Generalized parton distributions from lattice QCD and experimental data

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CHARTERED 1693

Overview

1. **Challenges of GPD phenomenology from experimental data**
2. The Hadstruc GPD calculation on the lattice: **arXiv:2405.10304**
3. Perspectives

Generalized parton distributions (GPDs): [Müller et al, 1994], [Radyushkin, 1996], [Ji, 1997]

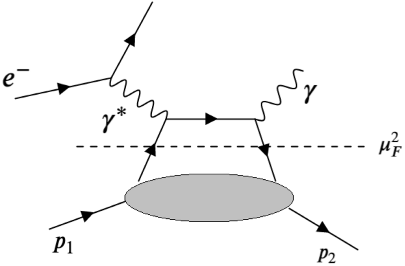
$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \left\langle P_2 \middle| \bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+\psi^q\left(\frac{z}{2}\right) \middle| P_1 \right\rangle \Big|_{z_\perp=0,\ z^+=0}$$

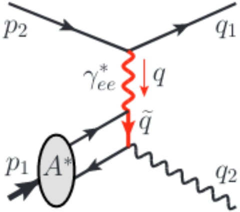
$$= \frac{1}{2P^+}\bar{u}(P_2)\left(H^q(x,\xi,t)\gamma^+ + E^q(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}\right)u(P_1)$$

$$\Delta = P_2 - P_1\,,\quad t = \Delta^2\,,\quad P = \frac{1}{2}(P_1 + P_2)\,,\quad \xi = \frac{P_1^+ - P_2^+}{P_1^+ + P_2^+} = -\frac{\Delta^+}{2P^+}$$

- 3D (x, ξ, t) vs 1D for PDFs
- more GPDs than PDFs
- $\rightarrow$ need to measure more (exclusive vs inclusive)
- $\rightarrow$ most current GPD extractions are not data-driven

The problem with DVCS





[Qiu, Yu, 2022]

$$\tilde{q}^2 = \frac{Q^2 + q_2^2}{2\xi}\left[x - \xi\left(\frac{1 - q_2^2/Q^2}{1 + q_2^2/Q^2}\right)\right] + \mathcal{O}(t/Q^2)$$

real photon $q_2^2 = 0$: x and ξ not entangled with the virtuality Q^2.
Entanglement of x and Q^2 only through perturbative radiation: “missing” variable

$$\xi, t, [Q^2] \qquad \text{vs} \qquad x, \xi, t, [\mu^2]$$

DVCS observables parametrized in terms of Compton form factors (CFFs) [Radyushkin, 1997], [Ji, Osborne, 1998], [Collins, Freund, 1999]

$$\mathcal{H}(\xi, t, Q^2) = \sum_a \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^a \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \frac{H^a(x, \xi, t, \mu^2)}{|x|^{p_a}}$$

Leading Log | Fixed order a_s^1 | Fixed order a_s^2

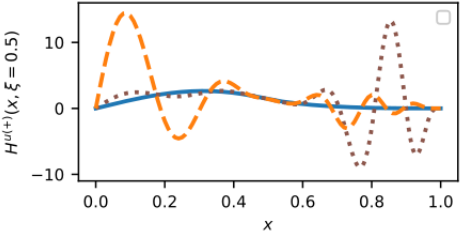
$$T_{LO} \otimes H(\mu^2)$$

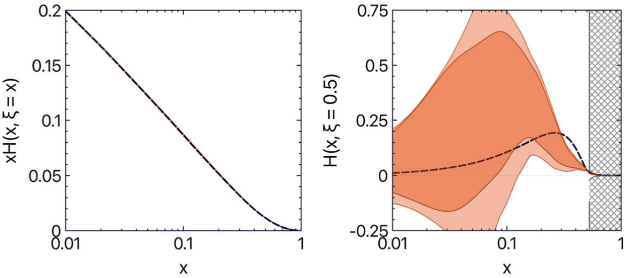
$$+ \alpha_s(\mu^2) T_{NLO} \otimes H(\mu^2) \quad + \ldots$$

$$-\alpha_s(\mu^2) \ln\left(\frac{Q^2}{\mu^2}\right) T_{LO} \otimes K_{LO} \otimes H(\mu^2) \quad + \ldots$$

$$+ \ldots$$

(caricatural) case of uncertainty propagation at NLO: needs to measure DVCS over a range from 1 to 100 GeV^2 with 10^{-5} relative accuracy to discriminate those GPDs. **[Bertone, HD, Mezrag, Moutarde, Sznajder, 2021]**





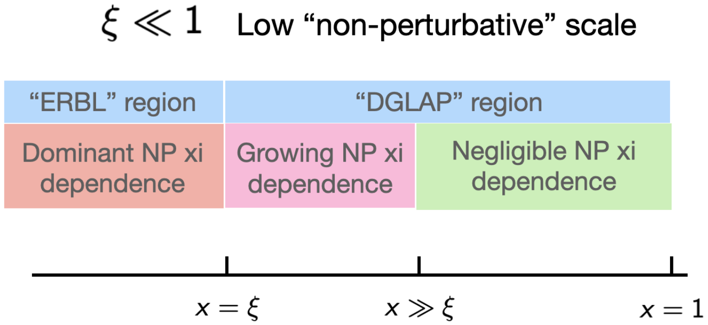
Positivity **[Pire, Soffer, Teryaev, 1998]** for $|x| > |\xi|$ with use of NN **[HD, Grocholski, Moutarde, Sznajder, 2021]**:

- NN model of DD (Lorentz sym)
- Fit DVCS observables produced by a model (no experimental uncertainty)
- Vary the architecture of the NN to probe the functional space

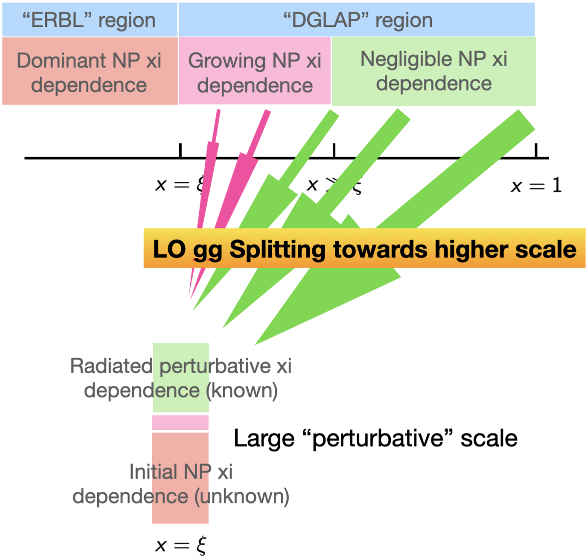
What about smaller x?

