

Towards the transverse SSA
for $ep^\uparrow \rightarrow hX$ at NLO and its
connection to $ep^\uparrow \rightarrow \gamma X$

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Introduction

- $pp^\uparrow \rightarrow hX$: Large transverse single-spin asymmetries

(TSSA) $A_{UT} \equiv \frac{\sigma(\vec{S}_T) - \sigma(-\vec{S}_T)}{\sigma(\vec{S}_T) + \sigma(-\vec{S}_T)}$ of up to $\sim 50\%$

- proper treatment \leftrightarrow collinear twist-3 formalism
 - however: very challenging
 - NLO so far not on the horizon

Klem, R. D., et al., 1976, [Phys. Rev. Lett. 36, 929](#)
Dragoset, W. H., et al., 1978, [Phys. Rev. D 18, 3939](#)
Antille, J., et al., 1980, [Phys. Lett. 94B, 523](#)
Apokin, V. D., et al., 1990, [Phys. Lett. B 243, 461](#)
Saroff, S., et al., 1990, [Phys. Rev. Lett. 64, 995](#)

J.-w. Qiu, G. Sterman, [Phys. Rev. Lett. 67, 2264 \(1991\)](#)
Y. Kanazawa, Y. Koike, [Phys. Lett. B478, 121 \(2000\)](#)
C. Kouvaris, J.-W. Qiu, W. Vogelsang, F. Yuan, [Phys. Rev. D74, 114013 \(2006\)](#)
F. Yuan, J. Zhou, [Phys. Rev. Lett. 103, 052001 \(2009\)](#)
Kanazawa, Y. Koike, A. Metz, D. Pitonyak, [Phys. Rev. D89, 111501 \(2014\)](#)

Introduction

Simpler similar processes?

$$lp^\uparrow \rightarrow lX \text{ (DIS)}$$

TSSA vanishes for one-photon exchange

- time-reversal symmetry
- non-zero TSSA for two-photon exchange

N. Christ, T. D. Lee, [Phys. Rev. 143, 1310 \(1966\)](#)

$$lp^\uparrow \rightarrow hX$$

Accessible at the EIC

- Computation at NLO
- Jet production

$$lp^\uparrow \rightarrow \text{jet } X$$

$$lp^\uparrow \rightarrow \gamma X$$

No twist-3 fragmentation, but interference

$$lp^\uparrow \rightarrow l\gamma X \text{ (\gamma SIDIS)}$$

W.S. Albaltan, A. Prokudin, M. Schlegel, [Phys. Lett. B 804, 135367 \(2020\)](#)

Collinear twist-3 factorization approach

Intrinsic

- higher twist, transverse-spin dependant terms from the familiar quark-quark correlator

Kinematical

- keep the transverse momentum k_T of the quark to first order
- first k_T -moment of TMDs, for example of the Sivers function

Dynamical

- correlation functions of three parton fields
- functions of two light-cone momentum fractions
- exist for both distribution and fragmentation

The notation in the following slides follows the conventions from K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak, M. Schlegel, [Phys. Rev. D 93, 054024 \(2016\)](#)

Intrinsic twist-3 distribution functions

Higher twist terms from the quark-quark correlator familiar from unpolarized and longitudinally polarized twist-2 calculations:

$$\Phi_{ij}^q(x) = \int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S | \bar{q}_j(0) q_i(\lambda n) | P, S \rangle$$

Wilson lines, like $\mathcal{W}[0; \lambda n]$ in this case, are omitted for simplicity!

$$\begin{aligned} \Phi^q(x) = & \frac{1}{2} \not{n} f_1^q(x) + \frac{1}{2} M e^q(x) - \frac{1}{2} M (S \cdot n) \not{n} \gamma_5 g_1^q(x) + \frac{1}{4} M^2 (S \cdot n) [\not{n}, \not{\not{n}}] \gamma_5 h_L^q(x) \\ & - \frac{1}{4} [\not{n}, \not{\not{n}}] \gamma_5 h_1^q(x) - \frac{1}{2} M (\not{\not{n}} - (S \cdot n) \not{n}) \gamma_5 g_T^q(x) \end{aligned}$$

Kinematical twist-3 distribution functions

Keeping a non-zero k_T one encounters the following k_T -dependant correlator:

$$\Phi_{ij}^q(x, k_T) = \int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi} \int \frac{d^{d-2}z_T}{(2\pi)^{d-2}} e^{i\lambda x + i k_T \cdot z_T} \langle P, S | \bar{q}_j(0) q_i(\lambda n + z_T) | P, S \rangle$$

This can appear in a general expression like:

$$\begin{aligned} \frac{E_h d\sigma}{d^{d-1}P_h} &\propto \int dx d^{d-2}k_T \int d(1/z) \Phi_{ij}^q(x, k_T) H_{ij}(x, k_T, z) D_1^q(z) \\ &\xrightarrow{O(k_T)} \int dx \int d(1/z) \left(\int d^{d-2}k_T k_T^\rho \Phi_{ij}^q(x, k_T) \right) \left(\frac{d}{dk_T^\rho} H_{ij}(x, k_T, z) \Big|_{k_T=0} \right) D_1^q(z) \end{aligned}$$

Kinematical twist-3 distribution functions

This leads to the definition of the kinematical twist-3 correlator:

$$\begin{aligned}\Phi_{\partial,ij}^{q,\rho}(x) &= \int d^2k_T k_T^\rho \Phi_{ij}^q(x, k_T) \\ &= \frac{1}{2} M \epsilon^{Pn\rho S} \not{p}_{ij} f_{1T}^{\perp(1),q}(x) + \dots\end{aligned}$$

Where the first k_T -moment of the Sivers function appears:

$$f_{1T}^{\perp(1),q}(x) = \int d^2k_T \frac{\vec{k}_T^2}{2M^2} f_{1T}^{\perp q}(x, k_T^2)$$

Dynamical twist-3 distribution functions

Multi-parton correlators:

$$\begin{aligned}\Phi_{F,ij}^{q,\rho}(x, x') &= \int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi} \int_{-\infty}^{+\infty} \frac{d\mu}{2\pi} e^{ix'\lambda + i(x-x')\mu} \langle P, S | \bar{q}_j(0) i g F^{n\rho}(\mu n) q_i(\lambda n) | P, S \rangle \\ &= \frac{M}{2} (\epsilon^{Pn\rho S} \not{n} i F_{FT}^q(x, x') - S_T^\rho \not{T} \gamma_5 G_{FT}^q(x, x') + \dots)\end{aligned}$$

