The perturbative tail of the TMD shape function in SIDIS

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STRONG 2020

HAS QCD HADRONIC STRUCTURE AND QUANTUM CHROMODYNAMICS

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In collaboration with:
J. Bor, D. Boer, C.Pisano & F. Yuan

Outline

- **Part I:** **TMD shape function** in **TMD factorization** for Quarkonium
- **Part II:** **Matching procedure** to access the **TMDShF perturbative tail**
  - Relevance of the hard amplitude **pole structure**
- **Part III:** the TMDShF depends on Q? **Process dependence**?
- **Part IV:** Opportunities at the **EIC** to investigate the **TMDShF**

Quarkonia & gluon TMDs

Processes involving Quarkonia are **sensitive to gluons**

hadron collisions

- $p + p \to \eta_Q + X$
- $p + p \to \chi_Q + X$
- $p + p \to J/\psi + J/\psi + X$
- $p + p \to J/\psi + X$ **?**

ep **collisions**

- $e + p \to e' + J/\psi + X$
- $e + p \to e' + J/\psi + \gamma + X$
- $e + p \to e' + J/\psi + \text{jet} + X$

and more…

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Processes involving Quarkonia are **sensitive to gluons**

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- $p + p \to \eta_Q + X$
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- $p + p \to J/\psi + J/\psi + X$
- $p + p \to J/\psi + X$ **?**

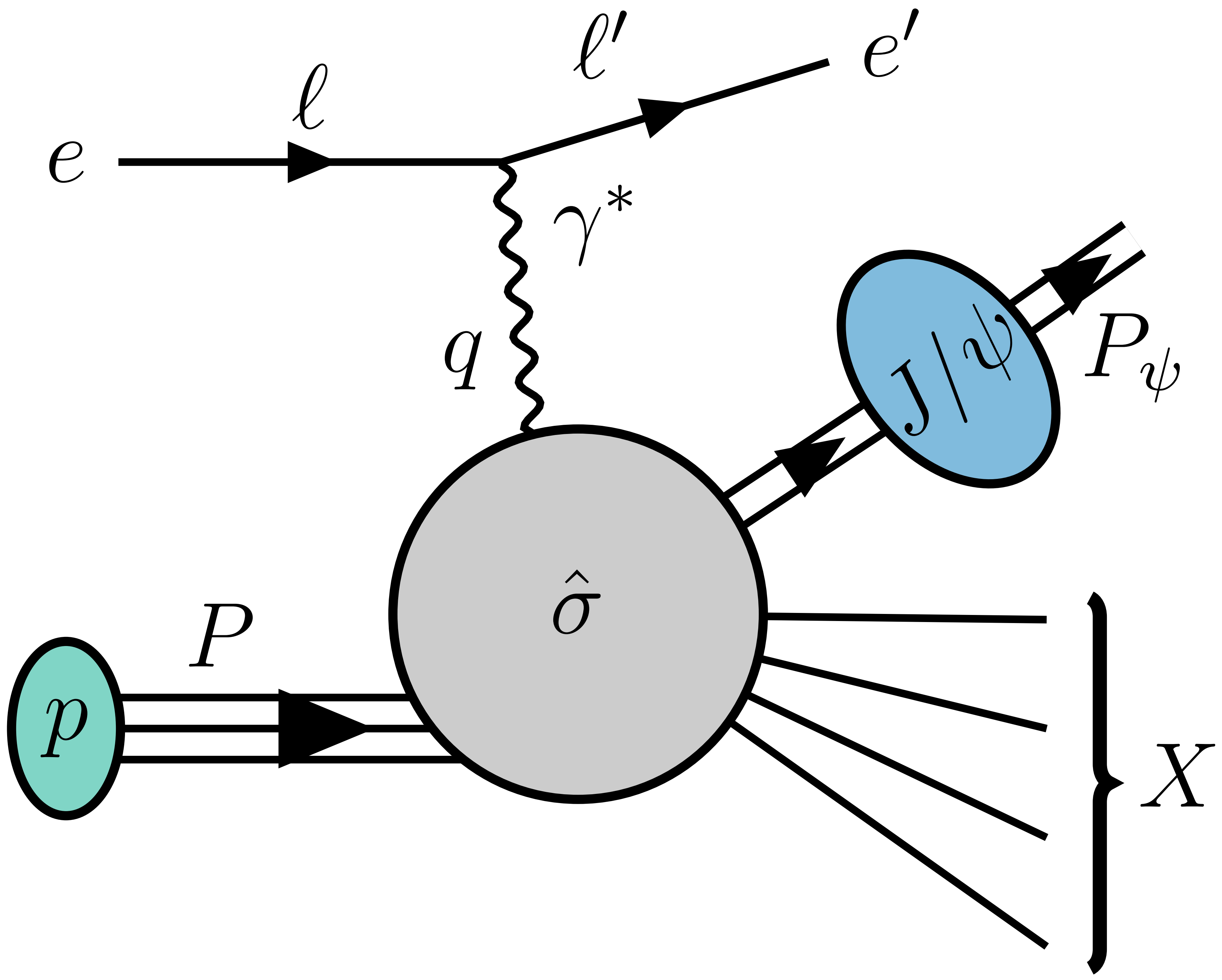
***ep* collisions**

- $\boxed{e + p \to e' + J/\psi + X}$
- $e + p \to e' + J/\psi + \gamma + X$
- $e + p \to e' + J/\psi + \mathrm{jet} + X$

and more…

Theoretical framework

$$e\,(\ell) + p\,(P) \to e'\,(\ell') + \gamma^*\,(q) + p\,(P) \to e'\,(\ell') + J/\psi\,(P_\psi) + X$$



SIDIS variables

$$q^2 = -Q^2,\ S \approx 2P\cdot\ell$$

$$x_B = \frac{Q^2}{2\cdot q},\ y = \frac{P\cdot q}{P\cdot \ell},\ z = \frac{P\cdot P_\psi}{P\cdot q}$$

Phase spaces

$$\frac{\mathrm{d}^3\ell'}{2E'} = 2\pi\, yS\, \mathrm{d}x_B \mathrm{d}y$$

$$\frac{\mathrm{d}^3 P_\psi}{2E_\psi} = \frac{\mathrm{d}z}{z}\mathrm{d}^2 P_{\psi\perp}\mathrm{d}\phi_\psi$$

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 02 (2007)

Sun, Zhang, EJPC 77 (2017)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\mathbf{P}^2_{\psi\perp}\,\mathrm{d}\phi_\psi} = \frac{\alpha}{yQ^2}\Big\{ \left[1+(1-y)^2\right] F_{UUT} + 4(1-y)\,F_{UUL} + (2-y)\sqrt{1-y}\,\cos\phi_\psi\,F_{UU}^{\cos\phi_\psi} + 4(1-y)\,\cos 2\phi_\psi\,F_{UU}^{\cos 2\phi_\psi}\Big\}$$

(Some) Model for Quarkonium formation

- **Colour Singlet Model**

Baier, Rückl, Z.Phys.C 19 (1983)

$$d\sigma[\mathcal{Q}] = \int d\xi_i \, d\xi_j \, f_i(\xi_i) f_j(\xi_j) \, d\hat{\sigma}_{i+j\to\mathcal{Q}+X} |R(0)|^2$$

(S-wave production)

- **Non Relativistic QCD**

(CS + CO mechanism)

Bodwin, Braaten, Lepage, PRD 51 (1997)

$$d\sigma[\mathcal{Q}] = \sum_n \int d\xi_i \, d\xi_j \, f_i(\xi_i) f_j(\xi_j) \, d\hat{\sigma}_{i+j\to QQ[n]+X} \langle \mathcal{O}_{\mathcal{Q}}[n] \rangle$$

Long-Distance Matrix Elements

(**universal** in principle)

(Improved)

- **Colour Evaporation Model**

Ma, Vogt, PRD 94 (2016)

$$\frac{d\sigma[\mathcal{Q}]}{dP_{\mathcal{Q}}} = F_{\mathcal{Q}} \int_{M_{\mathcal{Q}}}^{2M_H} dM \frac{d\sigma_{QQ}(M, P'_{\mathcal{Q}})}{dM dP_{\mathcal{Q}}}$$

