Dynamical cold nuclear effects on transverse momentum dependent Drell-Yan production in pA

QCD Evolution 2024, University of Pavia, Italy, May 30

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Based on WK, Vitev, PLB854(2024)138751
WK, Terry, Vitev 240X.XXXXX

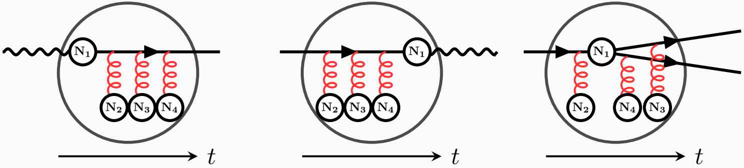
Intrinsic non-perturbative v.s. dynamical nuclear effects

Examples of intrinsic NP structure:

- Parton structure of a nucleus is different from that of a free nucleon.
- Nuclear structure effects and nucleon motion & correlations.

Examples of dynamical nuclear effects

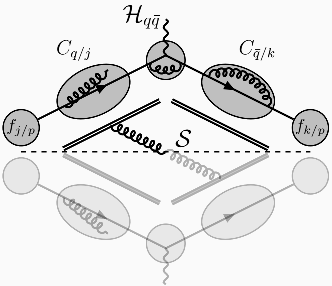
- Hadron level: hadronization & hadronic scatterings in the medium.
- Parton level: parton rescatterings with medium constituents



SIDIS **Drell Yan** **Hadron production**

TMD factorization for Drell-Yan

- In the TMD region $\Lambda_{\mathrm{QCD}}^2 < p_T^2 \ll Q^2$. Cross section is factorized into a hard function ($\mathcal{H}$), beam functions ($\mathcal{B}_{q/p}, \mathcal{B}_{\bar{q}/p}$) and a soft function ($\mathcal{S}$) The TMD Handbook arXiv:2304.03302.

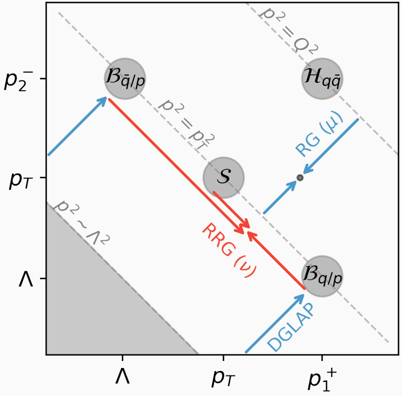


$$\frac{d\sigma}{dYdM^2d^2\mathbf{p}} = \sum_q \mathcal{H}_{q\bar{q}}(Q,\mu) \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{p}} \mathcal{B}_{q/p}\left(x_1, b; \mu, \frac{\zeta_1}{\nu^2}\right)$$
$$\mathcal{B}_{\bar{q}/p}\left(x_2, b; \mu, \frac{\zeta_2}{\nu^2}\right) \mathcal{S}(b,\mu,\nu) + [q \leftrightarrow \bar{q}] + \mathcal{O}\left(\frac{p_T^2}{Q^2}\right)$$

- At small $b \sim 1/p_T$, $\mathcal{B}_{q/p}$ is matched to collinear PDF

$$\mathcal{B}_{q/p}(x_1, b) = \int_{x_1}^1 \frac{dz}{z} C_{qj}\left(x, b; \mu, \frac{\zeta}{\nu^2}\right) f_{j/p}\left(\frac{x}{z}, \mu\right) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2 b^2\right)$$

Scale and rapidity evolution in TMD factorization



With the power counting parameter $\lambda \sim \frac{p_T}{Q}$, each sector has a unique momentum scaling

- Hard: $p \sim (1, 1, 1)Q$.
- Collinear $p \sim (1, \lambda^2, \lambda)Q$, $(\lambda^2, 1, \lambda)Q$.
- Soft $p \sim (\lambda, \lambda, \lambda)Q$.

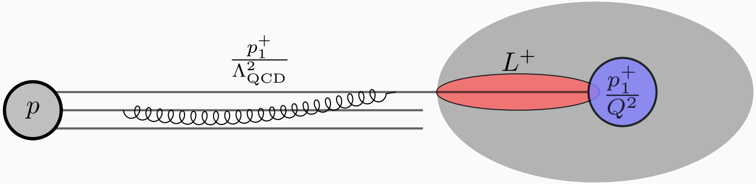
The renormalization scale (μ) and rapidity scale (ν) dependencies are described by RG and RRG equations

$$\frac{d\ln\mathcal{H}}{d\ln\mu} = \gamma_\mu^H(Q,\mu), \quad \frac{d\ln\mathcal{B}}{d\ln\mu} = \gamma_\mu^B\left(\mu, \frac{\zeta}{\nu^2}\right), \quad \frac{d\ln\mathcal{S}}{d\ln\mu} = \gamma_\mu^S\left(\mu, \frac{\mu}{\nu}\right)$$

$$\frac{d\ln\mathcal{B}}{d\ln\nu} = \gamma_\nu^B(b,\mu), \quad \frac{d\ln\mathcal{S}}{d\ln\nu} = \gamma_\nu^S(b,\mu)$$

