Recent results on GPDs and TMDs

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Extended Twisted Mass Collaboration

AQTIVATE

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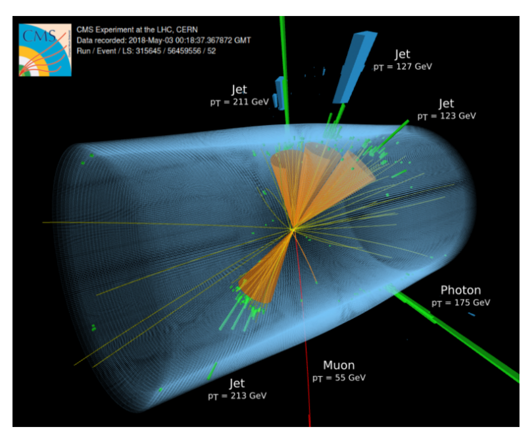
RESEARCH & INNOVATION FOUNDATION

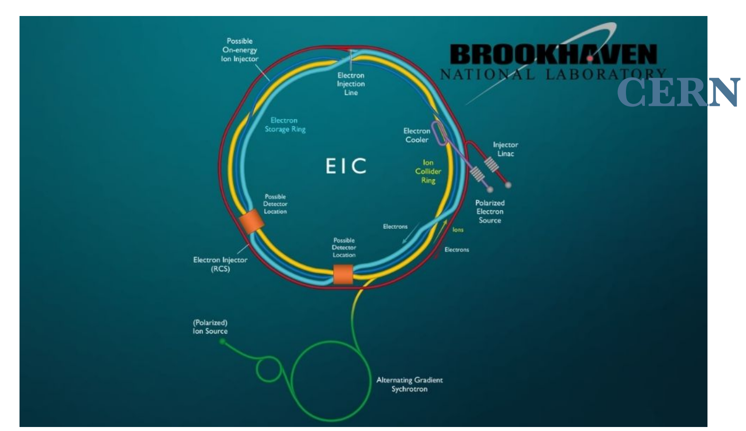
EXCELLENCE/0421/0043

Outline

- ❋Mellin moments - precision era of lattice QCD
- ❋Results on parton distribution functions (PDFs) & generalised parton distributions (GPDs)
- ❋Calculation of TMDs
- ❋Conclusions

Relevant for interpreting and providing input for on-going and future experiments

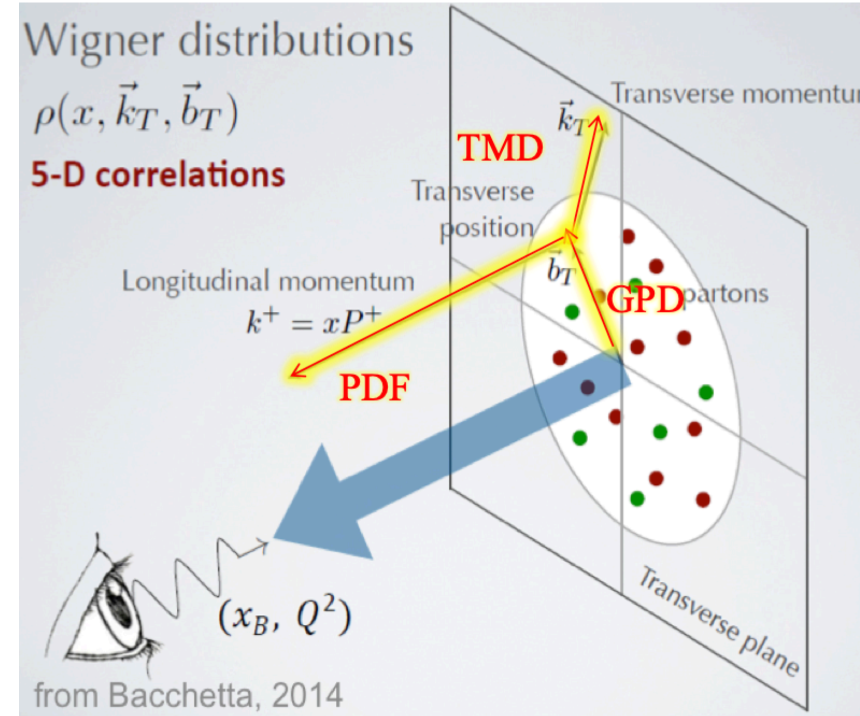


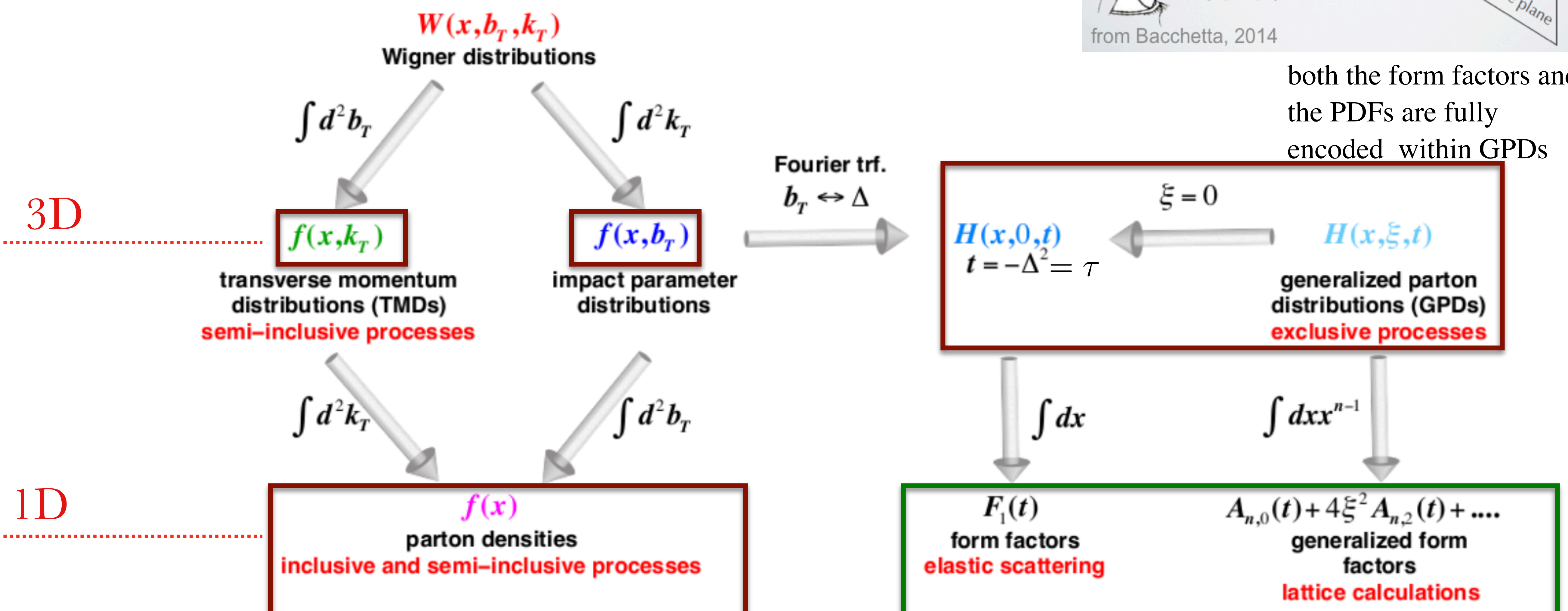


3D structure of hadrons

- The 3D-structure of the nucleon is major goal of on-going experiments and the future EIC

- Lattice QCD can contribute towards this goal - many recent developments to compute Mellin moments but also directly parton distributions



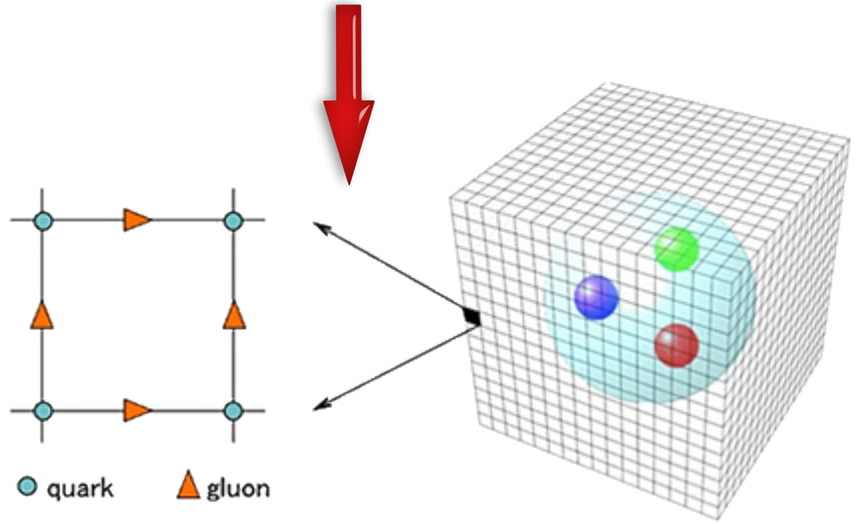


Studies in lattice QCD since the 1980s

EIC white paper, arXiv:1212.1701

Simulations of lattice QCD

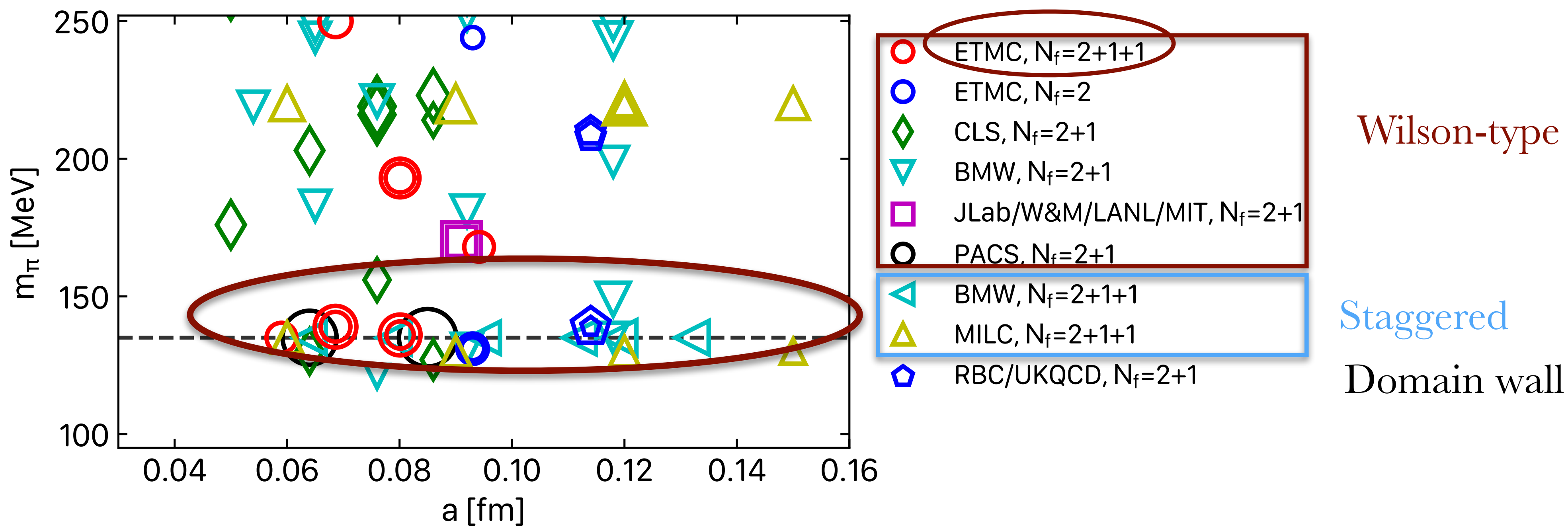
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$$



1. Simulation of gauge ensembles {U}:

$$P[U] = \frac{1}{Z} \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$$

ETMC: S. Bacchio, J. Finkenrath, R. Frezzotti, B. Kostrzewa, C. Urbach

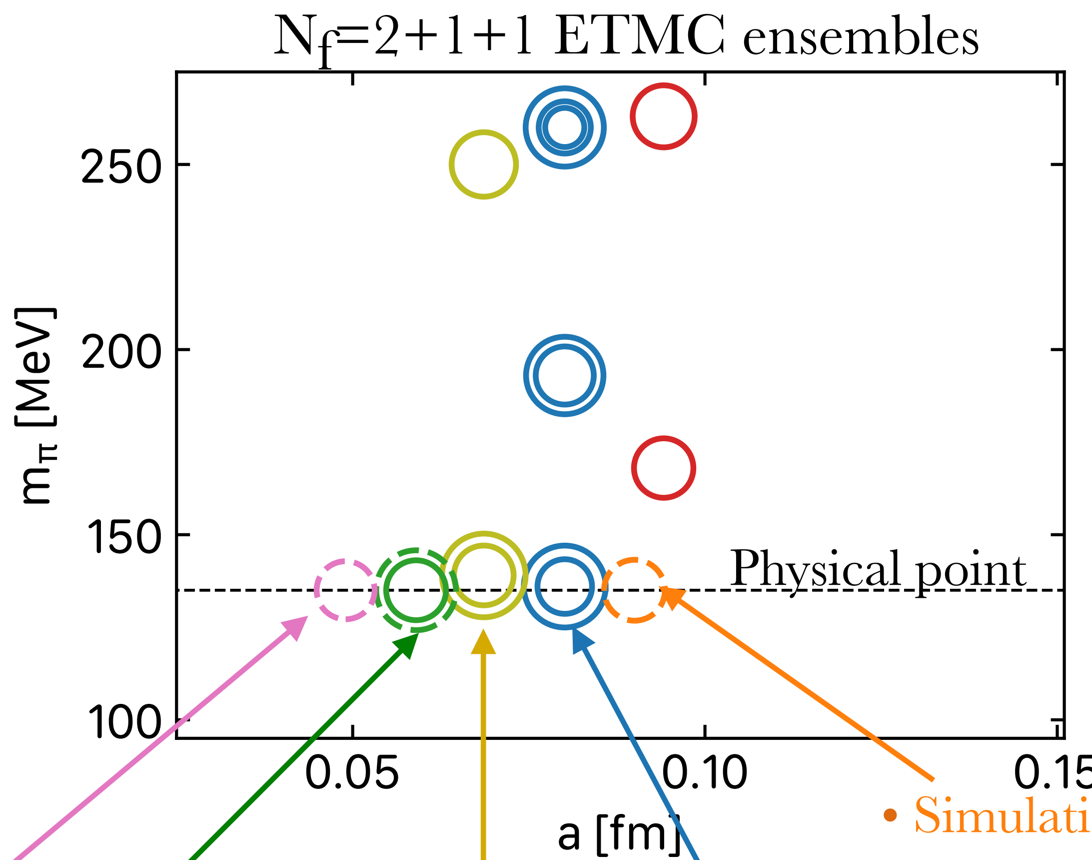


2. Quark propagators: inverse of Dirac matrix $D_f[U]$ using multi-grid solver DD-αAMG

Gauge ensembles generated by ETMC

5 ensembles completed and 3 under production at physical pion mass

- 5 lattice spacings 0.05<a<0.1 fm —> take continuum limit **directly at the physical point** avoiding chiral extrapolation removing a major systematic error in the baryon sector
- 2 volumes at a=0.08 fm, 0.07 fm and 0.06 fm of Lm_π~3.6 (5.1 fm) and Lm_π ~5.4 (7.7 fm) completed



- Simulation ongoing 48^3 x 96 a~0.09 fm
- Analysis completed for 64^3x128 a~0.08 fm
- Analysis ongoing for 96^3x192 a~0.08 fm
- Simulation ongoing for 112^3x224, a~0.05 fm
- Analysis completed for 80^3x160, a~0.07 fm
- Analysis ongoing for 112^3x224, a~0.07 fm
- Analysis completed for 96^3x192, a~0.06 fm
- Simulation ongoing for 112^3x224, a~0.06 fm

C. A. *et al.* (ETMC) Phys. Rev. D98 (2018) 054518

Results in this talk from the analysis of 3 physical point ensembles

- B-ensemble: 64^3 x 128, a~0.08 fm
- C-ensemble: 80^3x160, a~0.07 fm
- D-ensemble:96^3x192, a~0.06 fm

ETMC: S. Bacchio, J. Finkenrath, R. Frezzotti, B. Kostrzewa, C. Urbach

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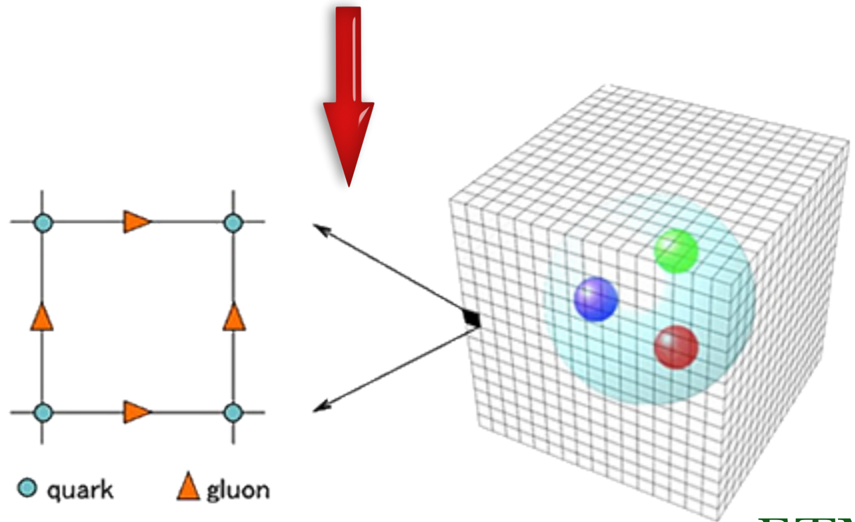