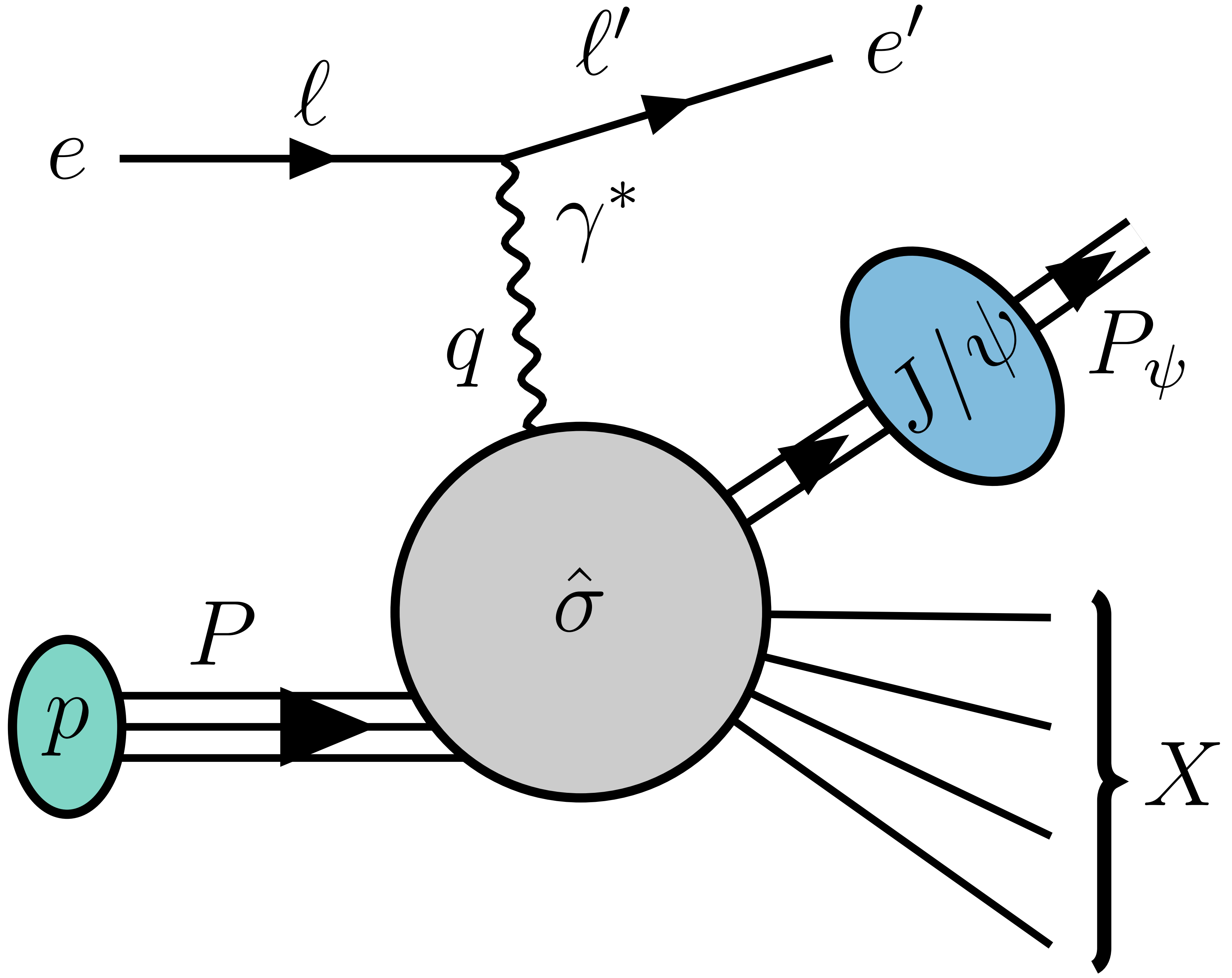
- **Fragmentation Functions**

Kang, Ma, Qiu, Sterman, PRD 90 (2014)

$$d\sigma[\mathcal{Q}] = \int d\xi_i \, d\xi_j \, dz \; f_i \, f_j \, d\hat{\sigma}_{i+j\to f+X} D_{f\to\mathcal{Q}}(z) + \int d\xi_i \, d\xi_j \, dz \; f_i \, f_j \, d\hat{\sigma}_{i+j\to QQ+X} D_{QQ\to\mathcal{Q}}(z)$$

The TMD shape function

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)



"light-hadron" SIDIS

$$\sigma^{ep\to e'hX} = \boxed{\hat{\sigma}^{[a]}(\mu_H)} \otimes \boxed{f_a(\hat{x};\mu_H)} \otimes \boxed{D_{a\to h}(\hat{z};\mu_H)}$$

Bodwin, Braaten, Lepage, PRD 51 (1997)

"Quarkonium" SIDIS (adopting NRQCD)

$$\sigma^{ep\to e'J/\psi X} = \boxed{\hat{\sigma}^{[n]}(\mu_H)} \otimes \boxed{f_a(\hat{x};\mu_H)} \otimes \boxed{\langle \mathcal{O}_\psi[n]\rangle\, \delta(\hat{z}-z)}$$

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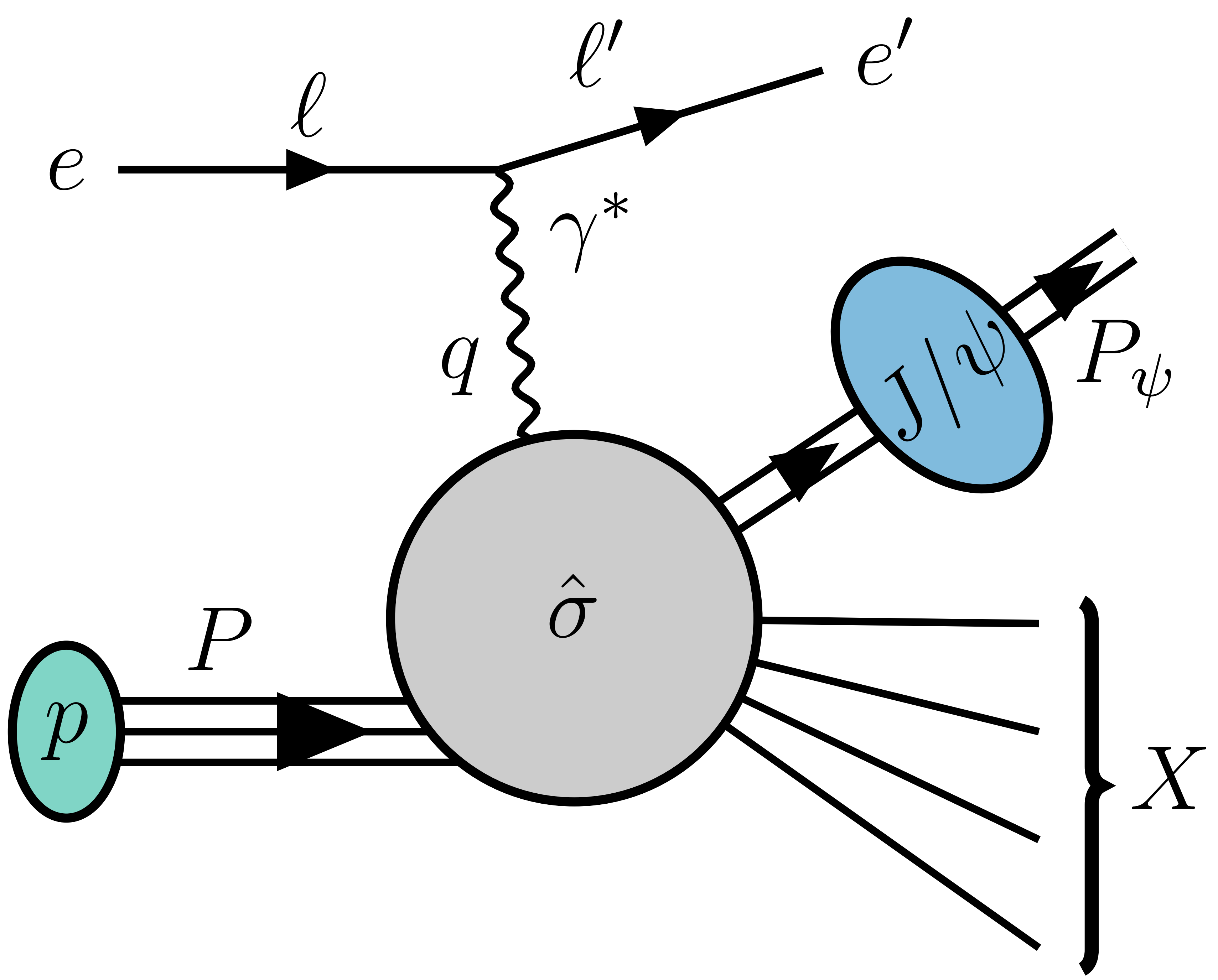