The space-time picture and scale separation in pA

Physics in medium is complicated, let's consider the simplest scenario:

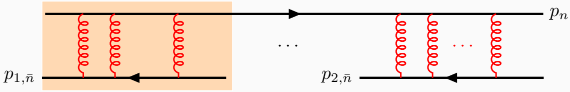


- NP time scale $\frac{p_1^+}{\Lambda_{\rm QCD}^2} \gg$ medium size L^+: to neglect hadronic scatterings effects in pA.
- $L^+ \gg$ time scale of hard process $\frac{p_1^+}{Q^2}$: separate medium effects from physics inside nucleon.

⋆ They translate into a hierarchy of energy scales $Q^2 \gg p_1^+/L^+ \gg \Lambda_{\rm QCD}^2$.

- The dynamical nuclear effects modify the proton-collinear beam function! The semi-hard scale p_1^+/L^+ is the foundation of a partly perturbative treatment.

Medium corrections analyzed in opacity expansion (for dilute medium)

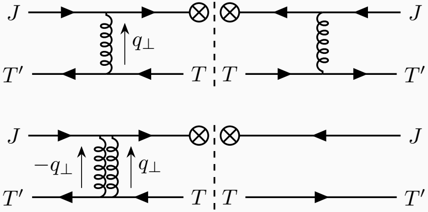


Opacity χ: number of independently interacting medium scattering centers.

Foward scattering between collinear parton & anti-collinear medium are mediated by Glauber gluons

$$q \sim (\lambda^a, \lambda^b, \lambda)Q, \;\; a + b > 2$$

For example: LO at first order in χ



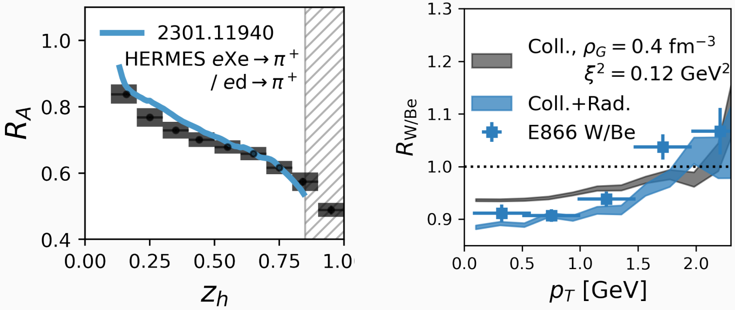
LO, all-opacity beam function $\mathcal{B}_{q/p}$ after propagation in medium Gyulassy, Levai, Vitev PRD66(2002)014005

$$\begin{aligned}
\mathcal{B}^{(0)}_{q/p} &= \mathcal{B}^{(0)}_{q/p,0} + \chi \mathcal{B}^{(0)}_{q/p,1} + \cdots \\
&= \mathcal{B}^{(0)}_{q/p}(x,b)\left[1 + \rho_N^- f_T L^+ \left(\Sigma^{(0)}_{FT}(b) - \Sigma^{(0)}_{FT}(0)\right) + \cdots\right] \\
&= \mathcal{B}^{(0)}_{q/p}(x,b) \exp\left\{\rho_N^- f_T L^+ \left(\Sigma^{(0)}_{FT}(b) - \Sigma^{(0)}_{FT}(0)\right)\right\}
\end{aligned}$$

with a screened LO forward cross section

$$\Sigma^{(0)}_{RT}(b) = \frac{g_s^2 C_R g_s^2 C_T}{d_A} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{e^{-i\mathbf{b}\cdot\mathbf{q}}}{(\mathbf{q}^2 + \xi^2)^2}$$

Looking for consistency of cold nuclear matter parameter in different processes



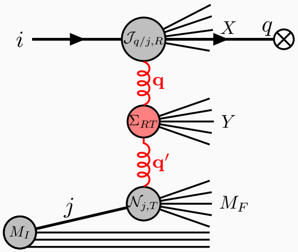
- CNM parameters ξ^2 and $\rho_G = \sum_T \rho_N f_T g_s^2 C_T / d_A$ determined from collinear SIDIS in *eA*: $\rho_G \approx 0.4$ fm^{-3}, $\xi^2 \approx 0.12$ GeV^2 WK, Vitev PLB854(2024)138751.
- The LO, all opacity calculation of TMD Drell-Yan with such parameters (gray band) is clearly inadequate to explain the observed p_T broadening.

