$1/Q^2$ power corrections to TMD factorization for Drell-Yan hadronic tensor

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Outline

1. Introduction: TMD factorization from rapidity factorization:
2. Classical fields from retarded propagators at $p_\perp^2/p_\parallel^2 \ll 1$.
3. Full list of power corrections for DY hadronic tensor at $\frac{1}{Q^2}$ leading-N_c level.
4. Back-of-the envelope estimates of angular asymmetries for Z-boson production.
5. Conclusions and outlook

TMD factorization

TMD factorization formula for particle production in hadron-hadron scattering looks like

$$\frac{d\sigma}{d\eta d^2q_\perp} = \sum_{\text{flavors}} e_f^2 \int d^2k_\perp \mathcal{D}_{f/A}(x_A,k_\perp)\mathcal{D}_{f/B}(x_B,q_\perp-k_\perp)C(q,k_\perp)$$
$$+ \text{ power corrections } + \text{ ``Y} - \text{terms''}$$

- $\mathcal{D}_{f/A}(x_A,k_\perp)$ is the TMD density of a parton f in hadron A with fraction of momentum x_A and transverse momentum $k_\perp$,
- $\mathcal{D}_{f/B}(x,q_\perp-k_\perp)$ is a similar quantity for hadron B,
- $C_i(q,k)$ are determined by the cross section $\sigma(ff\to\mu^+\mu^-)$ of production of DY pair of invariant mass q^2 in the scattering of two partons.

Examples: Drell-Yan process with Q being the mass of DY pair and Higgs production by gluon-gluon fusion

TMD approach is relevant when the transverse momentum $q_\perp \ll Q$

Long-term project: TMD factorization valid at small and moderate x

$$\frac{d\sigma}{d\eta d^2q_\perp} = \sum_{\text{flavors}} e_f^2 \int d^2k_\perp \mathcal{D}_{f/A}(x_A,k_\perp)\mathcal{D}_{f/B}(x_B,q_\perp-k_\perp)C(q,k_\perp)$$
$$+ \text{ power corrections } + \text{ ``Y} - \text{terms''}$$

The quantities $\mathcal{D}_{f/A}(x_A,k_\perp)$, $\mathcal{D}_{f/B}(x_B,q_\perp-k_\perp)$, and $C(q,k_\perp)$ are defined with cutoffs. The dependence on the cutoffs cancels in their product order by order in α_s.

At moderate x_A,x_B: CSS/SCET approach. The TMDs $\mathcal{D}_{f/A}(x_A,k_\perp)$ are defined with a combination of UV and rapidity cutoffs.

At $x_A,x_B \ll 1$: k_T-factorization approach. The TMDs are defined with rapidity-only cutoffs.

It is impossible to extend CSS to small x. (Recently: LO BFKL from SCET)

It *is* possible to study TMD factorization at moderate x using small-x methods (rapidity-only factorization etc.) (A. Tarasov, G. Chirilli, I.B, 2015-2023)

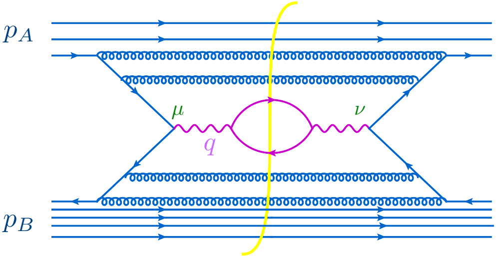
Example: full list of power corrections $\sim \frac{1}{Q^2}$ for DY hadronic tensor, see below.

They are not obtained (yet?) by CSS or SCET

Classical example: DY hadronic tensor

DY cross section is given by the product of leptonic tensor and hadronic tensor. The hadronic tensor $W_{\mu\nu}$ is defined as

$$W_{\mu\nu}(p_A, p_B, q) \;=\; \frac{1}{(2\pi)^4}\int d^4x\; e^{-iqx}\langle p_A, p_B|J_\mu(x)J_\nu(0)|p_A, p_B\rangle$$



p_A, p_B = hadron momenta, q = the momentum of DY pair, and J_μ is the electromagnetic or Z-boson current.

There are four tensor structures W_T, W_L, W_Δ, $W_{\Delta\Delta}$

TMD representation for W_i

The hadronic tensor in the Sudakov region $q^2 \equiv Q^2 \gg q_\perp^2$ can be studied by TMD factorization. For example, functions W_T and $W_{\Delta\Delta}$ can be represented as

$$W_i = \sum_{\text{flavors}} e_f^2 \int d^2k_\perp \mathcal{D}^{(i)}_{f/A}(x_A, k_\perp)\mathcal{D}^{(i)}_{f/B}(x_B, q_\perp - k_\perp) C_i(q, k_\perp) + \text{power corrections} + \text{Y} - \text{terms} \quad (1)$$

There is, however, a problem with Eq. (1) for the functions W_L and W_Δ.

W_T and $W_{\Delta\Delta}$ are determined by leading-twist quark TMDs, but W_Δ and W_L start from terms $\sim \frac{q_\perp}{Q}$ and $\sim \frac{q_\perp^2}{Q^2}$ determined by quark-quark-gluon TMDs.

