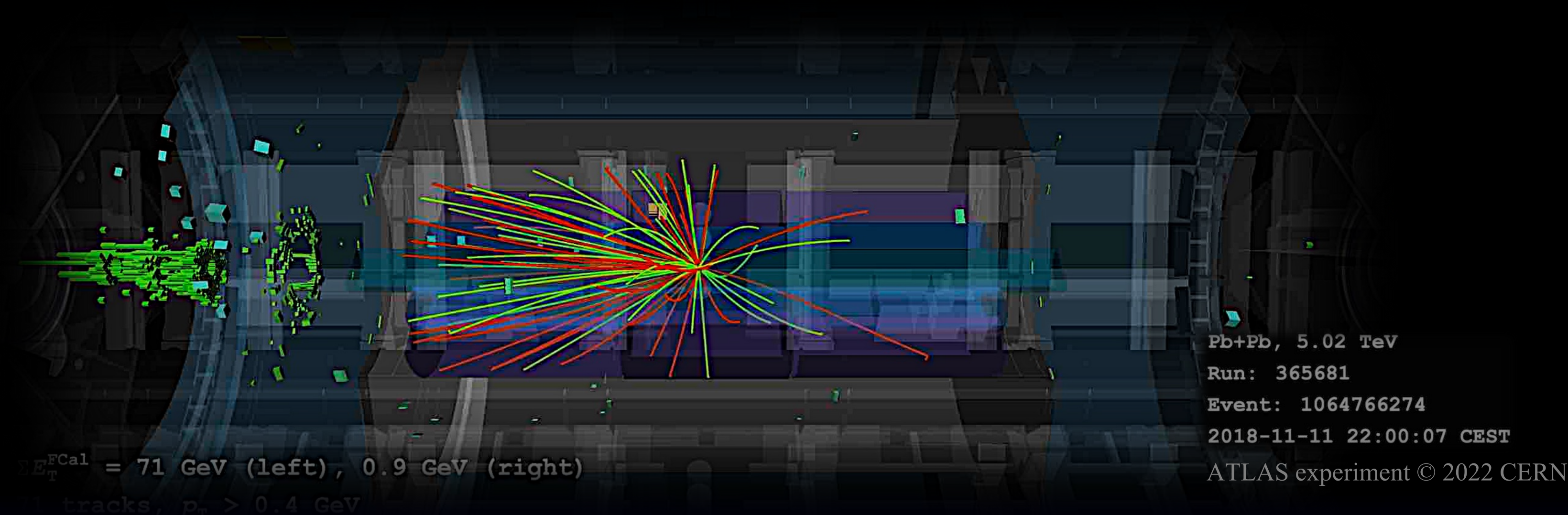
Factorization of *ep* diffraction

Stella Schindler

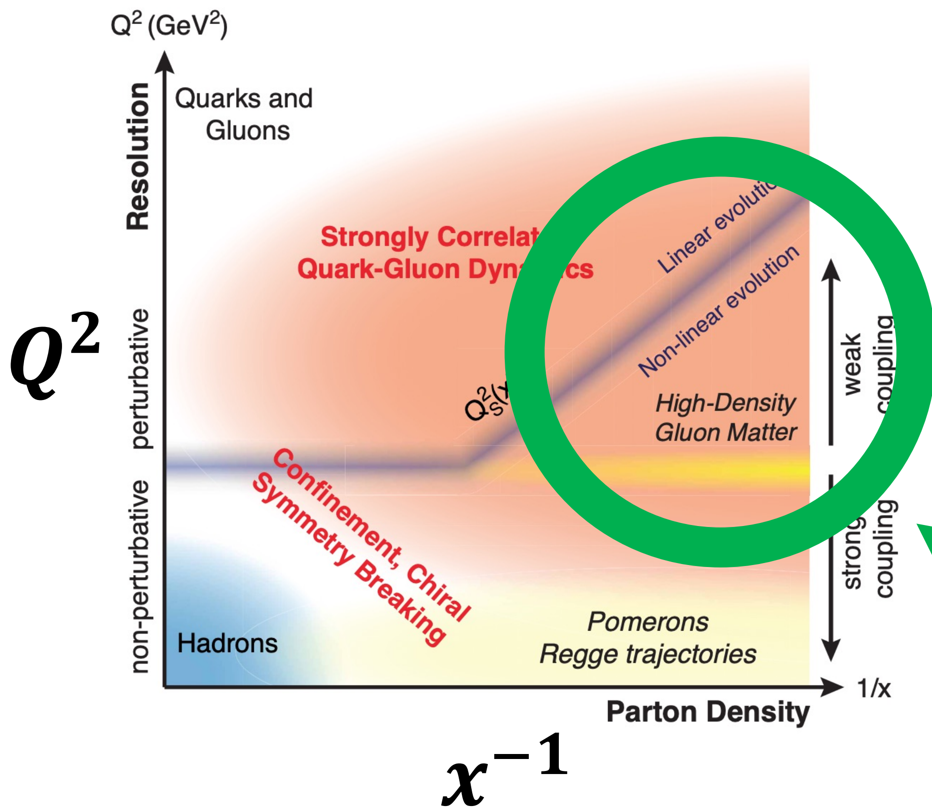


QCD Evolution Workshop
Università di Pavia
Friday, May 31, 2024

Support

Collaborators
Iain Stewart (MIT)
Kyle Lee (MIT)

Diffraction

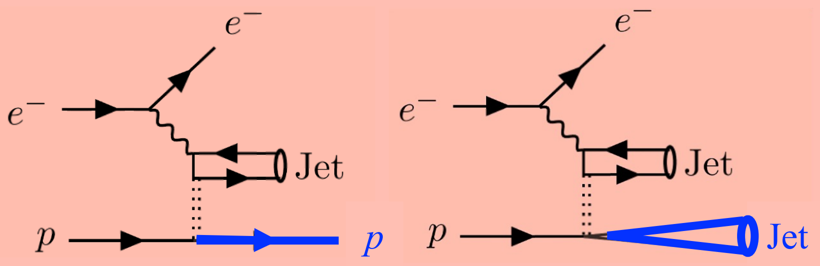


- Forward-scattering event with a **large rapidity gap**
- Double-digit percent of events at colliders
- Important probe of **small-*x* physics**

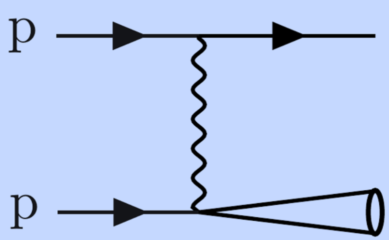
Pomeranchuk & Feinberg (1953, 1956). Review, e.g. Frankfurt et al., 2203.12289. Figure: 1708.01527.

Wide range of diffractive processes

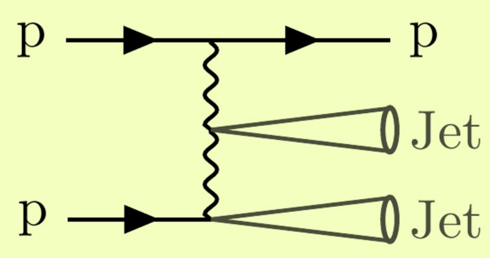
Coherent or incoherent



***eA*, *AA*, *ep*, *pp* collisions**



Single or multi jet/gap



Tagged final states

Heavy mesons

Dijet photoproduction

Etc.

How well do we understand diffraction?