Consider radiative correction to forward scattering in a medium

- Forward scattering formulated in SCET with Glauber operators Fleming PLB735(2014)266, Rothstein, Stewart JHEP08(2016)025. Applied to study jet function in a thermal plasma + open quantum system Vaidya JHEP11(2021)064, JHEP05(2024)028
- Jet broadening in quark-gluon plasma, anomalous diffusion from radiative corrections Wu, Liou, Mueller, Blaizot, Mehtar-Tani, 2011-2014. Caucal, Mehtar-Tani, PRD106(2022)L051501, JHEP09(2022)023, PRD108(2023)014008.
- NLO correction to nuclear higher-twist in SIDIS Kang, Wang, Wang, Xing, PRL112(2014)102001.

This work: follow SCET developments to handle multi-scale problem of TMD Drell-Yan in pA.

Factorize the forward scattering cross-section



At opacity-order one, we assume at small b, the problem can be reduced to the computation of a partonic forward scattering

$$\mathcal{B}_{q/p,1}(x,\mathbf{b}) = \sum_{i,j} \sigma_{ij\to q} \otimes f_{i/p} \otimes f_{j/N} \cdot \rho_N^- L^+$$

Partonic cross-section is further factorized into a collinear function $\mathcal{J}_R$, anti-collinear $\mathcal{N}_T$, and two-body cross section Σ_{RT}

$$\sigma_{ij\to q}(x,\mathbf{b}) \equiv \mathcal{J}_{q/i,R} \otimes \Sigma_{RT} \otimes \mathcal{N}_{j,T}\,.$$
$$= \sum_{T,R} \int \frac{d^2\mathbf{p}}{(2\pi)^2} \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{d^2\mathbf{q}'}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{p}} \mathcal{J}_{q/i,R}(x,\mathbf{p},\mathbf{q}) \Sigma_{RT}(\mathbf{q},\mathbf{q}') \mathcal{N}_{j,T}(\mathbf{q}')$$

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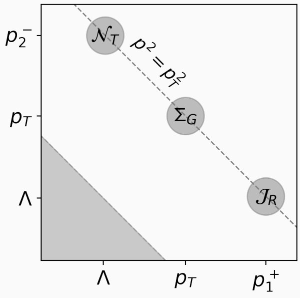

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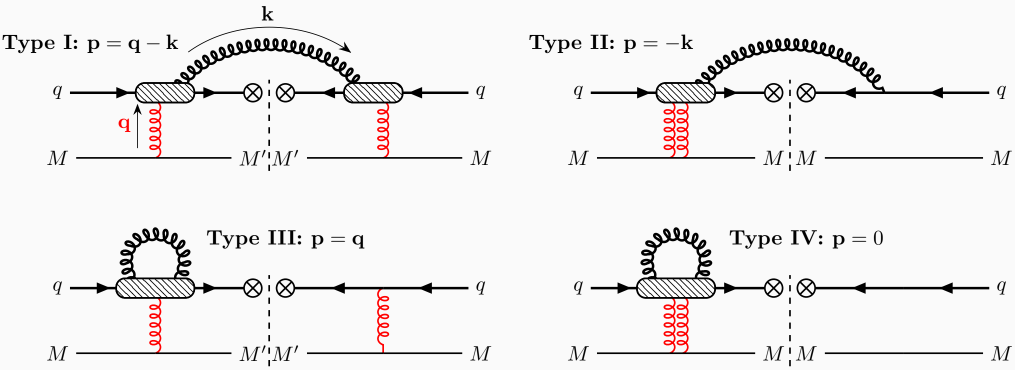
NLO collinear function $\mathcal{J}^{(1)}_{q/q,R}$ from SCET$_\mathrm{G}$

To obtain the collinear function at NLO, it is sufficient to the treat the Glauber gluon as coming from a background field (SCET$_\mathrm{G}$). At NLO, there are four types of diagrams with different transverse momentum recoils



The final result for $\mathcal{J}^{(1)}_{q/q,R}$

$$\mathcal{J}^{(1)}_{q/q,R}(x,\mathbf{p},\mathbf{q}) = \frac{g_s^2 C_F}{2\pi} P_{qq}(x) \int d^2\mathbf{k} \left[\delta^{(2)}(\mathbf{p}-\mathbf{q}+\mathbf{k})\mathcal{I}_R^{\mathrm{II}}(x,\mathbf{p},\mathbf{q}) + \delta^{(2)}(\mathbf{p}+\mathbf{k})\mathcal{I}_R^{\mathrm{II}}(x,\mathbf{p},\mathbf{q})\right]$$

$$+\frac{g_s^2 C_F}{2\pi}\delta(1-x)\int_0^1 dx' P_{qq}(x') \int d^2\mathbf{k} \left[\delta^{(2)}(\mathbf{p}-\mathbf{q})\mathcal{I}_R^{\mathrm{III}}(x',\mathbf{k},\mathbf{q}) + \delta^{(2)}(\mathbf{p})\mathcal{I}_R^{\mathrm{IV}}(x',\mathbf{k},\mathbf{q})\right].$$

