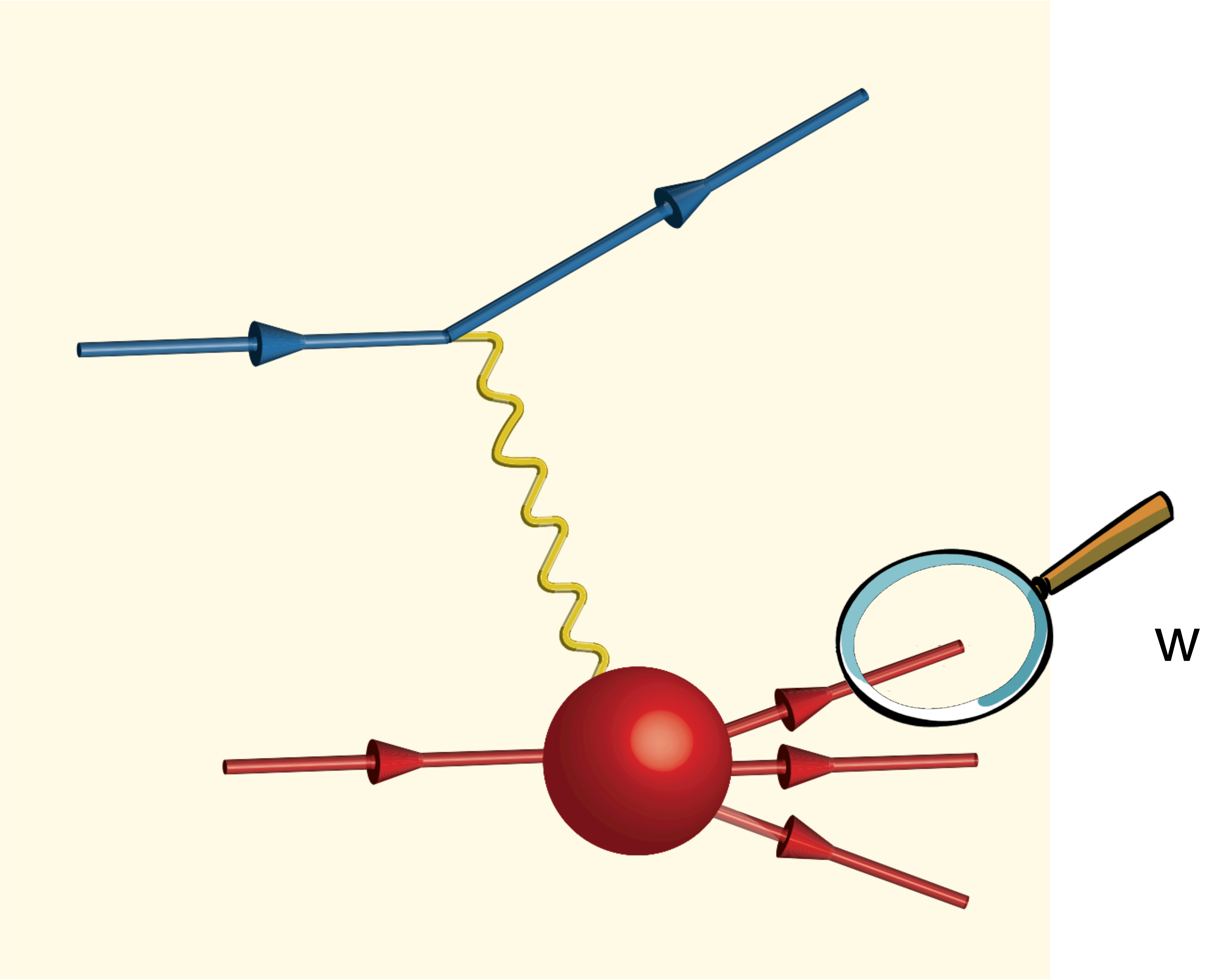
Semi-inclusive DIS at NNLO in QCD



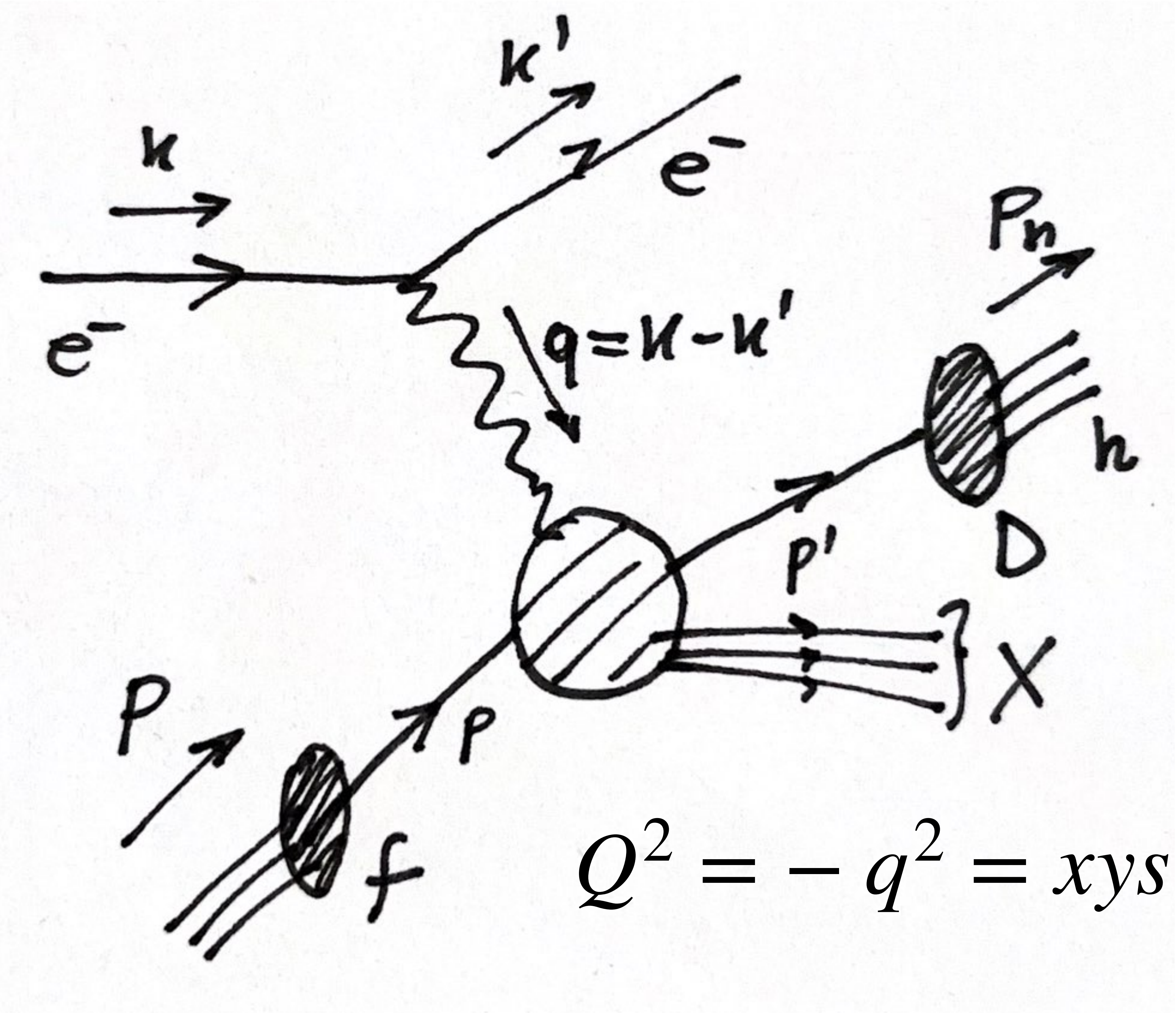
Giovanni Stagnitto
(Milano Bicocca University & INFN)

with Leonardo Bonino and Thomas Gehrmann
[2401.16281]
+ Markus Löchner and Kay Schönwald
[2404.08597]

QCD Evolution, Pavia, 27.05.2024

This talk: semi-inclusive DIS (SIDIS)

$$\ell(k) + p(P) \rightarrow \ell(k') + h(P_h) + X$$

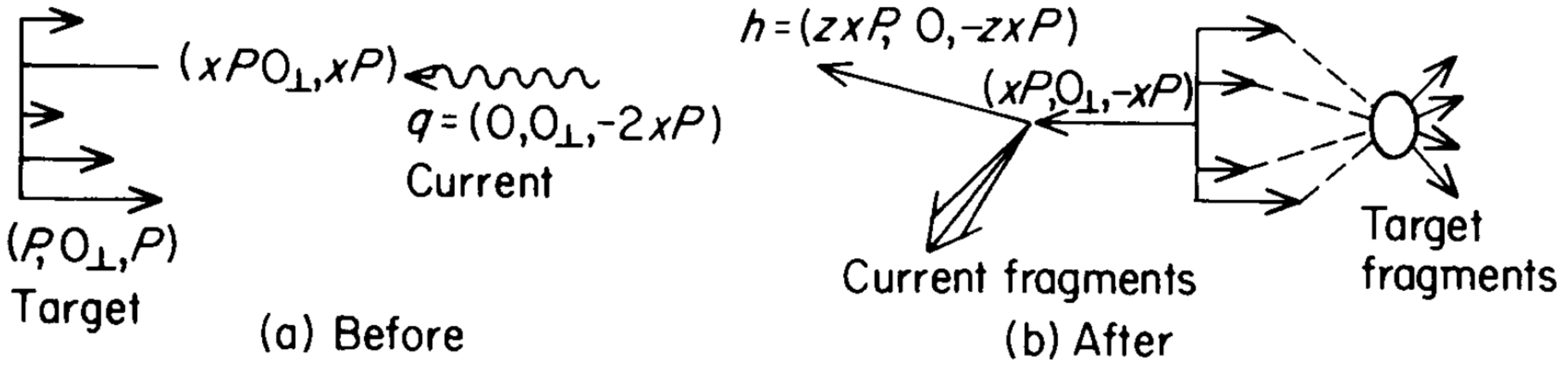


$$Q^2 = -q^2 = xys$$

$$x = \frac{Q^2}{2P \cdot q}$$ *Bjorken variable* (momentum fraction of the parton)

$$y = \frac{P \cdot q}{P \cdot k}$$ *Inelasticity* (energy transfer) (related to polarisation of virtual photon)

$$z = \frac{P \cdot P_h}{P \cdot q}$$ In Breit frame, is the fraction of the parton's longitudinal momentum carried of by the observed hadron



*we assume only photon exchange ($Q \ll M_Z$)

Outlook

Unpolarised

$$\ell(k)\, p(P) \to \ell(k')\, h(P_h)\, X$$

$$\frac{\mathrm{d}^3\sigma^h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathcal{F}_T^h(x,z,Q^2) + \frac{1-y}{y}\mathcal{F}_L^h(x,z,Q^2)\right]$$

Longitudinally polarised

$$\overrightarrow{\ell}(k)\, \vec{p}(P) \to \ell(k')\, h(P_h)\, X$$

$$\frac{1}{2}\left(\frac{\mathrm{d}^3\sigma^h(\uparrow\uparrow)}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} - \frac{\mathrm{d}^3\sigma^h(\uparrow\downarrow)}{\mathrm{d}x\mathrm{d}y\mathrm{d}z}\right) = \frac{4\pi\alpha^2}{Q^2}\frac{1-(1-y)^2}{2y}\mathcal{G}_1^h(x,z,Q^2)$$

Outlook

Unpolarised

$$\ell(k)\, p(P) \to \ell(k')\, h(P_h)\, X$$

$$\frac{d^3\sigma^h}{dx dy dz} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathscr{F}_T^h(x,z,Q^2) + \frac{1-y}{y}\mathscr{F}_L^h(x,z,Q^2)\right]$$

Longitudinally polarised

$$\overrightarrow{\ell}(k)\, \vec{p}(P) \to \ell(k')\, h(P_h)\, X$$

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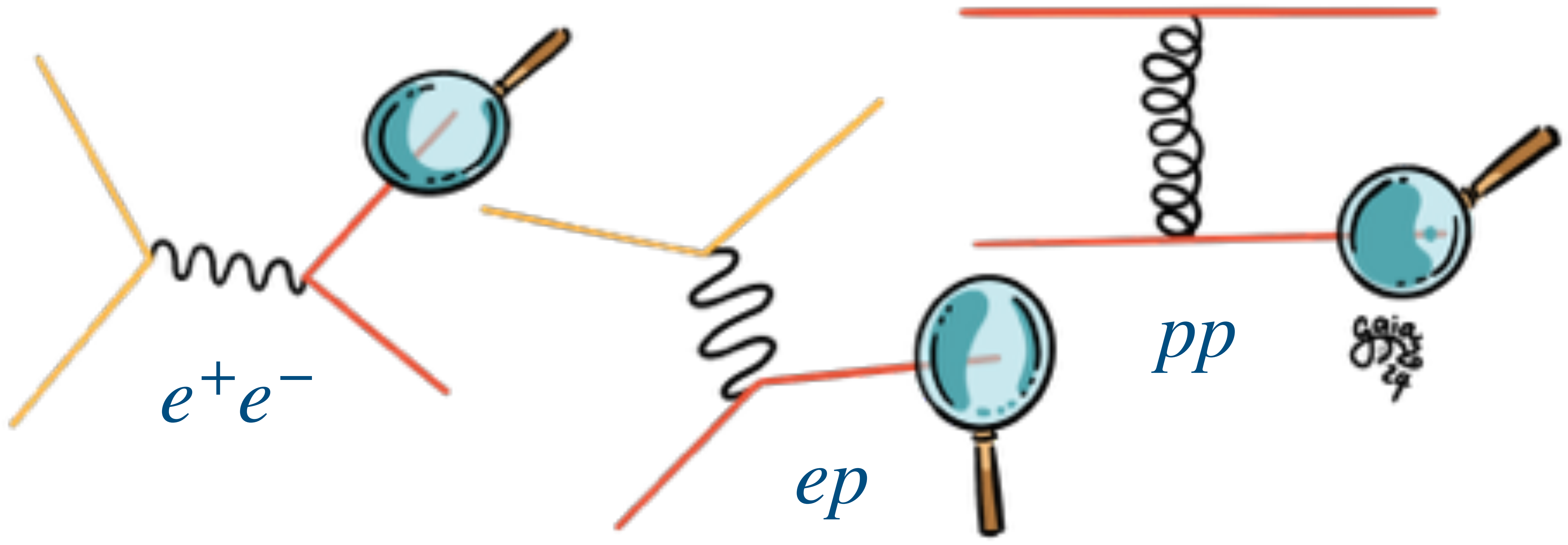
Motivation

Global fits of fragmentation functions (FFs)

One-particle inclusive processes are backbone of FF determination. Known up to:

- NNLO in e^+e^- (SIA) [Rijken, Van Neerven ’96,’97] [Mitov, Moch, Vogt ’06]
- NLO in ep (SIDIS) [Altarelli, Ellis, Martinelli, Pi ’79] [Baier, Fey ’79] (NNLO is this work)
- NLO in pp [Aversa, Chiappetta, Greco, Guillet ’89]



Therefore, fits at NNLO so far limited to e^+e^- data
[Bertone, Carrazza, Hartland, Nocera, Rojo ’17] [Anderle, Ringer, Stratmann ’15] [xFitter ’21]

Motivation

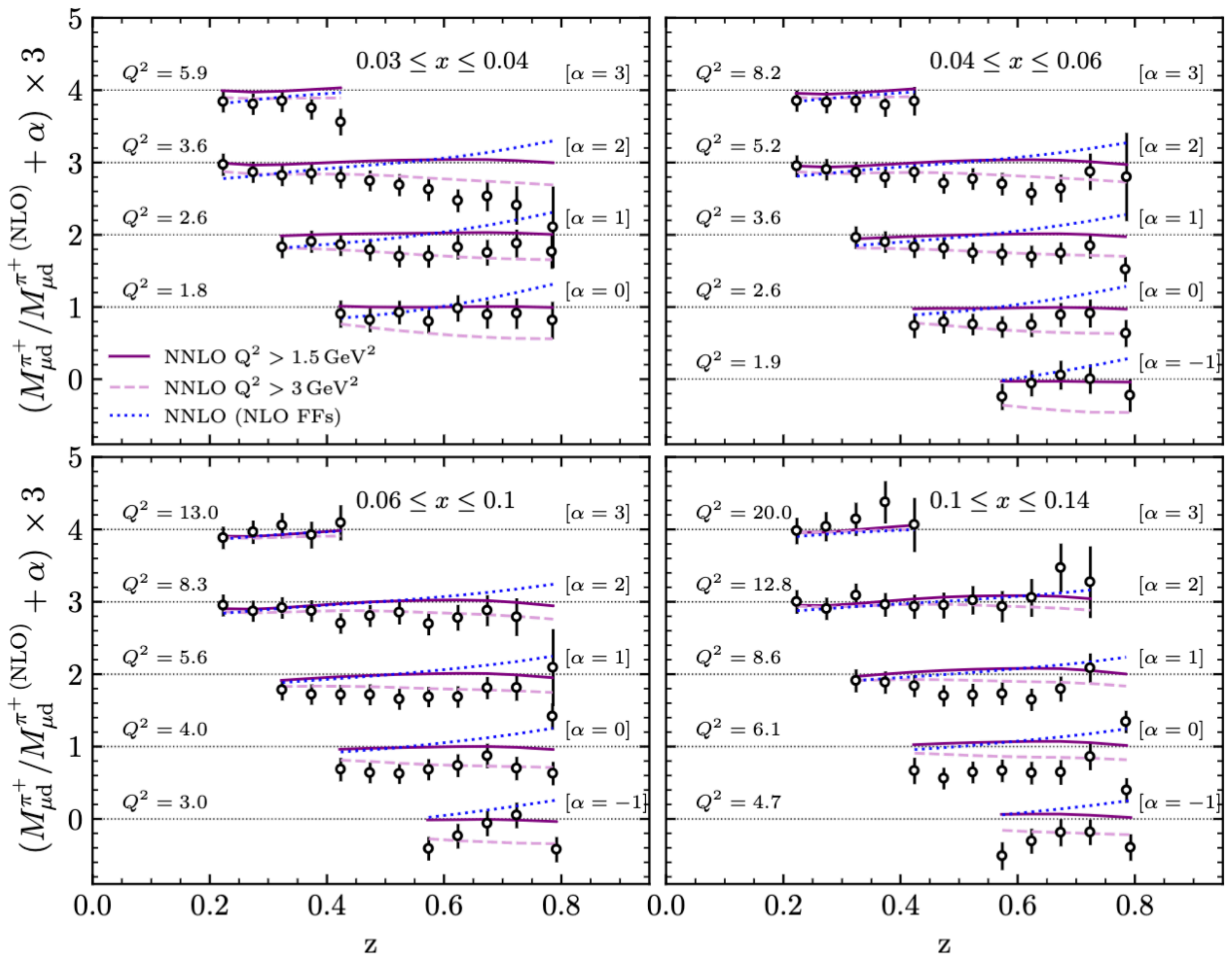
Global fits of fragmentation functions (FFs)

Recent fits at aNNLO with e^+e^- and ep data [Borsa, Sassot, de Florian, Vogelsang '22] [Abdul Khalek, Bertone, Khoudli, Nocera '22], exploiting approximate NNLO results for SIDIS obtained from threshold resummation [Abele, De Florian, Vogelsang '21,'22]

	Experiment	$Q^2 \geq 1.5\,\mathrm{GeV}^2$			$Q^2 \geq 2.0\,\mathrm{GeV}^2$			$Q^2 \geq 2.3\,\mathrm{GeV}^2$			$Q^2 \geq 3.0\,\mathrm{GeV}^2$		
		#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO
e^+e^-	SIA	288	1.05	0.96	288	0.91	0.87	288	0.90	0.91	288	0.93	0.86
ep	COMPASS	510	0.98	1.14	456	0.91	1.04	446	0.91	0.92	376	0.94	0.93
ep	HERMES	224	2.24	2.27	160	2.40	2.08	128	2.71	2.35	96	2.75	2.26
	TOTAL	1022	1.27	1.33	904	1.17	1.17	862	1.17	1.13	760	1.16	1.07

It is very encouraging that our NNLO analysis based on the approximate NNLO corrections for SIDIS shows an overall improvement in χ^2 relative to NLO once we go beyond $Q^2 = 2\,\mathrm{GeV}^2$. It is an interesting question, however, why the situation is opposite when the lower cut $Q^2 \geq 1.5\,\mathrm{GeV}^2$ is used. We first note that the lack

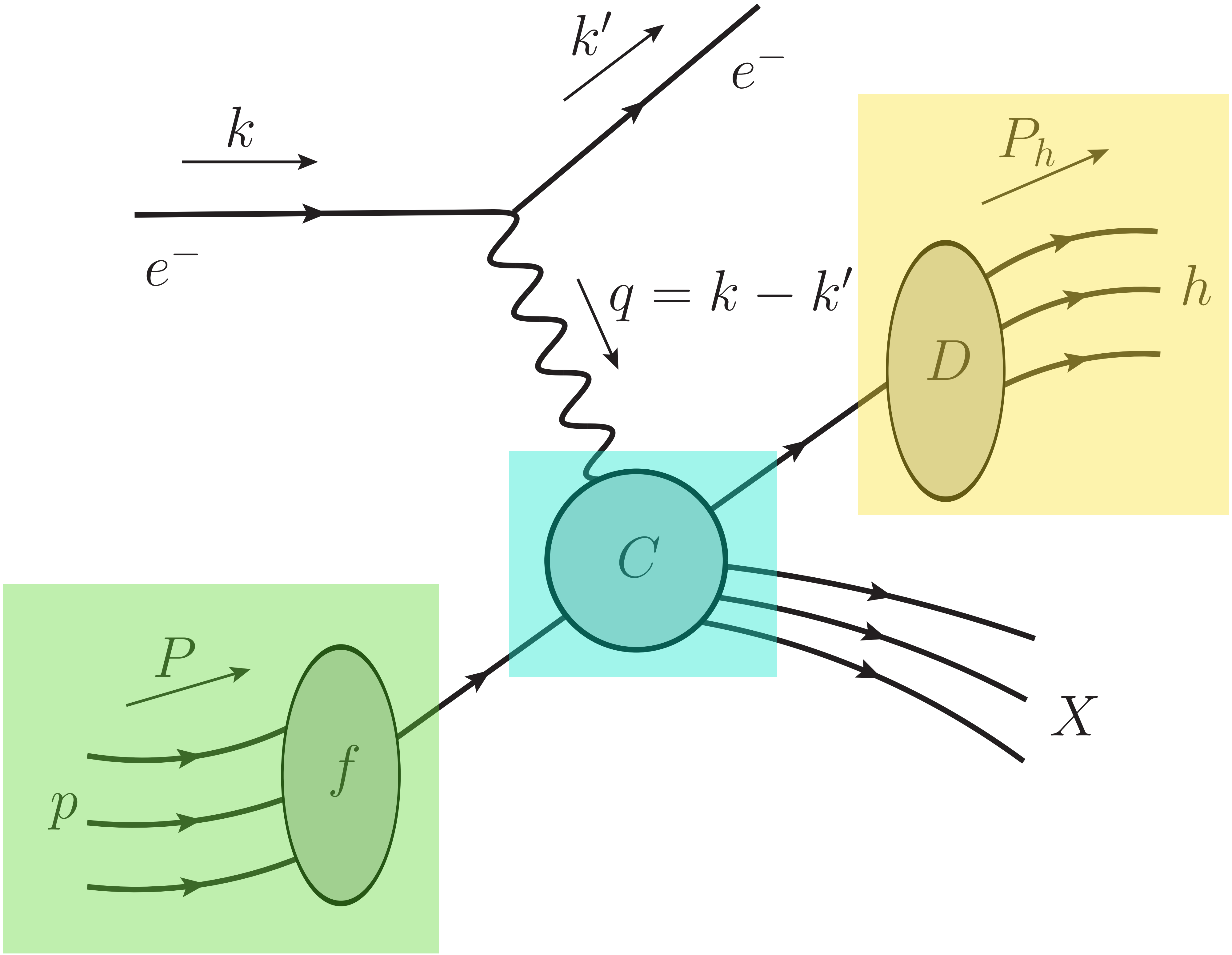
[2202.05060]



Our work will enable a consistent NNLO fit with SIDIS data.

