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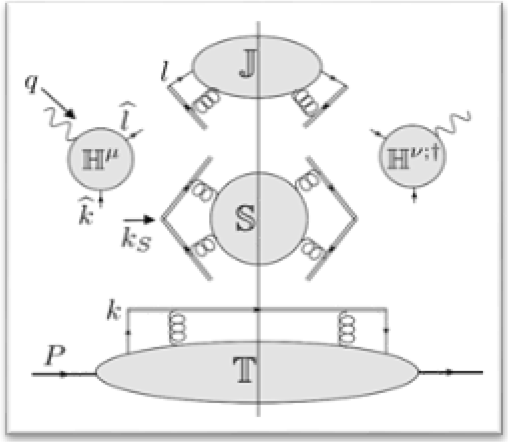
New Insights on DIS factorization at threshold

OLD DOMINION UNIVERSITY

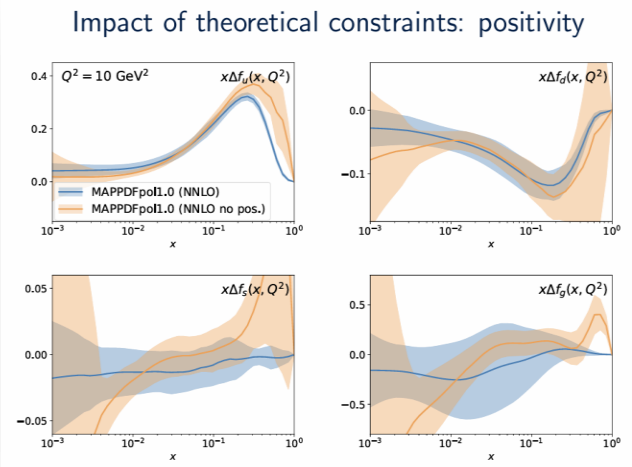
Jefferson Lab

Deep Inelastic Scattering at threshold

A "simple" framework where to test/explore factorization in hadron processes

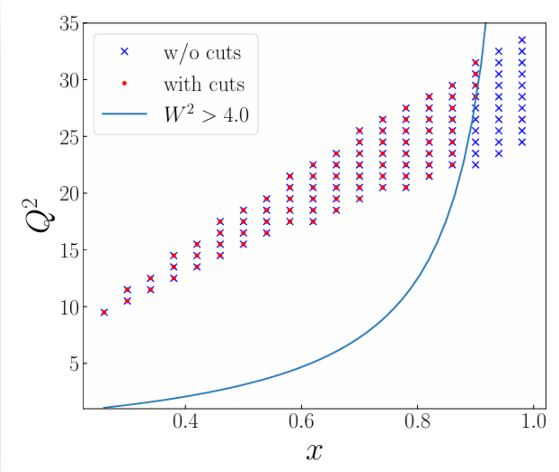


Relevant for better understanding hadron structure (ratio u/d, positivity bound etc..)



(From Emanuele Nocera's talk)

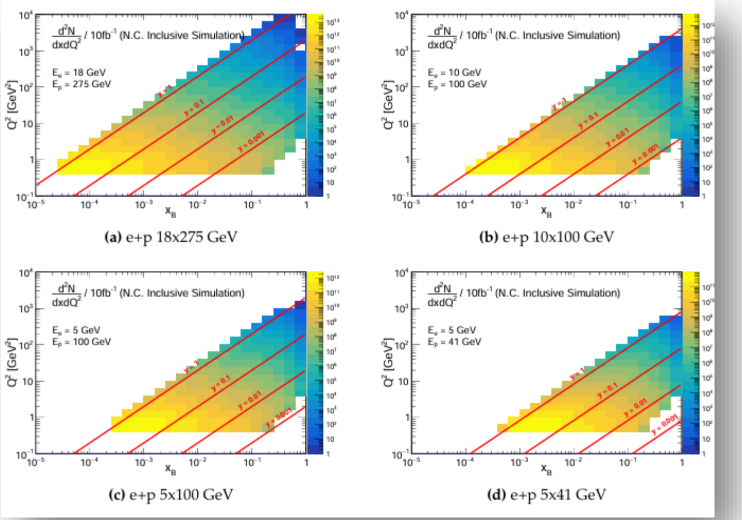
Fits and future data interpretation



Strong Interaction Physics at the Luminosity Frontier with 22 GeV Electrons at Jefferson Lab

e-Print: 2306.09360 [nucl-ex]

EIC Yellow Report



Widely studied in the past...

In QCD

- *Summation of Large Corrections to Short Distance Hadronic Cross-Sections*, **Sterman** (1986)
- *Resummation of the QCD Perturbative Series for Hard Processes,* **Catani, Trentadue** (1989)

….

In SCET

- *Factorization and Momentum-Space Resummation in Deep-Inelastic Scattering,* **Becher, Neubert, Pecjak** (2007)
- *Rapidity Divergences and Deep Inelastic Scattering in the Endpoint Region*, **Fleming, Ou Labun** (2012)
- *Proper factorization theorems in high-energy scattering near the endpoint*, **Chay, Kim** (2013)

….

...but there are still some *subtleties* not fully explored.

- Early QCD works use an "archaic" (poetic) language for the mechanism of soft subtractions

In the terminology of [13], any choice of soft subdiagram S is known as a "tulip". A "garden" K is any set $\{S\}$ of nested (non-overlapping and not disjoint) sets of tulips S. An example is shown in fig. 4.3. This construction allows one to show [13]

[Sterman (1986)]

- The **treatment of rapidity divergences** is never really addressed

Well, this was almost **40** years ago

- Debatable gauge choice $A^3 = 0$ in light of modern considerations

difficulties in a general treatment. Even the non-light-like case, with $n^2 \neq 0$, is not adequate for our later work, because the singularity at $k \cdot n = 0$ breaks standard analyticity rules for propagators that are needed in proofs of factorization; see Ch. 14.

