The Axial Current and its Divergence

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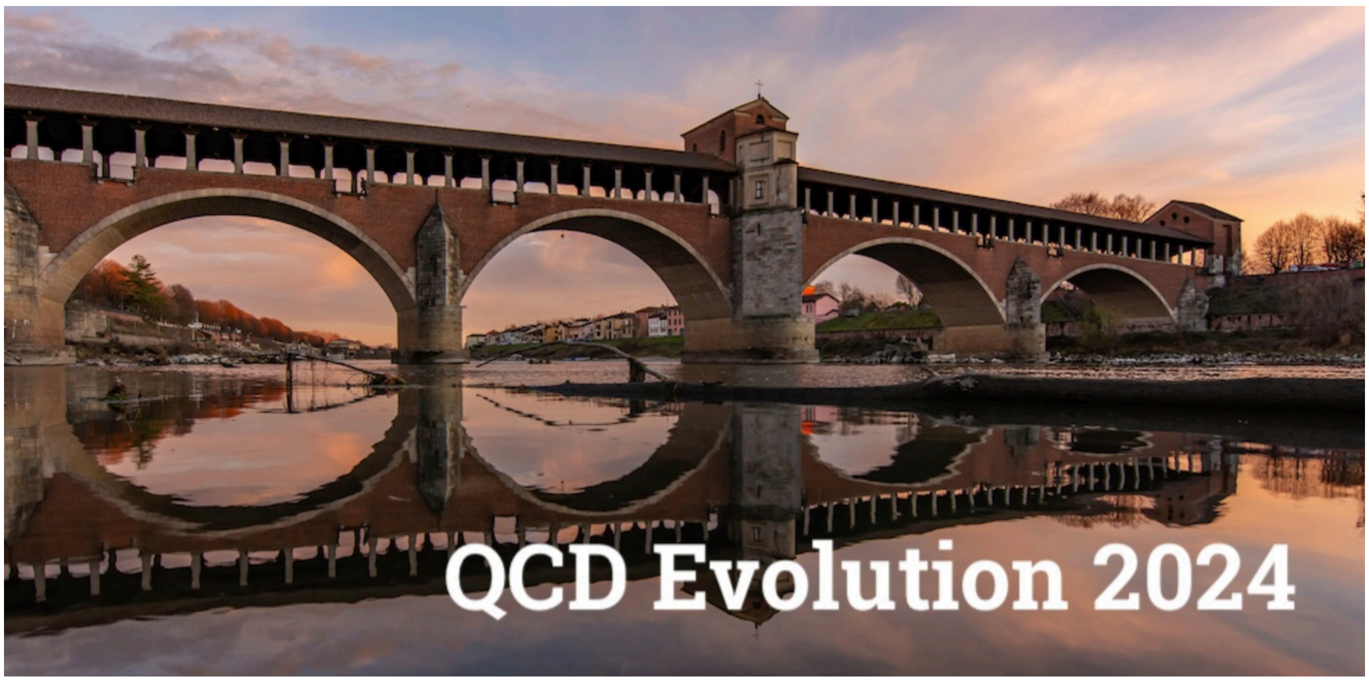

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Outline

- Introduction and motivation

- Perturbative calculations involving the axial current
  - parton distribution
  - local current and form factor
  - generalized parton distributions

- Summary

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Introduction and Motivation

- Throughout we mostly consider QED, with one fermion type
- Vector current

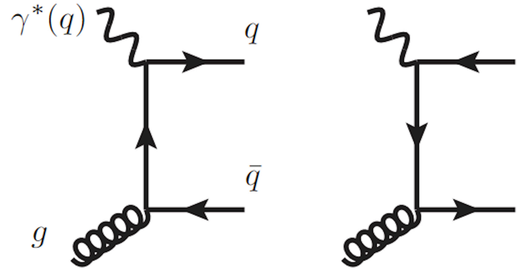
$$\begin{aligned} J^\mu(x) &= \bar{\psi}(x)\,\gamma^\mu\,\psi(x) \\ \partial_\mu J^\mu(x) &= 0 \end{aligned}$$

