SIDIS at small x:

Sudakov double logs from NLO corrections

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QCD Evolution 2024

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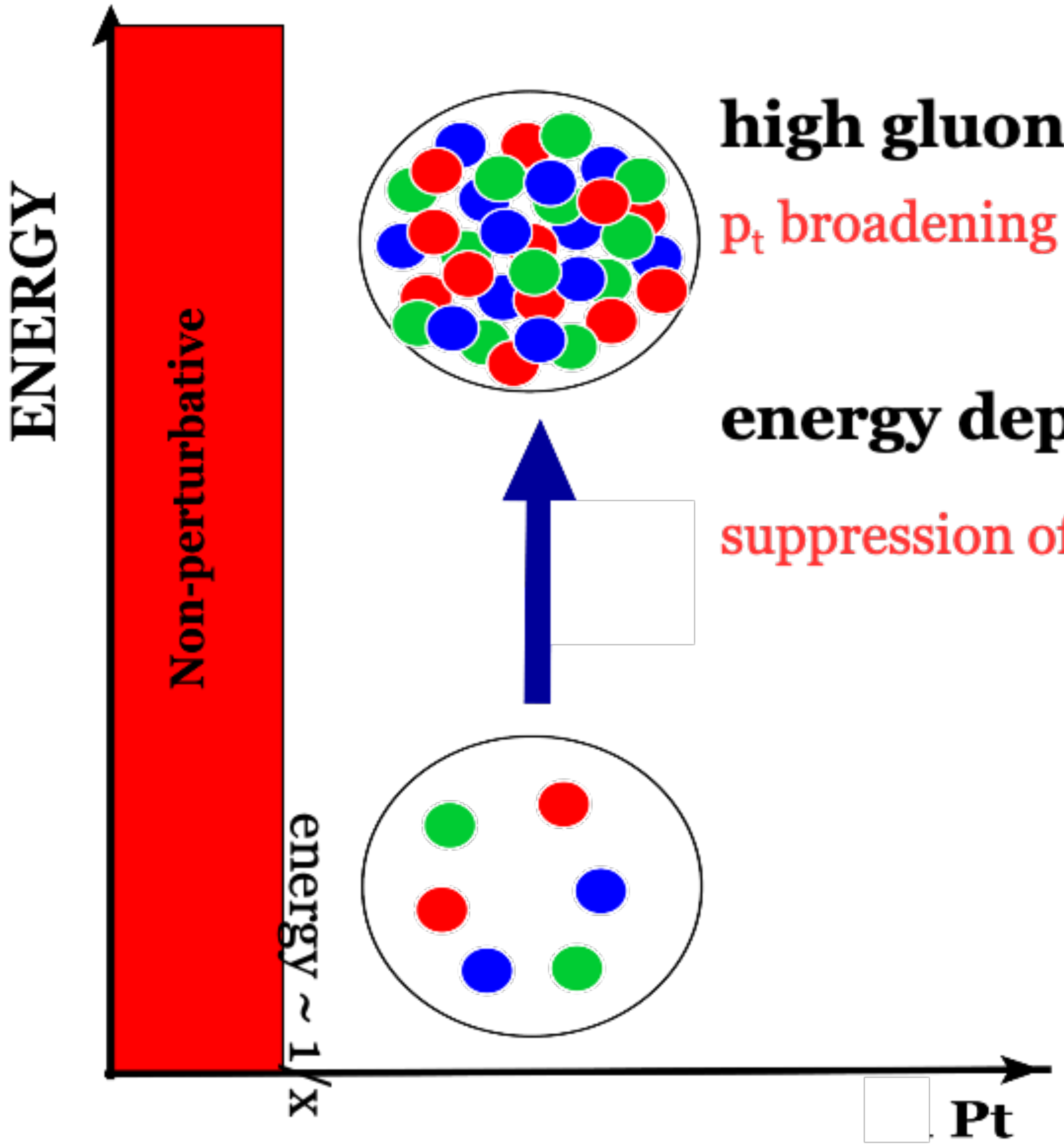
QCD at high energy: gluon saturation



high gluon density: Eikonal multiple scattering

p_t broadening

energy dependence: x-evolution via JIMWLK/BK

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

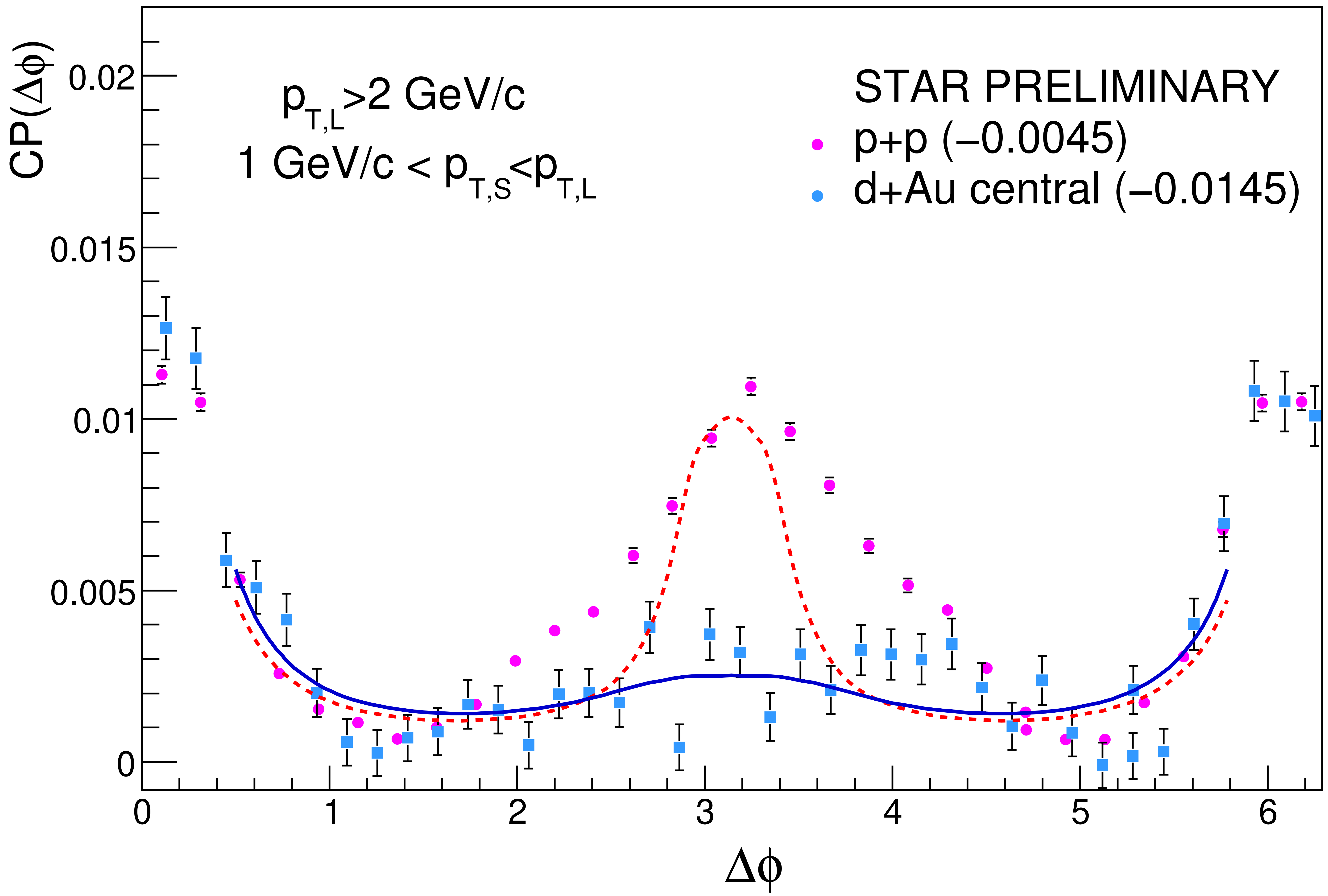
for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

Shadowing/Nuclear modification factor
Azimuthal angular correlations (dihadrons/dijets,...)
Long range rapidity correlations (ridge,...)
Connections to TMDs,...

$$\mathbf{x \leq 0.01}$$

Back to back limit in dihadron production in pA collisions at RHIC



Albacete and Marquet, PRL105 (2010) 162301

Probing CGC in high energy collisions

nucleus-nucleus collisions: “dense on dense” significant modeling/QGP

proton-nucleus collisions: “dilute on dense”