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As for $D_{a\to h}(\hat{z}) \to D_{a\to h}(\hat{z}, k_T)$, we have $\langle \mathcal{O}_\psi[n]\rangle \delta(\hat{z}-z) \to \Delta^{[n]}(\hat{z}, k_T)$

Echevarría, JHEP 144 (2019)

Fleming, Markis, Mehen, JHEP 112 (2020)

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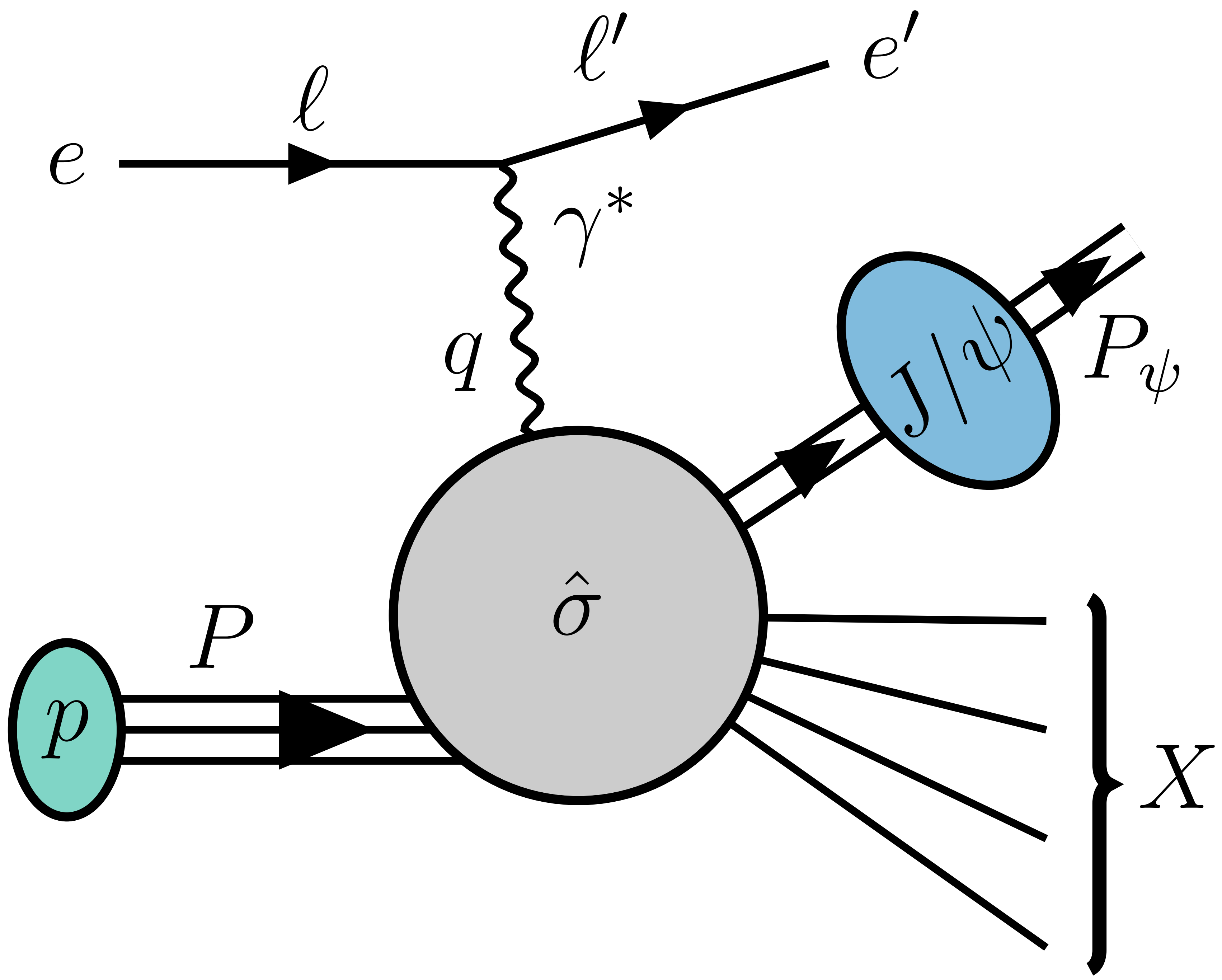

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As for $D_{a\to h}(\hat{z}) \to D_{a\to h}(\hat{z}, k_T)$, we have $\langle \mathcal{O}_\psi[n]\rangle \delta(\hat{z}-z) \to \Delta^{[n]}(\hat{z}, k_T)$

$\mathbf{\Delta^{[n]}}$ encodes hadronization
plus
exchange of soft gluons

Echevarría, JHEP 144 (2019)

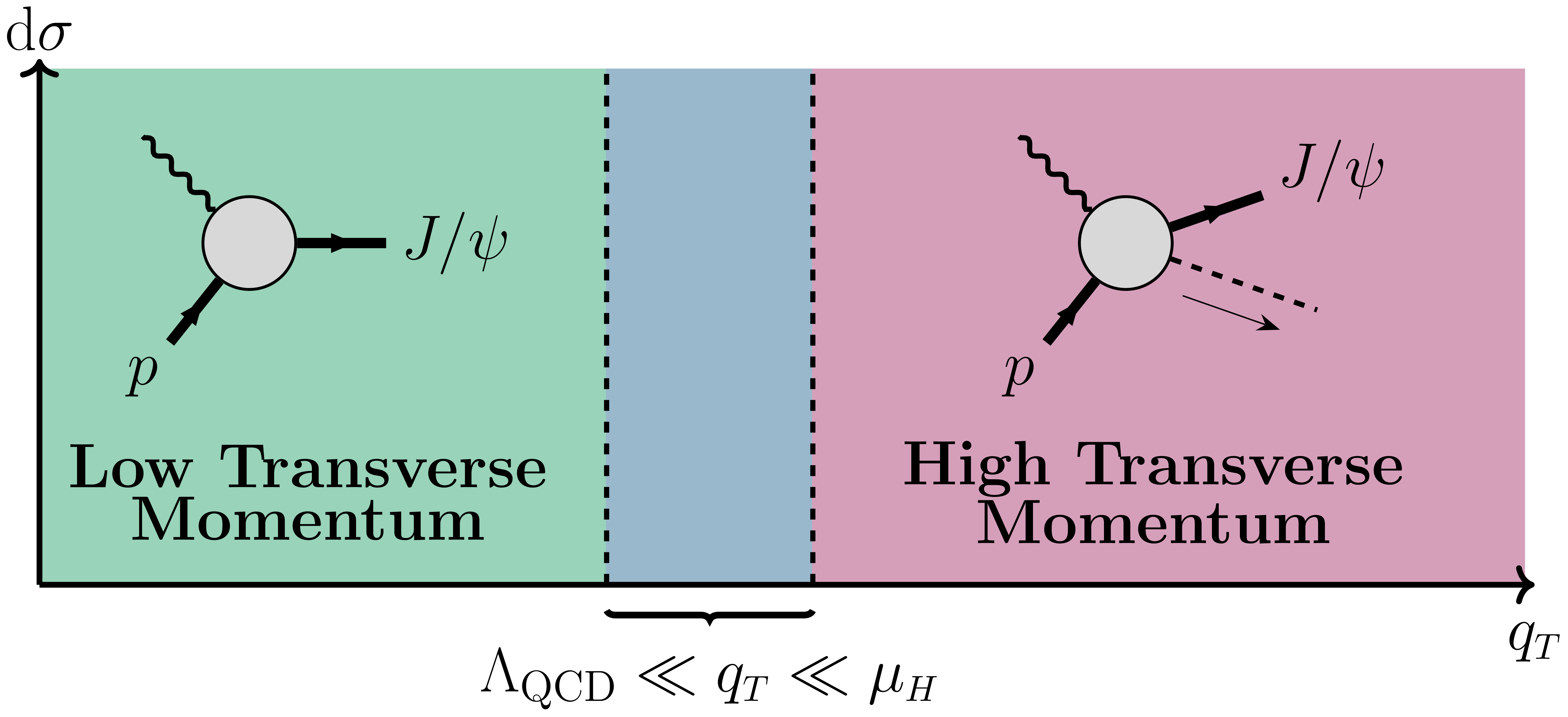
Fleming, Markis, Mehen, JHEP 112 (2020)