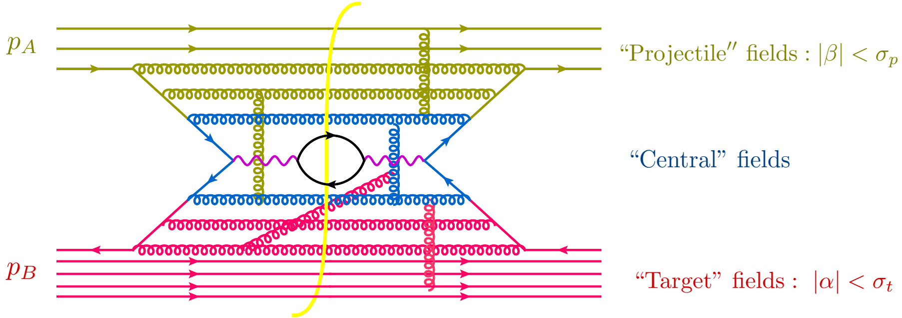
The power corrections $\sim \frac{q_\perp}{Q}$ were found more than two decades ago but there was no calculation of power corrections $\sim \frac{q_\perp^2}{Q^2}$ until recently.

Power corrections from tree diagrams in background fields

TMD factorization from rapidity factorization (A. Tarasov and I.B.)

Sudakov variables:

$$p \ = \ \alpha p_1 + \beta p_2 + p_\perp, \qquad p_1 \simeq p_A, \ \ p_2 \simeq p_B, \ \ p_1^2 = p_2^2 = 0$$



The result of the integration over "central" fields in the background of projectile and target fields is a series of TMD operators made from projectile (or target) fields multiplied by powers of $\frac{1}{Q^2} \Rightarrow$ power corrections

Double operator expansion

$$\hat{J}(x_1)\hat{J}(x_2) = \sum_{I,J}\int dz_1^- dz_2^- dw_1^+ dw_2^+ \mathfrak{C}_{IJ}(x_1,x_2;z_i^-,w_i^+;\sigma_p,\sigma_t) \times \hat{\mathcal{O}}_I^{\sigma_p}(z_2^-,x_{2_\perp};z_1^-,x_{1_\perp})\hat{\mathcal{O}}_J^{\sigma_t}(z_2^+,x_{2_\perp};z_1^+,x_{1_\perp})$$

$\hat{\mathcal{O}}_i^{\sigma_p}$ - “projectile” TMD operators, $\hat{\mathcal{O}}_i^{\sigma_p}$ - “target” TMD operators

To find relevant operators and coefficients, it is convenient to consider “matrix elements” of the l.h.s. and r.h.s. in suitable background field

Suitable field $\mathbb{A}$: solution of classical YM equations with boundary condition that at the remote past the field is a sum of projectile and target fields

$$\langle\hat{J}(x_1)\hat{J}(x_2)\rangle_{\mathbb{A}} = \sum_{I,J}\int dz_1^- dz_2^- dw_1^+ dw_2^+ \mathfrak{C}_{IJ}(x_1,x_2;z_i^-,w_i^+;\sigma_p,\sigma_t) \times \langle\hat{\mathcal{O}}_I^{\sigma_p}(z_2^-,x_{2_\perp};z_1^-,x_{1_\perp})\hat{\mathcal{O}}_J^{\sigma_t}(z_2^+,x_{2_\perp};z_1^+,x_{1_\perp})\rangle_{\mathbb{A}}$$

In the tree approximation

$$\langle\hat{\mathcal{O}}_I^{\sigma_p}\hat{\mathcal{O}}_J^{\sigma_t}\rangle_{\mathbb{A}} = \hat{\mathcal{O}}_I(\mathbb{A})\hat{\mathcal{O}}_J(\mathbb{A})$$

Classical solution $\mathbb{A}$ and $\psi_{\mathbb{A}}$

Solution of classical YM equations

$$\not{P}\psi_{\mathbb{A}} = 0, \quad \mathscr{D}^\nu \mathscr{F}^a_{\mu\nu} = \sum_f g\bar{\psi}_{\mathbb{A}} t^a \gamma_\mu \psi_{\mathbb{A}}$$

Boundary conditions :

$$\mathbb{A}_\mu(x) \overset{x^+\to-\infty}{=} \bar{A}_\mu(x^-, x_\perp), \quad \psi_{\mathbb{A}}(x) \overset{x^+\to-\infty}{=} \psi_a(x^-, x_\perp)$$
$$\mathbb{A}_\mu(x) \overset{x^-\to-\infty}{=} \bar{B}_\mu(x^+, x_\perp), \quad \psi_{\mathbb{A}}(x) \overset{x^-\to-\infty}{=} \psi_b(x^+, x_\perp)$$

The projectile and target fields satisfy YM equations

$$(\not{P} + m_f)\psi_a = 0, \quad D^\nu F^a_{\mu\nu} = g\bar{\psi}_a t^a \gamma_\mu \psi_a$$
$$(\not{P} + m_f)\psi_b = 0, \quad D^\nu F^a_{\mu\nu} = g\bar{\psi}_b t^a \gamma_\mu \psi_b$$

Method of solution:

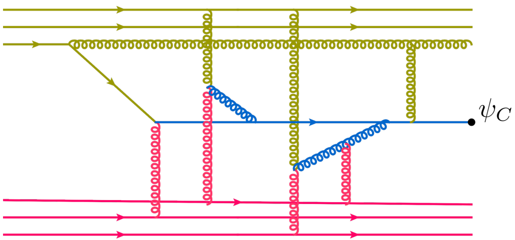
- Start with $\psi_A + \psi_B$ and $\bar{A}_\mu + \bar{B}_\mu$ in the gauge $A^+ = 0$, $A^- = 0$
- Correct by computing Feynman diagrams (with retarded propagators) with sources $(\not{P} + m)(\psi_A + \psi_B)$ and $J_\nu = D^\mu F^{\mu\nu}(U + V)$