Unpolarised SIDIS structure functions

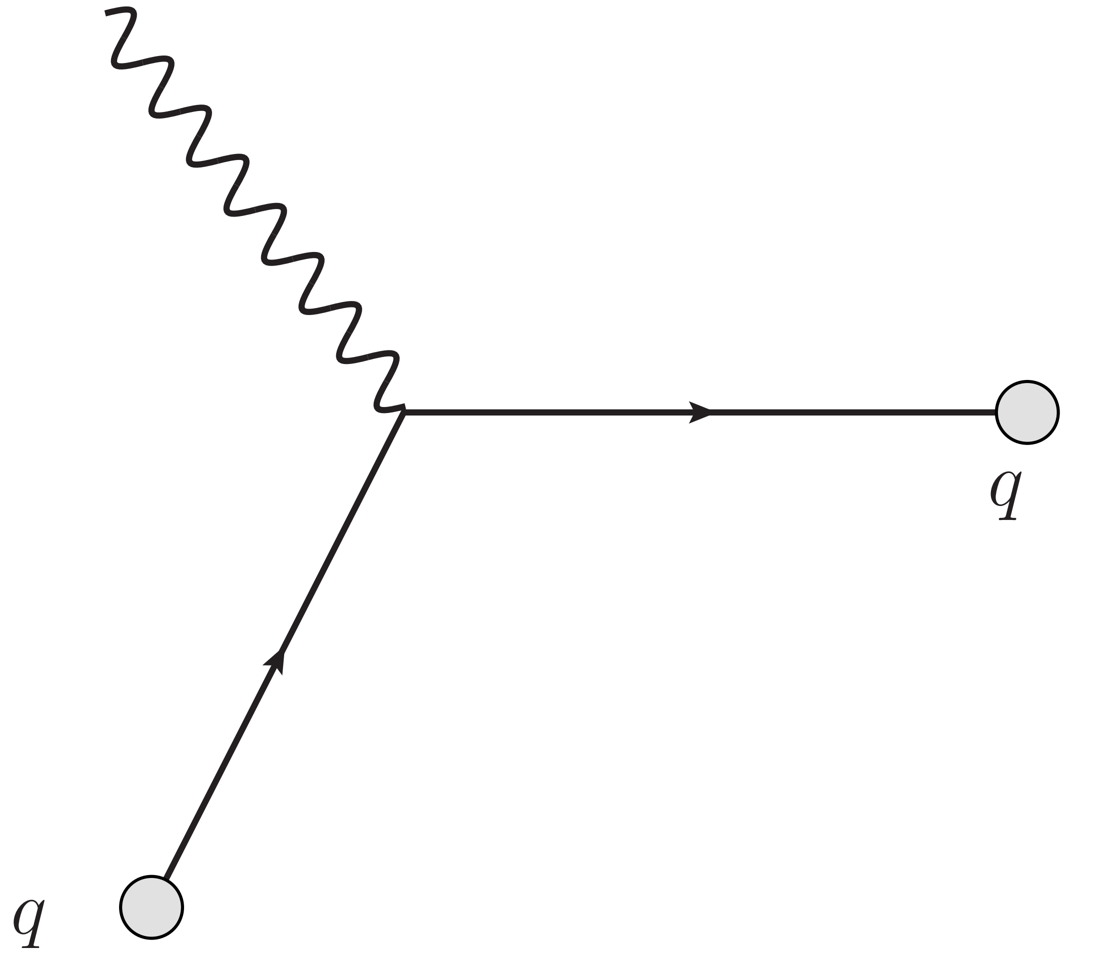
$$\mathcal{F}_i^h(x, z, Q^2) = \sum_{p,p'} \int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}} \int_z^1 \frac{\mathrm{d}\hat{z}}{\hat{z}} f_p\left(\frac{x}{\hat{x}}, \mu_F^2\right) D_{p'}^h\left(\frac{z}{\hat{z}}, \mu_A^2\right) \mathscr{C}^i_{p'p}\left(\hat{x}, \hat{z}, Q^2, \mu_R^2, \mu_F^2, \mu_A^2\right), \quad i = T, L$$



$$\mathscr{C}^i_{p'p} = C^{i,(0)}_{p'p} + \frac{\alpha_s(\mu_R^2)}{2\pi} C^{i,(1)}_{p'p} + \left(\frac{\alpha_s(\mu_R^2)}{2\pi}\right)^2 C^{i,(2)}_{p'p} + \mathcal{O}(\alpha_s^3)$$

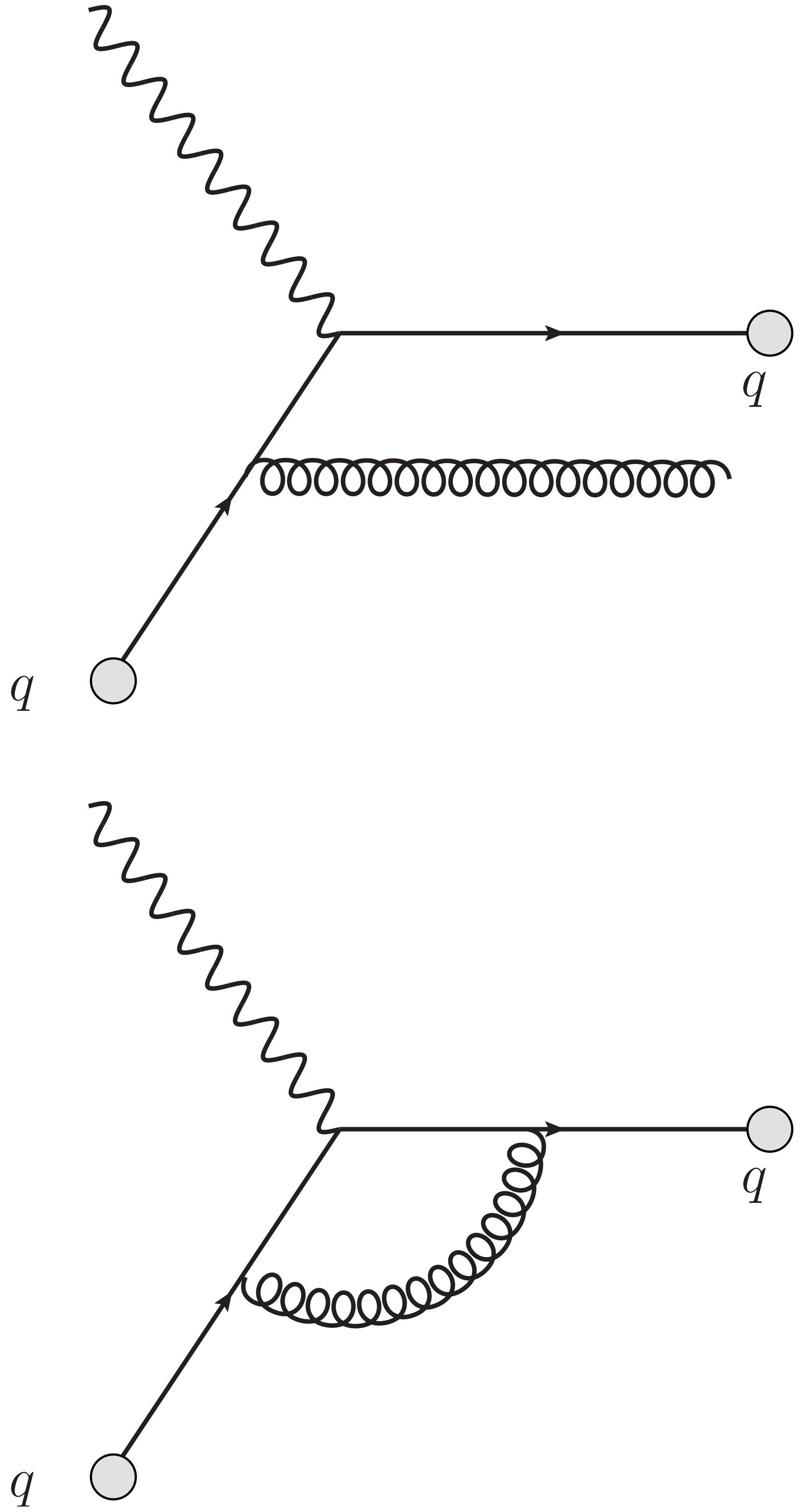
$$C^{T,(0)}_{qq} = e_q^2 \delta(1-\hat{x})\delta(1-\hat{z})$$

$$C^{L,(0)}_{qq} = 0$$



SIDIS @ NLO

[Altarelli, Ellis, Martinelli, Pi ’79][Baier, Fey ’79]



$$C_{qq}^{T,(1)}(\hat{x},\hat{z}) = e_q^2 C_F \Bigg[-8\delta(1-\hat{x})\delta(1-\hat{z})$$
$$+\delta(1-\hat{x})\left[\tilde{P}_{qq}(\hat{z})\ln\frac{Q^2}{\mu_F^2} + L_1(\hat{z}) + L_2(\hat{z}) + (1-\hat{z})\right]$$
$$+\delta(1-\hat{z})\left[\tilde{P}_{qq}(\hat{x})\ln\frac{Q^2}{\mu_F^2} + L_1(\hat{x}) - L_2(\hat{x}) + (1-\hat{x})\right]$$
$$+\frac{2}{(1-\hat{x})_+(1-\hat{z})_+} - \frac{1+\hat{z}}{(1-\hat{x})_+} - \frac{1+\hat{x}}{(1-\hat{z})_+}$$
$$+2(1+\hat{x}\hat{z})\Bigg], \qquad (49)$$

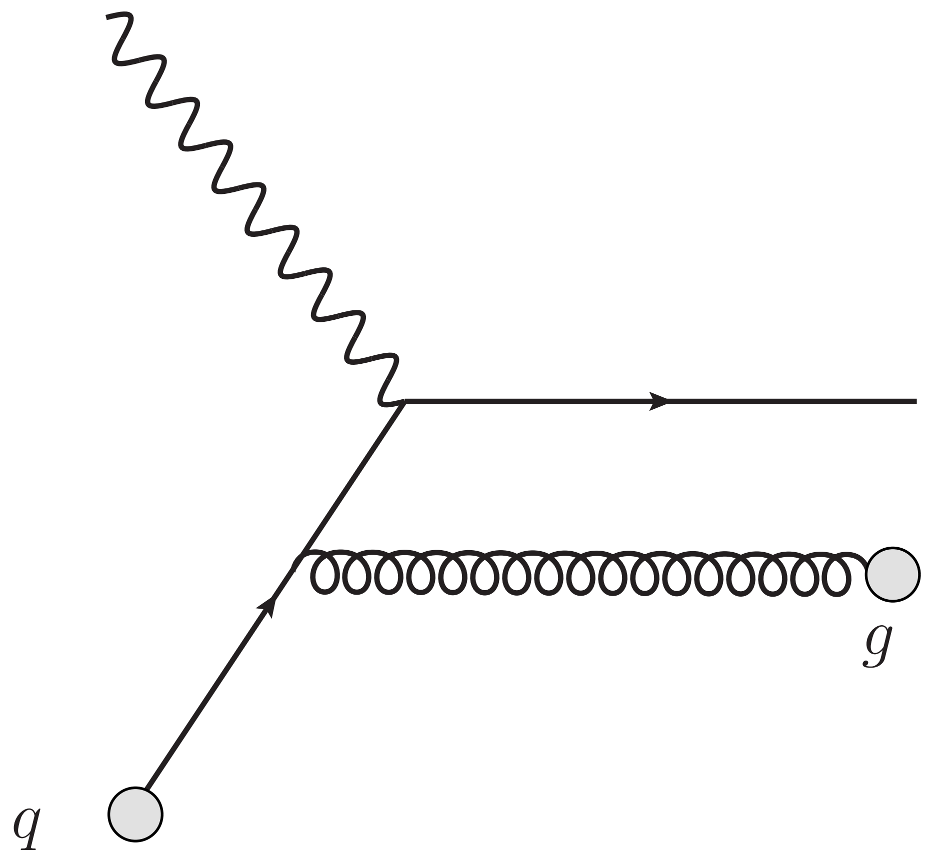
$$C_{qq}^{L,(1)}(\hat{x},\hat{z}) = 4e_q^2 C_F \hat{x}\hat{z},$$

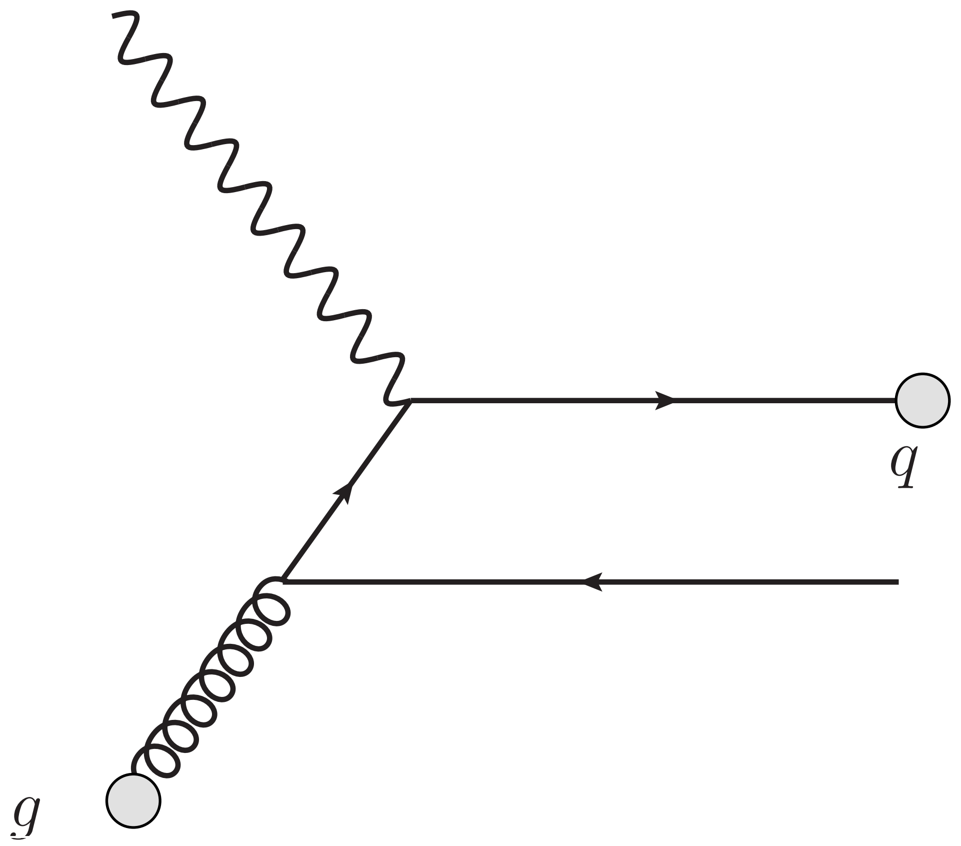
$$\tilde{P}_{qq}(\xi) = \frac{1+\xi^2}{(1-\xi)_+} + \frac{3}{2}\delta(1-\xi),$$
$$\tilde{P}_{gq}(\xi) = \frac{1+(1-\xi)^2}{\xi},$$
$$\tilde{P}_{qg}(\xi) = \xi^2 + (1-\xi)^2,$$
$$L_1(\xi) = (1+\xi^2)\left(\frac{\ln(1-\xi)}{1-\xi}\right)_+,$$
$$L_2(\xi) = \frac{1+\xi^2}{1-\xi}\ln\xi, \qquad (52)$$

Screenshots from
[Anderle, Ringer, Vogelsang ’12]

SIDIS @ NLO

[Altarelli, Ellis, Martinelli, Pi ’79][Baier, Fey ’79]





$$C_{gq}^{T,(1)}(\hat{x},\hat{z}) = e_q^2 C_F \Bigg[\tilde{P}_{gq}(\hat{z}) \left(\delta(1-\hat{x}) \ln\left(\frac{Q^2}{\mu_F^2} \hat{z}(1-\hat{z}) \right) + \frac{1}{(1-\hat{x})_+} \right) + \hat{z}\delta(1-\hat{x}) + 2(1+\hat{x}-\hat{x}\hat{z}) - \frac{1+\hat{x}}{\hat{z}} \Bigg], \tag{50}$$

$$C_{qg}^{T,(1)}(\hat{x},\hat{z}) = e_q^2 T_R \Bigg[\delta(1-\hat{z}) \left[\tilde{P}_{qg}(\hat{x}) \ln\left(\frac{Q^2}{\mu_F^2} \frac{1-\hat{x}}{\hat{x}} \right) + 2\hat{x}(1-\hat{x}) \right] + \tilde{P}_{qg}(\hat{x}) \left\{ \frac{1}{(1-\hat{z})_+} + \frac{1}{\hat{z}} - 2 \right\} \Bigg], \tag{51}$$

$$C_{gq}^{L,(1)}(\hat{x},\hat{z}) = 4 e_q^2 C_F \hat{x}(1-\hat{z}),$$

$$C_{qg}^{L,(1)}(\hat{x},\hat{z}) = 8 e_q^2 T_R \hat{x}(1-\hat{x}).$$

$$\begin{aligned}
\tilde{P}_{qq}(\xi) &= \frac{1+\xi^2}{(1-\xi)_+} + \frac{3}{2}\delta(1-\xi), \\
\tilde{P}_{gq}(\xi) &= \frac{1+(1-\xi)^2}{\xi}, \\
\tilde{P}_{qg}(\xi) &= \xi^2 + (1-\xi)^2, \\
L_1(\xi) &= (1+\xi^2)\left(\frac{\ln(1-\xi)}{1-\xi}\right)_+, \\
L_2(\xi) &= \frac{1+\xi^2}{1-\xi}\ln\xi,
\end{aligned} \tag{52}$$

Screenshots from
[Anderle, Ringer, Vogelsang ’12]