Type K	$\mathcal{I}_F^K(x,\mathbf{k},\mathbf{q})$	$\mathcal{I}_A^K(x,\mathbf{k},\mathbf{q})$
I	$\frac{1}{\mathbf{Q}_1^2} + 2\frac{\mathbf{Q}_2}{\mathbf{Q}_2^2}\cdot\left(\frac{\mathbf{Q}_2}{\mathbf{Q}_2^2}-\frac{\mathbf{Q}_1}{\mathbf{Q}_1^2}\right)\Phi_2$	$\frac{1}{\mathbf{Q}_3^2} - \frac{\mathbf{Q}_1}{\mathbf{Q}_1^2}\cdot\frac{\mathbf{Q}_3}{\mathbf{Q}_3^2} + \frac{\mathbf{Q}_2}{\mathbf{Q}_2^2}\cdot\left(\frac{\mathbf{Q}_1}{\mathbf{Q}_1^2}-\frac{\mathbf{Q}_3}{\mathbf{Q}_3^2}\right)\Phi_2$
II	$-\frac{1}{\mathbf{Q}_1^2}$	$\frac{\mathbf{Q}_1}{\mathbf{Q}_1^2}\cdot\left(\frac{\mathbf{Q}_1}{\mathbf{Q}_1^2}-\frac{\mathbf{Q}_3}{\mathbf{Q}_3^2}\right)(\Phi_1-1)$
III	$-2\frac{\mathbf{Q}_2}{\mathbf{Q}_2^2}\cdot\left(\frac{\mathbf{Q}_2}{\mathbf{Q}_2^2}-\frac{\mathbf{Q}_1}{\mathbf{Q}_1^2}\right)\Phi_2$	$-\frac{\mathbf{Q}_1\cdot\mathbf{Q}_2}{\mathbf{Q}_1^2\mathbf{Q}_2^2}\Phi_2 + \frac{\mathbf{Q}_2}{\mathbf{Q}_2^2}\cdot\frac{\mathbf{Q}_4}{\mathbf{Q}_4^2}\Phi_4$
IV	0	$-\frac{1}{\mathbf{Q}_1^2}\Phi_1 + \frac{\mathbf{Q}_1\cdot\mathbf{Q}_5}{\mathbf{Q}_1^2\mathbf{Q}_5^2}\Phi_5$

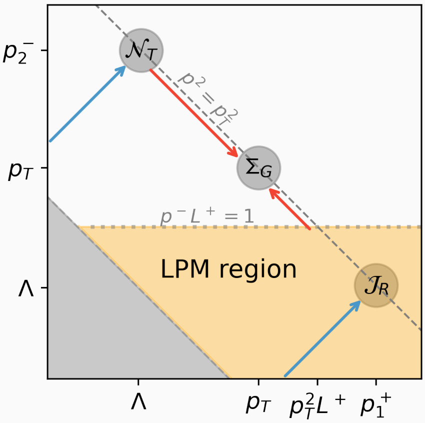
$$\mathbf{Q}_1 = x\mathbf{k}-(1-x)(\mathbf{p}_0-\mathbf{k}),\quad \mathbf{Q}_2 = x\mathbf{k}-(1-x)(\mathbf{p}_0-\mathbf{k}+\mathbf{q}),\quad \mathbf{Q}_3 = x(\mathbf{k}-\mathbf{q})-(1-x)(\mathbf{p}_0-\mathbf{k}+\mathbf{q}),$$

$$\mathbf{Q}_4 = x(\mathbf{k}+\mathbf{q})-(1-x)(\mathbf{p}_0-\mathbf{k}),\quad \mathbf{Q}_5 = x(\mathbf{k}-\mathbf{q})-(1-x)(\mathbf{p}_0-\mathbf{k}+\mathbf{q}),\quad \Phi_n = 1-\mathrm{sinc}\left(\frac{\mathbf{Q}_n^2}{2x(1-x)p^+/L^+}\right)$$

The LPM region and the Gunion-Bertsch region

Scale separation scenario I:

$\Lambda^2_{\rm QCD} \sim \xi^2 \lesssim \langle \Delta p_T^2 \rangle \quad p_T^2 \quad p_1^+/L^+ \quad Q^2$



Meaning of the phase factor Φ_n: whether a quantum fluctuation is long-lived to be aware of the hard vertex

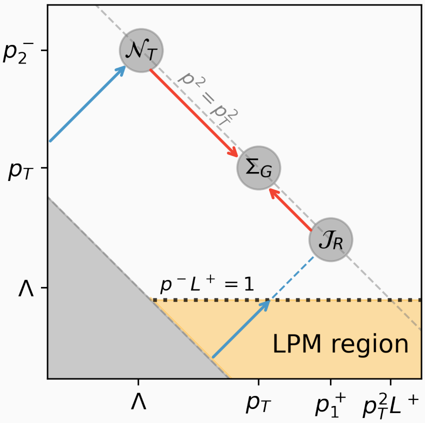
$$\Phi_n = 1 - \mathrm{sinc}\left(\frac{\mathbf{Q}_n^2}{2x(1-x)p^+/L^+}\right)$$
$$= 1 - \mathrm{sinc}\left(L^+/\tau_f^+\right), \quad \tau_f^+ = 1/p^- .$$