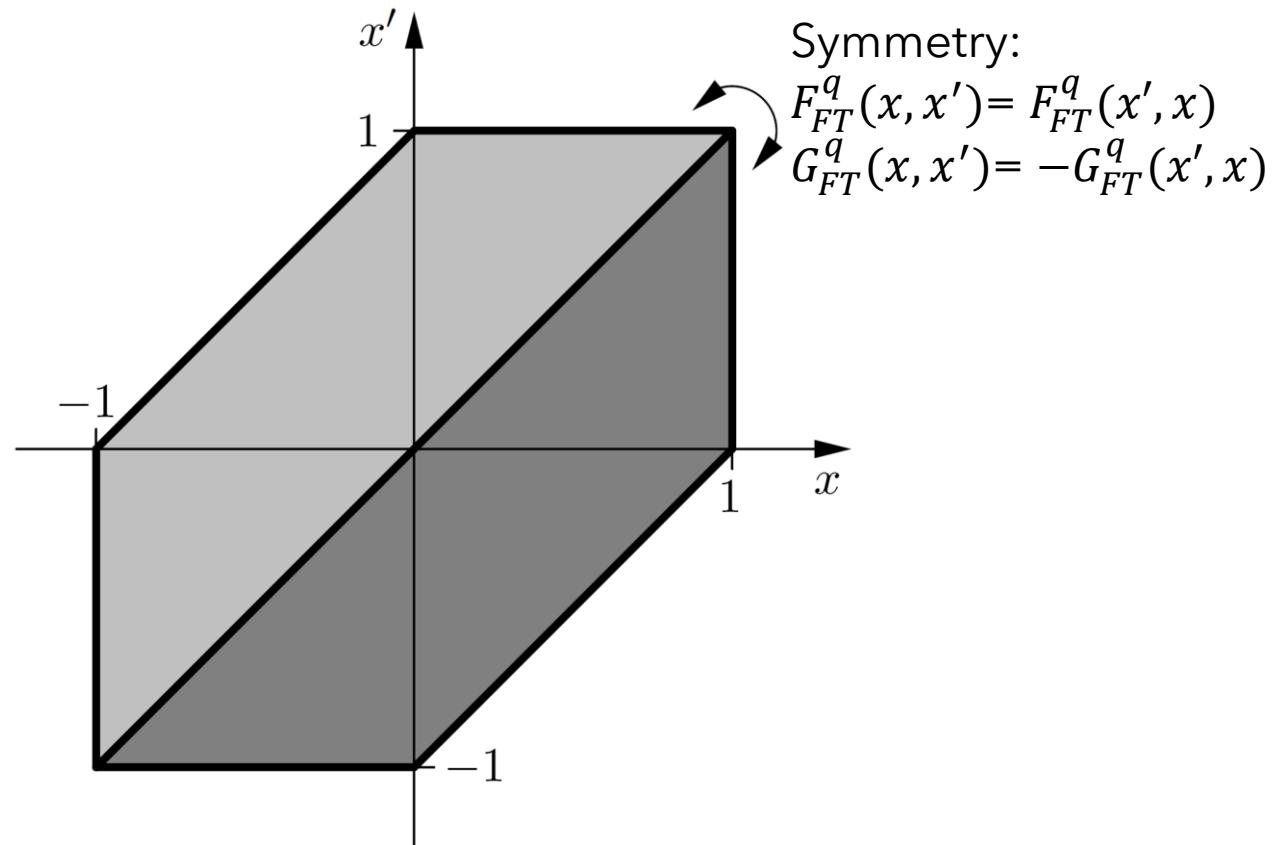
Connection with Sivers function

$$\pi F_{FT}^q(x, x) = f_{1T}^{\perp(1),q}(x)$$

D. Boer, P.J. Mulders, F. Pijlman, [Nucl. Phys. B 667 \(2003\)](#)

Dynamical twist-3 distribution functions

Support:
 $|x| \leq 1, |x'| \leq 1$ and
 $|x - x'| \leq 1$



Key features of the calculation

LO result for $e(l)p^\uparrow(P) \rightarrow h(P_h)X$:

Twist-3 effects on the fragmentation side, which we do not consider in this talk

$$E_h \frac{d\sigma_{LO}}{d^{d-1}P_h}(S) = \sigma_0(S) \int_{v_0}^{v_1} dv \int_{x_0}^1 \frac{dw}{w} \hat{\sigma}_{LO}(v, w) \sum_q e_q^2 \left[\left(1 - x \frac{d}{dx} \right) F_{FT}^q(x, x) D_1^q(z) \right] \Bigg|_{x=\frac{x_0}{w}}^{z=\frac{1-v_1}{1-v}} + h_1^q \otimes \mathfrak{S}[\hat{H}_{FU}^q]$$