SIDIS @ NNLO

ancillary.inc

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*************************************************************************************
** SIDIS coefficient functions up to NNLO from:
**
** Semi-inclusive deep-inelastic scattering at NNLO in QCD
** L. Bonino, T. Gehrmann and G. Stagnitto
**
** FORM readable format
**
** Notation, according to eq.(6) of the paper:
**          C[order][component][a2b][label] with
**          - order: 1 = NLO, 2 = NNLO
**          - component: L = Longitudinal, T = transverse
**          - a2b: means a -> b, for a and b partons
**          - label (it can be none, NS, PS, 1, 2, 3)
**
** Symbols:
**         NC = 3: number of colours
**         NF = 5: number of active flavours
**
** Scales:
**        LMUR = ln(muR^2/Q2)
**        LMUF = ln(muF^2/Q2)
**        LMUA = ln(muA^2/Q2)
**        with Q2 = -q2, invariant mass of the photon
**             muR: renormalisation scale
**             muF: initial-state factorisation scale
**             muA: final-state factorisation scale
**
** Functions:
**           Li2(a) = PolyLog(2,a)
**           Li3(a) = PolyLog(3,a)
**           sqrtxz1 = sqrt(1 - 2*z + z*z + 4*x*z)
**           poly2   = 1 + 2*x + x*x - 4*x*z
**           sqrtxz2 = sqrt(poly2)
**           sqrtxz3 = sqrt(x/z)
**           InvTanInt(x) = int_0^x dt arctan(t)/t : Arctangent integral
**           T(region): Heaviside Theta function
**
** Distributions:
**          Dd([1-x]) is the Dirac delta function of argument [1-x]
**          Dn(a,[1-x]) = (ln^a(1-x)/(1-x))_+ (plus-prescription) for a = 1,2,3
**          same for z
**
** Kinematic regions in (x,z)-plane: as defined in 2201.06982
**          ui = Ui for i = 1,2,3,4
**          ri = Ri, ti = Ti for i = 1,2
**          Ri, Ti and Ui defined in eq. (5.9), (5.12) and (5.16)
**
** Constants: pi, zeta3 = Zeta(3) with Zeta Riemann Zeta function
**
*************************************************************************************
```

1.1 MB of size

$$C^{i,(2)}_{qq} = C^{i,(2)}_{\bar{q}\bar{q}} = e_q^2 C^{i,\mathrm{NS}}_{qq} + \left(\sum_j e_{q_j}^2\right) C^{i,\mathrm{PS}}_{qq} ,$$

$$C^{i,(2)}_{\bar{q}q} = C^{i,(2)}_{q\bar{q}} = e_q^2 C^{i}_{\bar{q}q} ,$$

$$C^{i,(2)}_{q'q} = C^{i,(2)}_{\bar{q}'\bar{q}} = e_q^2 C^{i,1}_{q'q} + e_{q'}^2 C^{i,2}_{q'q} + e_q e_{q'} C^{i,3}_{q'q} ,$$

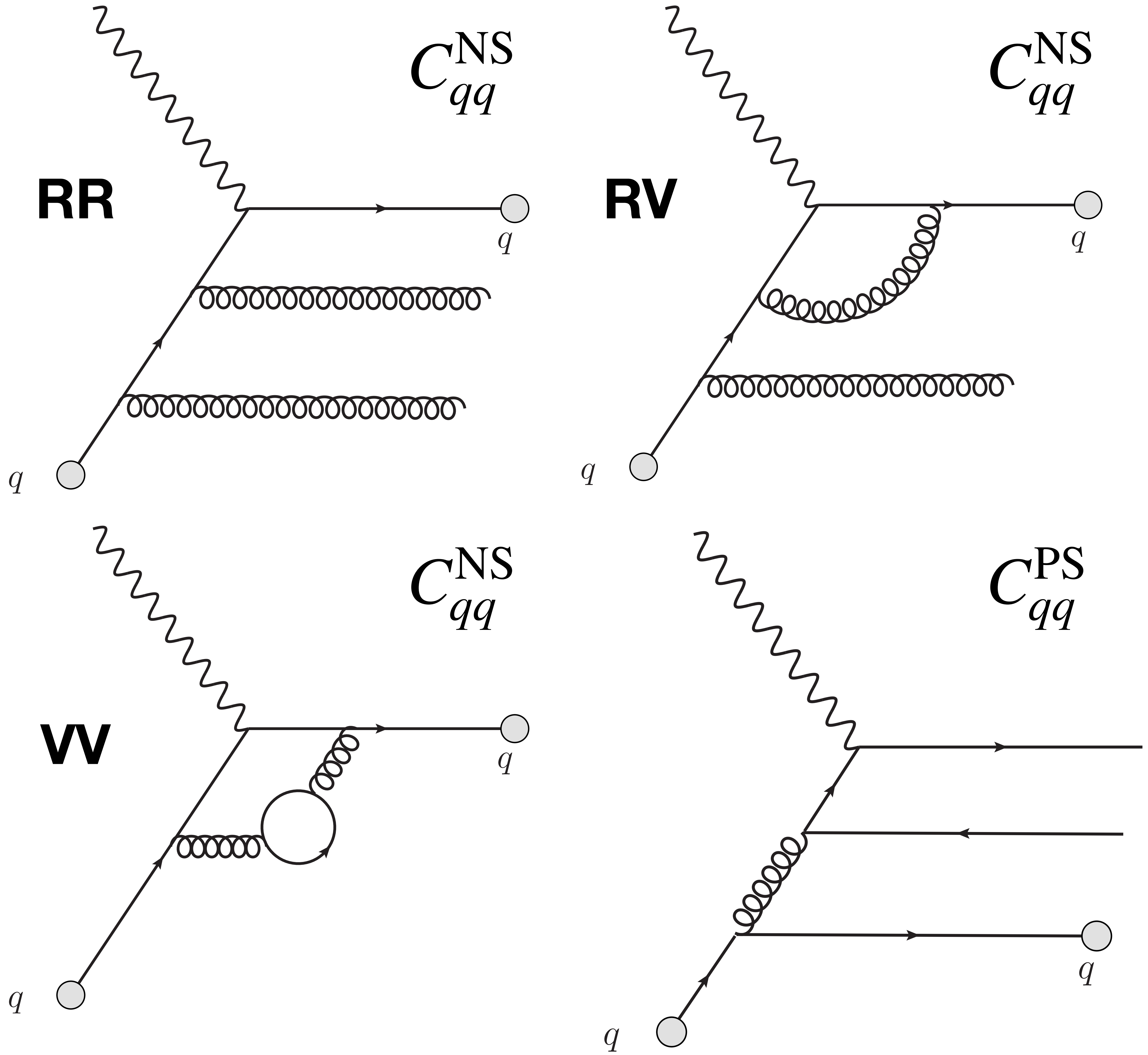
$$C^{i,(2)}_{\bar{q}'q} = C^{i,(2)}_{q'\bar{q}} = e_q^2 C^{i,1}_{q'q} + e_{q'}^2 C^{i,2}_{q'q} - e_q e_{q'} C^{i,3}_{q'q} ,$$

$$C^{i,(2)}_{gq} = C^{i,(2)}_{g\bar{q}} = e_q^2 C^{i}_{gq} ,$$

$$C^{i,(2)}_{qg} = C^{i,(2)}_{\bar{q}g} = e_q^2 C^{i}_{qg} ,$$

$$C^{i,(2)}_{gg} = \left(\sum_j e_{q_j}^2\right) C^{i}_{gg} ,$$

SIDIS @ NNLO



$$C_{qq}^{i,(2)} = C_{\bar{q}\bar{q}}^{i,(2)} = e_q^2 C_{qq}^{i,\rm NS} + \left(\sum_j e_{q_j}^2\right) C_{qq}^{i,\rm PS} ,$$

$$C_{\bar{q}q}^{i,(2)} = C_{q\bar{q}}^{i,(2)} = e_q^2 C_{\bar{q}q}^{i} ,$$

$$C_{q'q}^{i,(2)} = C_{\bar{q}'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} + e_q e_{q'} C_{q'q}^{i,3} ,$$

$$C_{\bar{q}'q}^{i,(2)} = C_{q'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} - e_q e_{q'} C_{q'q}^{i,3} ,$$

$$C_{gq}^{i,(2)} = C_{g\bar{q}}^{i,(2)} = e_q^2 C_{gq}^{i} ,$$

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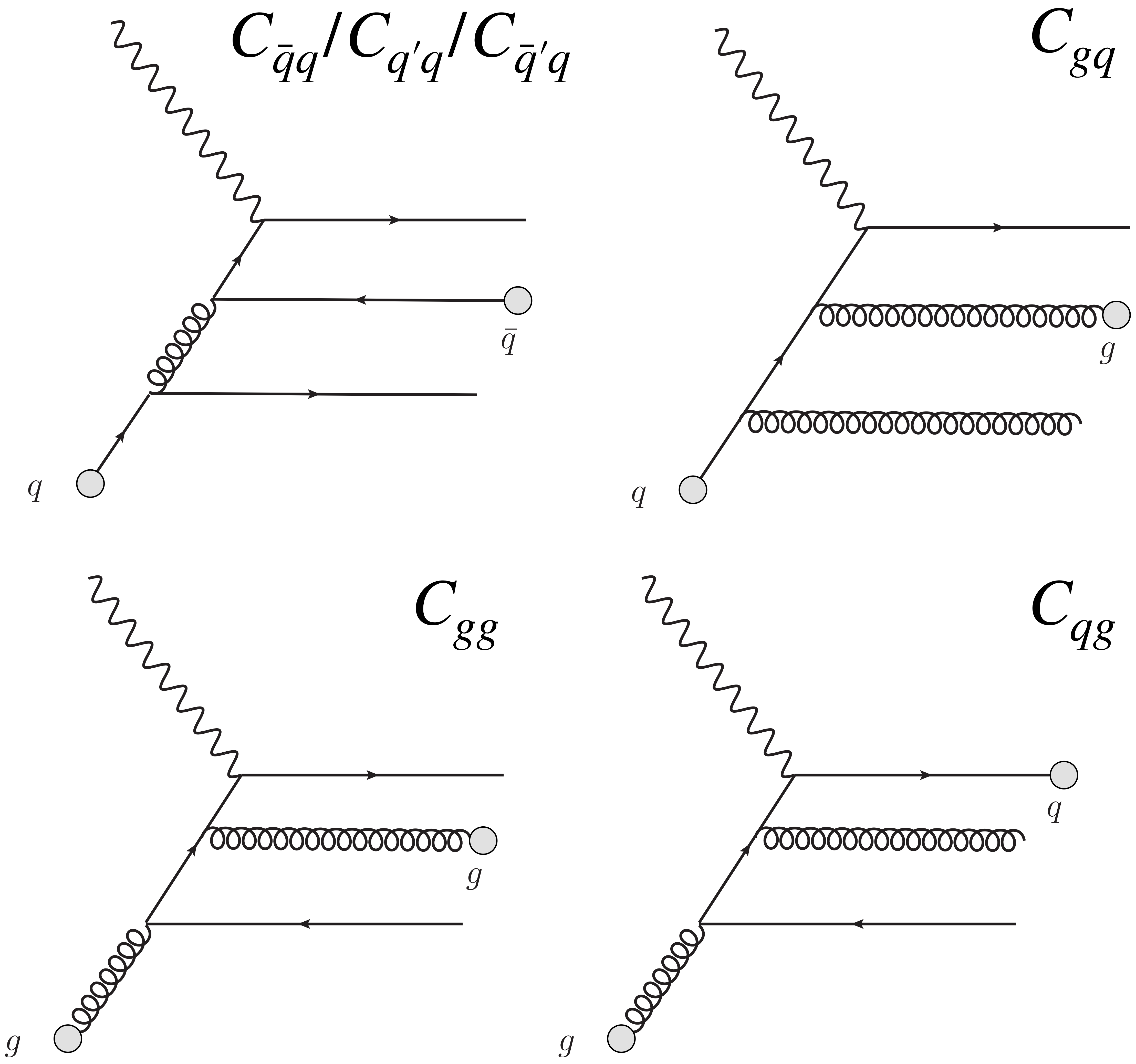
$$C_{gg}^{i,(2)} = \left(\sum_j e_{q_j}^2\right) C_{gg}^{i} ,$$



SIDIS @ NNLO

$$C^{i,(2)}_{qq} = C^{i,(2)}_{\bar{q}\bar{q}} = e_q^2 C^{i,\mathrm{NS}}_{qq} + \left(\sum_j e^2_{q_j}\right) C^{i,\mathrm{PS}}_{qq} \,,$$

$$C^{i,(2)}_{\bar{q}q} = C^{i,(2)}_{q\bar{q}} = e_q^2 C^{i}_{\bar{q}q} \,,$$

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$$C^{i,(2)}_{gg} = \left(\sum_j e^2_{q_j}\right) C^{i}_{gg} \,,$$

Details of the calculation

VV: well-known two-loop quark form factor in space-like kinematics

RV: one-loop squared matrix elements in terms of one-loop bubble and box integrals, which are known in exact form in ε. For fixed $\hat{x}$ and $\hat{z}$, the phase space integral is fully constrained:

$$C^{\mathrm{RV}}_{j\leftarrow i} \propto \int \mathrm{d}\Phi_2(k_j, k_k; k_i, q)\, \delta\left(z - x\frac{(k_i+k_j)^2}{Q^2}\right) \left|\mathcal{M}^{\mathrm{RV}}\right|^2 \propto \mathcal{J}(x,z) \, \left|\mathcal{M}^{\mathrm{RV}}\right|^2(x,z)$$

Only expansions in the end-point distributions $\hat{x} = 1$ and $\hat{z} = 1$ are required.

RR: integrations over three-particle phase space with multi-loop techniques:

$$C^{\mathrm{RR}}_{j\leftarrow i} \propto \int \mathrm{d}\Phi_3(k_j, k_k, k_l; k_i, q)\, \delta\left(z - x\frac{(k_i+k_j)^2}{Q^2}\right) \left|\mathcal{M}^{\mathrm{RR}}\right|^2$$

Reduction to master integrals using IBP identities, 13 integral families, 21 master integrals.
Solved using differential equations, boundary terms obtained by integrating over $\hat{z}$ and comparing to master integrals relevant to inclusive version.

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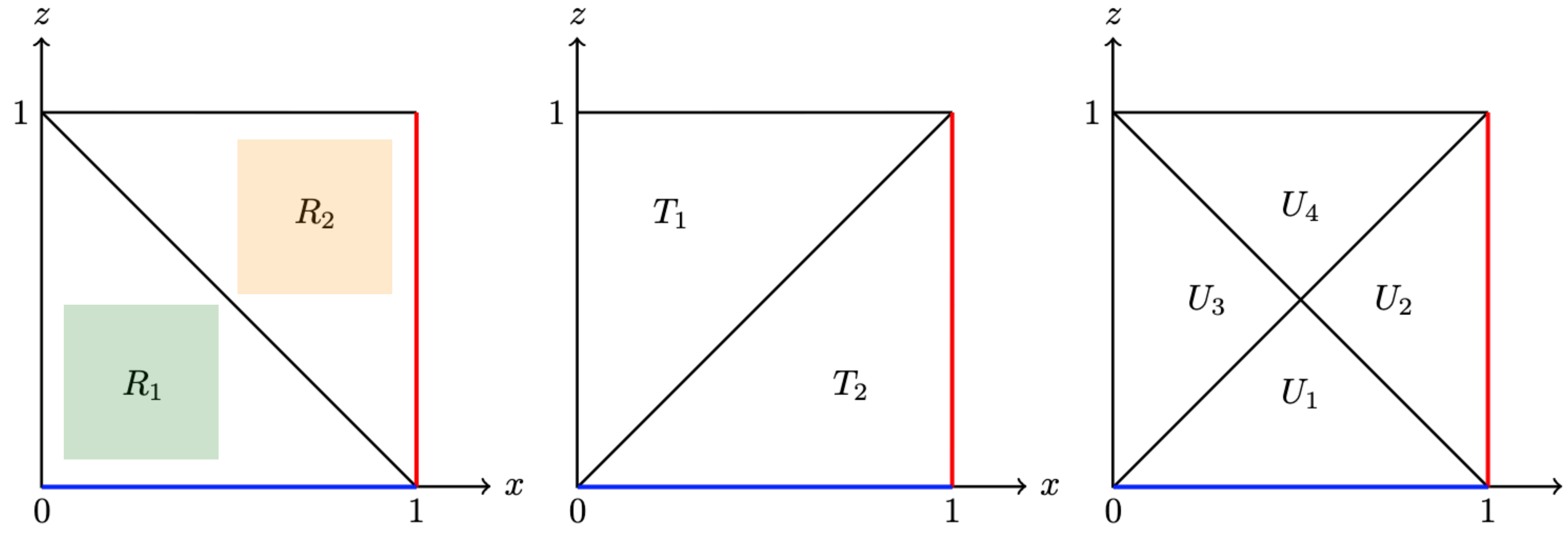
Analytic continuation in the real-virtual

[Gehrmann, Schürmann '22]

To avoid ambiguities associated with the analytic continuation of boxes, we segment the (x,z) plane into four sectors, where manifestly real-valued expressions are obtained.



$$\begin{aligned}
&\mathrm{Box}(s_{ij},s_{ik}) \\
&= \frac{2(1-2\epsilon)}{\epsilon} A_{2,LO} \frac{1}{s_{ij}s_{ik}} \\