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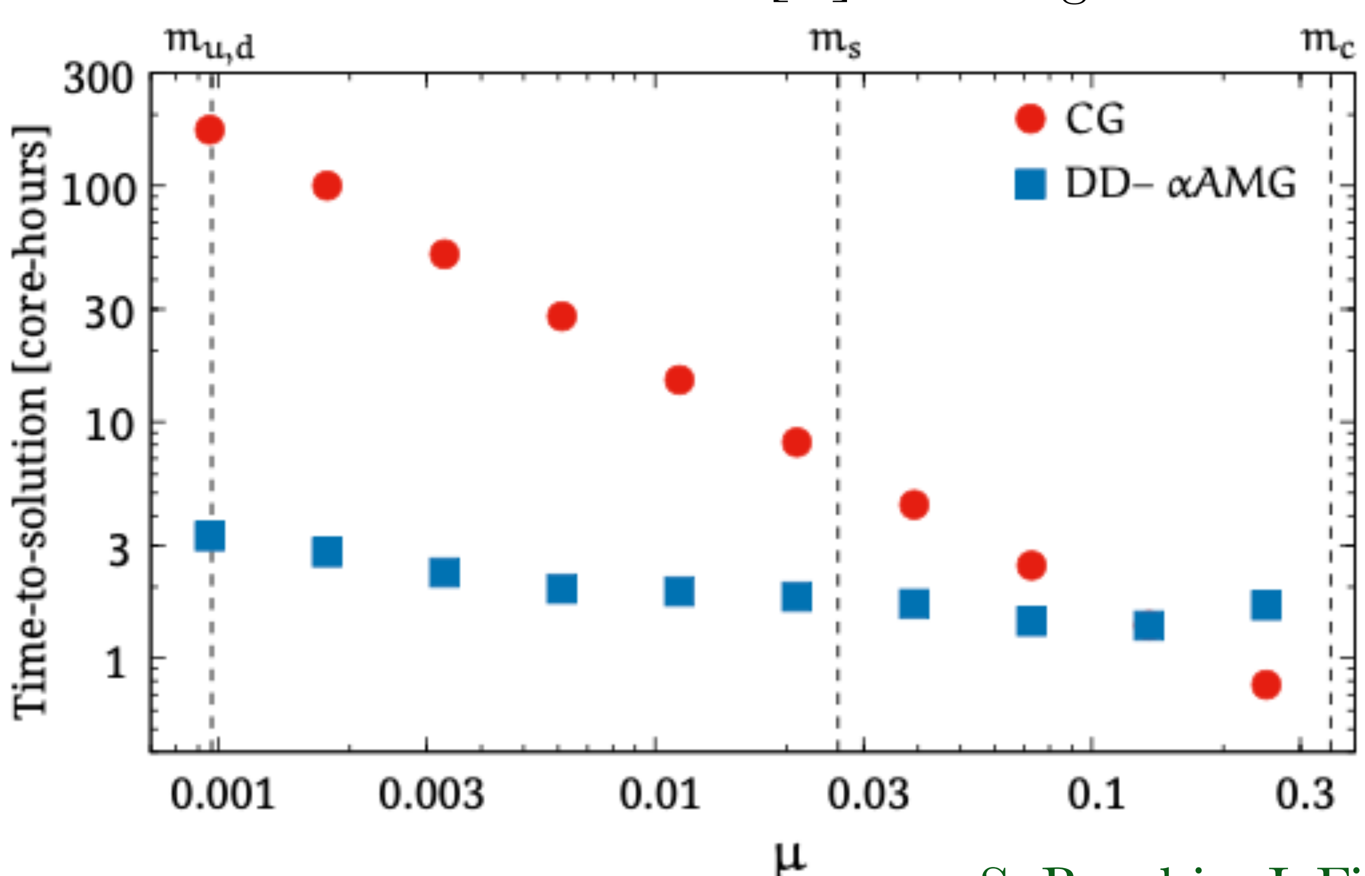
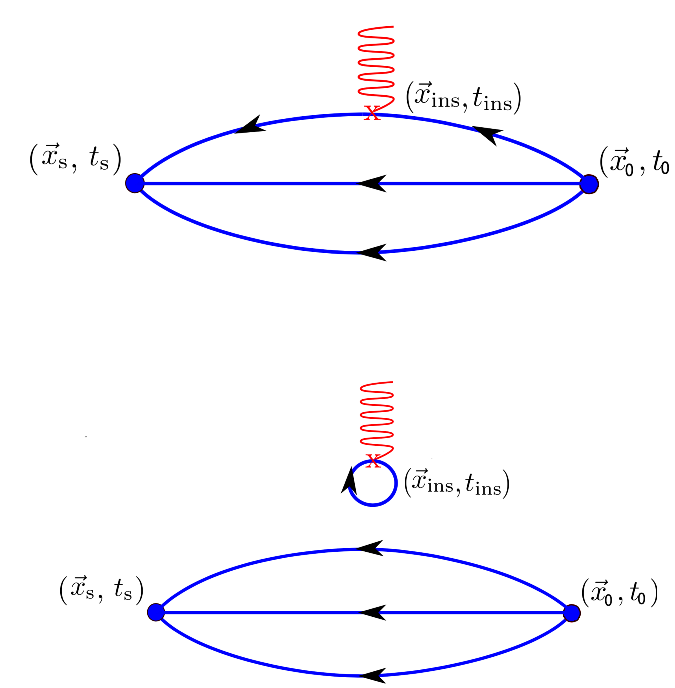
2. Quark propagators: inverse of Dirac matrix $D_f[U]$: Multi-grid solvers

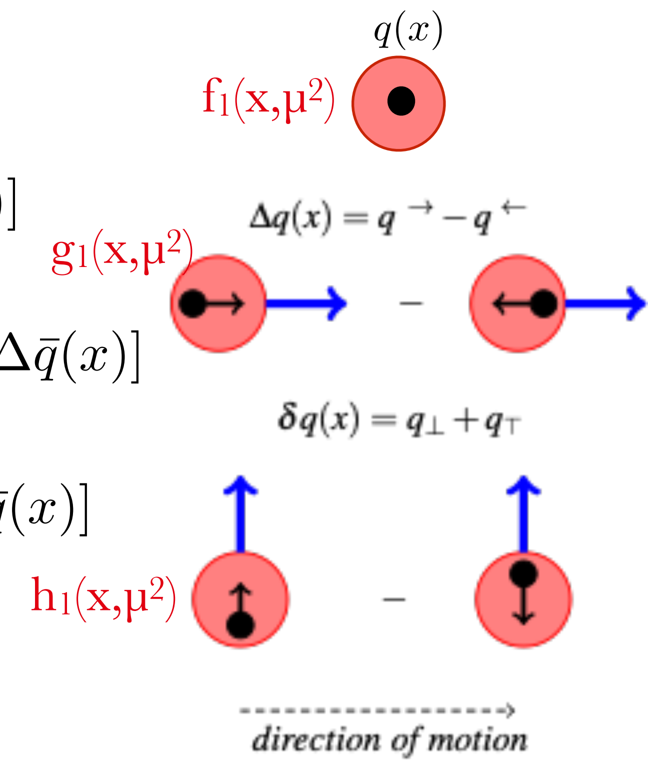




S. Bacchio, J. Finkenrath

Computation of Mellin moments of GPDs

- Light-cone matrix elements cannot be computed using a Euclidean lattice formulation of QCD
- Expansion of light-cone operator leads to a tower of local twist-2 operators —> connected to moments that can be computed in lattice QCD



Twist-2 PDFs

$$\mathcal{O}^{\mu_1\dots\mu_n} = \bar{\psi}\gamma^{\{\mu_1}iD^{\mu_2}\dots iD^{\mu_n\}}\psi \quad \xrightarrow{unpolarized} \quad \langle x^n\rangle_q = \int_0^1 dx\, x^n\,[q(x) - (-1)^n\bar{q}(x)]$$

$$\tilde{\mathcal{O}}^{\mu_1\dots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu_1}iD^{\mu_2}\dots iD^{\mu_n\}}\psi \quad \xrightarrow{helicity} \quad \langle x^n\rangle_{\Delta q} = \int_0^1 dx\, x^n\,[\Delta q(x) + (-1)^n\Delta\bar{q}(x)]$$

$$\mathcal{O}_T^{\rho\mu_1\dots\mu_n} = \bar{\psi}\sigma^{\rho\{\mu_1}iD^{\mu_2}\dots iD^{\mu_n\}}\psi \quad \xrightarrow{transversity} \quad \langle x^n\rangle_{\delta q} = \int_0^1 dx\, x^n\,[\delta q(x) - (-1)^n\delta\bar{q}(x)]$$

$$q = q_\downarrow + q_\uparrow, \quad \Delta q = q_\downarrow - q_\uparrow, \quad \delta q = q_\top + q_\perp$$

Ph. Hagler, Phys. Rept. 490 (2010) 49

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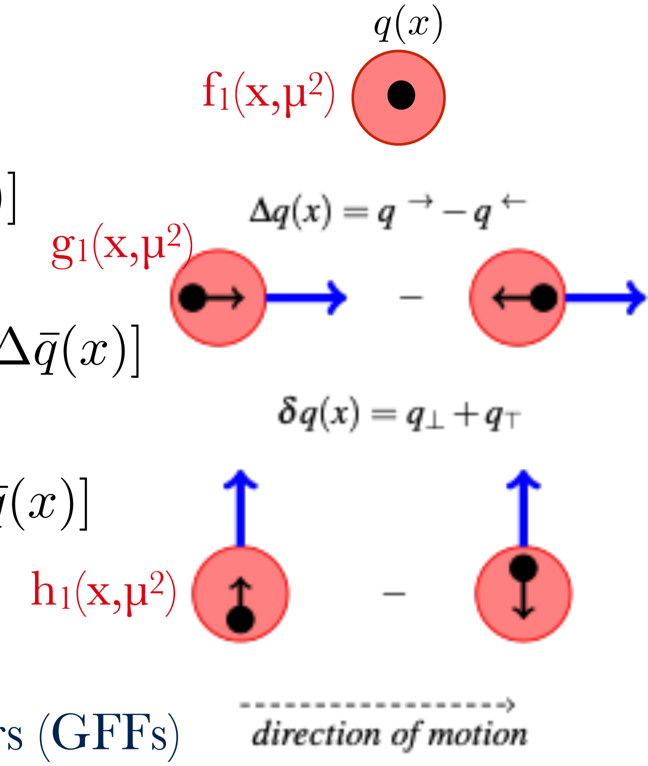

Twist-2 PDFs

- For off-diagonal matrix elements we obtain moments of GPDs or the generalised form factors (GFFs)

$$\int_{-1}^1 dx\, x^{n-1}H(x,\xi,\tau) = \sum_{i=0,2,\cdots}^{n-1}\left[(2\xi)^i A_{ni}(\tau) + \mathrm{mod}(n,2)(2\xi)^n C_{n0}(\tau)\right]$$

$$\int_{-1}^1 dx\, x^{n-1}E(x,\xi,\tau) = \sum_{i=0,2,\cdots}^{n-1}\left[(2\xi)^i B_{ni}(\tau) - \mathrm{mod}(n,2)(2\xi)^n C_{n0}(\tau)\right]$$

Ph. Hagler, Phys. Rept. 490 (2010) 49

Computation of Mellin moments of GPDs

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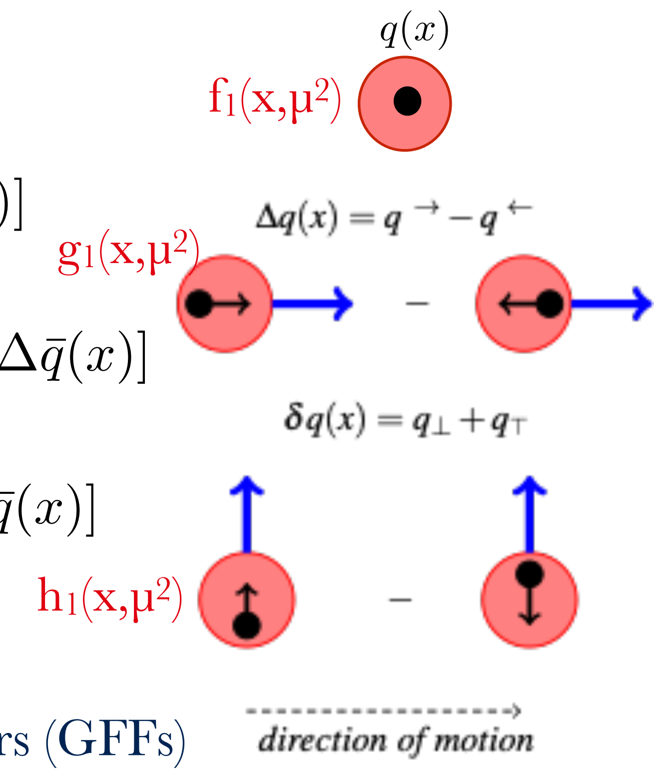

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Special cases: n=1,2 for the nucleon

- n=1: $\tau=0$ —> charges g_V, g_A, g_T
  $\tau \neq 0$ —> form factors: $A_{10}(\tau) = F_1(\tau)$, $B_{10}(\tau) = F_2(\tau)$, $\tilde A_{10}(\tau) = G_A(\tau)$, $\tilde B_{10}(\tau) = G_p(\tau)$
- n=2: generalised form factors: $A_{20}(\tau)$, $B_{20}(\tau)$, $C_{20}(\tau)$, $\tilde A_{20}(\tau)$, $\tilde B_{20}(\tau)$

$$\langle x\rangle_q = A_{20}(0), \quad \langle x\rangle_{\Delta q} = \tilde A_{20}(0), \quad \langle x\rangle_{\delta q} = A_{20}^T(0) \quad \text{and } J_q = \tfrac{1}{2}[A_{20}(0)+B_{20}(0)] = \tfrac{1}{2}\Delta\Sigma_q + L_q$$

- Spin and momentum sums: $\sum_q\left[\frac{1}{2}\Delta\Sigma_q + L_q\right] + J_g = \frac{1}{2}$, $\quad \sum_q\langle x\rangle_q + \langle x\rangle_g = 1$

Mellin moments - precision era of lattice QCD

First Mellin moments

- Moments for small n are readily accessible on the lattice from matrix elements of local operators
- Computation of the low Mellin moments has a long history, G. Martinelli and Ch. Sachradja Phys. Lett. B217 (1989) 319
- Only recently we have results directly at the physical point (i.e. simulations with $m_\pi \sim 135$ +/-10 MeV)

Nucleon isovector charges

$$g_V = \langle 1 \rangle_{u-d}$$
$$g_A = \langle 1 \rangle_{\Delta u - \Delta d}$$
$$g_T = \langle 1 \rangle_{\delta u - \delta d}$$

- $g_V = 1$
- $g_A = 1.2764 \pm 0.0006$ reproduce
- $g_T = 0.53 \pm 0.25$ M. Radici and A. Bacchetta. PRL 120 (2018) 192001

Determine for each quark flavour

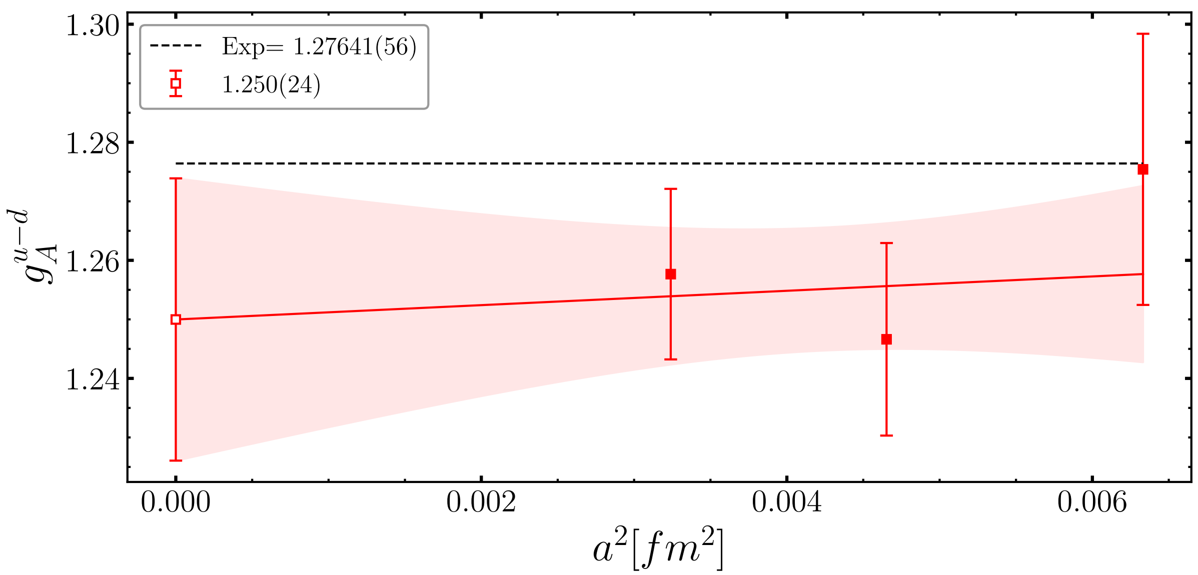
- e.g. $\Delta\Sigma_{q^+} = g_A^q$

$$\Delta\Sigma_{q_+}(\mu^2) = \int_0^1 dx \left[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)\right] = g_A^q$$