- Axial current

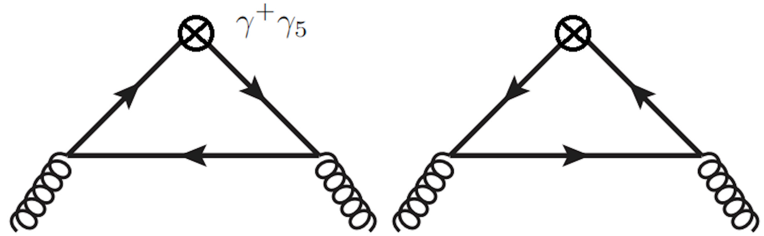
$$\begin{aligned} J_5^\mu(x) &= \bar{\psi}(x)\,\gamma^\mu\gamma_5\,\psi(x) \\ \partial_\mu J_5^\mu(x) &= 2im\,\bar{\psi}(x)\,\gamma_5\,\psi(x) - \frac{\alpha_{\rm em}}{2\pi}\,F^{\mu\nu}(x)\widetilde{F}_{\mu\nu}(x) \end{aligned}$$

  - axial current not conserved due to (i) nonzero fermion mass and (ii) chiral anomaly (Adler, 1969 / Bell, Jackiw, 1969 / Adler, Bardeen, 1969 / ...)
  - chiral anomaly can be derived, e.g., by evaluating $J_5^\mu(x)$ between photon states
  - chiral anomaly was intensively discussed in hadronic physics soon after discovery of nucleon spin crisis through DIS measurements

- Pioneering work (Altarelli, Ross, 1988 (AR) / Carlitz, Collins, Mueller, 1988 (CCM) / ...)
  - considering process $\gamma^* + g \to q + \bar{q}$



  - extracting leading power-term of $1/q^2$ expansion and integrating upon x
    $\to$ calculation of local axial current



  - overall conclusion: difference between measured $(\Delta\Sigma)$ and "intrinsic" $(\Delta\widetilde{\Sigma})$ quark-spin contributions

$$\Delta\Sigma = \Delta\widetilde{\Sigma} - \frac{\alpha_s N_f}{2\pi} \Delta G$$

    * term proportional to ΔG due to chiral anomaly (?)
    * explanation of nucleon spin crisis (?)

- Critique of pioneering papers (Jaffe, Manohar, 1989 / Bodwin, Qiu, 1989 / ...)
  - main concern: result depends on infrared (IR) regulator
    AR: nonzero quark mass m in denominators of propagators
    CCM: nonzero off-shellness p^2 of gluons / find zero if $m \neq 0$ used throughout
  - this concern, and need for very large ΔG, raised severe doubts

- Recent renewed interest in field
  (Tarasov, Venugopalan, 2021, 2022 / Bhattacharya, Hatta, Vogelsang, 2022, 2023)
  - considered also the x-dependence as opposed to x-integrated results only
  - statements include:
    * need off-forward kinematics to capture physics of anomaly
    * GPDs may have more robust connection to anomaly than PDFs
    * anomaly manifests in pole contribution for $t = \Delta^2 \to 0$
    * anomaly pole could challenge factorization (not stated in all papers)
  - papers reached important conclusions based on perturbative calculations

- Our motivations
  - revisit dependence of perturbative calculations on IR regulator
  - what role is played by fermion mass ?
  - relation between "classic papers" and more recent work ?