&\quad\times\Bigg[\left(\frac{s_{ij}s_{ik}}{s_{ij}-s_{ijk}}\right)^{-\epsilon} {}_2F_1\left(-\epsilon,-\epsilon;1-\epsilon;\frac{s_{ijk}-s_{ij}-s_{ik}}{s_{ijk}-s_{ij}}\right) \\
&\quad+\left(\frac{s_{ij}s_{ik}}{s_{ik}-s_{ijk}}\right)^{-\epsilon} {}_2F_1\left(-\epsilon,-\epsilon;1-\epsilon;\frac{s_{ijk}-s_{ij}-s_{ik}}{s_{ijk}-s_{ik}}\right) \\
&\quad-\left(\frac{-s_{ijk}s_{ij}s_{ik}}{(s_{ij}-s_{ijk})(s_{ik}-s_{ijk})}\right)^{-\epsilon} {}_2F_1\left(-\epsilon,-\epsilon;1-\epsilon;\frac{s_{ijk}(s_{ijk}-s_{ij}-s_{ik})}{(s_{ijk}-s_{ij})(s_{ijk}-s_{ik})}\right)\Bigg]
\end{aligned}$$

Example:
in $\mathrm{Box}(s_{12}, s_{23})$ we use

$$\begin{aligned}
a_1(s_{12},s_{23}) &= \frac{s_{123}-s_{12}-s_{23}}{s_{123}-s_{12}} = -\frac{z}{1-x-z}\,, \\
a_2(s_{12},s_{23}) &= \frac{s_{123}-s_{12}-s_{23}}{s_{123}-s_{23}} = z\,, \qquad R_1 \\
a_3(s_{12},s_{23}) &= \frac{s_{123}\,s_{13}}{(s_{13}+s_{23})(s_{12}+s_{13})} = -\frac{x\,z}{1-x-z}
\end{aligned}$$

$$\begin{aligned}
\tilde{a}_1(s_{12},s_{23}) &= 1-\frac{1}{a_1(s_{12},s_{23})} = \frac{1-x}{z}\,, \qquad R_2 \\
\tilde{a}_3(s_{12},s_{23}) &= 1-\frac{1}{a_3(s_{12},s_{23})} = \frac{(1-x)(1-z)}{xz}
\end{aligned}$$

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Real-real master integrals

[Bonino, Gehrmann, Schürmann, GS, in preparation]

Notation:

$$I[-3,7] \propto \int d^dk_j d^dk_l \delta(D_9)\delta(D_{10})\delta(D_{11})\delta(D_{12}) \frac{D_3}{D_7}$$

Integrals of families A, B, C already calculated in the context of antenna subtraction for photon fragmentation [Gehrmann, Schürmann '22]

Some of them derived in closed form, the others up to finite part in ϵ. Expansion in distributions after insertion of master integrals in reduced expressions.

$$\begin{aligned}
D_1 &= (q-k_j)^2\,,\\
D_2 &= (p+q-k_j)^2\,,\\
D_3 &= (p-k_l)^2\,,\\
D_4 &= (q-k_l)^2\,,\\
D_5 &= (p+q-k_l)^2\,,\\
D_6 &= (q-k_j-k_l)^2\,,\\
D_7 &= (p-k_j-k_l)^2\,,\\
D_8 &= (k_j+k_l)^2\,,\\
D_9 &= k_j^2\,,\\
D_{10} &= k_l^2\,,\\
D_{11} &= (q+p-k_j-k_l)^2\,,\\
D_{12} &= (p-k_j)^2 + Q^2\frac{z}{x}\,,
\end{aligned}$$

set of denominator factors

family	master	deepest pole	at $x=1$	at $z=1$
	$I[0]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
A	$I[5]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2,3,5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
B	$I[7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-2,7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-3,7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2,3,7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
C	$I[5,7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[3,5,7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
D	$I[1]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1,4]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1,3,4]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
E	$I[1,3,5]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
G	$I[1,3,8]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
H	$I[1,4,5]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
I	$I[2,4,5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
J	$I[4,7]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[3,4,7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
K	$I[3,5,8]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
L	$I[4,5,7]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
M	$I[4,5,8]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$

Table 1. Summary of the double real radiation master integrals.

Assembling and checking the result

The sum VV+VR+RR still contain UV and IR pole terms. They are removed by:

- renormalising the strong coupling (in $\overline{\mathrm{MS}}$ ren. scheme)
- adding the mass factorisation counterterms, both initial- and final-state (in $\overline{\mathrm{MS}}$ fac. scheme)

Checks:

- Scale dependent terms are found to be as predicted by RGE
- We used the underlying RR, RV and VV subprocess matrix elements to re-derive the inclusive NNLO coefficient functions.
- We integrated specific subprocess contributions over the final-state momentum $\hat{z}$ and we recovered the respective contributions to the inclusive coefficient function.
- Comparison to approximate results
- Comparison to partial results

Comparison to approximate results

[Abele, De Florian, Vogelsang '21,'22]

By expanding the NNLL threshold resummation (i.e. resummation of dominant terms in the $\hat{x} \to 1$ and/or $\hat{z} \to 1$ limit) , approximate corrections have been derived at NNLO and at N3LO

We then have for the leading-power part:

$$
\begin{aligned}
\Delta^{(2),C_F}_{qq,\mathrm{LP}} &= \frac{1}{2}\left(\delta_x\,\mathcal{D}_z^3+\delta_z\,\mathcal{D}_x^3\right)+\frac{3}{2}\left(\mathcal{D}_x^0\mathcal{D}_z^2+\mathcal{D}_z^0\mathcal{D}_x^2+2\,\mathcal{D}_x^1\mathcal{D}_z^1\right)\\
&- \left(4+\frac{\pi^2}{3}\right)\left(\mathcal{D}_x^0\mathcal{D}_z^0+\delta_x\,\mathcal{D}_z^1+\delta_z\mathcal{D}_x^1\right)+2\zeta(3)\left(\delta_x\,\mathcal{D}_z^0+\delta_z\,\mathcal{D}_x^0\right)\\
&+ \delta_x\,\delta_z\left(\frac{511}{64}-\frac{15\zeta(3)}{4}+\frac{29\pi^2}{48}-\frac{7\pi^4}{360}\right)\\
&+ \left[\delta_x\,\mathcal{D}_z^1+\delta_z\,\mathcal{D}_x^1+\mathcal{D}_x^0\,\mathcal{D}_z^0+\frac{3}{2}\left(\delta_x\,\mathcal{D}_z^0+\delta_z\,\mathcal{D}_x^0\right)+\delta_x\,\delta_z\left(\frac{9}{8}-\frac{\pi^2}{6}\right)\right]\ln^2\frac{\mu_F^2}{Q^2}\\
&+ \left[-\frac{3}{2}\left(\delta_x\,\mathcal{D}_z^2+\delta_z\,\mathcal{D}_x^2+2\,\mathcal{D}_x^0\,\mathcal{D}_z^1+2\,\mathcal{D}_z^0\,\mathcal{D}_x^1+\mathcal{D}_x^0\,\mathcal{D}_z^0+\delta_x\,\mathcal{D}_z^1+\delta_z\,\mathcal{D}_x^1\right)\right.\\