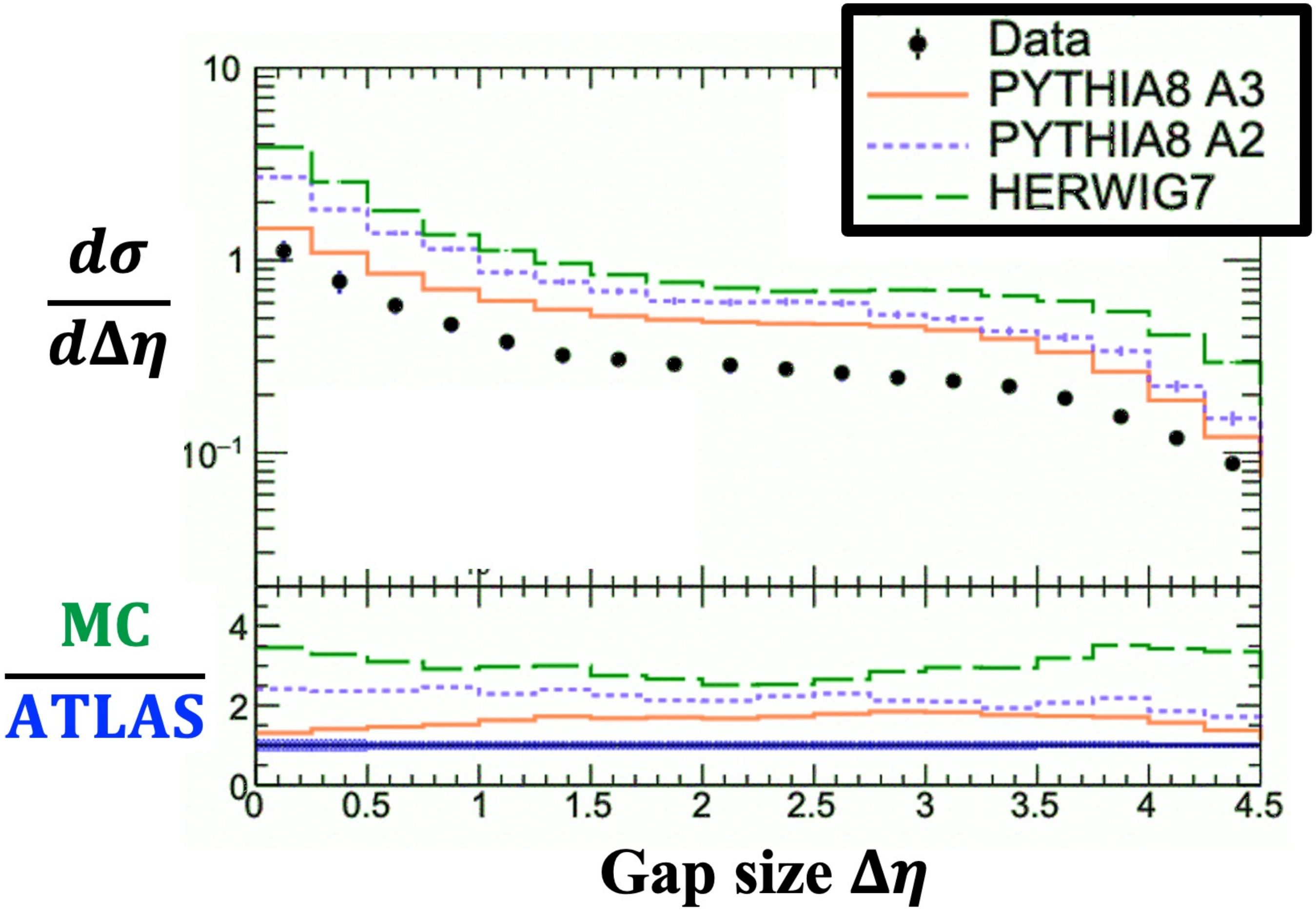
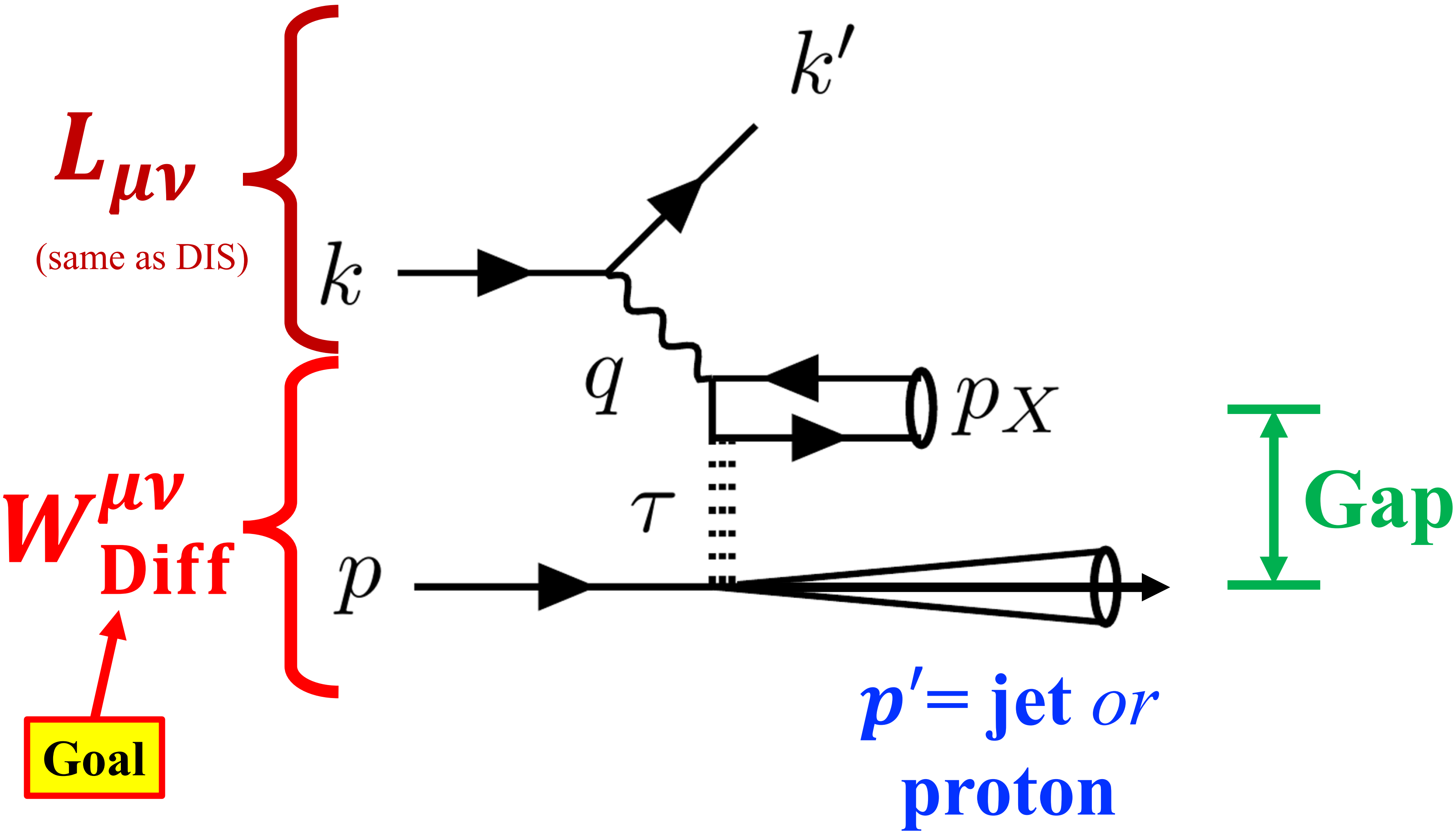


Figure: ATLAS, 1911.00453.