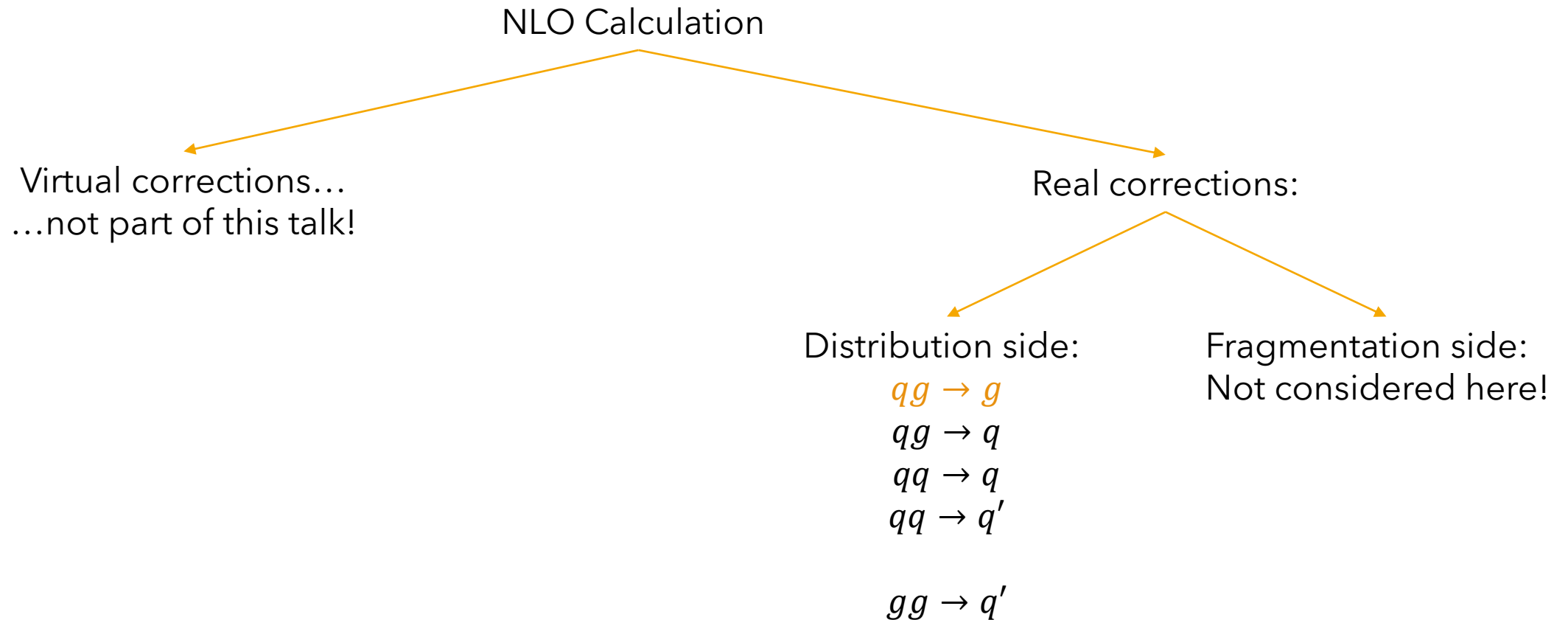
Nucleon-spin (S)
dependence

$$\sigma_0(S) = \left(\frac{2\alpha_{\text{em}}^2}{s(-u)} \right) \left(\frac{4\pi M \epsilon^{lPP_h S}}{s(-u)} \right), \quad \hat{\sigma}_{LO}(v, w) = \frac{1 + v^2}{(1 - v)^4} \delta(1 - w)$$

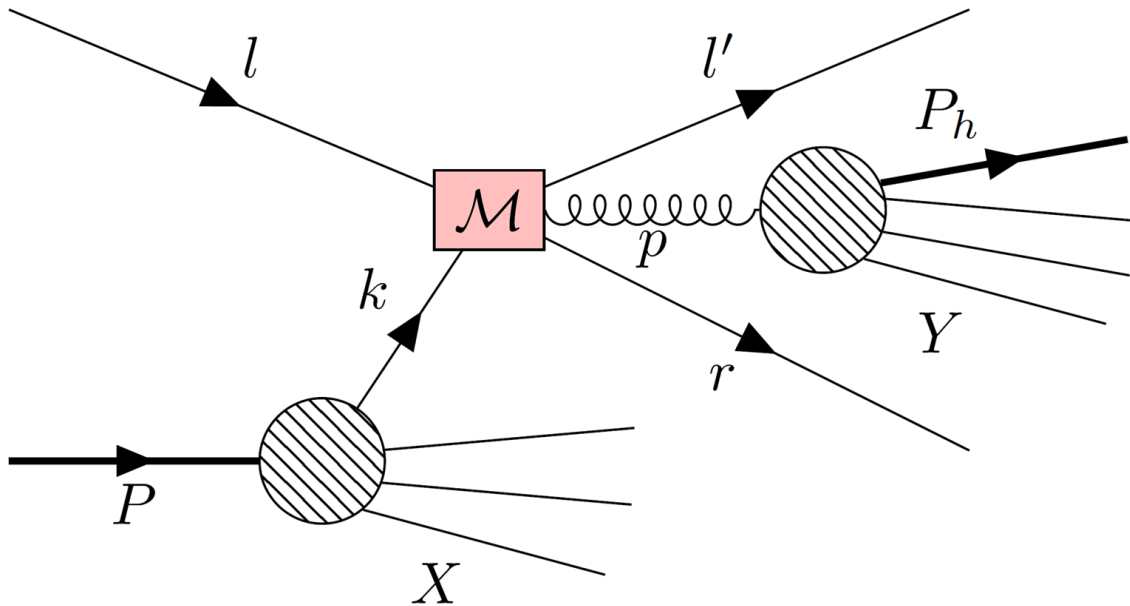
$$s = (l + P)^2 \cong 2l \cdot P, \quad t = (P - P_h)^2 \cong -2P \cdot P_h, \quad u = (l - P_h)^2 \cong -2l \cdot P_h$$

$$x_0(v) = \frac{1 - v u}{v t}, \quad v_0 = \frac{u}{t + u}, \quad v_1 = \frac{s + t}{s}$$

Key features of the calculation



Key features of the kinematical part

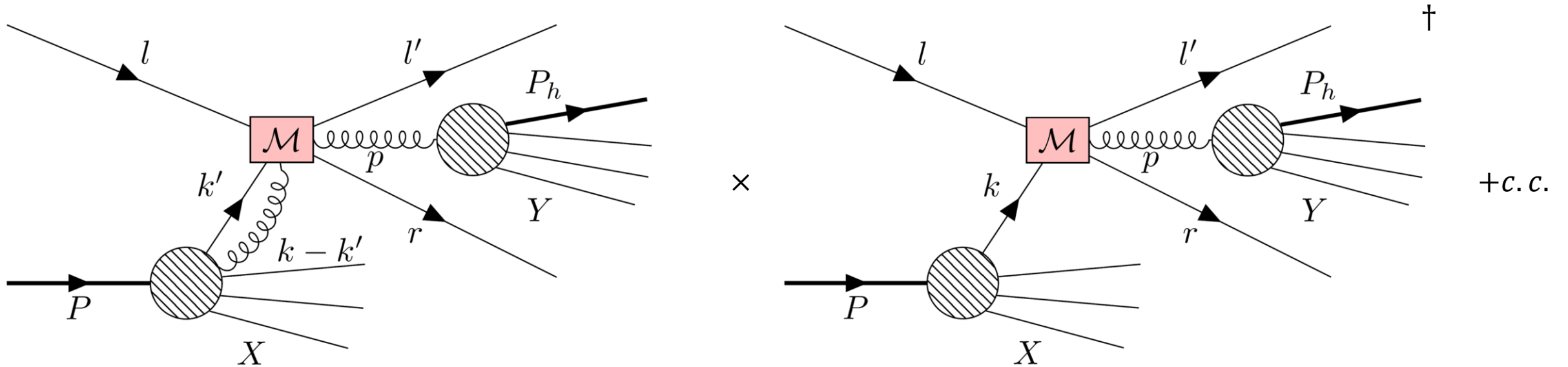


$$\frac{E_h d\sigma_{kin}}{d^{d-1}P_h} = \int dx \Phi_{\partial,ij}^{q,\rho}(x) \int d(1/z) D_1^q(z) \times \frac{d}{dk_T^\rho} H_{ij} \left(k = xP + k_T, p = \frac{P_h}{z} \right) \Big|_{k_T=0}$$

Where the hard scattering part has the form:

$$H_{ij} \left(k = xP + k_T, p = \frac{P_h}{z} \right) = \int d^d r \delta^+(r^2) \delta^+((l + xP + k_T - P_h/z - r)^2) \hat{\sigma}_{ij}(k, p)$$

Key features of the dynamical part



$$\frac{E_h d\sigma_{dyn}}{d^{d-1}P_h} = \int dx \int dx' \int d(1/z) z^{2\varepsilon} D_1^q(z) H_{ij} \left(k = xP, k' = x'P, p = \frac{P_h}{z} \right) \frac{i\Phi_{F,ij}^{q,\rho}(x, x')}{x' - x} + c.c.$$