DIS: (inclusive/diffractive) much less modeling

structure functions

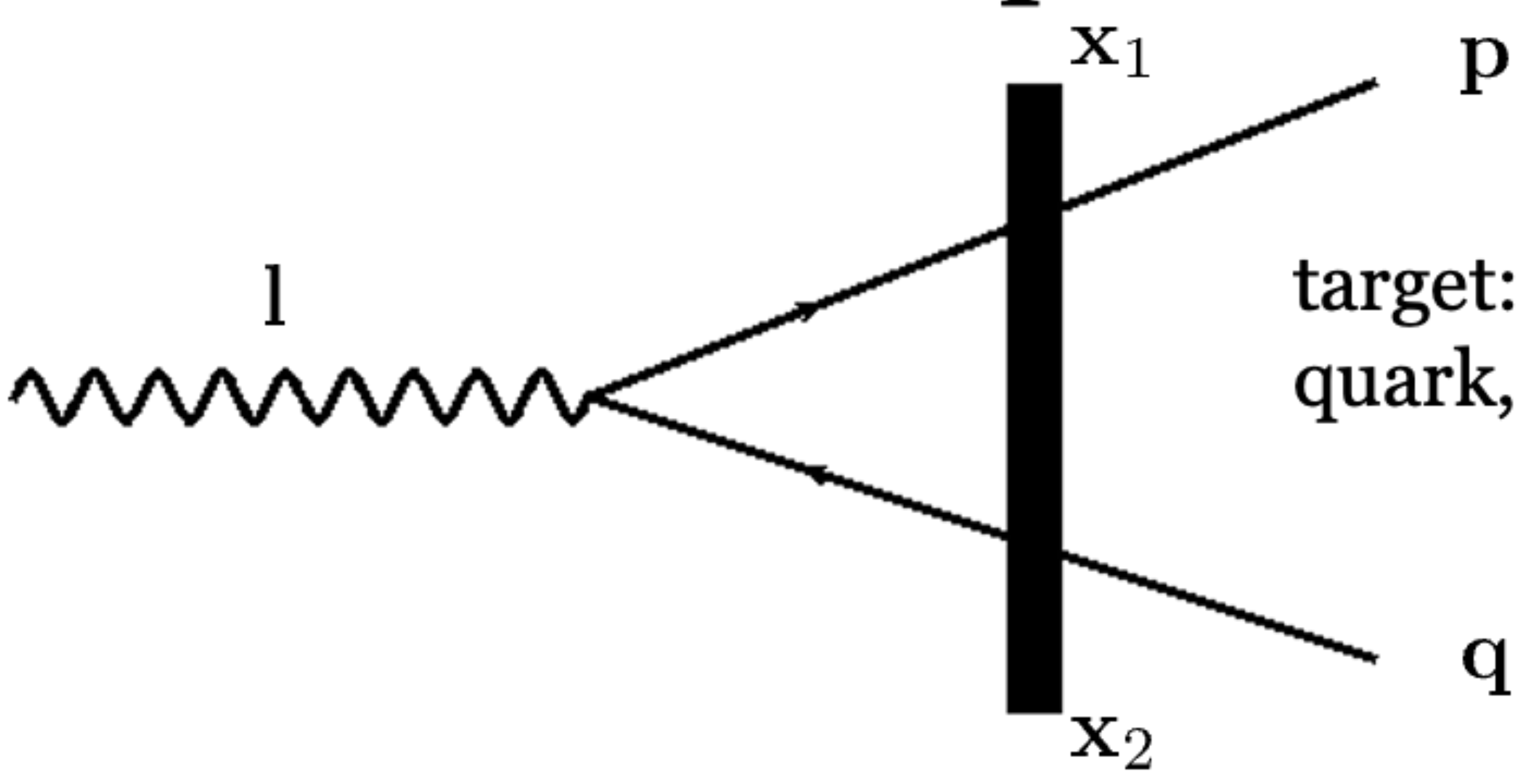
particle production

angular correlations

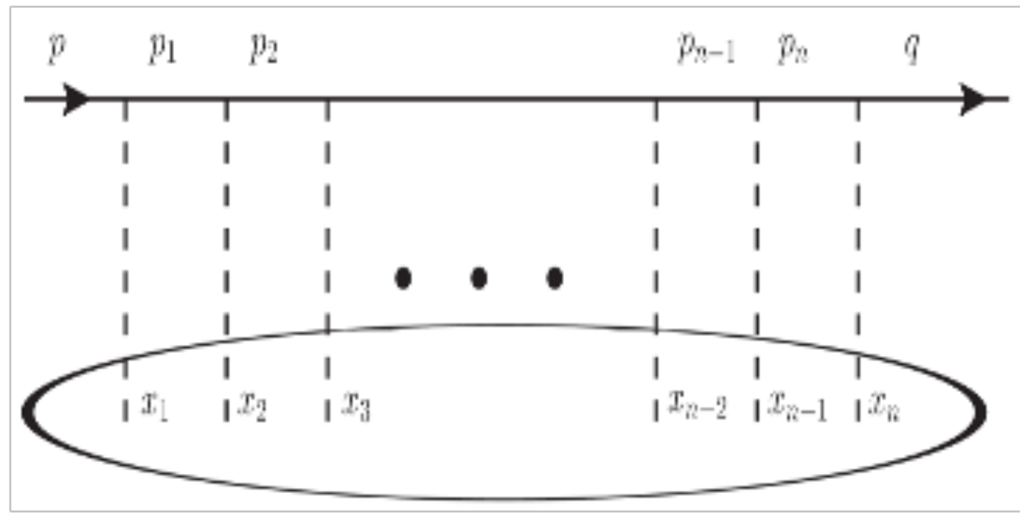
EIC

to start in ~ 8 years

Inclusive dihadron production in forward rapidity: LO



target: a classical color field
quark, antiquark multiply scatter on the target



$$\frac{d\sigma^{\gamma^* A \to q\bar{q} X}}{d^2p\, d^2q\, dy_1\, dy_2} = \frac{e^2 Q^2 (z_1 z_2)^2 N_c}{(2\pi)^7} \delta(1 - z_1 - z_2)$$

$$\int d^8 x_\perp e^{ip\cdot(x_1' - x_1)} e^{iq\cdot(x_2' - x_2)} \left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right]$$

$$\Big\{ 4 z_1 z_2 K_0(|x_{12}|Q_1) K_0(|x_{1'2'}|Q_1) +$$

$$(z_1^2 + z_2^2) \frac{x_{12} \cdot x_{1'2'}}{|x_{12}||x_{1'2'}|} K_1(|x_{12}|Q_1) K_1(|x_{1'2'}|Q_1) \Big\}$$

with

dipole $\mathbf{S}_{12} \equiv \frac{1}{N_c} Tr\, V(x_1)\, V^\dagger(x_2)$

$$\mathbf{x}_{12} \equiv \mathbf{x}_1 - \mathbf{x}_2$$

quadrupole

$$\mathrm{S}_{122'1'} \equiv \frac{1}{N_c} Tr\, V(\mathbf{x}_1)\, V^\dagger(\mathbf{x}_2)\, V(\mathbf{x}_{2'})\, V^\dagger(\mathbf{x}_{1'})$$

Only dipoles and quadrupoles contribute: DMXY, PRD 83 (2011) 105005

Toward precision CGC: inclusive DIS

NLO BK/JIMWLK evolution equations

Kovner, Lublinsky, Mulian (2013)

Balitsky, Chirilli (2007)

NLO corrections to structure functions

Beuf, Lappi, Paatelainen (2022)

Beuf (2017)

NLO corrections to SIDIS

Bergabo, JJM (2023, 2024)

Caucal, Ferrand, Salazar (2024)

NLO corrections to dihadron/dijets (+)

Bergabo, JJM (2022, 2023)

Iancu, Mulian (2023)

Caucal, Salazar, Schenke, Stebel, Venugopalan (2023), Caucal, Salazar, Schenke, Venugopalan (2022)

Taels, Altinoluk, Beuf, Marquet (2022), Taels (2023)

Caucal, Salazar, Venugopalan (2021)

Ayala, Hentschinski, JJM, Tejeda-Yeomans (2016,2017),............

Toward precision CGC: exclusive/diffractive DIS

NLO corrections to diffractive structure functions

Beuf, Hanninen, Lappi, Mulian, Mantyssari (2022)

……………………..

NLO corrections to diffractive dihadron/dijets (+)

Boussarie, Grabovsky, Szymanowski, Wallon (2016)

Iancu, Mueller, Triantafyllopoulos (2021, 2022)

Fucilla, Grabovsky, Li, Szymanowski, Wallon (2023)

……………………..

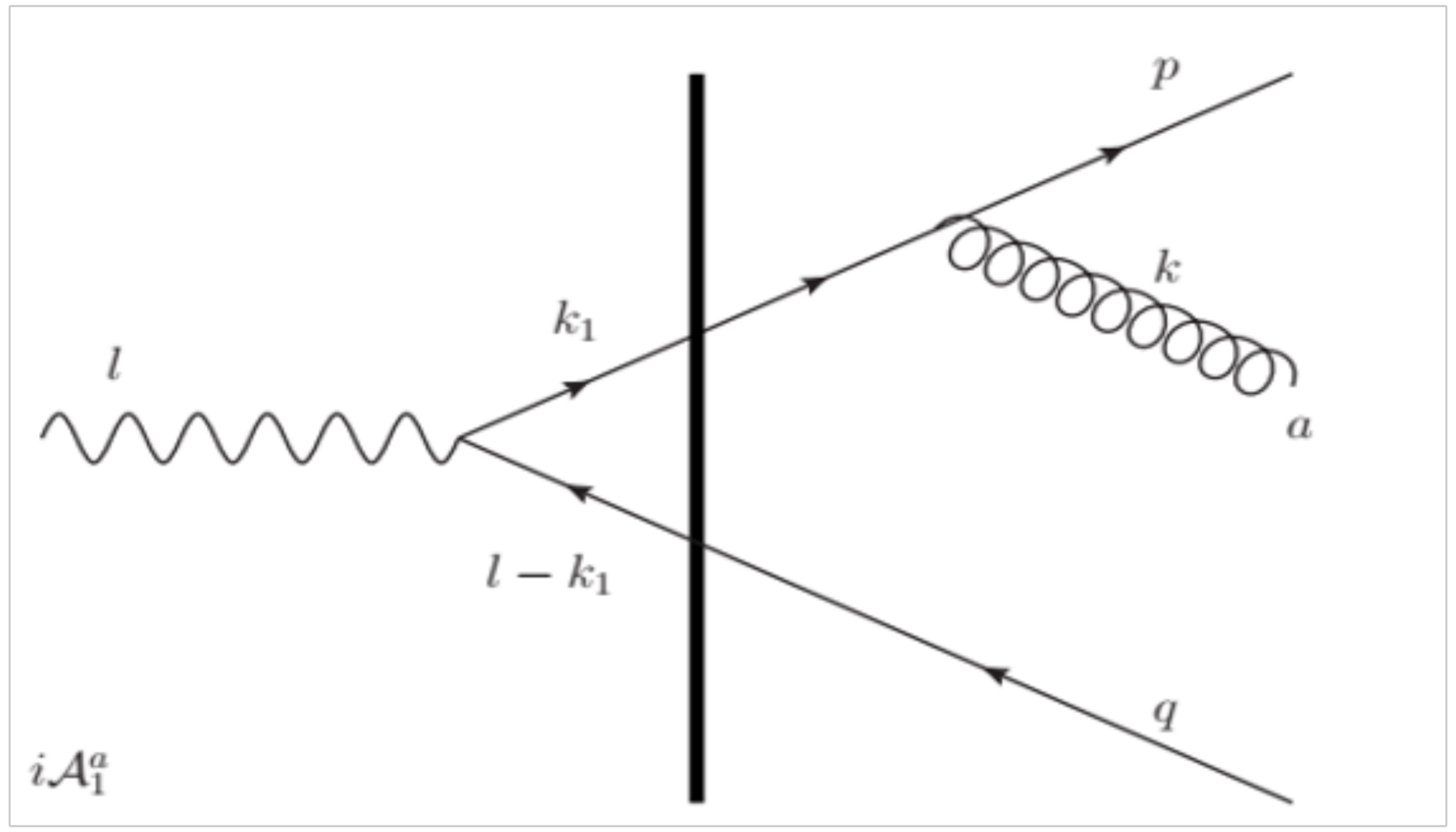
NLO corrections to exclusive light/heavy vector meson production (+)