Matching region

q_T → photon transverse momentum

$P_{\psi\perp}$ → J/ψ transverse momentum

$$q_T = \frac{P_{\psi\perp}}{\hat{z}}$$



TMD factorization $q_T \ll \mu_H$

Overlapping at $\Lambda_{\mathrm{QCD}} \ll q_T \ll \mu_H$

Collinear factorization $q_T \gg \Lambda_{\mathrm{QCD}}$

Matching region

q_T → photon transverse momentum

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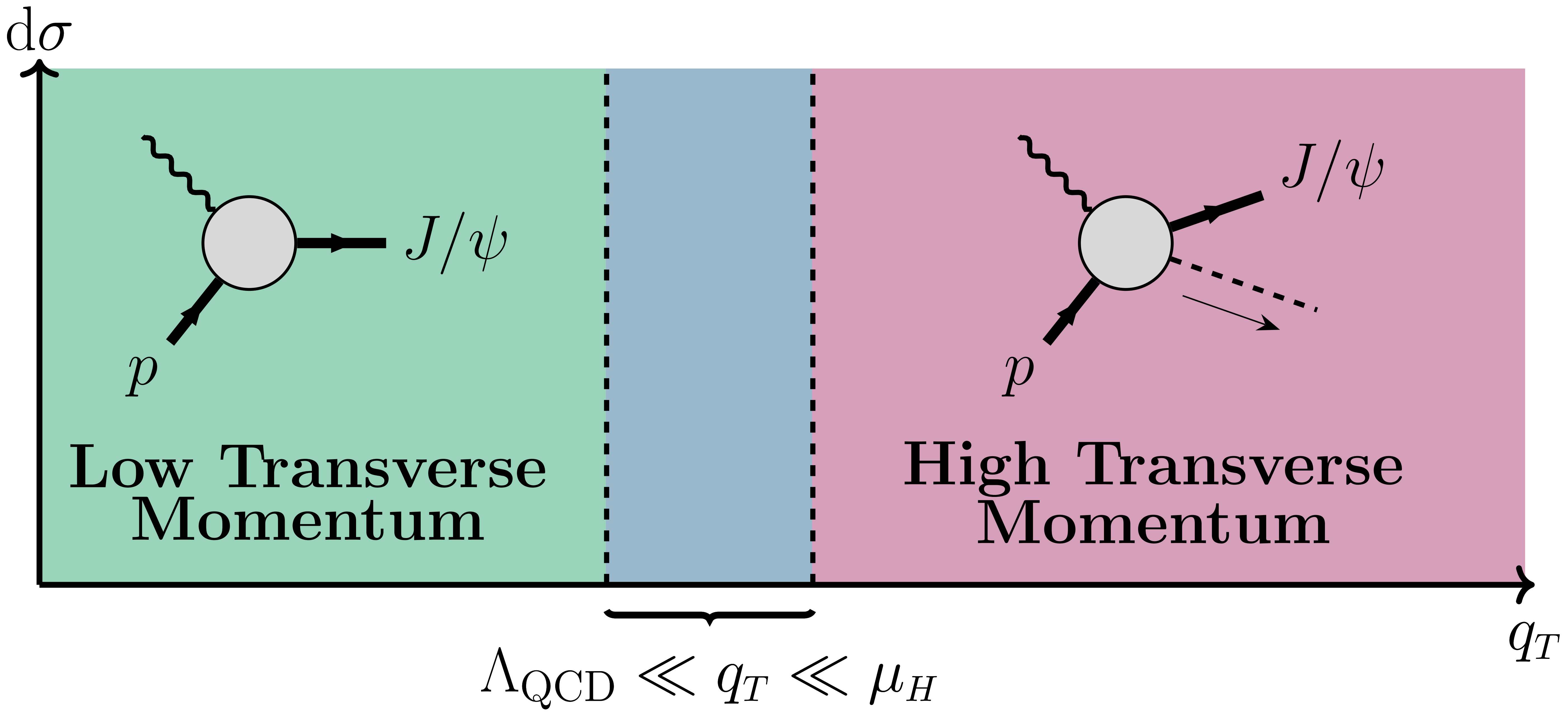
$$q_T = \frac{P_{\psi\perp}}{\hat{z}}$$



TMD factorization $\quad q_T \ll \mu_H$

Overlapping at $\quad \Lambda_{\mathrm{QCD}} \ll q_T \ll \mu_H$

Collinear factorization $\quad q_T \gg \Lambda_{\mathrm{QCD}}$

Description of the same dynamics?

Thus they should match!

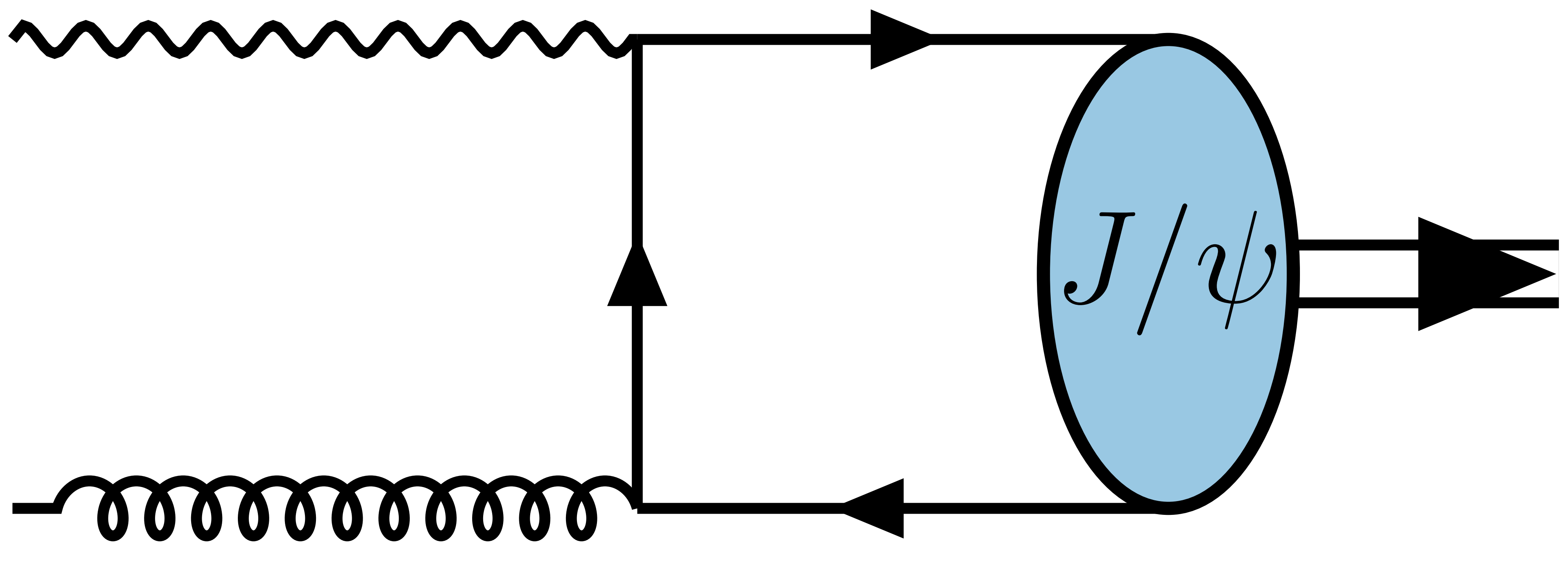
Bacchetta, Boer, Diehl, Mulders, JHEP 08 (2008)

Boer, D'Alesio, Murgia, Pisano, Taels, JHEP 09 (2020)

Structure function at small-q_T (TMD region)

J/ψ production at the lowest α_s-order: $\gamma^* + g \to c\bar{c}[n]$

Bacchetta, Boer, Pisano, Taels, EPJC 80 (2020)



Kinematic fixes most of the variables:

- $\hat{x} = x \quad \left(\text{where } x = x_B \dfrac{M_\psi^2 + Q^2}{Q^2}\right)$
- $\hat{z} = 1$
- $p_{aT} = q_T$