ψ_C in the tree approximation

It is convenient to choose projectile/target fields as

Projectile fields: $\beta = 0 \Rightarrow A(x^-, x_\perp),\ \psi_A(x^-, x_\perp)$

Target fields: $\alpha = 0 \Rightarrow B(x^+, x_\perp),\ \psi_B(x^-, x_\perp)$



Classical background fields: ψ_C , C_μ

ψ_C = sum of tree diagrams in external $A, \tilde{A}, \psi_A, \tilde{\psi}_A$ and $B, \tilde{B}, \psi_B, \tilde{\psi}_B$ fields with sources

$J_\psi = (\not{P} + m)(\psi_A + \psi_B)$, $\qquad J_\nu = D^\mu F^{\mu\nu}(A + B)$

and

$\tilde{J}_\psi = (\not{P} + m)(\tilde{\psi}_A + \tilde{\psi}_B)$, $\qquad \tilde{J}_\nu = D^\mu F^{\mu\nu}(\tilde{A} + \tilde{B})$

Classical solution $\equiv \sum$ tree diagrams with retarded propagators

The fields A, ψ and $\tilde{A}, \tilde{\psi}$ do not depend on $x^+ \Rightarrow$
if they coincide at $x^+ = \infty \Rightarrow$ they coincide everywhere.

Similarly,
B, ψ_b and $\tilde{B}, \tilde{\psi}_b$ do not depend on $x^- \Rightarrow$
if they coincide at $x^- = \infty$ they should be equal.

Since $\tilde{A} = A$ and $\tilde{B} = B$ the sources and background fields are the same to the left and to the right of the cut

$\Rightarrow$

ψ_C and C_μ are given by the sum of tree diagrams with *retarded* Green functions

Classical fields in the leading order in $p_\perp^2/p_\parallel^2 \sim q_\perp^2/Q^2$

The solution of such YM equations in general case is yet unsolved problem (goes under the name "glasma" $\Leftrightarrow$ scattering of two "color glass condensates").

Fortunately, for our case of particle production with $\frac{q_\perp}{Q} \ll 1$ we can use this small parameter and construct the approximate solution.

At the tree level transverse momenta are $\sim q_\perp^2$ and longitudinal are $\sim Q^2 \Rightarrow$

$$\psi, A \;=\; \text{series in } \frac{q_\perp}{Q}: \qquad \psi = \psi^{(0)} + \psi^{(1)} + ..., \qquad A = A^{(0)} + A^{(1)} + ...$$

NB: After the expansion

$$\frac{1}{p^2 + i\epsilon p_0} \;=\; \frac{1}{p_\parallel^2 - p_\perp^2 + i\epsilon p_0} \;=\; \frac{1}{p_\parallel^2} - \frac{1}{p_\parallel^2 + i\epsilon p_0} p_\perp^2 \frac{1}{p_\parallel^2 + i\epsilon p_0} \;+ ...$$

the dynamics in transverse space is trivial.

Fields are either at the point $x_\perp$ or at the point $0_\perp \Rightarrow$ TMDs

Leading-N_c power corrections

Power corrections are $\sim$ leading twist $\times\Big(\frac{q_\perp}{Q}$ or $\frac{q_\perp^2}{Q^2}\Big)\times\big(1+\frac{1}{N_c}+\frac{1}{N_c^2}\big)$.

NB: almost all $\bar{q}Gq$ TMDs not suppressed by $\frac{1}{N_c}$ can be rewritten in terms of $\bar{q}q$ TMDs due to QCD equations of motion

Leading twist: $\varrho\equiv\sqrt{s/2}$

$$\frac{1}{8\pi^3 s}\int dx^- d^2x_\perp\ e^{-i\alpha\varrho x^- + i(k,x)_\perp}\ \langle A|\hat{\bar{\psi}}(x^-,x_\perp)\ \not{p}_2\hat{\psi}(0)|A\rangle\ =\ f_1(\alpha,k_\perp^2)$$

Power correction:

$$\frac{1}{8\pi^3 s}\int dx^- dx_\perp\ e^{-i\alpha\varrho x^- + i(k,x)_\perp}$$
$$\times\ \langle A|\hat{\bar{\psi}}(x^-,x_\perp)\hat{\not{A}}(x^-,x_\perp)\not{p}_2\gamma_i\hat{\psi}(0)|A\rangle$$
$$=\ k_i f_1(\alpha,k_\perp) - \alpha k_i\big[f_\perp(\alpha,k_\perp) + ig^\perp(\alpha,k_\perp)\big],$$

(Mulders & Tangerman, 1996)