[Collins (2011)]

Need to uniform the language to modern formalism

- In SCET the factorization theorem is clear and simple:

$$F_2^{\rm ns}(x,Q^2) = \sum_q e_q^2\, |C_V(Q^2,\mu)|^2\, Q^2 \int_x^1 d\xi\, J\Big(Q^2\,\frac{\xi - x}{x},\mu\Big)\, \phi_q^{\rm ns}(\xi,\mu)\,.$$

[Becher, Neubert, Pecjak (2007)]

Different organization of subtractions

compare it with Sterman's

$$F(x,Q^2) = |H_{\rm DI}(Q^2)|^2 \int_x^1 (\mathrm{d}y/y)\phi(y,Q^2) \int_0^{y-x} (\mathrm{d}w/[1-w]) V(wQ)$$

$$\times J\left[Q^2(y-x-w)/2x, Q\right] + O(1-x)^0. \qquad (3.13)$$

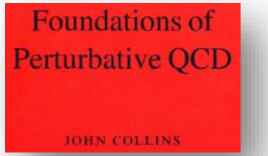
[Sterman (1986)]

- Difficulties/problems with rapidity divergences

[rapidity anomalous dimensions] reveal sensitivity to IR scales, which may signal a breakdown of rapidity factorization in $\mathrm{SCET_{II}}$.

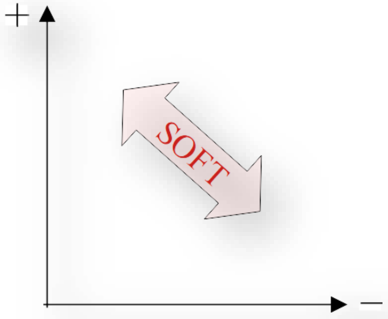
[Fleming, Labun (2012)]

Goals



- Uniform the language of early QCD works to modern factorization formalism (in QCD)
- Fix once and for all the subtleties related to the **rapidity divergences** and the subtraction mechanism
- Find (and exploit) analogies with processes *apparently* very different but actually sharing a similar physics

Two light-cone directions entangled by soft radiation

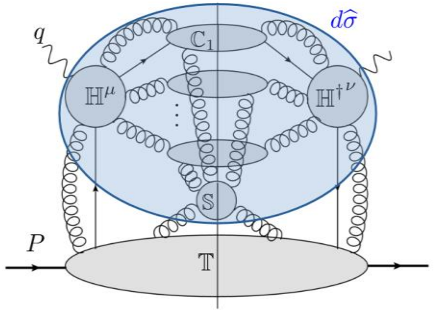


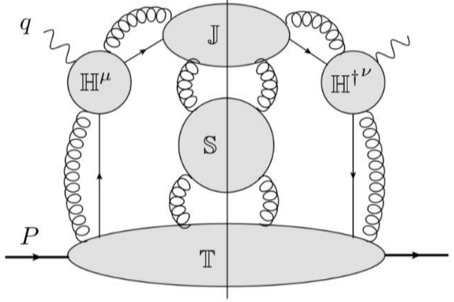
Analogies with TMD processes, event shape observables (thrust etc...)

- Obtain a factorization theorem that is consistent with the result of the past, completing and bridging all of them in a fully consistent and modern QCD treatment

Kinematics and Libby-Sterman analysis

DIS

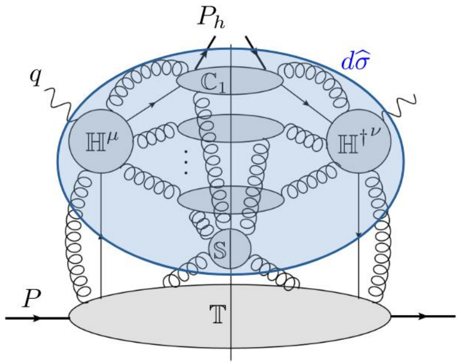


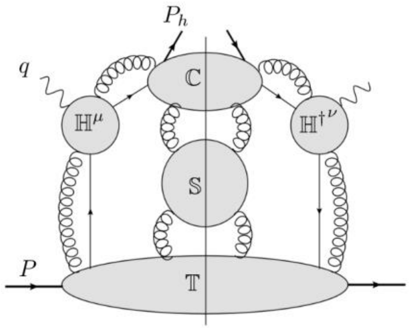


$x_{\rm bj}$

0 — 1

SIDIS (TMD)

