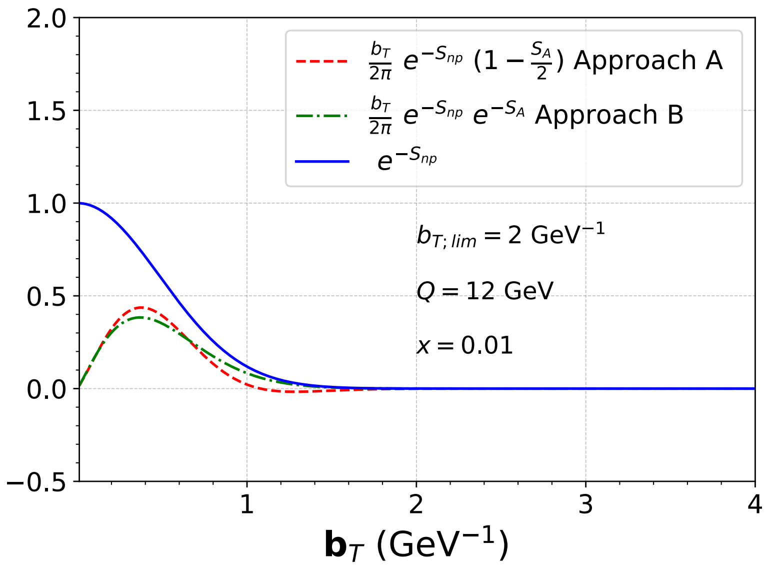
$b_{Tlim} = 2\ \mathrm{GeV}^{-1}$

$x = 0.01$

# Results

$$S_{NP}(x, b_T; Q) = \left[A \ln\left(\frac{Q}{Q_{NP}}\right) + B(x)\right] b_T^2$$