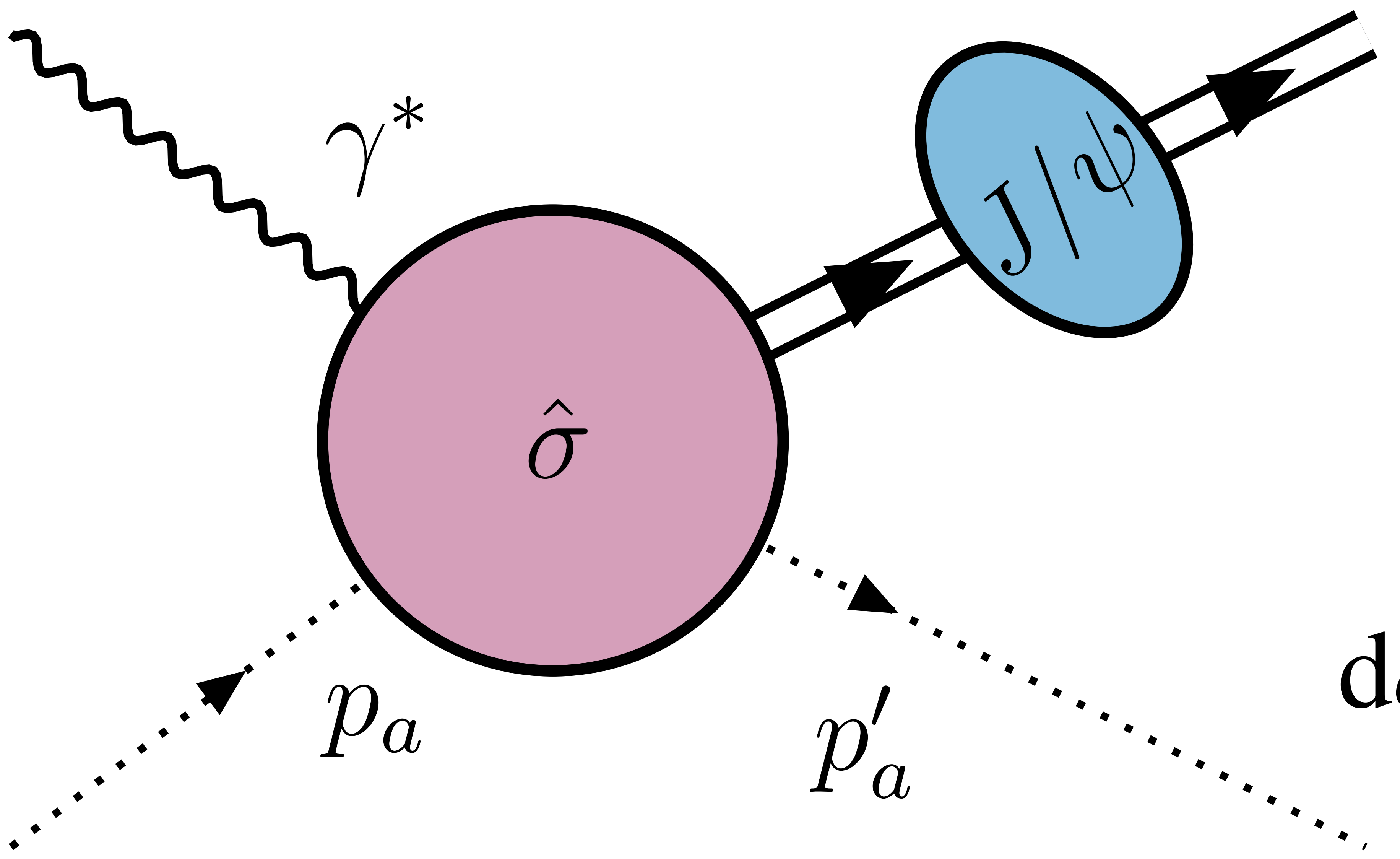
$$d\sigma|_{\mathrm{TMD}} = \frac{\alpha}{yQ^2}\left\{ \left[1 + (1-y)^2\right] \mathcal{F}_{UUT} + 4(1-y)\left(\mathcal{F}_{UUL} + \cos 2\phi\, \mathcal{F}_{UU}^{\cos 2\phi}\right) \right\}$$

Involves the convolutions:
$$\begin{cases} \mathcal{C}\left[f_1^g \Delta^{[n]}\right](x, q_T) \\ \mathcal{C}\left[w\, h_1^{\perp g} \Delta_h^{[n]}\right](x, q_T) \end{cases}$$

TMDShF needed to investigate gluon TMDs!

Structure function at high-q_T (collinear region)

J/ψ production at the lowest α_s-order: $\gamma^* + a \to c\bar{c}[n] + a$ $(a = q,\, \bar{q},\, g)$



$$\mathrm{d}\sigma^{ep\to e' J/\psi X} = \mathrm{d}\hat{\sigma}^{a\,[n]}(\mu_H) \otimes f_p^a(\hat{x};\mu_H) \otimes \langle \mathcal{O}_\psi[n]\rangle\, \delta(\hat{z} - z)$$

Lepton tensor from

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 02 (2007)

$$\hat{x}' = \frac{x_B}{\hat{x}}\frac{M_\psi^2 + Q^2}{Q^2}$$

$$\mathrm{d}\hat{\sigma}^{a\,[n]} \propto \int \frac{\mathrm{d}\hat{x}}{\hat{x}}\frac{\mathrm{d}\hat{z}}{\hat{z}}\frac{L^{\mu\nu}}{Q^4} H_\mu^{a\,[n]} H_\nu^{*\,a\,[n]}\, \delta(\hat{x}', \hat{z})$$

$$\delta(\hat{x}', \hat{z}) = \delta\left(\frac{(1-\hat{x}')(1-\hat{z})}{\hat{x}'\hat{z}} + \frac{1-\hat{z}}{\hat{z}}\frac{\hat{z}-\hat{x}'}{\hat{x}'\hat{z}}\frac{M_\psi^2}{Q^2} + \frac{q_T^2}{Q^2}\right)$$

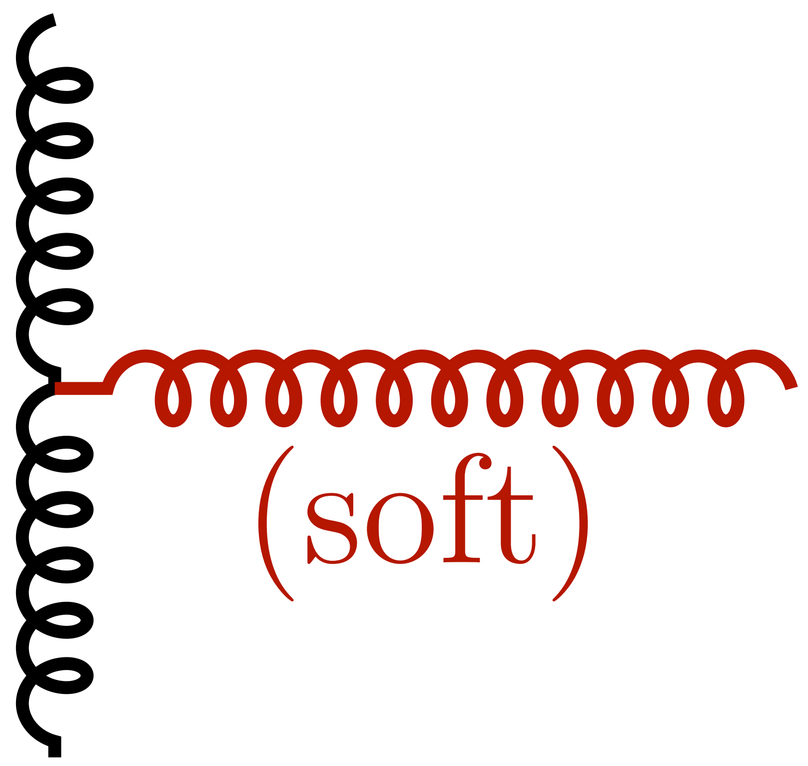
for $M_\psi \ll Q$ in agreement with

Meng, Olness, Soper JHEP 11 (2019)