Exclusive processes at small x_B (e.g. J/ψ photoproduction at the LHC) $\rightarrow$ sensitivity to gluon GPDs at $x \approx \xi \ll 1$ **Reconstructing the ξ dependence to relate to gluon PDFs is exactly our deconvolution problem**

Proposed solution [Shuvaev et al, 1999] based on LO evolution: a parton with (x, μ_0^2) radiates a lot of children $(y \ll x, \mu_1^2 \gg \mu_0^2)$



$\xi \ll 1$ Low "non-perturbative" scale



How small ξ and how large μ^2?

First order improvement compared to just using the PDF in lieu of the GPD. Model the tricky $x \approx \xi$ region using the analytical structure generated by LO pQCD.

- Larger μ^2 always better
- Smaller ξ only better if the rate of growth of the PDF at small x is not too high (otherwise increased radiation does not keep up with increasing NP content)

Our model-dependent estimate of uncertainty on relating GPDs to PDFs within a LO framework: 10 to 20% for J/ψ production, a few percents for Υ. **[HD, Winn, Bertone, 2023]**
So much better than at large x and ξ, as noted also in **[Moffat et al, 2023]**

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GPD matrix element [Bhattacharya et al, 2022]:

$$\langle P_2|\bar{\psi}\left(-\frac{z}{2}\right)\gamma^\mu\psi\left(\frac{z}{2}\right)|P_1\rangle = \bar{u}(P_2)\Big[\gamma^\mu A_1 + z^\mu A_2 + \sigma^{\mu\nu}z_\nu A_3 + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}A_4 + \frac{\Delta^\mu}{2m}A_5 + \frac{i\sigma^{\alpha\beta}z_\alpha\Delta_\beta}{2m}\left(P^\mu A_6 + \Delta^\mu A_7 + z^\mu A_8\right)\Big]u(P_1)$$

Identify the terms that survive in the light-cone limit (non-singlet case):

$$H(\nu,\xi,t,z^2) = A_1(\nu,\xi,t,z^2)$$
$$E(\nu,\xi,t,z^2) = A_4(\nu,\xi,t,z^2) + \nu A_6(\nu,\xi,t,z^2) - 2\xi\nu A_7(\nu,\xi,t,z^2)$$

Match to the light-cone limit

$$\begin{pmatrix} H \\ E \end{pmatrix}(\nu,\xi,t,\mu^2) = \int_{-1}^{1} d\alpha C(\alpha,\xi\nu,\mu^2 z^2)\begin{pmatrix} H \\ E \end{pmatrix}(\alpha\nu,\xi,t,z^2) + \text{power corrections}$$

Extracting each amplitude A_k requires to measure matrix elements with various combinations of helicity and gamma structure (kinematic matrix inversion)

Ioffe-time GPDs are great GPDs! **[Braun, Gornicki, Mankiewicz, 1995]**

x-dependent GPDs

non analytical form

triangular for x > xi (only needs larger momentum fractions)

needs all x range for x < xi

$$\frac{d}{d\ln(\mu^2)} H^q(x,\xi,\mu^2) = \int_0^1 \frac{dy}{y} C\left(\frac{x}{y}, \frac{\xi}{x}, \alpha_s(\mu^2)\right) H^q\left(y,\xi,\mu^2\right)$$

$$C\left(\alpha, \frac{\xi}{x}, \alpha_s(\mu^2)\right) = \delta(1-\alpha) + \frac{\alpha_s(\mu^2)C_F}{2\pi}\left\{\theta(1-\alpha)\left[\left(\frac{1+\alpha^2}{1-\alpha}\right)_+ + \mathcal{O}(\xi^2)\right] + \theta(x\leq\xi)\left[-\left(\frac{1}{1-y}\right)_{++} + \ldots\right]\right\}$$

Ioffe-time GPDs

simple analytical form

triangular (only needs smaller Ioffe time range)

$$\frac{d}{d\ln(\mu^2)} H^q(\nu,\xi,\mu^2) = \int_0^1 d\alpha\, K(\alpha,\xi\nu,\alpha_s(\mu^2)) H^q(\alpha\nu,\xi,\mu^2)$$

$$K(\alpha,\xi\nu,\alpha_s(\mu^2)) = \delta(1-\alpha) + \frac{\alpha_s(\mu^2)C_F}{2\pi}\left[\left(\frac{2\alpha}{1-\alpha}\right)_+ \cos\left(\bar{\alpha}\xi\nu\right) + \frac{\sin\left(\bar{\alpha}\xi\nu\right)}{\xi\nu} - \frac{\delta\left(1-\alpha\right)}{2}\right]$$

Conformal moments

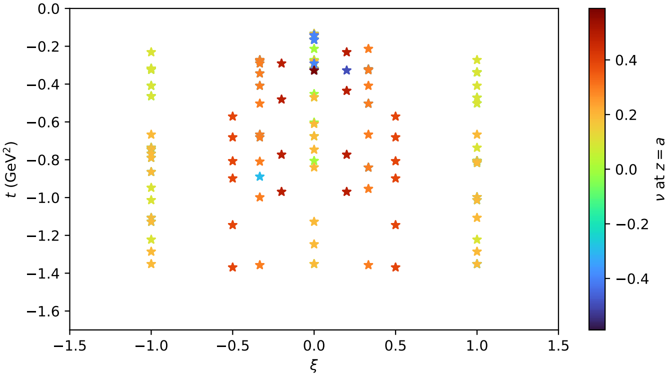
simple analytical form

diagonal (each moment evolves independently)

$$O_n(\xi,\mu^2) \propto \int_{-1}^1 dx\, C_n^{(3/2)}\left(\frac{x}{\xi}\right) H^q(x,\xi,\mu^2)$$

$$O_n(\xi,\mu^2) = O_n(\xi,\mu_0^2)\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_n/(2\pi\beta_0)}$$