Parton Distribution in Perturbation Theory

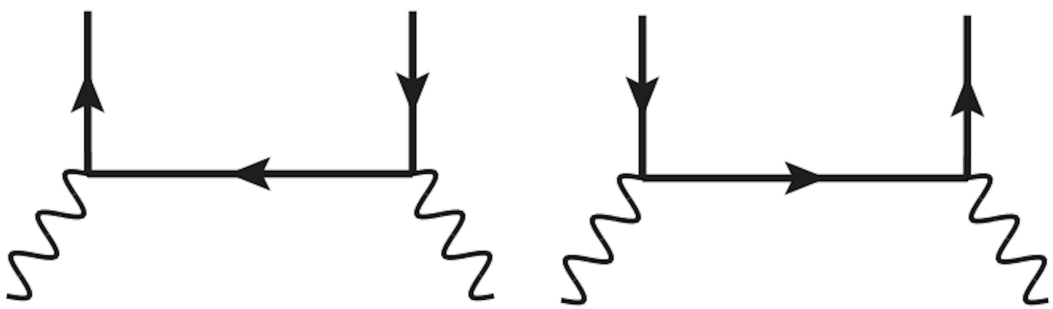
- Definition of PDF

$$F^{[\gamma^+\gamma_5]}_{\lambda,\lambda'}(x) = \int \frac{dz^-}{4\pi}\, e^{ik\cdot z}\, \langle \gamma(p,\lambda')|\bar{\psi}(-\tfrac{z}{2})\,\gamma^+\gamma_5\,\psi(\tfrac{z}{2})|\gamma(p,\lambda)\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= \frac{i}{p^+}\,\varepsilon^{+\,\varepsilon\,\varepsilon'\,p}\, g_1(x) = \frac{i}{p\cdot n}\,\varepsilon^{n\,\varepsilon\,\varepsilon'\,p}\, g_1(x)$$

$$g_1(x) \sim \left(F^{[\gamma^+\gamma_5]}_{+,+}(x) - F^{[\gamma^+\gamma_5]}_{-,-}(x)\right)$$ circularly polarized photons

- Leading-order diagrams



  – two diagrams contribute in different regions of x

- Result for $m \neq 0$ and off-shellness $p^2 < 0$, for $0 \leq x \leq 1$ $\quad (\bar{\mu}^2 = 4\pi e^{-\gamma_E}\mu^2)$

$$g_1(x,\mu) = \frac{\alpha_{\rm em}}{2\pi}\left[\left(\frac{1}{\varepsilon} + \ln\frac{\bar{\mu}^2}{m^2 - p^2 x(1-x)}\right)(1-2x) - \frac{p^2 x(1-x)}{m^2 - p^2 x(1-x)}\right] + \mathcal{O}(\varepsilon)$$

  - $\int dx\, g_1$ provides total spin contribution

- UV behavior
  - $g_1(x,\mu)$ UV-divergent, divergence regulated using dimensional regularization (DR)
  - $\int dx\, g_1$ UV-finite, does not depend on UV regulator

- IR behavior
  - $g_1(x,\mu)$ IR-divergent, divergence regulated using nonzero m and p^2
  - result well behaved for $m \neq 0$ and $p^2 = 0$
  - result well behaved for $m = 0$ and $p^2 \neq 0$, except for endpoints $x = 0, 1$
  - also DR could be used as IR regulator; in that case

$$\int d^{n-2}\vec{k}_\perp \frac{1}{\vec{k}_\perp^{\,2}} \sim \frac{1}{\varepsilon_{\rm UV}} - \frac{1}{\varepsilon_{\rm IR}}$$

    separation of UV and IR divergence needed, otherwise $g_1(x,\mu) = 0$

  - $\int dx\, g_1$ IR-finite, and does depend on IR regulator

- Integral upon x
  - full result

$$\int_{-1}^{1} dx\, g_1(x,\mu) = \frac{\alpha_{\rm em}}{\pi} \int_0^1 dx\, \frac{-p^2 x(1-x)}{m^2 - p^2 x(1-x)}$$

$$= \frac{\alpha_{\rm em}}{\pi} \left[1 - \int_0^1 dx\, \frac{2m^2(1-x)}{m^2 - p^2 x(1-x)} \right]$$

  - after $\alpha_{\rm em} \to \frac{1}{2}\,\alpha_{\rm s}\, N_f$, full agreement with CCM (1988)
  - special cases

$$\int_{-1}^{1} dx\, g_1(x,\mu)\Big|_{m\neq 0,\, p^2=0} = 0 \qquad \int_{-1}^{1} dx\, g_1(x,\mu)\Big|_{m=0,\, p^2\neq 0} = \frac{\alpha_{\rm em}}{\pi}$$

  - one can understand origin of

$$\Delta\Sigma = \Delta\widetilde{\Sigma} - \frac{\alpha_{\rm s}\, N_f}{2\pi}\, \Delta G$$

  - also result of AR (1988) can be obtained by computing $\int dx\, g_1$ in their scheme