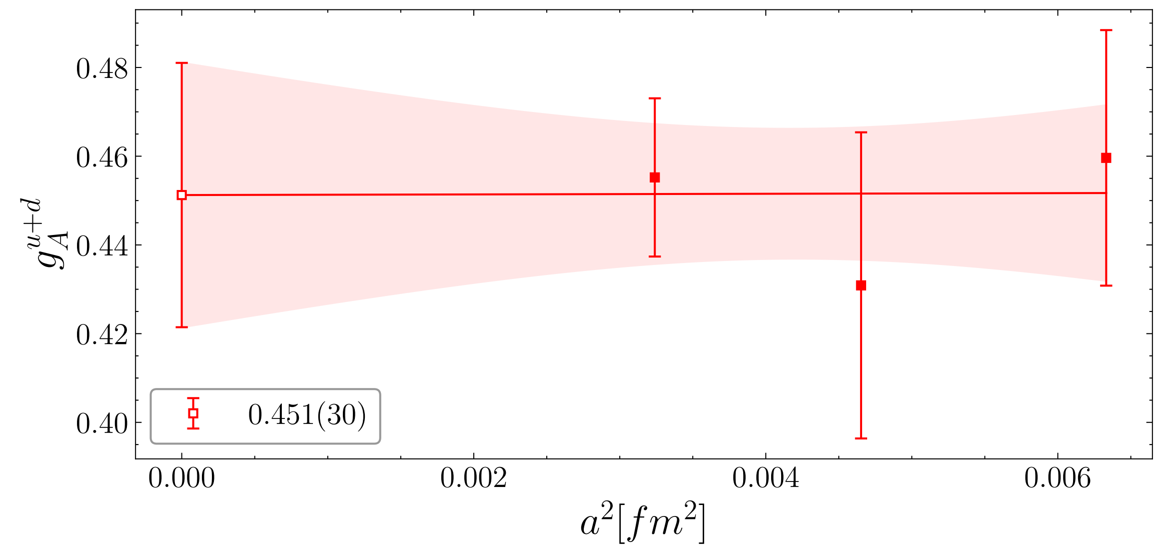
Continuum results

- Axial charges extracted directly from the forward matrix element

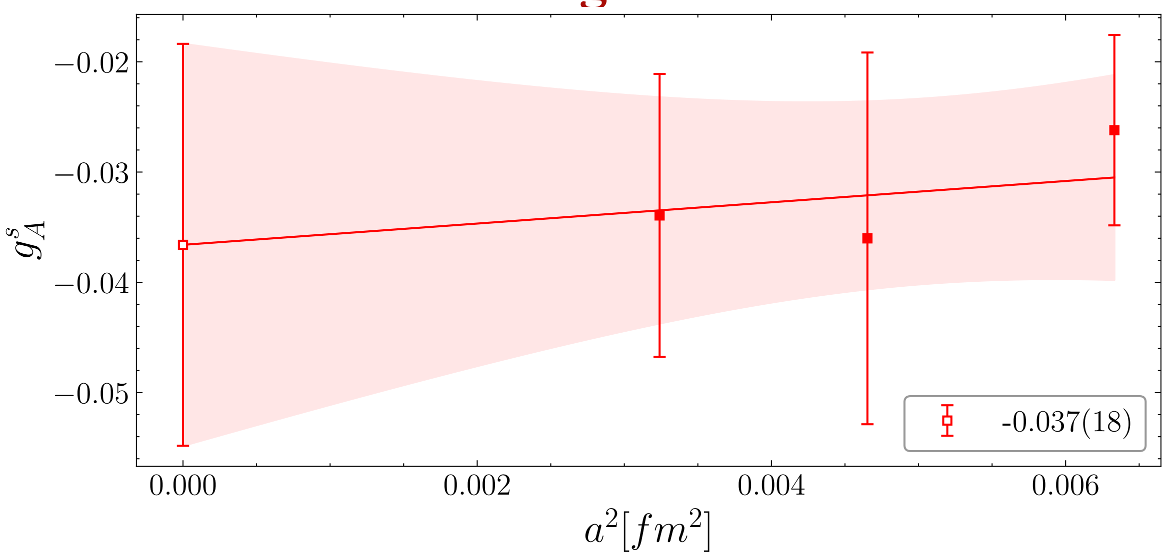
Isovector



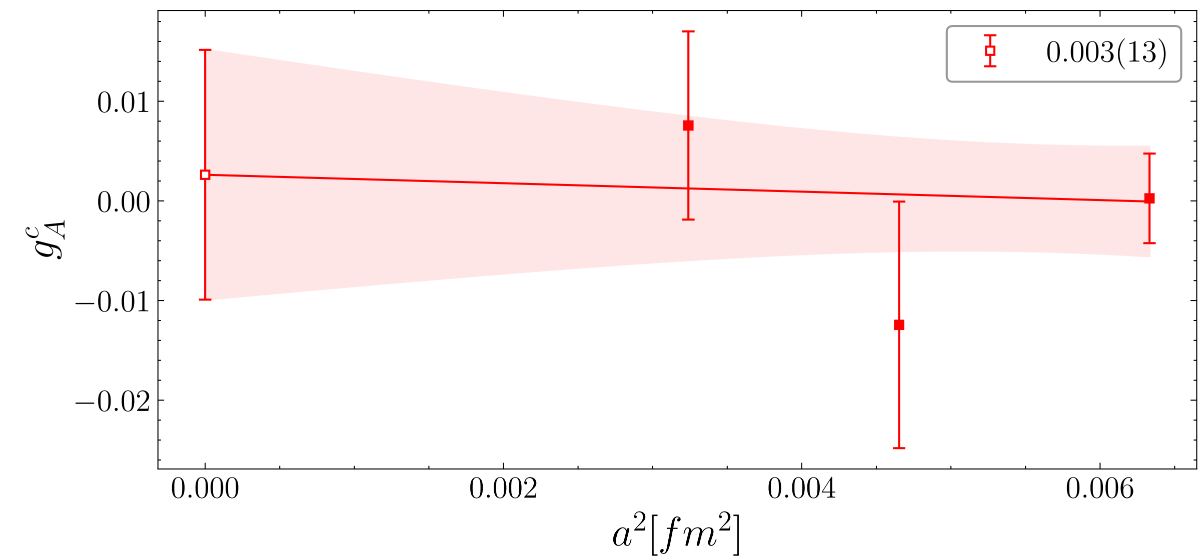
Isoscalar including disconnected



Strange

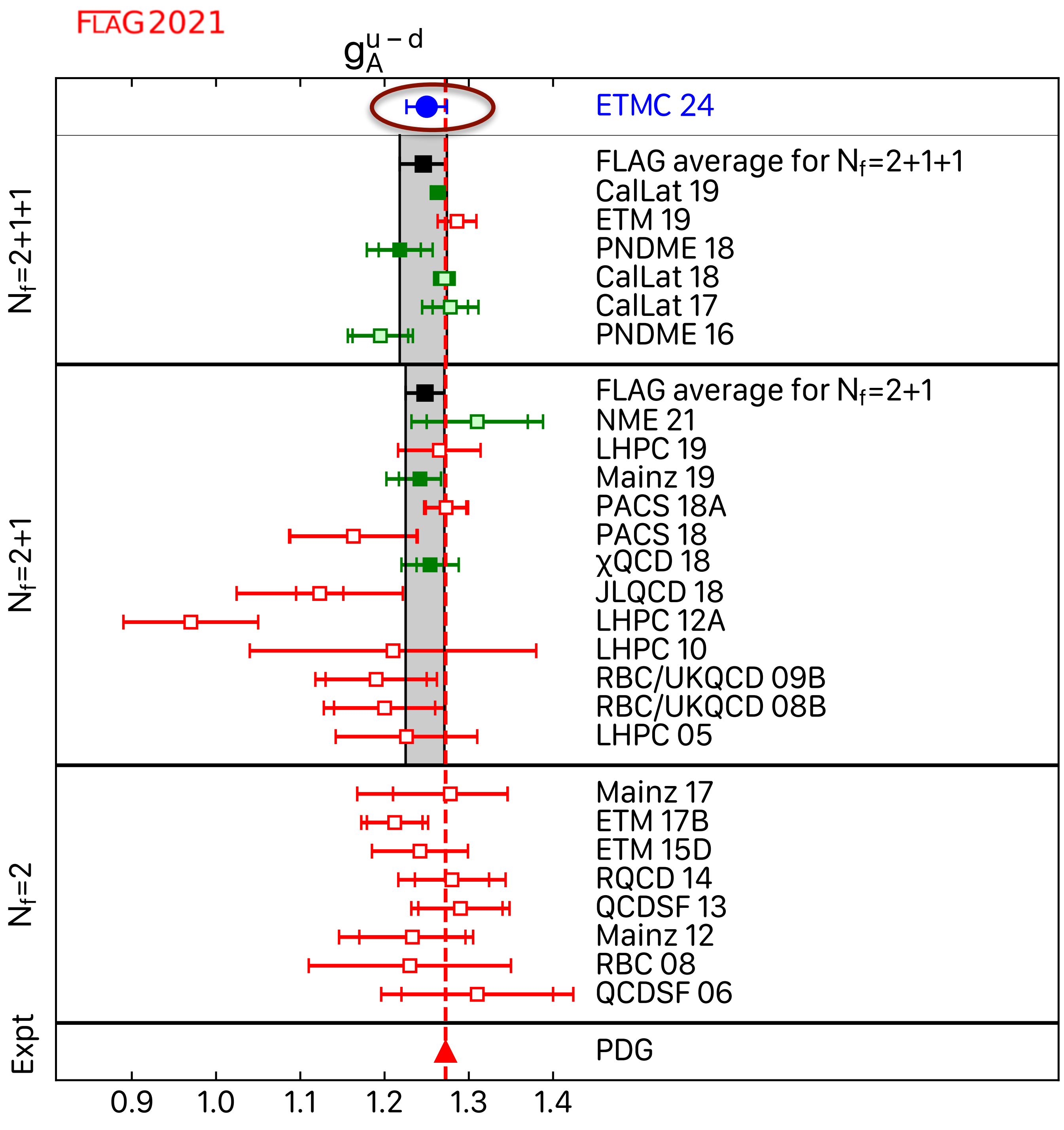


Charm



- Non-zero strangeness, upper limit on charmness of 0.013
- With our two additional lattice spacings we expect more stability in the results and reduced errors at the continuum limit

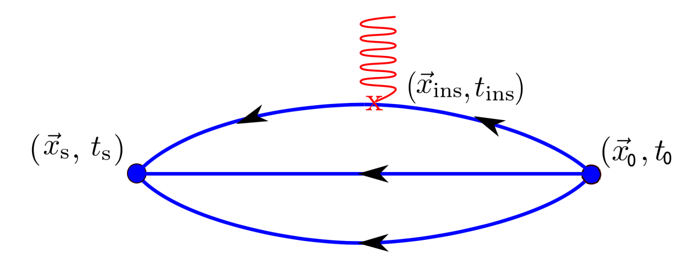
Nucleon isovector (u-d) axial charge

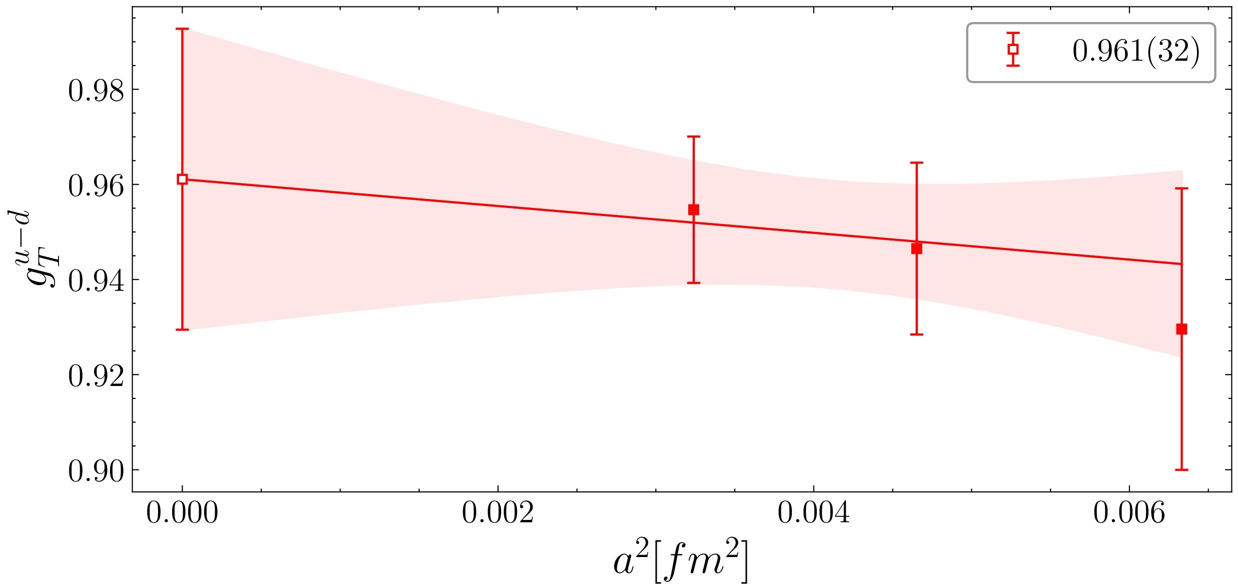


Lattice QCD results on g_A consistent with experimental value

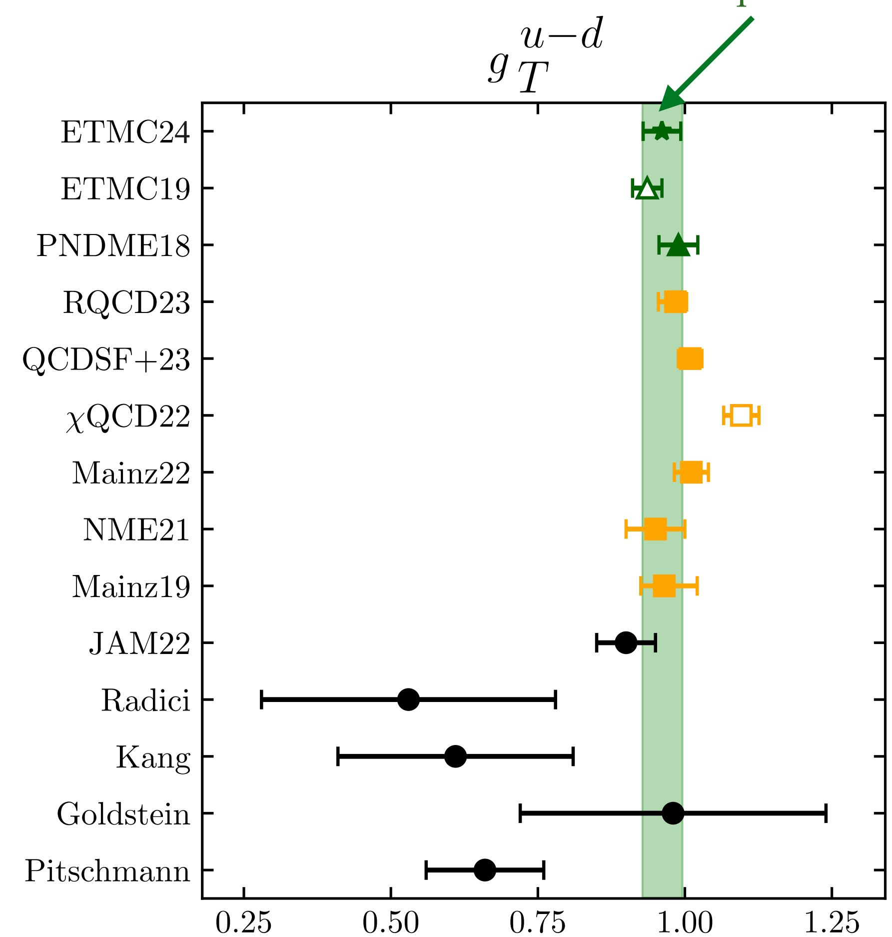
Nucleon isovector (u-d) tensor charge

❋Only connected contributions





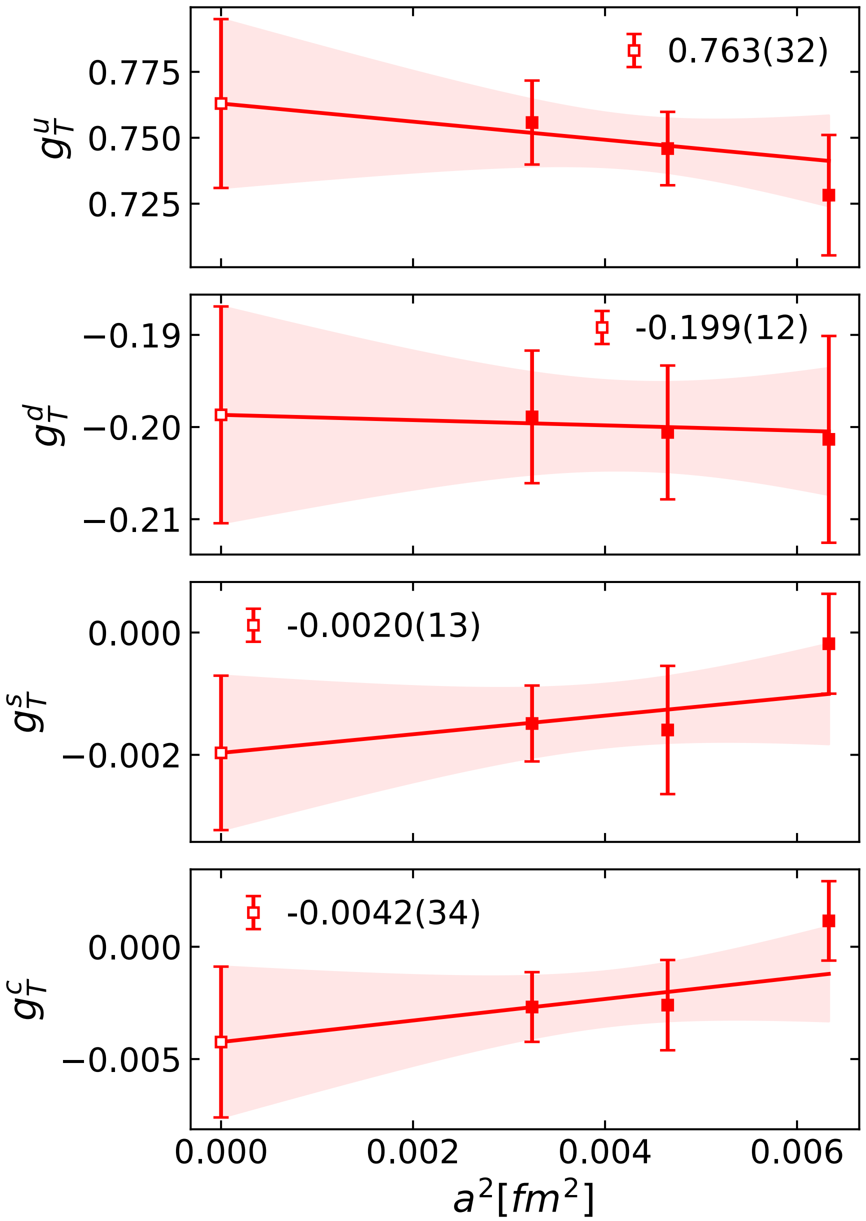
Continuum limit directly at physical pion mass