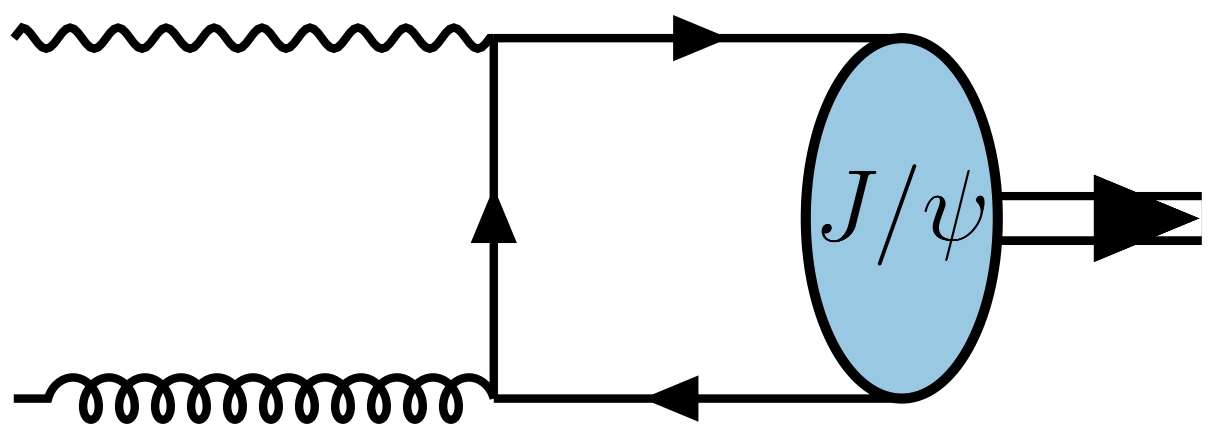
Pictorial view of the small-q_T limit

$\Lambda_{\mathrm{QCD}} \ll q_T \ll \mu_H$

$q_T \gg \Lambda_{\mathrm{QCD}}$

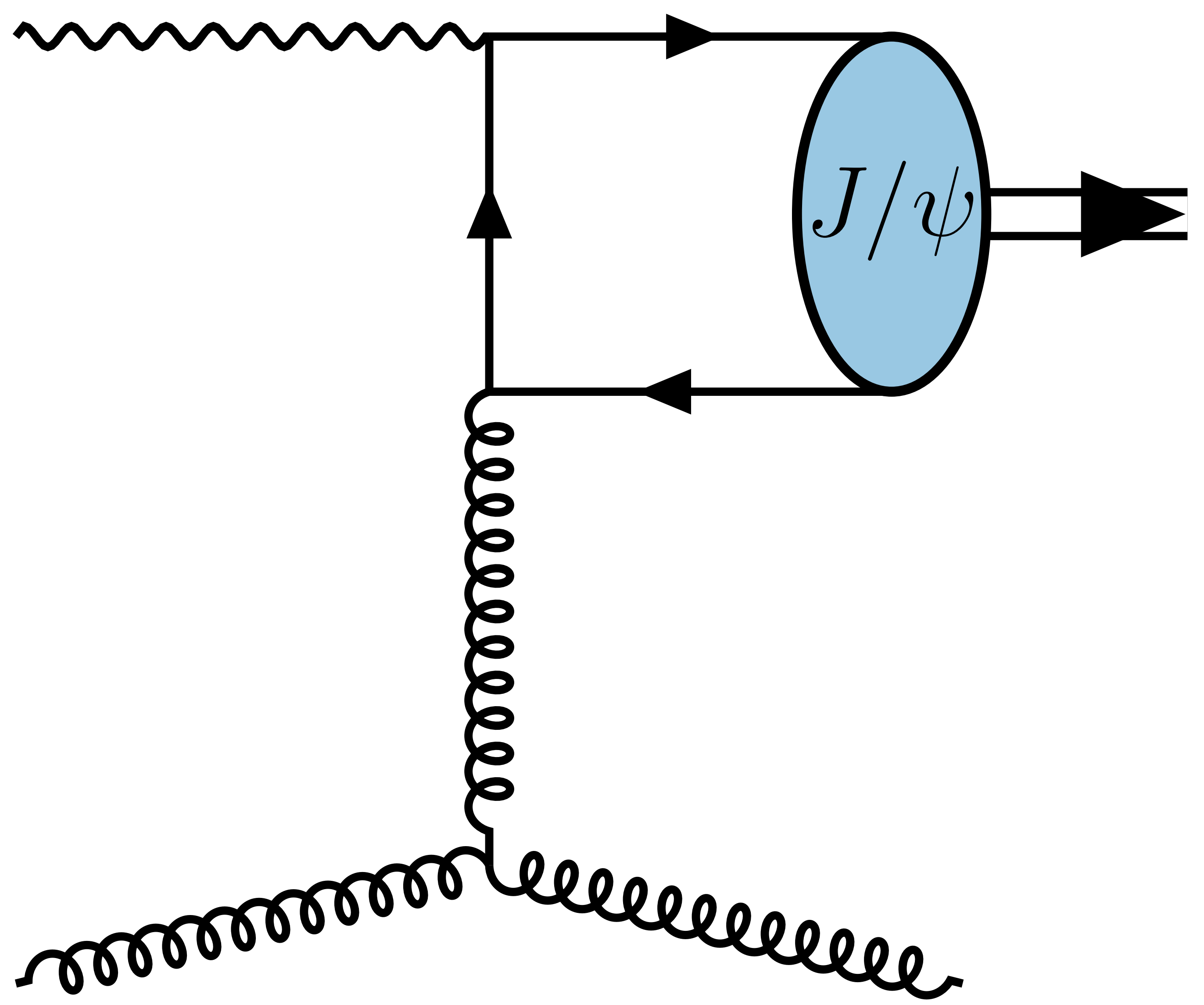


+



Divergent!

$2 \to 1$ **channel**



$2 \to 2$ **channel**

Limit is obtained by expanding $\delta(\hat{x}', \hat{z})$

$$\delta(\hat{x}', \hat{z}) \sim \frac{M_\psi^2 + Q^2}{M_\psi^2/\hat{z} + Q^2} \frac{\hat{z}}{(1-\hat{z})_+} \delta(1-\hat{x}') + \log \frac{M_\psi^2 + Q^2}{\boldsymbol{q}_T^2} \delta(1-\hat{x}')\,\delta(1-\hat{z}) + \frac{\hat{x}'}{(1-\hat{x})_+} \delta(1-\hat{z})$$

Analytical view of the small-q_T limit (delta expansion)

Boer, D'Alesio, Murgia, Pisano, Taels, JHEP 09 (2020)

$$I = \int_0^1 \mathrm{d}\hat{z}\, g(\hat{z}) \int_0^{\hat{x}_{\max}} \mathrm{d}\hat{x}\, f(\hat{x})\, \delta(\hat{x},\hat{z}) = \hat{x}_{\max} \int_0^1 \mathrm{d}\hat{z}\, \hat{z}^2\, g(\hat{z}) \int_0^1 \mathrm{d}\hat{x}'\, \hat{x}' f(\hat{x}')\; \delta\Big((1-\hat{x}')(1-\hat{z}) + (1-\hat{z})(\hat{z}-\hat{x}')\frac{M_\psi^2}{Q^2} + \frac{q_T^2}{Q^2}\Big)$$

continuous test functions

$$\hat{x}'_0 = \Big(1 + \frac{M_\psi^2}{Q^2}\Big)\left[1 + \frac{M_\psi^2}{\hat{z}\,Q^2} + \frac{\hat{z}}{1-\hat{z}}\frac{\boldsymbol{q}_T^2}{Q^2}\right]^{-1}$$

$$I = \int_0^1 \mathrm{d}\hat{z}\, \frac{(1-\hat{z})\,(\hat{z}\,Q^2 + M_\psi^2)}{(1-\hat{z})\,(\hat{z}\,Q^2 + M_\psi^2) + \hat{z}^2\,\boldsymbol{q}_T^2}\, \tilde{g}(\hat{z})\, \tilde{f}(\hat{x}'_0) = I_1 + I_2 + I_3$$

Obtained from:

$$\tilde{g}(\hat{z})\,\tilde{f}(\hat{x}'_0) = \big(\tilde{g}(\hat{z}) - \tilde{g}(1)\big)\,\tilde{f}(1) + \tilde{g}(1)\,\tilde{f}(1) + \tilde{g}(\hat{z})\,\big(\tilde{f}(\hat{x}'_0) - \tilde{f}(1)\big)$$

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$$\delta(\hat{x}',\hat{z}) \sim \frac{M_\psi^2 + Q^2}{M_\psi^2/\hat{z} + Q^2}\frac{\hat{z}}{(1-\hat{z})_+}\delta(1-\hat{x}') + \log\frac{M_\psi^2+Q^2}{\boldsymbol{q}_T^2}\delta(1-\hat{x}')\,\delta(1-\hat{z}) + \frac{\hat{x}'}{(1-\hat{x})_+}\delta(1-\hat{z})$$

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continuous test functions ← **Key word!**

$$\hat{x}_0' = \Big(1 + \frac{M_\psi^2}{Q^2}\Big)\left[1 + \frac{M_\psi^2}{\hat{z}\,Q^2} + \frac{\hat{z}}{1-\hat{z}}\frac{q_T^2}{Q^2}\right]^{-1}$$

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Are the hard amplitude continuous?

