Understanding the Energy Momentum Distribution with the Weizsäcker-Williams Method

Bo-Wen Xiao

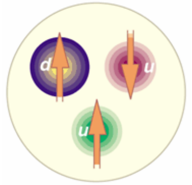
School of Science and Engineering, CUHK-Shenzhen

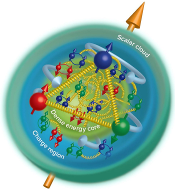
Y. Hagiwara, X.B. Tong, B.W. Xiao, arXiv: 2401.12840

Emergent Phenomena in QCD

Three pillars of EIC Physics:

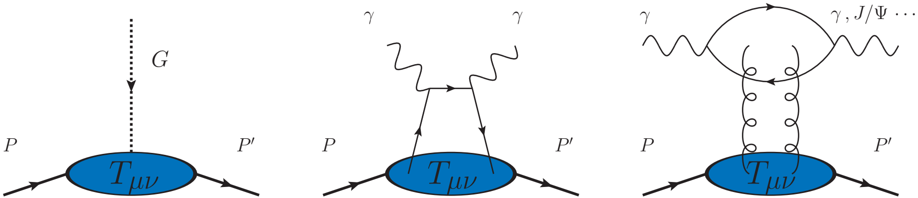
- How does proton mass arise/distribute? Mass gap: million dollar question.
- How does the spin of proton arise? (Spin puzzle)
- What are the emergent properties of dense gluon system?
- This talk: Mass distribution? three quark model VS Gluon Core?





- Determining the gluonic GFFs of the proton B. Duran, *et al.*, ▸ Nature **615** (2023) no.7954, 813-816.
  A mass radius that is notably smaller than the electric charge radius?

Energy-Momentum Tensor and Gravitational Form Factors

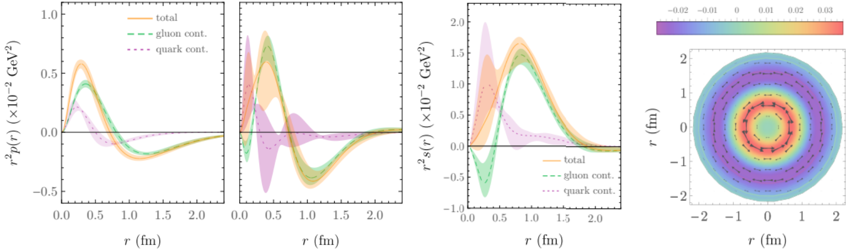


- Impossible to use graviton (spin 2) to probe proton mass distribution (GFF).
- [Ji, 97]; [Kharzeev, 96] Use two photons/gluons (spin 1) to study GPDs and GFFs, and probe quark and gluon parts, respectively.
- Usually near the production threshold, but what about high energy limit?

Pressure and Shear forces inside proton

Lattice perspective: [Shanahan, Delmold, 19]

$$T^{ij}(r) = \left(\frac{r^i r^i}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r) + \delta^{ij} p(r)$$



- The spatial of static EMT define the stress tensor. It can be decomposed in a traceless part associated with shear forces $s(r)$ and a trace associated with the pressure $p(r)$.
- $s(r)$ and $p(r)$ are computed in LQCD recently.

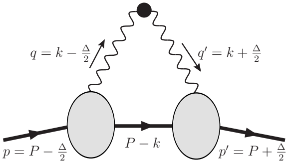
Energy-Momentum Tensor and Gravitational Form Factors

Consider the photon/gluon GFFs, defined from the associated EMT:

$$T^{\mu\nu}_{\gamma} = F^{\mu\alpha} F_{\alpha}^{\ \nu} + \frac{g^{\mu\nu}}{4} F^{\alpha\beta} F_{\alpha\beta}.$$

N.B. $T^{++} = F^{+\alpha} F_{\alpha}^{+}$

F.T. $\Rightarrow$ Off diagonal Matrix Elements.



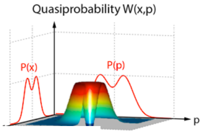
The photon/gluon GFFs for a spin-zero hadron [Polyakov, Schweitzer, 2018]:

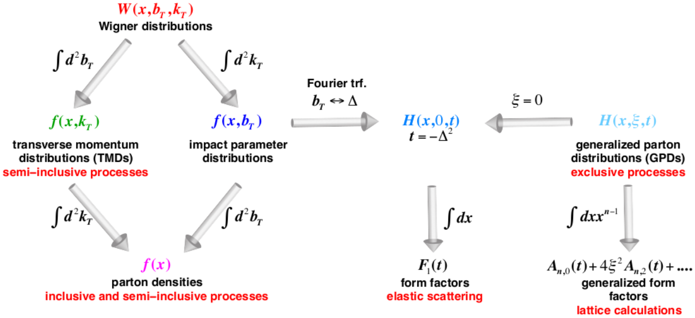
$$\text{spin-0}: \quad \langle p' | T^{\mu\nu}_{\gamma} | p \rangle = 2P^{\mu} P^{\nu} A_{\gamma}(t) + C_{\gamma}(t) \frac{\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^2}{2} + 2m^2 \bar{C}_{\gamma}(t) g^{\mu\nu}.$$

- A: mass/momentum distribution; C (or D): shear and pressure information.
- B (or J) are related to spin. $\bar{C}$ is due to non-conservation of individual part.
- Connection with parton distributions!