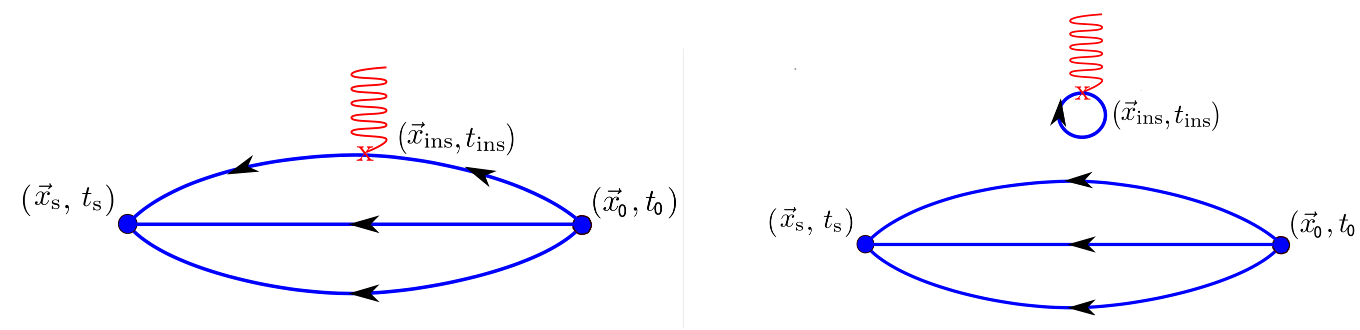


❋Precision results on the isovector tensor charge - input for phenomenology e.g. JAM3D-22 analysis

Phys.Rev.D 106 (2022) 3, 034014, arXiv:2205.00999

Flavor diagonal tensor charge

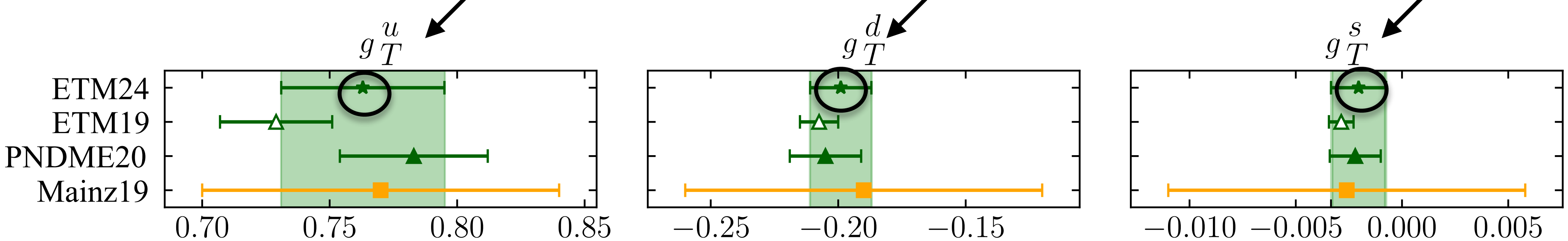




- Evaluate both connected and disconnected contributions
- Obtain flavor diagonal tensor charge for the first time in the continuum using only physical point ensembles - input for phenomenology
- JAM3D-22: g_T^u=0.78(11) and g_T^d=-0.12(11), arXiv:2306.12998

Thanks to Daniel Pitonyak

Only calculation in the continuum limit directly at the physical point



Precision era of lattice QCD for first Mellin moments including flavor diagonal

Second Mellin moments

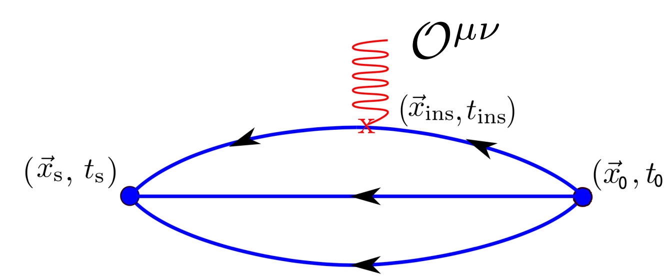
- Quark unpolarised moment $\mathcal{O}^{\mu\nu,q} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$

- Gluon unpolarised moment $\mathcal{O}^{\mu\nu,g} = F^{\{\mu\rho}F_\rho^{\nu\}}$ Field strength tensor

$$\langle N(p',s')|\mathcal{O}^{\mu\nu,q}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}^q(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^q(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}^q(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]u_N(p,s)$$

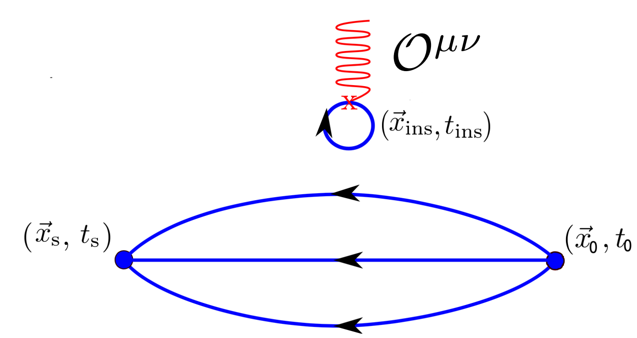
$$\langle x\rangle_q = A_{20}^q(0) \qquad J_q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$$

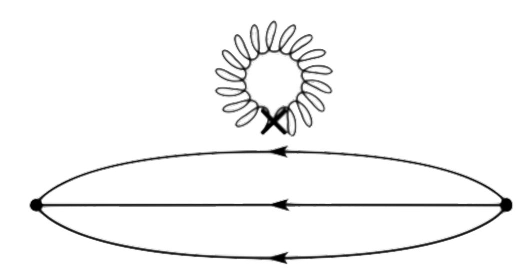
Momentum fraction carried by quark - best measured



- Equivalent expression for gluon

$$\langle x\rangle_g = A_{20}^g(0) \qquad J_g = \frac{1}{2}\left[A_{20}^g(0) + B_{20}^g(0)\right]$$





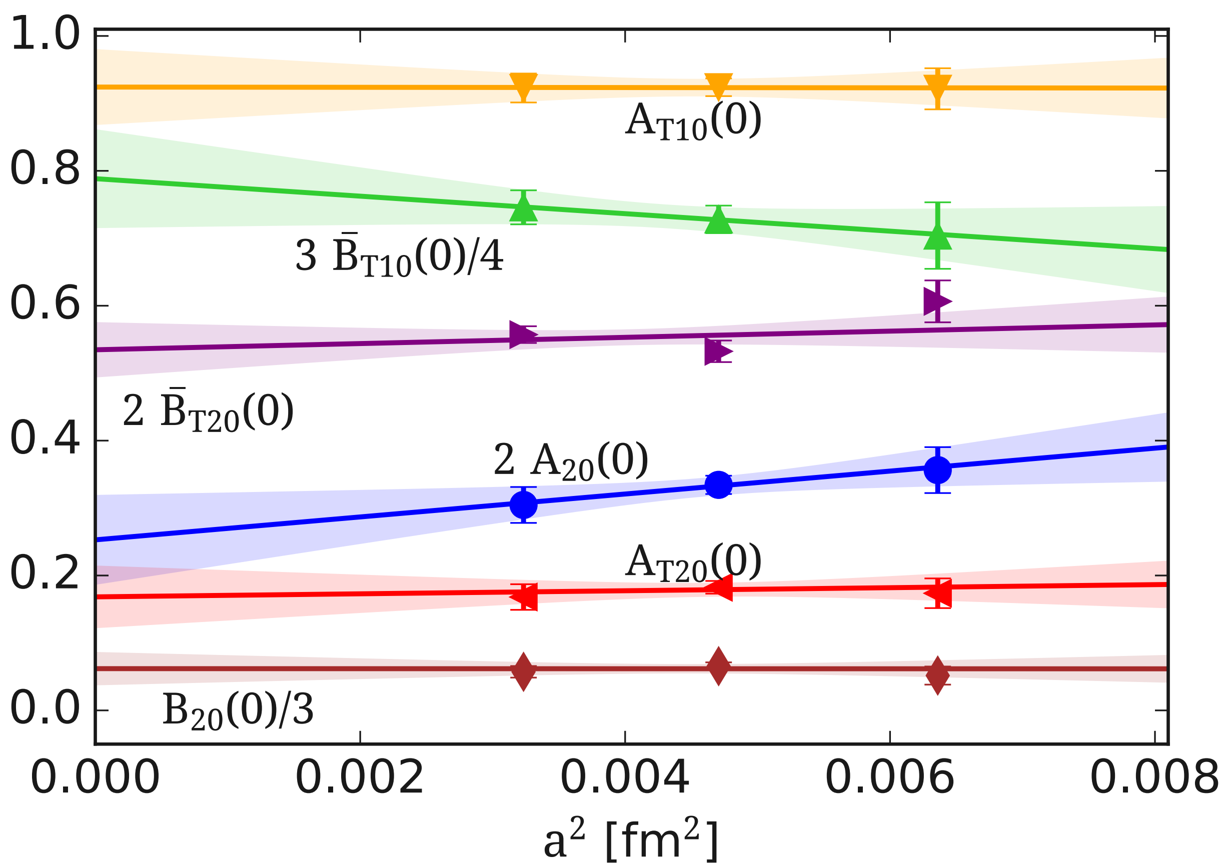
➡ Momentum sum: $\sum_q \langle x\rangle_q + \langle x\rangle_g = 1$

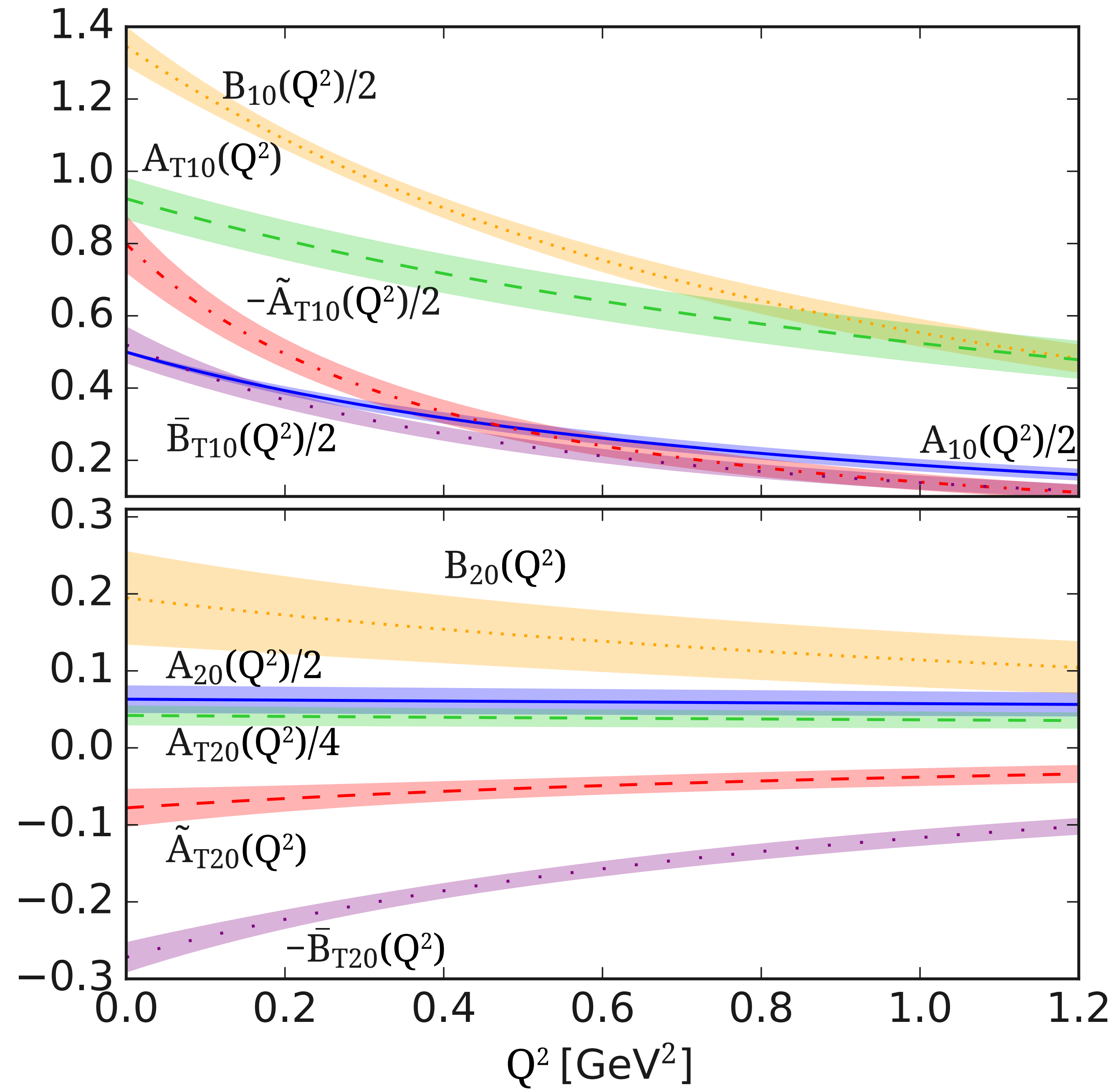
➡ Spin sum: $\sum_q \underbrace{\left[\frac{1}{2}\Delta\Sigma_q + L_q\right]}_{J_q} + J_g = \frac{1}{2}$

- Matrix elements of helicity and transversity one derivative operators yield $\langle x\rangle_{\Delta q}$, $\langle x\rangle_{\delta q}$

Unpolarised and transversity generalised form factors

❋Continuum extrapolate and fit the Q²-dependence to $F(Q^2) = \frac{F(0)}{(1+Q^2/m^2)^p}$





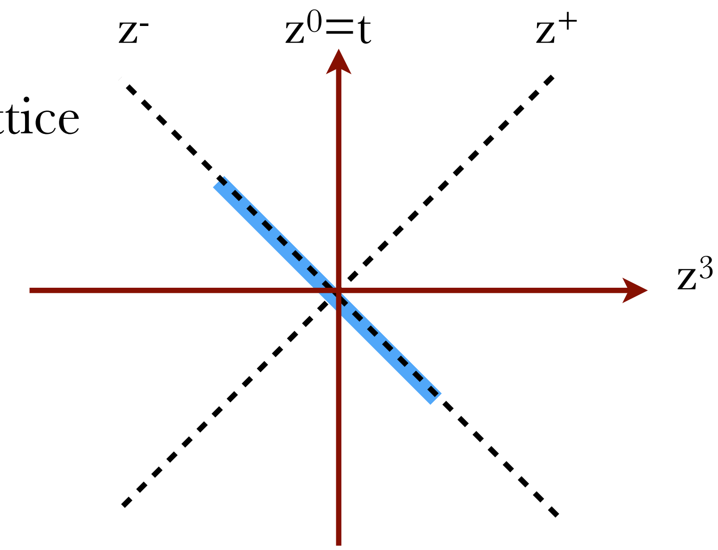
$\tilde{B}_{T10} = B_{T10} + 2\tilde{A}_{T10}$