&\quad\left.+\left(4+\frac{\pi^2}{3}\right)\left(\delta_x\,\mathcal{D}_z^0+\delta_z\,\mathcal{D}_x^0\right)+\delta_x\,\delta_z\left(-5\zeta(3)+\frac{\pi^2}{4}+\frac{93}{16}\right)\right]\ln\frac{\mu_F^2}{Q^2}\,,
\end{aligned}
$$

while the dominant NLP terms are given by

$$
\Delta^{(2),C_F}_{qq,\mathrm{NLP}} = -\frac{3}{2}\left(\mathcal{D}_x^2+\mathcal{D}_z^2+2\,\mathcal{D}_x^1\,\ell_z^1+2\,\mathcal{D}_z^1\,\ell_x^1+\mathcal{D}_x^0\,\ell_z^2+\mathcal{D}_z^0\,\ell_x^2\right)-\frac{1}{2}\left(\delta_x\,\ell_z^3+\delta_z\,\ell_x^3\right)\,.
$$

$$
\begin{aligned}
\Delta^{(2),C_A}_{qq} &= -\frac{11}{24}\left(\delta_x\,\mathcal{D}_z^2+\delta_z\,\mathcal{D}_x^2+2\,\mathcal{D}_x^0\,\mathcal{D}_z^1+2\,\mathcal{D}_z^0\,\mathcal{D}_x^1\right)+\left(\frac{67}{36}-\frac{\pi^2}{12}\right)\left(\mathcal{D}_x^0\,\mathcal{D}_z^0+\delta_x\,\mathcal{D}_z^1+\delta_z\,\mathcal{D}_x^1\right)\\
&+ \left(\delta_x\,\mathcal{D}_z^0+\delta_z\,\mathcal{D}_x^0\right)\left(\frac{7\zeta(3)}{4}+\frac{11\pi^2}{72}-\frac{101}{54}\right)+\delta_x\,\delta_z\left(\frac{43\zeta(3)}{12}+\frac{17\pi^4}{720}-\frac{1535}{192}-\frac{269\pi^2}{432}\right)\\
&+ \frac{11}{24}\left[\delta_x\,\mathcal{D}_z^0+\delta_z\,\mathcal{D}_x^0+\frac{3}{2}\,\delta_x\,\delta_z\right]\ln^2\frac{\mu_F^2}{Q^2}\\
&+ \left[-\left(\delta_x\,\mathcal{D}_z^0+\delta_z\,\mathcal{D}_x^0\right)\left(\frac{67}{36}-\frac{\pi^2}{12}\right)+\delta_x\,\delta_z\left(\frac{3\zeta(3)}{2}-\frac{11\pi^2}{36}-\frac{17}{48}\right)\right]\ln\frac{\mu_F^2}{Q^2}\,. \qquad (64)
\end{aligned}
$$

$$
\begin{aligned}
\Delta^{(2),N_f}_{qq} &= \frac{1}{12}\left(\delta_x\,\mathcal{D}_z^2+\delta_z\,\mathcal{D}_x^2+2\,\mathcal{D}_x^0\,\mathcal{D}_z^1+2\,\mathcal{D}_z^0\,\mathcal{D}_x^1\right)-\frac{5}{18}\left(\mathcal{D}_x^0\,\mathcal{D}_z^0+\delta_x\,\mathcal{D}_z^1+\delta_z\,\mathcal{D}_x^1\right)\\
&+ \left(\delta_x\,\mathcal{D}_z^0+\delta_z\,\mathcal{D}_x^0\right)\left(\frac{7}{27}-\frac{\pi^2}{36}\right)+\delta_x\,\delta_z\left(\frac{\zeta(3)}{6}+\frac{19\pi^2}{216}+\frac{127}{96}\right)\\
&- \frac{1}{12}\left[\delta_x\,\mathcal{D}_z^0+\delta_z\,\mathcal{D}_x^0+\frac{3}{2}\,\delta_x\,\delta_z\right]\ln^2\frac{\mu_F^2}{Q^2}\\
&+ \left[\frac{5}{18}\left(\delta_x\,\mathcal{D}_z^0+\delta_z\,\mathcal{D}_x^0\right)+\delta_x\,\delta_z\left(\frac{1}{24}+\frac{\pi^2}{18}\right)\right]\ln\frac{\mu_F^2}{Q^2}\,. \qquad (65)
\end{aligned}
$$

Full agreement with our result

Comparison to partial results

[Goyal, Moch, Pathak, Rana, Ravindran ’23]

Very recently, the leading colour contribution to the $q \to q$ non-singlet channel was computed.

e.g. piece contained in the transverse coefficient function with single distributions in x or z $\longrightarrow$

$$
\begin{aligned}
\mathcal{F}^{(2)}_{1,1} = C_F^2 \Big[& \delta_{\bar{x}} \Big\{ 2l_z^2(1-4\bar{z}) + 4(1-8\bar{z}) - 8\,\mathrm{Li}_3(\bar{z})\tilde{z} + \frac{25}{3} l_z^3 \tilde{z} - 4l_z l_{\bar{z}}^2 \tilde{z} - 4l_{\bar{z}}^3 \tilde{z} + 52\,\mathrm{S}_{12}(\bar{z})\tilde{z} + \mathrm{Li}_2(\bar{z})(4(1-6\bar{z}) \\
& + 40 l_z \tilde{z}) + \frac{1}{\bar{z}}\big(8\,\mathrm{Li}_3(\bar{z}) - 64\,\mathrm{Li}_2(\bar{z}) l_z - \frac{40}{3} l_z^3 + 12 l_z l_{\bar{z}}^2 - 88\,\mathrm{S}_{12}(\bar{z}) + l_{\bar{z}}\big(-8\,\mathrm{Li}_2(\bar{z}) - 12 l_z^2\big) + l_z\big(-64 + 24\zeta_2\big)\big) \\
& + l_{\bar{z}}\big(14 + 24\tilde{z} + 4l_z(1-2\bar{z}) + 8\,\mathrm{Li}_2(\bar{z})\tilde{z} + 10 l_z^2 \tilde{z} + 16\tilde{z}\zeta_2\big) + l_z\big(-2 + 38\tilde{z} - 16\tilde{z}\zeta_2\big) + 8\bar{z}\zeta_2 - 16\tilde{z}\zeta_3 \Big\} \\
& + \mathcal{D}_{x,0}\{12 + 24\tilde{z} + 4l_z(1-3\bar{z}) + 12\,\mathrm{Li}_2(\bar{z})\tilde{z} + 16 l_z^2 \tilde{z} - 4 l_z l_{\bar{z}} \tilde{z} - 12 l_{\bar{z}}^2 \tilde{z} - \frac{1}{\bar{z}}\big(16\,\mathrm{Li}_2(\bar{z}) + 24 l_z^2 - 16 l_z l_{\bar{z}}\big) + 16\tilde{z}\zeta_2\} \\
& + \mathcal{D}_{x,1}\{(4l_z\tilde{z} - 24 l_{\bar{z}}\tilde{z})\} - 12\tilde{z}\,\mathcal{D}_{x,2} + \delta_{\bar{z}}\Big\{ -4 - 48\bar{x} - 2l_x^2 + \frac{11}{3} l_x^3 \tilde{x} + 16 l_x l_{\bar{x}}^2 \tilde{x} - 4 l_{\bar{x}}^3 \tilde{x} - 24\,\mathrm{S}_{12}(\bar{x})\tilde{x} \\
& + \mathrm{Li}_2(\bar{x})(4 + 8\bar{x} - 12 l_x \tilde{x}) + \frac{1}{\bar{x}}\big(-8\,\mathrm{Li}_3(\bar{x}) + 16\,\mathrm{Li}_2(\bar{x}) l_x - 4 l_x^3 - 28 l_x l_{\bar{x}}^2 + 48\,\mathrm{S}_{12}(\bar{x}) + l_{\bar{x}}\big(8\,\mathrm{Li}_2(\bar{x}) + 32 l_x^2\big) \\
& + l_x\big(64 + 32\zeta_2\big)\big) + l_{\bar{x}}\big(14 + 26\tilde{x} + 4l_x - 20 l_x^2 \tilde{x} + 16\tilde{x}\zeta_2\big) + l_x\big(-8 - 34\tilde{x} - 20\tilde{x}\zeta_2\big) + 8\bar{x}\zeta_2 - 16\tilde{x}\zeta_3 \Big\} \\
& + \mathcal{D}_{z,0}\{12 + 28\tilde{x} + 4l_x(1+\bar{x}) - 4\,\mathrm{Li}_2(\bar{x})\tilde{x} - 12 l_x^2 \tilde{x} + 28 l_x l_{\bar{x}} \tilde{x} - 12 l_{\bar{x}}^2 \tilde{x} + \frac{1}{\bar{x}}\big(16\,\mathrm{Li}_2(\bar{x}) + 16 l_x^2 - 48 l_x l_{\bar{x}}\big) + 16\tilde{x}\zeta_2\} \\
& + \mathcal{D}_{z,1}\Big\{ -\frac{32}{\bar{x}} l_x + 20 l_x \tilde{x} - 24 l_{\bar{x}} \tilde{x} \Big\} - 12\tilde{x}\mathcal{D}_{z,2} \Big] + C_A C_F \Big[\delta_{\bar{x}} \Big\{ 4\,\mathrm{Li}_2(\bar{z})(1-2\bar{z}) + \frac{2}{3} l_z^2 (3 - 11\tilde{z}) + \frac{1}{27}(396 + 179\tilde{z}) \\