Local Axial Current in Perturbation Theory

- Divergence of axial current
  - recall operator

$$\partial_\mu J_5^\mu(x) = 2im\,\bar\psi(x)\,\gamma_5\,\psi(x) - \frac{\alpha_{\rm em}}{2\pi}\,F^{\mu\nu}(x)\widetilde{F}_{\mu\nu}(x)$$

  - matrix element of anomaly term $\quad (P = \frac{1}{2}(p+p'),\ \Delta = p' - p)$

$$-\frac{\alpha_{\rm em}}{2\pi}\,\langle\gamma(p',\lambda')|F^{\mu\nu}(0)\widetilde{F}_{\mu\nu}(0)|\gamma(p,\lambda)\rangle = \frac{2\,\alpha_{\rm em}}{\pi}\,\varepsilon^{\,\varepsilon\,\varepsilon'\,P\,\Delta} \;\to\; \Delta \neq 0 \ \ \text{needed}$$

  - matrix element of mass term $\quad (\tau = -\Delta^2/m^2 > 0)$

$$2im\,\langle\gamma(p',\lambda')|\bar\psi(0)\,\gamma_5\,\psi(0)|\gamma(p,\lambda)\rangle = -\frac{2\,\alpha_{\rm em}}{\pi}\,\varepsilon^{\,\varepsilon\,\varepsilon'\,P\,\Delta}\,\frac{1}{\tau}\,\ln^2\frac{\sqrt{\tau+4}-\sqrt{\tau}}{\sqrt{\tau+4}+\sqrt{\tau}}$$

$$\overset{\tau\to 0}{\longrightarrow} -\frac{2\,\alpha_{\rm em}}{\pi}\,\varepsilon^{\,\varepsilon\,\varepsilon'\,P\,\Delta} \quad \text{independent of } m\ (\neq 0)$$

$\to$ for $\Delta^2 = 0$, exact cancellation between anomaly term and fermion mass term

- General structure of axial current

$$\Gamma_5^\mu = \langle \gamma(p', \lambda') |\, J_5^\mu(0)\, | \gamma(p, \lambda) \rangle = \sum_{i=1}^{3} \widetilde{G}_i(\Delta^2)\, A_i^\mu$$

$$A_1^\mu = i\, \varepsilon^{\mu\, \varepsilon\, \varepsilon'\, P}$$

$$A_2^\mu = \frac{i}{2\Delta^2}\, \Delta^\mu\, \varepsilon^{\varepsilon\, \varepsilon'\, P\, \Delta}$$

$$A_3^\mu = \frac{i}{\Delta^2} \left(\varepsilon \cdot P\, \varepsilon^{\mu\, \varepsilon'\, P\, \Delta} + \varepsilon' \cdot P\, \varepsilon^{\mu\, \varepsilon\, P\, \Delta} \right)$$

A_2 and A_3 do not exhibit a pole for $\Delta^2 \to 0$

  - Schouten identity

$$A_2^\mu = -\frac{1}{2} A_1^\mu + A_3^\mu$$

  - Ward identities related to incoming/outgoing photon provide one more constraint

$$\Gamma_5^\mu = G(\Delta^2)\, A_2^\mu$$

$\to$ local current is parametrized through just one form factor

- Axial current in perturbation theory
  - consider axial (anomalous) Ward identity

$$\langle\, J_5^\mu(x)\,\rangle = \Gamma_5^\mu(x) = \Gamma_5^\mu\, e^{i\Delta\cdot x}$$

$$\to\ \partial_\mu\, \Gamma_5^\mu(x) = i\Delta_\mu\, \Gamma_5^\mu\, e^{i\Delta\cdot x} = \langle\, \partial_\mu J_5^\mu(0)\,\rangle\, e^{i\Delta\cdot x}$$

  - here, axial current fully determined by its divergence
  - final result ($m \neq 0$ as IR regulator)

$$\Gamma_5^\mu = G(\Delta^2)\,\frac{i}{2\Delta^2}\,\Delta^\mu\,\varepsilon^{\varepsilon\,\varepsilon'\,P\,\Delta}$$

$$G(\Delta^2) = \frac{4\,\alpha_{\rm em}}{\pi}\left[\frac{1}{\tau}\ln^2\frac{\sqrt{\tau+4}-\sqrt{\tau}}{\sqrt{\tau+4}+\sqrt{\tau}} - 1\right] \overset{\tau\to 0}{\to} 0$$