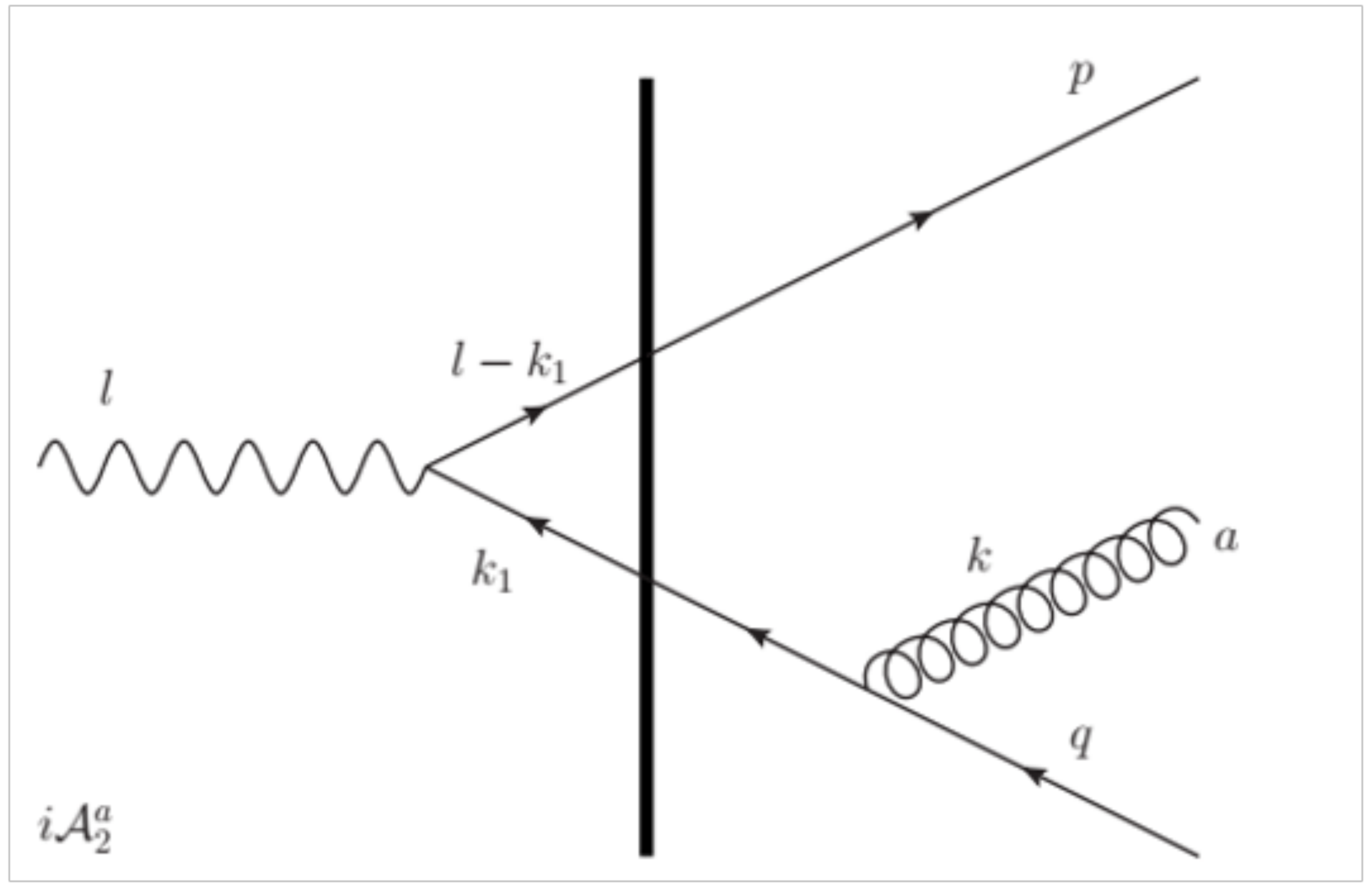
Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2016)

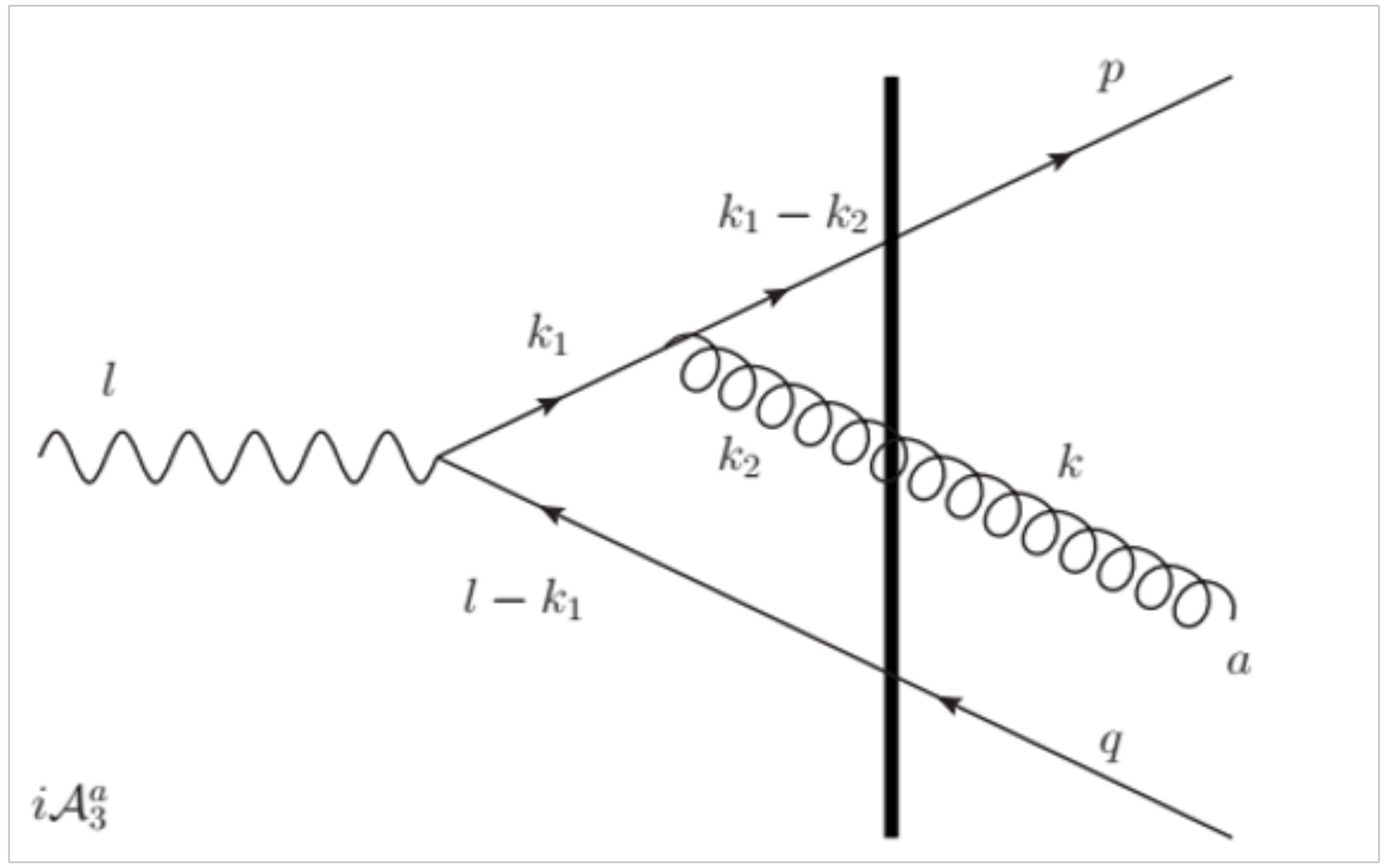
Mantyssari, Penttala (2021, 2022)

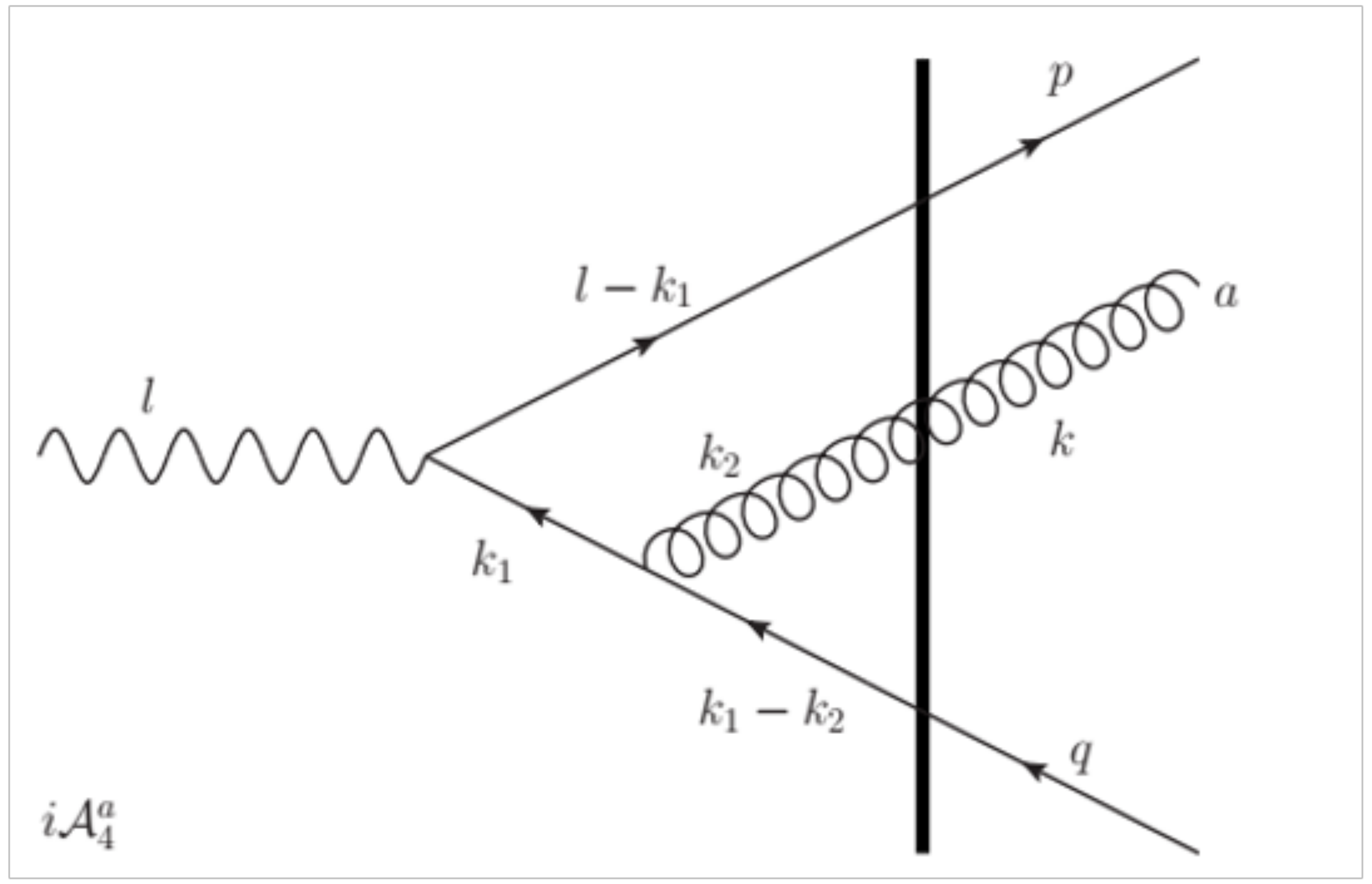
……………………..