3D Tomography of Proton

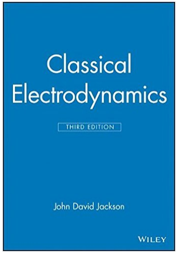
Wigner distributions [Belitsky, Ji, Yuan, 04] ingeniously encode all quantum information of how partons are distributed inside hadrons.





Classical Electrodynamics and Virtual Quanta

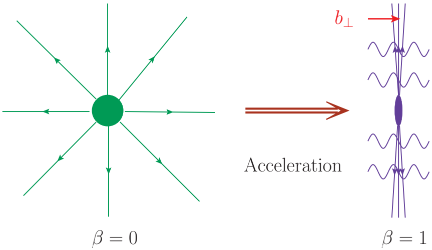




- Following Fermi[24], Weizsäcker [34] and Williams [35] discovered that the EM fields of a relativistically moving charged particle are almost transverse. Equivalent to Say:
- Charged particles carry a cloud of quasi-real photons, ready to be radiated if perturbed.
- Weizsäcker-Williams method of virtual quanta (Equivalent Photon Approximation).
- Application in QCD: WW gluon distribution. [McLerran, Venugopalan, 94; Kovchegov, 96; Jalilian-Marian, Kovner, McLerran and Weigert, 97]

EPA and Weizsäcker-Williams Photon Distribution

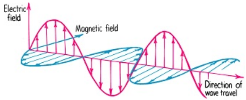
Boost Static Field to infinite momentum frame ($\gamma \to \infty$): [Jackiw, Kabat and Ortiz, 92; Jackson]



$$A^+_{Cov} = -\frac{q}{\pi} \ln(\lambda b_\perp)\delta(t-z),$$

$$\vec{E} = \frac{q}{2\pi}\frac{\vec{b}_\perp}{b_\perp^2}\delta(t-z), \quad \vec{B} = \frac{q}{2\pi}\frac{\hat{v}\times\vec{b}_\perp}{b_\perp^2}\delta(t-z),$$

$$\vec{A}^{LC}_\perp = -\frac{q}{2\pi}[\vec{\nabla}_\perp \ln(\lambda b_\perp)]\theta(t-z).$$

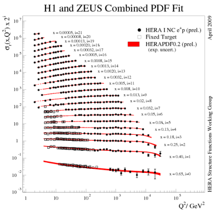


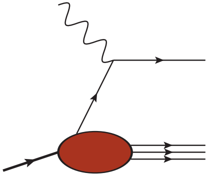
Static E fields $\Rightarrow$ EM Wave

$\Rightarrow$ EM pulses are equivalent to photons

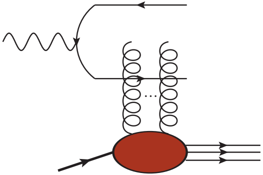
- A_μ in Covariant gauge and LC gauge are related by a gauge transformation.
- Classical EM: transverse EM fields $\Leftrightarrow$ QM: Co-moving Quasi-real photons.
- Quantization $\Rightarrow$ photon distribution

Dual Descriptions of Deep Inelastic Scattering

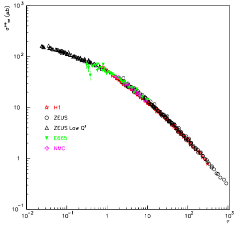




Bjorken frame



Dipole frame



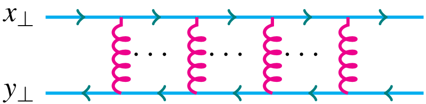
- Bjorken: partonic picture is manifest. Saturation shows up as limit of number density.
- Dipole: the partonic picture is no longer manifest. Saturation appears as the unitarity limit for scattering. Convenient to resum the multiple gluon interactions.

$$F_2(x,Q^2) = \sum_f e_f^2 \frac{Q^2}{4\pi^2\alpha_{\text{em}}} S_\perp \int_0^1 \mathrm{d}z \int \mathrm{d}^2 r_\perp \left|\psi\left(z, r_\perp, Q\right)\right|^2 \left[1 - S^{(2)}\left(Q_s r_\perp\right)\right]$$