Kinematics

Single-gap diffractive *ep* scattering



Lorentz invariants (7 are independent)

Shared with DIS

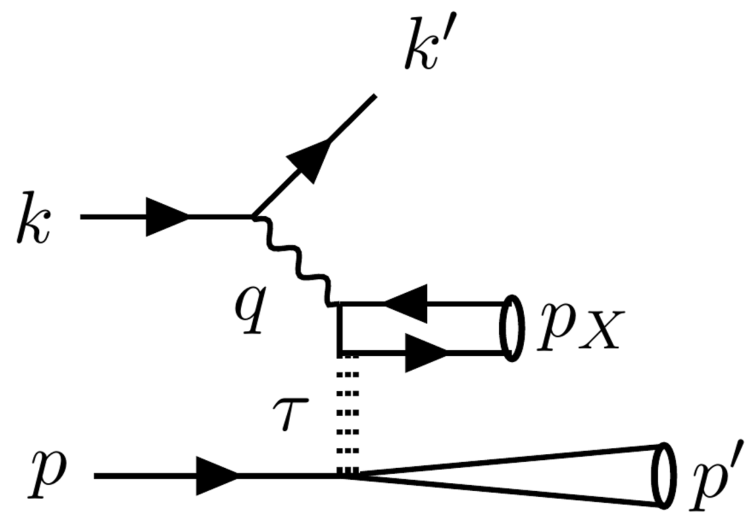
$$Q^2 = -q^2$$

$$W^2 = (p+q)^2$$

$$s = (p+k)^2$$

$$x = \frac{Q^2}{2p \cdot q}$$

$$y = \frac{p \cdot q}{p \cdot k}$$



Specific to diffraction (use *p*′)

$$m_J^2 = p'^2 > 0$$

$$m_X^2 = p_X^2 > 0$$

$$t = \tau^2 < 0$$

$$\beta = \frac{Q^2}{2q \cdot \tau}$$

$$\xi = \frac{q \cdot \tau}{q \cdot p}$$

$$z = \frac{p \cdot p'}{p \cdot q}$$

$$\bar{x} = \frac{k \cdot \tau}{k \cdot p}$$

Diffractive phase space

DIS invariants

$$0 < x < 1, \quad 0 < y < 1, \quad 0 < Q^2 < s$$

Diffractive invariants

$$\frac{1}{1 + \frac{-t}{Q^2} + \left(\frac{\bar{x}}{x} - y\right)(1 - z)} < \beta < \frac{1}{1 + \frac{-t}{Q^2}}$$

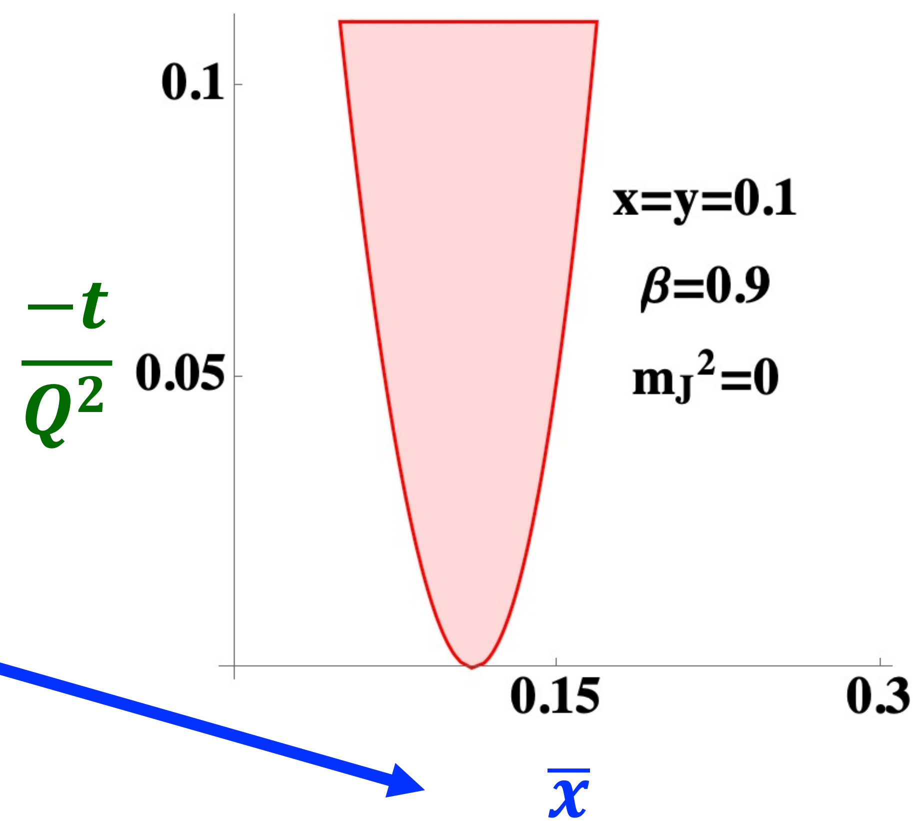
$$\Lambda_{QCD}^2 \lesssim m_J^2 < \frac{1 - \bar{x}}{\bar{x}}(-t)$$

$$\frac{\bar{x}}{1 - \bar{x}} \frac{m_J^2}{Q^2} < \frac{-t}{Q^2} < \frac{1 - \beta}{\beta}$$

$$-1 < \frac{\beta x y z - 2\beta x z + x - \beta \bar{x}}{2\sqrt{\beta x z (1 - y)(\beta x z - x + \beta)}} < 1$$

Etc.

Longitudinal momentum fraction $\bar{x} = \frac{k \cdot \tau}{k \cdot p}$



Lee, **Schindler**, & Stewart, in preparation.

Measure p, p', q $\Rightarrow$ 4 structure functions

Like DIS (no p')

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \boldsymbol{F_1^D} + \frac{1}{2x} U^\mu U^\nu \boldsymbol{F_2^D}$$

$$+ \frac{1}{x}\left(X^\mu X^\nu - \frac{1}{2} U^\mu U^\nu\right) \boldsymbol{F_3^D} + \frac{1}{2x}(U^\mu X^\nu + X^\mu U^\nu) \boldsymbol{F_4^D}$$

Orthonormal basis:

- $\boldsymbol{q^\mu}$
- $\boldsymbol{U^\mu} = \frac{2x}{Q}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)$
- $\boldsymbol{X^\mu} = \frac{1}{(U\cdot V)^2 - V^2}\left(V^\mu - \frac{U\cdot V}{V^2} U^\mu\right)$
- Auxiliary: $V^\mu = \frac{2x}{Q}\left(p'^\mu - \frac{p'\cdot q}{q^2} q^\mu\right)$

Typically only talk about F_2 & F_L

Punchline: $\boldsymbol{F_3}$ & $\boldsymbol{F_4}$ are big!

Arens et al., hep-ph/9605376.
Also see e.g. Blumlein & Robaschik hep-ph/0106037.

Coefficients of structures

$$L_{\mu\nu}W^{\mu\nu} = -\frac{Q^2}{x}\boldsymbol{F_L^D} + Q^2\frac{1+(1-y)^2}{xy^2}\boldsymbol{F_2^D} + \frac{4(\boldsymbol{k}\cdot\boldsymbol{X})^2+Q^2}{x}\boldsymbol{F_3^D} + \frac{4(\boldsymbol{k}\cdot\boldsymbol{X})(k\cdot U)}{x}\boldsymbol{F_4^D}$$

$\bar{\boldsymbol{x}}$ & y are in coefficients, not in

$$\boldsymbol{F_i^D}(x, Q^2, \beta, m_J^2, t)$$

How to miss $\boldsymbol{F_3}$, $\boldsymbol{F_4}$:

- Integrate over $\bar{x}$
- Set $p' \propto p$

Coefficients:

- $k\cdot X = Q^2\frac{x-\bar{x}\beta-(2-y)xz\beta}{2N_X xy\beta}$
- $k\cdot U = \frac{Q(2-y)}{2y}$

Auxiliary:

- $N_X^2 = -t + z^2Q^2 - \frac{zQ^2}{\beta}$
- $z = \frac{x}{Q^2}(m_J^2 - t)$

Lee, **Schindler**, & Stewart, in preparation.