ID	a (fm)	m_π (MeV)	β	$m_\pi L$	$L^3 \times N_T$	N_{cfg}	N_{srcs}	rk ($\mathcal{D}$)
a094m358	0.094(1)	358(3)	6.3	5.4	$32^3 \times 64$	348	4	64



polynomiality of moments of GPDs (non-singlet case):

$$\int_{-1}^{1} x\, x^{n-1} H(x,\xi,t) = \sum_{k=0 \text{ even}}^{n-1} A_{n,k}(t)\,\xi^k$$

$$A_{1,0}(t) = F_1(t) \quad \text{(elastic form factor)}$$

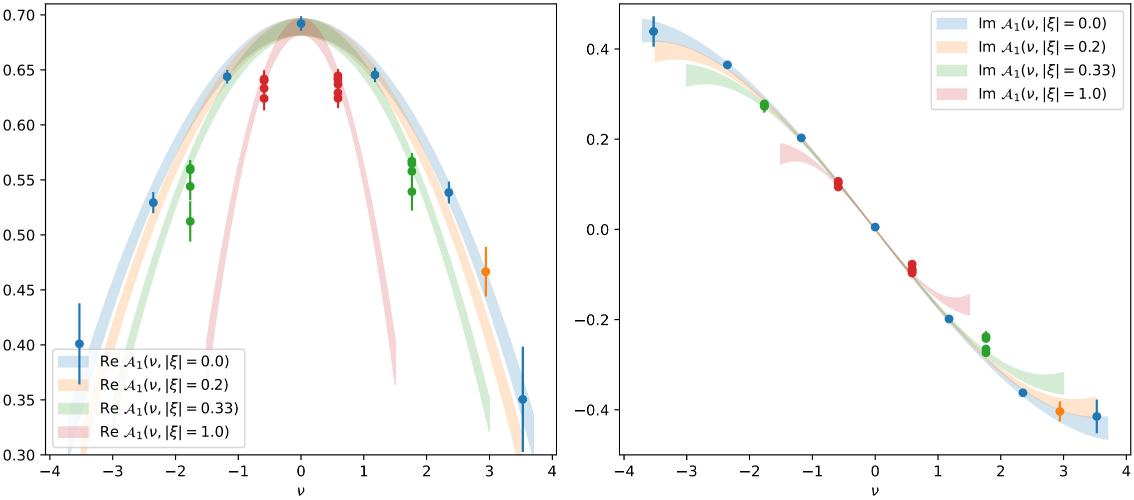
Hence small Ioffe-time behavior:

$$H(\nu,\xi,t) = \int dx\, e^{-ix\nu} H(x,\xi,t)$$

$$= F_1(t) - i\nu A_{2,0}(t) - \frac{\nu^2}{2}[A_{3,0}(t) + \xi^2 A_{3,2}(t)] + ... + \text{power corrections}$$

With momenta up to 1.4 GeV used in this study, we have signal up to $A_{4,0}$ and $A_{4,2}$.

$$\text{Dipole fit:} \;\; A_{n,k}(t) = A_{n,k}(t=0)\left(1 - \frac{t}{\Lambda_{n,k}^2}\right)^{-2}$$



Re $\mathcal{A}_1(\nu, |\xi| = 0.0)$
Re $\mathcal{A}_1(\nu, |\xi| = 0.2)$
Re $\mathcal{A}_1(\nu, |\xi| = 0.33)$
Re $\mathcal{A}_1(\nu, |\xi| = 1.0)$
Im $\mathcal{A}_1(\nu, |\xi| = 0.0)$
Im $\mathcal{A}_1(\nu, |\xi| = 0.2)$
Im $\mathcal{A}_1(\nu, |\xi| = 0.33)$
Im $\mathcal{A}_1(\nu, |\xi| = 1.0)$
ν
ν

Our results

Pion mass = 0.36 GeV - Proton mass = 1.12 GeV
No continuum limit - signs of discretization errors / light-cone uncertainty
Matching at 2 GeV with leading logarithmic accuracy

Value at t = 0

GPD H^{u-d}

$A_{1,0}$	0.97(2)	
$A_{2,0}$	0.204(4)	
$A_{3,0}$ / $A_{3,2}$	0.062(4)	0.42(7)
$A_{4,0}$ / $A_{4,2}$	0.06(1)	0.5(2)

GPD E^{u-d}

$B_{1,0}$	3.44(4)	
$B_{2,0}$	0.36(2)	
$B_{3,0}$ / $B_{3,2}$	0.07(2)	0.9(6)
$B_{4,0}$ / $B_{4,2}$	0.06(4)	1.2(9)

Dipole mass (GeV)

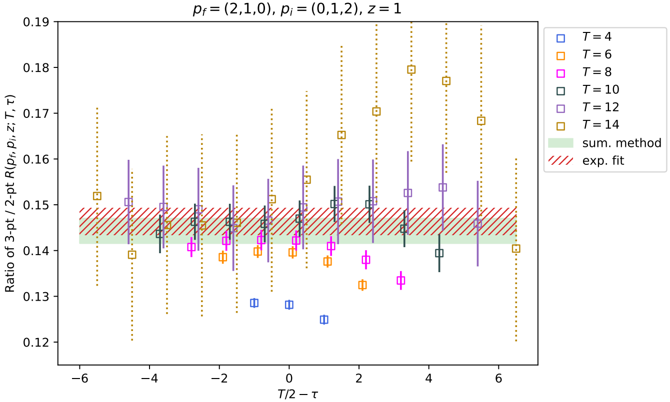
GPD H^{u-d}

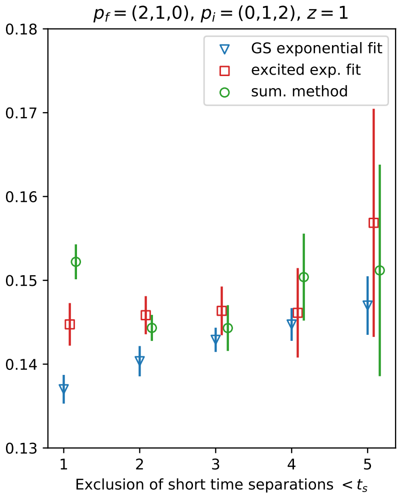
$A_{1,0}$	1.25(2)	
$A_{2,0}$	1.86(6)	
$A_{3,0}$ / $A_{3,2}$	2.2(4)	1.07(9)
$A_{4,0}$ / $A_{4,2}$	Unreliable	1.2(2)