One loop corrections - real diagrams



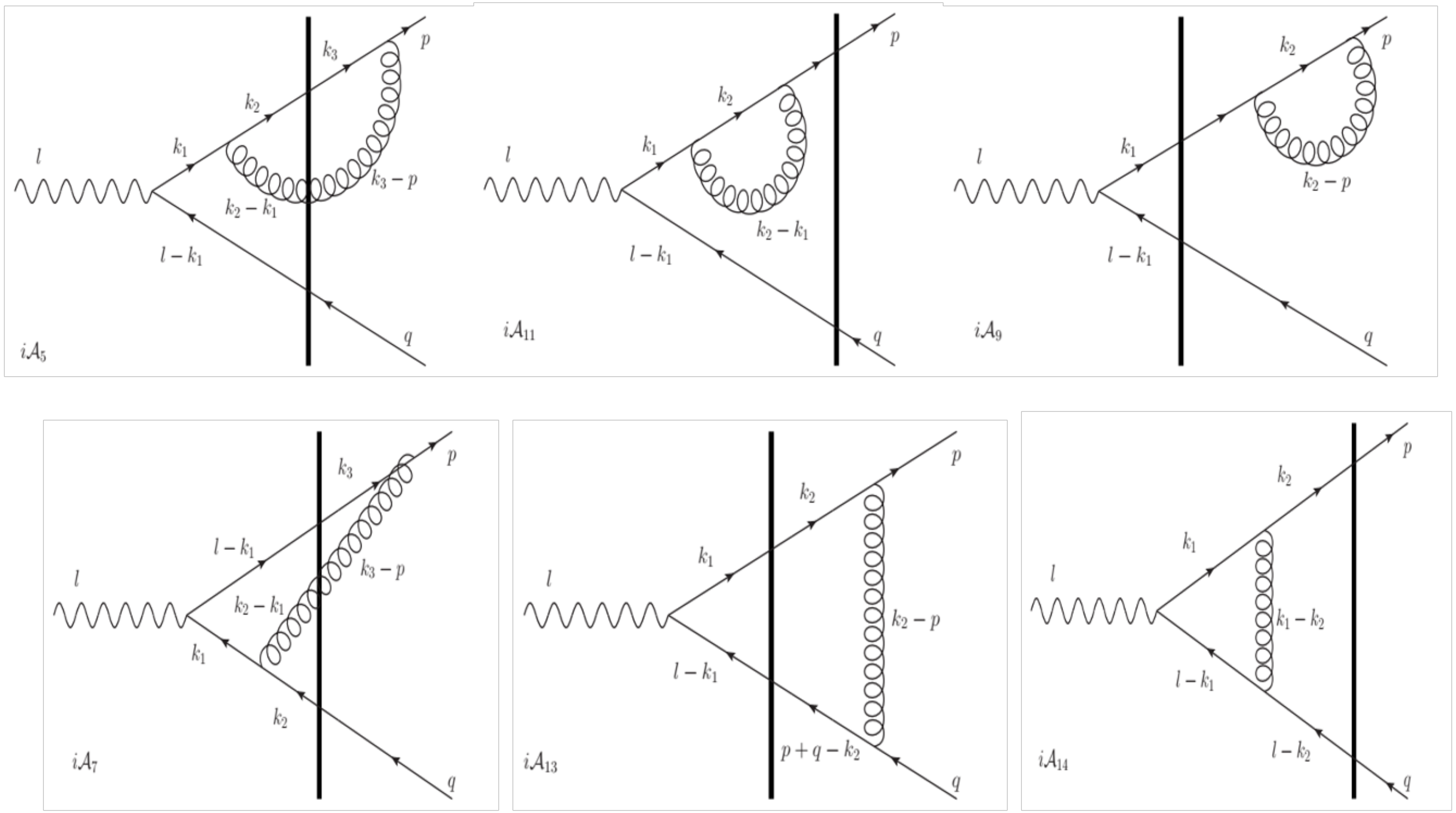






3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans
PLB 761 (2016) 229 and NPB 920 (2017) 232

One loop corrections – virtual diagrams



F. Bergabo and JJM, dihadrons, 2207.03606
P. Taels et al., dijets, 2204.11650
P. Caucal et al., dijets, 2108.06347

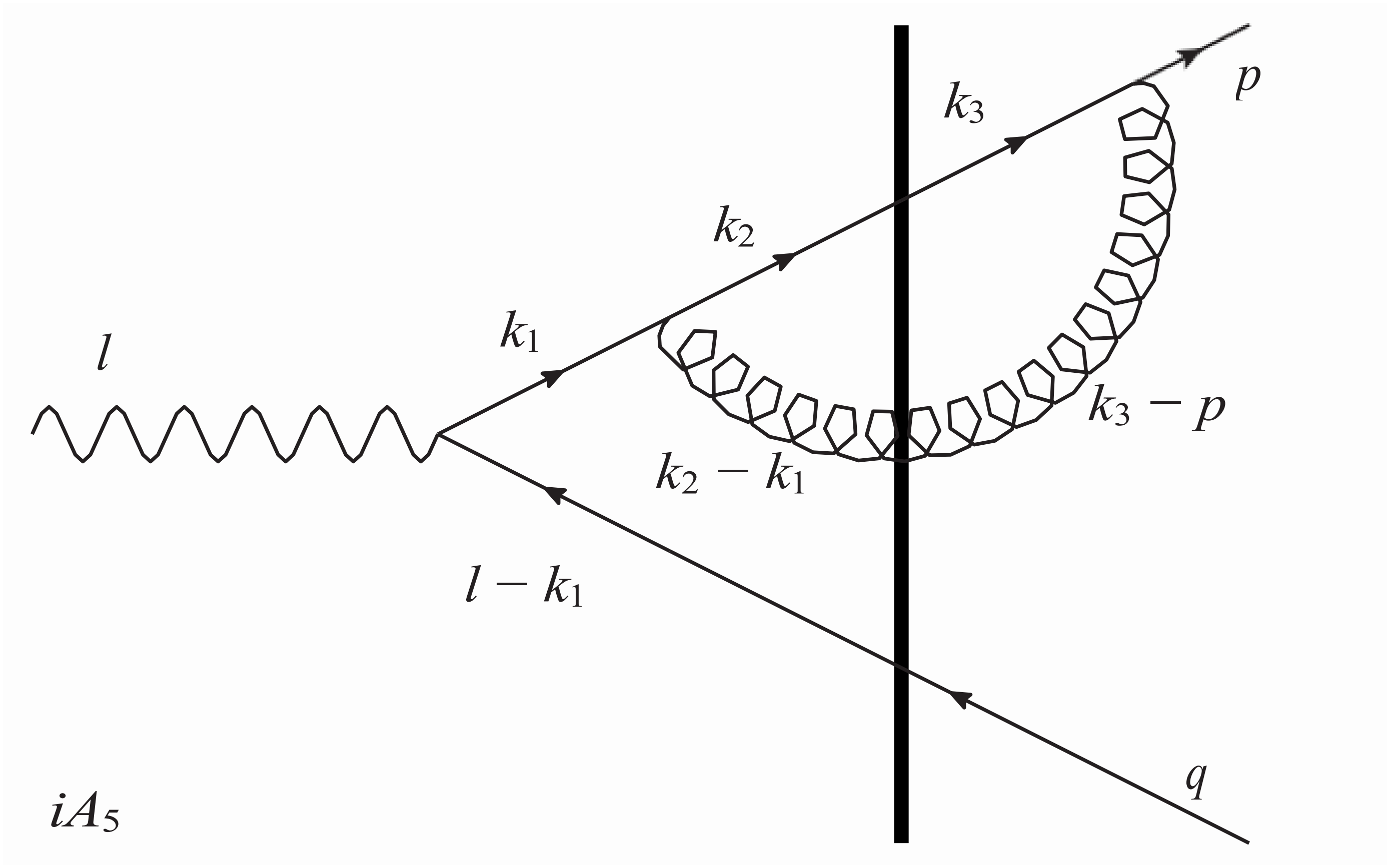
divergences

- ***Ultraviolet:***

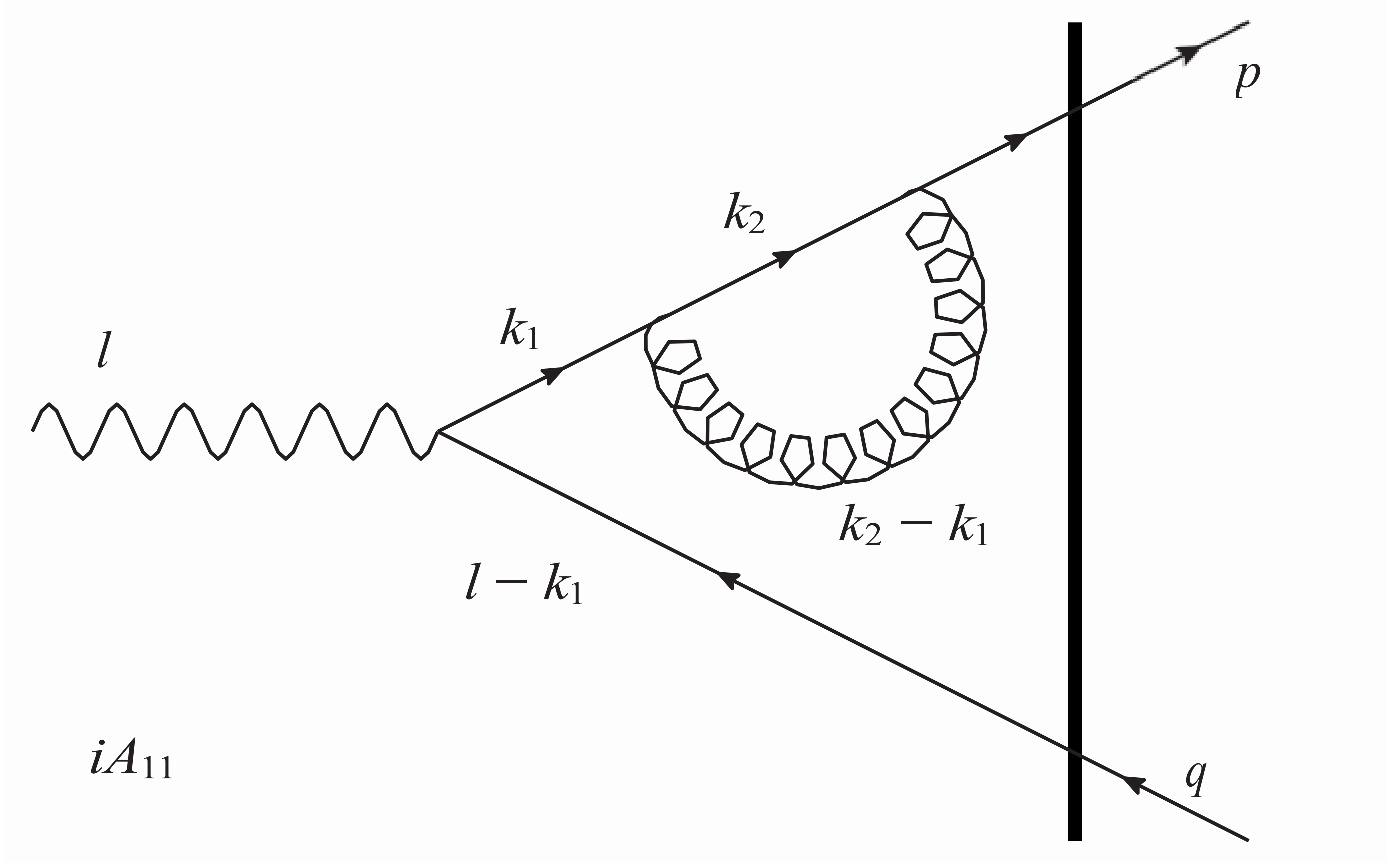
Real corrections are UV finite

UV divergences cancel among virtual corrections $\qquad \mathbf{k} \to \infty \quad \text{or} \quad \mathbf{x_3} \to \mathbf{x_i}$

$$\big(d\sigma_5 + d\sigma_{11}\big)_{UV} = 0$$
$$\big(d\sigma_6 + d\sigma_{12}\big)_{UV} = 0$$
$$\big(d\sigma_9 + d\sigma_{10} + d\sigma_{14(1)} + d\sigma_{14(2)}\big)_{UV} = 0$$