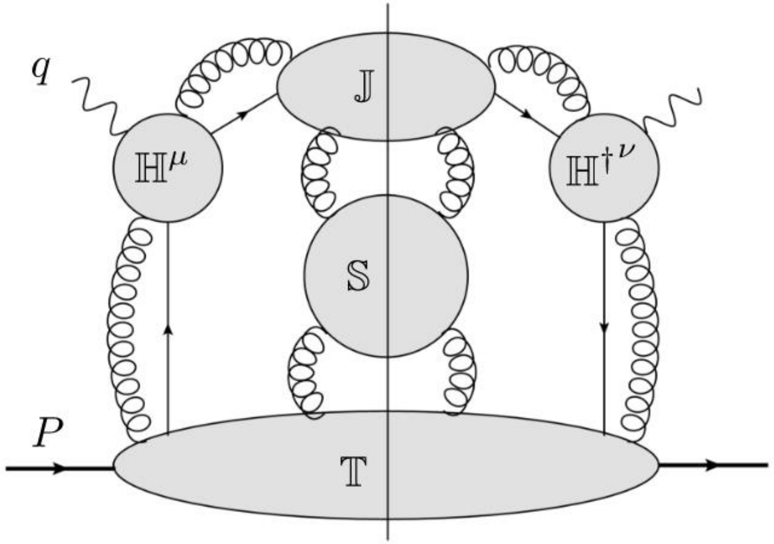




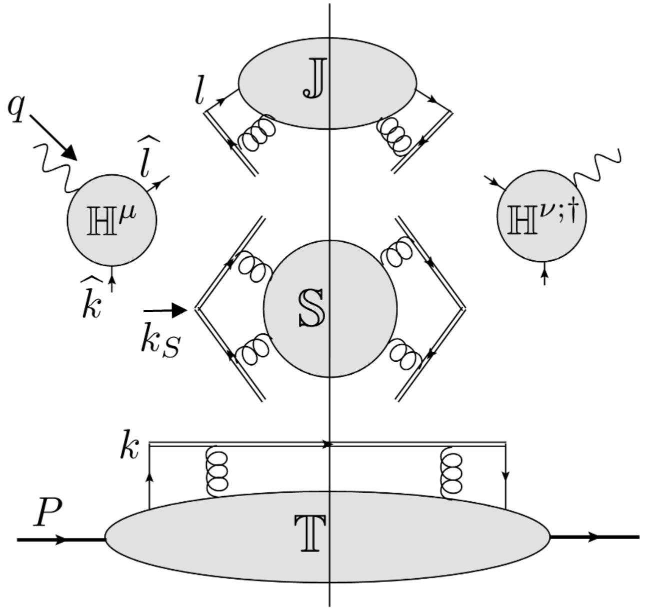
q_T/Q

1 — 0

Factorization



- Power counting
- Definition of kinematic approximators
- Ward identities

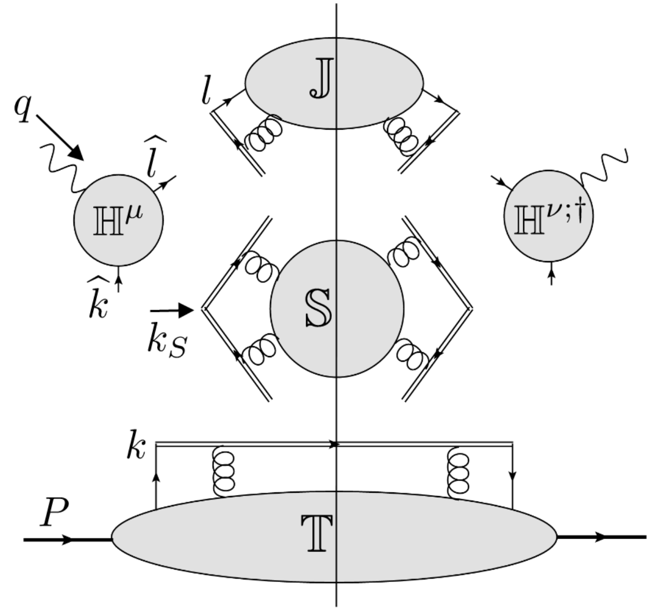


$$F_1 = |V(q^2)|^2 \frac{N_C}{x_{bj}} \int_{x_{bj}}^{1} \frac{d\xi}{\xi} \int_0^{\xi - x_{bj}} d\rho\, \phi(\xi; m) S\left(\frac{\rho}{\xi}\right) J\left(\frac{\xi - x - \rho}{x}\right) + \text{p.s.}$$

Same as Sterman (1987)

$$F(x,Q^2) = |H_{\mathrm{DI}}(Q^2)|^2 \int_x^1 (\mathrm{d}y/y)\phi(y,Q^2) \int_0^{y-x} (\mathrm{d}w/[1-w])V(wQ) \times J[Q^2(y-x-w)/2x, Q] + O(1-x)^0. \qquad (3.13)$$

Factorization



$$F_1 = |V(q^2)|^2 \frac{N_C}{x_{bj}} \int_{x_{bj}}^{1} \frac{d\xi}{\xi} \int_0^{\xi - x_{bj}} d\rho\, \phi_{\text{sub.}}(\xi; m) S\left(\frac{\rho}{\xi}\right) \times J_{\text{sub.}}\left(\frac{\xi - x - \rho}{x}\right) + \text{p.s.}$$

1. Dealing with rapidities:
   Kinematic approximations are defined in such a way that light-cone directions are reached only *as a limit.*

$$\lim_{\text{tilt}\to 0} \left(\begin{array}{l}\text{Factorization}\\ \text{off light-cone}\end{array}\right) = \left(\begin{array}{l}\text{Factorization}\\ \text{on the light-cone}\end{array}\right)$$

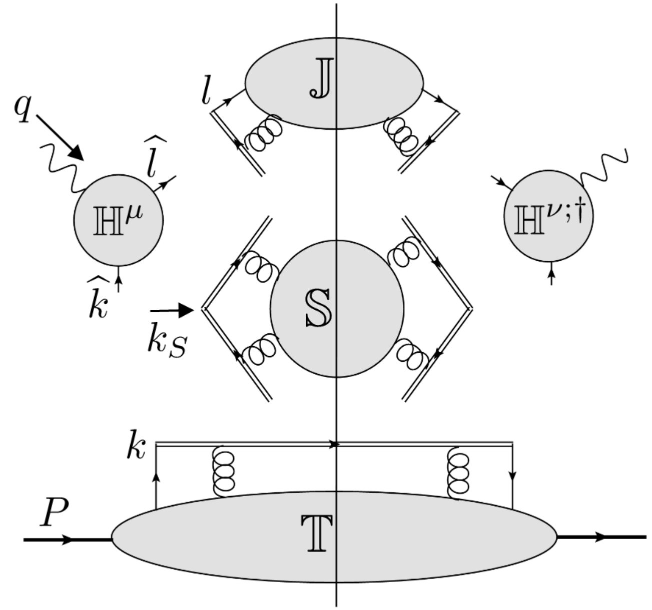
$$(1, 0, \vec{0}_T) \to \lim_{y_1 \to \infty} (1, -e^{-2y_1}, \vec{0}_T)$$

$$(0, 1, \vec{0}_T) \to \lim_{y_2 \to -\infty} (-e^{2y_2}, 1, \vec{0}_T)$$

BUT $$\lim_{\text{tilt}\to 0} \left(\begin{array}{l}\text{Operator}\\ \text{off light-cone}\end{array}\right) \neq \left(\begin{array}{l}\text{Operator}\\ \text{on the light-cone}\end{array}\right)$$

$$\left(e^{-2y_1}\right)^{-\varepsilon} \to \log y_1$$

Factorization



$$F_1 = |V(q^2)|^2 \frac{N_C}{x_{bj}} \int_{x_{bj}}^1 \frac{d\xi}{\xi} \int_0^{\xi - x_{bj}} d\rho \, \phi_{\text{sub.}}(\xi; m) S\left(\frac{\rho}{\xi}\right) \times J_{\text{sub.}}\left(\frac{\xi - x - \rho}{x}\right) + \text{p.s.}$$