GPD E^{u-d}

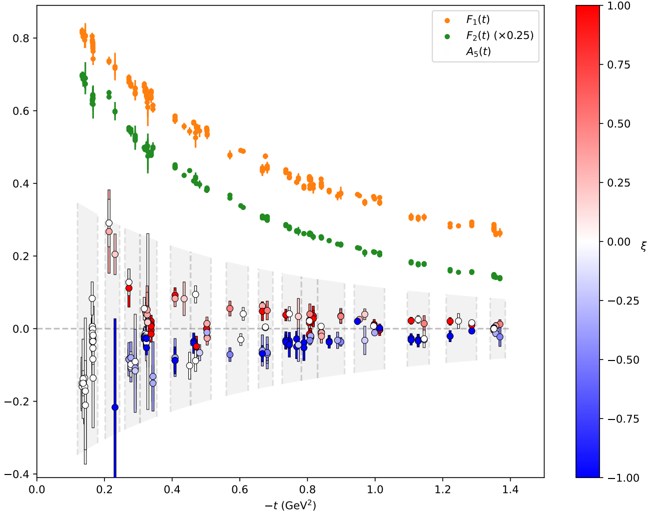
$B_{1,0}$	0.982(6)	
$B_{2,0}$	1.41(8)	
$B_{3,0}$ / $B_{3,2}$	2.4(9)	1.0(3)
$B_{4,0}$ / $B_{4,2}$	Unreliable	1.1(2)

Excited state contamination



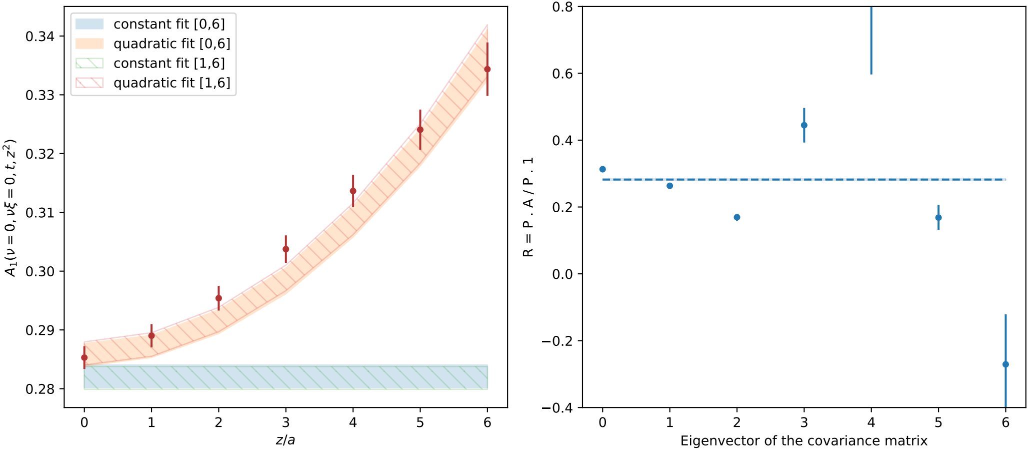


$$\langle p'|\bar{\psi}^q\gamma^\mu\psi^q|p\rangle\Big|_{z=0} = \bar{u}(p')\left[F_1^q(t)\gamma^\mu + F_2^q(t)\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} + A_5^q(t)\frac{\Delta^\mu}{2m}\right]u(p)$$



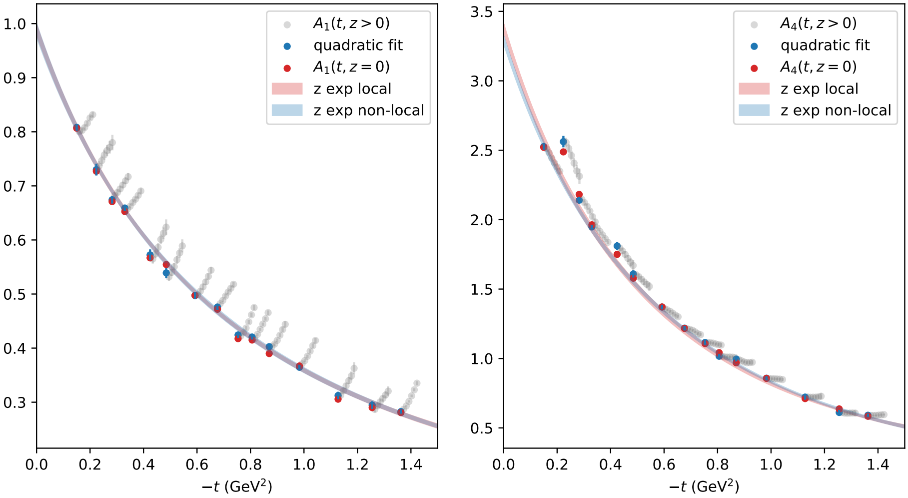
probable sign of lattice discretization in A_5 + enhanced sensitivity to excited state contamination

If $p_{f,z} = p_{i,z} = 0$, then $\nu = 0$ and $\nu\xi = 0$, so we have non-local data with signal only of the EFF