Factorization

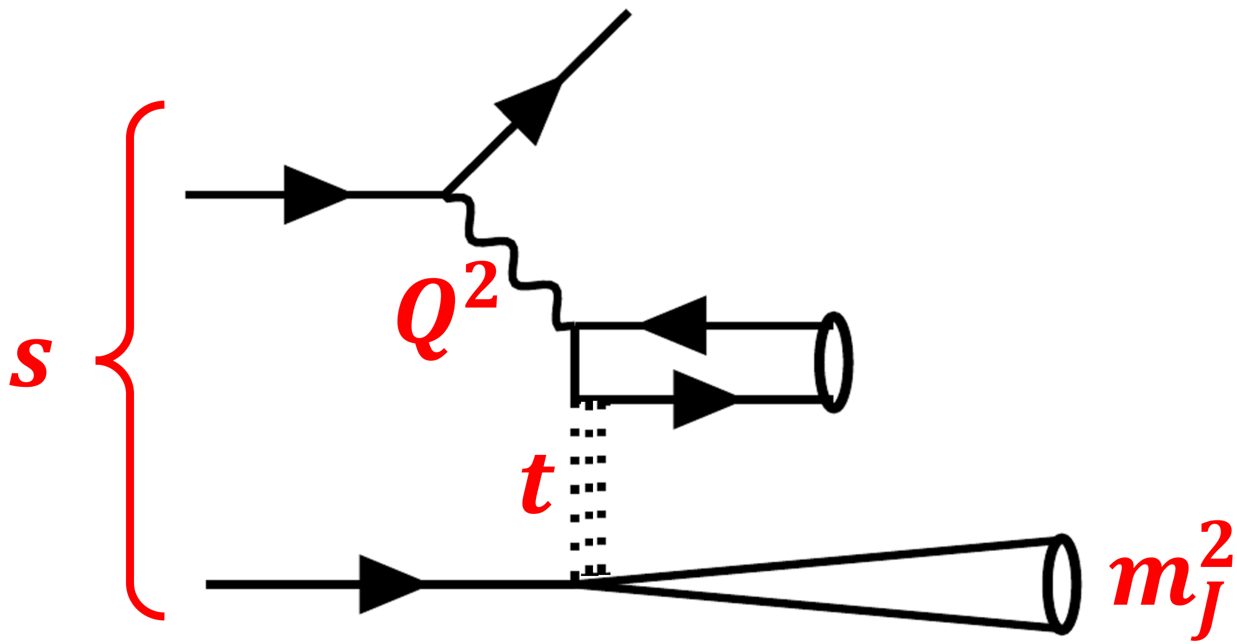
Many power expansions in diffraction

Focus today:

$$\lambda = \frac{Q}{\sqrt{s}} \qquad \lambda_t = \frac{\sqrt{-t}}{Q}$$

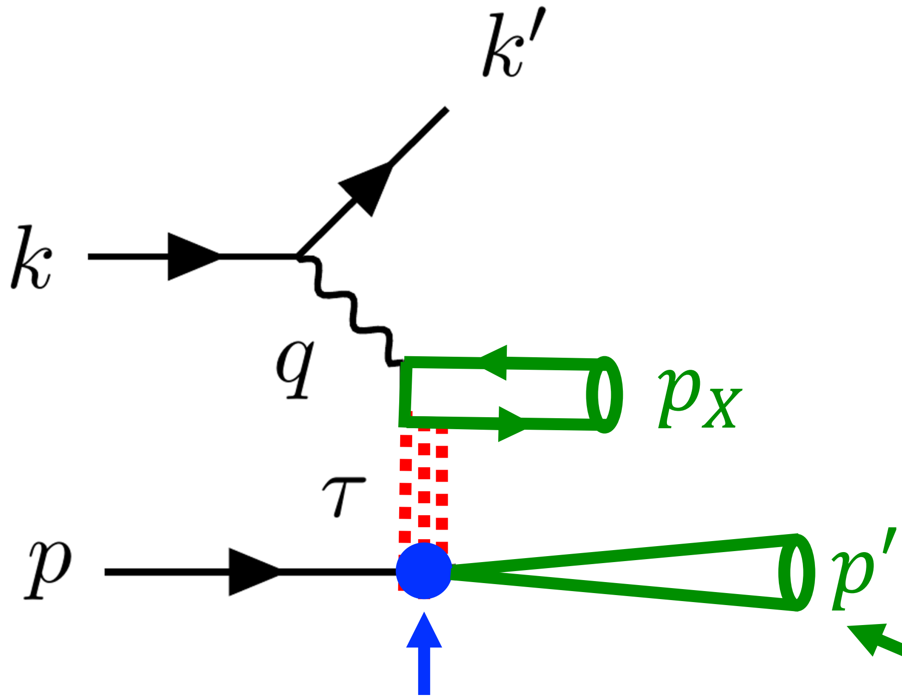
Other options:

$$\lambda_\Lambda = \frac{\Lambda_{\text{QCD}}}{Q} \qquad \rho = \frac{m_J}{\sqrt{-t}}$$



Lee, **Schindler**, & Stewart, in preparation.

Constraints on diffraction



Rapidity gap:

$$\ln \frac{p_X^-}{p_X^+} \ll \ln \frac{p'^-}{p'^+}$$

Forward scattering:

$$-t \ll W^2$$

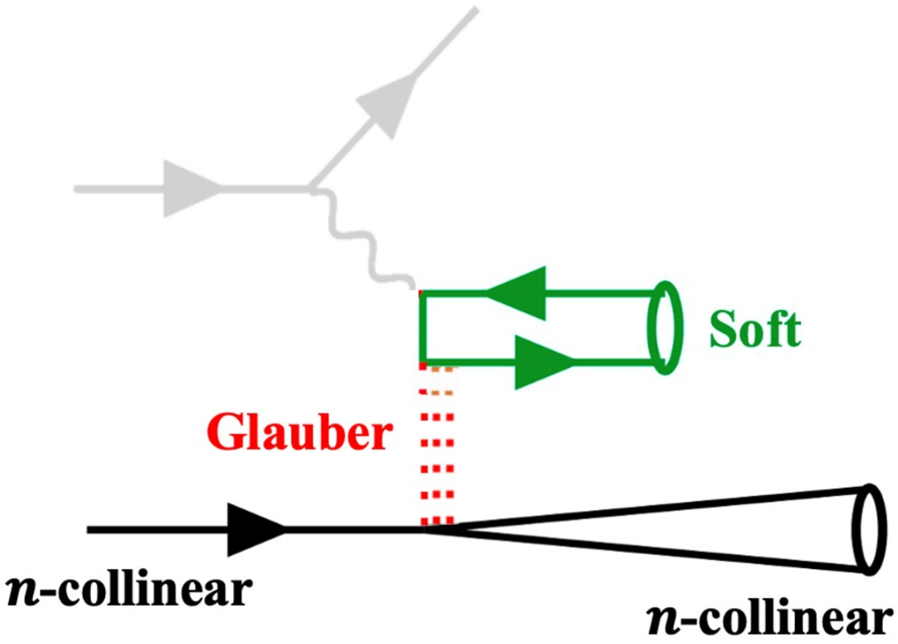
Collimated jets:

$$\Lambda_{\mathrm{QCD}}^2 \ll m_{\mathrm{J}}^2, m_X^2 \ll W^2$$

Forward + Gap + Jets $\Longrightarrow$ $\boldsymbol{\lambda = \frac{Q}{\sqrt{s}} \ll 1}$

Diffraction from SCET

Forward + Gap + Jets $\Longrightarrow$ Scaling of momenta



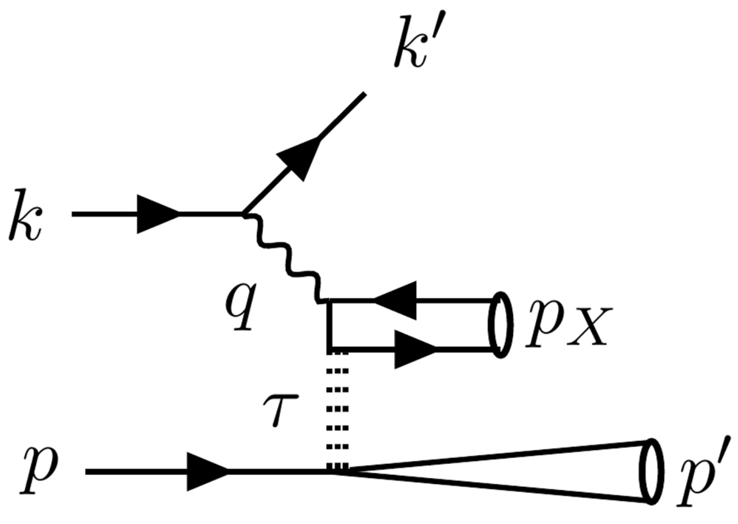
Simple case: only $\lambda = \frac{Q}{\sqrt{s}} \ll 1$