- $\Phi_n \to 0$ for $\tau_f^+ \gg L^+$: the Landau-Pomeranchuk -Migdal (LPM) destructive interference.
- $\Phi_n \sim 1$ for $\tau_f^+ \ll L^+$: the Gunion-Bertsch region.
- Transition happens around $p^- L^+ = 1$.
- Two possible scenarios for scale separation: $p_T^2 \ll p^+/L^+$ or $p_T^2 \gg p^+/L^+$

The LPM region and the Gunion-Bertsch region

Scale separation scenario II:

$\Lambda^2_{\rm QCD} \sim \xi^2 \lesssim \langle \Delta p_T^2 \rangle \quad p_1^+/L^+ \quad p_T^2 \quad Q^2$



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Medium-induced collinear divergences

- With dimensional regularization $d = 4 - 2\epsilon$, we can identify an collinear divergences as $1/\epsilon$

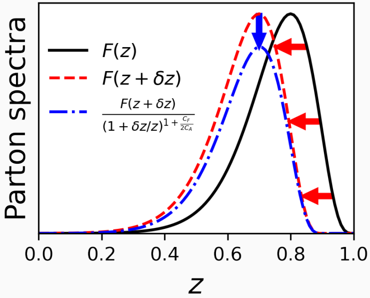
$$
\begin{aligned}
&\rho_N^- L^+ f_T \otimes f_{q/p} \otimes \mathcal{J}^{(1)}_{q/q,R} \otimes \Sigma^{(0)}_{RT} \otimes \mathcal{N}^{(0)}_T \\
&\supset \frac{\rho_G L \alpha_s^2(\mu^2)}{8p_1^+/L^+} \cdot \left[\frac{\mu^2}{2p^+/L^+}\right]^{2\epsilon} B_\epsilon \left(\frac{\mu_b^2}{2p^+/L^+}\right) \int_0^1 \frac{dx'}{x'} \frac{P^{(0)}_{qq}(x')}{[x'(1-x')]^{1+2\epsilon}} f_{q/p}\left(\frac{x}{x'}\right) \times [\text{Color factors}] \\
&= \frac{\alpha_s^2(\mu^2) B_0(\mu_b^2 L/2p^+) \rho_G L}{8p^+/L^+} \left(\frac{1}{2\epsilon} + \ln \frac{\mu^2}{\min\{\mu_b^2, 2p^+/L^+\}}\right) 2C_F \left[2C_A\left(-\frac{d}{dz} + \frac{1}{z}\right) + \frac{C_F}{z}\right] z f_q(z)
\end{aligned}
$$

- The LPM effect modifies the endpoint behavior $\Rightarrow$ an IR pole. Can be canceled by the UV divergences in the collinear-soft sector at $p^2 \sim \xi^2, p^- L^+ = 1$ (work in progress WK, Vitev).

- Because the realistic collinear-soft scale is NP, We introduce an in-medium counter term as a simple model to replace its effect $\left(1 + \frac{1}{\epsilon} M^{(1)}_{qq}\right) \otimes B^{(1)}_{q/q}$.

In-medium renormalization and RG evolution

The associated in-medium RGE is set of partial-differential eqs for $F_i(\tau, x) = xf_{i/p}(x, \mu^2)$ Ke, Vitev 2301.11940 with a redfined evolution variable $\tau(\mu^2) = \frac{4\pi}{\beta_0}\frac{B\rho_G L}{8p_1^+/L^+}\left[\alpha_s(\mu^2) - \alpha_s\left(\frac{\chi p_1^+}{L^+}\right)\right]$,



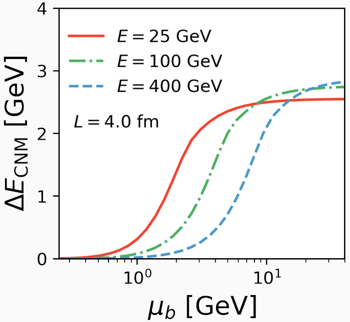
$$\frac{\partial F_{q-\bar{q}}}{\partial \tau} = \left(4C_F C_A \frac{\partial}{\partial x} - \frac{4C_F C_A}{x} - \frac{2C_F^2}{x}\right) F_{q-\bar{q}}$$

$$\frac{\partial F_{q+\bar{q}}}{\partial \tau} = \left(4C_F C_A \frac{\partial}{\partial x} - \frac{4C_F C_A}{x} - \frac{2C_F^2}{x}\right) F_{q+\bar{q}} + \frac{2C_F T_F}{x} F_g,$$

$$\frac{\partial F_g}{\partial \tau} = \left(4C_A^2 \frac{\partial}{\partial x} - \frac{2N_f C_F}{x}\right) F_g + \sum_q \frac{2C_F^2}{x} F_{q+\bar{q}}.$$

- Encodes parton energy loss in medium and conversion between collinear quarks and gluons.
- The leading-log ($\ln \frac{p^+/L^+}{\xi^2}$) behavior is the same as medium-modified DGLAP equations.