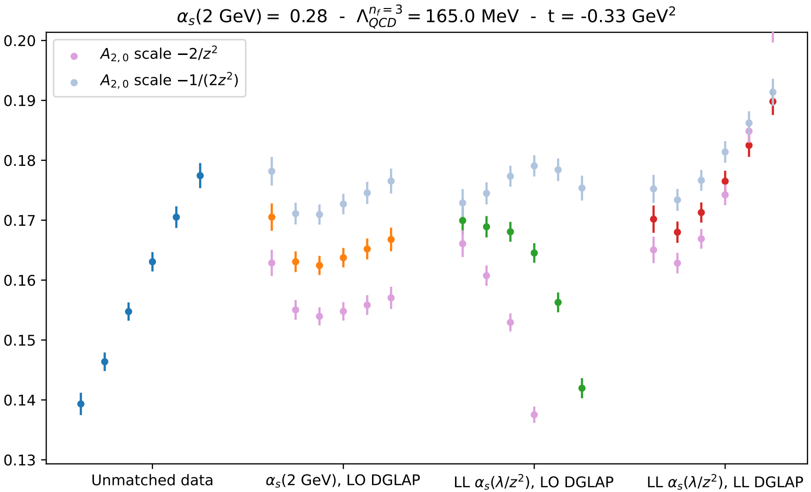


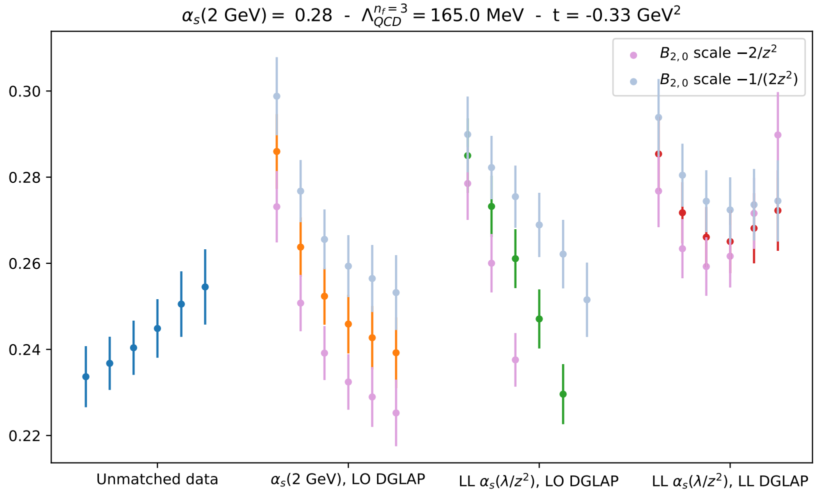
Candidates: lattice discretization + power corrections?

Full non-local EFF extraction perfectly compatible with the local one (with excited state uncertainty + binning).

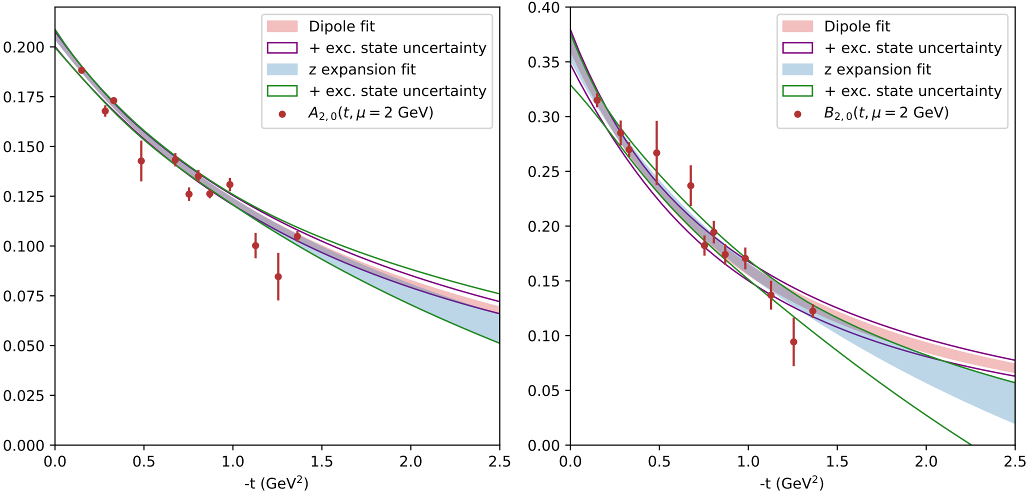


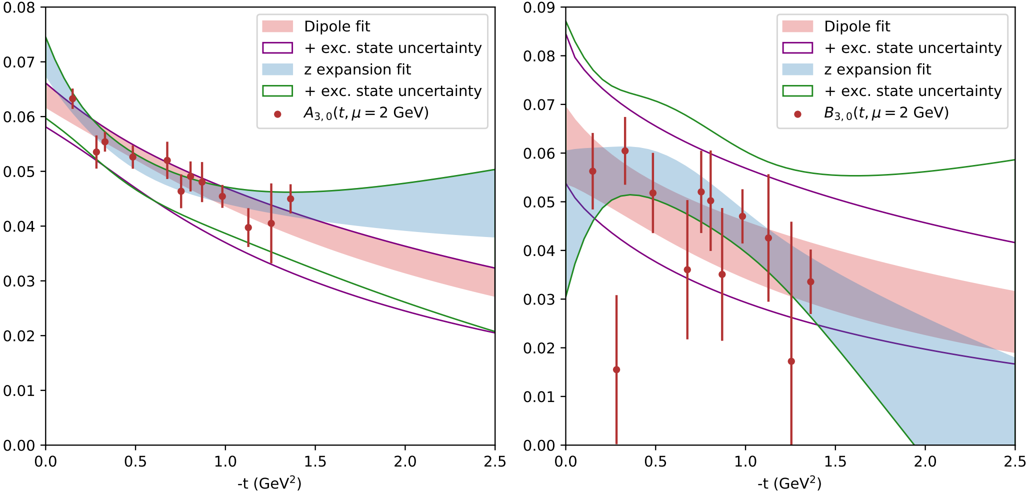
Perturbative matching uncertainty: if there is a strong leading-twist dominance up to separations of 1 fm, what is the perturbative matching kernel worth in this region? also the curse of precision