Mode	Momentum
Soft	$\sqrt{s}(\lambda, \lambda, \lambda)$
Glauber	$\sqrt{s}(\lambda^a, \lambda^b, \lambda)$
Collinear	$\sqrt{s}(\lambda^2, 1, \lambda)$

$$\mathcal{L}_{\text{SCET}}^{(0)} = \mathcal{L}_{\text{hard}} + \mathcal{L}_{\text{collinear}} + \mathcal{L}_{\text{soft}} + \mathcal{L}_{\text{Glauber}}$$

Estimating coefficients of the F_i^D's

Momentum	Scaling
k, k'	$(1, \lambda^2, \lambda)$
q	$(\lambda, \lambda^2, \lambda)$
p_X	$(\lambda, \lambda, \lambda)$
τ	$(\lambda^2, \lambda, \lambda)$
p, p'	$(\lambda^2, 1, \lambda)$



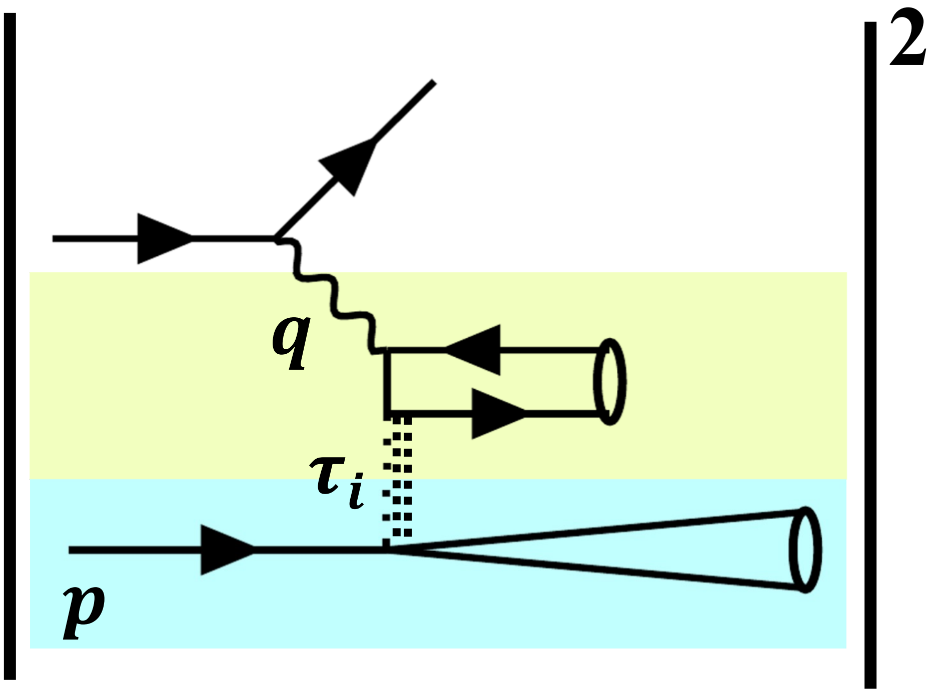
$$\boldsymbol{L_{\mu\nu} W^{\mu\nu}} = -\frac{Q^2}{x} F_L^D + Q^2 \frac{1+(1-y)^2}{xy^2} F_2^D + \frac{4(k\cdot X)^2 + Q^2}{x} F_3^D + \frac{4(k\cdot U)(k\cdot X)}{x} F_4^D$$

$$\sim \lambda\, F_L^D + \lambda^{-1} F_2^D + \lambda^{-1} F_3^D + \lambda^{-1} F_4^D$$

How big is each F_i^D ?

$\lambda \ll 1$ factorization

$$W_D^{\mu\nu} = S^{\mu\nu} \otimes_\perp B$$



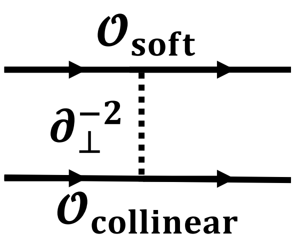
Soft tensor $S^{\mu\nu}(q, \tau_i)$:

- Vacuum-to-jet matrix element of SCET operators

Beam function $B(p, \tau_i)$:

- ***Coherent:*** color-singlet proton matrix element
- ***Incoherent:*** proton to jet
  (Can include color channel projector)

Define matrix elements using SCET operators, e.g.



Comparison to literature

Factorization from EFT

$$F_j^D(x, Q^2, \beta, t, m_J^2) = \mathcal{P}_j^{\mu\nu} \, \boldsymbol{S_{\mu\nu}(Q^2, \beta, t, \tau_{i\perp})} \otimes_\perp \boldsymbol{B(t, m_J^2, \tau_{i\perp})}$$

Projects out structure function F_j^D

- Need $\boldsymbol{\lambda = Q/\sqrt{s} \ll 1}$ for **forward + jet + gap**

Comparison to literature

Factorization from EFT

$$F_j^D\left(x,Q^2,\beta,t,m_J^2\right) = \mathcal{P}_j^{\mu\nu}\, \boldsymbol{S_{\mu\nu}\left(Q^2,\beta,t,\tau_{i\perp}\right)} \otimes_\perp \boldsymbol{B\left(t,m_J^2,\tau_{i\perp}\right)}$$

Collins' hard scattering approach

$$F_{2/L}^D(x,Q^2,\beta,t) = \sum_i \int_\beta^1 \frac{d\zeta}{\zeta}\, \boldsymbol{H_{2/L}^{(i)}\left(\frac{\beta}{\zeta},Q^2\right)}\, \boldsymbol{f_i^D\left(\zeta,Q^2,\frac{x}{\beta},t\right)}$$

"Diffractive PDF" (dPDF)

- Imposes *only* $\boldsymbol{\lambda_t = \sqrt{-t}/Q \ll 1}$
- EFT also agrees with this for $\boldsymbol{\lambda_t}$ and $\boldsymbol{\lambda \ll 1}$

Berera/Soper, hep-ph/9509239. Collins, hep-ph/9709499. Frankfurt et al., 2203.12289.

Comparison to literature

Factorization from EFT

$$F_j^D(x, Q^2, \beta, t, m_J^2) = \mathcal{P}_j^{\mu\nu}\, \boldsymbol{S_{\mu\nu}(Q^2, \beta, t, \tau_{i\perp})} \otimes_\perp \boldsymbol{B(t, m_J^2, \tau_{i\perp})}$$

Collins' hard scattering approach

$$F_{2/L}^D(x, Q^2, \beta, t) = \sum_i \int_\beta^1 \frac{d\zeta}{\zeta}\, \boldsymbol{H_{2/L}^{(i)}\left(\frac{\beta}{\zeta}, Q^2\right)}\, \underline{\boldsymbol{f_i^D\left(\zeta, Q^2, \frac{x}{\beta}, t\right)}}$$

Ingelman-Schlein model

$$\boldsymbol{f_i^D\left(\zeta, Q^2, \frac{x}{\beta}, t\right)} = \boldsymbol{f_{i/\mathbb{P}}(\zeta, Q^2)} \times \boldsymbol{f_{\mathbb{P}/p}\left(\frac{x}{\beta}, t\right)}$$

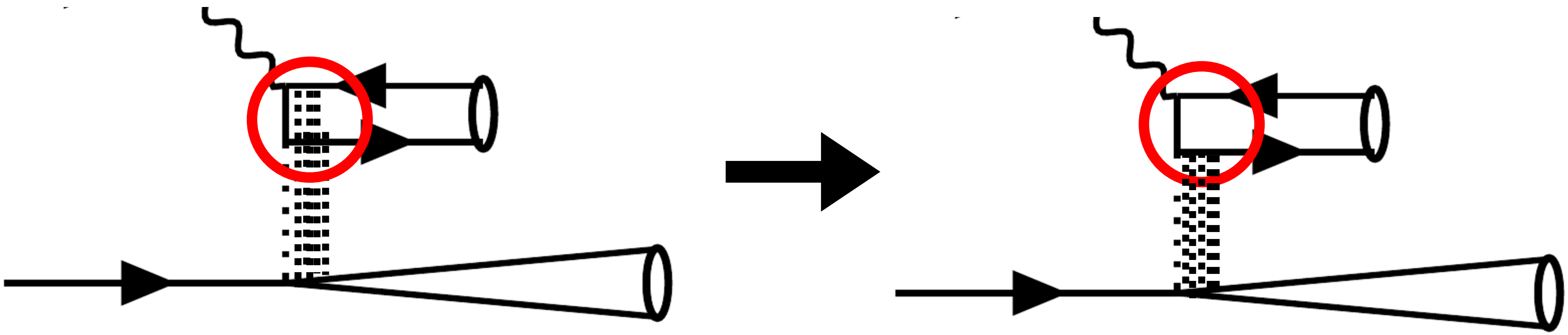
- Different # of arguments

Ingelman/Schlein (1984). Frankfurt et al., 2203.12289.