& + \frac{1}{9} l_z (1 + 70\bar{z}) - 4\,\mathrm{Li}_3(\bar{z})\tilde{z} - \frac{5}{3} l_z^3 \tilde{z} + \frac{11}{3} l_{\bar{z}}^2 \tilde{z} + 6\,\mathrm{S}_{12}(\bar{z})\tilde{z} + \frac{1}{\bar{z}}\big(6\,\mathrm{Li}_2(\bar{z}) + 8\,\mathrm{Li}_3(\bar{z}) + \frac{62}{9} l_z + \frac{49}{6} l_z^2 + \frac{10}{3} l_z^3 + 6 l_z l_{\bar{z}} \\
& - 12\,\mathrm{S}_{12}(\bar{z})\big) + l_{\bar{z}}\Big(\frac{4}{9}(15 - 41\tilde{z}) - 6 l_z \tilde{z} + 4\tilde{z}\zeta_2\Big) - \frac{4}{3}(3 + 4\tilde{z})\zeta_2 - 14\tilde{z}\zeta_3 \Big\} + \mathcal{D}_{x,0}\Big\{ \frac{2}{9}(39 - 82\tilde{z}) - 4\,\mathrm{Li}_2(\bar{z})\tilde{z} - 6 l_z \tilde{z} \\
& - 2 l_z^2 \tilde{z} + \frac{22}{3} l_{\bar{z}} \tilde{z} + \frac{1}{\bar{z}}\big(8\,\mathrm{Li}_2(\bar{z}) + 6 l_z + 4 l_z^2\big) + 4\tilde{z}\zeta_2 \Big\} + \frac{22}{3}\tilde{z}\mathcal{D}_{x,1} + \delta_{\bar{z}}\Big\{ \frac{46}{3} + \frac{197}{27}\tilde{x} + 8\,\mathrm{Li}_3(\bar{x})\tilde{x} + \frac{55}{6} l_x^2 \tilde{x} + \frac{11}{3} l_{\bar{x}}^2 \tilde{x} \\
& + 2\,\mathrm{S}_{12}(\bar{x})\tilde{x} + \mathrm{Li}_2(\bar{x})\Big(-4 l_x \tilde{x} - \frac{4}{3}(3 + 4\tilde{x})\Big) + l_x\Big(\frac{1}{3}(-13 + 77\tilde{x}) - 4\tilde{x}\zeta_2\Big) + \frac{1}{\bar{x}}\Big(-16\,\mathrm{Li}_3(\bar{x}) - \frac{83}{6} l_x^2 \\
& + \frac{2}{3}\,\mathrm{Li}_2(\bar{x})(13 + 12 l_x) + \frac{70}{3} l_x l_{\bar{x}} - 4\,\mathrm{S}_{12}(\bar{x}) + l_x\Big(-\frac{116}{3} + 8\zeta_2\Big)\Big) + l_{\bar{x}}\Big(-\frac{44}{3} l_x \tilde{x} - \frac{4}{9}(-15 + 41\tilde{x}) + 4\tilde{x}\zeta_2\Big) \\
& - \frac{4}{3}(3 + 4\tilde{x})\zeta_2 - 14\tilde{x}\zeta_3 \Big\} + \mathcal{D}_{z,0}\Big\{ 4\,\mathrm{Li}_2(\bar{x})\tilde{x} - \frac{44}{3} l_x \tilde{x} + \frac{22}{3} l_{\bar{x}} \tilde{x} + \frac{26}{9}(3 - 7\tilde{x}) + \frac{1}{\bar{x}}\big(-8\,\mathrm{Li}_2(\bar{x}) + \frac{70}{3} l_x\big) + 4\tilde{x}\zeta_2 \Big\} \\
& + \frac{22}{3}\tilde{x}\mathcal{D}_{z,1} \Big] + \frac{1}{3} C_F n_F \Big[\delta_{\bar{x}} \Big\{ 4 - \frac{2}{3}\frac{1}{\bar{z}} l_z (10 + 3 l_z) - \frac{74}{9}\tilde{z} + l_z^2 \tilde{z} - 2 l_{\bar{z}}^2 \tilde{z} + \frac{8}{3} l_{\bar{z}}(-3 + 4\tilde{z}) + \frac{2}{3} l_z (-12 + 11\tilde{z}) + 4\tilde{z}\zeta_2 \Big\} \\
& + \mathcal{D}_{x,0}\Big\{ \frac{32}{3}\tilde{z} - 8 - 4 l_{\bar{z}} \tilde{z} \Big\} - 4\tilde{z}\mathcal{D}_{x,1} + \delta_{\bar{z}}\Big\{ 4\,\mathrm{Li}_2(\bar{x})\tilde{x} - 4 - \frac{38}{9}\tilde{x} - 5 l_x^2 \tilde{x} - 2 l_{\bar{x}}^2 \tilde{x} + 2 l_x (2 - 7\tilde{x}) + \frac{1}{\bar{x}}\big(20 l_x + 10 l_x^2 \\
& - 8\,\mathrm{Li}_2(\bar{x}) - 16 l_x l_{\bar{x}}\big) + l_{\bar{x}}\Big(\frac{32}{3}\tilde{x} - 8 + 8 l_x \tilde{x}\Big) + 4\tilde{x}\zeta_2 \Big\} + \mathcal{D}_{z,0}\Big\{ \frac{32}{3}\tilde{x} - 8 - \frac{16}{\bar{x}} l_x + 8 l_x \tilde{x} - 4 l_{\bar{x}} \tilde{x} \Big\} - 4\tilde{x}\mathcal{D}_{z,1} \Big]. \quad (10)
\end{aligned}
$$

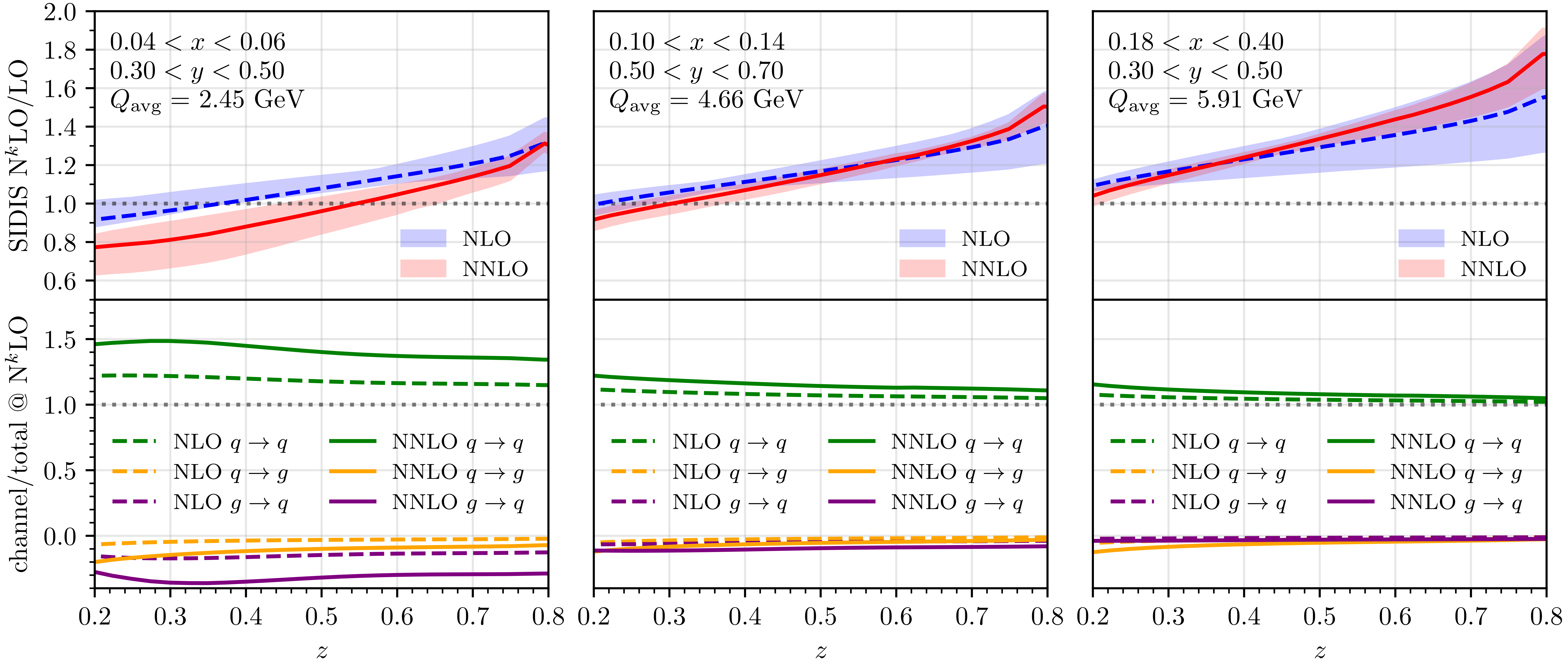
We found analytical agreement for all terms involving distributions, and numerical agreement for the regular parts

Impact of NNLO corrections

Focus on COMPASS 2016 data for SIDIS charged pion production

(fixed-target experiment, muon beam scattering off an isoscalar target at $\sqrt{s} \simeq 17.35$ GeV)

COMPASS cuts: $Q^2 = xys > 1$ GeV^2, $\sqrt{(P+q)^2}$ (invariant mass of the hadronic system) 5 GeV



Note: FF adopted are the ones of [Borsa, Sassot, De Florian, Stratmann, Vogelsang '22]

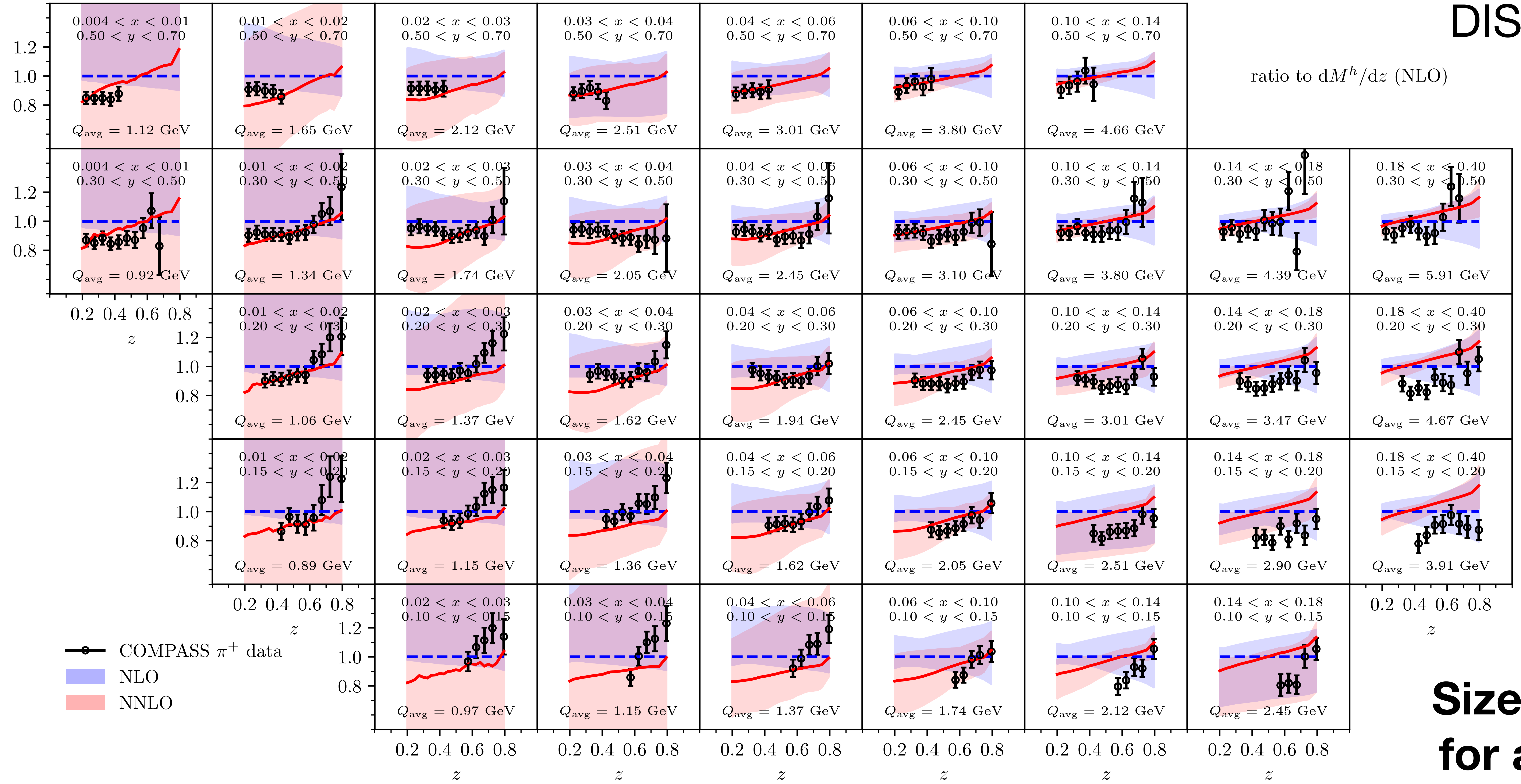
Fit on e^+e^- and SIDIS data (including this dataset) at NNLO, using the approximate NNLO for SIDIS

Hadron multiplicities

"ratio of SIDIS over DIS"

$$\frac{\mathrm{d}M^h}{\mathrm{d}z} = \frac{\mathrm{d}^3\sigma^h/\mathrm{d}x\mathrm{d}y\mathrm{d}z}{\mathrm{d}^2\sigma/\mathrm{d}x\mathrm{d}y}$$

integrated over bins in x and y
DIS from APFEL [Bertone '17]



NNLO improves data description in some bins, but makes it worse in others

Size of NNLO corrections call for a new global fit to assess the impact of SIDIS data

Outlook

Unpolarised

$$\ell(k)\, p(P) \to \ell(k')\, h(P_h)\, X$$

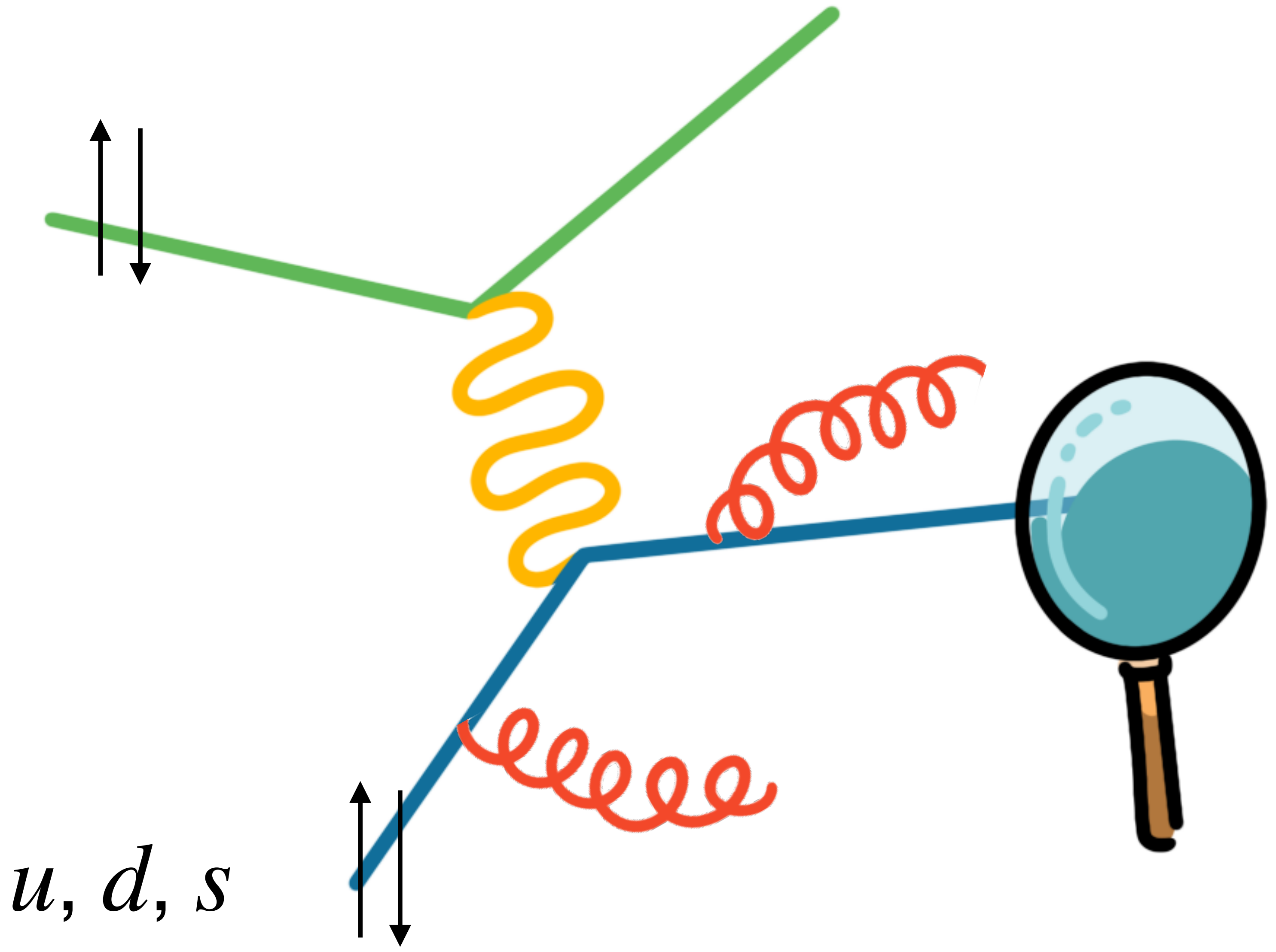
$$\frac{\mathrm{d}^3\sigma^h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathcal{F}_T^h(x,z,Q^2) + \frac{1-y}{y}\mathcal{F}_L^h(x,z,Q^2)\right]$$

Longitudinally polarised

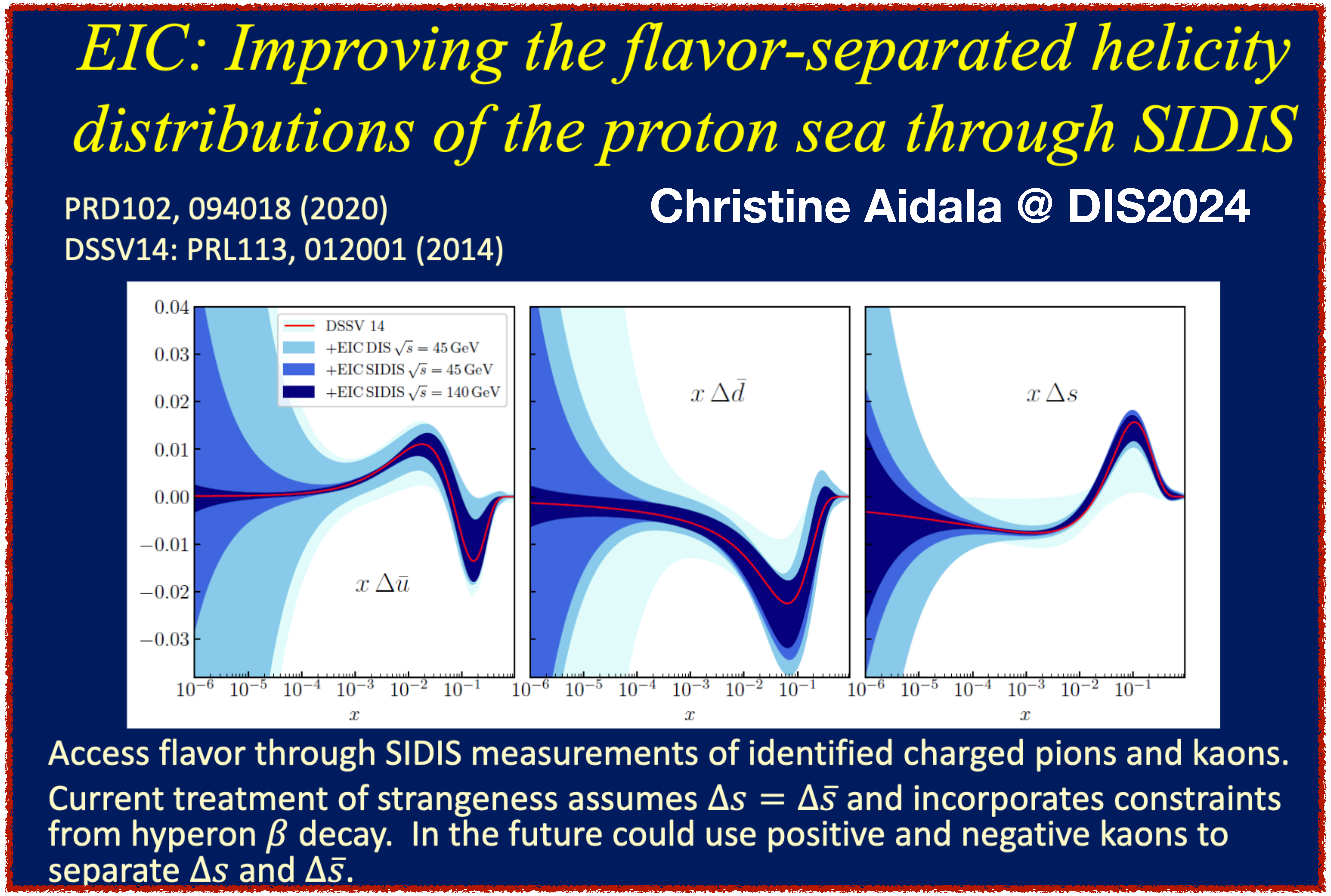
$$\overrightarrow{\ell}(k)\, \vec{p}(P) \to \ell(k')\, h(P_h)\, X$$

$$\frac{1}{2}\left(\frac{\mathrm{d}^3\sigma^h(\uparrow\uparrow)}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} - \frac{\mathrm{d}^3\sigma^h(\uparrow\downarrow)}{\mathrm{d}x\mathrm{d}y\mathrm{d}z}\right) = \frac{4\pi\alpha^2}{Q^2}\frac{1-(1-y)^2}{2y}\mathcal{G}_1^h(x,z,Q^2)$$

Motivation



Identified hadrons with polarised beams at the EIC are great handles on accessing individual quark helicity PDFs



Polarised SIDIS structure function

Unpolarised

$$\mathcal{F}_i^h(x,z,Q^2) = \sum_{p,p'} \int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}} \int_z^1 \frac{\mathrm{d}\hat{z}}{\hat{z}} f_p\left(\frac{x}{\hat{x}},\mu_F^2\right) D_{p'}^h\left(\frac{z}{\hat{z}},\mu_A^2\right) \mathcal{C}_{p'p}^i\left(\hat{x},\hat{z},Q^2,\mu_R^2,\mu_F^2,\mu_A^2\right) , \quad i = T, L$$

Longitudinally polarised

$$\mathcal{G}_1^h(x,z,Q^2) = \sum_{p,p'} \int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}} \int_z^1 \frac{\mathrm{d}\hat{z}}{\hat{z}} \Delta f_p\left(\frac{x}{\hat{x}},\mu_F^2\right) D_{p'}^h\left(\frac{z}{\hat{z}},\mu_A^2\right) \Delta\mathcal{C}_{p'p}\left(\hat{x},\hat{z},Q^2,\mu_R^2,\mu_F^2,\mu_A^2\right)$$