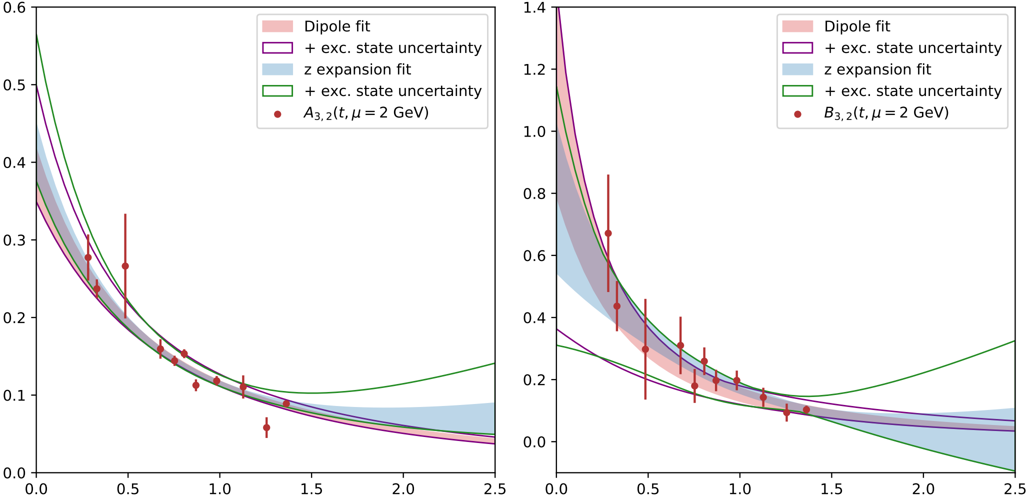


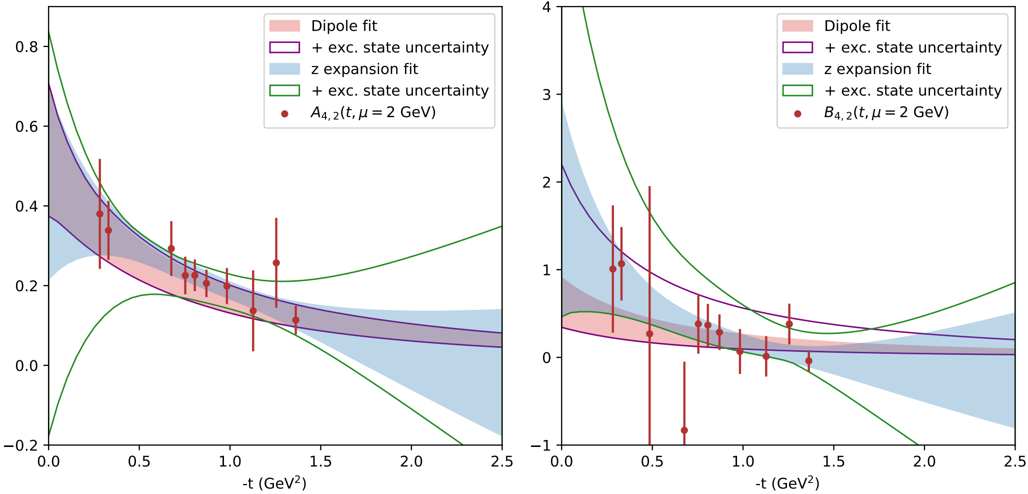


α_s(2 GeV) = 0.28 - $\Lambda_{QCD}^{n_f=3}$ = 165.0 MeV - t = -0.33 GeV^2
$B_{2,0}$ scale $-2/z^2$
$B_{2,0}$ scale $-1/(2z^2)$
0.30
0.28
0.26
0.24
0.22
Unmatched data
α_s(2 GeV), LO DGLAP
LL $\alpha_s(\lambda/z^2)$, LO DGLAP
LL $\alpha_s(\lambda/z^2)$, LL DGLAP


Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{2,0}(t, \mu = 2\text{ GeV})$
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
0.0
0.5
1.0
1.5
2.0
2.5
-t (GeV^2)
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$B_{2,0}(t, \mu = 2\text{ GeV})$
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.5
1.0
1.5
2.0
2.5
-t (GeV^2)

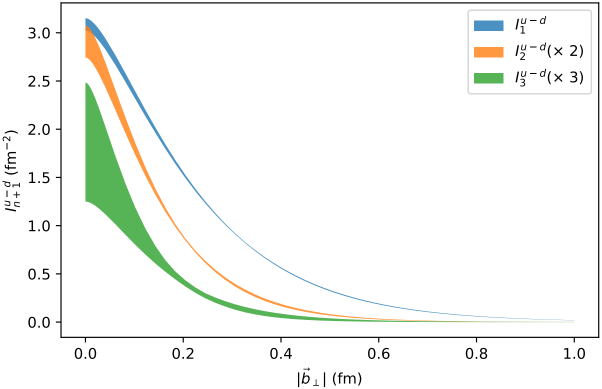

Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{3,0}(t, \mu = 2\text{ GeV})$
$B_{3,0}(t, \mu = 2\text{ GeV})$
-t (GeV^2)
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.0
0.5
1.0
1.5
2.0
2.5


Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{3,2}(t, \mu = 2\text{ GeV})$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.0
0.5
1.0
1.5
2.0
2.5
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$B_{3,2}(t, \mu = 2\text{ GeV})$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.0
0.5
1.0
1.5
2.0
2.5


Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{4,2}(t, \mu = 2$ GeV)
-t (GeV^2)
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$B_{4,2}(t, \mu = 2$ GeV)
-t (GeV^2)

Impact parameter distribution: number density of unpolarized quarks in an unpolarized proton with mom. fraction x and radial distance to the center of longitudinal momentum $\vec{b}_\perp$: **model dependence!**

$$I(x, \vec{b}_\perp) = \int \frac{\mathrm{d}^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H(x, \xi = 0, t = -\vec{\Delta}_\perp^2)$$