Perturbative predictions

λ & $\lambda_t \ll 1$ at leading-soft order

$$W_D^{\mu\nu} = S^{\mu\nu} \otimes_\perp B \quad \longrightarrow \quad W_D^{\mu\nu} = S'^{\mu\nu} \times B'$$



- $\lambda, \lambda_t \ll 1$: Glaubers only attach to **bottom** quark line
  - Glaubers attached to the same lines "collapse" → universal B
- **Convolution** → **multiplication**
- $\boldsymbol{B'}$ absorbs Glauber propagators & color factors

LO predictions for F_i ratios

Take $Q^2, t \gg \Lambda^2_{\text{QCD}}$

$$\frac{F_i}{F_2} = \frac{\mathcal{P}_{i,\mu\nu} W_D^{\mu\nu}}{\mathcal{P}_{2,\mu\nu} W_D^{\mu\nu}} \underset{\text{LO}}{\approx} \frac{\mathcal{P}_{i,\mu\nu} \left(S'^{\mu\nu} \times B'\right)}{\mathcal{P}_{2,\mu\nu} \left(S'^{\mu\nu} \times B'\right)} = \frac{\mathcal{P}_{i,\mu\nu} S'^{\mu\nu}}{\mathcal{P}_{2,\mu\nu} S'^{\mu\nu}}$$

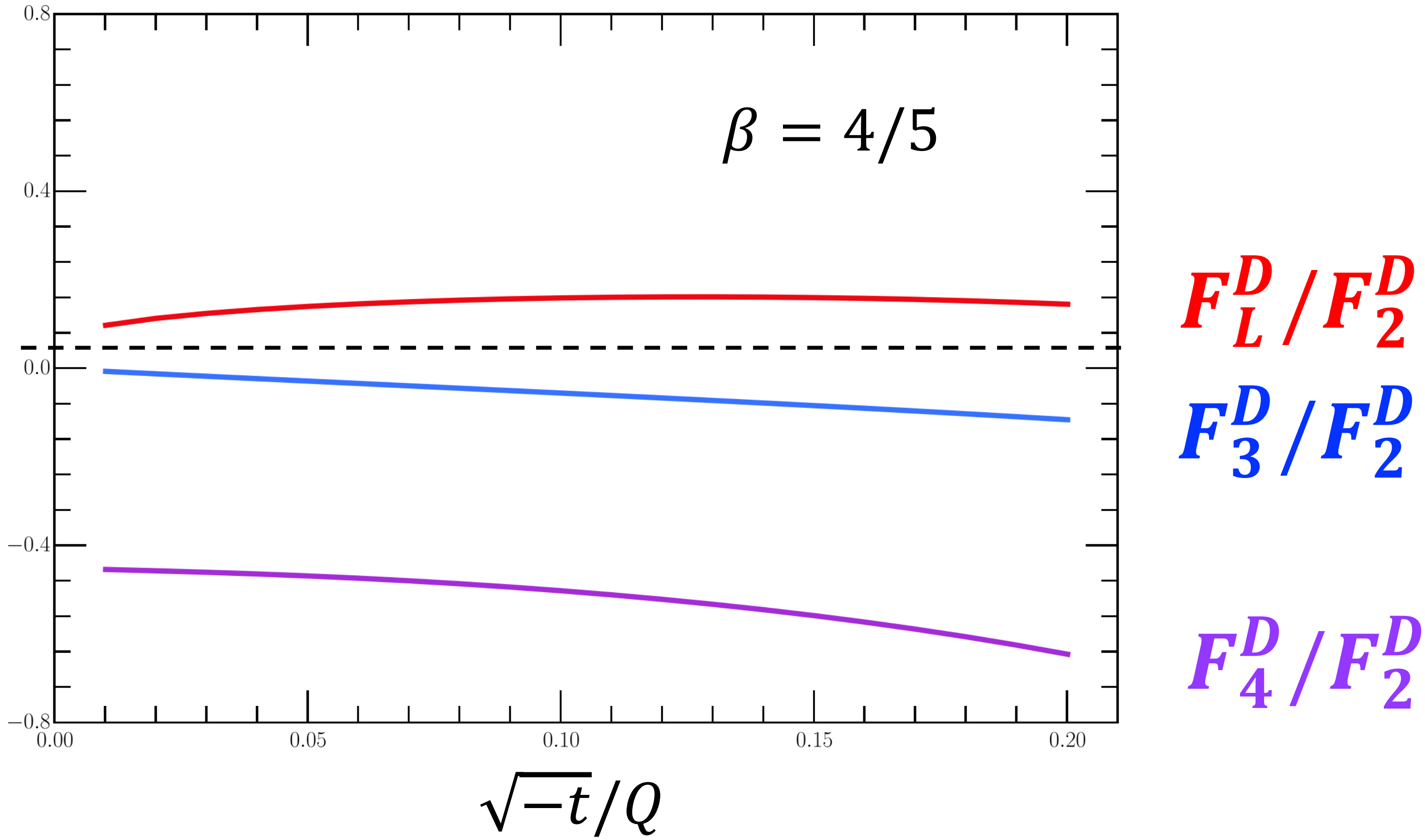
$f\left(\beta, \frac{t}{Q^2}\right)$

Same for every F_i^D

Perturbative

Ratios of F_i^D's are perturbatively calculable!

LO ratio prediction



F_3^D and F_4^D may be visible at the EIC & HERA in appropriate regions of parameter space

Conclusions and outlook

Our contributions to diffraction

- Importance of F_3^D and F_4^D for testing diffraction
- Factorization in Glauber SCET
- Perturbative predictions for HERA & the EIC
- Universal hadronic functions & systematically calculable soft function

Next steps

Full details of other cases:

- Coherent vs. incoherent
- pp, eA, & AA collisions
- (Adding spin is straightforward)
- Connection to saturation

Towards precision diffraction:

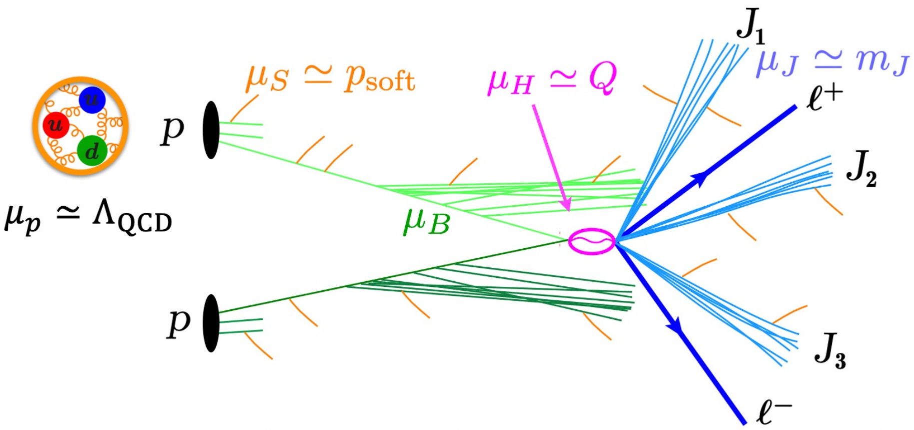
- Beam refactorization
- Resummation (small x, DGLAP)
- Higher-order predictions