Key features of the dynamical part

$\propto C_F$

$$\frac{1}{(l+k'-p-r)^2 + i\eta}$$

$$= \frac{x}{\hat{s} + \hat{t} - 2k \cdot r} \left(\mathcal{P} \frac{1}{x' - x} - i\pi\delta(x' - x) \right)$$

$\propto C_F - \frac{N_C}{2}$

$$\frac{1}{(l+k'-r)^2 + i\eta}$$

$$= \frac{x}{\hat{s} - 2k \cdot r} \left(\mathcal{P} \frac{1}{x' - \tilde{x}} - i\pi\delta(x' - \tilde{x}) \right)$$

$\tilde{x} \equiv \frac{2x l \cdot r}{\hat{s} - 2k \cdot r}$

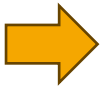
$\propto -\frac{N_C}{2}$

$$\frac{1}{(k'-p)^2 + i\eta}$$

$$= \frac{x}{\hat{t}} \left(\mathcal{P} \frac{1}{x'} + i\pi\delta(x') \right)$$

Key features of the dynamical part

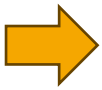
$$\frac{\delta(x' - x)}{x' - x}$$



$$F_{FT}^q(x, x), (G_{FT}^q(x, x) = 0), \\ \frac{d}{dx} F_{FT}^q(x, x), \frac{\partial}{\partial x'} G_{FT}^q(x, x') \Big|_{x'=x}$$

Soft Gluon Pole (SGP)

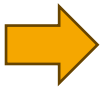
$$\delta(x')$$



$$F_{FT}^q(x, 0), G_{FT}^q(x, 0)$$

Soft Fermion Pole (SGP)

$$\delta(x' - \tilde{x})$$



$$F_{FT}^q(x, \tilde{x}), G_{FT}^q(x, \tilde{x})$$

Hard Pole (HP)

Key features of the dynamical part

But Careful! $\tilde{x} = \frac{2x l \cdot r}{\hat{s} - 2k \cdot r}$ so $F_{FT}^q(x, \tilde{x})$, $G_{FT}^q(x, \tilde{x})$ depend on the momentum r that is integrated over!



Keep the imaginary part $i\eta$ in the corresponding propagator and perform the phase space integral first, then take the imaginary part of the result



Extract imaginary part via
$$\log[x \pm i\eta] \xrightarrow{\eta \rightarrow 0^+} \log[|x|] \pm \Theta(-x)i\pi$$

Key features of the dynamical part

Convenient change of variable $\zeta = \frac{x'}{x}$; The result has a discontinuity at $\zeta = w$ and poles at $\zeta = 0, \zeta = 1$
 → Need to regularize these poles!

$$\frac{E_h d\sigma_{dyn}}{d^{d-1}P_h} \propto \int_{v_0}^{v_1} dv \int_{x_0}^1 \frac{dw}{w} \int_0^1 d\zeta \left[\left(\hat{\sigma}(v, w, \zeta) \frac{F_{FT}^q(x, x\zeta)}{\zeta(1-\zeta)^2} + \hat{\sigma}_5(v, w, \zeta) \frac{G_{FT}^q(x, x\zeta)}{\zeta(1-\zeta)^2} \right) z^{2\varepsilon} D_1^q(z) \right] \Bigg|_{x=\frac{x_0}{w}}^{z=\frac{1-v_1}{1-v}}$$

Finite over the whole
integration domain

Contains all poles and needs to be
regularized

Key features of the dynamical part

Possible ways to regularize (add and subtract suitable terms):

$$\frac{F_{FT}^q(x, x\zeta)}{\zeta(1-\zeta)^2} \rightarrow - \frac{(1-\zeta)^2 F_{FT}^q(x, 0) + \zeta(2-\zeta) F_{FT}^q(x, x) - \frac{x}{2} \zeta(1-\zeta) \frac{d}{dx} F_{FT}^q(x, x) - F_{FT}^q(x, x\zeta)}{\zeta(1-\zeta)^2}$$

Extra SFP
Extra SGP

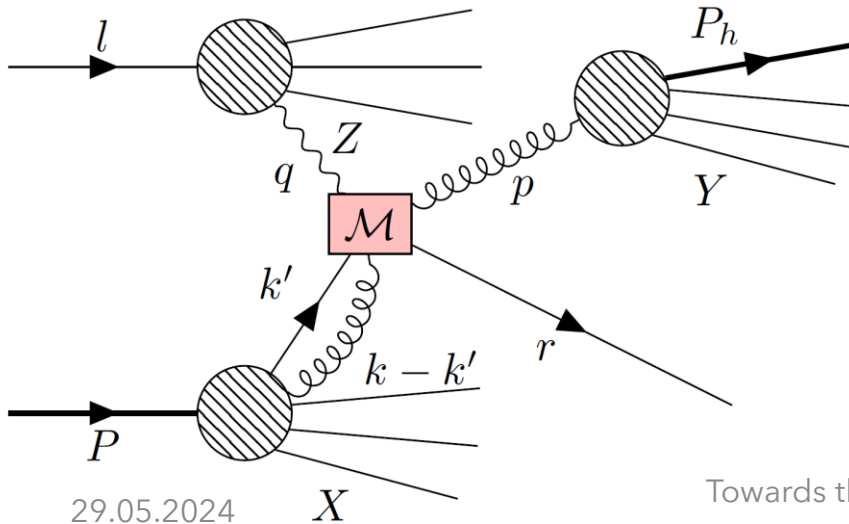
$$\frac{G_{FT}^q(x, x\zeta)}{\zeta(1-\zeta)^2} \rightarrow - \frac{(1-\zeta)^2 G_{FT}^q(x, 0) - x \zeta(1-\zeta) \frac{\partial}{\partial x'} G_{FT}^q(x, x')|_{x'=x} - G_{FT}^q(x, x\zeta)}{\zeta(1-\zeta)^2}$$

There is no ζ -dependence in the added F_{FT}^q, G_{FT}^q functions \rightarrow apply the delta function $\delta(x - \tilde{x})$ for the extra SFP/SGP contributions

Key features of the dynamical part

There are two types of collinear singularities left at this point

Singularities from neglecting the electron mass
 → removed by Weizsäcker-Williams contribution



Singularities from collinear emission of the gluon from the quark
 → removed by fragmentation function renormalization

$$\hat{\sigma}_{LO} \otimes \frac{\alpha_s}{2\pi} \frac{S_\epsilon}{\epsilon} \int_z^1 \frac{dw}{w} P_{gq}(w) D_1^q\left(\frac{z}{w}\right)$$