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Perspectives on the lattice

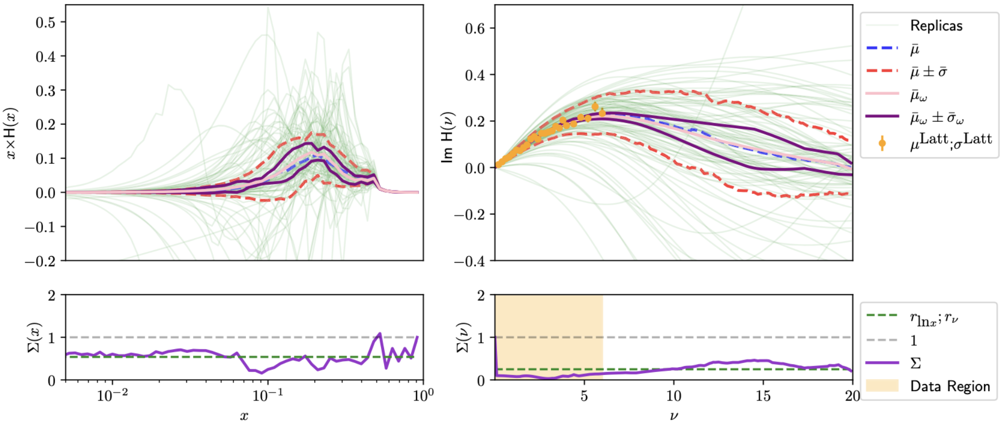
Uncertainties in lattice QCD:

- **Lattice discretization / power corrections**: evidence of issues in the EFFs, needs a continuum limit
- **Excited state / range in Ioffe time**: need a better assurance (GEVP) in order to produce reliable x-reconstruction / high-order moments using large momentum boost
- **Matching uncertainty / power corrections**: proposal to identify a regime of validity of factorization without the use of perturbation theory **[HD, Karpie, Monahan, Orginos, Zafeiropoulos, 2023]**
- **Pion mass / finite volume effects**

A joint lattice - experimental phenomenology

Modest example: using a model whose uncertainty is purely theoretical deconvolution uncertainty (not experimental) and pseudo-lattice data **[Riberdy, HD, Mezrag, Sznajder, 2023]**

$\xi = 0.5$



Conclusion

- We have a framework with excellent signal and very large kinematic coverage for GPDs
- There is still a lot of work on purely lattice systematics (but that's ok), power corrections and matching uncertainty is a bigger concern that we start to address
- We have quite a few innovative ideas to explore beyond the dumb powering through the various limits, stay tuned!

Thank you for your attention!

- **Hadron tomography** [Burkardt, 2003]:

$$I(x,\mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\Delta_\perp} H^q(x,\xi=0,t=-\Delta_\perp^2)$$

- **Gravitational form factors** [Polyakov, 2003], [Lorcé et al, 2017]: radial energy / pressure

$$\langle P_2|T_a^{\mu\nu}|P_1\rangle = \bar{u}(P_2)\Bigg\{\frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C_a(t) + M\eta^{\mu\nu}\bar{C}_a(t)$$
$$+\frac{P^{\{\mu}i\sigma^{\nu\}\rho}\Delta_\rho}{4M}\left[A_a(t)+B_a(t)\right] + \frac{P^{[\mu}i\sigma^{\nu]\rho}\Delta_\rho}{4M} D_a^{GFF}(t)\Bigg\}u(P_1)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, H^q(x,\xi,t,\mu^2) = A_q(t,\mu^2) + 4\xi^2 C_q(t,\mu^2)$$

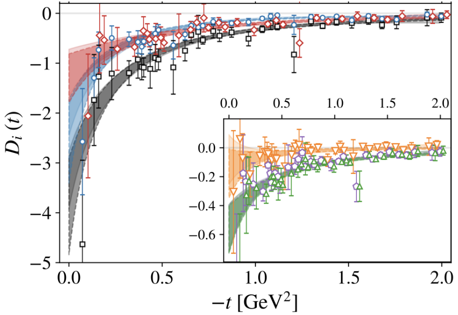
- **Proton's spin decomposition** [Ji, 1997]:

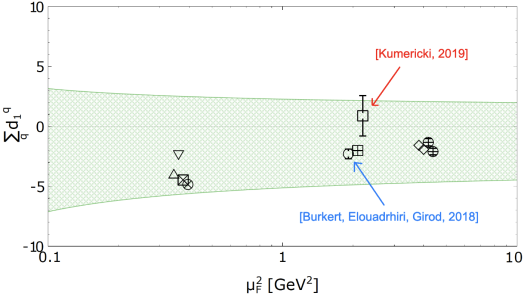
$$\frac{1}{2} = \sum_q \frac{1}{2}\int_{-1}^{1} dx\, x\left[H^q + E^q\right]\Big|_{t=0} + \frac{1}{2}\int_{-1}^{1} dx\left[H^g + E^g\right]\Big|_{t=0}$$

The GFFs can be accessed from the local matrix element

$$\langle P_2|\bar{\psi}D^{\{\mu}\gamma^{\nu\}}\psi|P_1\rangle$$

Calculations on the lattice have been available for 20 years **[Hagler, 2003]**. Recent result at $m_\pi = 170$ MeV **[Hackett, Pefkou, Shanahan, 2023]** vs experimental extraction **[HD, Lorcé, Moutarde, Sznajder, Trawinski, Wagner, 2021]**





DIS hadronic tensor (one-photon exchange approximation)

$$W^{\mu\nu} = \int d^4z\, e^{iq.z} \langle P | J^\mu(z) J^\nu(0) | P \rangle \propto \sum_X |\mathcal{M}(\gamma^* P \to X)|^2$$