Backup slides

Soft-Collinear Effective Theory (SCET)

Mode	Momentum in $(+,-,\perp)$
Hard	$(1,0,0)$
Collinear	$(1,\lambda^2,\lambda)$ or $(\lambda^2,1,\lambda)$
Soft	$(\lambda,\lambda,\lambda)$
Glauber	$(\lambda^a,\lambda^b,\lambda)$ for $a+b>2$

$\lambda \ll 1$: power counting



$$\mathcal{L}_{\rm SCET}^{(0)} = \mathcal{L}_{\rm hard} + \mathcal{L}_{\rm collinear} + \mathcal{L}_{\rm soft} + \mathcal{L}_{\rm Glauber}$$

- **Hard:** connects different sectors but only occurs once
- **Glauber:** Talks between soft & collinear sectors

Bauer et al., hep-ph/0005275, 0011336, 0107001, & 0109045. Rothstein & Stewart, 1601.04695.

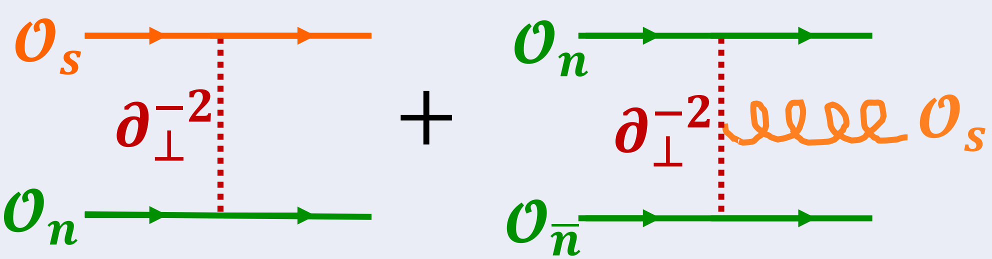
SCET Lagrangian

Like copies of QCD for specific modes

$$\mathcal{L}^{(0)}_{\text{collinear}} = \bar{\xi}_n \left(i n \cdot D + i \not{D}_{n\perp} \frac{1}{i \bar{n} \cdot D_n} i \not{D}_{n\perp} \right) \frac{\not{\bar{n}}}{2} \xi_n + \cdots$$

$$\mathcal{L}^{(0)}_s = \bar{\psi}_s (i \not{D} - m) \psi_s + \frac{1}{4} G_{s\mu\nu} G_s^{\mu\nu}$$

$$\mathcal{L}^{(0)}_{\text{Glauber}} = \mathcal{O}_n \frac{1}{\partial_\perp^2} \mathcal{O}_s + \mathcal{O}_n \frac{1}{\partial_\perp^2} \mathcal{O}_s \frac{1}{\partial_\perp^2} \mathcal{O}_{\bar{n}}$$

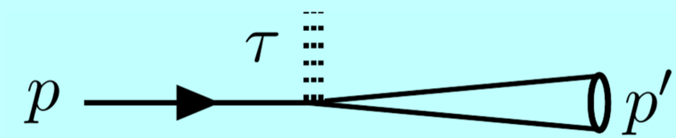
= 

$$\mathcal{O}_n^q = \frac{1}{2} \bar{\chi}_n T^B \not{\bar{n}} \chi_n \qquad \mathcal{O}_s^q = 4\pi\alpha_s (\psi_s^{\bar{n}} T^B n \psi_s^{\not{n}}) \qquad \mathcal{O}_n^g = [\ldots]$$

Rothstein & Stewart, 1601.04695.

Color exchange

Forward scattering



Coherent: Always color singlet

Incoherent: Studied in amplitudes community

- Mediated by color-singlet "Pomeron" **or** color non-singlet "Reggeon"
- Different use of term Reggeon from diffraction community (≠ color singlet)

Diffraction

- Experiments always impose momentum cut to classify process as gapped (e.g. 200-800 MeV) [ATLAS, 1201.2808.]
- Non-singlet ***not*** ruled out

The bottom line

- Cannot *a priori* distinguish color singlet exchange from color non-singlet background
- SCET can handle both cases

Lee, **Schindler**, & Stewart, in preparation.

Resummation & evolution

- Much work already in the classic diffraction literature
- In SCET, recent studies of small-x DIS give us tools to:
  - Carry out small-x resummation (difference: in diffraction, t is fixed)
  - Bridge from BFKL to the saturation regime

Full QCD
Pomeron ladders
Reggeization

→

Glauber SCET
RG evolution
Sum rapidity logs

Frankfurt et al., 2203.12289. SCET: e.g. Neill, Pathak, Stewart 2303.13710. Stewart & Vaidya, 2305.16393.

Diffraction has same # of momenta as SIDIS

	DIS	**Diffraction**
Measurement	q, p	$q, p,$ **and** $\boldsymbol{p'}$
Lorentz invariants	3	7
Unpolarized structure functions	2	4
Total structure functions	4	18

Arens, Nachtmann, Diehl, and Landshoff, hep-ph/9605376.

Projecting out structure functions

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \boldsymbol{F_1^D} + \frac{1}{2x} U^\mu U^\nu \boldsymbol{F_2^D}$$

$$+ \frac{1}{x} X^\mu X^\nu \boldsymbol{F_3^D} + \frac{1}{2x} (U^\mu X^\nu + X^\mu U^\nu) \boldsymbol{F_4^D}$$

Projectors

$$\boldsymbol{F_i^D} = \boldsymbol{\mathcal{P}_{i,\mu\nu}} W^{\mu\nu}$$

$$\boldsymbol{\mathcal{P}_{L,\mu\nu}} = 2x U^\mu U^\nu$$

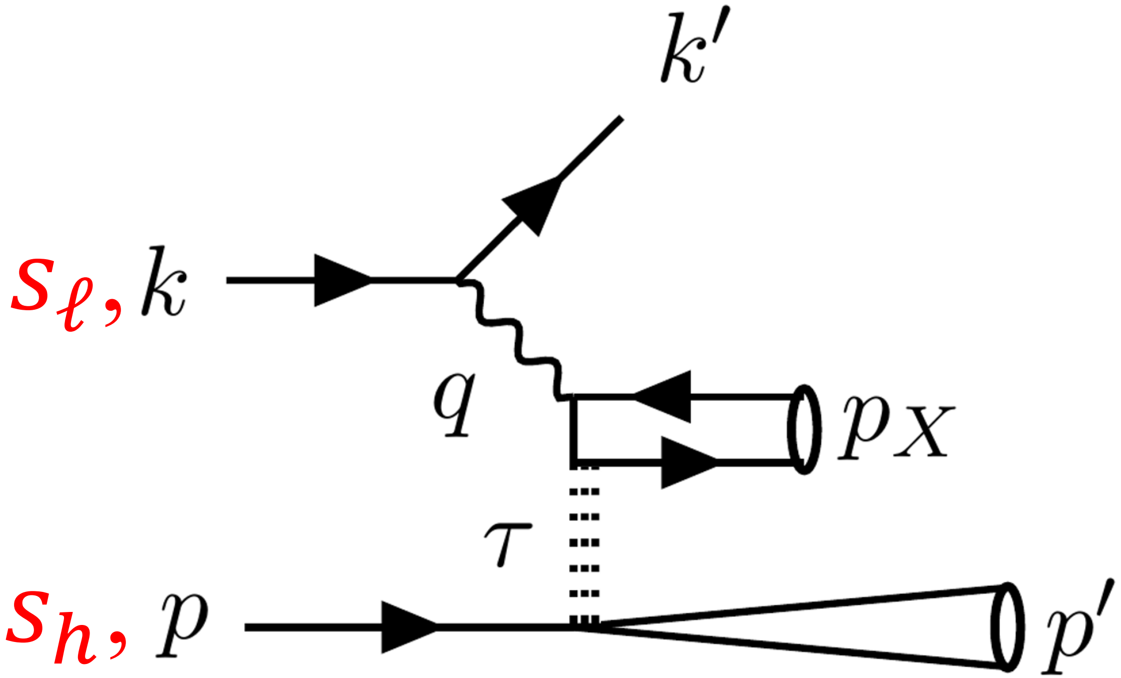
$$\boldsymbol{\mathcal{P}_{2,\mu\nu}} = 2x(-g^{\mu\nu} - X^\mu X^\nu + 2U^\mu U^\nu)$$

$$\boldsymbol{\mathcal{P}_{3,\mu\nu}} = x(g^{\mu\nu} + 2X^\mu X^\nu - U^\mu U^\nu)$$

$$\boldsymbol{\mathcal{P}_{4,\mu\nu}} = -x(U^\mu X^\nu + X^\mu U^\nu)$$

Lee, **Schindler**, & Stewart, in preparation.