Quark-gluon splitting function

Towards the transverse SSA for $ep \rightarrow hX$ at NLO and its connection to $ep \rightarrow \gamma X$

Final result

$$E_h \frac{d\sigma}{d^3P_h}(S) = E_h \frac{d\sigma^{SGP}}{d^3P_h}(S) + E_h \frac{d\sigma^{SFP}}{d^3P_h}(S) + E_h \frac{d\sigma^{int}}{d^3P_h}(S)$$

$$\sigma_1(S) \equiv \frac{8\pi\alpha_{em}^2 \alpha_s M \epsilon^{lPP_h S}}{s^2 2\pi u^2}, \quad (\partial_{x'} G_{FT}^q)(x, x) \equiv \frac{\partial}{\partial x'} G_{FT}^q(x, x') \Big|_{x'=x}$$

$$E_h \frac{d\sigma^{int}}{d^3P_h} = \sigma_1(S) \sum_q e_q^2 \int_{v_0}^{v_1} dv \int_{x_0}^1 \frac{dw}{w} \int_0^1 d\zeta \left(\left[C_F \hat{\sigma}_{CF}^{int}(v, w, \zeta) + \frac{N_C}{2} \hat{\sigma}_{NC}^{int}(v, w, \zeta) \right] \left[\frac{(1-\zeta)^2 F_{FT}^q(x, 0) + \zeta(2-\zeta) F_{FT}^q(x, x) - \frac{x}{2} \zeta(1-\zeta) \frac{d}{dx} F_{FT}^q(x, x) - F_{FT}^q(x, x\zeta)}{\zeta(1-\zeta)^2} D_1^g(z) \right] \right. \\ \left. + \left[C_F \hat{\sigma}_{5,CF}^{int}(v, w, \zeta) + \frac{N_C}{2} \hat{\sigma}_{5,NC}^{int}(v, w, \zeta) \right] \left[\frac{(1-\zeta)^2 G_{FT}^q(x, 0) - x \zeta(1-\zeta) (\partial_{x'} G_{FT}^q)(x, x) - G_{FT}^q(x, x\zeta)}{\zeta(1-\zeta)^2} D_1^g(z) \right] \right) \Bigg|_{x=\frac{x_0}{w}}^{z=\frac{1-v_1}{1-v}}$$

$$E_h \frac{d\sigma^{SFP}}{d^3P_h} = \sigma_1(S) \sum_q e_q^2 \int_{v_0}^{v_1} dv \int_{x_0}^1 \frac{dw}{w} \left(\left[C_F \hat{\sigma}_{CF}^{SFP}\left(v, w, \frac{su}{tm^2}\right) + \frac{N_C}{2} \hat{\sigma}_{NC}^{SFP}\left(v, w, \frac{su}{tm^2}\right) \right] [F_{FT}^q(x, 0) D_1^g(z)] + \left[C_F \hat{\sigma}_{5,CF}^{SFP}\left(v, w, \frac{su}{tm^2}\right) + \frac{N_C}{2} \hat{\sigma}_{5,NC}^{SFP}\left(v, w, \frac{su}{tm^2}\right) \right] [G_{FT}^q(x, 0) D_1^g(z)] \right) \Bigg|_{x=\frac{x_0}{w}}^{z=\frac{1-v_1}{1-v}}$$

$$E_h \frac{d\sigma^{SGP}}{d^3P_h} = \sigma_1(S) \sum_q e_q^2 \int_{v_0}^{v_1} dv \int_{x_0}^1 \frac{dw}{w} \left(\left[C_F \hat{\sigma}_{CF}^{SGP}\left(v, w, \frac{su}{t\mu^2}\right) + \frac{N_C}{2} \hat{\sigma}_{NC}^{SGP}\left(v, w, \frac{su}{tm^2}\right) \right] [F_{FT}^q(x, x) D_1^g(z)] + \left[C_F \hat{\sigma}_{5,CF}^{SGP}(v, w) + \frac{N_C}{2} \hat{\sigma}_{5,NC}^{SGP}(v, w) \right] [x (\partial_{x'} G_{FT}^q)(x, x) D_1^g(z)] \right) \Bigg|_{x=\frac{x_0}{w}}^{z=\frac{1-v_1}{1-v}}$$

Connection to $ep^\uparrow \rightarrow \gamma X$ - model ansatz

$$\begin{aligned}
 F_{FT}^q(r, \varphi) &= \left(\frac{1}{2\pi} \left(f_{1T}^{\perp(1),q} + f_{1T}^{\perp(1),\bar{q}} \right) \left(\frac{r}{\sqrt{2}} \right) + \frac{1}{2\pi} \left(f_{1T}^{\perp(1),q} - f_{1T}^{\perp(1),\bar{q}} \right) \left(\frac{r}{\sqrt{2}} \right) \cos \varphi \right. \\
 &\quad \left. + \sum_{n=1}^{\infty} a_{2n}^q(r) (\cos(2n\varphi) - 1) + \sum_{n=1}^{\infty} a_{2n+1}^q(r) (\cos((2n+1)\varphi) - \cos \varphi) \right) \\
 &\quad \times \left(\frac{(1-x^2)(1-(x')^2)}{(1-xx')^2} \right)^\delta (1-(x-x')^2)^\epsilon \Theta(1-|x|) \Theta(1-|x'|) \Theta(1-|x-x'|) \left. \begin{array}{l} x' = r \sin(\varphi + \frac{\pi}{4}) \\ x = r \cos(\varphi + \frac{\pi}{4}) \end{array} \right) \\
 G_{FT}^q(r, \varphi) &= -\frac{1}{\pi} \left(f_{1T}^{\perp(1),q} + f_{1T}^{\perp(1),\bar{q}} \right) \left(\frac{r}{\sqrt{2}} \right) \sum_{n=1}^{\infty} b_n^q(r) \sin(n\varphi) \\
 &\quad \times \left(\frac{(1-x^2)(1-(x')^2)}{(1-xx')^2} \right)^\delta (1-(x-x')^2)^\epsilon \Theta(1-|x|) \Theta(1-|x'|) \Theta(1-|x-x'|) \left. \begin{array}{l} x' = r \sin(\varphi + \frac{\pi}{4}) \\ x = r \cos(\varphi + \frac{\pi}{4}) \end{array} \right)
 \end{aligned}$$