$+$



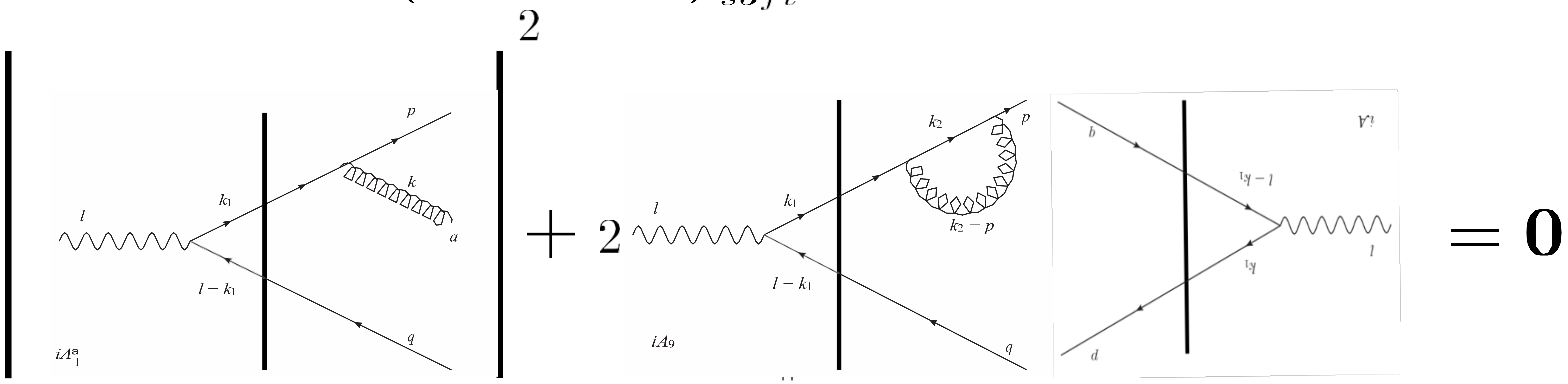
$= 0$

divergences

- ***Soft:*** $\mathbf{k^\mu \to 0\ \ (x_3 \to \infty\ \ AND\ \ z \to 0)}$

Soft divergences cancel between real and virtual corrections

$$(d\sigma_{1-1} + d\sigma_9)_{soft} = 0,$$
$$\left(d\sigma_{1-2} + d\sigma_{13}^{(1)} + d\sigma_{13}^{(2)}\right)_{soft} = 0$$
$$(d\sigma_{3-3} + d\sigma_{4-4} + d\sigma_{3-4})_{soft} = 0$$
$$(d\sigma_{1-3} + d\sigma_{1-4})_{soft} = 0$$
$$(d\sigma_{2-3} + d\sigma_{2-4})_{soft} = 0$$
$$(d\sigma_5 + d\sigma_7)_{soft} = 0$$
$$\left(d\sigma_{11} + d\sigma_{14}^{(1)}\right)_{soft} = 0$$



divergences

- ***Rapidity:*** $\mathbf{z} \to \mathbf{0}$ ***, but finite*** $\boldsymbol{k_t}$

$$\int_0^1 \frac{dz}{z} = \int_0^{z_f} \frac{dz}{z} + \int_{z_f}^1 \frac{dz}{z}$$

rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\frac{d\sigma^L_{\rm NLO}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, y_2} = \frac{2e^2 g^2 Q^2 N_c^2 (z_1 z_2)^3}{(2\pi)^{10}} \delta(1 - z_1 - z_2) \int_0^{z_f} \frac{dz}{z} \int d^{10}\mathbf{x}\, K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1)$$

$$e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \Big\{ \left(\tilde\Delta_{12} + \tilde\Delta_{22'} - \tilde\Delta_{12'}\right) S_{132'1'} S_{23} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{22'} - \tilde\Delta_{21'}\right) S_{1'321} S_{2'3}$$

$$+ \left(\tilde\Delta_{12} + \tilde\Delta_{11'} - \tilde\Delta_{21'}\right) S_{322'1'} S_{13} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{11'} - \tilde\Delta_{12'}\right) S_{32'21} S_{1'3}$$

$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} + \tilde\Delta_{12} + \tilde\Delta_{1'2'}\right) S_{122'1'} - \left(\tilde\Delta_{12} + \tilde\Delta_{1'2'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{12} S_{1'2'}$$

$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{11'} S_{22'} - 2\tilde\Delta_{12} \left(S_{13} S_{23} - S_{12}\right) - 2\tilde\Delta_{1'2'} \left(S_{1'3} S_{2'3} - S_{1'2'}\right) \Big\}$$

JIMWLK evolution of quadrupoles

JIMWLK evolution of dipoles

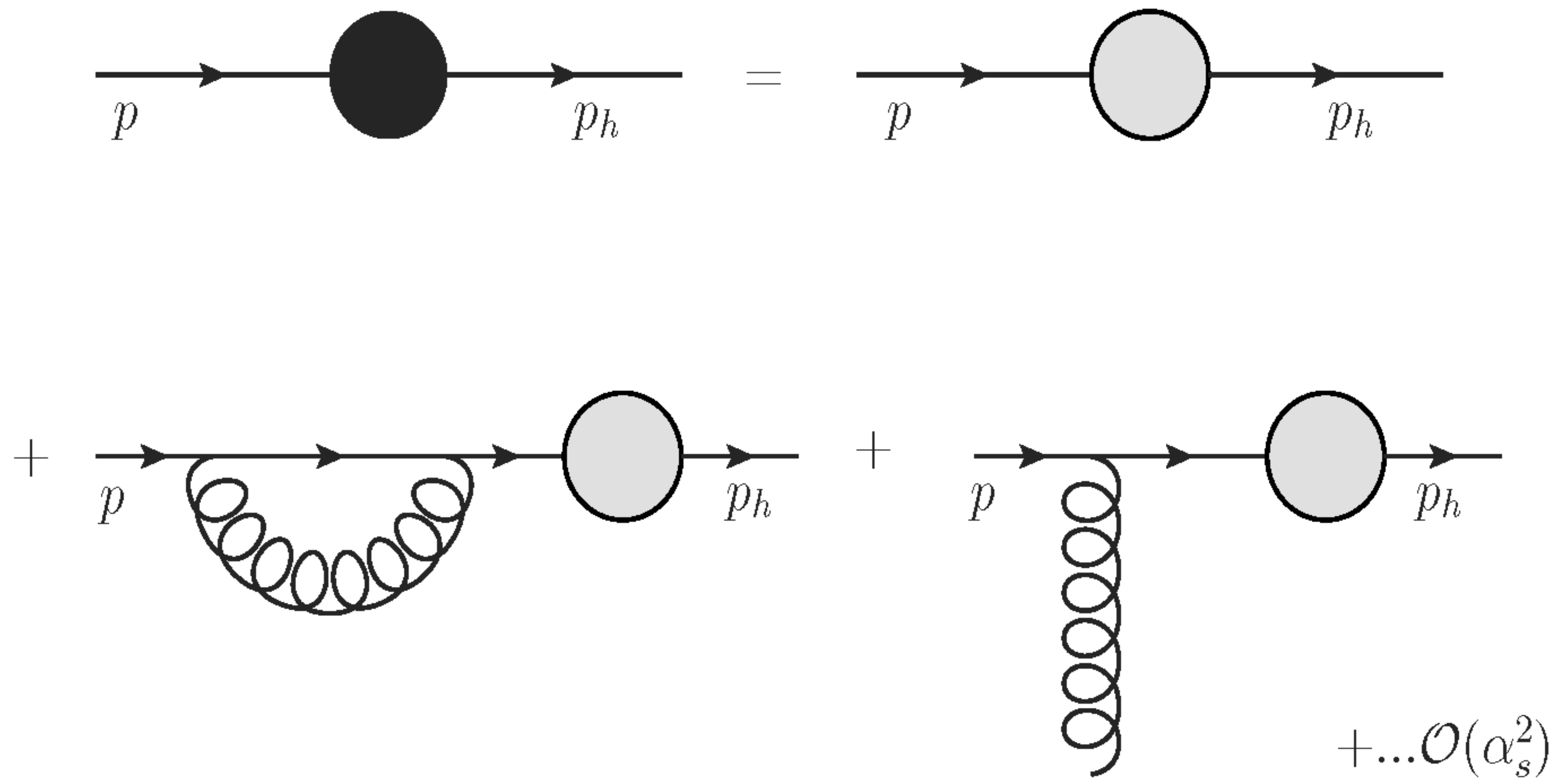
$$\tilde{\boldsymbol{\Delta}}_{\mathbf{12}} \equiv \frac{(\mathbf{x_1} - \mathbf{x_2})^2}{(\mathbf{x_1} - \mathbf{x_3})^2 (\mathbf{x_2} - \mathbf{x_3})^2}$$

divergences

- ***Collinear:***

$$\frac{1}{(p+k)^2} = \frac{1}{|\vec{p}||\vec{k}|(1-cos\,\theta)} \longrightarrow \infty \quad as \quad \theta \to 0$$

Collinear divergences are absorbed into evolution of parton-hadron fragmentation functions