Result for $W_{\mu\nu}$ for unpolarized hadrons

Result:

$$W_{\mu\nu}(q) \;=\; W^1_{\mu\nu}(q) + W^2_{\mu\nu}(q) + W^3_{\mu\nu}(q)$$

The first, gauge-invariant, part is a "gauge completion" of leading-twist result

$$\begin{aligned}
W^1_{\mu\nu}(q) \;&=\; W^{1F}_{\mu\nu}(q) + W^{1H}_{\mu\nu}(q),\\
W^{1F}_{\mu\nu}(q) \;&=\; \sum_f e_f^2 W^{fF}_{\mu\nu}(q), \quad W^{fF}_{\mu\nu}(q) \;=\; \frac{1}{N_c}\int d^2k_\perp \{f_1\bar{f}_1 + \bar{f}_1 f_1\}\mathcal{W}^F_{\mu\nu}(q,k_\perp),\\
W^{1H}_{\mu\nu}(q) \;&=\; \sum_f e_f^2 W^{fH}_{\mu\nu}(q), \quad W^{fH}_{\mu\nu}(q) \;=\; \frac{1}{N_c}\int d^2k_\perp \{h_1^\perp\bar{h}_1^\perp + \bar{h}_1^\perp h_1^\perp\}\mathcal{W}^H_{\mu\nu}(q,k_\perp)
\end{aligned}$$

where ($\alpha_q \equiv x_A$, $\beta_q \equiv x_B$)

$$\begin{aligned}
\{f_1\bar{f}_1 + \bar{f}_1 f_1\} \;&\equiv\; f_1(\alpha_q,k_\perp)\bar{f}_1\big(\beta_q,(q-k)_\perp\big) \;+\; f_1 \leftrightarrow \bar{f}_1\\
\{h_1^\perp\bar{h}_1^\perp + \bar{h}_1^\perp h_1^\perp\} \;&\equiv\; h_1^\perp(\alpha_q,k_\perp)\bar{h}_1^\perp(\beta_q,(q-k)_\perp) \;+\; h_1^\perp \leftrightarrow \bar{h}_1^\perp
\end{aligned}$$

Gauge-invariant structures $q^\mu W^F_{\mu\nu} = q^\mu W^H_{\mu\nu} = 0$

$$\mathcal{W}^F_{\mu\nu}(q,k_\perp) \ = \ \mathrm{LT} + \text{“gauge} - \text{invariant completion}''$$

$$= \ -g^\perp_{\mu\nu} + \frac{1}{Q^2}(q^\parallel_\mu q^\perp_\nu + q^\parallel_\nu q^\perp_\mu) + \frac{q^2_\perp}{Q^4} q^\parallel_\mu q^\parallel_\nu + \frac{\tilde{q}_\mu \tilde{q}_\nu}{Q^2}[q^2_\perp - 4(k,q-k)_\perp]$$

$$- \ \Big[\frac{\tilde{q}_\mu}{Q^2}\Big(g^\perp_{\nu i} - \frac{q^\parallel_\nu q_i}{Q^2}\Big)(q-2k)^i_\perp + \mu \leftrightarrow \nu\Big]$$

$$\tilde{q} \equiv \alpha_q p_1 - \beta_q p_2$$

$$m^2 \mathcal{W}^H_{\mu\nu}(q,k_\perp)$$

$$= \ -\big[k^\perp_\mu (q-k)^\perp_\nu + k^\perp_\nu (q-k)^\perp_\mu + g^\perp_{\mu\nu}(k,q-k)_\perp\big] + 2\frac{\tilde{q}_\mu \tilde{q}_\nu - q^\parallel_\mu q^\parallel_\nu}{Q^4} k^2_\perp (q-k)^2_\perp$$

$$- \ \Big(\frac{q^\parallel_\mu}{Q^2}\big[k^2_\perp (q-k)^\perp_\nu + k^\perp_\nu (q-k)^2_\perp\big] + \ \frac{\tilde{q}_\mu}{Q^2}\big[k^2_\perp (q-k)^\perp_\nu - k^\perp_\nu (q-k)^2_\perp\big] + \mu \leftrightarrow \nu\Big)$$

$$- \ \frac{\tilde{q}_\mu \tilde{q}_\nu + q^\parallel_\mu q^\parallel_\nu}{Q^4}\big[q^2_\perp - 2(k,q-k)_\perp\big](k,q-k)_\perp - \ \frac{q^\parallel_\mu \tilde{q}_\nu + \tilde{q}_\mu q^\parallel_\nu}{Q^4}(2k-q,q)_\perp (k,q-k)_\perp$$