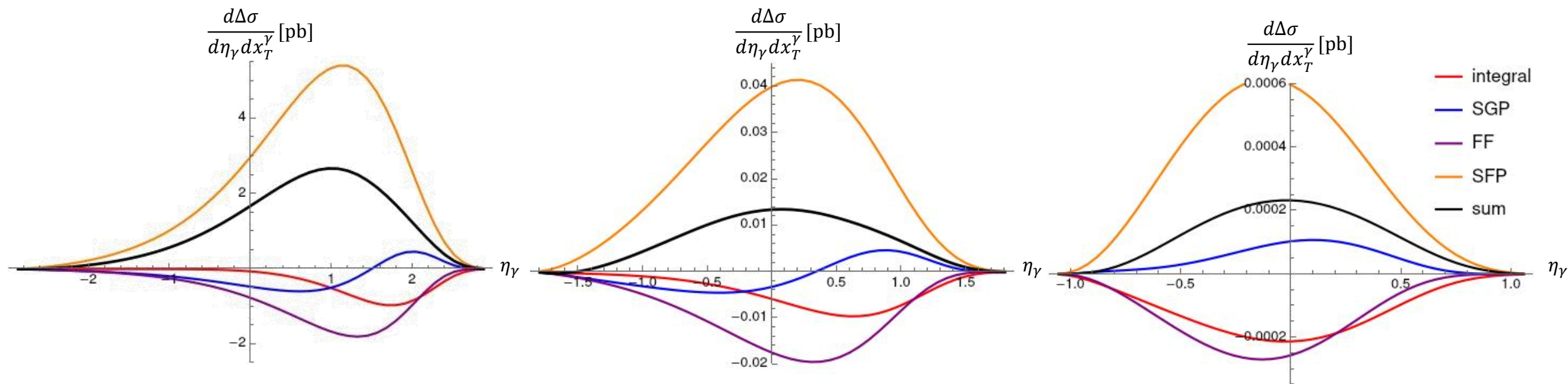
Rapidity plots for $ep^\uparrow \rightarrow \gamma X$

$$x_T^\gamma = \frac{2p_T^\gamma}{\sqrt{s}}$$

$\sqrt{s} = 100 \text{ GeV}, x_T^\gamma = 0.1, \mu = p_T^\gamma = 5 \text{ GeV}$

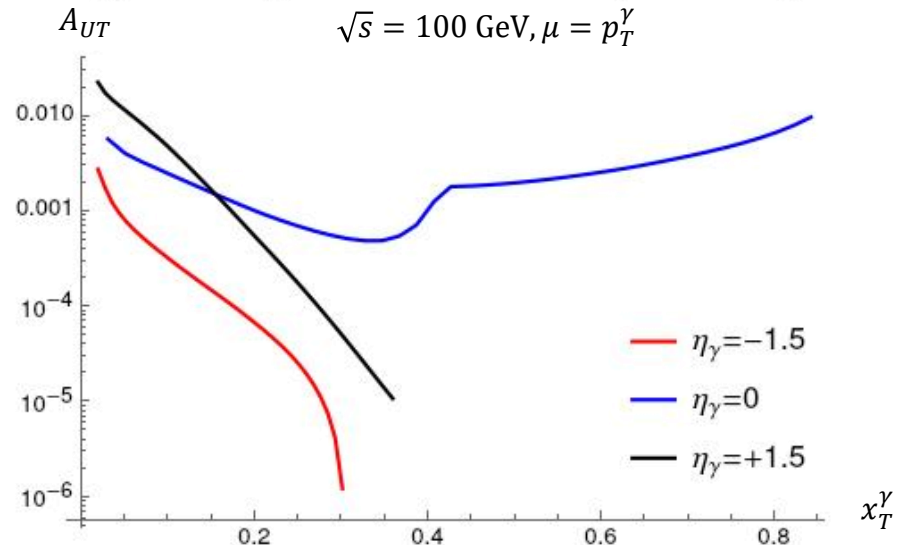
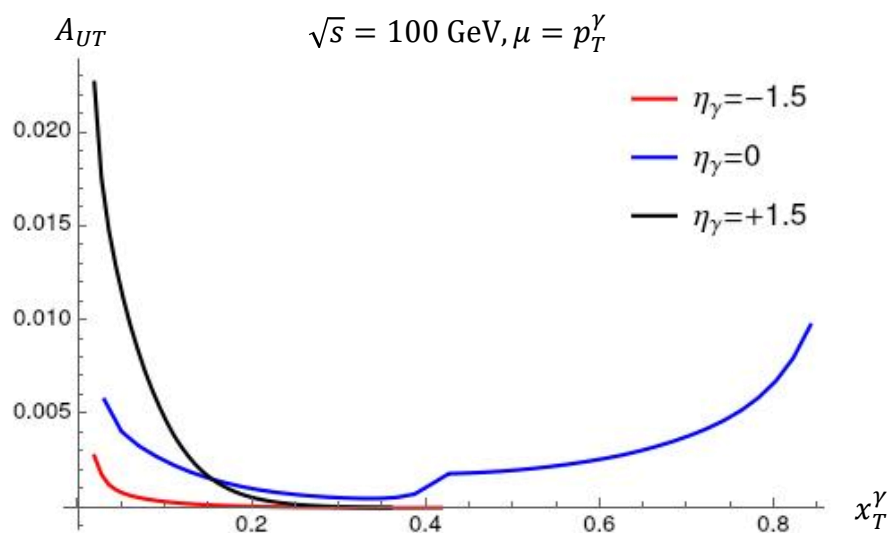
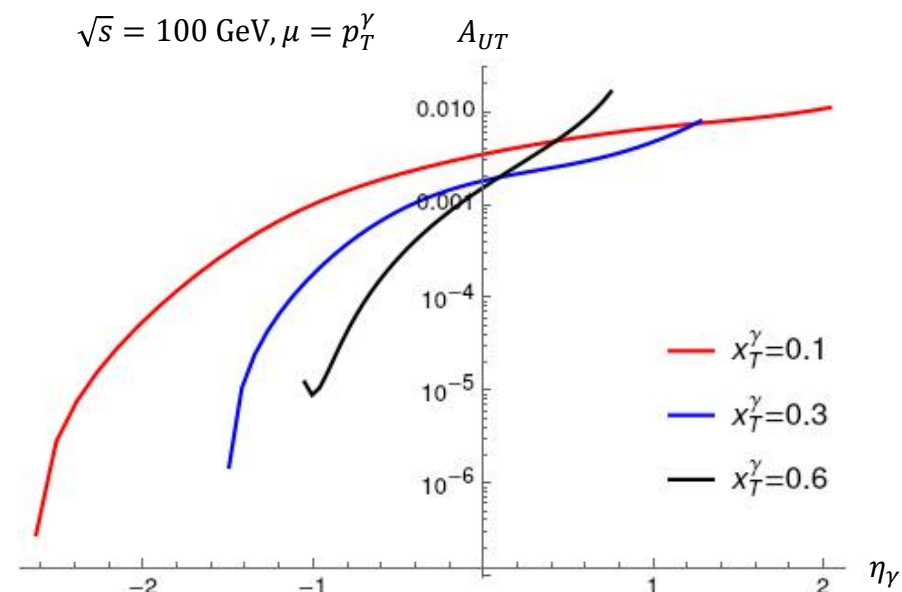
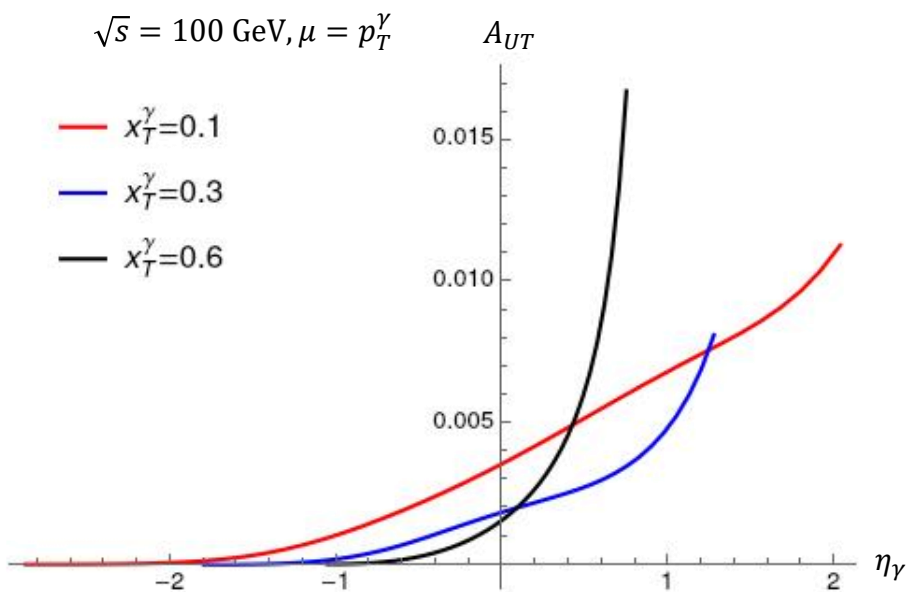
$\sqrt{s} = 100 \text{ GeV}, x_T^\gamma = 0.3, \mu = p_T^\gamma = 15 \text{ GeV}$