2. Dealing with subtractions:
   Overlapping between soft and collinear momentum regions

$$J\left(\frac{c-x}{x}\right) = \int_0^{c-x} \frac{d\alpha}{c} S_J\left(\frac{\alpha}{c}\right) J_{\text{sub.}}\left(\frac{c-x-\alpha}{x}\right)$$

$$\phi(\xi) = \int_\xi^1 \frac{d\rho}{\rho} \phi_{\text{sub.}}(\rho) \, S_T\left(1 - \frac{\xi}{\rho}\right)$$

Mellin Space: Convolutions $\Longrightarrow$ Products

$$\widehat{F}_1 = \int_0^1 dx_{bj}\, x_{bj}^{N-1}\, F_1(x_{bj})$$
$$= N_C |V(q^2)|^2 \widehat{\phi}_{\text{sub.}}(N; y_1)\widehat{S}(N; y_1, y_2)\widehat{J}_{\text{sub.}}(N; y_2) + \text{p.s.}$$

$$\left\{\begin{aligned} \widehat{\phi}(N) &= \int_0^1 d\xi\, \xi^{N-1}\phi(\xi) \\ \widehat{S}(N) &= \int_0^1 dw\, (1-w)^{N-1} S(w) \\ \widehat{J}(N) &= \int_0^1 dx\, x^{N-1}\frac{1}{x} J\left(\frac{1-x}{x}\right) \end{aligned}\right.$$

Also, dealing with rapidities and subtractions is much easier:

$$\widehat{J}_{\text{sub.}}(N, y_2) = \lim_{\widehat{y}_1 \to \infty} \frac{\widehat{J}(N, \widehat{y}_1)}{\widehat{S}(N, \widehat{y}_1, y_2)}$$

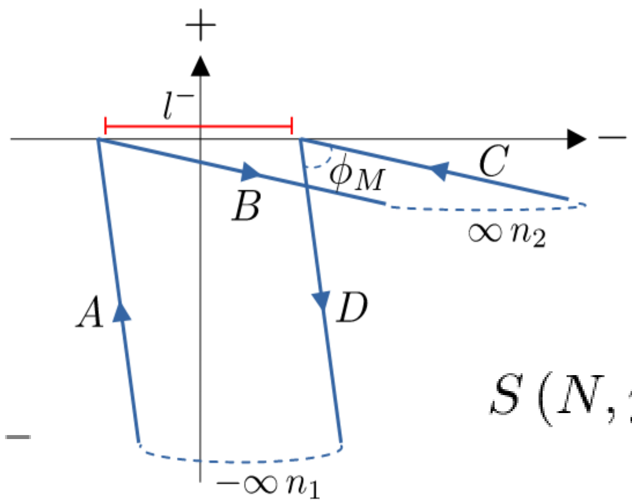
$$\widehat{\phi}_{\text{sub.}}(N, y_1) = \lim_{\widehat{y}_2 \to -\infty} \frac{\widehat{\phi}(N, \widehat{y}_2)}{S(N; y_1, \widehat{y}_2)}$$

The dependence on the rapidity regulators cancel out at the level of structure functions i.e. in the final factorization theorem

Soft Function

$$\cosh\phi_M = \frac{n_1 \cdot n_2}{\sqrt{-n_1^2}\sqrt{-n_2^2}}$$

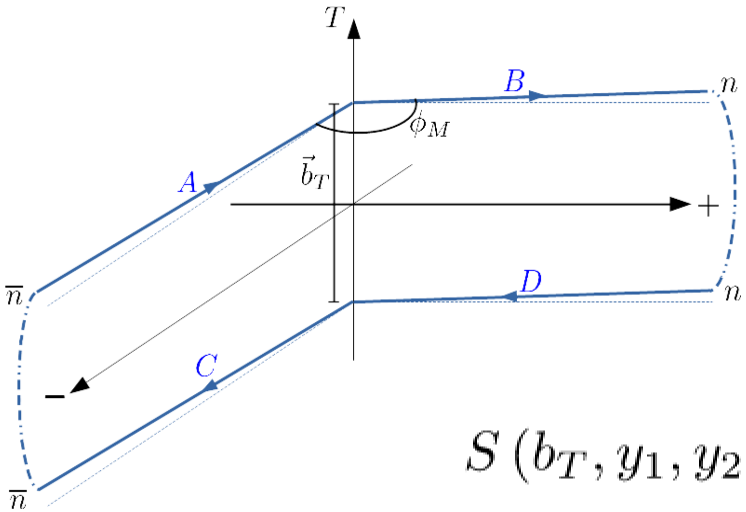
DIS



$N = iP^+ l^-$

$$S\left(N, y_1, y_2\right) = Z_S^{\mathrm{UV}} \frac{\mathrm{Tr}_C}{N_C} \langle 0 | \mathcal{P}\exp\left\{-ig_0 \int_{\Gamma_{\mathrm{DIS}}} dx^\mu A_\mu^{(0)}(x)\right\} |0\rangle$$

SIDIS (TMD)



$$S\left(b_T, y_1, y_2\right) = Z_S^{\mathrm{UV}} \frac{\mathrm{Tr}_C}{N_C} \langle 0 | \mathcal{P}\exp\left\{-ig_0 \int_{\Gamma_{\mathrm{TMD}}} dx^\mu A_\mu^{(0)}(x)\right\} |0\rangle$$