Correlation between parton energy loss and the transverse momentum



- The leading-L^2 contribution to energy loss come from the LPM region. The evolution equation resums radiations from $\mu^2 \sim \xi^2$ to $\mu^2 \sim \min\left(\mu_b^2, p^+/L^+\right)$.

$$\Delta E_{CNM} = \frac{C_F C_A}{2} B\left(\frac{\mu_b^2 L^+}{2p^+}\right) \rho_G L^2 \frac{4\pi}{\beta_0}\left[\alpha_s(\xi^2) - \alpha_s(\min\{\mu_b^2, \frac{p^+}{L^+}\})\right]$$

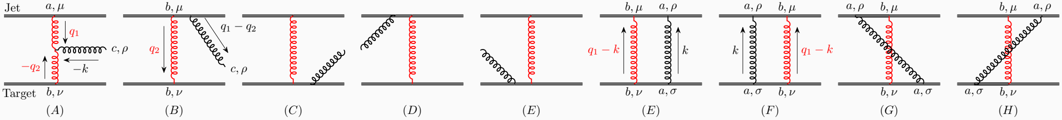
- Partons with small $\mu_b \sim p_T$ tend to lose less energy! **A survival bias.**

The rapidity divergence at opacity one

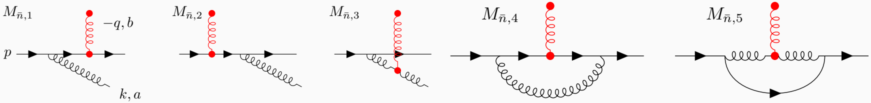
- Rapidity divergence cancels among the NLO correction of $\mathcal{J}_{q/q,R}$, $\mathcal{N}_T$ and Σ_{RT} and lead to the BFKL evolution on the rapidity scale

  [Fleming PLB735(2014)266; Rothstein, Stewart, JHEP08(2016)025; Vaidya 2107.00029, 2109.11568]

- Subset of NLO diagrams of Σ_{RT} that contains rapidity divergence.

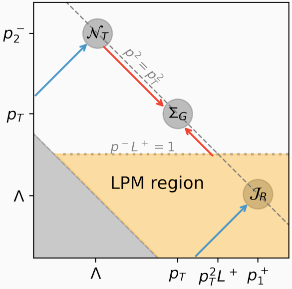


- Non-zero diagrams for NLO $\mathcal{N}_T$



- We checked explicitly that the rapidity divergences cancel in $\mathcal{J}_{q/q,R} \otimes \Sigma_{RT} \otimes \mathcal{N}_T$ at NLO.

The RRG equation: BFKL



$$\frac{\partial V(\mathbf{b},\nu)}{\partial \ln \nu} = \frac{\alpha_s C_A}{\pi^2}\left\{\int_{|\mathbf{b}-\mathbf{b}'|<|\mathbf{b}|} d^2\mathbf{b}' \frac{V(\mathbf{b}') - V(\mathbf{b})}{|\mathbf{b}-\mathbf{b}'|^2} + \int_{|\mathbf{b}-\mathbf{b}'|>|\mathbf{b}|} d^2\mathbf{b}' \frac{V(\mathbf{b}')}{|\mathbf{b}-\mathbf{b}'|^2}\right\},$$

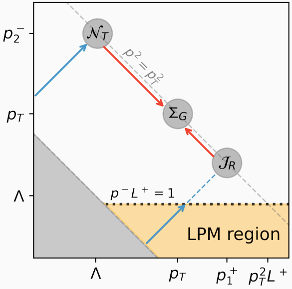
with initial condition $V(\mathbf{b},\nu_0) = g_s^2 \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{e^{-i\mathbf{b}\cdot\mathbf{q}}}{\mathbf{q}^2+\xi^2}$. The evolved Glauber cross-section

$$\Sigma_{RT}\left(\mathbf{b}, \ln\frac{\nu}{\nu_0}\right) = \frac{C_R C_T}{d_A} \int d^2\mathbf{b}' V(\mathbf{b}+\mathbf{b}', \nu_0) V(\mathbf{b}', \nu).$$

⋆ Below the line $p^- L^+ = 1$, the rapidity-log is destroyed by the LPM effect. Therefore, depending on the two scenarios of scale separation, the final rapidity log enhancement is

$$\mathcal{L} = \ln\frac{\sqrt{\zeta_1}}{\nu^2} + \ln\frac{\sqrt{\zeta_2}}{\nu^2} - \ln\frac{\nu^2}{\mu_b^2} = \min\left\{\ln\frac{p_1^+ p_2^-}{\mu_b^2},\quad \ln\frac{\mu_b^2 L^+ \cdot p_2^-}{\mu_b^2} \sim \ln r_0 m_N A^{1/3}\right\}$$