Wilson Lines in Color Glass Condensate Formalism

The Wilson loop (color singlet dipole) in McLerran-Venugopalan (MV) model

$$S^{(2)}(r_\perp) = \frac{1}{N_c}\left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp) \right\rangle = e^{-\frac{Q_s^2(x_\perp - y_\perp)^2}{4}}$$



- IP-Sat Model and Glauber-Mueller formula for $S^{(2)}(r_\perp)$

$$Q_s^2(x, b_\perp) = \frac{2\pi^2}{N_c}\alpha_s x g(x, \mu^2) T(b_\perp), \quad \text{with} \quad T(b_\perp) = \frac{1}{2\pi B_G} e^{-b_\perp^2/(2B_G)}.$$

- MV model for large nuclei with uniform nucleon distribution

$$T_A(b_\perp) = \frac{3A}{2\pi R_A^3}\sqrt{R_A^2 - b_\perp^2}.$$

- In general, unitarity and color transparency imply $S^{(2)}(r_\perp) = 1 - Q_s^2 r_\perp^2/4 + \cdots$.

Photon Gravitational Form Factors

For a charge with F.F. $F(q^2)$, compute photon A-GFF from definition with WW method:

$$A_\gamma(t) = 4\alpha \int_0^1 dx \int \frac{d^2k_\perp}{(2\pi)^2} \frac{q_\perp \cdot q'_\perp}{q^2 q'^2} F(q^2) F(q'^2)$$

$$\text{Connection with GTMD:} \quad = \int_0^1 dx \int d^2k_\perp \, x f_\gamma(x, k_\perp, \Delta_\perp).$$

- Mass Radius (2/3 D): Expansion at small-$|t|$ for a pointlike charge ($F = 1$)

$$\langle b_\perp^2 \rangle_\gamma = \frac{4}{A_\gamma(0)} \frac{dA_\gamma(t)}{dt}\Big|_{t=0} \quad \text{with} \quad A_\gamma(t) = \frac{\alpha}{\pi}\left[\text{U.V.} + \frac{t}{m^2}\left(\frac{3}{16}\frac{m\pi^2}{\sqrt{-t}} - \frac{1}{3}\right) + \cdots\right].$$

- Divergent radius due to the long-range tail of the Coulomb field.
- IR and UV divergences disappear for a charge neutral dipole with a distribution.

Relating GFFs to Scattering matrices

New Relation between GFF $A(t)$ and the Laplacian of Dipole Scattering Amplitude $S(r_\perp)$.

$$\begin{aligned} A_g(t) &= \int_0^1 dx \int d^2k_\perp x\mathcal{G}_x(k_\perp, \Delta_\perp), \\ &= \frac{N_c}{\alpha_s} \int_0^1 dx \int \frac{d^2b_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} \underbrace{\vec{\nabla}^2_{r_\perp} \left[1 - S_x(b_\perp, r_\perp)\right] \Big|_{r_\perp=0}}_{\text{Slope of } 1-S = \# r_\perp^2 \text{ at small } r_\perp^2} . \end{aligned}$$

- Two GTMDs (WW and Dipole) reduce to the same GPD and GFF.
- Conjecture: similar relations between other GFFs and non-eikonal S-matrices. Trento Workshop on ▸ Beyond-Eikonal Methods in High-Energy Scattering, May 2024, 2024

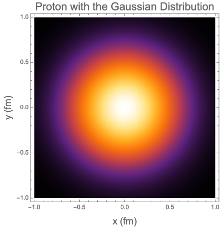
Understanding GFFs in the small-x formalism

- Gaussian Ansatz. $S_x(b_\perp, r_\perp) = \exp[-\frac{r_\perp^2}{4} Q_s^2(x, b_\perp)]$.

$$A_g(t) = \frac{N_c}{\alpha_s} \int_0^1 dx \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} Q_s^2(x, b_\perp), \qquad \Rightarrow \text{Probe } Q_s.$$

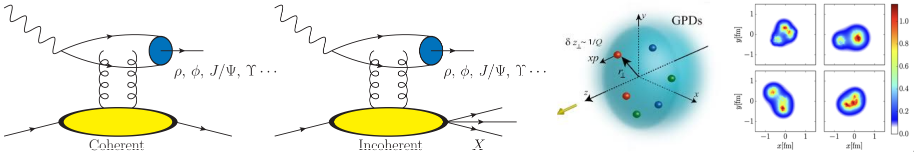
- IP-Sat Model: $Q_s^2(x, b_\perp) = \frac{2\pi^2}{N_c} \alpha_s x g(x, \mu^2) T(b_\perp)$, with $T(b_\perp) = \frac{1}{2\pi B_G} e^{-b_\perp^2/(2B_G)}$.

$$\begin{aligned} A_g(t) &= A_g(0) \int d^2 b_\perp e^{-i\Delta_\perp \cdot b_\perp} T(b_\perp) \\ &= A_g(0) e^{-B_G|t|/2}, \\ \text{with} \quad A_g(0) &= \int_0^1 dx x g(x, \mu^2) \to \frac{4C_F}{4C_F + n_f}. \end{aligned}$$



- Gluon Mass Radius $\langle b_\perp^2 \rangle_g = 2B_G$ independent of $A_g(0)$.