Soft Function

The **Collins-Soper kernel** appears in DIS operators!

$$L_N = \log\left(\frac{\mu}{\sqrt{2}P^+}Ne^{\gamma_E}\right)$$

DIS

$$S(N,\mu;y_1,y_2) = \exp\left\{\int_{y_2}^{y_1} dy\, K\left(a_S(\mu), L_N + y\right) + \frac{1}{2}\left[P\left(a_S(\mu), L_N + y_1 + \frac{i\pi}{2}\right) + P\left(a_S(\mu), L_N + y_2 + \frac{i\pi}{2}\right)\right] + \mathcal{O}\left(e^{-2y_1}, e^{2y_2}\right)\right\}$$

SIDIS (TMD)

$$L_b = \log\left(\frac{\mu\, b_T}{2e^{-\gamma_E}}\right)$$

$$S(b_T,\mu;y_1,y_2) = \exp\left\{(y_1 - y_2)K\left(a_S(\mu), L_b\right) + P\left(a_S(\mu), L_b\right) + \mathcal{O}\left(e^{-2y_1}, e^{2y_2}\right)\right\}$$

Subtracted Target Function

$$\phi(\xi) = Z_{\mathrm{T}}^{\mathrm{UV}} \left[\int \frac{dw^-}{2\pi} e^{-i\xi P^+ w^-} \lim_{\widehat{y}_2 \to -\infty} \langle P | \overline{\psi}^{(0)}(w) W^\dagger(w \to \infty \widehat{n}_2) W(0 \to \infty \widehat{n}_2) \frac{\gamma^+}{2} \psi^{(0)}(0) | P \rangle \right]_{\xi \to 1}$$

Parton Density at threshold tilted off the light-cone

$$\widehat{\phi}_{\mathrm{sub.}}(N, y_1) = \lim_{\widehat{y}_2 \to -\infty} \frac{\widehat{\phi}(N, \widehat{y}_2)}{S(N; y_1, \widehat{y}_2)}$$

$$\widehat{\phi}_{\mathrm{sub.}}(N, \mu, y_1) = \widehat{\phi}_{\mathrm{sub.}}(N, \mu, L_N) \exp\left\{ - \int_{L_N}^{y_1} dy\, K\left(a_S(\mu), L_N + y\right) - \frac{1}{2} P\left(a_S(\mu), L_N + y_1 + \frac{i\pi}{2} \right) \right\}$$

$$\frac{d}{dy_1} \left(\widehat{S}(N, \mu; y_1, y_2) \times \widehat{\phi}_{\mathrm{sub.}}(N, \mu, y_1) \right) \equiv 0$$

Cancelled out in the product with the soft function

Subtracted Jet Function

$$J\left(l^2/Q^2\right) = Z_{\rm J}^{\rm UV} \quad \frac{Q^2}{2\pi l^-} \int \frac{d^4\eta}{(2\pi)^4} e^{-il\cdot\eta} \lim_{\hat{y}_1\to\infty} \frac{{\rm Tr}_C}{N_c} \frac{{\rm Tr}_D}{4} \langle 0|\gamma^- \overline{\psi}^{(0)}(\eta) W^\dagger(\eta \to -\infty\hat{n}_1) W(0 \to -\infty\hat{n}_1)\psi^{(0)}(0)|0\rangle$$

$$\hat{J}_{\rm sub.}(N, y_2) = \lim_{\hat{y}_1\to\infty} \frac{\hat{J}(N,\hat{y}_1)}{\hat{S}(N,\hat{y}_1,y_2)}$$

$$\hat{J}_{\rm sub.}(N,\mu,y_2) = \hat{J}_{\rm sub.}(N,-L_N) \exp\left\{ -\int_{y_2}^{-L_N} dy\, K\left(a_S(\mu), L_N + y\right) - \frac{1}{2} P\left(a_S(\mu), L_N + y_2 + \frac{i\pi}{2}\right)\right\}$$

$$\frac{d}{dy_2}\left(\hat{J}_{\rm sub.}(N,\mu,y_2) \times \hat{S}\left(N,\mu;y_1,y_2\right)\right) \equiv 0$$

Cancelled out in the product with the soft function

The factorization theorem is then:

$$\widehat{F}_1 = \lim_{\substack{\widehat{y}_1 \to \infty \\ \widehat{y}_2 \to -\infty}} N_C |V(q^2)|^2 \, \frac{\widehat{\phi}(N;\widehat{y}_2)}{\widehat{S}(N; y_1, \widehat{y}_2)} \, \widehat{S}(N; y_1, y_2) \, \frac{\widehat{J}(N;\widehat{y}_1)}{\widehat{S}(N;\widehat{y}_1, y_2)} + \text{p.s.}$$

- All operators are rapidity finite.
- The dependence on the rapidity regulators y_1, y_2 vanishes in the final result.
- The P-terms (off light-cone effects) cancel out in the final result.

$$\widehat{F}_1 = N_C \left|V(q^2)\right|^2 \, \widehat{\phi}_{\text{sub.}}(N, \mu, L_N) \exp\left\{ \int_{-L_N}^{L_N} dy \, K\left(a_S(\mu), L_N + y\right) \right\} \widehat{J}_{\text{sub.}}(N, \mu, -L_N) + \text{ p.s.}$$

The **Collins-Soper kernel** appears
in DIS structure functions!

The factorization theorem is then:

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The **Collins-Soper kernel** appears
in DIS structure functions!