$\tilde{B}_{T10}(0)$ is the anomalous tensor magnetic moment

C.A. *et al* (ETMC) Phys.Rev.D 107 (2023) 5, 054504, arXiv: 2202.09871

Direct computation of x-dependence of parton distributions

Large momentum effective theory(LaMET)



- PDFs light-cone correlation matrix elements - cannot be computed on a Euclidean lattice

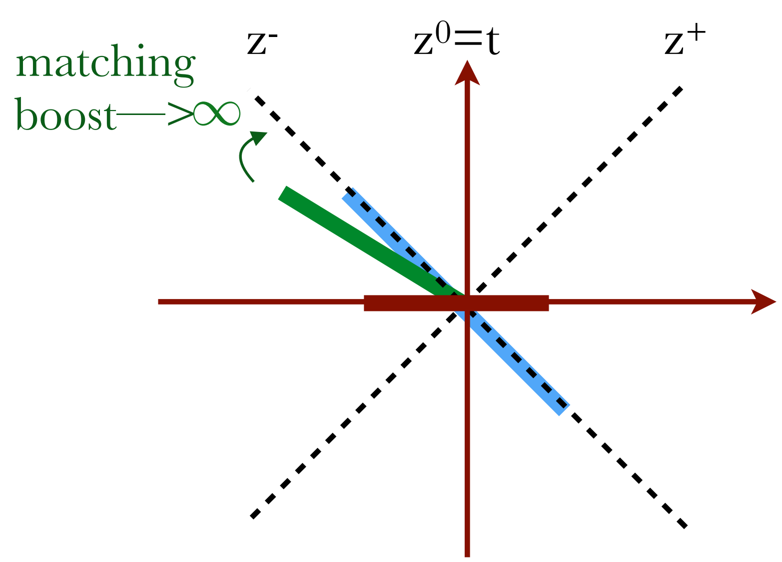
$$F_\Gamma(x) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-}\langle N(p)|\bar\psi(-z/2)\Gamma W(-z/2,z/2)\psi(z/2)|N(p)\rangle|_{z^+=0,\vec{z}=0}$$

- Define spatial correlators e.g. along z^3 and boost nucleon state to large momentum —> quasi PDFs (have same IR behaviour)

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

- Match to the infinite momentum frame using the matching kernel computed in perturbation theory (possible due to asymptotic freedom of QCD)

- Allow momentum transfer —> generalised parton distributions



Computation of quasi-PDFs

X. Ji, Phys. Rev. Lett. 110 (2013) 262002 [arXiv:1305.1539]

- Compute space-like matrix elements for boosted nucleon states and take the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z} \langle P_3 | \overline{\psi}(0)\, \Gamma W(0,z)\, \psi(z) | P_3 \rangle|_\mu$$

Renormalise non-perturbatively, $Z(z,\mu)$

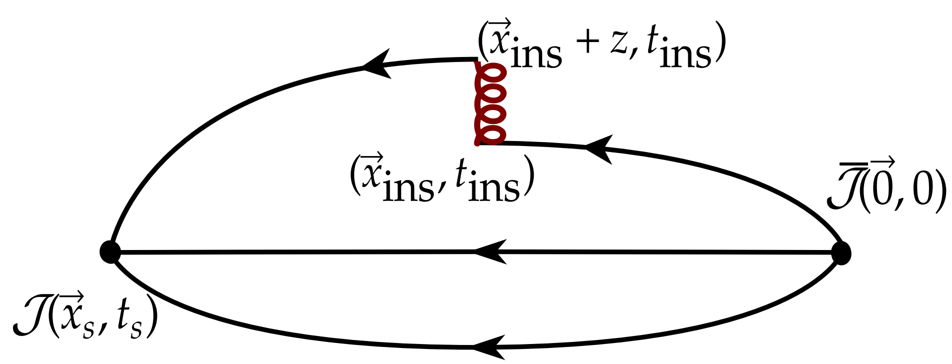
Need to eliminate both UV and exponential divergences

- Match using LaMET

Perturbative kernel

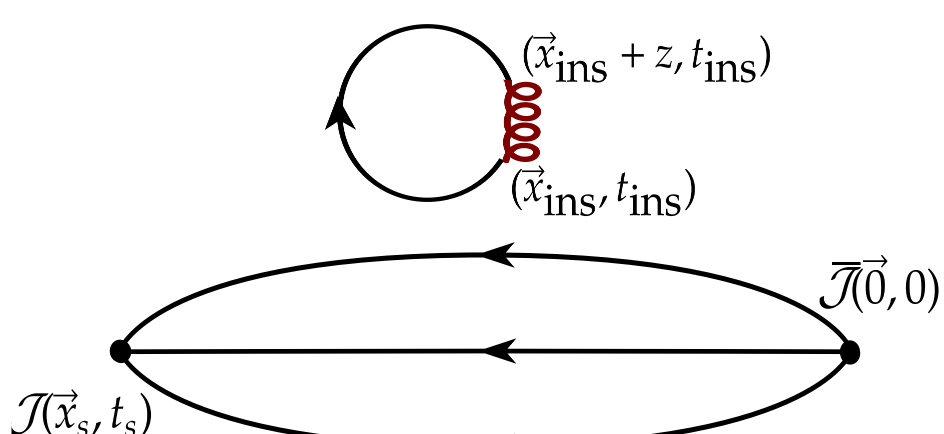
$$\tilde{F}_\Gamma(x, P_3, \mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y, \mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{\rm QCD}^2}{P_3^2}\right)$$

Isovector (u-d) and isoscalar (u+d) connected

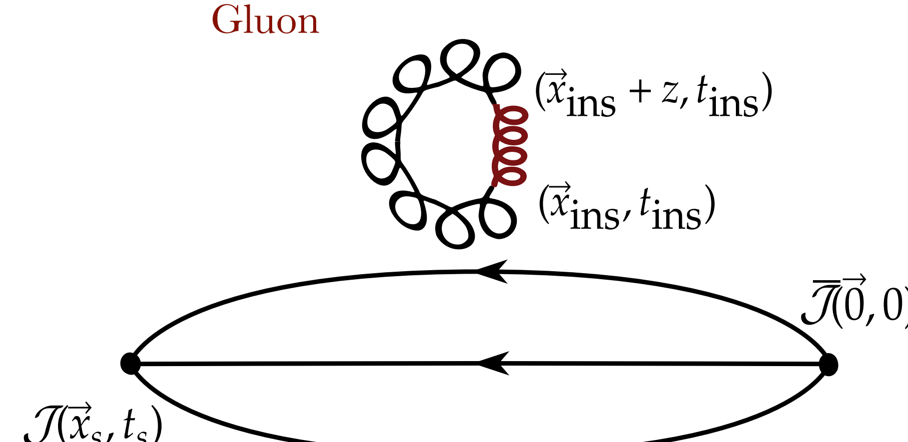


$\Gamma =$	γ_0	unpolarised
	$\gamma_5\gamma_3$	helicity
	$\sigma_{3i}, i = 1, 2$	transversity

Isoscalar (u+d) disconnected, s and c



Gluon



Direct computation of PDFs

- Compute space-like matrix elements for boosted nucleon states and take the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z} \langle P_3 | \overline{\psi}(0)\, \Gamma W(0,z)\, \psi(z) | P_3 \rangle |_\mu$$

Renormalise non-perturbatively, $Z(z,\mu)$

Need to eliminate both UV and exponential divergences

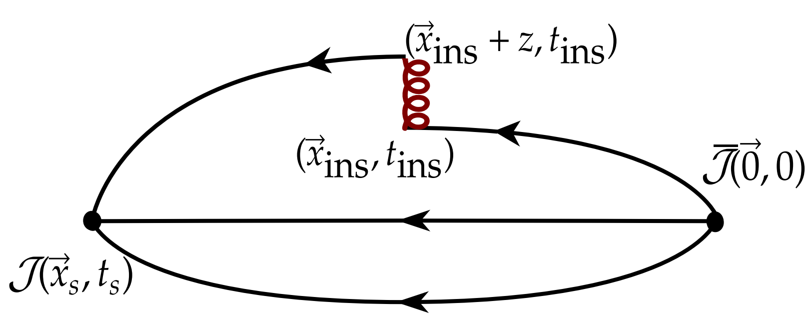
- Match using LaMET

Perturbative kernel

$$\tilde{F}_\Gamma(x, P_3, \mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y, \mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{\rm QCD}^2}{P_3^2}\right)$$

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

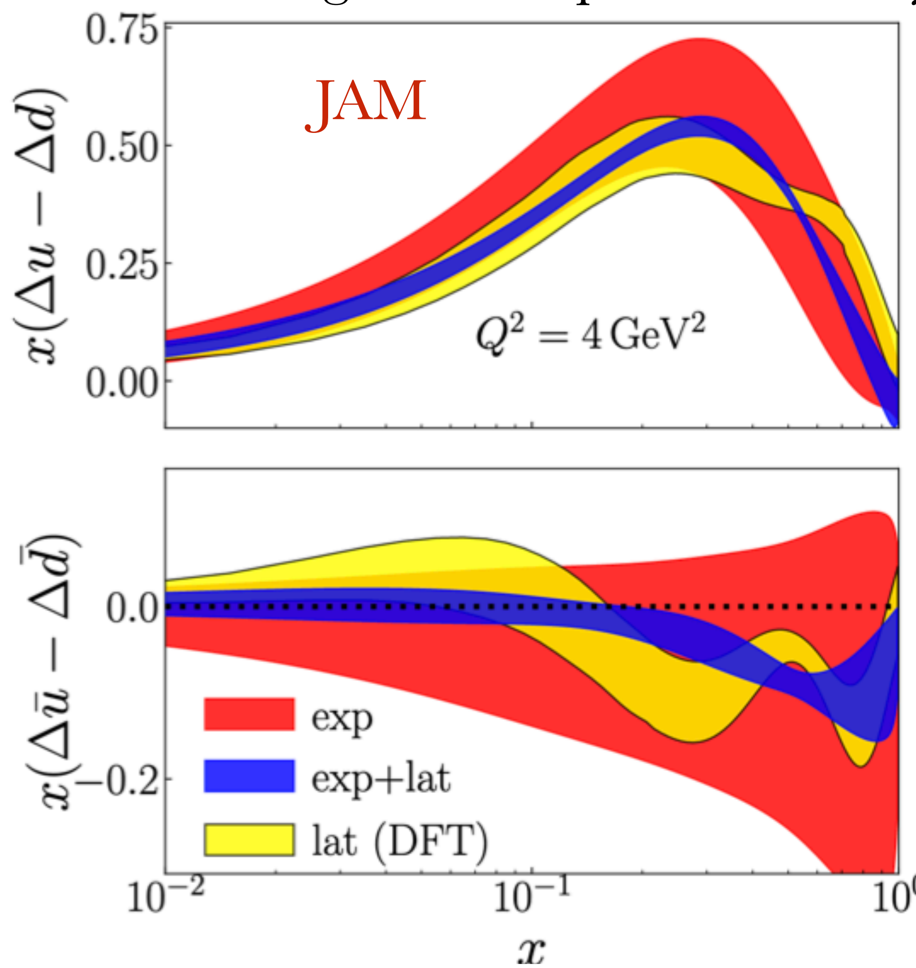
Isovector (u-d)



$\Gamma =$	γ_0	unpolarised
	$\gamma_5\gamma_3$	helicity
	$\sigma_{3i}, i=1,2$	transversity

C.A. et al. (ETMC) Phys. Rev. Lett. **121**, 112001 (2018) , arXiv:1803.02685

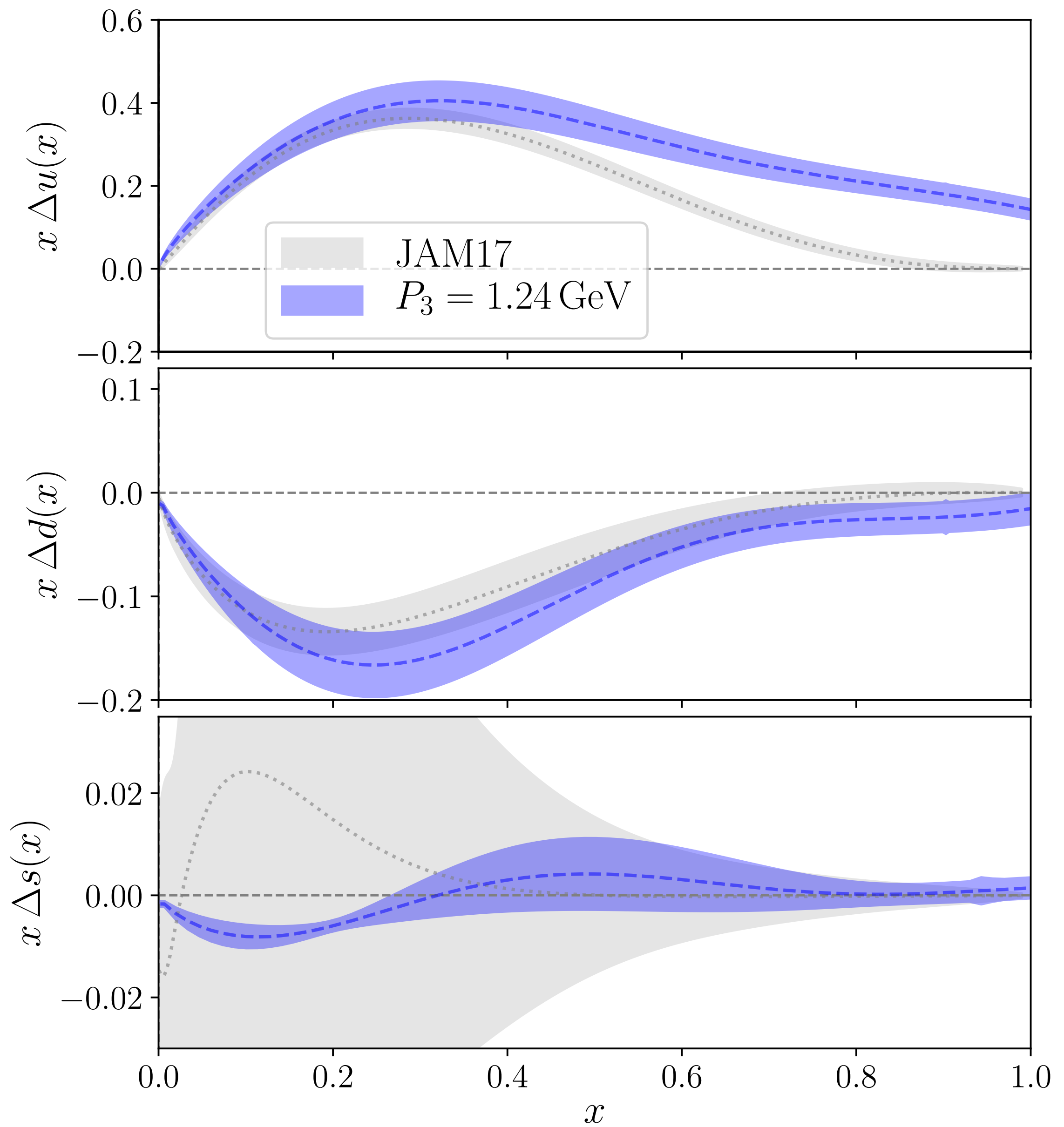
Combining lattice input for helicity



J. Bringewatt *et al.* (JAM) Phys.Rev.D 103 (2021) 1, 016003 , arXiv:2010.00548

Helicity distributions

$32^3 \times 64$ $L = 3.0$ fm	a=0.0938(3)(2) fm $\quad m_N = 1.050(8)$ GeV $m_\pi \approx 260$ MeV $\quad m_\pi L \approx 4.0$