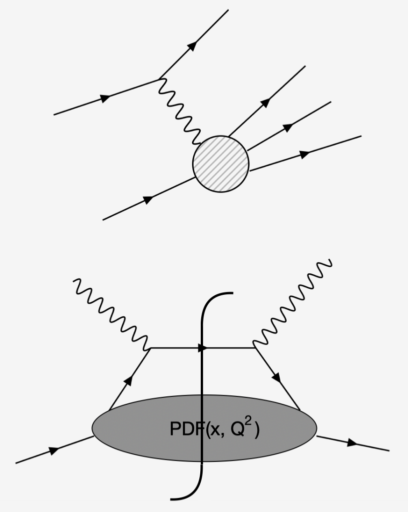
In the Bjorken limit $-q^2 \to \infty$ with $x_B = -q^2/2P \cdot q$ fixed, stationary phase for

$$z^2 \leq \mathcal{O}(1/Q^2)\,, \quad q \cdot z \approx x_B P \cdot z$$

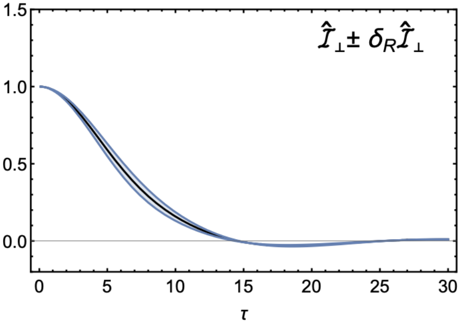
Hence a DIS-scheme definition of the PDF as:

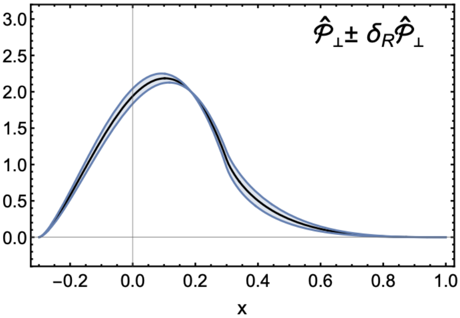
$$q(x_B) = \int dz^-\, e^{ix_B P^+ z^-} \langle P | J^\mu(z^-) J^\nu(0) | P \rangle$$

Study of the analytical properties of $\mathcal{M}(\gamma^* P \to \gamma^* P)$ gives integrals of a PDF on all x_B in the Bjorken limit can be deformed to $1/x_B \to 0$ and therefore $z \to 0 \to$ **OPE of moments (and other integrals of x) with local operators**

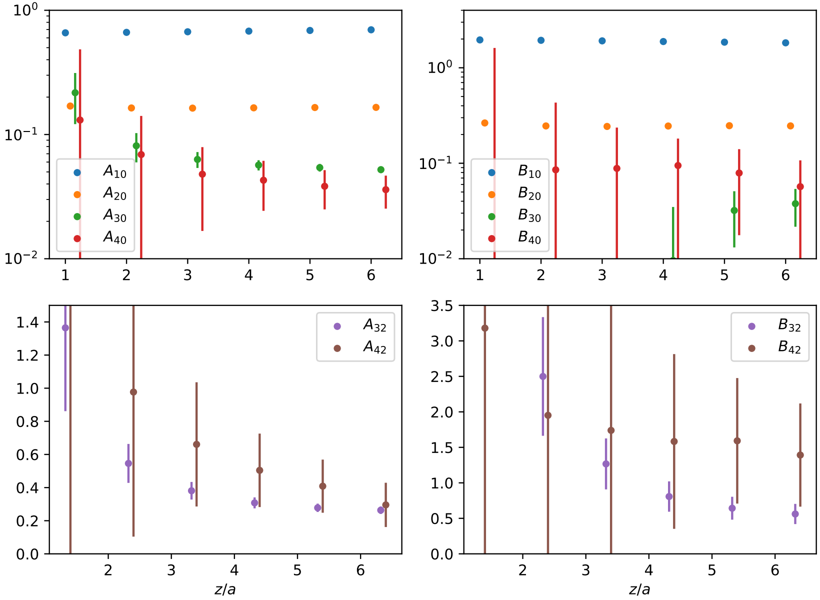


Even with a space-like separation of 1 fm, the power-corrections at $\xi = 0.3$ might be very small (model-dependent estimate of non-perturbative higher-twist contributions through renormalon ambiguity) [Braun, Koller, Schoenleber, 2024]:

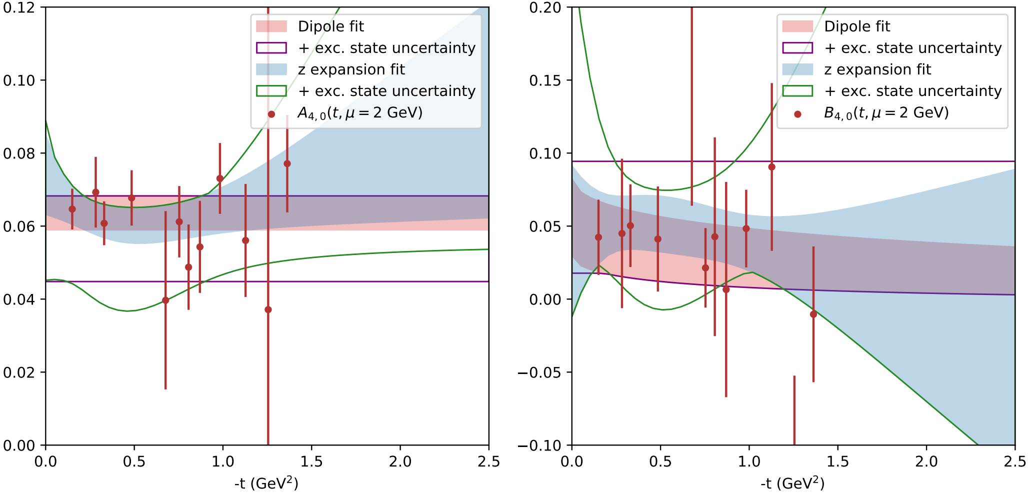




Situation much more complicated for LaMET formalism, with potentially divergent power-corrections when approaching $x \approx \xi$.



A_{10}
A_{20}
A_{30}
A_{40}
B_{10}
B_{20}
B_{30}
B_{40}
A_{32}
A_{42}
B_{32}
B_{42}
z/a
z/a


Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{4,0}(t, \mu = 2$ GeV)
$B_{4,0}(t, \mu = 2$ GeV)
-t (GeV^2)
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.20
0.15
0.10
0.05
−0.05
−0.10
0.0
0.5
1.0
1.5
2.0
2.5