Optimal definition (square root) for the Target Function

$$\widehat{\phi}^{\,\text{sqrt.}}_{\text{sub.}}(N, y_n) = \lim_{\substack{\widehat{y}_1 \to \infty \\ \widehat{y}_2 \to -\infty}} \widehat{\phi}\,(N, \widehat{y}_2) \sqrt{\frac{\widehat{S}(N, \widehat{y}_1, y_n)}{\widehat{S}(N, \widehat{y}_1, \widehat{y}_2)\, \widehat{S}(N, y_n, \widehat{y}_2)}}$$

$$\begin{cases} \dfrac{\partial \log \phi^{\text{sqrt.}}_{\text{sub.}}}{dy_n} = -K\left(a_S(\mu), L_N + y_n\right), \\[2ex] \dfrac{\partial \log \phi^{\text{sqrt.}}_{\text{sub.}}}{d \log \mu} = \gamma_\phi\left(a_S(\mu), L_T - L_N + y_n\right) \end{cases}$$

With: $L_T = \log\left(\dfrac{\mu^2}{2(P^+)^2} N e^{\gamma_E}\right)$

$$\phi^{\text{sqrt.}}_{\text{sub.}}(N; \mu, y_n) = \phi^{\text{sqrt.}}_{\text{sub.}}(N; \mu_0, y_0) \exp\left\{\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_f(a_S(\mu')) + \gamma_K(a_S(\mu'))\,(L'_T - L'_N + y_0)\right]\right\}$$
$$\times \exp\left\{-\int_{y_0}^{y_n} dy\, K\left(a_S(\mu), L_N + y\right)\right\}$$

BONUS (matching): $\phi^{\text{sqrt.}}_{\text{sub.}}(N; \mu, -L_N) = C^{-1}\left(a_S(\mu)\right)\, f^{\text{thr}}(N, \mu)$ ← PDF operator (defined on the light-cone) in the threshold region

Optimal definition (square root) for the Jet Function

$$\widehat{J}^{\text{sqrt.}}_{\text{sub.}}(N, y_n) = \lim_{\substack{\widehat{y}_1 \to \infty \\ \widehat{y}_2 \to -\infty}} \widehat{J}(N, \widehat{y}_1) \sqrt{\frac{\widehat{S}(N, y_n, \widehat{y}_2)}{\widehat{S}(N, \widehat{y}_1, \widehat{y}_2)\, \widehat{S}(N, \widehat{y}_1, y_n)}}$$

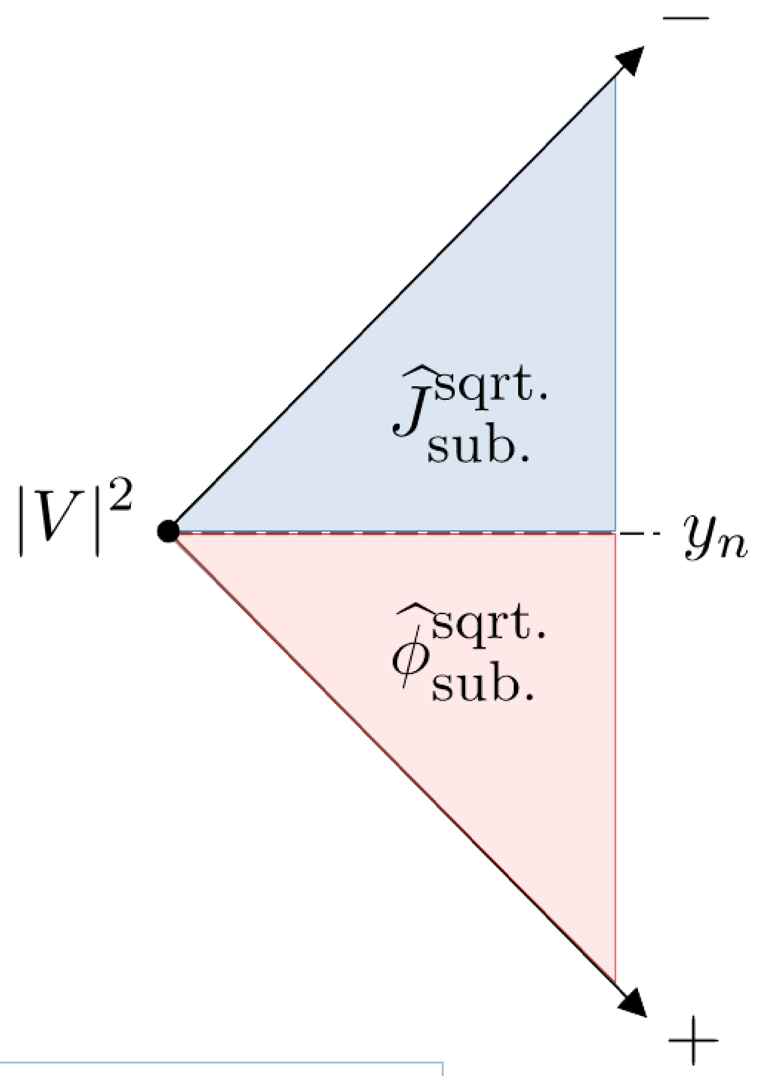
$$\begin{cases} \dfrac{\partial \log J^{\text{sqrt}}_{\text{sub.}}}{dy_n} = K\left(a_S(\mu), L_N + y_n\right), \\[2ex] \dfrac{\partial \log J^{\text{sqrt}}_{\text{sub.}}}{d\log\mu} = \gamma_J\left(a_S(\mu), L_J - L_N - y_n\right) \end{cases}$$

With: $L_J = \log\left(\dfrac{\mu^2}{Q^2} N e^{\gamma_E}\right)$

$$J^{\text{sqrt.}}_{\text{sub.}}(N; \mu, y_n) = J^{\text{sqrt.}}_{\text{sub.}}(N; \mu_0, y_0) \exp\left\{\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_j(a_S(\mu')) + \gamma_K(a_S(\mu'))\,(L'_J - L'_N - y_0)\right]\right\}$$
$$\times \exp\left\{\int_{y_0}^{y_n} dy\, K\left(a_S(\mu), L_N + y\right)\right\}$$

BONUS (matching): $J^{\text{sqrt.}}_{\text{sub.}}(N; \mu, -L_N) = C\left(a_S(\mu)\right)\, \mathcal{J}(N, \mu)$ ⟵ Standard Jet Function Operator (defined on the light-cone)