Divergences

- Ultraviolet

  real corrections are UV finite

  UV divergences cancel among virtual diagrams

- Soft

  soft divergences cancel between real and virtual diagrams

- Collinear

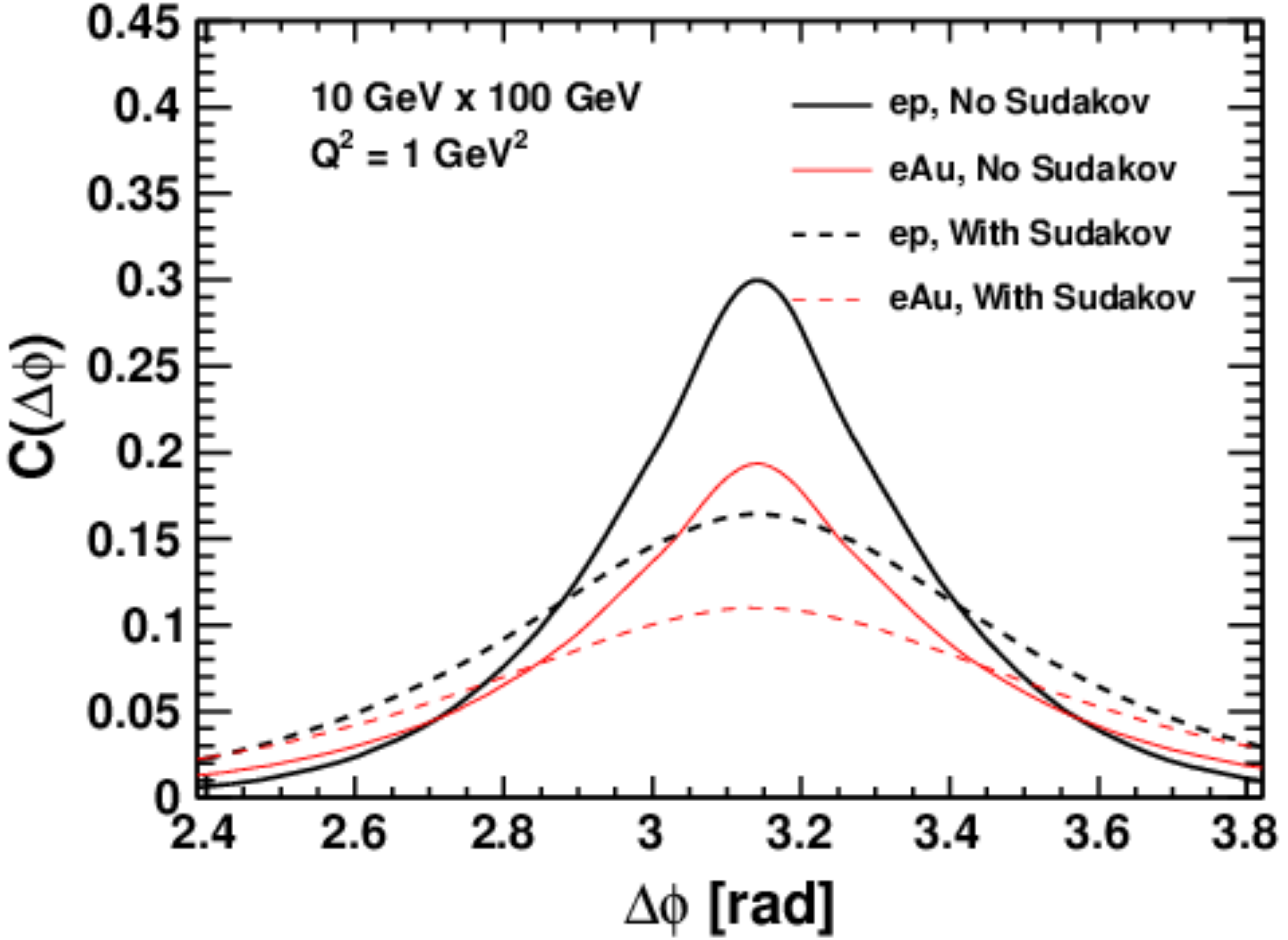
  collinear divergences are absorbed into fragmentation functions

- Rapidity

  Rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

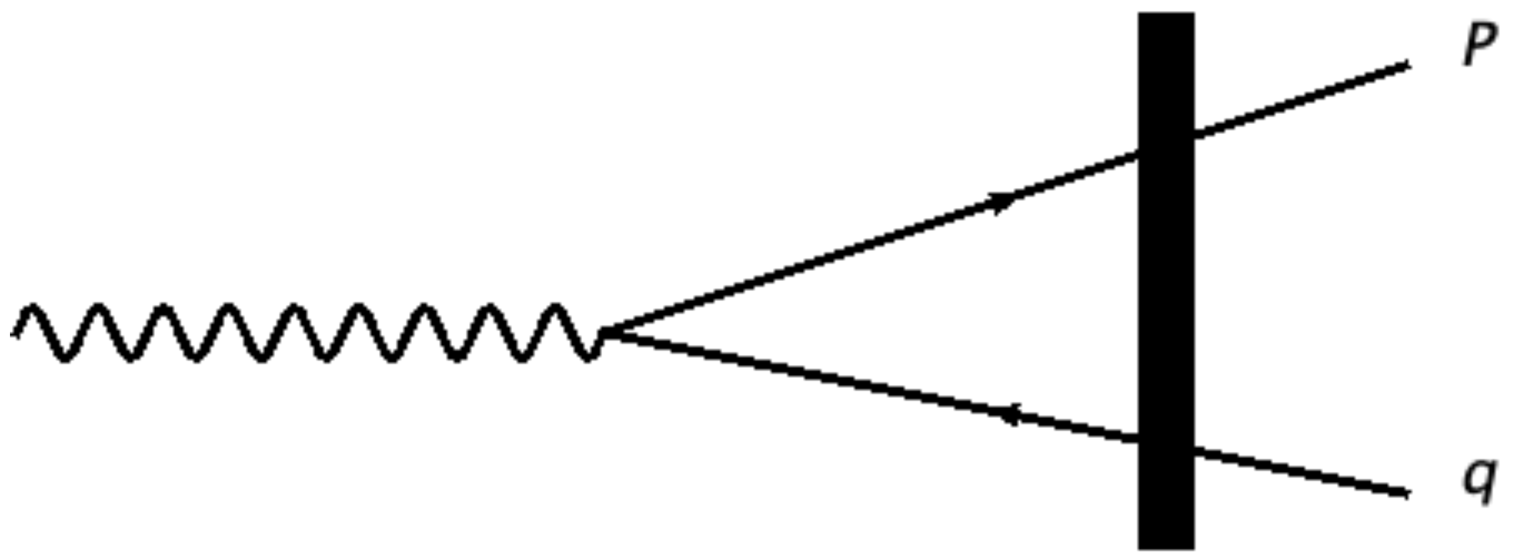
$$\sigma^{\gamma^* A \to h_1 h_2 X} = \sigma_{LO} \otimes \text{JIMWLK} + \sigma_{LO} \otimes D_{h/q}(z_h, \mu^2) \otimes D^{(0)}_{h/q}(z_h) + \sigma^{\text{finite}}_{NLO}$$

Back to back limit: deep connections to physics of TMDs, Sudakov effect,....



Zheng, Aschenauer, Lee, Xiao, PRD89 (2014) 074037

Inclusive dihadron production in DIS at small x: back to back limit



Quadrupole

$$\frac{1}{N_c} Tr\, V(\mathbf{x}_1)V^\dagger(\mathbf{x}_2)V(\mathbf{x}_2')V^\dagger(\mathbf{x}_1')$$

$$\mathbf{P}_\perp = \mathbf{p} - \mathbf{q}$$

$$\mathbf{K}_\perp = \mathbf{p} + \mathbf{q}$$

$$\mathbf{K}_\perp \longrightarrow 0$$

for large $\mathbf{P}_\perp \sim |\mathbf{p}| \sim |\mathbf{q}|$ one can get large (Sudakov) logs $\log \frac{\mathbf{P}_\perp^2}{\mathbf{K}_\perp^2}$

Sudakov double logs in dijets production in DIS at small x:

Taylor expansion of Wilson lines around "center of mass" coordinate ⟶ Weizsacker-Williams field

CGC calculations include Sudakov double logs, but with the wrong sign! $(+)\left[\log \mathbf{P}_\perp^2 \Delta \mathbf{b}_\perp^2\right]^2$

impose a kinematic constraint on life time of gluon radiation

(the usual strong ordering in + momenta is not sufficient)

Taels, Altinoluk, Beuf, Marquet, JHEP 10 (2022) 184

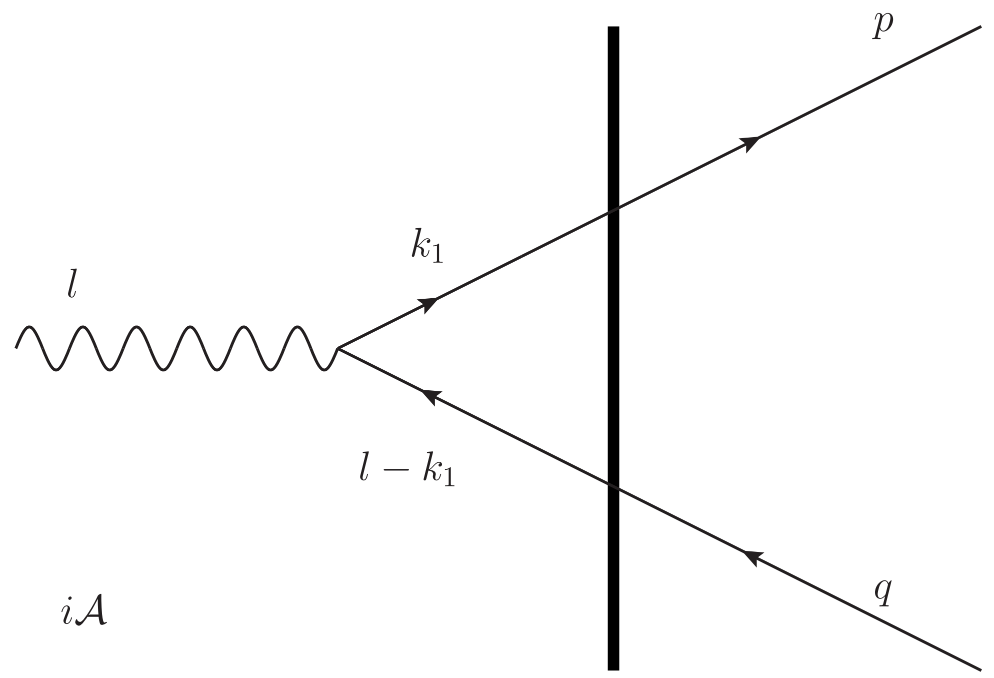
Caucal, Salazar, Schenke, Venugopalan, JHEP 11 (2022) 169

Single Inclusive hadron production in DIS at small x

larger kinematic phase space at EIC than dihadrons

Sudakov effects can be minimized

dipoles only (at leading N_c)