Second gauge-invariant part

$$
\begin{aligned}
W^2_{\mu\nu}(q) \;=\; & \frac{2}{N_cQ^2}\int d^2k_\perp\Big\{\Big[\tilde{q}_\mu(q-k)_\nu + \frac{2}{\beta_q s}\tilde{q}_\mu p_{1\nu}(k,q-k)_\perp + \frac{2}{\alpha_q s}\tilde{q}_\mu p_{2\nu}(q-k)^2_\perp \;+\; \mu\leftrightarrow\nu\Big] \\
& \times\Big(\beta_q\{f_1\bar{f}_\perp + \bar{f}_1 f_\perp\} - \alpha_q\{h\bar{h}_1^\perp + \bar{h}h_1^\perp\}\Big) \\
& + \Big[\tilde{q}_\mu k^\perp_\nu + \frac{2}{s\beta_q}k^2_\perp\tilde{q}_\mu p_{1\nu} + \frac{2}{s\alpha_q}(k,q-k)_\perp\tilde{q}_\mu p_{2\nu} \;+\; \mu\leftrightarrow\nu\Big] \\
& \times\Big(-\alpha_q\{f_\perp\bar{f}_1 + \bar{f}_\perp f_1\} + \beta_q\{h_1^\perp\bar{h} + \bar{h}_1^\perp h\}\Big) \\
& + \frac{4\tilde{q}_\mu\tilde{q}_\nu}{Q^2}\Big[m^2\Big(\alpha_q^2\{f_3\bar{f}_1 + \bar{f}_3 f_1\} + \beta_q^2\{f_1\bar{f}_3 + \bar{f}_1 f_3\} + \alpha_q\beta_q\big[\{e\bar{e} + \bar{e}e\} + \{h\bar{h} + \bar{h}h\}\big]\Big) \\
& + (k,q-k)_\perp\Big(-\alpha_q\beta_q\big[\{f_\perp\bar{f}_\perp + \bar{f}_\perp f_\perp\} + \{g_\perp\bar{g}_\perp + \bar{g}_\perp g_\perp\}\big] \\
& + \beta_q^2\{h_1^\perp\bar{h}_3^\perp + \bar{h}_1^\perp h_3^\perp\} + \alpha_q^2\{h_3^\perp\bar{h}_1^\perp + \bar{h}_3^\perp h_1^\perp\}\Big)\Big] \\
& + \frac{1}{m^2}\mathcal{W}^\perp_{\mu\nu}(q,k_\perp)\Big[\frac{2}{\alpha_q}\{h_1^\perp\Re\bar{h}_{1\mathcal{G}} + \bar{h}_1^\perp\Re h_{1\mathcal{G}}\} + \frac{2}{\beta_q}\{\Re h_{1\mathcal{G}}\bar{h}_1^\perp + \Re\bar{h}_{1\mathcal{G}}h_1^\perp\} + \;\Re\big(\{h^\perp_{1G}\bar{h}^\perp_{1G} + \bar{h}^\perp_{1G}h^\perp_{1G}\}\big)\Big]
\end{aligned}
$$

where $\mathcal{W}^\perp_{\mu\nu}(q_\perp,k_\perp)$ is a transverse gauge-invariant structure

$$
\begin{aligned}
\mathcal{W}^\perp_{\mu\nu}(q_\perp,k_\perp) \;\equiv\; & g^\perp_{\mu\nu}(k,q-k)^2_\perp - g^\perp_{\mu\nu}k^2_\perp(q-k_\perp)^2 \\
& + [k^\perp_\mu(q-k)^\perp_\nu + \mu\leftrightarrow\nu](k,q-k)_\perp - k^2_\perp(q-k)^\perp_\mu(q-k)^\perp_\nu - \;(q-k_\perp)^2k^\perp_\mu k^\perp_\nu
\end{aligned}
$$

$f_\perp, f_3, h, h_3^\perp, g_\perp, e$ are the quark-antiquark TMDs of a non-leading twist,

$h^\perp_{1G}$ - quark-antiquark-gluon TMD.

Third part

$$W^3_{\mu\nu}(q) \;=\; \frac{2}{N_cQ^2}\int d^2k_\perp$$
$$\times\,\Big\{g^\perp_{\mu\nu}\times\big[\text{a bunch of quark–antiquark}\;\;\text{and}\;\;\text{quark–antiquark–gluon TMDs}\big]$$
$$+\,\big[k^\perp_\mu(q-k)^\perp_\nu + k^\perp_\nu(q-k)^\perp_\mu + g^\perp_{\mu\nu}(k,q-k)_\perp\big]\times\text{same}$$
$$+\,\big[2(q-k)^\perp_\mu(q-k)^\perp_\nu + g^\perp_{\mu\nu}(q-k)^2_\perp\big]\times\text{same}$$
$$+\,\big[2(q-k)^\perp_\mu(q-k)^\perp_\nu + g^\perp_{\mu\nu}(q-k)^2_\perp\big]\times\text{same}\Big\}$$

Similarly to LT contribution, is not EM gauge invariant ⇒ needs "gauge completion" by $\frac{1}{Q^3}$ and $\frac{1}{Q^4}$ power corrections.
Still, it is gauge invariant at the $\frac{1}{Q^2}$ level

$$q^\mu W^3_{\mu\nu} \;=\; O\Big(\frac{1}{Q^2}\Big)$$

This is the result of cancellations of $O\big(\frac{1}{Q}\big)$ corrections due to QCD equations