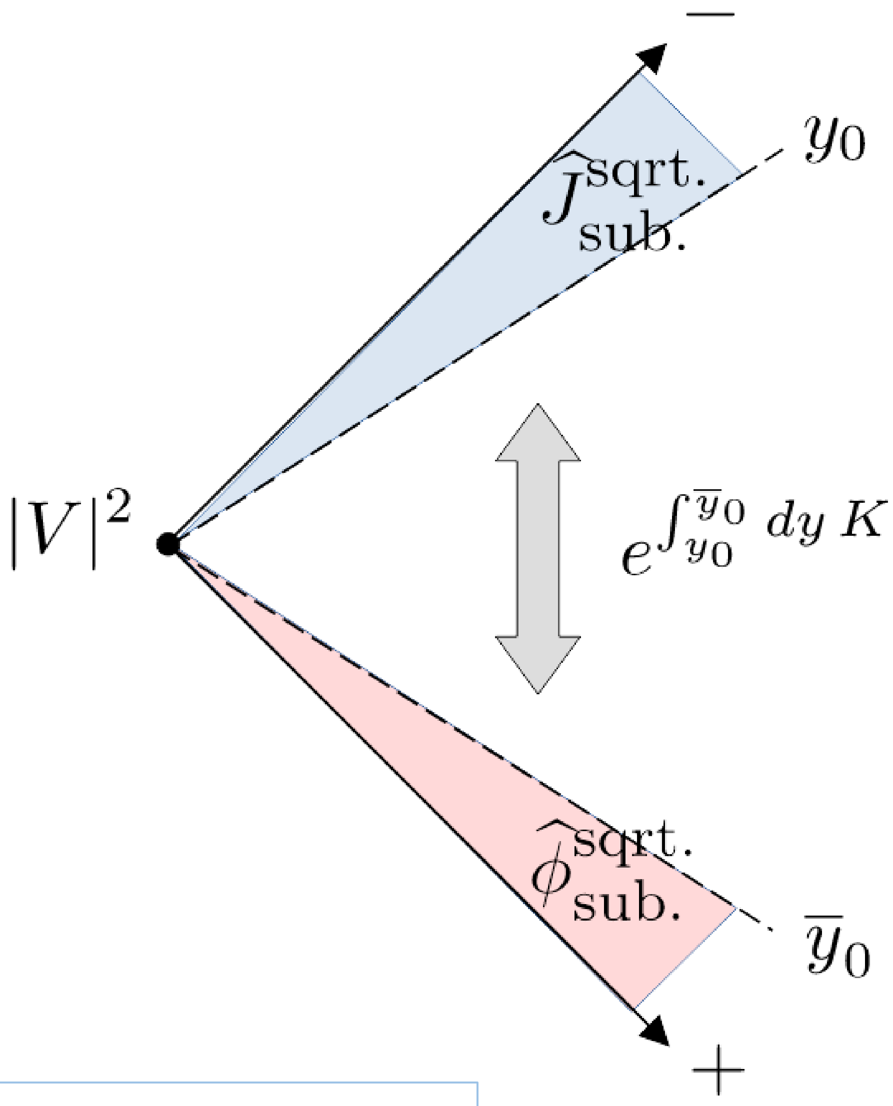
$$\widehat{F}_1 = N_C |V(q^2)|^2 \, \widehat{\phi}^{\,\mathrm{sqrt.}}_{\mathrm{sub.}}(N;\mu,y_n)\, \widehat{J}^{\,\mathrm{sqrt.}}_{\mathrm{sub.}}(N;\mu,y_n)$$



← Separation of left and right moving quanta

Or, exploiting evolution...

$$\widehat{F}_1 = N_C |V(q^2)|^2 \, \widehat{\phi}^{\text{sqrt.}}_{\text{sub.}}(N;\mu,\overline{y}_0) e^{\int_{y_0}^{\overline{y}_0} dy K(a_S(\mu), L_N + y)} \, \widehat{J}^{\text{sqrt.}}_{\text{sub.}}(N;\mu,y_0)$$



Typical jet rapidity scale (large and negative)

$$e^{\int_{y_0}^{\overline{y}_0} dy\, K(a_S(\mu), L_N + y)}$$

[rapidity anomalous dimensions] reveal sensitivity to IR scales, which may signal a breakdown of rapidity factorization in SCET_{II} .

[Fleming, Labun (2012)]

Rapidity evolution is sensitive to IR scales through the Collins-Soper kernel. There is no breakdown of factorization, and the separation of rapidity scales is totally analogous to TMD processes.

Typical target rapidity scale (large and positive)

There are two natural choices for the reference rapidity scales:

1. Threshold choice: $y_0 = -L_N = \overline{y}_0$

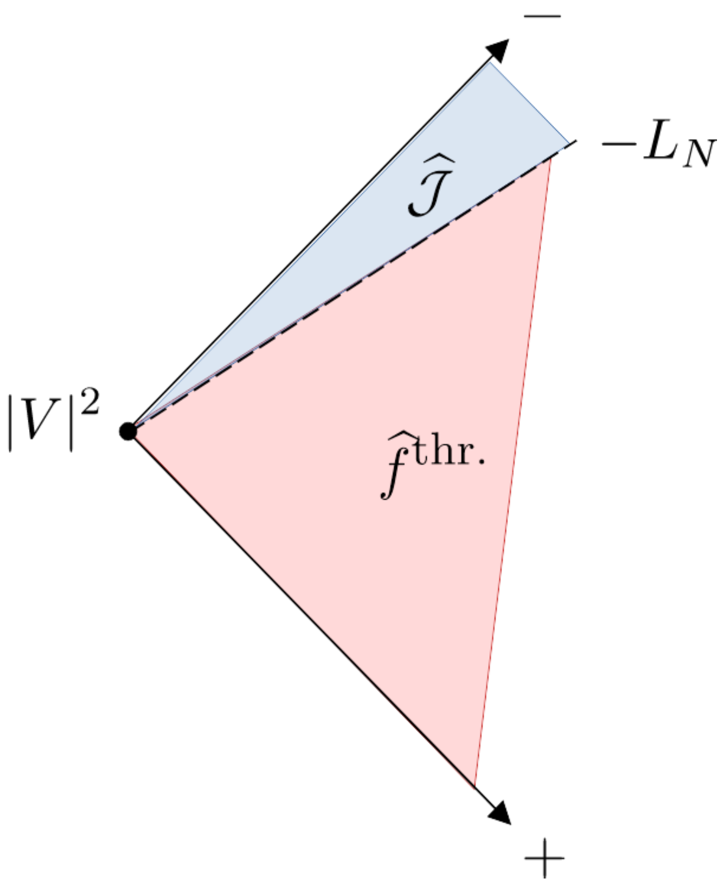
   - Simplifies the evolution (and the subtractions):