LO: start with quark-antiquark production and integrate out the antiquark

hadronization via fragmentation function

$$\frac{\mathrm{d}\sigma^{\gamma^* A \to h(\mathbf{p}_h, y_h) X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h} = 2\frac{e^2 N_c}{(2\pi)^5}\int_{z_h}^{1}\frac{dz_1}{z_h}D_{h/q}(z_h/z_1)\; z_1 Q_1^2 \int \mathrm{d}^6\mathbf{x}\left[S_{11'} - S_{12} - S_{1'2} + 1\right] e^{i(z_1/z_h)\mathbf{p}_h\cdot\mathbf{x}_{1'1}}$$
$$\left\{\frac{4Q_1^2}{Q^2}K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2}|Q_1) + \left[z_1^2 + (1-z_1)^2\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}K_1(|\mathbf{x}_{12}|Q_1)K_1(|\mathbf{x}_{1'2}|Q_1)\right\}$$

with $Q_1^2 \equiv z_1(1-z_1)Q^2$

will need the high $\mathbf{Q}^2 \gg \mathbf{P}_h^2$ limit for Sudakov

Sudakov double logs: high Q^2 limit

Altinoluk, JJM, Marquet

work in progress

problem at end points: use a trick introduced by Marquet, Xiao, Yuan, PLB682 (2009) 207

$$Q_1^{2n}\, K_{(0,1)}(|\mathbf{x}_{12}|Q_1)\, K_{(0,1)}(|\mathbf{x}_{1'2}|Q_1) = \int_0^{Q^2/4} d\bar{Q}^2\, (\bar{Q}^2)^n K_{(0,1)}(|\mathbf{x}_{12}|\bar{Q}) K_{(0,1)}(|\mathbf{x}_{1'2}|\bar{Q})\, \delta\left[\bar{Q}^2 - z_1(1-z_1)Q^2\right]$$

and

$$\delta\left[\bar{Q}^2 - z_1(1-z_1)Q^2\right] = \frac{\delta(z_1 - z_+)}{Q^2|1-z_+|} + \frac{\delta(z_1 - z_-)}{Q^2|1-z_-|} \quad \text{with} \quad z_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 - 4\bar{Q}^2/Q^2}\right)$$

then

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A \to h(\mathbf{p}_h, y_h) X}}{\mathrm{d}^2\mathbf{p}_h\, \mathrm{d}y_h}\right|_{T,LP} = \frac{1}{Q^2}\frac{e^2 N_c}{(2\pi)^5}\frac{D_{h/q}(z_h)}{z_h}\int \mathrm{d}^6\mathbf{x}\left[S_{11'} - S_{12} - S_{1'2} + 1\right] e^{i(\mathbf{p}_h/z_h)\cdot\mathbf{x}_{1'1}}$$
$$\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int_0^\infty d\bar{Q}^2 \bar{Q}^2\, K_1(|\mathbf{x}_{12}|\bar{Q}) K_1(|\mathbf{x}_{1'2}|\bar{Q})$$

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A \to h(\mathbf{p}_h, y_h) X}}{\mathrm{d}^2\mathbf{p}_h\, \mathrm{d}y_h}\right|_{L,LP} = \frac{1}{Q^2}\frac{e^2 N_c}{(2\pi)^5}\frac{D_{h/q}(z_h)}{z_h}\int \mathrm{d}^6\mathbf{x}\left[S_{11'} - S_{12} - S_{1'2} + 1\right] e^{i(\mathbf{p}_h/z_h)\cdot\mathbf{x}_{1'1}}$$
$$\frac{4}{Q^2}\int_0^\infty d\bar{Q}^2 \bar{Q}^4 K_0(|\mathbf{x}_{12}|\bar{Q}) K_0(|\mathbf{x}_{1'2}|\bar{Q})$$

contribution of longitudinal photons is suppressed

SIDIS at small x: NLO

Bergabo, JJM, JHEP 01 (2023) 095, and in progress

Caucal, Ferrand, Salazar, JHEP 05 (2024) 110 (jets)

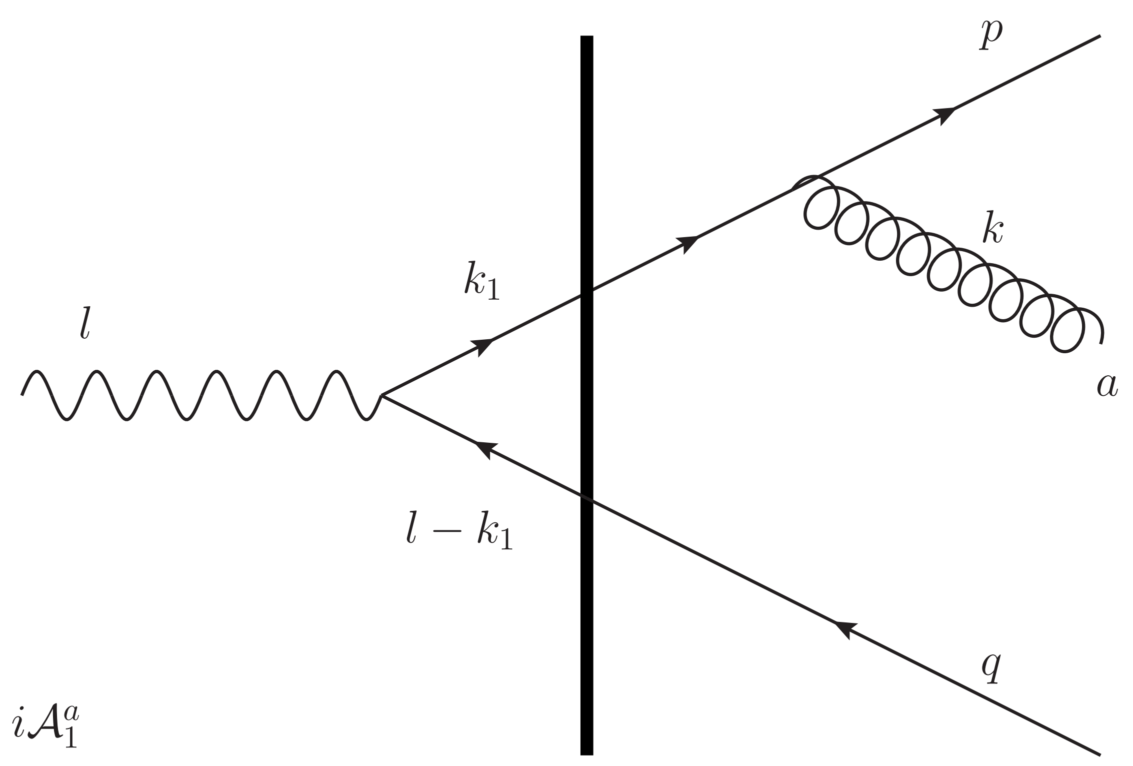
integrate out all partons except one

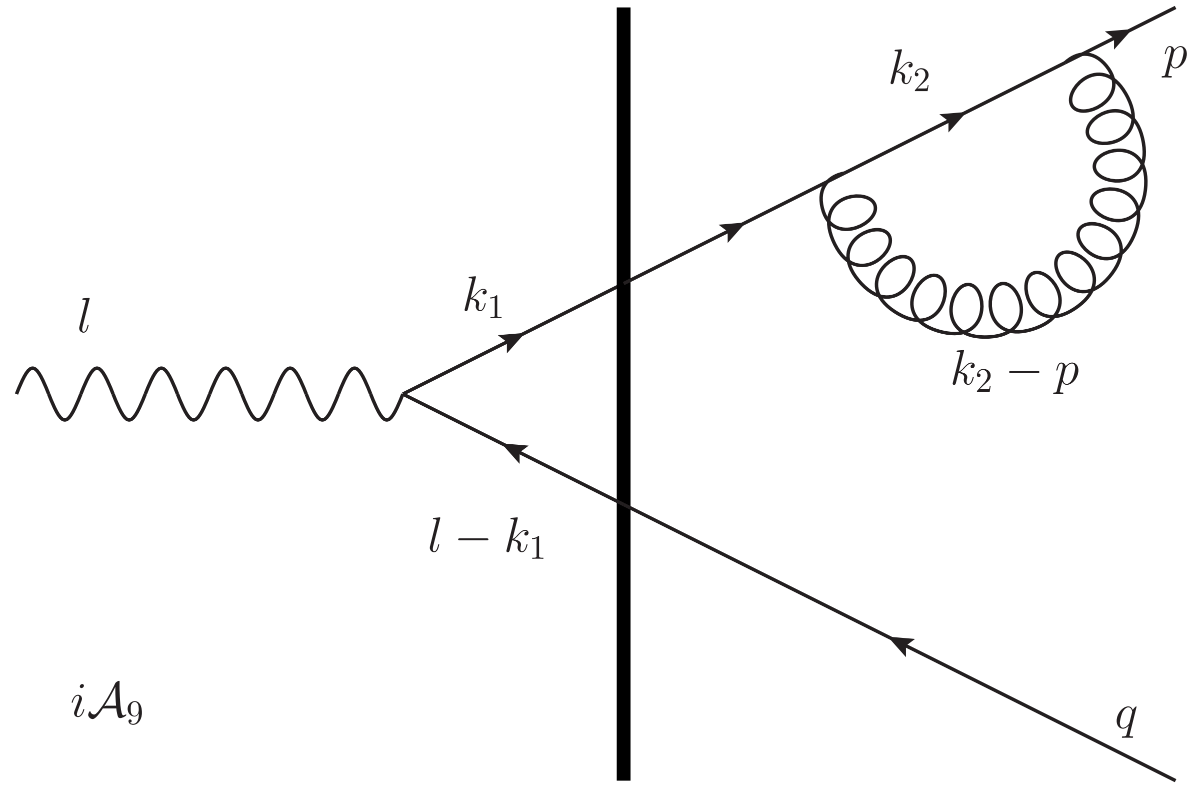
UV and soft divergences cancel

rapidity divergences lead to BK/JIMWLK evolution of dipoles

collinear divergences lead to DGLAP evolution of fragmentation functions

focus on diagrams leading to Sudakov double logs





and take the high $\mathbf{Q}^2$ limit

SIDIS at small x: NLO at high $\mathbf{Q}^2$

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}_{LO+NLO}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = \frac{1}{Q^2}\,\frac{e^2 N_c}{(2\pi)^5}\,\frac{1}{z_h}\int \mathrm{d}^4\mathbf{x}\, S_{11'}\int \mathrm{d}^2\mathbf{k}\, e^{i\mathbf{k}\cdot\mathbf{x}_{11'}}\int_0^\infty \mathrm{d}\bar{\mathbf{Q}}^2$$

$$\times\Bigg\{\left[\frac{\mathbf{p}_h/z_h+\mathbf{k}}{\bar{\mathbf{Q}}^2+(\mathbf{p}_h/z_h+\mathbf{k})^2}-\frac{\mathbf{p}_h/z_h}{\bar{\mathbf{Q}}^2+(\mathbf{p}_h/z_h)^2}\right]^2\left[1-\frac{\alpha_s C_F}{2\pi^2}\int\frac{\mathrm{d}z}{z}\,[1+(1-z)^2]\int\frac{\mathrm{d}^2\mathbf{p}}{(\mathbf{p}-z\,\mathbf{k})^2}\right]D_{h/q}(z_h)$$

$$+\frac{\alpha_s C_F}{2\pi^2}\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\,\frac{[1+(1-z)^2]}{1-z}\,D_{h/q}(\frac{z_h}{1-z})\int\frac{\mathrm{d}^2\mathbf{p}}{(\mathbf{p}-z\,\mathbf{k})^2}e^{-i(\mathbf{p}-z\,\mathbf{k})\cdot\mathbf{x}_{11'}}$$

$$\left[\frac{\mathbf{p}_h/z_h+\mathbf{k}}{\bar{\mathbf{Q}}^2+(\mathbf{p}_h/z_h+\mathbf{k})^2}-\frac{\mathbf{p}_h/z_h+\mathbf{p}-z\,\mathbf{k}}{\bar{\mathbf{Q}}^2+(\mathbf{p}_h/z_h+\mathbf{p}-z\,\mathbf{k})^2}\right]^2\Bigg\}+\cdots$$

add and subtract the collinear divergent piece of the real corrections: DGLAP evolution of frag. function

introduce a factorization scale to isolate the rapidity divergence

$$k^+ < k_f^+ \;\; (z\equiv\frac{k^+}{l^+})$$

introduce the kinematic constraint on k$^-$ (constrained BK evolution)

$$k^- = \frac{\mathbf{k}^2}{2k^+} > k_f^- = \frac{\mathbf{Q}^2}{2k_f^+}$$

$$\int_0^{z_f}\frac{\mathrm{d}z}{z}\longrightarrow\int_0^{z_f}\frac{\mathrm{d}z}{z}\left[1-\Theta\left(z_f\,\frac{\mathbf{p}^2}{Q^2}-z\right)\right]=\ln\left(\frac{Q^2}{\mathbf{p}^2}\right)$$