Third part: 4 transverse structures and $\sim 10\ \bar{q}qG$ TMDs

$$
\begin{aligned}
W^3_{\mu\nu}(q) \;=\; & \frac{2}{N_c Q^2}\int d^2k_\perp \Big[g^\perp_{\mu\nu}\Big\{ m^2\alpha_q\beta_q\big(\{h\bar{h}+\bar{h}h\} - \{e\bar{e}+\bar{e}e\}\big) \\
&+ \alpha_q\beta_q m^2\big[\Re f_D(\alpha_q,k_\perp)\bar{f}_1'(\beta_q,q_\perp-k_\perp) + \Re\bar{f}_D(\alpha_q,k_\perp)f_1'(\beta_q,q_\perp-k_\perp) + f_1'(\alpha_q,k_\perp)\Re\bar{f}_D(\beta_q,q_\perp-k_\perp) + \bar{f}_1'(\alpha_q,k_\perp)\Re f_D \\
&\quad - m^2\beta_q\big(\{f_1\Re\bar{f}_{2\mathcal{G}} + \bar{f}_1\Re f_{2\mathcal{G}}\} + \{f_1\Re\bar{f}_{3\mathcal{G}} + \bar{f}_1\Re f_{3\mathcal{G}}\}\big) - m^2\alpha_q\big(\{\Re\bar{f}_{2\mathcal{G}}f_1 + \Re f_{2\mathcal{G}}\bar{f}_1\} + \{\Re\bar{f}_{3\mathcal{G}}f_1 + \Re f_{3\mathcal{G}}\bar{f}_1\}\big) \\
&\quad + (k,q-k)_\perp\Big(-\beta_q\{h_1^\perp\bar{h} + \bar{h}_1^\perp h\} - \alpha_q\{h\bar{h}_1^\perp + \bar{h}h_1^\perp\} + 2\beta_q^2\{h_1^\perp\bar{h}_3^\perp + \bar{h}_1^\perp h_3^\perp\} \\
&\quad + 2\alpha_q^2\{h_3^\perp\bar{h}_1^\perp + \bar{h}_3^\perp h_1^\perp\} + 2\beta_q\{\Im\acute{e}_G\bar{h}_1^\perp + \Im\bar{\acute{e}}_G h_1^\perp\} + 2\alpha_q\{h_1^\perp\Im\bar{\acute{e}}_G + \bar{h}_1^\perp\Im\acute{e}_G\} \\
&\quad - \beta_q\{f_1\Re\bar{\acute{f}}_{1\mathcal{G}} + \bar{f}_1\Re\acute{f}_{1\mathcal{G}}\} - \alpha_q\{\Re\bar{\acute{f}}_{1\mathcal{G}}f_1 + \Re\acute{f}_{1\mathcal{G}}\bar{f}_1\} - \alpha_q\beta_q\big[\{f_\perp\Re\bar{\grave{f}}_{1\mathcal{G}} + \bar{f}_\perp\Re\grave{f}_{1\mathcal{G}}\} \\
&\quad + \{g_\perp\Im\bar{\grave{f}}_{1\mathcal{G}} - \bar{g}_\perp\Im\grave{f}_{1\mathcal{G}}\} + \{\Re\bar{\grave{f}}_{1\mathcal{G}}f_\perp + \Re\grave{f}_{1\mathcal{G}}\bar{f}_\perp\} + \{\Im\bar{\grave{f}}_{1\mathcal{G}}g_\perp - \Im\grave{f}_{1\mathcal{G}}\bar{g}_\perp\}\big]\Big\} \\
&+ \big[g^\perp_{\mu\nu}(k,q-k)_\perp + k^\perp_\mu(q-k)^\perp_\nu + k^\perp_\nu(q-k)^\perp_\mu\big]\Big\{\alpha_q\beta_q\big[\{f_\perp\bar{f}_\perp + \bar{f}_\perp f_\perp\} - \{g_\perp\bar{g}_\perp + \bar{g}_\perp g_\perp\}\big] \\
&+ \alpha_q\beta_q\big[\Re h_D(\alpha_q,k_\perp)\bar{h}'^\perp_1(\beta_q,q_\perp-k_\perp) + \Re\bar{h}_D(\alpha_q,k_\perp)h'^\perp_1(\beta_q,q_\perp-k_\perp) \\
&\qquad + h'^\perp_1(\alpha_q,k_\perp)\Re\bar{h}_D(\beta_q,q_\perp-k_\perp) + \bar{h}'^\perp_1(\alpha_q,k_\perp)\Re h_D(\beta_q,q_\perp-k_\perp)\big] \\
&+ \beta_q\{h_1^\perp\Re\bar{h}_{3\mathcal{G}} + \bar{h}_1^\perp\Re h_{3\mathcal{G}}\} + \beta_q\{h_1^\perp\Im\bar{h}_{4\mathcal{G}} + \bar{h}_1^\perp\Im h_{4\mathcal{G}}\} \\
&\qquad + \alpha_q\{\Re\bar{h}_{3\mathcal{G}}h_1^\perp + \Re h_{3\mathcal{G}}\bar{h}_1^\perp\} + \alpha_q\{\Im\bar{h}_{4\mathcal{G}}h_1^\perp + \Im h_{4\mathcal{G}}\bar{h}_1^\perp\}\Big\} \\
&- \big[g^\perp_{\mu\nu}k_\perp^2 + 2k_\mu k_\nu\big]\Big(\frac{(q-k)_\perp^2}{\alpha_q m^2}\{h_1^\perp\Re\bar{h}_{2\mathcal{G}} + \bar{h}_1^\perp\Re h_{2\mathcal{G}}\} + \{\Re h_{1\mathcal{G}}\bar{h} + \Re\bar{h}_{1\mathcal{G}}h\} + \{\Im h_{1\mathcal{G}}\bar{e} - \Im\bar{h}_{1\mathcal{G}}e\}\Big) \\
&- \big[2(q-k)^\perp_\mu(q-k)^\perp_\nu + g^\perp_{\mu\nu}(q-k)_\perp^2\big]\Big(\frac{k_\perp^2}{\beta_q m^2}\{\Re h_{2\mathcal{G}}\bar{h}_1^\perp + \Re\bar{h}_{2\mathcal{G}}h_1^\perp\} + (\{h\Re\bar{h}_{1\mathcal{G}} + \bar{h}\Re h_{1\mathcal{G}}\} + \{\bar{e}\Im h_{1\mathcal{G}} - e\Im\bar{h}_{1\mathcal{G}}\}\Big)\Big]
\end{aligned}
$$