    * anomaly makes form factor vanish for $\Delta^2 = 0$
    * matrix element Γ_5^μ vanishes for $\Delta = 0$ for on-shell photons
  - considering $\langle\, \partial_\mu J_5^\mu(0)\,\rangle$ is the easiest way to compute $G(\Delta^2)$

- Considering the forward limit
  - relation between PDF g_1 and form factor G

$$\int_{-1}^{1} dx\, g_1(x,\mu) = -\frac{1}{4}\, G(0)$$

  - by computing $G(0)$ (for nonzero m and p^2) we (again) find result of CCM (1988)

$$\int_{-1}^{1} dx\, g_1(x,\mu) = \frac{\alpha_{\rm em}}{\pi}\left[1 - \int_0^1 dx\, \frac{2m^2(1-x)}{m^2 - p^2 x(1-x)}\right]$$

  - result for $\int dx\, g_1$ depends on anomaly and fermion mass term in $\partial_\mu J_5^\mu$
  - for $m \neq 0$ and $p^2 = 0$, anomaly leads to $\Delta\Sigma = \Delta\widetilde{\Sigma}$
  - in scheme of AR (1988), one would neglect quark mass term in $\partial_\mu J_5^\mu$

$$\int_{-1}^{1} dx\, g_1(x,\mu) = \frac{\alpha_{\rm em}}{\pi}$$

$\rightarrow$ explanation why (nonzero) results by AR (1988) and CCM (1988) agree (?)

Generalized Parton Distributions in Perturbation Theory

- Definition (using Schouten identities and Ward identities)

$$F^{[\gamma^+\gamma_5]}_{\lambda,\lambda'}(x,\Delta) = \int \frac{dz^-}{4\pi}\, e^{ik\cdot z}\, \langle \gamma(p',\lambda')|\bar{\psi}(-\tfrac{z}{2})\,\gamma^+\gamma_5\,\psi(\tfrac{z}{2})|\gamma(p,\lambda)\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$= B_1\,\widetilde{H}_1(x,\xi,\Delta^2) + B_2\,\widetilde{H}_2(x,\xi,\Delta^2)$$

  - agreement with BHV (2022, 2023) about number of independent terms

  - structures B_i and gauge invariance (Ward identities)

$$B_1 \overset{\Delta\to 0}{\to} \frac{1}{P^+}\,A_1^+ \qquad\qquad B_2 = \frac{i}{2\Delta^2}\,\frac{\Delta\cdot n}{P\cdot n}\,\varepsilon^{\varepsilon\,\varepsilon'\,P\,\Delta} = \frac{1}{P^+}\,A_2^+$$

$$B_i(\varepsilon\to p) = B_i(\varepsilon'\to p') = 0$$

  - to extract two GPDs, one can use circularly and linearly polarized photons

- Usage of nonzero Δ: (i) IR regulator; (ii) generates new structure
  - if no other IR regulator, one cannot recover forward limit of matrix element

- Forward limit, using (additional) IR regulator

$$\lim_{\Delta \to 0} F^{[\gamma^+ \gamma_5]}_{\lambda, \lambda'}(x, \Delta) = F^{[\gamma^+ \gamma_5]}_{\lambda, \lambda'}(x)$$

$$\widetilde{H}_1(x, 0, 0) = g_1(x)$$

- Comparison with local current (form factor)
  (see also Tarasov, Venugopalan, 2021, 2022 / Bhattacharya, Hatta, Vogelsang, 2022, 2023)

$$\int_{-1}^{1} dx \, \widetilde{H}_1(x, \xi, \Delta^2) = 0$$

$$\int_{-1}^{1} dx \, \widetilde{H}_2(x, \xi, \Delta^2) = \frac{1}{2} G(\Delta^2) \to \text{relation with anomaly}$$