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

Previous derivation relies on the hard amplitude to be continuous, but…

✓ ✗

$$F_{UU}(\hat{x}', \hat{z}) = F_{UU}^{(0)}(\hat{x}', \hat{z}) + \sum_{k=1} \left(\frac{1-\hat{z}}{1-\hat{x}'} \right)^k F_{UU}^{(k)}(\hat{x}', \hat{z})$$

(general notation)

Continuous functions of $\hat{x}'$ and $\hat{z}$

✓ Delta expansion is applicable

✗ Delta expansion is not applicable

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Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

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$$F_{UU}(\hat{x}', \hat{z}) = F_{UU}^{(0)}(\hat{x}', \hat{z}) + \sum_{k=1} \boxed{\left(\frac{1-\hat{z}}{1-\hat{x}'}\right)^k F_{UU}^{(k)}(\hat{x}', \hat{z})}$$

Relevant for $\gamma^* g$ in $F_{UUT}^{(k)}$ and $F_{UUL}^{(k)}$ with $k = 1,\ 2$

Impact on the double delta

$$\delta(\hat{x}', \hat{z}) \sim \frac{\hat{x}'}{(1-\hat{x})_+} \delta(1-\hat{z}) + \cancel{\log \frac{M_\psi^2 + Q^2}{q_T^2}} \delta(1-\hat{x}')\, \delta(1-\hat{z})$$

$$\log \frac{M_\psi^2 + Q^2}{q_T^2} \to \frac{1}{2}\left(\log \frac{M_\psi^2 + Q^2}{q_T^2} - 1 - \log \frac{M_\psi^2}{M_\psi^2 + Q^2}\right)$$

Large logarithms from the eikonal method

Same term is found by explicitly considering the soft gluon emission

$$x_g = \frac{p_a \cdot p_g}{p_a \cdot q}$$

$$\mathrm{d}\sigma_1 \propto \int_{\frac{-p_{g\perp}^2}{M_\psi^2 + Q^2}}^{1} \frac{\mathrm{d}x_g}{x_g} \left[2\, S_g(p_a, P_\psi) + S_g(P_\psi, P_\psi) \right]$$

$$S_g(v_1, v_2) = \frac{v_1 \cdot v_2}{(v_1 \cdot p_g)(v_2 \cdot p_g)}$$

from momentum conservation
and $2p_g^+ p_g^- = -p_{g\perp}^2 = P_{\psi\perp}^2$

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Relation to quark-pair Fragmentation Function?

Kang, Ma, Qiu, Sterman, PRD 90 (2014) & PRD 91 (2015)

Ma, Qiu, Sterman, Zhang, PRL 113 (2014)

$$\mathrm{d}\sigma[\mathcal{Q}] = \int \mathrm{d}\xi_i\,\mathrm{d}\xi_j\,\mathrm{d}z\; f_i\, f_j\, \mathrm{d}\hat{\sigma}_{i+j\to f+X}\, D_{f\to\mathcal{Q}}(z) + \int \mathrm{d}\xi_i\,\mathrm{d}\xi_j\,\mathrm{d}z\; f_i\, f_j\, \mathrm{d}\hat{\sigma}_{i+j\to QQ+X}\, \boldsymbol{D_{QQ\to\mathcal{Q}}}(z)$$

The TMD shape function perturbative tail

Comparison at $\Lambda_{\rm QCD} \ll q_T \ll \mu_H$

TMD-PDFs evolved according to

Echevarria, Kasemets, Mulders, Pisano, JHEP 07 (2015)

Sun, Xiao, Yuan, PRD 84 (2011)

$$\mathcal{F}_{UU}^{\cos 2\phi}\big|_{\rm TMD} = F_{UU}^{\cos 2\phi}\big|_{\rm coll} \quad \Longrightarrow \quad \Delta_{h,\psi}^{[n]} = \delta^{(2)}(k_T^2)\,\langle \mathcal{O}_\psi[n]\rangle\,\delta(1-z)$$

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

$$\left.\begin{aligned} \mathcal{F}_{UUT}\big|_{\rm TMD} &\neq F_{UUT}\big|_{\rm coll} \\ \mathcal{F}_{UUL}\big|_{\rm TMD} &\neq F_{UUL}\big|_{\rm coll} \end{aligned}\right\} \quad \Longrightarrow \quad \Delta_\psi^{[n]} = -\frac{\alpha_s}{2\pi^2 k_T^2} C_A \left(1 + \log\frac{M_\psi^2}{M_\psi^2+Q^2}\right)\langle \mathcal{O}_\psi[n]\rangle\,\delta(1-z)$$

Up to the precision considered, bulk of the expression driven by $\underbrace{\textbf{CO}}_{{}^1S_0^{(8)}\quad {}^3P_J^{(8)}}$ **waves**

Scale dependence of the TMD shape function

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

Previous equation is obtained for $\mu_H \equiv \sqrt{M_\psi^2 + Q^2}$

(in b_T-space)

$$\tilde{\Delta}_\psi^{[n]}\left(z, b_T; \sqrt{M_\psi^2 + Q^2}\right) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log\frac{M_\psi^2}{(M_\psi^2 + Q^2)}\right)\log\frac{M_\psi^2 + Q^2}{\mu_b^2}\right]\langle \mathcal{O}[n]\rangle\, \delta(1-z)$$

$$\mu_b = \frac{2\,\mathrm{e}^{-\gamma_E}}{b_T}$$

for a general (hard) scale μ_H

$$\tilde{\Delta}_{ep}^{[n]}(z, b_T; \mu_H) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log\frac{M_\psi^2\,\mu_H^2}{(M_\psi^2 + Q^2)^2}\right)\log\frac{\mu_H^2}{\mu_b^2}\right]\langle \mathcal{O}[n]\rangle\, \delta(1-z)$$

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! It is process dependent !

Note that a Q^2-dependent soft factor is present in the open-quark production too

Zhu, Sun, Yuan, Phys. Lett. B 727 (2013)

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$$\mu_b = \frac{2\, e^{-\gamma_E}}{b_T}$$

for a general (hard) scale μ_H

Consequence:

The **TMDShF depends on a process-induced quantity** (photon virtuality Q) **unrelated to** neither a specific **hard scale** or **rapidity regulator choice**, as it usually happens for other TMDs!

The problem of the TMDShF Q dependence

[Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)](#)

$$\tilde{\Delta}^{[n]}_{ep}(z, b_T; Q, \mu_H) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log \frac{M_\psi^2\, \mu_H^2}{(M_\psi^2 + Q^2)^2}\right) \log \frac{\mu_H^2}{\mu_b^2}\right] \langle \mathcal{O}[n] \rangle\, \delta(1 - z)$$

Reasons to split-up this term:

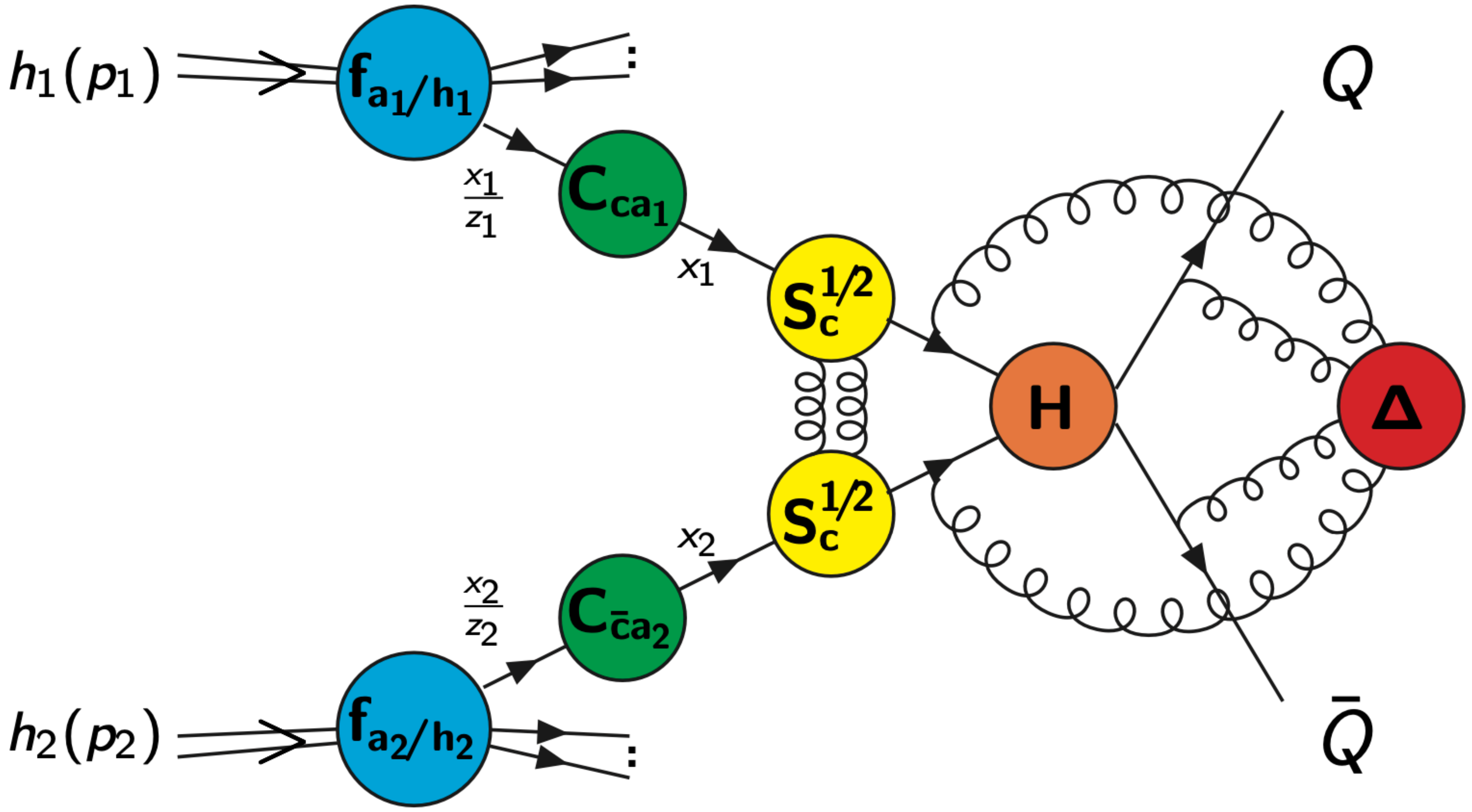
1. A purely quarkonium quantity should depend on M_ψ solely