$\sqrt{s} = 100 \text{ GeV}, x_T^\gamma = 0.6, \mu = p_T^\gamma = 30 \text{ GeV}$



Asymmetries for $ep^{\uparrow} \rightarrow \gamma X$

$$x_T^{\gamma} = \frac{2p_T^{\gamma}}{\sqrt{s}}$$



Summary/Outlook

- Twist-3 formalism - plethora of different contributions + distribution functions
- Combine using EoM-relations and/or lorentz invariance relations
→ express result for the $qg \rightarrow g$ channel entirely in terms of F_{FT}^q, G_{FT}^q and twist-2 FFs
- We also obtain the result for $ep^\uparrow \rightarrow \gamma X$
→ model predictions show potential for large A_{UT} in certain kinematic regions
- Finish the calculation for remaining real correction channel $gg \rightarrow q'$
- Derive corresponding results for $ep^\uparrow \rightarrow jet X$
- Include twist-3 fragmentation effects

Backup

The phase space integral can be solved analytically, choosing a frame where $\vec{l} + \vec{k} - \vec{p} = \vec{0}$ one finds

$$J \equiv \frac{1}{(2\pi)^{d-1}} \int d^d r \delta^+(r^2) \delta^+((l + xP + k_T - P_h/z - r)^2) f(r)$$

$$\rightarrow \frac{1}{4(2\pi)^3} S_\varepsilon \left(\frac{-u(1-w) + 2wk_T \cdot l}{w\mu^2} \right)^{-\varepsilon} \frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \int_0^\pi \sin^{1-2\varepsilon}(\theta) d\theta \int_0^\pi \sin^{-2\varepsilon}(\phi) d\phi f(\theta, \phi)$$

$\downarrow \frac{\partial}{\partial k_T}; k_T \rightarrow 0$

$$\varepsilon(1-w)^{-1-\varepsilon}$$

$$(1-w)^{-1-a\varepsilon} = -\frac{1}{a\varepsilon} \delta(1-w) + \frac{1}{(1-w)_+} + \mathcal{O}(\varepsilon)$$

For further evaluation of the angular integrals we followed:

V.E. Lyubovitskij, F. Wunder, A.S. Zhevnikov,
[J. High Energ. Phys. 2021, 66 \(2021\)](#)

Backup

Alternative approach (this was used for $lp^\dagger \rightarrow l\gamma X$ (γ SIDIS) in W.S. Albaltan, A. Prokudin, M. Schlegel, [Phys. Lett. B 804, 135367 \(2020\)](#)):
pull the k_T -derivative inside the phase space integral:

$$\begin{aligned} & \frac{\partial}{\partial k_T^\rho} \delta((l + xP + k_T - P_h/z - r)^2) \hat{\sigma}_{ij}(x, k_T, z) \Big|_{k_T=0} \\ &= \delta((l + k - p - r)^2) \left[\frac{\partial \hat{\sigma}_{ij}}{\partial k_T^\rho} \Big|_{k_T=0} - \frac{2x(l-r)_\rho}{\hat{s} + \hat{t} - 2k \cdot r} \left(\frac{\partial \hat{\sigma}_{ij}}{\partial x} \Big|_{k_T=0} + \hat{\sigma}_{ij} \Big|_{k_T=0} \frac{\partial}{\partial x} \right) \right] \end{aligned}$$

Leading to:

$$\begin{aligned} \frac{E_h d\sigma_{kin}}{d^{d-1}P_h} &= \int dx \int d(1/z) D_1^q(z) \int d^d r \delta^+(r^2) \delta^+((l + k - p - r)^2) \\ &\times \left[\Phi_{\partial,ij}^{q,\rho}(x) \left(\frac{\partial \hat{\sigma}_{ij}}{\partial k_T^\rho} \Big|_{k_T=0} - \frac{2x(l-r)_\rho}{\hat{s} + \hat{t} - 2k \cdot r} \frac{\partial \hat{\sigma}_{ij}}{\partial x} \Big|_{k_T=0} \right) - \frac{\partial \Phi_{\partial,ij}^{q,\rho}(x)}{\partial x} \frac{2x(l-r)_\rho}{\hat{s} + \hat{t} - 2k \cdot r} \hat{\sigma}_{ij} \Big|_{k_T=0} \right] \end{aligned}$$

$k = xP$

Backup

Another possible regularization: $\frac{\hat{\sigma}'(v,w,\zeta)F_{FT}^q(x,x\zeta)}{\zeta(w-\zeta)(1-\zeta)}$, $\frac{\hat{\sigma}_5'(v,w,\zeta)G_{FT}^q(x,x\zeta)}{\zeta(w-\zeta)(1-\zeta)}$ with

$$\frac{F_{FT}^q(x,x\zeta)}{\zeta(w-\zeta)(1-\zeta)} \rightarrow - \frac{\frac{(1-\zeta)(w-\zeta)}{w} F_{FT}^q(x,0) - \frac{\zeta(w-\zeta)}{1-w} F_{FT}^q(x,x) + \frac{\zeta(1-\zeta)}{w(1-w)} F_{FT}^q(x,xw) - F_{FT}^q(x,x\zeta)}{\zeta(w-\zeta)(1-\zeta)}$$