$\Delta\mathcal{C}_{p'p}$ known up to NLO (see e.g. [De Florian, Stratmann, Vogelsang ’97])

Calculation: polarised vs. unpolarised case

Prescription for γ_5

Problem: projector of the hadronic tensor to isolate the $g_1 = \mathscr{G}_1/2$ structure function is:

$$P_{g_1}^{\mu\nu} = \frac{i}{(D-2)(D-3)} \frac{2x}{Q^2} \varepsilon^{\mu\nu\rho\sigma} p_\rho q_\sigma$$

explicit Levi-Civita tensor

Required a consistent treatment in dim. reg.

In addition, we have polarised quark or gluon in the initial state: spin sum with explicit γ_5 or Levi-Civita

We adopt the Larin prescription: setting $\gamma_\mu \gamma_5 = \frac{i}{3!} \varepsilon_{\mu\nu\rho\sigma} \gamma^\nu \gamma^\rho \gamma^\sigma$

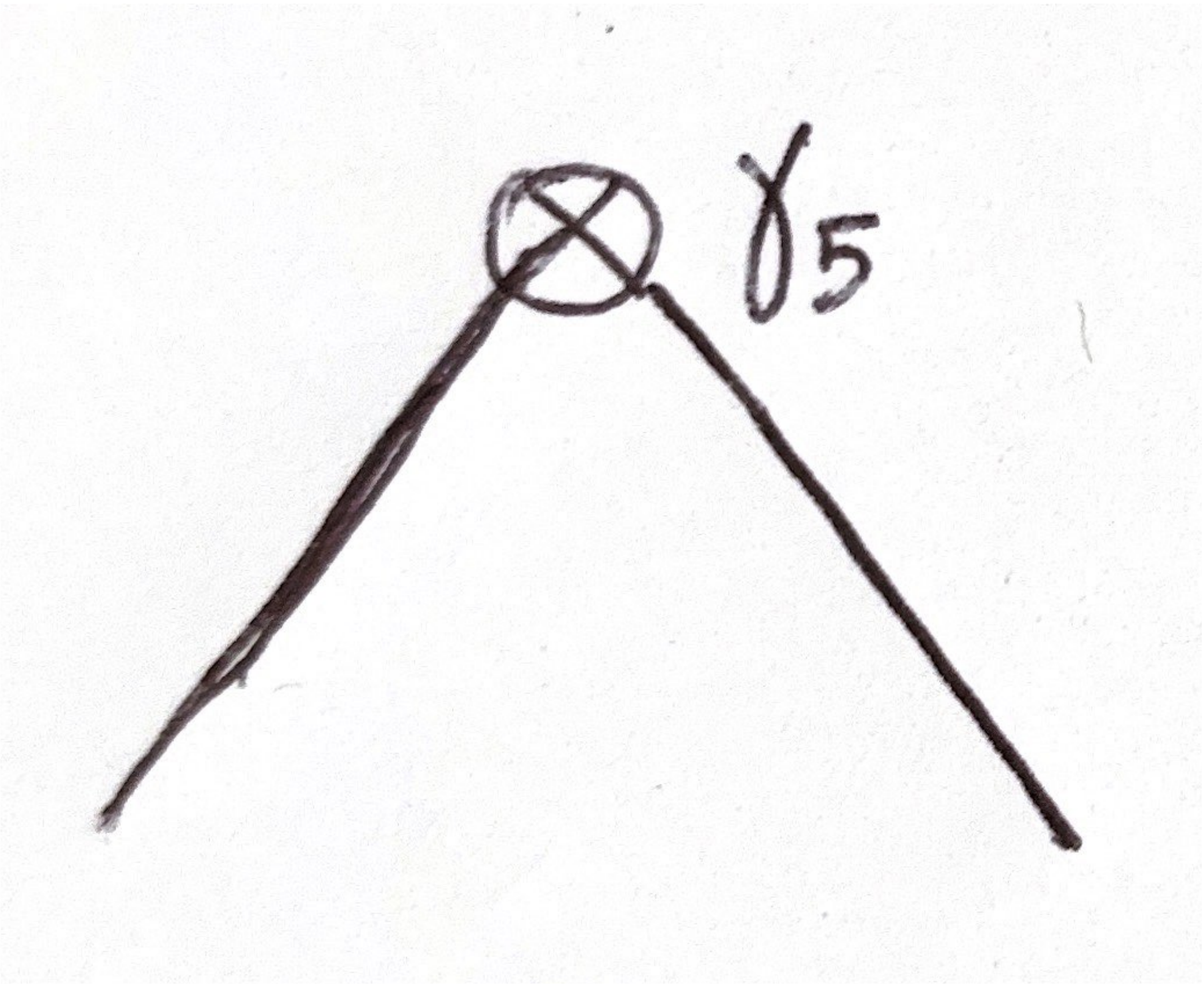
And evaluating traces in D dimensions, and contracting the two Levi-Civita into D-dim metric tensors.

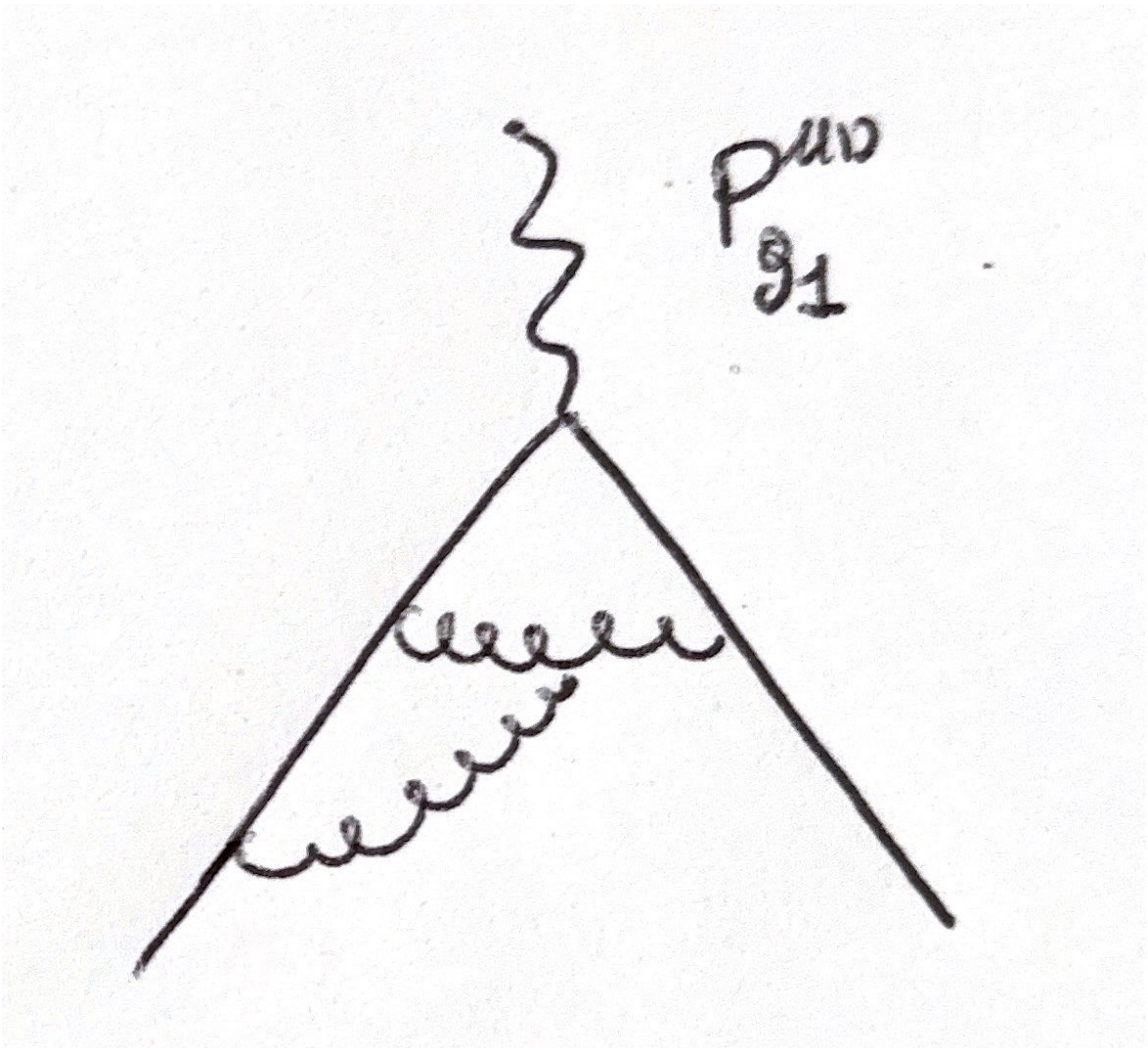
We carry out mass factorization with Larin space-like splitting functions and at the end in order to restore Ward identities we apply the transformation:

$$\begin{aligned} g_1 &= \Delta \mathcal{C}^{\overline{\mathrm{MS}}} \otimes \Delta f^{\overline{\mathrm{MS}}} \\ &= (\Delta \mathcal{C}^{\mathrm{L}} \otimes Z^{-1}) \otimes (Z \otimes \Delta f^{\mathrm{L}}) = \Delta \mathcal{C}^{\mathrm{L}} \otimes \Delta f^{\mathrm{L}} \end{aligned}$$

Calculation: polarised vs. unpolarised case

Purely virtual contributions (virtual and double-virtual)

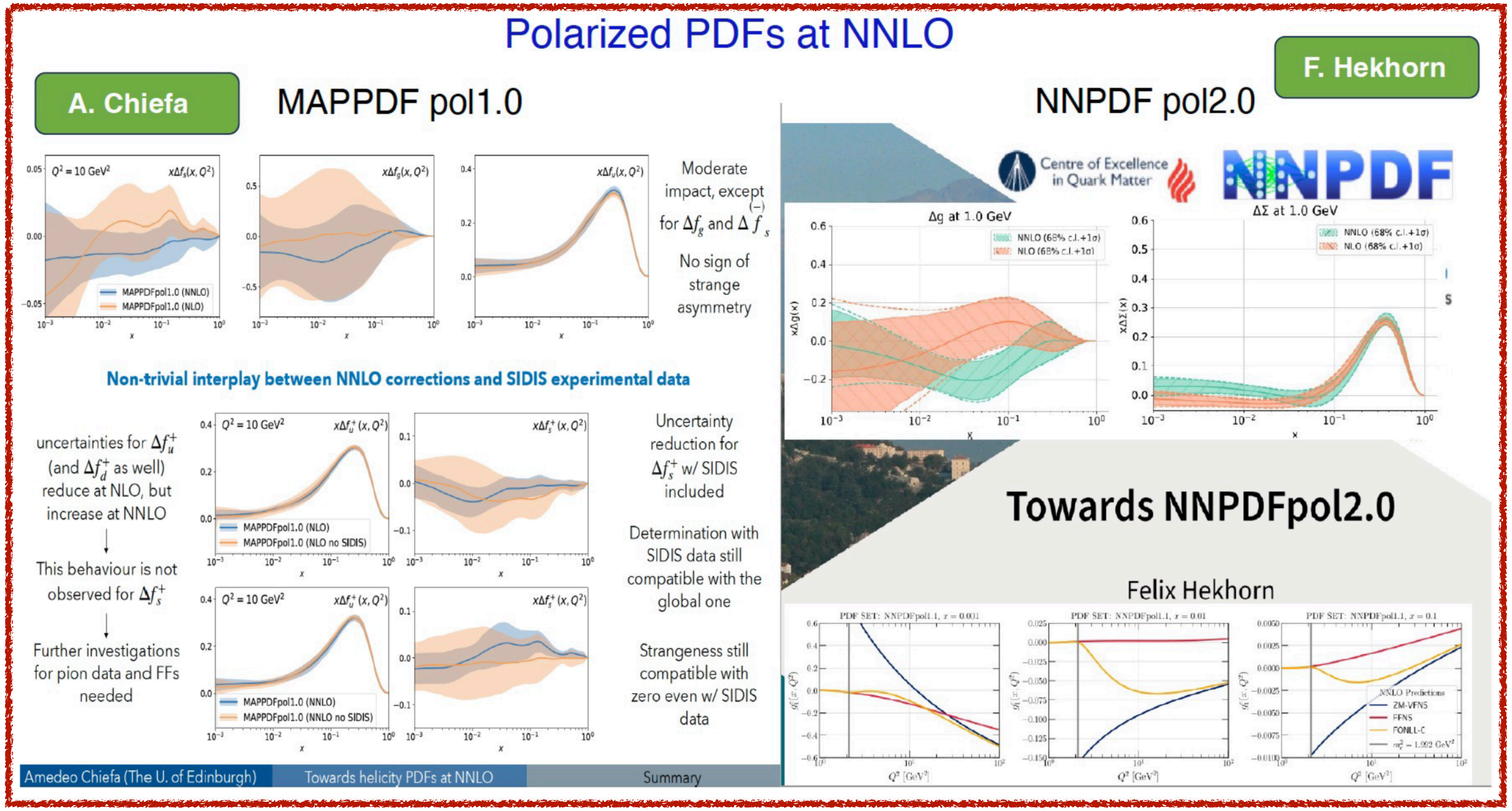




In operator matrix elements calculations, the g_1 projector of the photon is usually absorbed into an operator insertion, rendering the photon coupling axial.

Instead, in our case, **the photon coupling is always vectorial** and traces of quark-loops coupling to the polarized photon can be carried out consistently in $D = 4 - 2\epsilon$ dimensions.

New NNLO polarised PDFs

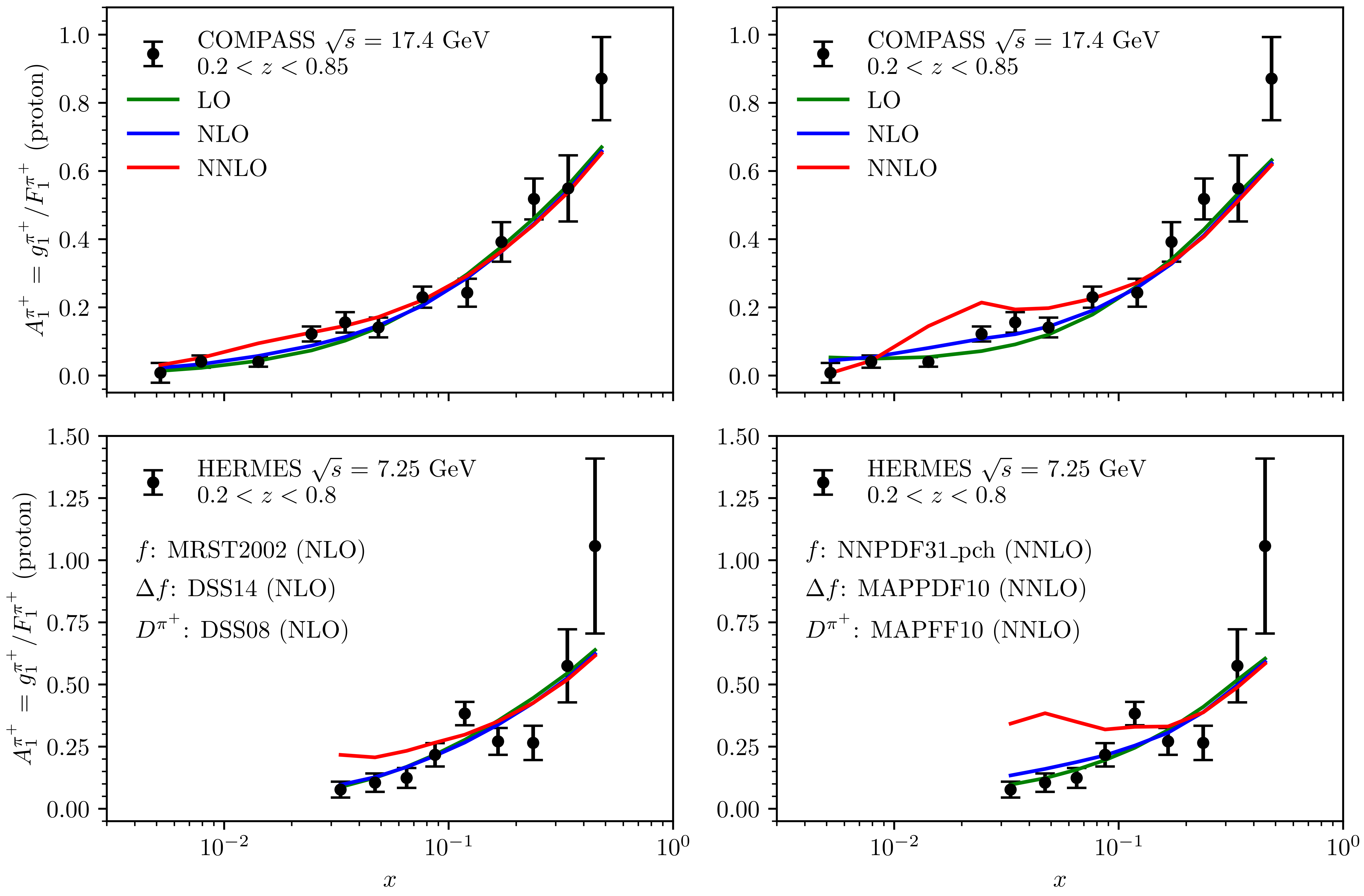


WG5 summary
@ DIS2024

(they exploit approximate NNLO corrections for SIDIS predictions)
(note that the approximate NNLO corrections are the same for polarised and unpolarised SIDIS)

Numerical results

Comparison to experimental points



Data points from CERN COMPASS and DESY HERMES for identified π^+ produced over a range of z-values.

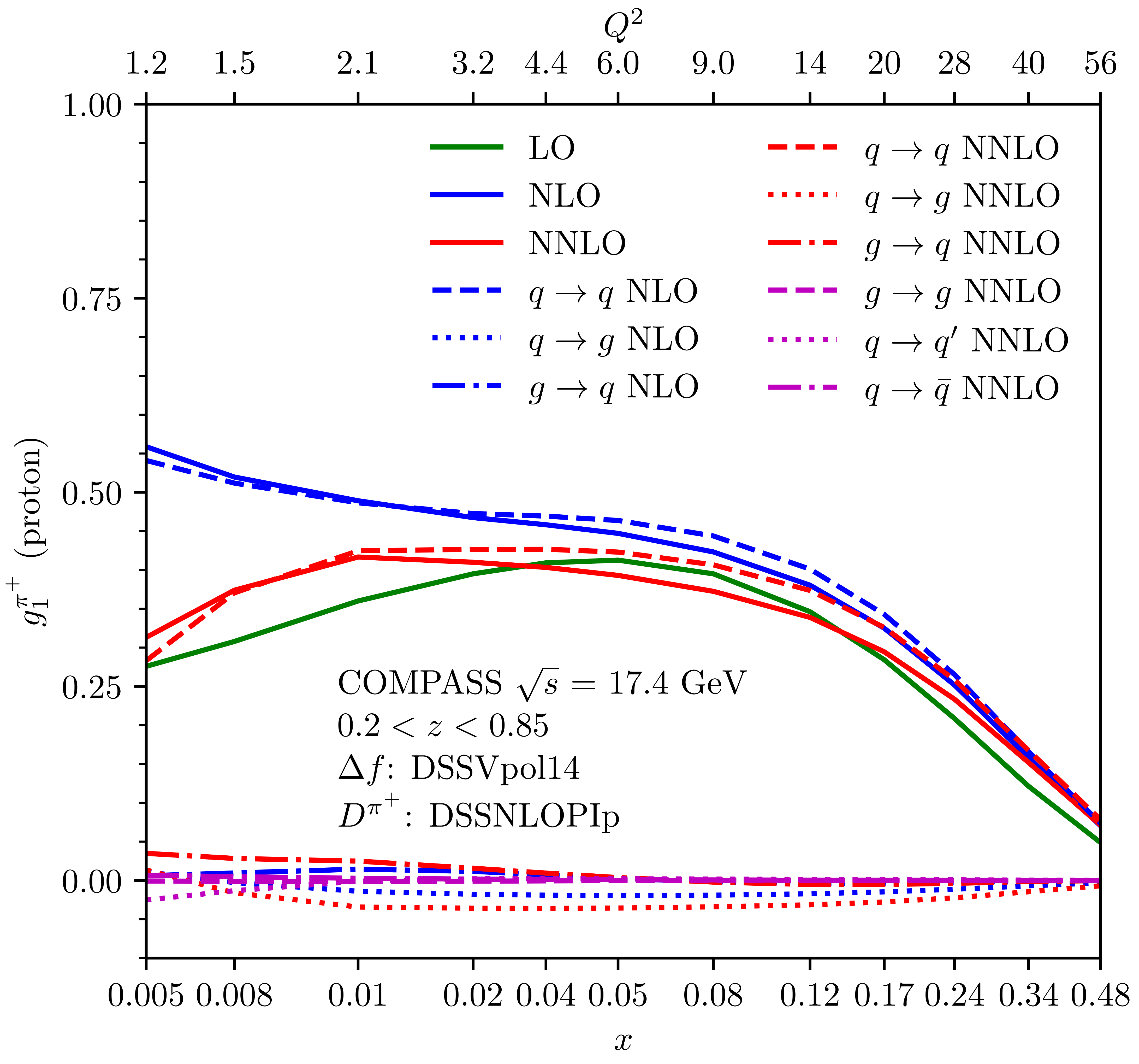
2-dim data points in (x, Q^2):
larger (smaller) x implies larger (smaller) Q^2

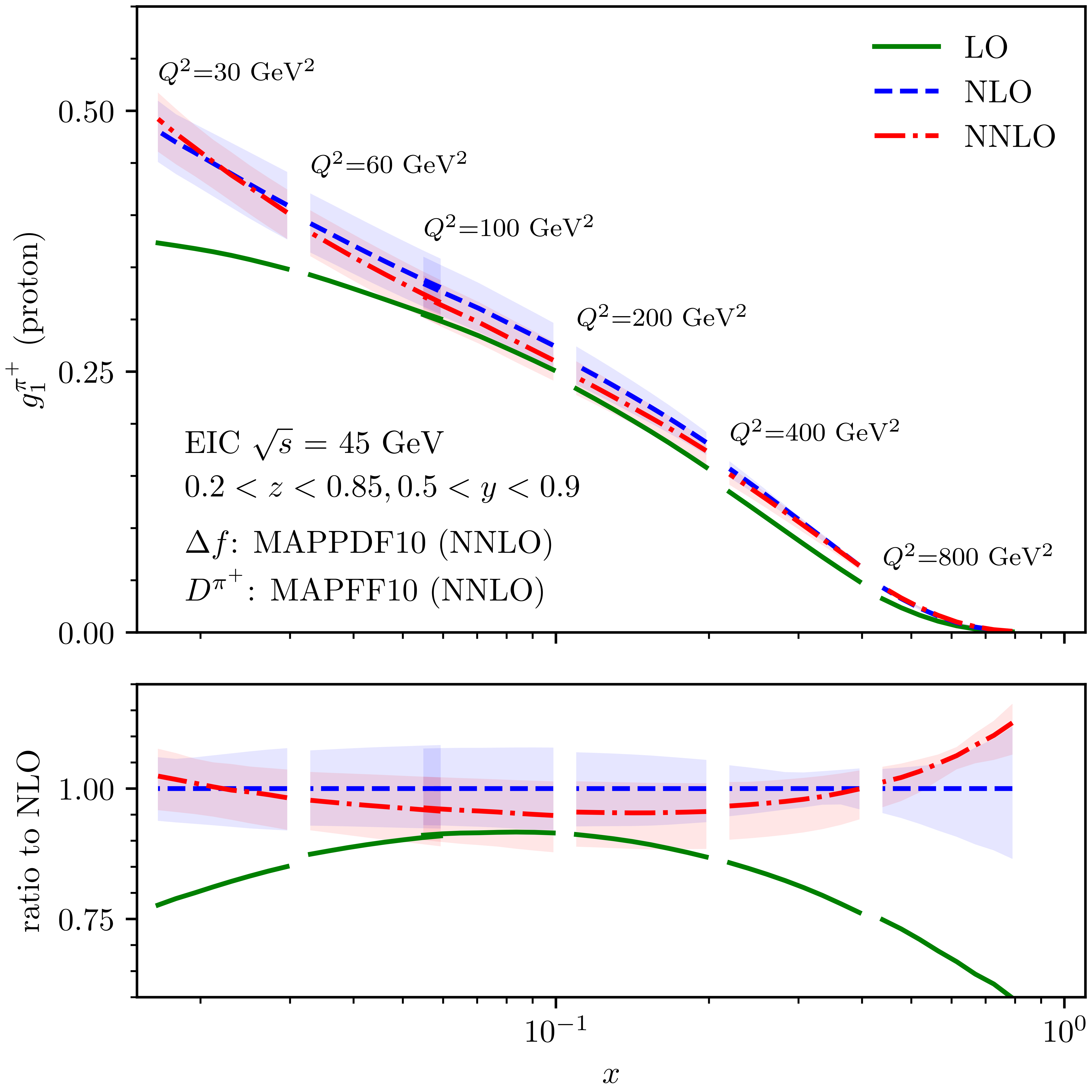
DSSV14 includes hadron collider data from RHIC that constrain the gluon PDF (dataset not included in MAPPDF10)

NNLO corrections can be sizeable, especially at low-x

Numerical results

Channel decomposition and perturbative convergence





Conclusions

- We computed the NNLO QCD corrections to SIDIS coefficient functions in analytical form, both for unpolarised and longitudinally polarised beam and target. Our results allow for NNLO global fits of fragmentation functions and helicity PDFs.

- After our paper, ref. [Goyal, Lee, Moch, Pathak, Rana, Ravindran 2404.09959] appeared, with calculation of NNLO polarised SIDIS. Comparison of results in progress

- Bonus: in the antenna subtraction formalism for fully differential NNLO calculations, matrix elements of simpler processes are used as subtraction terms. In case of an identified particle in the final state, **we can recycle the SIDIS coefficient functions as integrated subtraction terms**! Work in progress towards antenna subtraction for identified particles

Thank you!

BACKUP

Motivation

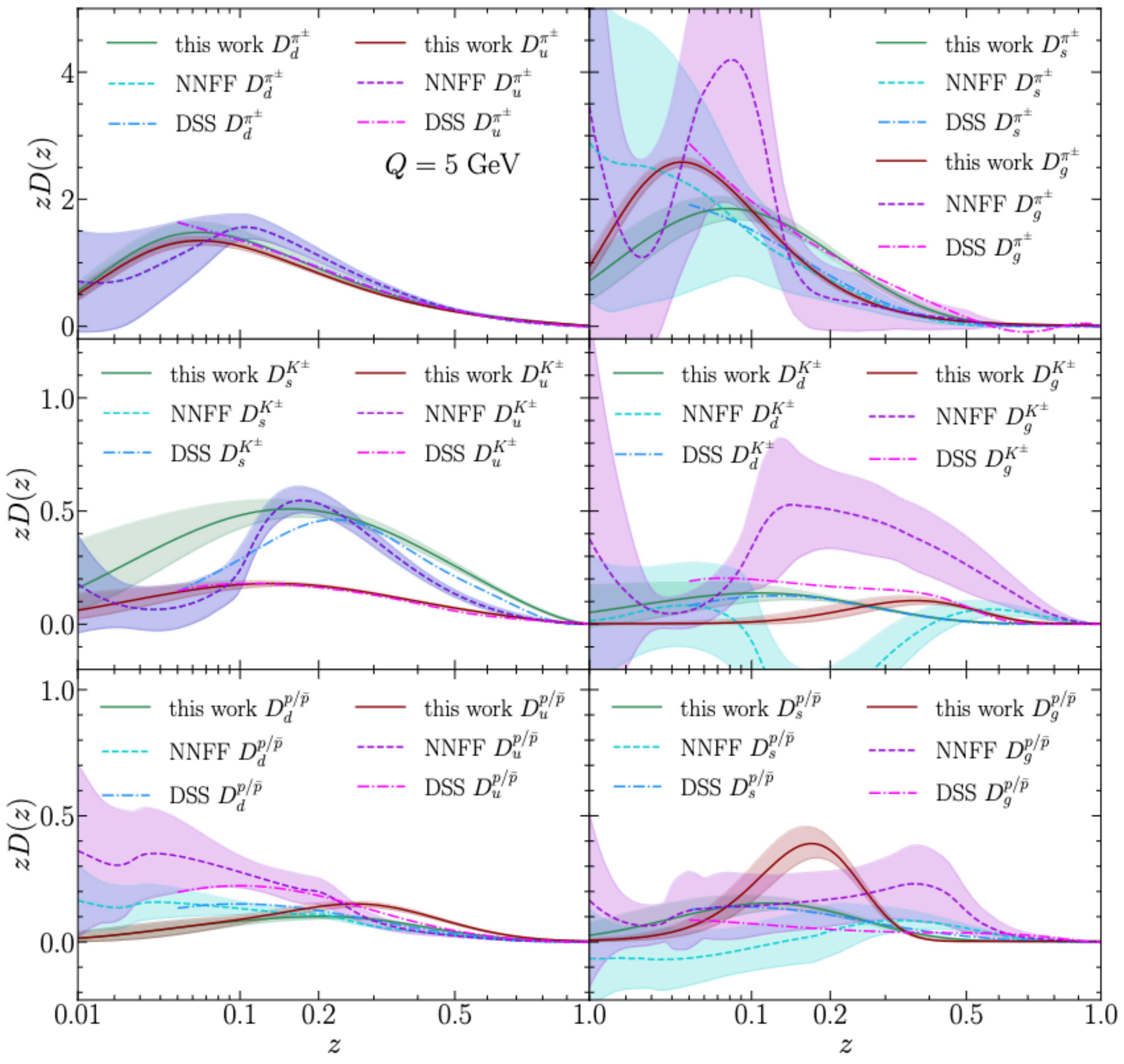
Global fits of fragmentation functions (FFs)

	Experiments	N_{pt}	χ^2	χ^2/N_{pt}
	ATLAS jets †	446	350.8	0.79
	ATLAS Z/γ+jet †	15	31.8	2.12
	CMS Z/γ+jet †	15	17.3	1.15
	LHCb Z+jet	20	30.6	1.53
	ALICE inc. hadron	147	150.6	1.02
	STAR inc. hadron	60	42.2	0.70
pp	pp sum	703	623.3	0.89
	TASSO	8	7.0	0.88
	TPC	12	11.6	0.97
	OPAL	20	16.3	0.81
	OPAL (202 GeV) †	17	24.2	1.42
	ALEPH	42	31.4	0.75
	DELPHI	78	36.4	0.47
	DELPHI (189 GeV)	9	15.3	1.70
	SLD	198	211.6	1.07
e^+e^-	SIA sum	384	353.8	0.92
	H1 †	16	12.5	0.78
	H1 (asy.) †	14	12.2	0.87
	ZEUS †	32	65.5	2.05
	COMPASS (06I)	124	107.3	0.87
	COMPASS (16p)	97	56.8	0.59
ep	SIDIS sum	283	254.4	0.90
	Global total	1370	1231.5	0.90

[2401.02781]

Fits routinely done at NLO by different groups, using data from e^+e^-, ep and pp colliders e.g. very recent global fit by [Gao, Liu, Shen, Xing, Zhao '24].

It exploits a new code FMNLO [Liu, Shen, Zhou, Gao '23], a wrapper around MG5 aMC@NLO, to compute arbitrary processes at the LHC with fragmentation at NLO.



Polarised SIDIS structure function

$$\mathcal{G}_1^h(x,z,Q^2) = \sum_{p,p'} \int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}} \int_z^1 \frac{\mathrm{d}\hat{z}}{\hat{z}} \Delta f_p\left(\frac{x}{\hat{x}},\mu_F^2\right) D_{p'}^h\left(\frac{z}{\hat{z}},\mu_A^2\right) \Delta\mathcal{C}_{p'p}\left(\hat{x},\hat{z},Q^2,\mu_R^2,\mu_F^2,\mu_A^2\right)$$

$\Delta\mathcal{C}_{p'p}$ known up to NLO (see e.g. [De Florian, Stratmann, Vogelsang '97])

What the experiments measure is the longitudinal double-spin asymmetry $A_{\|}$

$$A_{\|} = \frac{\mathrm{d}\sigma^{\uparrow\uparrow} - \mathrm{d}\sigma^{\uparrow\downarrow}}{\mathrm{d}\sigma^{\uparrow\uparrow} + \mathrm{d}\sigma^{\uparrow\downarrow}} \simeq D\,A_1$$

with a (known) kinematical factor D.

A_1 is related to the photoabsorption cross sections σ_{J_z},

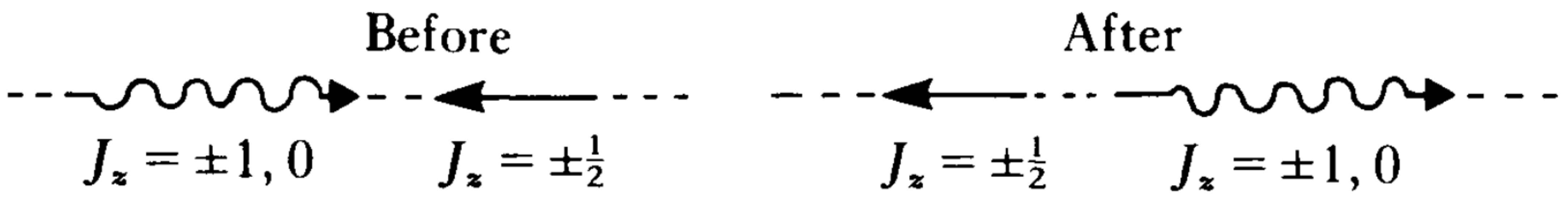