$$\frac{\partial \log \widehat{\phi}^{\,\text{sqrt.}}_{1,\text{sub.}}(N,\mu,y_n)}{\partial y_n} = -K\left(a_S(\mu), L_N + y_n\right)$$

$$\frac{\partial \log \widehat{J}^{\,\text{sqrt.}}_{\text{sub.}}(N,\mu,y_n)}{\partial y_n} = K\left(a_S(\mu), L_N + y_n\right)$$

   - Leads to standard SCET result and naïve (on the light-cone) factorization approach.

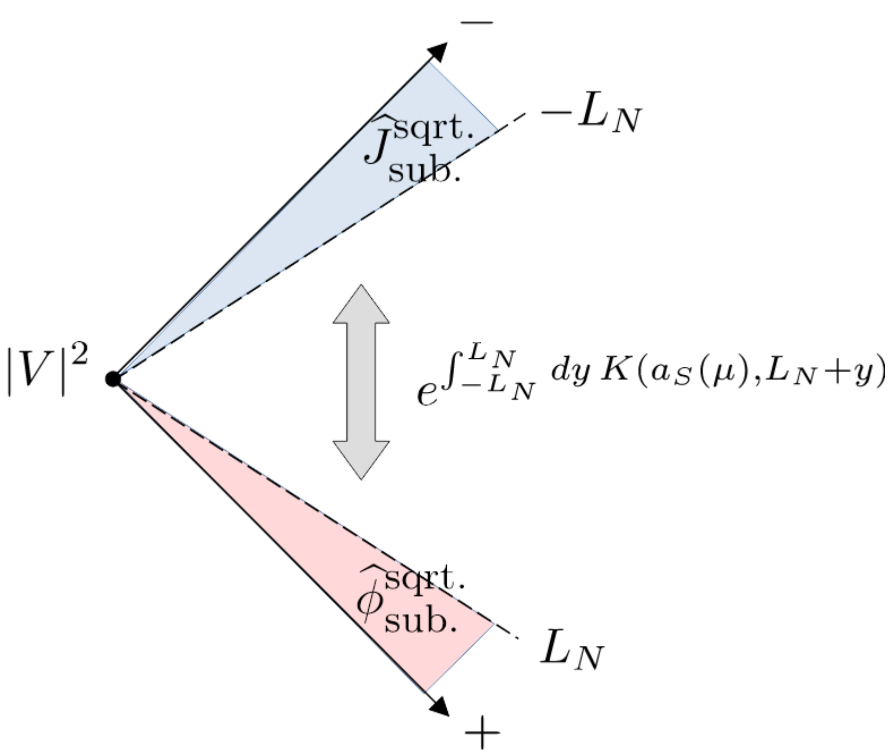
$$\widehat{F}_1 = H\left(a_S(\mu)\right)\, f^{\text{thr.}}(N,\mu)\, \mathcal{J}(a_S(\mu, \log\left(\mu/Q_J\right))$$



There are two natural choices for the reference rapidity scales:

2. TMD choice: $y_0 = -L_N; \quad \overline{y}_0 = L_N$

- Natural separation of scales: large and positive for target, large and negative for jet.
- Leads to a result in which the **CS-kernel is explicit**!



$$\widehat{F}_1 = H\left(a_S(\mu)\right) \widehat{\phi}^{\text{sqrt}}_{\text{sub.}}(N, \mu, L_N)\, C\left(a_S(\mu)\right) \widehat{\mathcal{J}}(N, \mu) e^{\int_{-L_N}^{L_N} dy\, K(a_S(\mu), L_N+y)}$$

$$\phi(N; \mu, L_N) = C^{-1}\left(a_S(\mu_N)\right) f^{\text{thr.}}(N, \mu_N) \exp\left\{\int_{\mu_N}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_f(a_S(\mu')) + \gamma_K(a_S(\mu')) \log\left(\frac{\mu'}{\sqrt{2}P^+}\right)\right]\right\}$$
$$\times \exp\left\{-\int_0^{L_N} dy\, K\left(a_S(\mu), L_N + y\right)\right\}$$

Conclusions

- Re-derivation of the factorization of DIS at threshold in full QCD and with a modern approach. All past results are included into a more general treatment.
- All the subtleties related to the rapidity divergences and the separation of the rapidity scales have been addressed in detail and clarified.
- The analogies with TMD physics are transparent and, more generally, they are the evidence for structures shared by processes only apparently very different.
- In particular, the Collins-Soper kernel plays a crucial role in entangling opposite directions on the light-cone. This feature is related to its geometrical nature, beyond the sole framework of the TMD physics.

$$\widehat{F}_1 = H\left(a_S(\mu)\right)\, \widehat{\phi}^{\text{sqrt}}_{\text{sub.}}(N,\mu,L_N)\, C\left(a_S(\mu)\right) \widehat{\mathcal{J}}(N,\mu) e^{\int_{-L_N}^{L_N} dy\, K(a_S(\mu),L_N+y)}$$

Thank You!