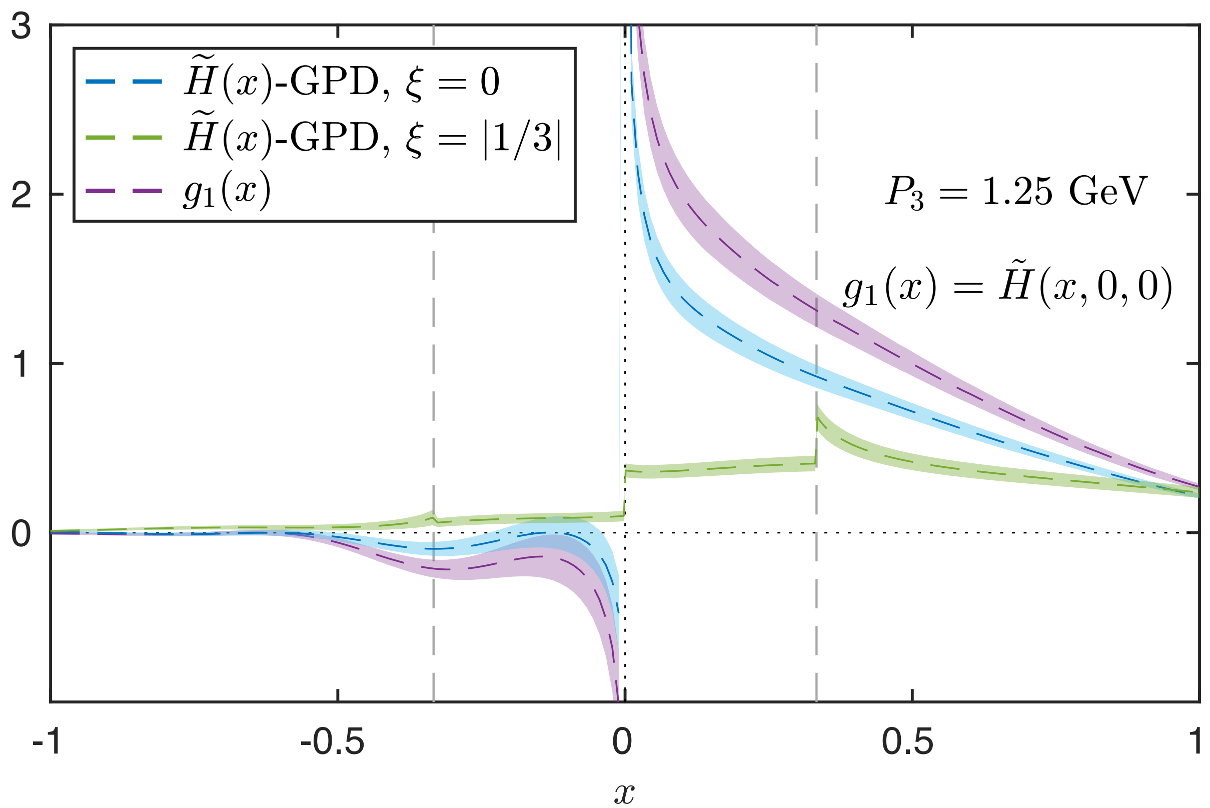
C. A., M. Constantinou, K. Jansen, F. Manigrasso, Phys. Rev. Lett. 126 (2021) 10, 102003, arXiv:2009.1306

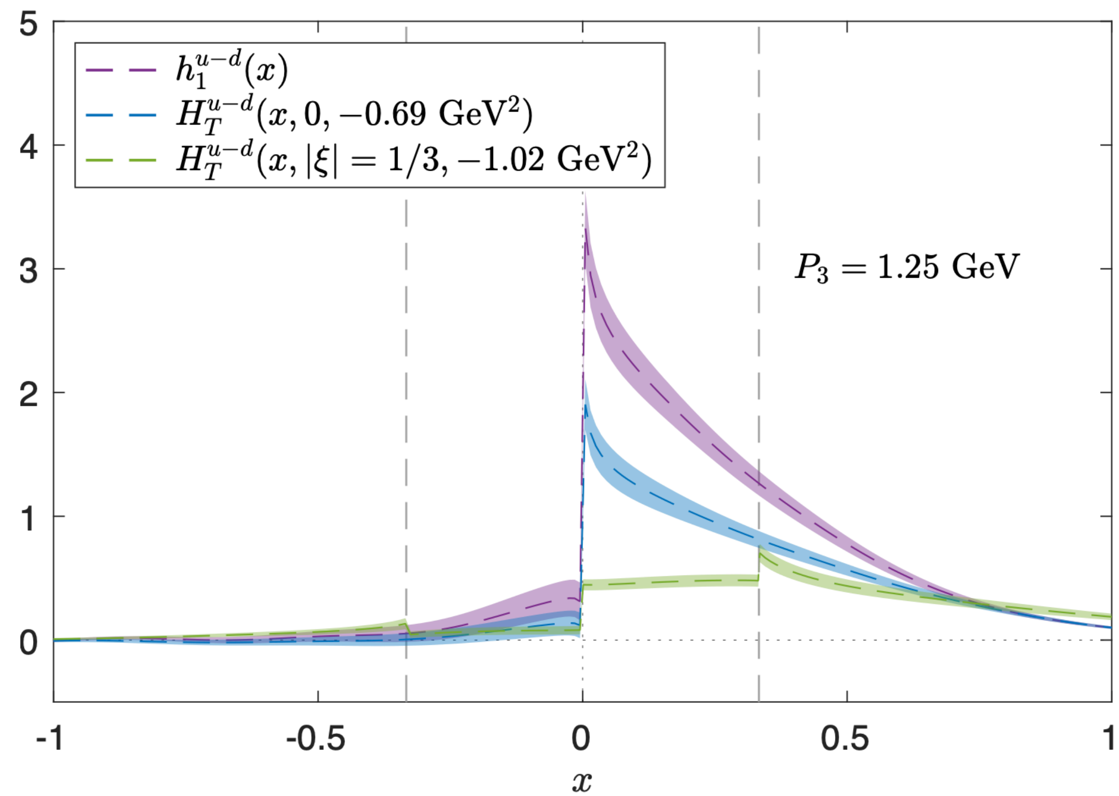
C.A., G. Iannelli, K. Jansen, F. Manigrasso, Phys. Rev. D 102 (2020) 9, 094508, arXiv:2007.13800

Helicity & transversity GPDs

$32^3 \times 64$ $L = 3.0$ fm	a=0.0938(3)(2) fm $\quad m_N = 1.050(8)$ GeV $m_\pi \approx 260$ MeV $\quad m_\pi L \approx 4.0$

Q^2=0.69 GeV^2





C. A. *et al.* (ETMC) Phys. Rev. Lett. 125 (2020) 262001,2008.10573
C.A. *et al.*(ETMC) , Phys.Rev.D 105 (2022) 3, 034501, 2108.10789

New developments of expressing GPDs in terms of Lorentz invariant amplitudes allows easier access to a range of momentum transfers in lattice QCD calculations

S. Bhattacharya *et al.* Phys. Rev. D 106 (2022) 114512, 2209.05373 for unpolarized
S. Bhattacharya *et al.* Phys. Rev. D 109 (2024) 034508, 2310.13114 for helicity

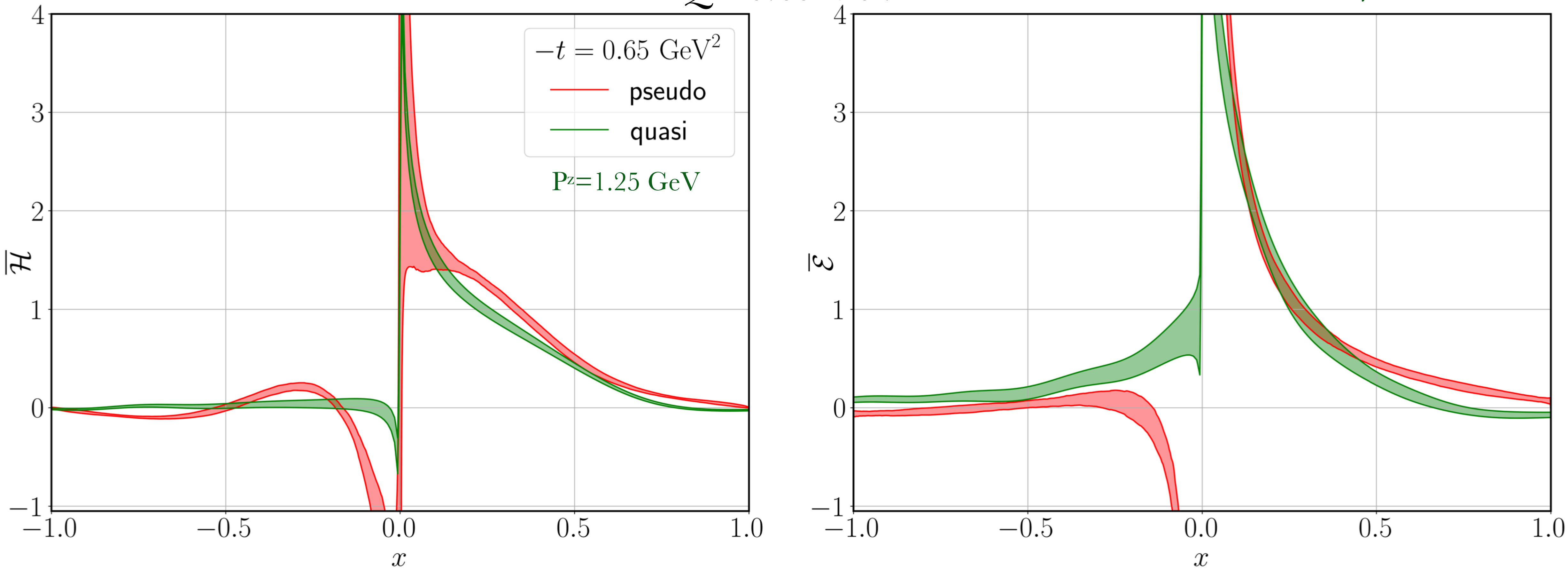
Quasi and pseudo-GPD approaches

$32^3 \times 64$	a=0.0938(3)(2) fm	$m_N = 1.050(8)$ GeV
$L = 3.0$ fm	$m_\pi \approx 260$ MeV	$m_\pi L \approx 4.0$

Zero-skewness unpolarised isovector GPDs in $\overline{\text{MS}}$ at 2 GeV

Q^2=0.69 GeV^2

S. Bhattacharya *et al.* 2405.04414



- Pseudo-GPDs bare matrix elements are renormalised using the double ratio
- Involve, match and take FT in ν to extract the GPDs
- Comparison provides a measure of systematics

$$\mathcal{M}(\nu, z) = \frac{M(\nu, z)}{f(0, z)} \frac{f(0,0)}{f(\nu, 0)}$$

Pseudo-GPDs, A. Radyushkin, Int. J. Mod. Phys. A 35, 2030002 (2020), arXiv:1912.04244

Towards TMD PDFs in lattice QCD

X. Ji, *et al.* Phys. Rev. D 99 (2019) 114006, 1801.05930

M. A. Ebert, I. W. Stewart, Y. Zhao, Phys.Rev.D 99 (2019) 3, 034505, 1811.00026; JHEP 09 (2019) 037, 1901.03685; JHEP 03 (2020) 099 ,1910.08569

- Quasi-TMDs formulated in the LaMET approach

- First results obtained for the unpolarised nucleon TMD PDF by the Lattice Parton Collaboration (LPC)

X. Ji *et al.* (LPC) 2211.02340

$$f^{\mathrm{TMD}}(x,\vec{b}_T,\mu,\zeta) = H\Big(\frac{\zeta_z}{\mu^2}\Big)\, e^{-\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_T,\mu)}\, \tilde{f}(x,\vec{b}_T,\mu,\zeta_z)\sqrt{S_r(b_T,\mu)} + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_T^2\zeta_z}\right)$$

perturbative matching kernel

Collins-Soper kernel, which is non-perturbative for $q_T \sim 1/b_T \sim \Lambda_{QCD}$

Rapidity independent reduced soft function

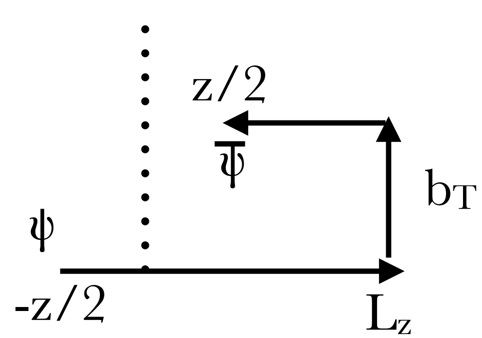
- $\zeta_z=(2xP^z)^2$ is the Collins-Soper scale of the quasi-TMD

- Quasi-TMD PDF is given as $\tilde{f}^{\mathrm{TMD}}(x,\vec{b}_T,\mu,\zeta_z) = \int \frac{dz}{2\pi}\, e^{-iz\zeta_z}\, \frac{P^z}{E_{\vec{P}}} B_\Gamma(z,\vec{b}_T,\mu,P^z)$

Renormalised beam function obtained from the bare

$$\tilde{B}_{0,\Gamma}(z,\vec{b}_T,L,P^z;1/a) = \langle N(P^z)|\bar{\psi}(z/2,\vec{0}_T)\Gamma\mathcal{W}(z,\vec{b}_T,L\hat{z})q(-z/2,\vec{b}_T)|N(P^z)\rangle$$

$$\mathcal{W}(z,\vec{b}_T,L\hat{z}) =$$



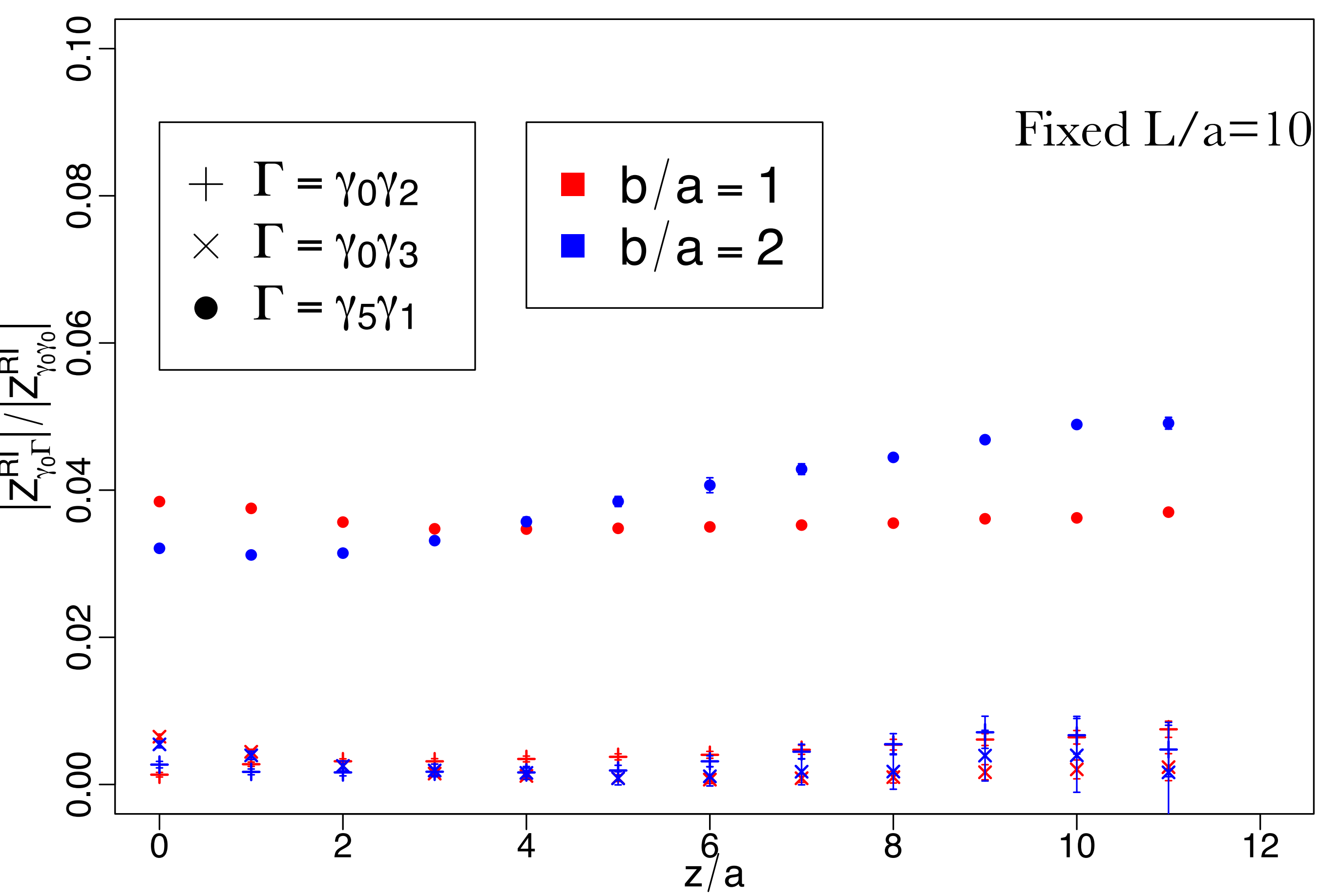
Renormalisation of quark bilinear with asymmetric staples