with J_z is the spin of the intermediate photon-nucleon system:

$$A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} = \frac{\mathcal{G}_1}{\mathcal{F}_T}$$

Why $A_1 = \mathcal{G}_1/\mathcal{F}_T$? Physical argument

from [F. Close, *An Introduction to Quarks and Partons*, 1979]

TABLE 13.1
Forward Compton helicity amplitudes



	Initial state γ_V	Initial state P	Intermediate state J_z	Final state γ_V	Final state P
(A)	$+1$	$+\frac{1}{2}$	$+\frac{3}{2}$	$+1$	$+\frac{1}{2}$
(B)	$+1$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+1$	$-\frac{1}{2}$
(C)	$+1$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$
$(\tilde{C})$	0	$+\frac{1}{2}$	$+\frac{1}{2}$	1	$-\frac{1}{2}$
(D)	0	$+\frac{1}{2}$	$+\frac{1}{2}$	0	$+\frac{1}{2}$

$$\gamma(J_z = +1): \quad \gamma^\uparrow + P^\uparrow \to \sigma_{3/2}$$

$$\gamma(J_z = -1): \quad \gamma_\downarrow + P^\uparrow \to \sigma_{1/2}$$

Quark moving along the z-axis
(i.e. $k_T = 0$)

$$\gamma^\uparrow + q_\downarrow \to q^\uparrow$$

$$\gamma_\downarrow + q^\uparrow \to q_\downarrow$$

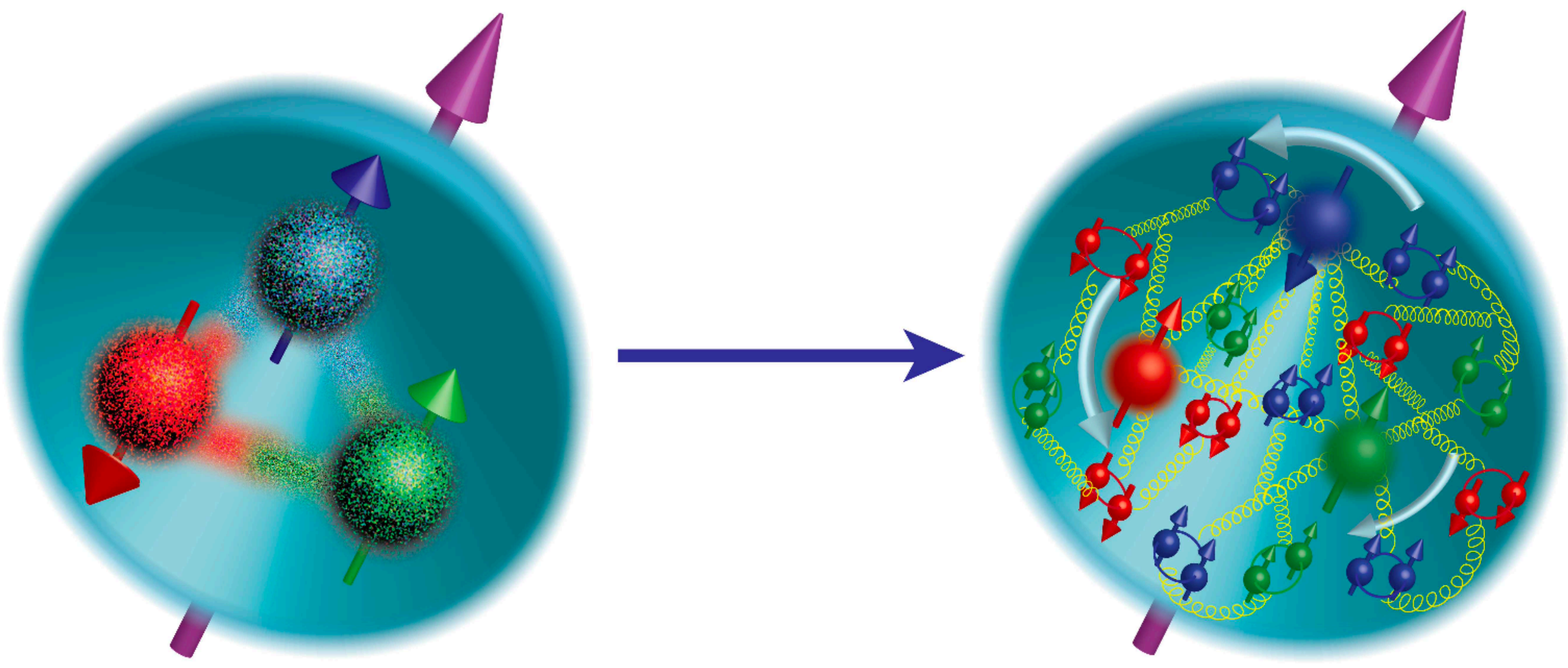
$$q^\uparrow = \sqrt{\left(\frac{E+m}{2E}\right)} \begin{pmatrix} \chi^\uparrow \\ \dfrac{P_z\chi^\uparrow + (P_x + iP_y)\chi_\downarrow}{E+m} \end{pmatrix} \qquad \gamma_\pm = \begin{pmatrix} 0 & \sigma_\pm \\ -\sigma_\pm & 0 \end{pmatrix}$$

$$\sigma_{1/2} \sim \gamma_\downarrow P^\uparrow \sim \sum_i e_i^2 q^\uparrow \qquad \sigma_{3/2} \sim \gamma^\uparrow P^\uparrow \sim \sum_i e_i^2 q_\downarrow$$

$$A \equiv \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} = \frac{\sum_i e_i^2 [q_i^\uparrow - q_{i\downarrow}]}{\sum_i e_i^2 [q_i^\uparrow + q_{i\downarrow}]}$$

Motivation

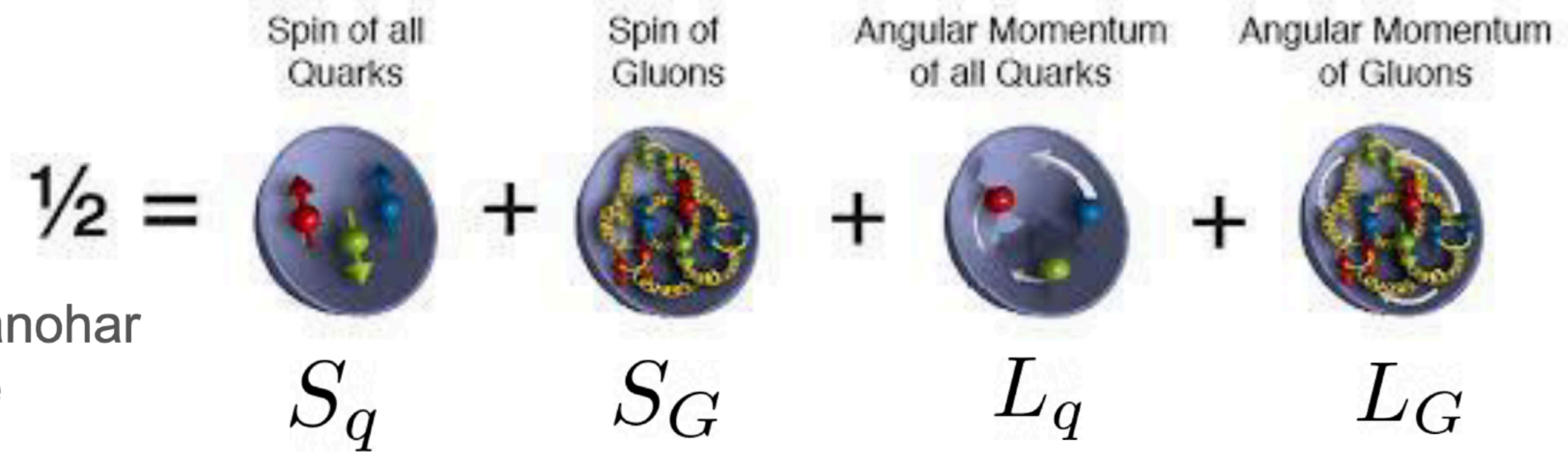
The proton spin puzzle



EMC 'spin crisis' (1987):
contribution of quark and anti-quark spins constitute only a small fraction of the proton spin (~ 10%)

Where is the rest?

Jaffe-Manohar sum rule



$$S_q(Q^2) = \frac{1}{2}\int_0^1 \Delta\Sigma(x,Q^2)dx \equiv \frac{1}{2}\int_0^1 \left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)(x,Q^2)dx$$

$$S_g(Q^2) = \int_0^1 \Delta g(x,Q^2)dx\ ,$$

polarised PDFs

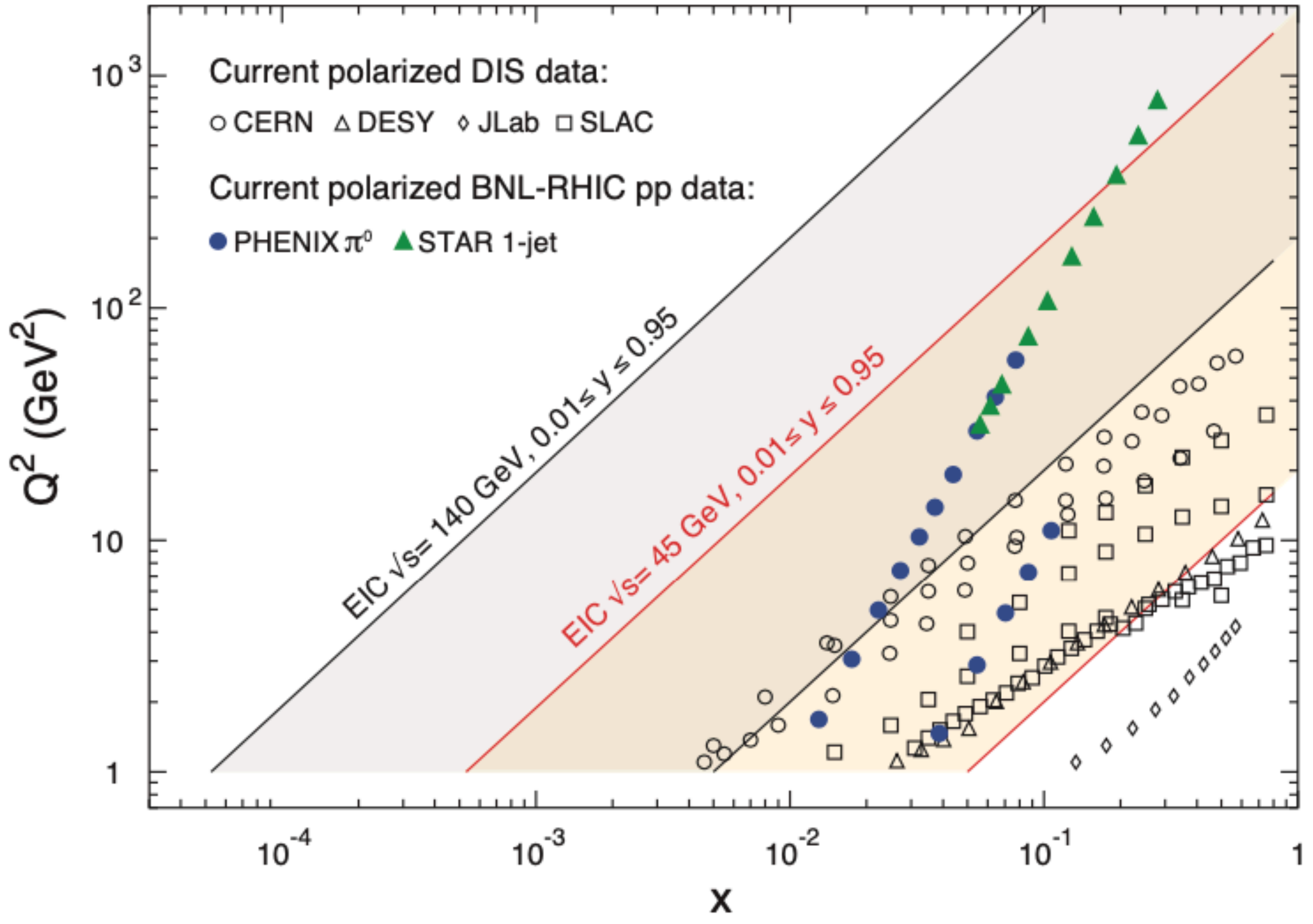
$$\Delta f(x,Q^2) \equiv f^+(x,Q^2)\ -\ f^-(x,Q^2)$$

Determined routinely at NLO through global fits
e.g. [NNPDFpol1.0 '14] [DSSV '14]

Motivation

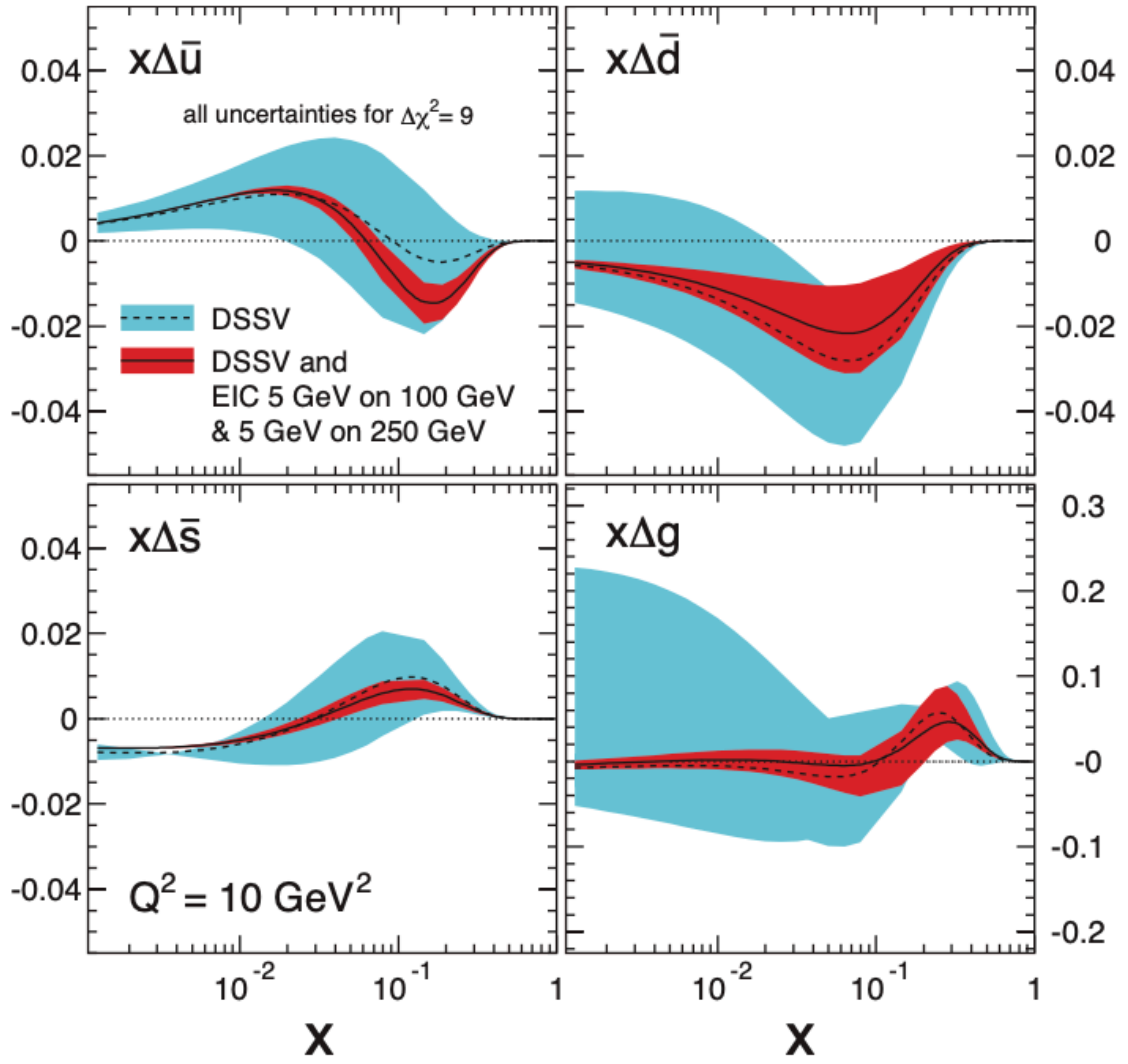
The proton spin puzzle

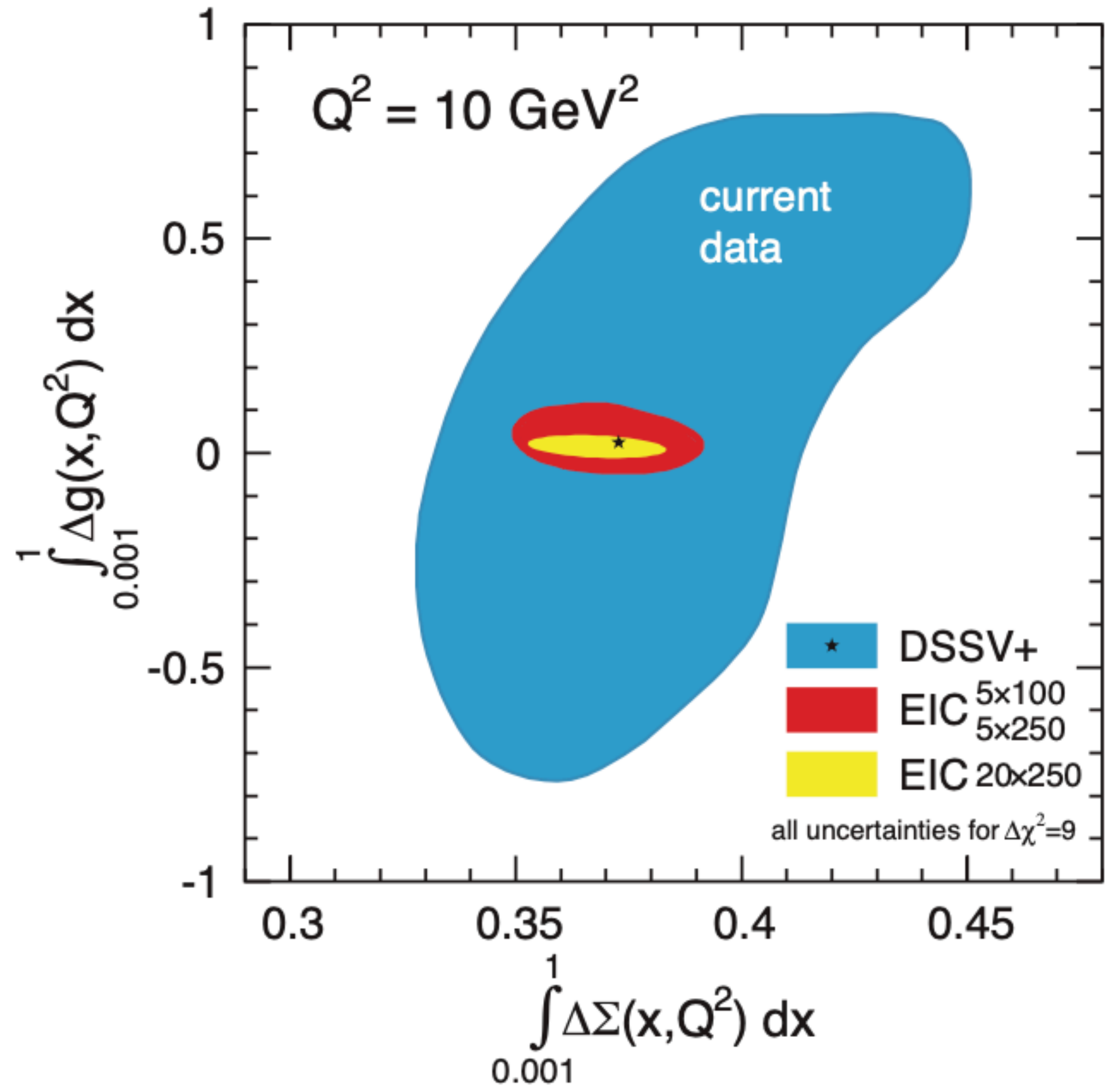
[EIC White Paper, 1212.1701]



EIC will significantly extend the kinematical region covered by previous spin experiments

[Aschenauer, Stratmann, Sassot '12]





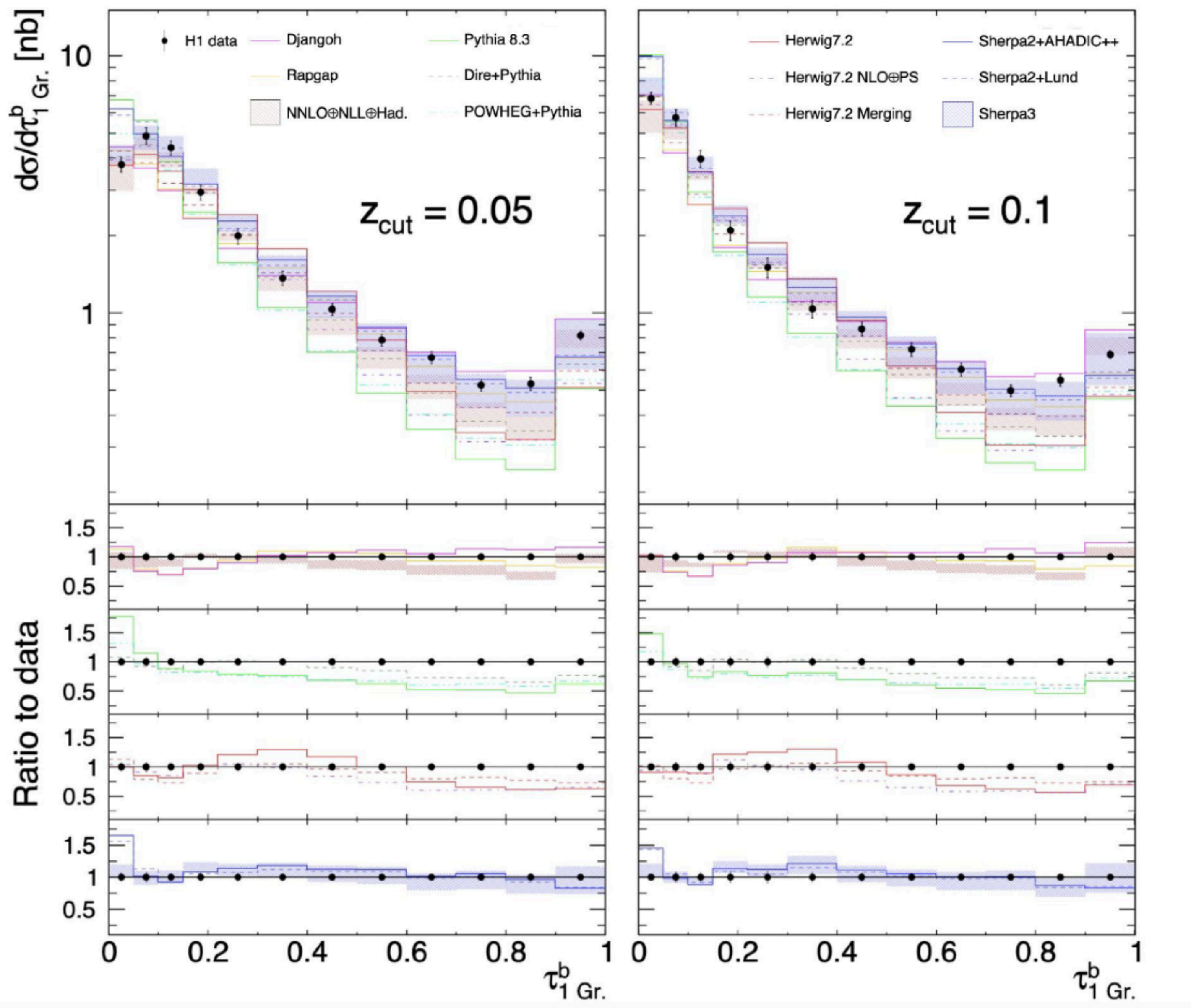
EIC has potential to greatly constrain helicity distributions and first moments!

DIS: a second youth?

A fresh look at HERA data with the expertise gained from LHC

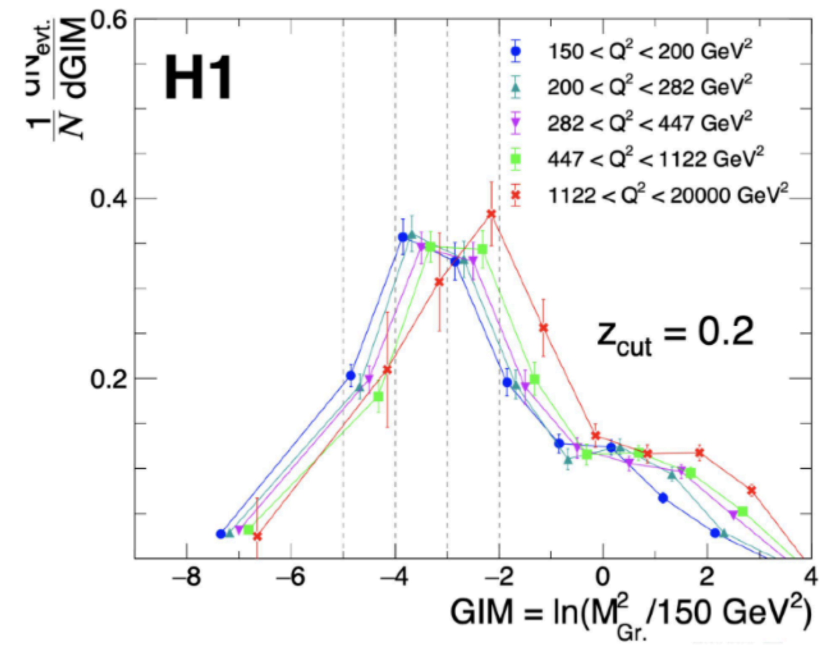
Groomed event shapes in DIS at H1

Henry Klest



Grooming with Soft Drop (remove soft wide-angle radiation), based on Centauro measure (keep radiation hard and/or collinear to current direction in Breit frame)

Focus on **groomed 1-jettiness $\tau^{b}_{1\,Gr.}$** and **groomed invariant mass $M_{Gr.}$**

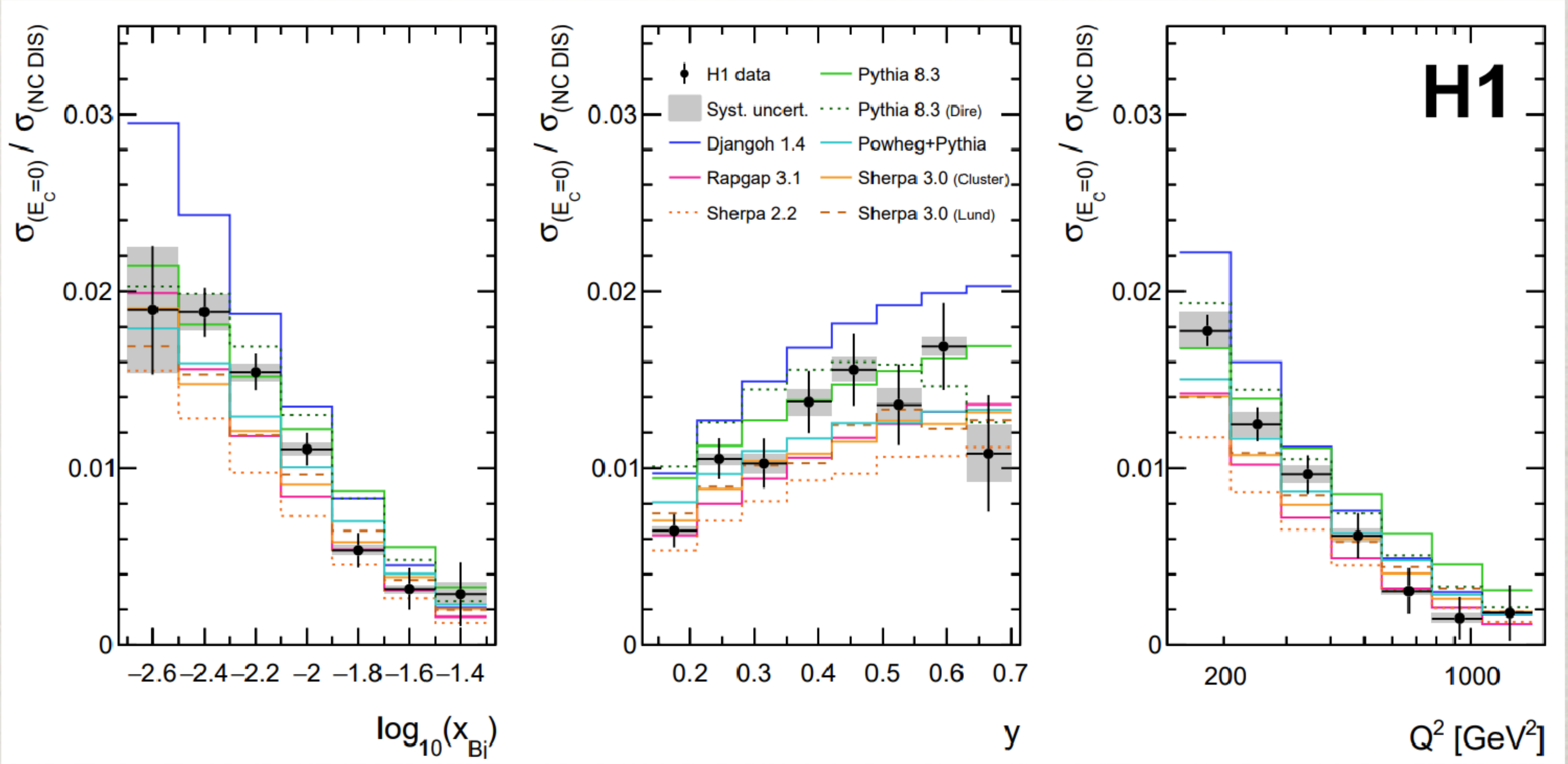


Almost Q2-independence predicted by SCET!

WG4 summary talk @ DIS2024

Empty current hemisphere

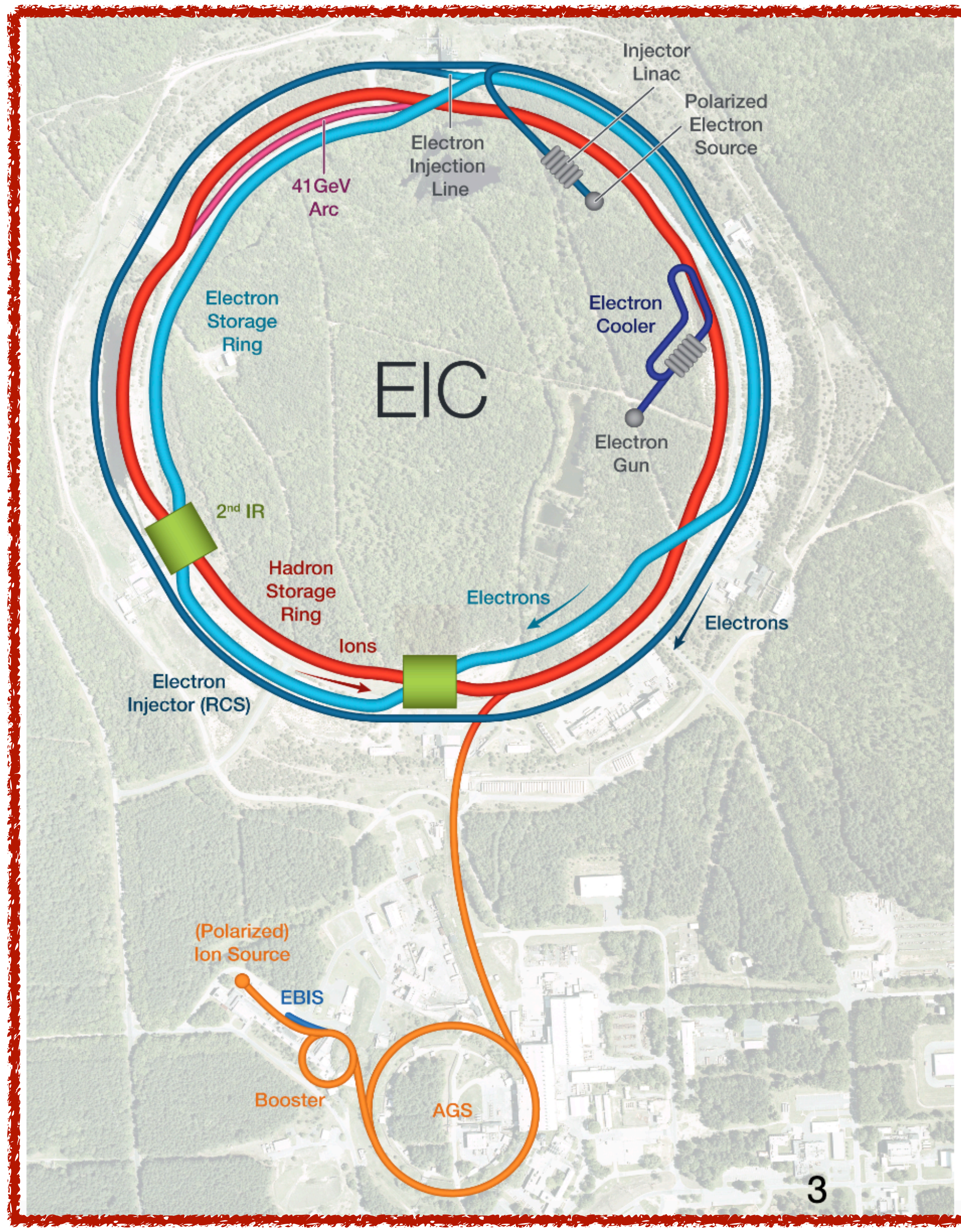
- H1 measured the fraction of events where the current hemisphere in the Breit frame is empty
- Our new POWHEG generator agrees with all distributions within experimental errors



Andrea Banfi @ DIS2024

DIS: a second youth?

Enthusiasm driven by the future Electron-Ion Collider (EIC)



- Based on RHIC:
  - use existing hadron storage ring energy: 41–275 GeV
  - add electron storage ring in RHIC tunnel energy: 5–18 GeV

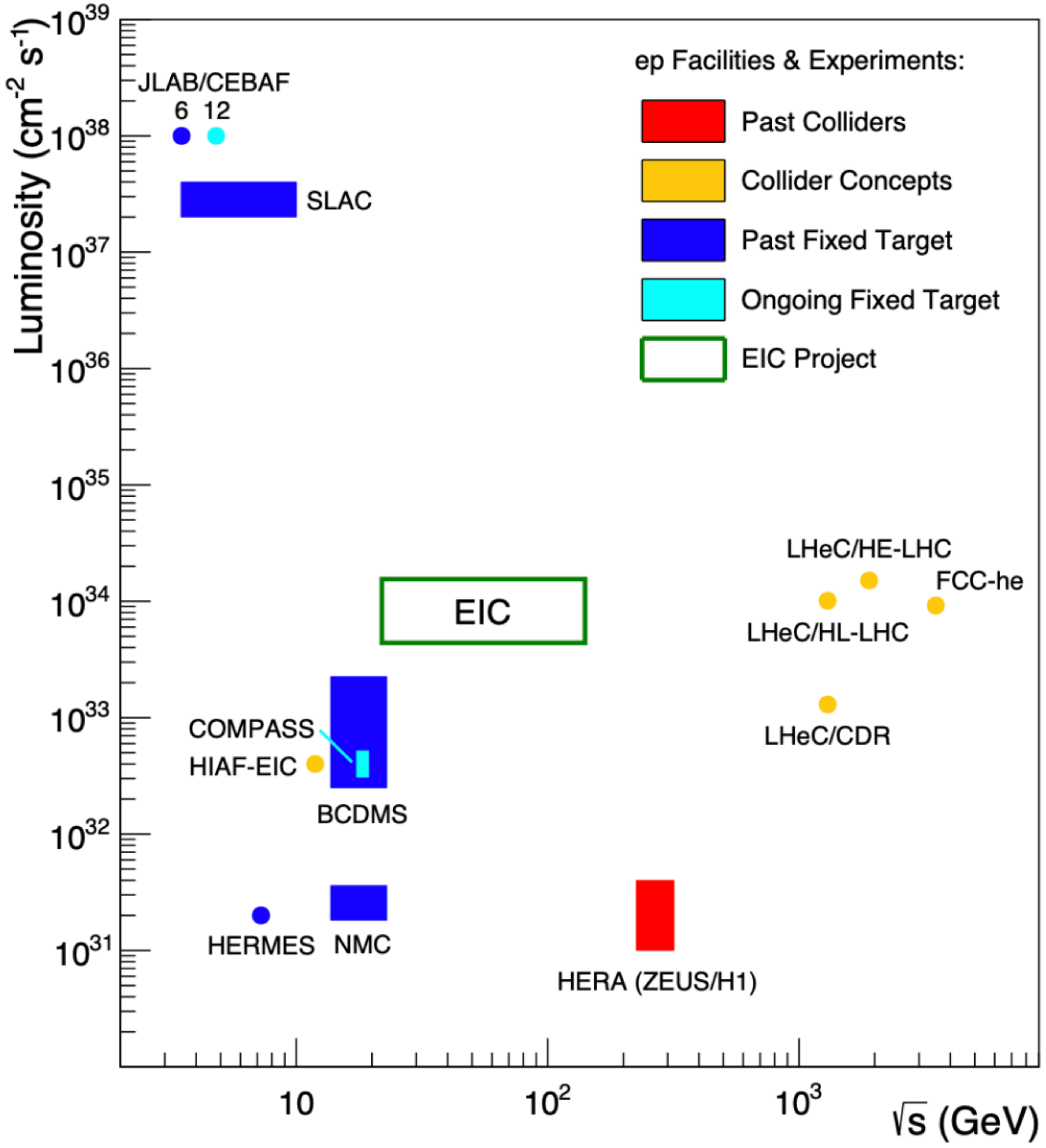
  $\rightarrow \sqrt{s} = 29 - 141$ GeV
- $\vec{e} + \vec{p}^{\uparrow}, \vec{d}^{\uparrow}, \vec{He}^{\uparrow}$, unpolarised ions up to U

  ~ 70% polarisation
- $\mathscr{L} = 10^{33-34}$ cm^{-2} s^{-1}

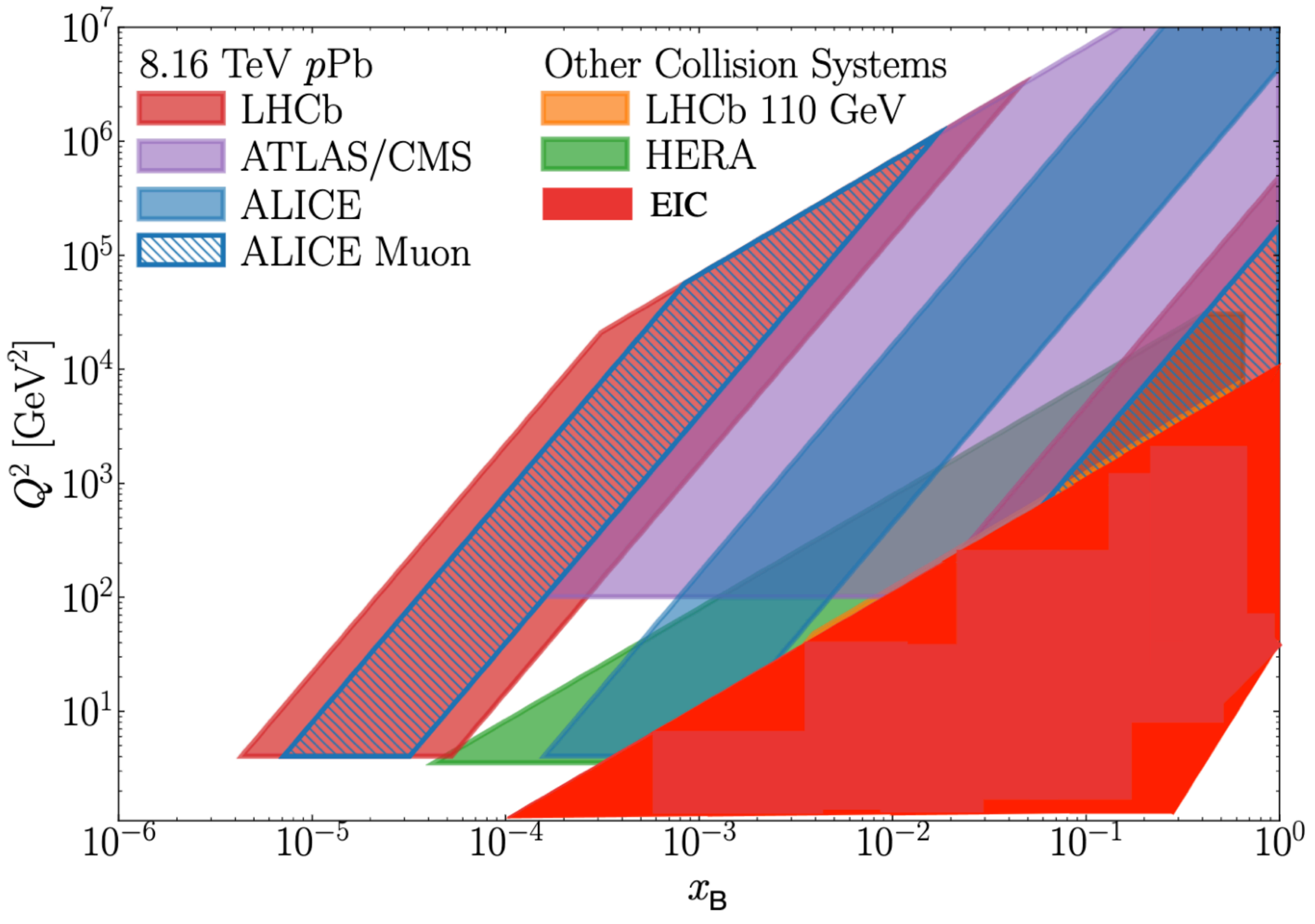
  $\leftrightarrow \mathscr{L}_{\text{int}} = 10 - 100$ fb^{-1}/year

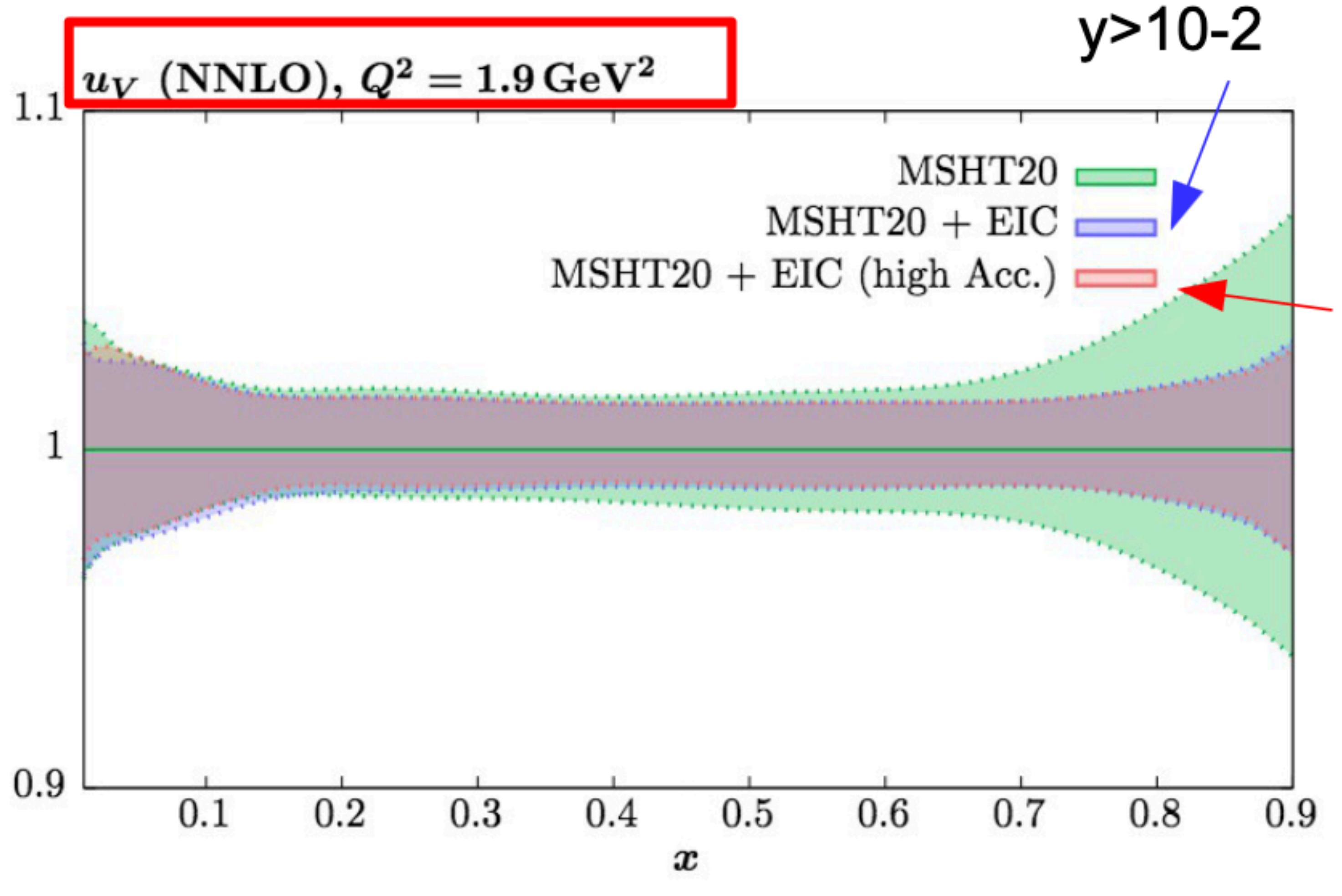
Construction ~ 2025-2034

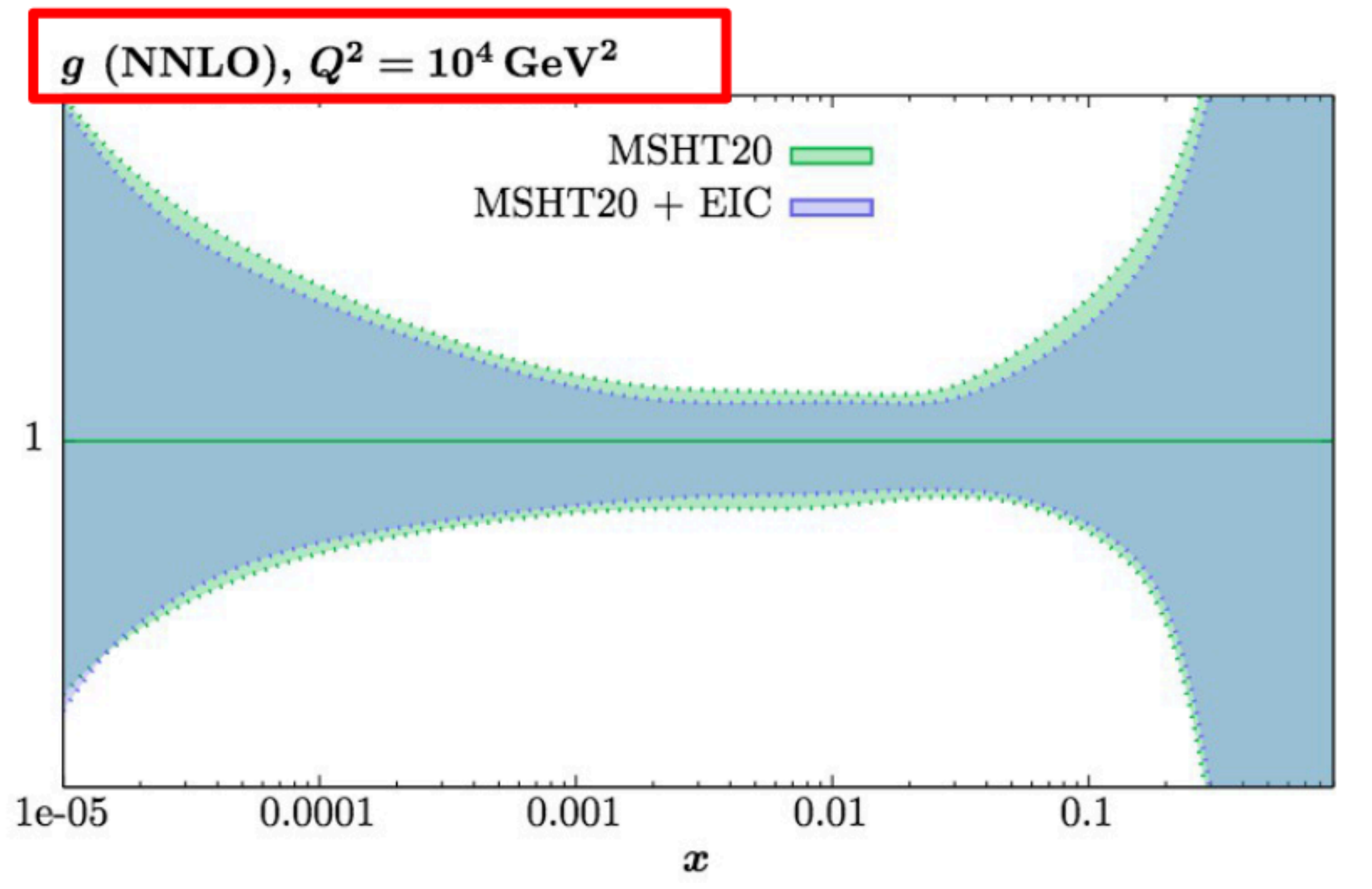
Charlotte Van Hulse @ DIS2024



EIC impact on collinear PDFs







Impact of EIC data on global fits @NNLO

- Improvement significantly reduced compared with HERAPDF2.0
- Still significant effects present
  biggest impact on up-valence distribution
  small but valuable improvement on all parton species visible at all x and Q^2 values

K. Wichmann @ DIS 2024

10

from [R. Roberts, *The structure of the proton*, 1993]

$$MW_1(\nu, Q^2) \longrightarrow F_1(x)$$
$$\nu W_2(\nu, Q^2) \longrightarrow F_2(x)$$

$$A = \frac{d\sigma(\uparrow\downarrow - \uparrow\uparrow)}{d\sigma(\uparrow\downarrow + \uparrow\uparrow)} = D[A_1 + \eta A_2]$$

$$\frac{\nu}{M} G_1(\nu, Q^2) \longrightarrow g_1(x), \quad \frac{\nu^2}{M^2} G_2(\nu, Q^2) \longrightarrow g_2(x)$$

$$D = \frac{1-(1-y)\epsilon}{1+\epsilon R}, \quad \eta = \frac{2M\epsilon\sqrt{Q^2}}{s[1-(1-y)\epsilon]}$$

$$\epsilon^{-1} = 1 + 2\left(1+\frac{\nu^2}{Q^2}\right)\left[\frac{(s-M^2)(s-2M\nu-M^2)}{M^2Q^2} - 1\right]^{-1}$$

$$A_1 \longrightarrow \frac{g_1(x)}{F_1(x)}, \quad A_2 \longrightarrow 0$$

$$A_1 = \frac{\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}}{\sigma_{\frac{1}{2}} + \sigma_{\frac{3}{2}}} = \frac{M\nu G_1 - Q^2 G_2}{M^3 W_1}$$