The RRG equation: BFKL



$$\frac{\partial V(\mathbf{b},\nu)}{\partial \ln \nu} = \frac{\alpha_s C_A}{\pi^2} \left\{ \int_{|\mathbf{b}-\mathbf{b}'|<|\mathbf{b}|} d^2\mathbf{b}' \frac{V(\mathbf{b}') - V(\mathbf{b})}{|\mathbf{b}-\mathbf{b}'|^2} + \int_{|\mathbf{b}-\mathbf{b}'|>|\mathbf{b}|} d^2\mathbf{b}' \frac{V(\mathbf{b}')}{|\mathbf{b}-\mathbf{b}'|^2} \right\},$$

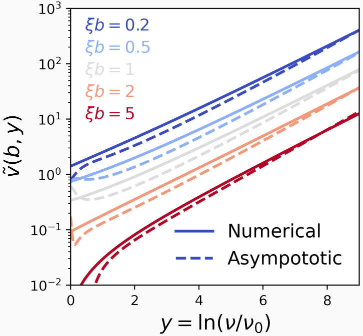
with initial condition $V(\mathbf{b},\nu_0) = g_s^2 \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{e^{-i\mathbf{b}\cdot\mathbf{q}}}{\mathbf{q}^2+\xi^2}$. The evolved Glauber cross-section

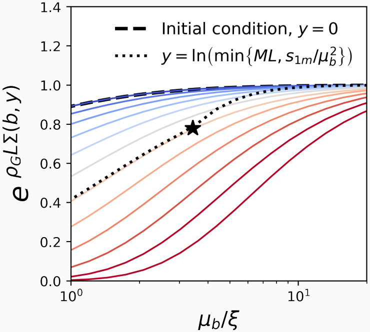
$$\Sigma_{RT}\left(\mathbf{b}, \ln\frac{\nu}{\nu_0}\right) = \frac{C_R C_T}{d_A} \int d^2\mathbf{b}' V(\mathbf{b}+\mathbf{b}',\nu_0) V(\mathbf{b}',\nu).$$

$\star$ Below the line $p^- L^+ = 1$, the rapidity-log is destroyed by the LPM effect. Therefore, depending on the two scenarios of scale separation, the final rapidity log enhancement is

$$\mathcal{L} = \ln\frac{\sqrt{\zeta_1}}{\nu^2} + \ln\frac{\sqrt{\zeta_2}}{\nu^2} - \ln\frac{\nu^2}{\mu_b^2} = \min\left\{ \ln\frac{p_1^+ p_2^-}{\mu_b^2},\quad \ln\frac{\mu_b^2 L^+ \cdot p_2^-}{\mu_b^2} \sim \ln r_0 m_N A^{1/3} \right\}$$

Numerical solution to the BFKL and the momentum broadening factor



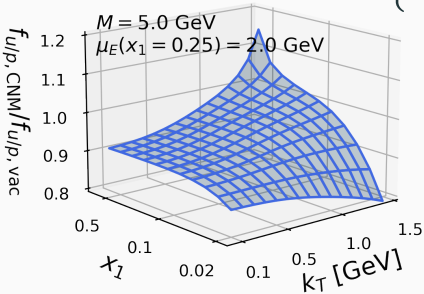


- Because $\ln r_0 m_N A^{1/3}$, we solve the equation numerically. It forgets the initial condition only at very large ν and approaches to the double-log asymptotic solution Kovchegov, Levin.
- Because it only renormalizes the Glauber interaction with a single scattering center, we **assume** that the momentum broadening is still the exponentiation of opacity-one result.

The final formula for in-medium beam function in a dilute medium upto NLO

$$\mathcal{B}^{\text{vac+med}}_{q/p}\left(x, b, \mu, \frac{\zeta_1}{\nu}\right) = \int_x^1 \frac{dx'}{x'} \left\{\mathcal{B}^{\text{vac}}_{q/j}\left(\frac{x}{x'}, b; \mu, \frac{\zeta_1}{\nu^2}\right) + \Delta B^{(1)}_{q/j,1}\right\} f_{j/p}\left(x', \mu_b, \min\left\{\mu_b, \sqrt{\frac{p^+}{L^+}}\right\}\right)$$

$$\times \exp\left\{\sum_T \rho_N^- L^+ \left[\Sigma_{FT}(b, \mathcal{L}) - \Sigma_{FT}(0, \mathcal{L})\right]\right\}$$

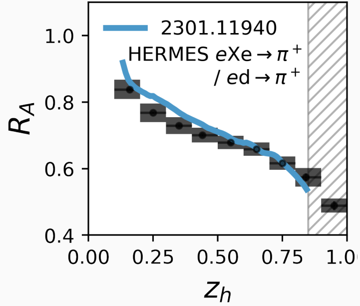


- $f_{j/p}(x, \mu_b, \mu_2)$ are evolved by in-medium RGE from ξ^2 to μ_2. $\Rightarrow$ parton energy losss and in-medium conversion.
- Σ_{FT} are evolved in BFKL, $\mathcal{L} = \min\left\{\ln \frac{p_1^+ p_2^-}{\mu_b^2}, \ln A^{1/3}\right\}$ $\Rightarrow$ radiative recoils summed by RRG of Glauber.