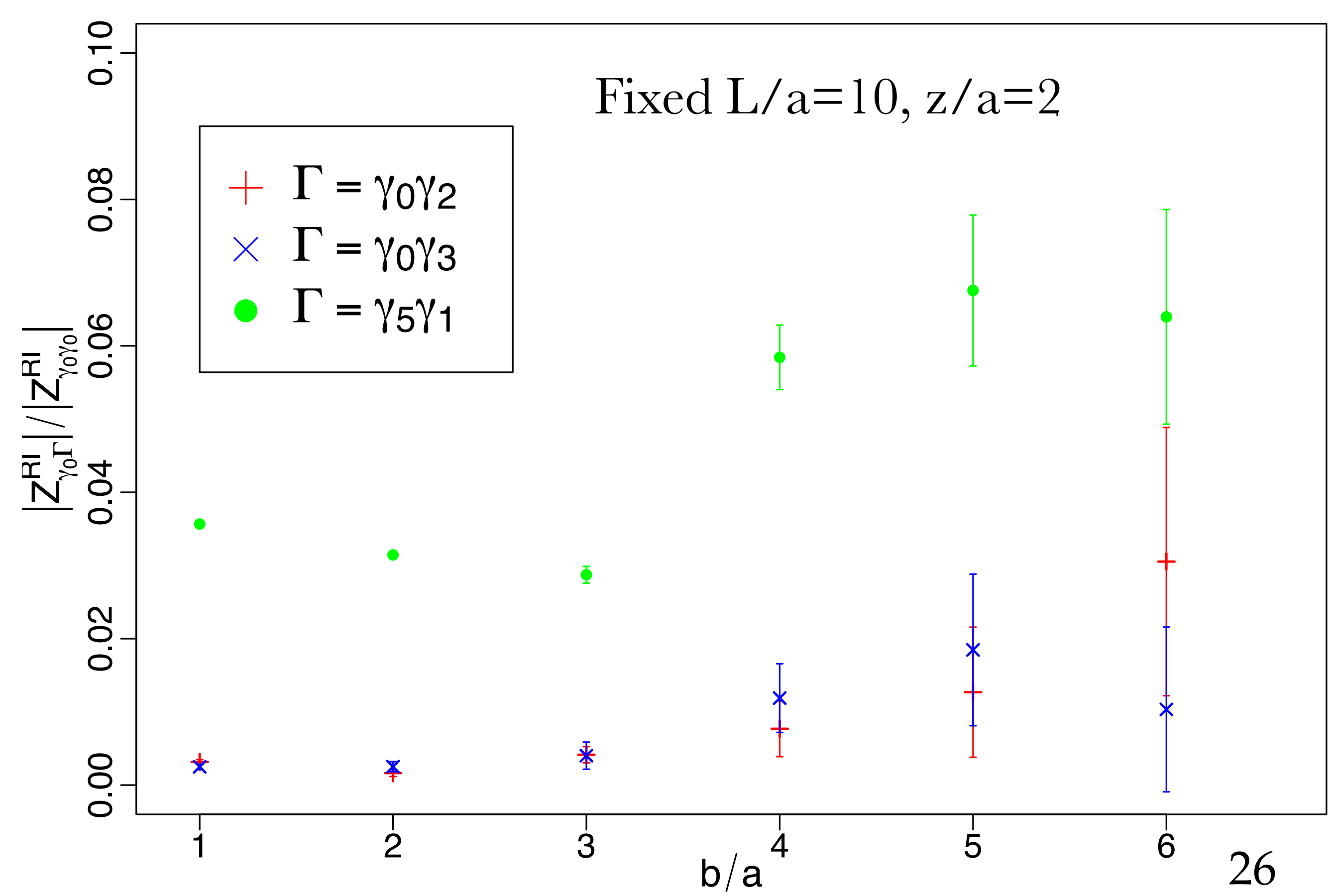
- Linear divergences from the Wilson line [2L+z+b_T]
- Logarithmic divergences due to the cusps and end points of the staple $[\sim \ln(a^2 b_T^2), \ln(a^2 z^2), \ln(a^2 p^2)]$
- Pinch-pole singularities $L \to \infty\, [\sim (L/b_T) \tan^{-1}(L/b_T)]$
- Mixing analysed using symmetries

C. Alexandrou *et al.*, Phys. Rev. D 108 (2023) 114503, arXiv:2305.11824

$$(\mathcal{O}_{\gamma_3}, \mathcal{O}_{\gamma_2}, \mathcal{O}_{\gamma_2\gamma_3}, \mathcal{O}_{\mathbb{1}}), \quad (\mathcal{O}_{\gamma_0}, \mathcal{O}_{\gamma_5\gamma_1}, \mathcal{O}_{\gamma_0\gamma_2}, \mathcal{O}_{\gamma_0\gamma_3}),$$
$$(\mathcal{O}_{\gamma_5\gamma_3}, \mathcal{O}_{\gamma_5\gamma_2}, \mathcal{O}_{\gamma_0\gamma_1}, \mathcal{O}_{\gamma_5}), \quad (\mathcal{O}_{\gamma_5\gamma_0}, \mathcal{O}_{\gamma_1}, \mathcal{O}_{\gamma_1\gamma_2}, \mathcal{O}_{\gamma_1\gamma_3}).$$

For b_T<6a and z mixing is small

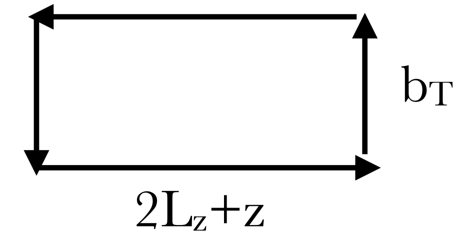




Renormalised beam function

✷ Intermediate renormalisation schemes

Cancel the exponential, cusps and pinch-pole divergences with the vacuum expectation value of a rectangular Wilson loop Z_E



$$B_\Gamma(z, \vec{b}_T, P^z; 1/a) = \lim_{L\to\infty} \frac{\tilde{B}_{0,\Gamma}(z, \vec{b}_T, L.P^z; 1/a)}{\sqrt{Z_E(\vec{b}_T, 2L+z; 1/a)}}$$

1. Short distance ratio **ignoring mixing**:

X. Ji et al. Phys. Rev. D 104 (2021) 94510

- Cancel UV divergences by taking ratio with same matrix element with e.g. zero momentum at a fixed short distance

$$B_\Gamma^{SDR}(z, b_T, P^z) = \frac{B_\Gamma(z, b_T, P^z; 1/a)}{B_\Gamma(z_0, b_{T0}, 0; 1/a)}$$

2. Short distance RI-MOM (RI-short):

- Define a vertex free of divergences in z and b_T

$$\Lambda(z, \vec{b}_T, p; 1/a) = \lim_{L\to\infty} \frac{\Lambda_0(z, \vec{b}_T, p; 1/a)}{\sqrt{Z_E(\vec{b}_T, 2L+z; 1/a)}}$$

- Compute renormalisation functions as in RI-scheme but at some fixed z_0, $\vec{b}_T^0$

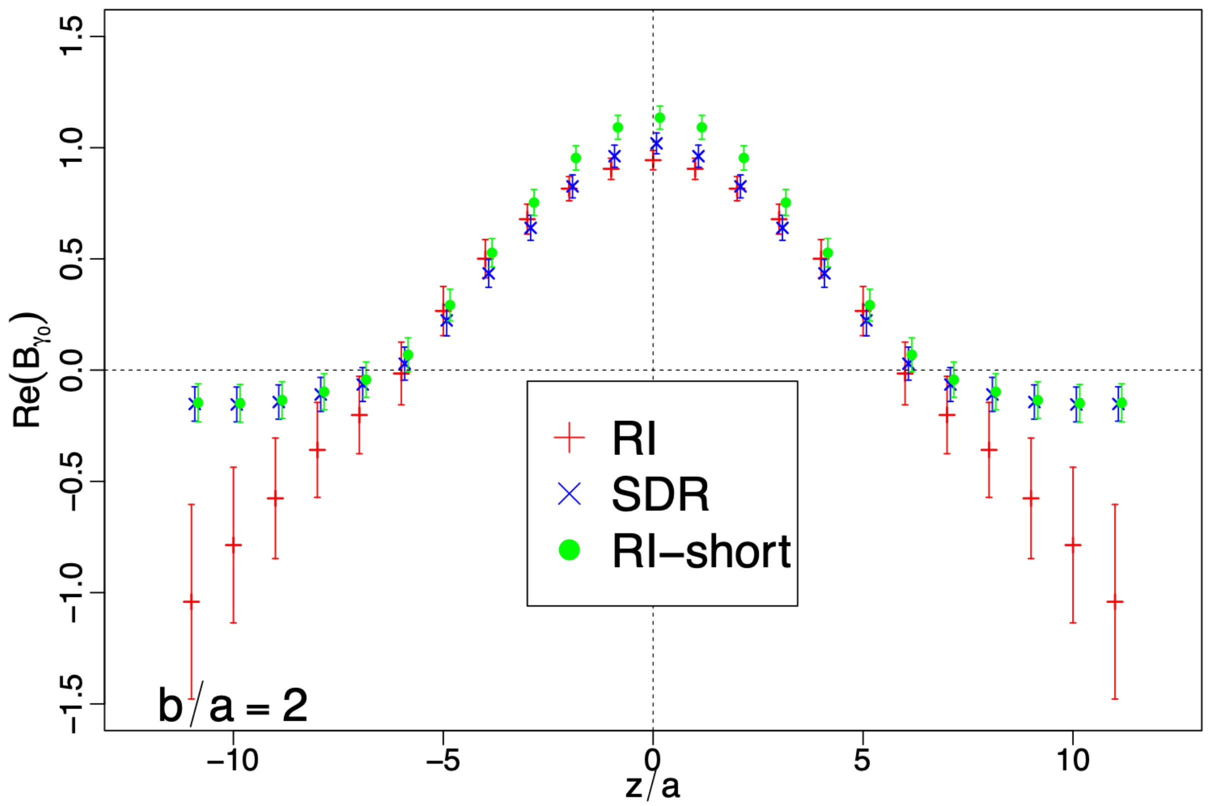
$$B_\Gamma^{\mathrm{RI-short}}(z, b_T, P^z) = \left[Z_{\mathcal{O}}^{\mathrm{RI-short}}(z_0, b_{T0}, \mu_0; 1/a)\right]_{\Gamma\Gamma'} B_{\Gamma'}(z, b_T, P^z; 1/a)$$

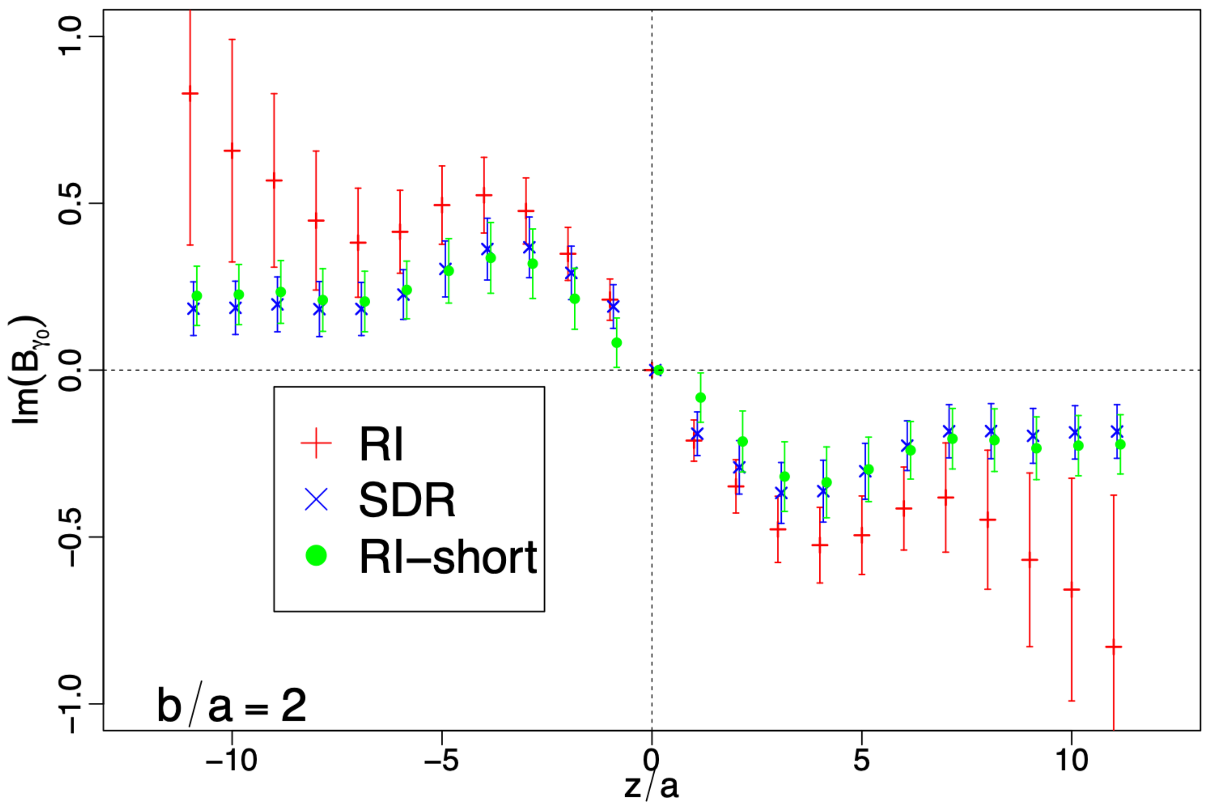
✷ Convert to $\overline{\mathrm{MS}}$ scheme at 2 GeV: $Z^{\overline{\mathrm{MS}}} = Z^{\overline{\mathrm{MS}},X} Z^X$, $Z^{\overline{\mathrm{MS}},X}$ calculated to one-loop

G. Spanoudes *et al*., arXiv:2401.01182

Comparison of renormalised beam function

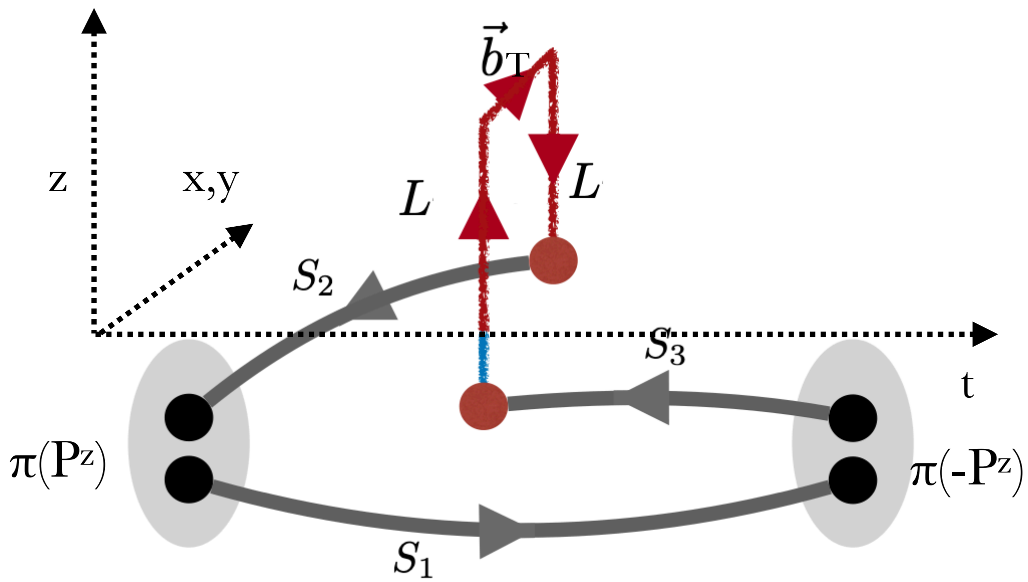
- Results obtained using an ensemble with pion mass 350 MeV, lattice spacing 0.093 fm and P^z=1.7GeV
- One-loop perturbation is used to covert to for all schemes $\overline{\mathrm{MS}}$



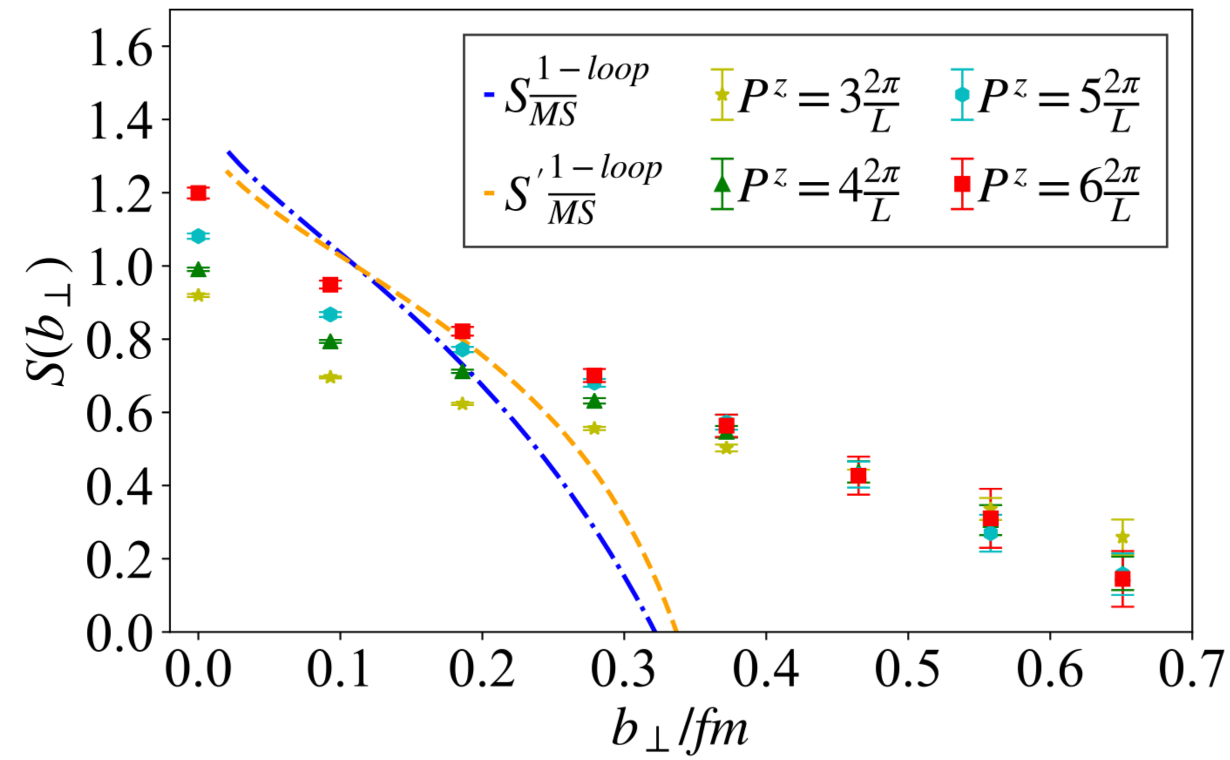


Reduced soft function

❋S_r can be computed from the matrix element of a meson and the quasi-TMD wave function