Sudakov double logs

next, do the transverse momentum integration

$$\int^{Q^2} \frac{\mathrm{d}^2\mathbf{p}}{\mathbf{p}^2}\left[e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}}-1\right]\ln\left(\frac{Q^2}{\mathbf{p}^2}\right) = 4\pi\int_0^{Q|\mathbf{x}_{11'}|}\frac{\mathrm{d}\tau}{\tau}\left[J_0(\tau)-1\right]\ln\left(\frac{Q|\mathbf{x}_{11'}|}{\tau}\right) = -\frac{\pi}{2}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right)+\cdots$$

with $c_0 = 2\,e^{-\gamma_E}$

add the LO term $\quad 1-\frac{\alpha_s C_F}{2\pi}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right) = 1 - S_{sud}(\mathbf{x}_{11'})$

assume/hope for exponentiation

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}_{LO+NLO}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = \frac{e^2}{Q^2}\,\frac{D_{h/q}(z_h,\mu^2)}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})\,e^{-S_{sud}(\mathbf{x}_{11'})} + d\sigma_{NLO-rap-finite}$$

with $\quad xq(x,\mathbf{p}) = \frac{N_c}{(2\pi)^5}\int \mathrm{d}^6\mathbf{x}\;e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}}\;\left[S_{11'}-S_{12}-S_{1'2}+1\right]\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int_0^\infty d\bar{Q}^2\bar{Q}^2\,K_1(|\mathbf{x}_{12}|\bar{Q})K_1(|\mathbf{x}_{1'2}|\bar{Q})$

Summary

QCD at high energy

dense hadron/nucleus: gluon saturation, strong color fields - CGC

strong hints from RHIC, LHC,...

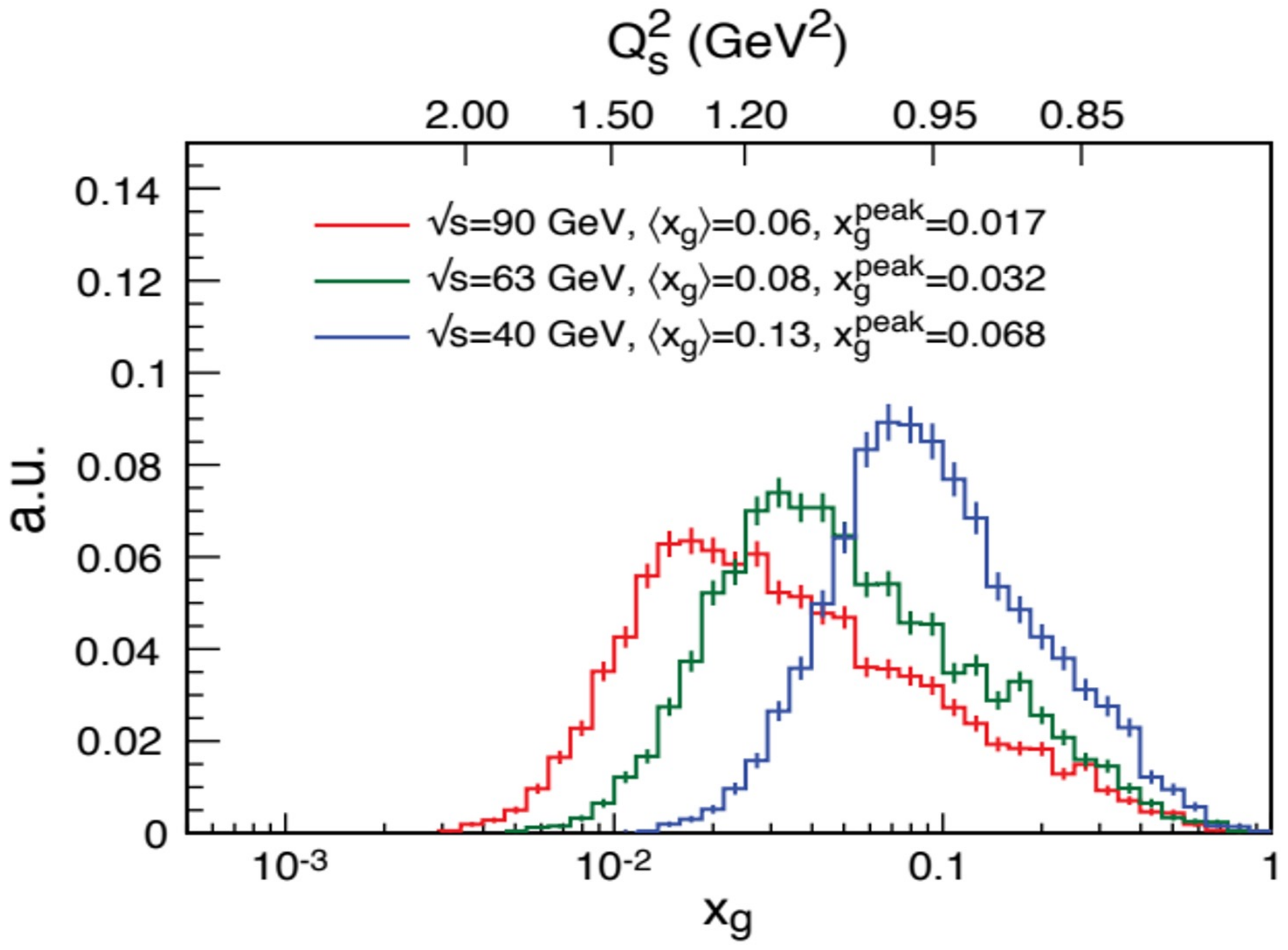
to be probed precisely at EIC

toward precision: NLO, beyond-eikonal corrections, ...

deep connections to TMD, Sudakov physics, ...

EIC

kinematics of inclusive dihadron production



Aschenauer et al. arXiv:1708.01527

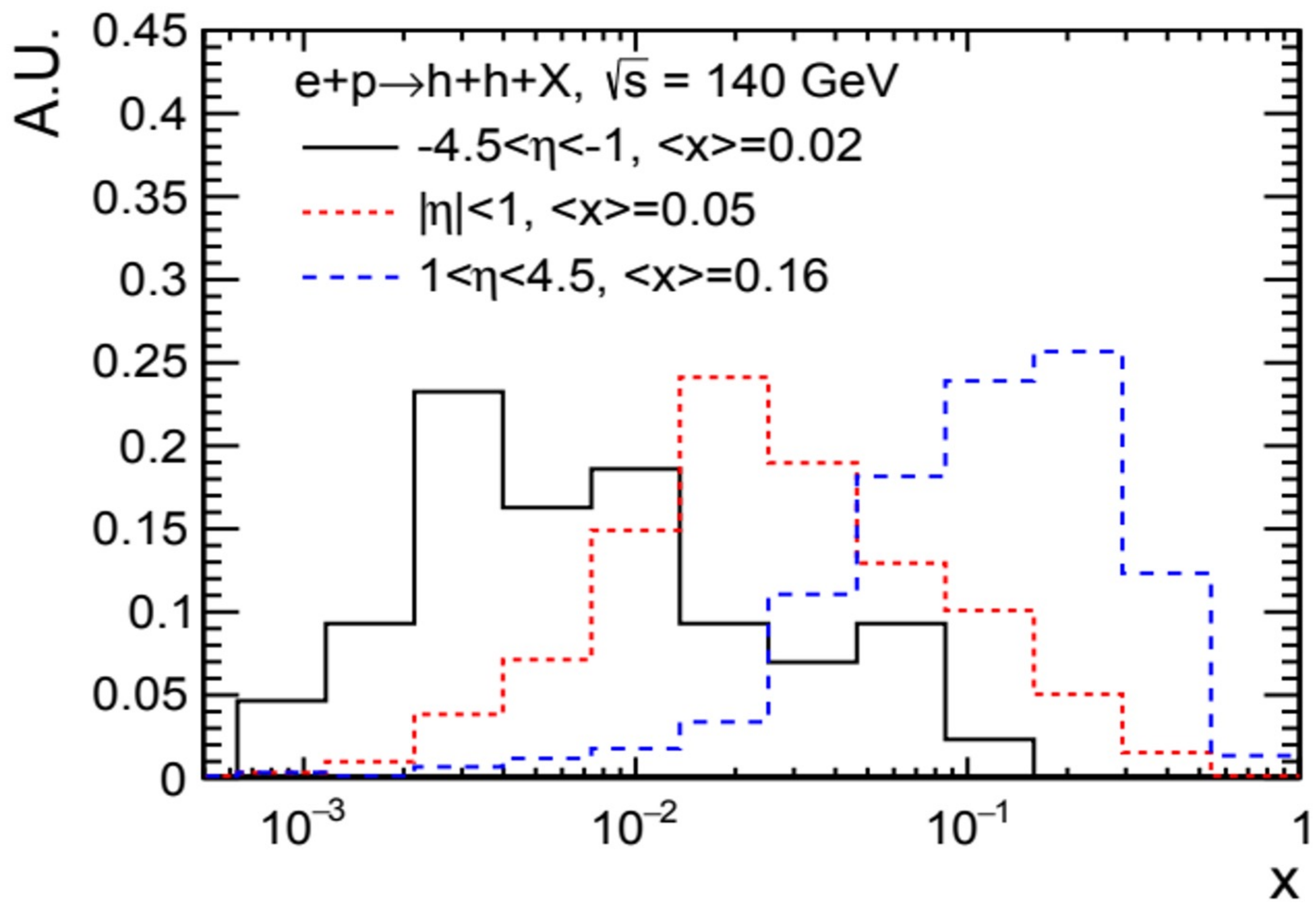


Fig. courtesy of Xiaoxuan Chu

transition region: from large x to small x