Basis of operators for $\frac{1}{Q^2}$

Power corrections $\sim \frac{1}{Q}$ are unique but corrections $\sim \frac{1}{Q^2}$ depend on the chosen basis of TMD operators because $\varrho \equiv \sqrt{s/2}$

$$EOM: \quad \begin{aligned} \not{A}_\perp(x)\psi(x) &= -i\not{\partial}_\perp\psi(x) - i\frac{1}{\varrho}\not{p}_1\partial_+\psi(x) - i\frac{1}{\varrho}\not{p}_2 D_-\psi(x) \\ \not{B}_\perp(x)\psi(x) &= -i\not{\partial}_\perp\psi(x) - i\frac{1}{\varrho}\not{p}_2\partial_-\psi(x) - i\frac{1}{\varrho}\not{p}_1 D_+\psi(x) \end{aligned}$$

r.h.s. = my choice, l.h.s = Vladimirov, Scimemi *et al*

Example:

$$\frac{1}{8\pi^3 s}\int dx^- dx_\perp \, e^{-i\alpha\varrho x^- + i(k,x)_\perp} \times \langle A|\hat{\bar{\psi}}(x^-, x_\perp)\hat{\not{A}}(x^-, x_\perp)\not{p}_2\gamma_i\hat{\psi}(0)|A\rangle$$

$$= k_i f_1(\alpha, k_\perp) - \alpha k_i\big[f_\perp(\alpha, k_\perp) + ig^\perp(\alpha, k_\perp)\big]$$

The terms $\sim \frac{1}{\varrho}\not{p}_1 D_\pm\psi(x)$ cancel in final expressions for $\frac{1}{Q^2}$ power corrections due to $\bar{q}qG$ operators.
(Of course they are still present in the kinematical corrections)

Application: angular coefficients of Z-boson production

In CMS and ATLAS experiments $s = 8$ TeV, $Q = 80 - 100$ GeV and $Q_\perp$ varies from 0 to 120 GeV.

Our analysis is valid at $Q_\perp = 10 - 30$ GeV and $Y \simeq 0$ ($x_A \sim x_B \sim 0.1$) so that power corrections are small but sizable.

Angular distribution of DY leptons in the Collins-Soper frame ($c_\phi \equiv \cos\phi$, $s_\phi \equiv \sin\phi$ etc.)

$$\frac{d\sigma}{dQ^2 dy d\Omega_l} = \frac{3}{16\pi}\frac{d\sigma}{dQ^2 dy}\Big[(1+c_\theta^2) + \frac{A_0}{2}(1-3c_\theta^2) + A_1 s_{2\theta}c_\phi + \frac{A_2}{2}s_\theta^2 c_{2\phi}$$
$$+ A_3 s_\theta c_\phi + A_4 c_\theta + A_5 s_\theta^2 s_{2\phi} + A_6 s_{2\theta} s_\phi + A_7 s_\theta s_\phi\Big]$$

Back-of-the envelope estimation: take only f_1 contribution at large N_c, use "factorization hypothesis" for TMD $f_1(x,k_\perp) \simeq f(x)g(k_\perp)$ and calculate integrals over $k_\perp$ in the leading log approximation using $f_1(x,k_\perp^2) \simeq \frac{f(x)}{k_\perp^2}$

Result with $\frac{1}{Q^2}$, large-N_c and "f_1" accuracy

$$
\begin{aligned}
\mathbb{W}(q,l,l') &= c_e^2 c_f^2 \frac{Q^4}{|m_Z^2 - Q^2|^2 + \Gamma_Z^2 m_Z^2} \\
\times \sum_f \Big\{ & (a_e^2+1)(a_f^2+1)\Big(\Big[\mathcal{W}^{\rm Ff} - \frac{Q_\perp^2}{2Q^2}(\mathcal{W}^{\rm Ff} - \mathcal{W}_L^{\rm Ff})\Big](1+\cos^2\theta) \\
&+ \frac{Q_\perp^2}{2Q^2}\mathcal{W}_L^{\rm Ff}(1-3\cos^2\theta) + \frac{Q_\perp}{Q}\mathcal{W}_1^{\rm Ff}\sin 2\theta\cos\phi + \frac{Q_\perp^2}{2Q^2}\mathcal{W}^{\rm Ff}\sin^2\theta\cos 2\phi\Big]\Big) \\
&+ 8a_e a_f\Big[\frac{Q_\perp}{Q}\mathcal{W}_3^{\rm Ff}\sin\theta\cos\phi + \mathcal{W}_4^{\rm Ff}\cos\theta\Big]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{W}^{\rm Ff}(q) &= \int d^2k_\perp F^f(q,k_\perp), \qquad \mathcal{W}_L^{\rm Ff}(q) = \int dk_\perp \frac{(q-2k)_\perp^2}{q_\perp^2} F^f(q,k_\perp) \\
\mathcal{W}_1^{\rm Ff}(q) &= \int d^2k_\perp \frac{(q,q-2k)_\perp}{q_\perp^2} F^f(q,k_\perp) \\
\mathcal{W}_3^{\rm Ff}(q) &= \int d^2k_\perp \frac{(q,q-2k)_\perp}{q_\perp^2} \mathcal{F}^f(q,k_\perp), \qquad \mathcal{W}_4^{\rm Ff}(q) = \int d^2k_\perp \mathcal{F}^f(q,k_\perp),
\end{aligned}
$$

$$
\mathcal{F}^f(q,k_\perp) = f_1^f(\alpha_q,k_\perp)\bar{f}_1^f(\beta_q,(q-k)_\perp) - f_1^f \leftrightarrow \bar{f}_1^f
$$

Comparison with LHC results

$$\mathbb{W} \sim \sum_f \mathcal{W}^{\mathrm{Ff}}\Big\{(a_e^2+1)(a_f^2+1)\Big(\Big[1-\frac{Q_\perp^2}{2m_Z^2}+\frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}\Big](1+\cos^2\theta)$$
$$+\frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}(1-3\cos^2\theta)+\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_1^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}\sin 2\theta\cos\phi+\frac{Q_\perp^2}{2m_Z^2}\sin^2\theta\cos 2\phi]\Big)$$
$$+8a_ea_f\Big[\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_3^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}\sin\theta\cos\phi+\frac{\mathcal{W}_4^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}\cos\theta\Big]\Big\}$$

We can easily estimate A_0 and A_2 which depend on $\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}$.