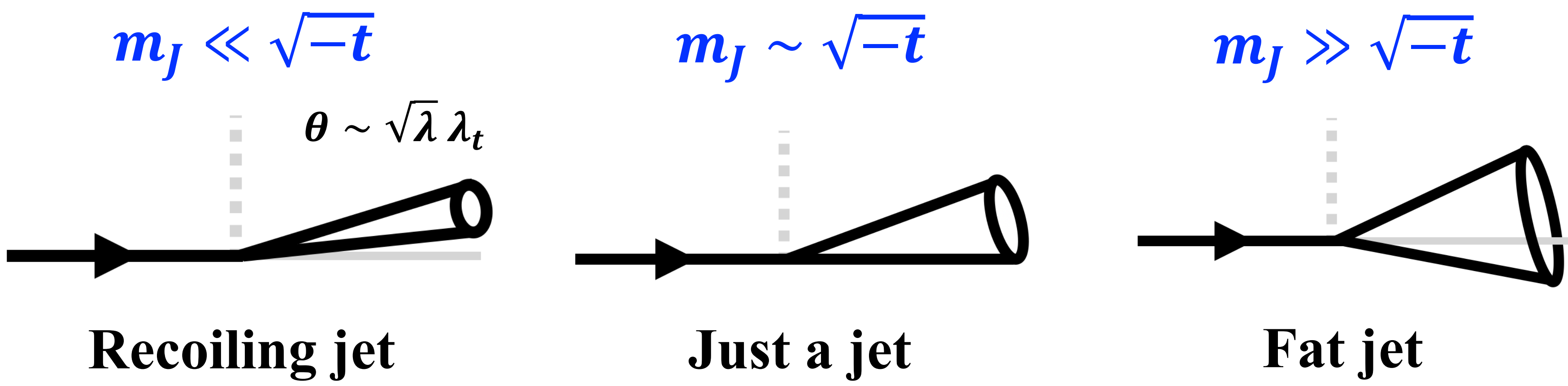
Spin dependence in diffraction for $\lambda, \lambda_t \ll 1$

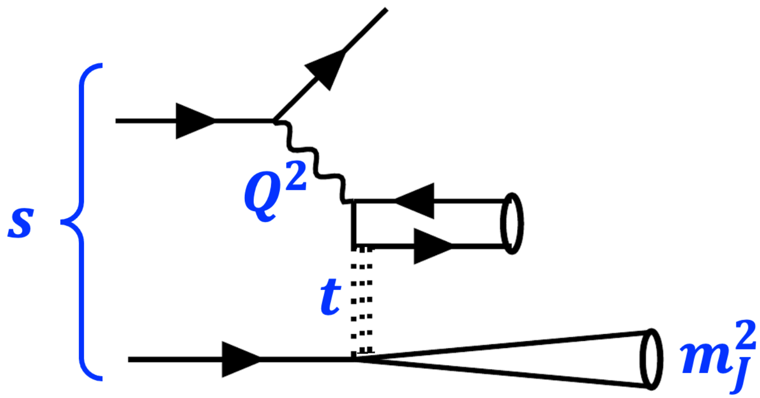


- Similar analysis to unpolarized case
- Four unpolarized + four polarized structure functions are nonzero at leading power, out of 18 possible

Lee, **Schindler**, & Stewart, in preparation.

What is the diffractive analogue of a PDF?

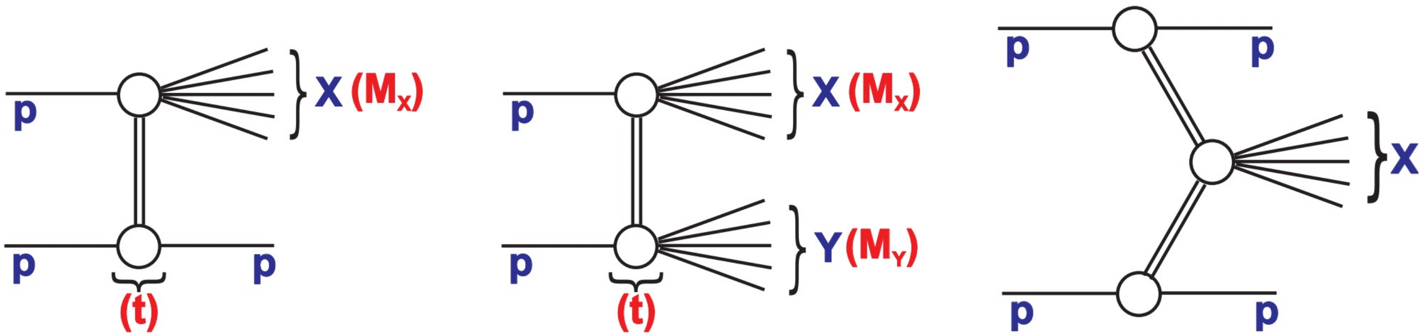




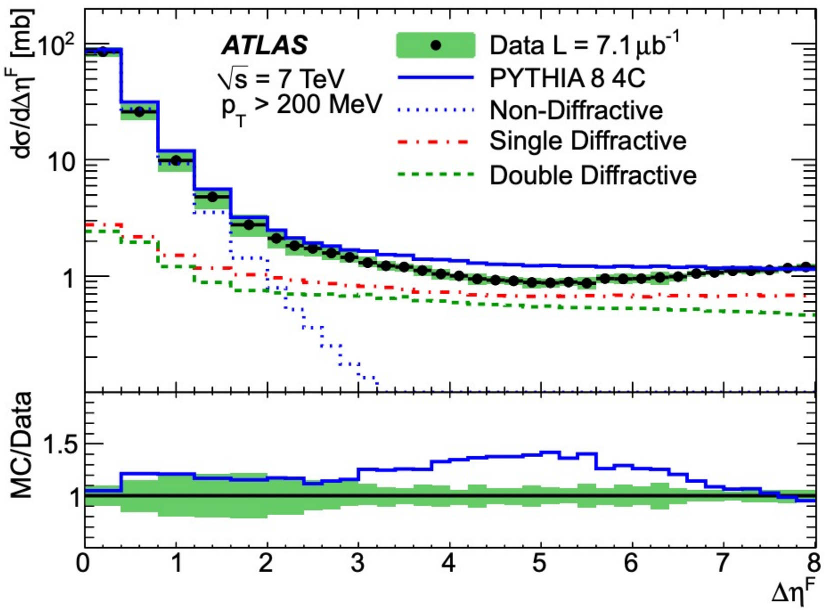
- Must refactorize beam function
- Not just a GPD in incoherent case, clear from presence of a jet
- Precise structure of the “dPDF” depends on parameter regime

Lee, **Schindler**, & Stewart, in preparation.

Diffraction at the LHC?



(a) (b) (c)



- Hadrons and/or central region can become a jet
- Mediated by Glaubers
- Universal with *ep* case or does factorization break?
  (Factorization breaking also is described by Glaubers!)

Figures: ATLAS, 1201.2808.