- Our perturbative GPD results satisfy quoted constraints

- GPD results for $m = 0$ and $\Delta_\perp \neq 0$ (shown for $\xi \leq x \leq 1$ only)

$$\widetilde{H}_1(x,\xi,\Delta^2) \sim \frac{\alpha_{\rm em}}{2\pi}\Big[\frac{1-2x+\xi^2}{1-\xi^2}\Big(\frac{1}{\varepsilon} - \ln\Big(-\frac{\Delta^2}{\bar{\mu}^2}\Big) - \ln\frac{(1-x)^2}{1-\xi^2}\Big) - 2\,\frac{1-x}{1-\xi^2}\Big]$$

$$\widetilde{H}_2(x,\xi,\Delta^2) \sim \frac{\alpha_{\rm em}}{2\pi}\Big[-2\,\frac{1-x}{1-\xi^2}\Big]$$

  - full agreement with results of BHV (2023)
  - presence of $\ln(-\Delta^2/\bar{\mu}^2)$ prevents one from taking forward limit
  - upon integration, results satisfy

$$\int_{-1}^{1} dx\, \widetilde{H}_1(x,\xi,\Delta^2) = 0 \qquad \int_{-1}^{1} dx\, \widetilde{H}_2(x,\xi,\Delta^2) = \frac{1}{2}\,G(\Delta^2)$$

  - $G(\Delta^2) = G(0)$ anomaly effect that was computed by CCM (1988) for $p^2 \neq 0$
  - term $\sim \widetilde{H}_2(x,\xi,\Delta^2)$ in $F^{[\gamma^+\gamma_5]}_{\lambda,\lambda'}(x,\Delta)$ has no pole

- GPD results for $m \neq 0$ and $\Delta_\perp = 0$ (implies $\Delta^2 = 0$)

$$\widetilde{H}_1^{\mathrm{DGLAP}}(x,\xi,0) = \frac{\alpha_{\mathrm{em}}}{2\pi}\left(\frac{1}{\varepsilon} + \ln\frac{\bar{\mu}^2}{m^2}\right)\frac{1-2x+\xi^2}{1-\xi^2} \qquad (\xi \leq x \leq 1)$$

$$\widetilde{H}_1^{\mathrm{ERBL}}(x,\xi,0) = \frac{\alpha_{\mathrm{em}}}{2\pi}\left(\frac{1}{\varepsilon} + \ln\frac{\bar{\mu}^2}{m^2}\right)\frac{(1-\xi)(x+\xi)}{2\xi(1+\xi)} + (x \to -x)$$

$$\widetilde{H}_2^{\mathrm{DGLAP}}(x,\xi,0) = \widetilde{H}_2^{\mathrm{ERBL}}(x,\xi,0) = 0$$

  - for $\xi \to 0$, we recover result for $g_1(x)$
  - $\int dx\, \widetilde{H}_1 = \int dx\, \widetilde{H}_2 = 0$, in agreement with $G(0)|_{m\neq 0} = 0$

- GPD results for $m \neq 0$ and $\Delta_\perp \neq 0$
  - for $\Delta_\perp \to 0$, we recover results above
  - again, we find agreement with

$$\int_{-1}^{1} dx\, \widetilde{H}_1(x,\xi,\Delta^2) = 0 \qquad \int_{-1}^{1} dx\, \widetilde{H}_2(x,\xi,\Delta^2) = \frac{1}{2}\, G(\Delta^2)$$

Summary

- Potential imprints of chiral anomaly in polarized DIS and DVCS have been discussed in literature

- We confirm "classic" results by AR (1988) and CCM (1988) for DIS

$$\Delta\Sigma = \Delta\widetilde{\Sigma} - \frac{\alpha_s N_f}{2\pi} \Delta G$$

- Perturbative results (for PDF, FF, GPDs) depend on IR scheme

- Going from $m = 0$ to $m \neq 0$ can qualitatively change results

- How to embed anomaly-related perturbative results in full process has been a matter of debate

- Additional contribution arises for $\Delta \neq 0$
  (Tarasov, Venugopalan, 2021, 2022 / Bhattacharya, Hatta, Vogelsang, 2022, 2023)

- Additional contribution has no pole for $\Delta \to 0$ (and no challenge for factorization)

- Perturbative calculations show that imprints of anomaly can be seen by (i) using off-shell photons and/or (ii) going to off-forward kinematics