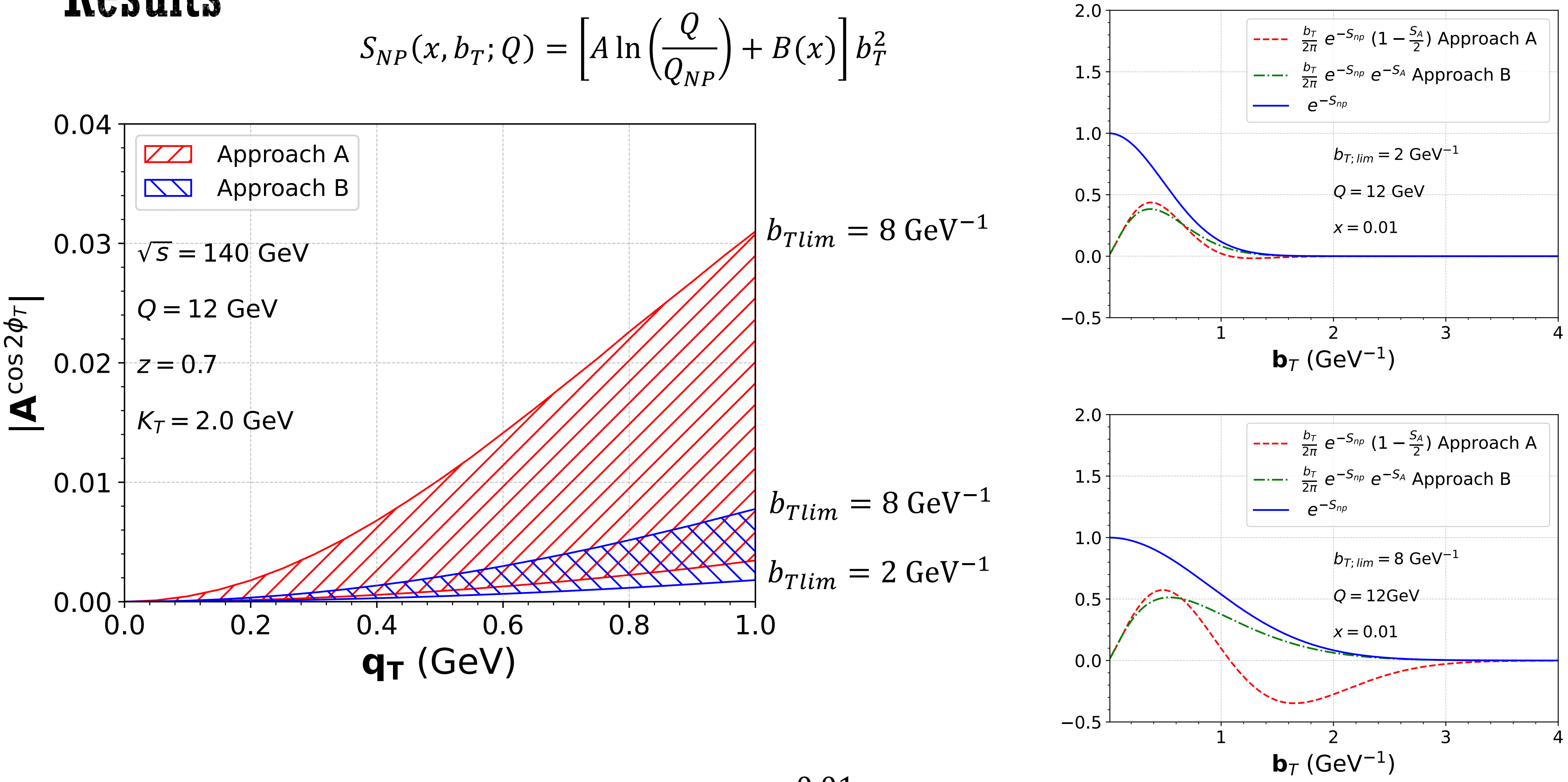



$x = 0.01$

# Results

$$S_{NP}(x, b_T; Q) = \left[A \ln\left(\frac{Q}{Q_{NP}}\right) + B(x)\right] b_T^2$$




$x = 0.01$

# Summary

We calculated the $\cos 2\phi_t$ azimuthal asymmetry in a back-to-back $J/\psi$ - $jet$ pair production shows a promising channel to prove poorly known linearly polarized gluon TMD at the future proposed EIC.

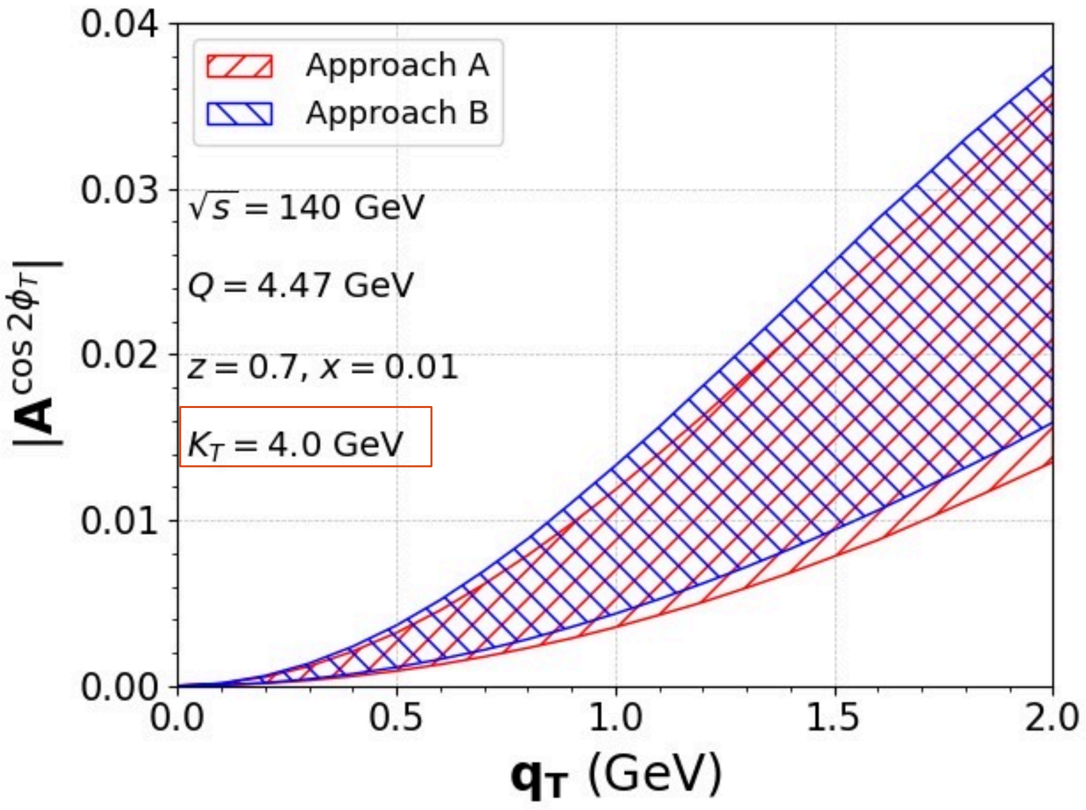
We consider the standard NRQCD framework for $J/\psi$ production. However, for the full TMDs factorization for $J/\psi$ - $jet$ pair production process may require for introduction of shape function.

We show the effect of TMD evolution on the asymmetry in two different parameterization for the perturbative tails of the TMDs and found they differ at large scale $Q$.

The parameterization of nonperturbative factor, particularly in the large $b_T$ region, show a significant role on the evolution of TMDs and hence on the $cos2\phi$ asymmetry.

# Backup Results

$$S_{NP}(x, b_T; Q) = \left[A \ln\left(\frac{Q}{Q_{NP}}\right) + B(x)\right] b_T^2$$




$b_{Tlim} = 8\ \mathrm{GeV}^{-1}$

$b_{Tlim} = 2\ \mathrm{GeV}^{-1}$

$x = 0.01$