2. In open-quark production the soft-factor may produce azimuthal dependeces

   [Catani, Grazzini, Torre, Nucl.Phys. B 890 (2014)](#)

From Ferrera's talk
@ Heavy-Quark Hadroproduction from
Collider to Astroparticle Physics (2019)

- Soft-factor $\mathbf{\Delta}(\mathbf{b}, M; \mathbf{\Omega})$ consistent with breakdown (in weak form) of TMD factorization (additional process-dependent non-perturbative factor needed) [Collins,Qiu('07)].



The TMDShF process-dependence

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

$$\tilde{\Delta}_{ep}^{[n]}(z, b_T; Q, \mu_H) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi}C_A\left(1 + \log\frac{M_\psi^2\,\mu_H^2}{(M_\psi^2 + Q^2)^2}\right)\log\frac{\mu_H^2}{\mu_b^2}\right]\langle\mathcal{O}[n]\rangle\,\delta(1-z)$$

split up: $\boldsymbol{\Delta_{ep}^{[n]} = \Delta_\psi^{[n]} \times S_{ep}}$

$$\Delta_\psi^{[n]}(z, b_T; \mu_H) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi}C_A\left(1 + \log\frac{M_\psi^2}{\mu_H^2}\right)\log\frac{\mu_H^2}{\mu_b^2}\right]\langle\mathcal{O}[n]\rangle\,\delta(1-z)$$ $\longrightarrow$ **Universal**

$$S_{ep}(b_T; Q, \mu_H) = 1 + \frac{\alpha_s}{2\pi}C_A\left(2\log\frac{\mu_H^2}{M_\psi^2 + Q^2}\right)\log\frac{\mu_H^2}{\mu_b^2}$$ $\longrightarrow$ **Process dependent**

A phenomenological test: single vs double scales processes

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

Quarkonium production in:

SIDIS (2 hard scales)

$$S_{ep}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(2 \log \frac{\mu_H^2}{M_\psi^2 + Q^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

hadron collisions (1 hard scale)

$$S_{pp}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(3 \log \frac{\mu_H^2}{M_\psi^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

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Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

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$\Downarrow$

$S_{pp}(M_\psi) \approx 0$ is a reasonable assumption

Easier extraction of $\Delta_\psi^{[n]}(M_\psi)$

A phenomenological test: single vs double scales processes

Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)

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$$\Delta_\psi^{[n]}\left(\sqrt{M_\psi^2 + Q^2}\right)$$

can be used combiantion with S_{ep}

hadron collisions (1 hard scale)

$$S_{pp}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(3 \log \frac{\mu_H^2}{M_\psi^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

$S_{pp}(M_\psi) \approx 0$ is a reasonable assumption

Easier extraction of

$\Delta_\psi^{[n]}(M_\psi)$

Evolution to

A phenomenological test: single vs double scales processes

[Boer, Bor, LM, Pisano, Yuan, JHEP 08 (2023)](#)

Quarkonium production in:

SIDIS (2 hard scales)

$$S_{ep}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(2 \log \frac{\mu_H^2}{M_\psi^2 + Q^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

$$\Delta_\psi^{[n]}\left(\sqrt{M_\psi^2 + Q^2}\right)$$

can be used combiantion with S_{ep}

Evolved and tested at higher scales, e.g. Υ production

hadron collisions (1 hard scale)

$$S_{pp}(\mu_H) = 1 + \frac{\alpha_s}{2\pi} C_A \left(3 \log \frac{\mu_H^2}{M_\psi^2} \right) \log \frac{\mu_H^2}{\mu_b^2}$$

$S_{pp}(M_\psi) \approx 0$ is a reasonable assumption

Easier extraction of $\Delta_\psi^{[n]}(M_\psi)$

Evolution to

Summary of the talk

- Factorization involves the presence of TMD shape functions
- We present a matching procedure to extract the TMDShF perturbative tail
- TMD shape functions separated in universal and process-dependent components

What to expect next?

- Perturbative tail at higher order → Relevant for $\Delta_h^{[n]}$
- Non-perturbative dependence
- Role of the TMD shape function in other processes
- The EIC is a promising playground to study the TMD shape function

Accessing gluon TMDs through Quarkonium observables

Back-up slides

Scale dependence of the TMD-PDFs (a comparison)

TMD-PDF matching coefficients taken from Echevarria, Kasemets, Mulders, Pisano, JHEP 07 (2015)

$$\tilde{f}_1^{g/A}(x, b_T; \zeta, \mu) = \int_x^1 \frac{\mathrm{d}\bar{x}}{\bar{x}} \, \tilde{C}_{g/j} \, f_{j/A}(\bar{x}/x; \mu)$$

(Only terms relevant in region $\Lambda_{\mathrm{QCD}} \ll \mu$)

gluon-gluon splitting function

$$\tilde{C}_{g/g} = \delta(1-x) + \frac{\alpha_s}{2\pi}\left[-\frac{C_A}{2}\log^2\frac{Q^2}{\mu_b^2}\delta(1-x) - \log\frac{Q^2}{\mu_b^2}\left(P_{g/g} - \delta(1-x)\frac{\beta_0}{2}\right)\right]$$

$$\beta_0 = \frac{11}{3}C_F - \frac{4}{3}T_F n_f$$

$$\mu_b = \frac{2\,\mathrm{e}^{-\gamma_E}}{b_T}$$

$$\tilde{C}_{g/q} = \frac{\alpha_s}{2\pi}\left[-\log\frac{Q^2}{\mu_b^2}P_{g/q}\right]$$

$$\mu_b = \frac{2\,\mathrm{e}^{-\gamma_E}}{b_T}$$

quark-gluon splitting function

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$$\tilde{C}_{g/g} = \delta(1-x) + \frac{\alpha_s}{2\pi}\left[-\frac{C_A}{2}\log^2\frac{\boxed{Q^2}}{\mu_b^2}\delta(1-x) - \log\frac{\boxed{Q^2}}{\mu_b^2}\left(P_{g/g} - \delta(1-x)\frac{\beta_0}{2}\right)\right]$$

$$\tilde{C}_{g/q} = \frac{\alpha_s}{2\pi}\left[-\log\frac{\boxed{Q^2}}{\mu_b^2}P_{g/q}\right]$$

Also the TMD-PDF displays a dependence over Q generated by the hardscale choice ($\mu = Q$)

Scale dependence of the TMD-PDFs (a comparison)

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(Only terms relevant in region $\Lambda_{\mathrm{QCD}} \ll \mu$ **and for general μ**)

$$\tilde{C}_{g/g} = \delta(1-x) + \frac{\alpha_s}{2\pi}\left[C_A\left(-\frac{1}{2}\log^2\frac{\mu^2}{\mu_b^2} + \log\frac{\mu^2}{\boxed{Q^2}}\log\frac{\mu^2}{\mu_b^2}\right)\delta(1-x) - \log\frac{\mu^2}{\mu_b^2}\left(P_{g/g} - \delta(1-x)\frac{\beta_0}{2}\right)\right]$$

$$\tilde{C}_{g/q} = \frac{\alpha_s}{2\pi}\left[-\log\frac{\mu^2}{\mu_b^2}P_{g/q}\right]$$

Comes from the rapidity regulator choice ($\zeta = Q^2$)