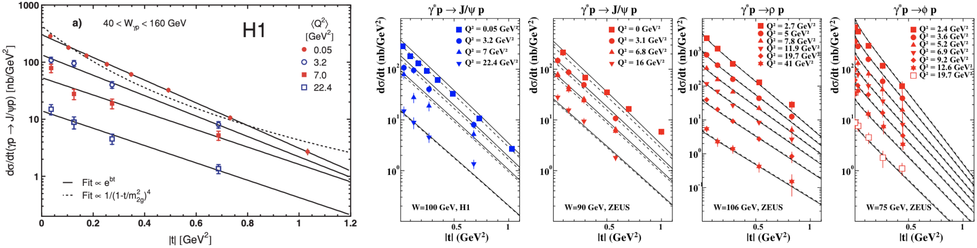
Diffractive vector meson production



- Analogy to Fraunhofer Diffraction [QCD at high energy, Kovchegov and Levin, 12]. Coherent and incoherent Diffractive scattering ⇒ gluon spatial distribution.
- DVMP is sensitive to the proton GPD and fluctuating shape. (Variance) [Mantysaari, Schenke, 16; Mantysaari, Roy, Salazar, Schenke, 20]

Exclusive diffraction at HERA

- IP-Sat Model: [Bartels, Golec-Biernat and H. Kowalski, 02; Kowalski, Teaney, 03; Kowalski, Motyka and Watt, 06; Watt, Kowalski, 07; Caldwell and Kowalski, 2010] [Rezaeian, Siddikov, Van de Klundert, Venugopalan, 13, ...] $B_G = 4.0 \pm 0.4\ \mathrm{GeV}^{-2}$



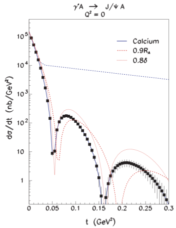
- Gluon Mass Radius in the proton (Originally called two gluon radius)

$$\sqrt{\langle b_\perp^2 \rangle_g} = \sqrt{2B_G} \approx 0.56\,fm \quad \text{and} \quad \sqrt{\langle r^2 \rangle_g} \approx 0.61\,fm \quad \text{with} \quad r_g^2 \simeq 3b_\perp^2/2.$$

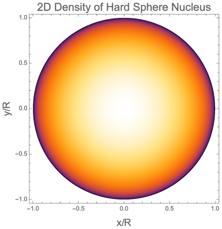
Application of GFF

- Comparison with Jlab experiment B. Duran, *et al.*, ▸ Nature **615** (2023) no.7954, 813-816. and lattice (MIT group) calculation [Hackett, Pefkou and Shanahan, 23] ⇒ Gluon core!
- Assume uniform gluon in p and n. ⇒ Neutron Radius in Nuclei at the EIC!

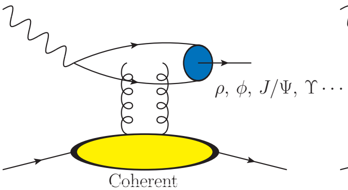
$$T_A(b_\perp) = \rho_p(b_\perp) + \rho_n(b_\perp) \quad \Rightarrow \quad \langle b_\perp^2 \rangle_g = \frac{Z}{A}\langle b_\perp^2 \rangle_p + \frac{A-Z}{A}\langle b_\perp^2 \rangle_n \ .$$

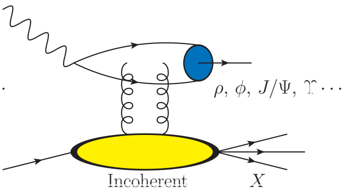


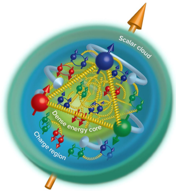
e+A sim



Summary







- WW method provides analytic insights into gluon GFFs and radii.
- A new relation between the gluon A-GFF and the Laplacian of dipole amplitude.
- Understanding dense gluon core inside Proton! B. Duran, *et al.* Nature **615** (2023) no.7954, 813-816.
- A-GFF of nuclei ⇒ nuclear gluon distribution – the charge distribution ⇒ neutron distribution for large nuclei.
- Measurements of GFFs at the upcoming EIC and EicC.