Logarithmic estimate of $\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}}$: if $k_\perp^2 \gg m_N^2$ we can approximate

$$f_1(x,k_\perp^2) \simeq \frac{f(x)}{k_\perp^2} \quad \Rightarrow \quad F(q,k_\perp) \simeq \frac{f(\alpha_q)\bar{f}(\beta_q)+f\leftrightarrow\bar{f}}{k_\perp^2(q-k)_\perp^2}$$

Performing integration over $k_\perp$ in logarithmical approximation, one obtains

$$\frac{\mathcal{W}_L^{\mathrm{Ff}}}{\mathcal{W}^{\mathrm{Ff}}} \simeq 1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}$$

Comparison of A_0 with LHC results

Logarithmic estimate of A_0 (m_z -Z-boson mass, m - proton mass)

$$A_0 \;=\; \frac{Q_\perp^2}{m_z^2}\,\frac{1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}}{1+\frac{Q_\perp^2}{m_z^2}\,\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}} \qquad (*)$$

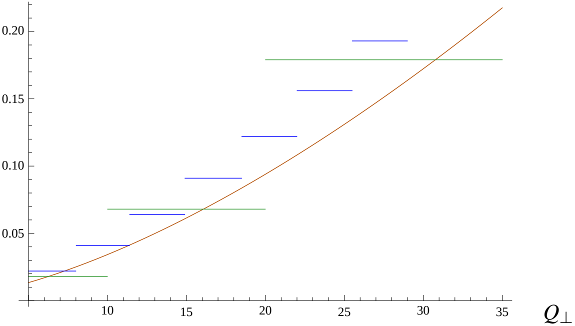


Figure: Comparison of prediction (∗) with lines depicting angular coefficient A_0 in bins of $Q_\perp$ and $Y < 1$ from CMS (arXiv:1504.03512) and ATLAS (arXiv1606.00689)

Comparison of A_2 with LHC results

Logarithmic estimate of A_2

$$A_2 = \frac{Q_\perp^2}{m_z^2}\frac{1}{1+\frac{Q_\perp^2}{m_z^2}\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}} \qquad (**)$$

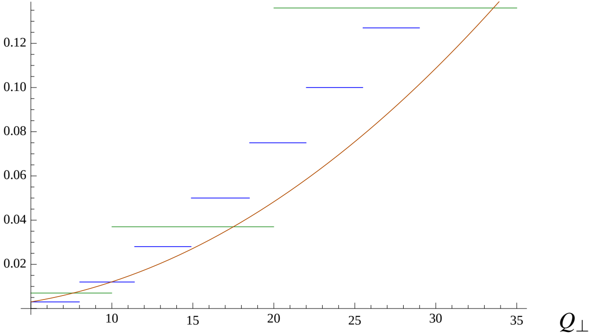


Figure: Comparison of prediction $(**)$ with lines depicting angular coefficient A_2 in bins of $Q_\perp$ and $Y < 1$ from CMS (arXiv:1504.03512) and ATLAS (arXiv1606.00689)

Qualitative checks

$$\mathbb{W} \sim \sum_f r^f \mathcal{W}^{\rm Ff}\Big\{1+\cos^2\theta + \frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}(1-3\cos^2\theta)$$
$$+\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_1^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}\sin 2\theta\cos\phi + \frac{Q_\perp^2}{2m_Z^2 r^f}\sin^2\theta\cos 2\phi]$$
$$+\frac{8a_e a_f}{(a_e^2+1)(a_f^2+1)}\Big[\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_3^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}\sin\theta\cos\phi + \frac{\mathcal{W}_4^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}\cos\theta\Big]\Big\}$$

$r^f \equiv 1 - \frac{Q_\perp^2}{2m_Z^2} + \frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}}$

Qualitative checks:

- Factorization of TMD $f_1(x,k_\perp^2) \simeq f(x)f(k_\perp^2) \Rightarrow \mathcal{W}_1^{Ff}(q) = 0$
  $\Rightarrow A_1$ is smaller than A_2
- A_4 does not depend on $Q_\perp$ and increases with rapidity
- A_3 is smaller than A_4
- A_5, A_6, A_7 are order of magnitude smaller than A_0, A_2, A_4

Conclusions and outlook

1. Conclusions
   - Power corrections $\sim \frac{1}{Q^2}$ for DY hadronic tensor $\Rightarrow$ “gauge-invariant completion” of the LT result.
   - Bookkeeping: full list of $\frac{1}{Q^2}$ power corrections.
   - Back-of-the-envelope estimates of angular distributions for DY Z-boson production are in good agreement with LHC data.
2. Outlook
   - Power corrections for SIDIS

Thank you for attention!