◁ Momentum broadening dominates large xp^+. At small xp^+, energy loss dominate over momentum broadening.

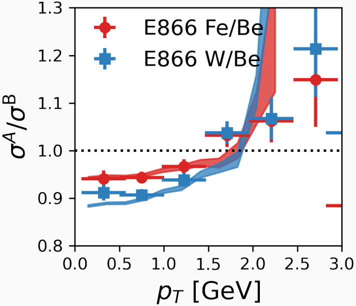
Towards a consistent set of CNM inputs in different processes

SIDIS in *eA*



HERMES, NPB 780(2007)1-27

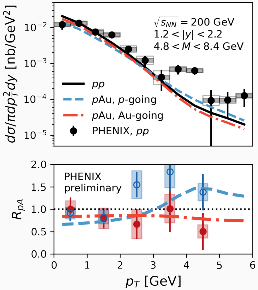
TMD Drell-Yan in *pA*



FNAL E866/NuSea Collab

PRL83(1999)2304-2307

TMD Drell-Yan in *pA*, collider



PHENIX *pp*: PRD99(2019)072003.

Prelim *p*Au: Leung PoS(HP2018)160.

The vacuum TMD calculation: NLO+LL w/o Y-term.

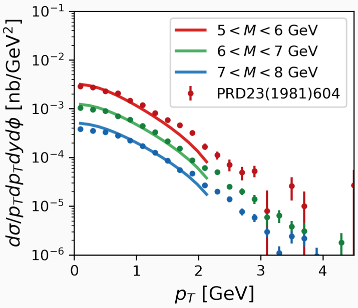
The medium effect: same set of cold nuclear matter parameters for Drell-Yan and SIDIS. **Only collinear nuclear PDFs are used!**

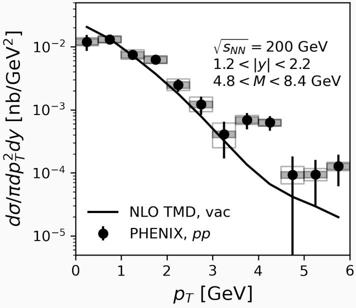
Summary

- Understanding dynamical nuclear effect is important for interpreting nuclear data.
- An EFT analysis is possible when $Q^2 \gg p_T^2 \gg p^+/L^+ \gg \langle \Delta p_T^2 \rangle \gtrsim \xi^2 \sim \Lambda_{\rm QCD}^2$,
  $Q^2 \gg p^+/L^+ \gg p_T^2 \gg \langle \Delta p_T^2 \rangle \gtrsim \xi^2 \sim \Lambda_{\rm QCD}^2$
- Apply SCET to opacity one, the NLO calculation of the proton beam function contains both collinear and rapidity divergences. Their renomalization lead to
  - The emergence of parton energy loss and flavor conversion in cold nuclear matter.
  - A resummation of soft-gluon recoil contribution to the momentum broadening.
- A consistent set of CNM parameters give a reasonable description of the p_T differential Drell-Yan data in pA and collinear SIDIS data in eA.
- **Future:** Generalize to TMD hadron productions in eA and pA.
  Include dynamical corrections into current global fitting of nuclear TMDPDF/TMDFF for example, to the framework in Alrashed, Anderle, Kang, Terry, Xing PRL129(2022)242001

Questions?

The pp baseline





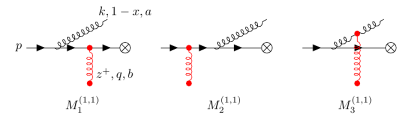
Fixed target PRD23(1981)604 Collider PHENIX PRD99(2019)072003

Reasonable agreement at low p_T (not including the Y terms yet).

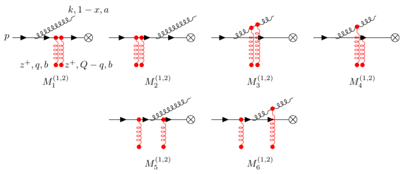
CT18nlo proton PDF PRD103(2021)014013; EPPS21 nPDF EPJC82(2022)5, 413; NP inputs for TMD Sun, Isaacson, Yuan, Yuan IJMPA33(2018)11,1841006, Echevarria, Kang, Terry JHEP01(2021)126.

Amplitudes (in light-cone gauge) to compute opacity-one collinear function

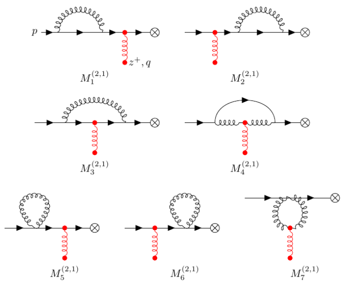
Radiative & collisional recoil



Radiative recoil



Collisional recoil



No recoil