„genuine“ hard pole

$$\frac{G_{FT}^q(x,x\zeta)}{\zeta(w-\zeta)(1-\zeta)} \rightarrow - \frac{\frac{(1-\zeta)(w-\zeta)}{w} G_{FT}^q(x,0) + \frac{\zeta(1-\zeta)}{w(1-w)} G_{FT}^q(x,xw) - G_{FT}^q(x,x\zeta)}{\zeta(w-\zeta)(1-\zeta)}$$

Backup

Extraction of Sivers function at scale $\mu = 1.55 \text{ GeV}$ (Anselmino et al., [Eur. Phys. J. A 39, 89-100 \(2009\)](#))

$$f_{1T}^{\perp(1),q}(x) = -\frac{1}{2} \mathcal{N}^q(x) f_1^q(x) \sqrt{2} e \frac{M_1^3 \langle k_T^2 \rangle}{M_p (M_1^2 + \langle k_T^2 \rangle)^2}$$

$$\mathcal{N}^q(x) = N^q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{\alpha_q + \beta_q}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

$$M_p = 0.93827 \text{ GeV}; \langle k_T^2 \rangle = 0.25 \text{ GeV}; M_1 = 0.583 \text{ GeV}; \beta = \beta_q = 3.46; \alpha_u = 0.73; \alpha_d = 1.08; \alpha_{sea} = 0.79;$$
$$N^u = 0.35; N^d = -0.9; N^s = -0.24; N^{\bar{u}} = 0.04; N^{\bar{d}} = -0.4; N^{\bar{s}} = 1$$

For the plots:

NLO unpolarized parton distributions MSTW2008 A. D. Martin, W. J. Stirling, R. S. Thorne, G. Watt, [Eur. Phys. J. C 63, 189 \(2009\)](#)

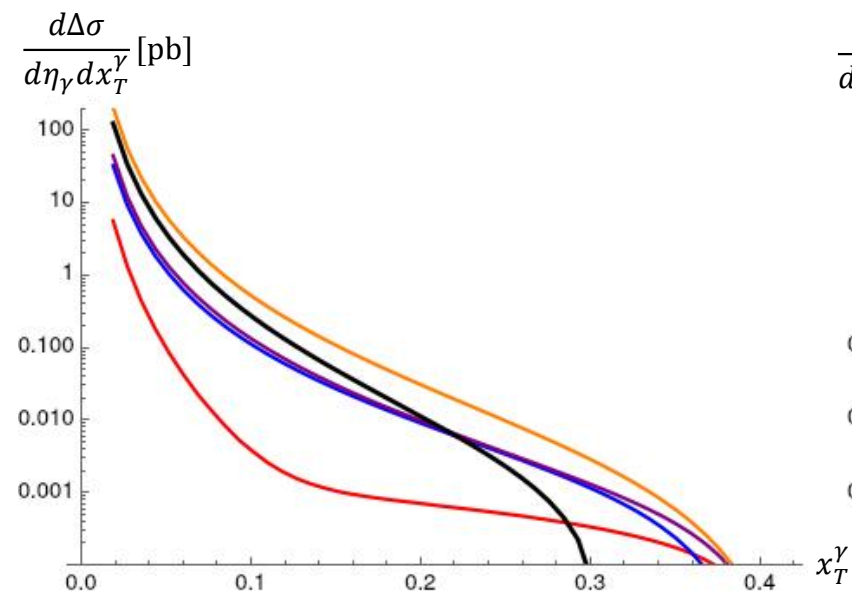
Parton-to-photon fragmentation function GRV NLO M. Gluck, E. Reya, A. Vogt, [Phys. Rev. D 48, 116 \(1993\)](#) [Erratum: [Phys.Rev.D 51, 1427 \(1995\)](#)]

$$\epsilon = \delta = 0.01; a_{2-7}^u = \left(-\frac{1}{2}, 0, 1, 0, \frac{1}{3}, 0 \right); a_{2-7}^d = a_{2-7}^s = 0; b_{1-6}^u = \left(0, -\frac{1}{2}, 0, 1, 0, \frac{1}{3} \right); b_{1-6}^d = b_{1-6}^s = 0$$

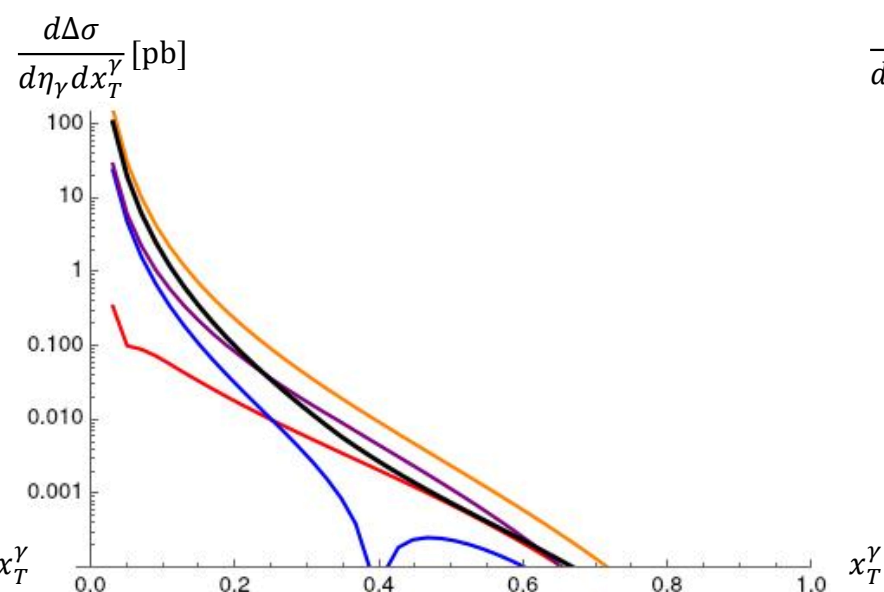
Backup

$$x_T^\gamma = \frac{2p_T^\gamma}{\sqrt{s}}$$

$\sqrt{s} = 100 \text{ GeV}, \eta_\gamma = -1.5, \mu = p_T^\gamma$



$\sqrt{s} = 100 \text{ GeV}, \eta_\gamma = 0, \mu = p_T^\gamma$



$\sqrt{s} = 100 \text{ GeV}, \eta_\gamma = -1.5, \mu = p_T^\gamma$