- X. Ji, Y. Liu, Y.-Sh. Liu, Nucl.Phys.B 955 (2020) 115054,1910.11415
- Q.-A. Zhang *et al.* (PLC) Phys. Rev. Lett. 125 (2020) 19, 192001, 2005.14572
- M.-H. Chu *et al.* (PLC), JHEP 08 (2023) 172, 2306.06488
- Y. Li *et al.* (ETMC), Phys. Rev. Lett. 128 (2022) 6, 062002, 2106.13027



$$F(\vec{b}_T, P^z) = \langle \pi(-P^z)|\bar{u}(\vec{b}_T, t_i)\Gamma u(\vec{b}_T, t_i)\bar{d}(\vec{0}_T, t_i)\Gamma d(\vec{0}_T, t_i)|\pi(P^z)\rangle$$

$$F(\vec{b}_T, P^z) = \lim_{P^z\to\infty} S_r(\vec{b}_T, \mu)\int_0^1 dx dx' H(x, x', P^z, \mu)\Phi^\dagger(x', \vec{b}_T, -P^z)\Phi(x, \vec{b}_T, P^z)$$

Perturbative hard kernel

Apply ratio renormalisation using ME with $P^z=0$

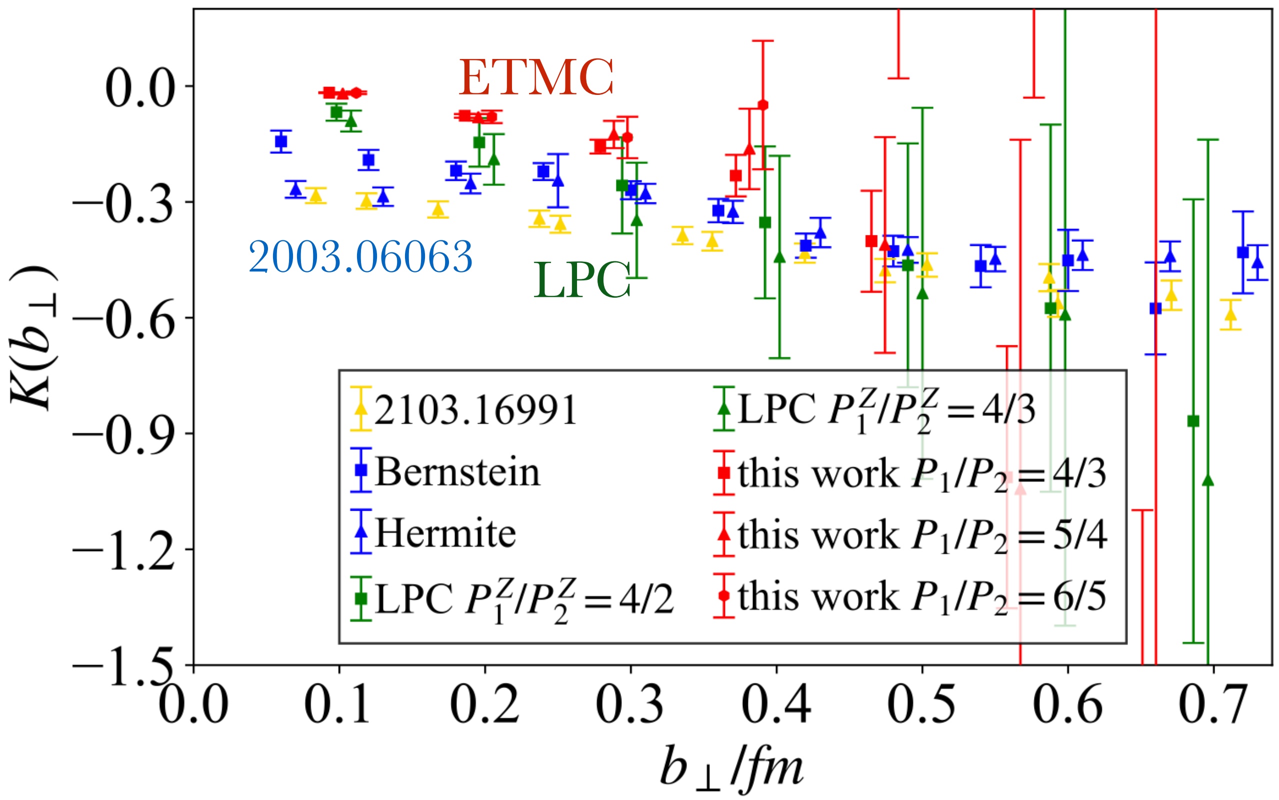
$$\Phi(x, \vec{b}_T, \pm P^z) = \lim_{L\to\infty}\int \frac{dz}{2\pi}P^z\, e^{\pm i(x-\frac{1}{2})zP^z}\,\phi(\pm z, \vec{b}_T, \pm L\hat{z}, \pm P^z)$$

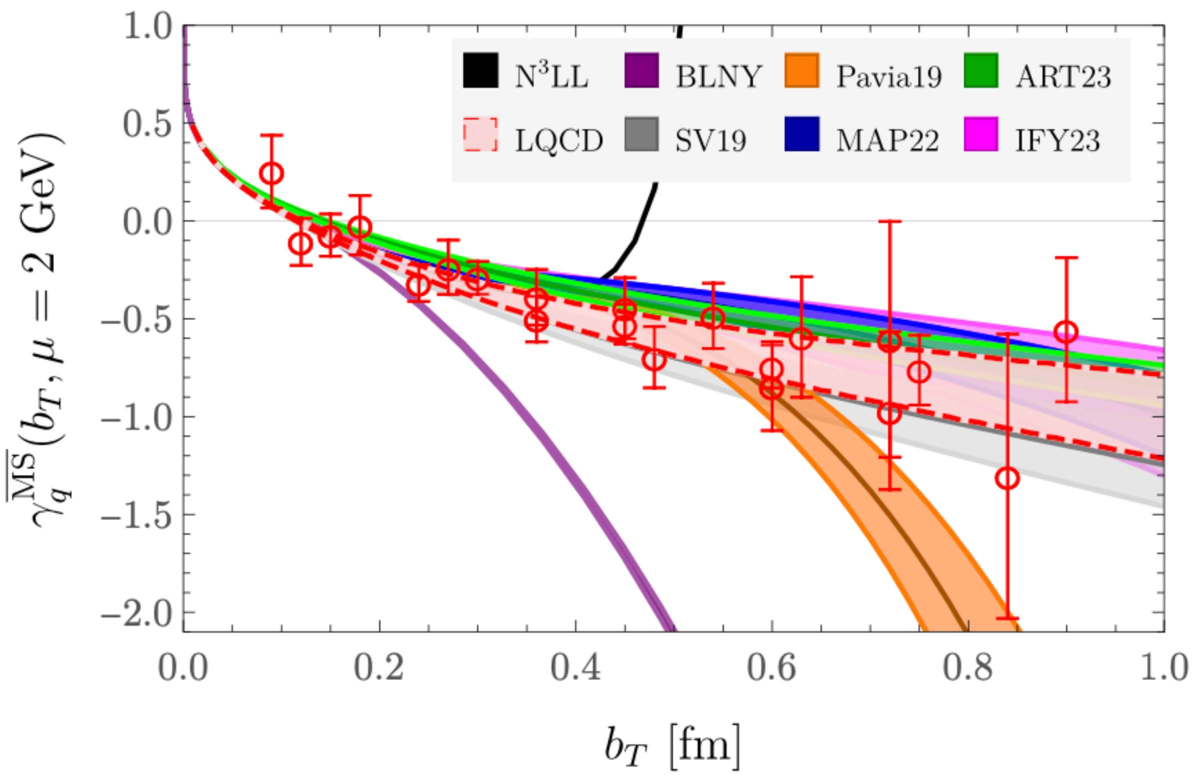
$$\phi(z, \vec{b}_T, L\hat{z}, P^z) = e^{E_\pi t}\,\langle 0|\bar{u}(t, z/2, \vec{b}_T)\Gamma'\mathcal{W}(z, \vec{b}_T, L\hat{z})d(t, -z/2, \vec{0}_T)|\pi(P^z)\rangle$$

Collins-Soper kernel

- Can be obtained as a ratio of quasi-TMD wave functions

$$K(\vec{b}_T,\mu) = \lim_{L\to\infty} \frac{1}{\ln(P_1^z/P_2^z)} \ln\left|\frac{\phi(\vec{b}_T, L\hat{z}, P_1^z)E_2}{\phi(\vec{b}_T, L\hat{z}, P_2^z)E_1}\right|$$





- Y. Li *et al.* (ETMC), Phys. Rev. Lett. 128 (2022) 6, 062002,
- M. Schemer *et al.* *JHEP* 08 (2021) 004, arXiv:2103.16991
- Q.-A. Zhang et al. (LPC) Phys. Rev. Lett. 125,192001 (2020), arXiv: 2005.14572

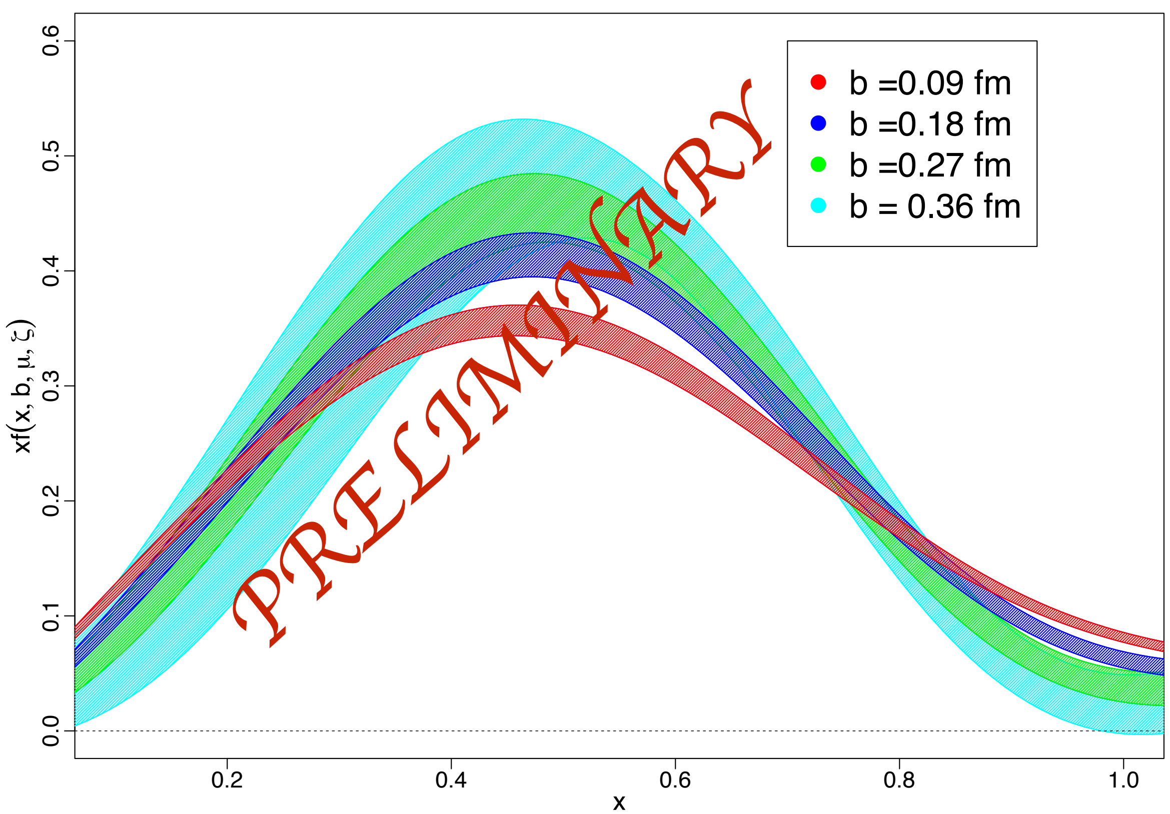
- Ph. Shanahan, M. Wagman, Y. Zhao, Phys. Rev. D 102 (2020) 1, 014511, 2003.06063; Phys. Rev. D 104 (2021) 114502, 2107.11930
- A. Avkhadiev, Ph. E. Shanahan, M. Wagman, Y. Zhao, 2402.06725

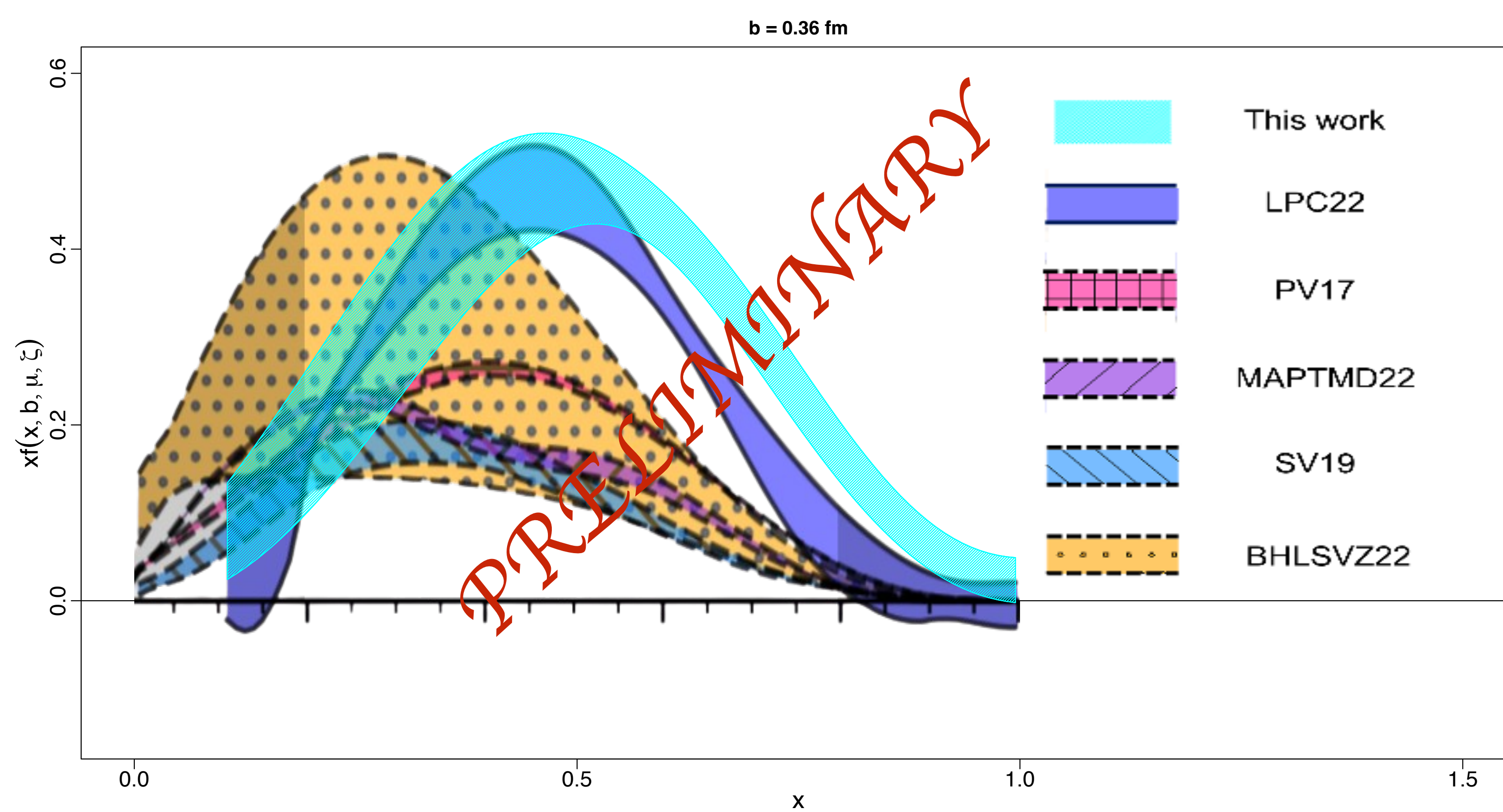
Nucleon unpolarised isovector TMD PDF

- LPC published the first results modelling the momentum dependence and taking the chiral and continuum limits

Jin-Chen He et al. (LPC) arXiv:2211.02340

- ETMC has preliminary results at one lattice spacing (0.093 fm) and heavier than physical pion mass (350 MeV), renormalised with the ratio scheme





Conclusions

- **Precision era of lattice QCD**: Moments of PFDs can be extracted precisely - we can extract a lot of interesting physics and also reconstruct the PDFs
- Results on isovector and gluon PDFs using simulations with physical pion mass using various approaches (**quasi-distributions**, pseudo-distributions, current-current correlates, etc)
- Calculations of GPDs using a suitable for lattice frame and extraction of Lorentz invariant amplitudes
- The calculation of sea quark contributions is feasible providing valuable input e.g. for the determination of strange helicity
- Exploratory studies of TMDs

  ✦ Way forward: continuum limit, larger boosts, volume effects,…